# **Strong decays of charmed baryons in heavy hadron chiral perturbation theory**

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(Received 24 October 2006; published 8 January 2007)

Strong decays of charmed baryons are analyzed in the framework of heavy hadron chiral perturbation theory (HHChPT) in which heavy quark symmetry and chiral symmetry are synthesized. HHChPT works excellently for describing the strong decays of *s*-wave charmed baryons. For  $L = 1$  orbitally excited states, two of the unknown couplings, namely,  $h_2$  and  $h_{10}$ , are determined from the resonant  $\Lambda_c^+ \pi \pi$  mode produced in the  $\Lambda_c(2593)$  decay and the width of  $\Sigma_c(2800)$ , respectively. Predictions for the strong decays of the *p*-wave charmed baryon states  $\Lambda_c(2625)$ ,  $\Xi_c(2790)$  and  $\Xi_c(2815)$  are presented. Since the decay  $\Lambda_c(2593)^+ \to \Lambda_c^+ \pi \pi$  receives nonresonant contributions, our value for  $h_2$  is smaller than the previous estimates. We also discuss the first positive-parity excited charmed baryons. We conjecture that the charmed baryon  $\Lambda_c(2880)$  with  $J^P = \frac{5}{2} + i$  is an admixture of  $\Lambda_{c2}(\frac{5}{2}, +)$  with an  $\tilde{\Lambda}_c''(\frac{5}{2}, +)$ ; both are  $L = 2$ orbitally excited states. The potential model suggests  $J^P = \frac{5}{2} - \text{ or } \frac{3}{2} + \text{ for } \Lambda_c(2940)^+$ . Measurements of the ratio of  $\Sigma_c^* \pi / \Sigma_c \pi$  will enable us to discriminate the *J<sup>P</sup>* assignments for  $\Lambda_c$  (2940). We advocate that the  $J^P$  quantum numbers of  $\Xi_c(2980)$  and  $\Xi_c(3077)$  are  $\frac{1}{2}^+$  and  $\frac{5}{2}^+$ , respectively. Under this  $J^P$ assignment, it is easy to understand why  $\Xi_c(2980)$  is broader than  $\Xi_c(3077)$ .

DOI: [10.1103/PhysRevD.75.014006](http://dx.doi.org/10.1103/PhysRevD.75.014006) PACS numbers: 14.20.Lq, 12.39.Fe, 12.39.Hg, 13.75.Gx

#### **I. INTRODUCTION**

In the past years many new excited charmed baryon states have been discovered by *BABAR*, Belle and CLEO. In particular, *B* factories have provided a very rich source of charmed baryons both from *B* decays and from the continuum  $e^+e^- \rightarrow c\bar{c}$ . A new era for the charmed baryon spectroscopy is opened by the rich mass spectrum and the relatively narrow widths of the excited states. Experimentally and theoretically, it is important to identify the quantum numbers of these new states and understand their properties. Since the pseudoscalar mesons involved in the strong decays of charmed baryons are soft, the charmed baryon system offers an excellent ground for testing the ideas and predictions of heavy quark symmetry of the heavy quarks and chiral symmetry of the light quarks.

The observed mass spectra and decay widths of charmed baryons are summarized in Tables I and II. Several new excited charmed baryon states such as  $\Lambda_c(2765)^+$ ,  $\Lambda_c(2880)^+$ ,  $\Lambda_c(2940)^+$ ,  $\Xi_c(2815)$ ,  $\Xi_c(2980)$  and  $E_c(3077)$  have been measured recently and they are still not on the particle listings of 2006 Review of Particle Physics by the Particle Data Group [[7](#page-13-0)]. By now, the  $J^P =$  $\frac{1}{2}$ <sup>+</sup> and  $\frac{1}{2}$ <sup>-</sup> antitriplet states:  $(\Lambda_c^+, \Xi_c^+, \Xi_c^0)$ ,  $(\Lambda_c(2593)^+, \Lambda_c^0)$  $E_c(2790)^+$ ,  $\Xi_c(2790)^0$ ), and  $J^P = \frac{1}{2}$ <sup>+</sup> and  $\frac{3}{2}$ <sup>+</sup> sexter

TABLE I. Masses (first entry) and decay widths (second entry) in units of MeV for the excited charmed baryons  $\Lambda_c(2880)$ ,  $\Lambda_c(2940)$ ,  $\Xi_c(2980)^{+,0}$ ,  $\Xi_c(3077)^{+,0}$  and  $\Omega_c(2768)$ .

State	$BABAR$ $[1-3]$	Belle $[4,5]$	CLEO $[6]$	Average
$\Lambda_c(2880)^+$	$2881.9 \pm 0.1 \pm 0.5$ $5.8 \pm 1.5 \pm 1.1$	$2881.2 \pm 0.2_{-0.3}^{+0.4}$ $5.5^{+0.7}_{-0.5} \pm 0.4$	$2882.5 \pm 2.2$ < 8	$2881.5 \pm 0.3$ $5.5 \pm 0.6$
$\Lambda_c(2940)^+$	$2939.8 \pm 1.3 \pm 1.0$ $17.5 \pm 5.2 \pm 5.9$	$2937.9 \pm 1.0^{+1.8}_{-0.4}$ $10 \pm 4 \pm 5$		$2938.8 \pm 1.1$ $13.0 \pm 5.0$
$\Xi_c(2980)^+$	$2967.1 \pm 1.9 \pm 1.0$ $23.6 \pm 2.8 \pm 1.3$	$2978.5 \pm 2.1 \pm 2.0$ $43.5 \pm 7.5 \pm 7.0$		$2971.1 \pm 1.7$ $25.2 \pm 3.0$
$\Xi_c(2980)^0$		$2977.1 \pm 8.8 \pm 3.5$ $43.5$ (fixed)		$2977.1 \pm 9.5$ 43.5
$\Xi_c(3077)^+$	$3076.4 \pm 0.7 \pm 0.3$ $6.2 \pm 1.6 \pm 0.5$	$3076.7 \pm 0.9 \pm 0.5$ $6.2 \pm 1.2 \pm 0.8$		$3076.5 \pm 0.6$ $6.2 \pm 1.1$
$\Xi_c(3077)^0$		$3082.8 \pm 1.8 \pm 1.5$ $5.2 \pm 3.1 \pm 1.8$		$3082.8 \pm 2.3$ $5.2 \pm 3.6$
$\Omega_c(2768)^0$	$2768.3 \pm 3.0$			$2768.3 \pm 3.0$

TABLE II. Mass spectra and decay widths (in units of MeV) of charmed baryons. Experimental values are taken from the Particle Data Group [\[7\]](#page-13-0) and Table I.

State	$J^P$	$S_{\ell}$	$L_{\ell}$	$J^{P_\ell}_\ell$	Mass	Width	Decay modes
$\Lambda_c^+$	$\frac{1}{2}$ + $\frac{1}{2}$ - $\frac{3}{2}$ -	$\mathbf{0}$	$\overline{0}$	$0^+$	$2286.46 \pm 0.14$		weak
$\Lambda_c(2593)^+$		$\mathbf{0}$	1	$1-$	$2595.4 \pm 0.6$	$3.6^{+2.0}_{-1.3}$	$\Sigma_c \pi$ , $\Lambda_c \pi \pi$
$\Lambda_c(2625)^+$		$\theta$	$\mathbf{1}$	$1-$	$2628.1 \pm 0.6$	< 1.9	$\Lambda_c \pi \pi$ , $\Sigma_c \pi$
$\Lambda_c(2765)^+$	$2^2$	$\gamma$	$\gamma$	$\overline{\cdot}$	$2766.6 \pm 2.4$	50	$\Sigma_c \pi$ , $\Lambda_c \pi \pi$
$\Lambda_c(2880)^+$	$\frac{5}{2}$ +	$\gamma$	$\gamma$	$\overline{\mathcal{L}}$	$2881.5 \pm 0.3$	$5.5 \pm 0.6$	$\Sigma_c^{(*)}\pi$ , $\Lambda_c\pi\pi$ , $D^0p$
$\Lambda_c(2940)^+$	$2^2$	$\overline{?}$	$\gamma$	$\overline{\mathcal{L}}$	$2938.8 \pm 1.1$	$13.0 \pm 5.0$	$\Sigma_c^{(*)}\pi$ , $\Lambda_c\pi\pi$ , $D^0p$
$\Sigma_c(2455)^{++}$		$\mathbf{1}$	$\mathbf{0}$	$1^+$	$2454.02 \pm 0.18$	$2.23 \pm 0.30$	$\Lambda_c \pi$
$\Sigma_c(2455)^+$		1	$\mathbf{0}$	$1^+$	$2452.9 \pm 0.4$	$<$ 4.6	$\Lambda_c \pi$
$\Sigma_c(2455)^0$		$\mathbf{1}$	$\boldsymbol{0}$	$1^+$	$2453.76 \pm 0.18$	$2.2 \pm 0.4$	$\Lambda_c \pi$
$\Sigma_c(2520)^{++}$		1	$\boldsymbol{0}$	$1^+$	$2518.4 \pm 0.6$	$14.9 \pm 1.9$	$\Lambda_c \pi$
$\Sigma_c(2520)^+$		1	$\mathbf{0}$	$1^+$	$2517.5 \pm 2.3$	$<$ 17	$\Lambda_c \pi$
$\Sigma_c(2520)^0$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{3}{2}$ $\frac{3}{2}$ $\frac{3}{2}$ $\frac{3}{2}$	1	$\mathbf{0}$	$1^+$	$2518.0 \pm 0.5$	$16.1 \pm 2.1$	$\Lambda_c \pi$
$\Sigma_c(2800)^{++}$	$-2$	1	1	$2^{-}$	$2801^{+4}_{-6}$	$75^{+22}_{-17}$	$\Lambda_c \pi$ , $\Sigma_c^{(*)} \pi$ , $\Lambda_c \pi \pi$
$\Sigma_c(2800)^+$	$rac{3}{2}$ $rac{3}{2}$ $-2$	1	1	$2^{-}$	$2792^{+14}_{-5}$	$62^{+60}_{-40}$	$\Lambda_c \pi$ , $\Sigma_c^{(*)} \pi$ , $\Lambda_c \pi \pi$
$\Sigma_c(2800)^0$	$rac{3}{2}$ $-2$	1	1	$2^{-}$	$2802^{+4}_{-7}$	$61^{+28}_{-18}$	$\Lambda_c \pi$ , $\Sigma_c^{(*)} \pi$ , $\Lambda_c \pi \pi$
$\Xi_c^+$	$+$	$\theta$	$\theta$	$0^+$	$2467.9 \pm 0.4$		weak
$\Xi_c^0$		$\boldsymbol{0}$	$\boldsymbol{0}$	$0^+$	$2471.0 \pm 0.4$		weak
$\Xi^{\prime +}_c$		$\mathbf{1}$	$\theta$	$1^+$	$2575.7 \pm 3.1$		$\Xi_c \gamma$
$\Xi_c^{\prime 0}$		1	$\mathbf{0}$	$1^+$	$2578.0 \pm 2.9$		$\Xi_c \gamma$
$\Xi_c(2645)^+$		$\mathbf{1}$	$\mathbf{0}$	$1^+$	$2646.6 \pm 1.4$	< 3.1	$\Xi_c\pi$
$\Xi_c(2645)^0$		1	$\boldsymbol{0}$	$1^+$	$2646.1 \pm 1.2$	< 5.5	$\Xi_c \pi$
$\Xi_c(2790)^+$		$\theta$	1	$1-$	$2789.2 \pm 3.2$	<15	$\Xi_c^{\prime} \pi$
$\Xi_c(2790)^0$		$\overline{0}$	$\mathbf{1}$	$1-$	$2791.9 \pm 3.3$	<12	$\Xi_c^\prime \pi$
$\Xi_c(2815)^+$		$\mathbf{0}$	$\mathbf{1}$	$1-$	$2816.5 \pm 1.2$	< 3.5	$\Xi_c^*\pi$ , $\Xi_c\pi\pi$ , $\Xi_c'\pi$
$\Xi_c(2815)^0$	$\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{2}{3} \frac{3}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{3}{2} \frac{3}{2} \frac{1}{2}$	$\overline{0}$	$\mathbf{1}$	$1-$	$2818.2 \pm 2.1$	< 6.5	$\Xi_c^*\pi$ , $\Xi_c\pi\pi$ , $\Xi_c'\pi$
$\Xi_c(2980)^+$	$2^2$	$\gamma$	$\gamma$	$\overline{\cdot}$	$2971.1 \pm 1.7$	$25.2 \pm 3.0$	see Table VII
$\Xi_c(2980)^0$	$2^?$	$\overline{?}$	$\gamma$	$\overline{\mathcal{L}}$	$2977.1 \pm 9.5$	43.5	see Table VII
$\Xi_c(3077)^+$	$2^2$	$\overline{\mathcal{L}}$	$\overline{?}$	$\overline{\mathcal{L}}$	$3076.5 \pm 0.6$	$6.2 \pm 1.1$	see Table VII
$\Xi_c(3077)^0$	2 <sup>2</sup>	$\overline{?}$	$\overline{?}$	$\overline{\mathcal{L}}$	$3082.8 \pm 2.3$	$5.2 \pm 3.6$	see Table VII
$\Omega_c^0$		$\mathbf{1}$	$\boldsymbol{0}$	$1^+$	$2697.5 \pm 2.6$		weak
$\Omega_c(2768)^0$	$\frac{1}{2}$ + $\frac{3}{2}$ +	$\mathbf{1}$	$\boldsymbol{0}$	$1^+$	$2768.3 \pm 3.0$		$\Omega_c\gamma$

states:  $(\Omega_c, \Sigma_c, \Xi_c')$ ,  $(\Omega_c^*, \Sigma_c^*, \Xi_c'^*)$  are established. Notice that except for the parity of the lightest  $\Lambda_c^+$  and the spinparity of  $\Lambda_c(2880)^+$ , none of the other  $J^P$  quantum numbers given in Table II has been measured. One has to rely on the quark model to determine the  $J<sup>P</sup>$  assignments.

This work is organized as follows. In Sec. II, the experimental status of the charmed baryon spectroscopy is reviewed. The *p*-wave charmed baryons and the first positive-parity excitations are discussed. In Sec. III we first present the relevant chiral Lagrangians which combine heavy quark and chiral symmetries. Then we proceed to the phenomenological implications to the strong decays of *s*-wave and *p*-wave charmed baryons as well as first positive-parity excited charmed baryon states. Conclusions are presented in Sec. IV.

## **II. SPECTROSCOPY**

Charmed baryon spectroscopy provides an ideal place for studying the dynamics of the light quarks in the environment of a heavy quark. The charmed baryon of interest contains a charmed quark and two light quarks, which we will often refer to as a diquark. Each light quark is a triplet of the flavor SU(3). Since  $3 \times 3 = \overline{3} + 6$ , there are two different SU(3) multiplets of charmed baryons: a symmetric sextet **6** and an antisymmetric antitriplet **3**. The spinflavor-space wave functions of baryons are totally symmetric since the color wave function is totally antisymmetric. For the ground-state *s*-wave baryons in the quark model, the symmetries in the flavor and spin of the diquarks are thus correlated. Consequently, the diquark in the

**TABLE III.** The *p*-wave charmed baryons and their quantum numbers, where  $S_{\ell}(J_{\ell})$  is the total spin (angular momentum) of the two light quarks. The quantum number in the subscript labels  $J_{\ell}$ , while the quantum number in parentheses is referred to the spin of the baryon. In the quark model, the upper (lower) eight multiplets have even (odd) orbital wave functions under the permutation of the two light quarks. That is,  $L_{\ell}$  for the former is referred to the orbital angular momentum between the diquark and the charmed quark, while  $L_{\ell}$  for the latter is the orbital angular momentum between the two light quarks. The explicit quark model wave functions for *p*-wave charmed baryons can be found in [\[8](#page-13-4)]. The states antisymmetric in orbital wave functions are denoted by a tilde, while the superscript prime is reserved for the  $\Xi_c$  charmed baryons to distinguish between the sextet and antitriplet SU(3) flavor states.

State	$SU(3)_F$	$S_{\ell}$ $L_{\ell}$ $J_{\ell}^{P_{\ell}}$	State	$SU(3)_F$	$S_{\ell}$ $L_{\ell}$ $J_{\ell}^{P_{\ell}}$	
$\Lambda_{c1}(\frac{1}{2}^{-},\frac{3}{2}^{-})$	$\bar{3}$ 0 1 1		$\Xi_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)$	$\bar{3}$ 0 1 1		
$\sum_{c} \left(\frac{1}{2} - \right)$	6	$1 \t 1 \t 0^-$	$\Xi_{c0}^{\prime}(\frac{1}{2}^{-})$	6 1 1 $0^-$		
$\sum_{c_1}(\frac{1}{2}^{-},\frac{3}{2}^{-})$	6 1 1 $1^{-}$		$\Xi_{c1}^{\prime}(\frac{1}{2}^{-},\frac{3}{2}^{-})$	6 $1 \t 1 \t 1^-$		
$\Sigma_{c2}(\frac{3}{2}^{-},\frac{5}{2}^{-})$	6	$1 \t 1 \t 2^-$	$\Xi_{c2}^{\prime}(\frac{3}{2}^{-},\frac{5}{2}^{-})$	6 $1 \t 1 \t 2^-$		
$\tilde{\Sigma}_{c1}(\frac{1}{2}^{-},\frac{3}{2}^{-})$	6 0 1 1 <sup>-1</sup>		$\tilde{\Xi}'_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)$		$0 \t 1 \t 1^-$	
$\tilde{\Lambda}_{c0}(\frac{1}{2}^{-})$	$\bar{3}$ 1 1 0 <sup>-1</sup>		$\tilde{\Xi}_{c0}(\frac{1}{2}^{-})$	$\bar{3}$ 1 1 0 <sup>-1</sup>		
$\tilde{\Lambda}_{c1}(\frac{1}{2}^{-},\frac{3}{2}^{-})$	$\bar{3}$ 1 1 1 $1^{-}$		$\Xi_{c1}(\frac{1}{2}^{-},\frac{3}{2}^{-})$	$\bar{3}$ 1 1 1 $^{-1}$		
$\tilde{\Lambda}_{c2}(\frac{3}{2}^-, \frac{5}{2}^-)$	$\bar{3}$ 1 1 2 <sup>-1</sup>		$\Xi_{c2}(\frac{3}{2}^{-},\frac{5}{2}^{-})$	$\bar{3}$ 1 1 2 <sup>-1</sup>		

flavor-symmetric sextet has spin 1, while the diquark in the flavor-antisymmetric antitriplet has spin 0. When the diquark combines with the charmed quark, the sextet contains both spin- $\frac{1}{2}$  and spin- $\frac{3}{2}$  charmed baryons. However, the antitriplet contains only spin- $\frac{1}{2}$  ones. More specifically, the  $\Lambda_c^+$ ,  $\Xi_c^+$  and  $\Xi_c^0$  form a  $\overline{3}$  representation and they all decay weakly. The  $\Omega_c^0$ ,  $\Xi_c^{\prime +}$ ,  $\Xi_c^{\prime 0}$  and  $\Sigma_c^{\dotsc +, +, 0}$  form a 6 representation; among them, only  $\Omega_c^0$  decays weakly. Note that we follow the Particle Data Group (PDG) [[7](#page-13-0)] to use a prime to distinguish the  $\Xi_c$  in the 6 from the one in the  $\bar{3}$ .

The lowest-lying orbitally excited baryon states are the *p*-wave charmed baryons with their quantum numbers listed in Table III. Although the separate spin angular momentum  $S_{\ell}$  and orbital angular momentum  $L_{\ell}$  of the light degrees of freedom are not well defined, they are included for guidance from the quark model. In the heavy quark limit, the spin of the charmed quark  $S_c$  and the total angular momentum of the two light quarks  $J_{\ell} = S_{\ell} + L_{\ell}$ are separately conserved. It is convenient to use them to enumerate the spectrum of states. There are two types of  $L_{\ell} = 1$  orbital excited charmed baryon states: states with the unit of orbital angular momentum between the diquark and the charmed quark, and states with the unit of orbital angular momentum between the two light quarks. The orbital wave function of the former (latter) is symmetric (antisymmetric) under the exchange of two light quarks. To see this, one can define two independent relative momenta  $\mathbf{k} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$  and  $\mathbf{K} = \frac{1}{2}(\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{p}_c)$  from the two light quark momenta  $\mathbf{p}_1$ ,  $\mathbf{p}_2$  and the heavy quark momentum  $\mathbf{p}_c$ . (In the heavy quark limit,  $\mathbf{p}_c$  can be set to zero.) Denoting the quantum numbers  $L_k$  and  $L_k$  as the eigen-

values of  $\mathbf{L}_k^2$  and  $\mathbf{L}_K^2$  the *k*-orbital momentum  $L_k$  describes relative orbital excitations of the two light quarks, and the *K*-orbital momentum  $L_K$  describes orbital excitations of the center of the mass of the two light quarks relative to the heavy quark [\[10\]](#page-13-2). The *p*-wave heavy baryon can be either in the  $(L_k = 0, L_k = 1)$  *K*-state or the  $(L_k = 1)$ 1,  $L_K = 0$ ) *k*-state. It is obvious that the orbital *K*-state (*k*-state) is symmetric (antisymmetric) under the interchange of  $\mathbf{p}_1$  and  $\mathbf{p}_2$ .

There are seven lowest-lying *p*-wave  $\Lambda_c$  arising from combining the charmed quark spin  $S_c$  with light constituents in  $J_{\ell}^{\bar{P}_{\ell}} = 1^-$  state: three  $J^{\bar{P}} = \frac{1}{2}^-$  states, three  $J^P =$  $\frac{3}{2}$  <sup>-</sup> states and one  $J^P = \frac{5}{2}$  <sup>-</sup> state. They form three doublets  $\Lambda_{c1}(\frac{1}{2}, \frac{3}{2}), \tilde{\Lambda}_{c1}(\frac{1}{2}, \frac{3}{2}), \tilde{\Lambda}_{c2}(\frac{3}{2}, \frac{5}{2})$  and one singlet  $\tilde{\Lambda}_{c0}(\frac{1}{2})$ , where we have used a tilde to denote the multiplets antisymmetric in the orbital wave functions under the exchange of two light quarks. In terms of *K*- and *k*-states introduced above, the doublets  $\Lambda_{c1}$ ,  $\tilde{\Lambda}_{c1}$ ,  $\tilde{\Lambda}_{c2}$  and the singlet  $\tilde{\Lambda}_{c0}$  are sometimes denoted by  $\Lambda_{cK1}$ ,  $\Lambda_{ck1}$ ,  $\Lambda_{ck2}$  and  $\Lambda_{ck0}$ , respectively, in the literature [\[10\]](#page-13-2). Quark models [[9\]](#page-13-3) indicate that the untilde states for  $\Lambda$ - and  $\Sigma$ -type charmed baryons with symmetric orbital wave functions lie about 150 MeV below the tilde ones. The two states in each doublet with *J*  $J_{\ell} \pm \frac{1}{2}$  are nearly degenerate; their masses split only by a chromomagnetic interaction.

Since the spin-parity of the newly measured  $\Lambda_c(2880)^+$ was recently pinned down to be  $\frac{5}{2}$  <sup>+</sup> by Belle [[4](#page-12-2)], we shall

<sup>&</sup>lt;sup>1</sup>In the notation of [[9](#page-13-3)],  $L_k$  and  $L_K$  correspond to  $\ell_\rho$  and  $\ell_\lambda$ , respectively.

TABLE IV. The first positive-parity excitations of charmed baryons and their quantum numbers. States with antisymmetric orbital wave functions (i.e.  $L_K = L_k = 1$ ) under the interchange of two light quarks are denoted by a tilde. A prime is used to distinguish between the sextet and antitriplet SU(3) flavor states of the excited  $\Xi_c$ .

<b>State</b>	$SU(3)_F$	$S_{\ell}$	$L_{\ell}$	$J^{P_\ell}_\ell$	<b>State</b>	$SU(3)_F$	$S_{\ell}$	$L_{\ell}$	$J^{P_\ell}_\ell$
$\Lambda_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$	$\bar{3}$	$\mathbf{0}$	2	$2^+$	$\Sigma_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$	6	$\mathbf{1}$	2	$1^+$
$\tilde{\Lambda}_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$	$\bar{3}$	$\mathbf{1}$	$\boldsymbol{0}$	$1^+$	$\Sigma_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$	6	$\mathbf{1}$	$\overline{2}$	$2^+$
$\tilde{\Lambda}_{c0}^{\prime}(\frac{1}{2}^{\ +})$	$\bar{3}$	$\mathbf{1}$	$\mathbf{1}$	$0^+$	$\sum_{c3}(\frac{5}{2}^+, \frac{7}{2}^+)$	6	$\mathbf{1}$	$\overline{2}$	$3^+$
$\tilde{\Lambda}'_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$	$\bar{3}$	$\mathbf{1}$	$\mathbf{1}$	$1^+$	$\tilde{\Sigma}_{c0}({\textstyle \frac{1}{2}\,}^+)$	6	$\overline{0}$	$\boldsymbol{0}$	$0^+$
$\tilde{\Lambda}_{c2}^{\prime}(\frac{3}{2}^+, \frac{5}{2}^+)$	$\bar{3}$	$\mathbf{1}$	$\mathbf{1}$	$2^+$	$\tilde{\Sigma}_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$	6	$\overline{0}$	$\mathbf{1}$	$1^+$
$\tilde{\Lambda}''_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$	$\bar{3}$	$\mathbf{1}$	$\overline{2}$	$1^+$	$\tilde{\Sigma}_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$	6	$\overline{0}$	$\overline{c}$	$2^+$
$\tilde{\Lambda}_{c2}''(\frac{3}{2}^+, \frac{5}{2}^+)$	$\bar{3}$	1	$\overline{2}$	$2^+$					
$\tilde{\Lambda}_{c3}''(\frac{5}{2}^+, \frac{7}{2}^+)$	$\bar{3}$	1	2	$3^+$					
$\Xi_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$	$\overline{3}$	$\boldsymbol{0}$	2	$2^+$	$\Xi'_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$	6	1	2	$1^+$
$\tilde{\Xi}_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$	$\bar{3}$	$\mathbf{1}$	$\mathbf{0}$	$1^+$	$\Xi'_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$	6	$\mathbf{1}$	$\overline{2}$	$2^+$
$\tilde{\Xi}''_{c0}(\frac{1}{2}$ +)	$\bar{3}$	$\mathbf{1}$	$\mathbf{1}$	$0^+$	$\Xi'_{c3}(\frac{5}{2}^+, \frac{7}{2}^+)$	6	$\mathbf{1}$	$\overline{2}$	$3^+$
$\tilde{\Xi}''_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$	$\bar{3}$	$\mathbf{1}$	1	$1^+$	$\tilde{\Xi}^{\prime}_{c0}({\frac{1}{2}}^{\ +})$	6	$\boldsymbol{0}$	$\boldsymbol{0}$	$0^+$
$\tilde{\Xi}''_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$	$\bar{3}$	$\mathbf{1}$	$\mathbf{1}$	$2^+$	$\tilde{\Xi}'_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$	6	$\boldsymbol{0}$	$\mathbf{1}$	$1^+$
$\tilde{\Xi}'''_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$	$\bar{3}$	$\mathbf{1}$	$\overline{c}$	$1^+$	$\tilde{\Xi}'_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$	6	$\boldsymbol{0}$	$\overline{c}$	$2^+$
$\tilde{\Xi}'''_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$	$\overline{3}$	1	$\overline{2}$	$2^+$					
$\tilde{\Xi}'''_{c3}(\frac{5}{2}^+, \frac{7}{2}^+)$	$\bar{3}$	$\mathbf{1}$	$\overline{2}$	$3^+$					

briefly discuss the positive-parity excitations of charmed baryons. Referring to the orbital angular momentum quantum numbers  $L_K$  and  $L_k$ , the first positive-parity excitations are those states with  $L_K + L_k = 2$ . For  $L_K = 2$ ,  $L_k = 0$ ,  $L = 2$  or  $L_k = 0$ ,  $L_k = 2$ ,  $L = 2$ , there is one multiplet for positive-parity excited  $\Lambda_c$  and three multiplets for  $\Sigma_c$  as tabulated in Table IV.<sup>2</sup> The orbital states of these multiplets are symmetric under the interchange of the two light quarks. For the case of  $L_K = L_k = 1$ , the total orbital angular momentum  $L_{\ell}$  of the diquark is 2, 1 or 0. Since the orbital states are antisymmetric under the interchange of two light quarks, we shall use a tilde to denote the  $L_K = L_k = 1$  states. The Fermi-Dirac statistics for baryons yields seven more multiplets for positive-parity excited  $\Lambda_c$  states and three more multiplets for  $\Sigma_c$  baryons. The reader is referred to [[11\]](#page-13-5) for more details.

In the following we discuss some of the new excited charmed baryon states:

# $A. \Lambda_c$

 $\Lambda_c(2593)^+$  and  $\Lambda_c(2625)^+$  form a doublet  $\Lambda_{c1}(\frac{1}{2}^{-}, \frac{3}{2}^{-})$ [\[12\]](#page-13-6). The dominant decay mode is  $\Sigma_c \pi$  in an *S* wave for  $\Lambda_{c1}(\frac{1}{2})$  and  $\Lambda_c \pi \pi$  in a *P* wave for  $\Lambda_{c1}(\frac{3}{2})$ . (The twobody mode  $\Sigma_c \pi$  is a *D*-wave in  $\Lambda_c(\frac{3}{2})$  decay.) This explains why the width of  $\Lambda_c(2625)^+$  is narrower than that of  $\Lambda_c(2593)^+$ .

 $\Lambda_c(2765)^+$  is a broad state ( $\Gamma \approx 50$  MeV) first seen in  $\Lambda_c^+ \pi^+ \pi^-$  by CLEO [\[6](#page-13-1)]. It appears to resonate through  $\Sigma_c$ and probably also  $\Sigma_c^*$ . However, whether it is a  $\Lambda_c^+$  or a  $\Sigma_c^+$ or whether the width might be due to overlapping states are not known. The Skyrme model [[13\]](#page-13-7) and the quark model [\[9\]](#page-13-3) suggest a  $J^P = \frac{1}{2} + \Lambda_c$  state with a mass 2742 and 2775 MeV, respectively. Therefore,  $\Lambda_c(2765)^+$  could be a first positive-parity excitation of  $\Lambda_c$ .

The state  $\Lambda_c(2880)^+$  first observed by CLEO [\[6](#page-13-1)] in  $\Lambda_c^+ \pi^+ \pi^-$  was also seen by *BABAR* in the  $D^0 p$  spectrum [\[1\]](#page-12-0). It was originally conjectured that, based on its narrow width,  $\Lambda_c(2880)^+$  might be a  $\tilde{\Lambda}_{c0}^{+}(\frac{1}{2}^{-})$  state [\[6\]](#page-13-1). Recently, Belle has studied the experimental constraint on the  $J<sup>P</sup>$ quantum numbers of  $\Lambda_c(2880)^+$  [\[4\]](#page-12-2). The angular analysis of  $\Lambda_c(2880)^+$   $\rightarrow \Sigma_c^{0,++} \pi^{\pm}$  indicates that  $J = \frac{5}{2}$  is favored over  $J = \frac{1}{2}$  or  $\frac{3}{2}$ , while the study of the resonant structure of  $\Lambda_c(2880)^+$   $\rightarrow \Lambda_c^+ \pi^+ \pi^-$  implies the existence of the  $\Sigma_c^* \pi$ intermediate states and  $\Gamma(\Sigma_c^* \pi^{\pm}) / \Gamma(\Sigma_c \pi^{\pm}) = (24.1 \pm$  $6.4^{+1.1}_{-4.5}$ %. This value is in agreement with heavy quark symmetry predictions [\[14](#page-13-8)] and favors the  $\frac{5}{2}$  + over the  $\frac{5}{2}$  assignment. We shall return back to this point in Sec. III C. It is interesting to notice that, based on the diquark idea, the quantum numbers  $J^P = \frac{5}{2}$  <sup>+</sup> have already been predicted in [\[15\]](#page-13-9) for the  $\Lambda_c(2880)$  before the Belle experiment.

<sup>&</sup>lt;sup>2</sup>Strictly spaking, there are two multiplets for positive-parity excited  $\Lambda_c$  and six multiplets for  $\Sigma_c$  coming from two different orbital states  $L_K = 2$ ,  $\hat{L_k} = 0$  and  $\hat{L}_K = 0$ ,  $L_k = 2$ . For simplicity, here we will not distinguish between them.

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The highest  $\Lambda_c(2940)^+$  was first discovered by *BABAR* in the  $D^0 p$  decay mode [\[1\]](#page-12-0) and confirmed by Belle in the decays  $\Sigma_c^0 \pi^+$ ,  $\Sigma_c^{++} \pi^-$  which subsequently decay into  $\Lambda_c^+ \pi^+ \pi^-$  [[4](#page-12-2),[16](#page-13-10)]. Since the mass of  $\Lambda_c(2940)^+$  is barely below the threshold of  $D^{*0}p$ , this observation has motivated the authors of [\[17\]](#page-13-11) to suggest an exotic molecular state of  $D^{*0}$  and *p* with a binding energy of order 6 MeV for  $\Lambda_c(2940)^+$ . Its quantum numbers  $J^P$  could be  $\frac{3}{2}$  + or  $\frac{5}{2}$  as suggested by the quark model calculation [\[9](#page-13-3)].

# **B.**  $\Sigma_c$

The highest isotriplet charmed baryons  $\Sigma_c(2800)^{+,+,+,0}$ decaying to  $\Lambda_c^+ \pi$  were first measured by Belle [\[18\]](#page-13-12). They are most likely to be the  $J^P = \frac{3}{2} - \sum_{c}$  states because the  $\sum_{c2}(\frac{3}{2})$  baryon decays principally into the  $\Lambda_c \pi$  system in a *D*-wave, while  $\Sigma_{c1}(\frac{3}{2})$  decays mainly to the two-pion system  $\Lambda_c \pi \pi$  in a *P*-wave. The state  $\Sigma_{c0}(\frac{1}{2})$  can decay into  $\Lambda_c \pi$  in an *S*-wave, but it is very broad with width of order 406 MeV (see Sec. III C).

# $C. \ \Xi_c$

The states  $\Xi_c(2790)$  and  $\Xi_c(2815)$  form a doublet  $\Xi_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)$ . Since the diquark transition  $1^- \rightarrow 0^+ + \pi$ is prohibited,  $\Xi_{c1}(\frac{1}{2}^{-}, \frac{3}{2}^{-})$  cannot decay to  $\Xi_{c}\pi$ . The dominant decay mode is  $[\Xi_c' \pi]_S$  for  $\Xi_{c1}(\frac{1}{2})$  and  $[\Xi_c^* \pi]_S$  for  $\Xi_{c1}(\frac{3}{2})$  where  $\Xi_c^*$  stands for  $\Xi_c(2645)$ .

The new charmed strange baryons  $\Xi_c(2980)^+$  and  $\Xi_c(3077)^+$  that decay into  $\bar{\Lambda}_c^+ K^- \pi^+$  were first observed by Belle [[5](#page-12-3)] and confirmed by *BABAR* [[2](#page-12-4)]. In the recent *BABAR* measurement [[2\]](#page-12-4), the  $\Xi_c(2980)^+$  is found to decay resonantly through the intermediate state  $\sum_{c}(2455)^{++}K^$ with 4.9 $\sigma$  significance and nonresonantly to  $\Lambda_c^+ K^- \pi^+$ with  $4.1\sigma$  significance. With  $5.8\sigma$  significance, the  $\Xi_c(3077)^+$  is found to decay resonantly through  $\sum_{c}(2455)^{++}K^{-}$ , and with  $4.6\sigma$  significance, it is found to decay through  $\Sigma_c(2520)^{++}K^-$ . The significance of the signal for the nonresonant decay  $\Xi_c(3077)^+ \to \Lambda_c^+ K^- \pi^+$ is  $1.4\sigma$ .

# $\mathbf{D}$ .  $\mathbf{\Omega}_c$

At last, the  $J^P = \frac{3}{2} + \Omega_c(2768)$  charmed baryon was recently observed by *BABAR* in the decay  $\Omega_c(2768)^0 \rightarrow$  $\Omega_c^0 \gamma$  [\[3\]](#page-12-1). With this new observation, the  $\frac{3}{2}$  <sup>+</sup> sextet is finally completed. However, it will be very difficult to measure the electromagnetic decay rate because the width of  $\Omega_c^*$ , which is predicted to be of order 0.9 keV [\[19\]](#page-13-13), is too narrow to be experimentally resolvable.

#### **III. STRONG DECAYS**

Because of the rich mass spectrum and the relatively narrow widths of the excited states, the charmed baryon system offers an excellent ground for testing the ideas and predictions of heavy quark symmetry and light flavor SU(3) symmetry. The pseudoscalar mesons involved in the strong decays of charmed baryons such as  $\Sigma_c \rightarrow \Lambda_c \pi$ are soft. Therefore, heavy quark symmetry of the heavy quark and chiral symmetry of the light quarks will have interesting implications for the low-energy dynamics of heavy baryons interacting with the Goldstone bosons.

The strong decays of charmed baryons are most conveniently described by the heavy hadron chiral Lagrangians in which heavy quark symmetry and chiral symmetry are incorporated [\[20](#page-13-14)[,21\]](#page-13-15). The Lagrangian involves two coupling constants  $g_1$  and  $g_2$  for *P*-wave transitions between *s*-wave and *s*-wave baryons [\[20\]](#page-13-14), six couplings  $h_2-h_7$  for the *S*-wave transitions between *s*-wave and *p*-wave baryons, and eight couplings  $h_8-h_{15}$ for the *D*-wave transitions between *s*-wave and *p*-wave baryons [[8\]](#page-13-4).

Since the general chiral Lagrangian for heavy baryons coupling to the pseudoscalar mesons can be expressed compactly in terms of superfields, we first introduce the superfields for *s*-wave baryons given by

$$
S_{\mu}^{ij} = \frac{1+\rlap{\hspace{0.02cm}/}{p}}{2} B_{6\mu}^{*ij} + \frac{1}{\sqrt{3}} (\gamma_{\mu} + \nu_{\mu}) \gamma_{5} \frac{1+\rlap{\hspace{0.02cm}/}{p}}{2} B_{6}^{ij},
$$
  
\n
$$
\bar{S}_{\mu}^{ij} = \bar{B}_{6\mu}^{*ij} \frac{1+\rlap{\hspace{0.02cm}/}{p}}{2} - \frac{1}{\sqrt{3}} \bar{B}_{6}^{ij} \frac{1+\rlap{\hspace{0.02cm}/}{p}}{2} \gamma_{5} (\gamma_{\mu} + \nu_{\mu}), \qquad (3.1)
$$
  
\n
$$
\mathcal{T}_{i} = \frac{1+\rlap{\hspace{0.02cm}/}{p}}{2} (\Xi_{c}^{0}, -\Xi_{c}^{+}, \Lambda_{c}^{+})_{i} = \frac{1}{2} \epsilon_{ijk} \frac{1+\rlap{\hspace{0.02cm}/}{p}}{2} (B_{\bar{3}})_{jk}.
$$

where the matrices  $B_6$  and  $B_{6\mu}^*$  are defined in [\[20\]](#page-13-14)

$$
(B_6)_{ij} = \begin{pmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}} \Sigma_c^+ & \frac{1}{\sqrt{2}} \Xi_c^{\prime+} \\ \frac{1}{\sqrt{2}} \Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}} \Xi_c^0 \\ \frac{1}{\sqrt{2}} \Xi_c^{+'} & \frac{1}{\sqrt{2}} \Xi_c^0 & \Omega_c^0 \end{pmatrix}_{ij},
$$
  
\n
$$
(B_{\bar{3}})_{ij} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}_{ij}.
$$
 (3.2)

<span id="page-4-0"></span>The superfield for *p*-wave **3** multiplets symmetric in orbital wave functions such as  $\Lambda_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)$  and  $\Xi_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)$ is given by

$$
\mathcal{R}^{i}_{\mu} = \frac{1}{\sqrt{3}} (\gamma_{\mu} + v_{\mu}) \gamma_{5} R^{i} + R^{*i}_{\mu}, \tag{3.3}
$$

with

$$
R_{i} = \frac{1+\rlap{\hspace{0.02cm}/}{2}}{2} (\Xi^{0}_{c_{1}}, -\Xi^{+}_{c_{1}}, \Lambda^{+}_{c_{1}})_{i},
$$
\n
$$
R^{*i}_{\mu} = \frac{1+\rlap{\hspace{0.02cm}/}{2}}{2} (\Xi^{*0}_{c_{1}}, -\Xi^{*+}_{c_{1}}, \Lambda^{*+}_{c_{1}})_{\mu i},
$$
\n(3.4)

where  $\mathcal{B}_{c}^{*}$  denotes a spin- $\frac{3}{2}$  charmed baryon. Note that  $v \cdot$  $S = v \cdot \mathcal{R} = 0.$ 

There are three other *p*-wave sextet multiplets with antisymmetric orbital states and with quantum numbers  $J_{\ell}^{P_{\ell}} = 0^{-}$ , 1<sup>-</sup>, 2<sup>-</sup>. Their *I* = 1 members are  $\Sigma_{c0}(\frac{1}{2}^{-})$ ,

 $\Sigma_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)$  and  $\Sigma_{c2}(\frac{3}{2}^-, \frac{5}{2}^-)$ , while the corresponding  $I = \frac{1}{2}$  members are  $\Xi_{c0}^{7}(\frac{1}{2}^{-})$ ,  $\Xi_{c1}^{7}(\frac{1}{2}^{-}, \frac{3}{2}^{-})$  and  $\Xi_{c2}^{7}(\frac{3}{2}^{-})$ ,  $\frac{5}{2}$ <sup>-</sup>) (see Table III), The  $J_{\ell}^{P_{\ell}} = 0$ <sup>-</sup> multiplet will be represented as a symmetric matrix  $(U)_{ii}$  defined in the same manner as  $(B_6)_{ij}$ 

$$
U_{ij} = \begin{pmatrix} \Sigma_{c0}^{++} & \frac{1}{\sqrt{2}} \Sigma_{c0}^{+} & \frac{1}{\sqrt{2}} \Xi_{c0}^{\prime+} \\ \frac{1}{\sqrt{2}} \Sigma_{c0}^{+} & \Sigma_{c0}^{0} & \frac{1}{\sqrt{2}} \Xi_{c0}^{\prime0} \\ \frac{1}{\sqrt{2}} \Xi_{c0}^{\prime+} & \frac{1}{\sqrt{2}} \Xi_{c0}^{\prime0} & \Omega_{c0}^{0} \end{pmatrix}_{ij}, \qquad (3.5)
$$

The  $J_{\ell}^{P_{\ell}} = 1^-$  multiplet will be represented as a superfield similar to ([3.3](#page-4-0)) but with a symmetric matrix  $\mathcal{V}_{\mu}^{ij}$ 

$$
\mathcal{V}_{\mu}^{ij} = \frac{1}{\sqrt{3}} (\gamma_{\mu} + v_{\mu}) \gamma_5 V^{ij} + V_{\mu}^{*ij}, \tag{3.6}
$$

where  $V_{ij}$  has the same expression as  $U_{ij}$  except for the replacement of the superscript ''*c*0'' by ''*c*1''. The superfield corresponding to the  $J_{\ell}^{P_{\ell}} = 2^{-}$  baryons is constructed as [\[22\]](#page-13-16)

$$
\mathcal{X}^{ij}_{\mu\nu} = X^{*ij}_{\mu\nu} + \frac{1}{\sqrt{10}} \{ (\gamma_{\mu} + \nu_{\mu}) \gamma_5 g^{\alpha}_{\nu} \n+ (\gamma_{\nu} + \nu_{\nu}) \gamma_5 g^{\alpha}_{\mu} \} X^{ij}_{\alpha},
$$
\n(3.7)

with  $X_{\mu\nu}^{*ij}$  a spin- $\frac{5}{2}$  Rarita-Schwinger field and  $X_{\alpha}^{ij}$  its spin- $\frac{3}{2}$ heavy quark symmetry partner.

The *p*-wave states with antisymmetric orbital wave functions can be constructed in complete analogy to the symmetric ones. Following [\[8\]](#page-13-4), we use the superfield  $\tilde{\mathcal{R}}_{\mu}^{ij}$ constructed in analogy to  $S_{\mu}^{ij}$  to represent the two sextets  $\sum_{c=1}^{\infty}$  $(\frac{1}{2} - \frac{3}{2} -)$  and  $\sum_{c=1}^{\infty}$  $(\frac{1}{2} - \frac{3}{2} -)$ . Likewise, we use the superfields  $\tilde{u}^i_\mu$ ,  $\tilde{\gamma}^i_\mu$ ,  $\tilde{\chi}^i_{\mu\nu}$  to denote the antitriplets:  $\tilde{\Lambda}^+_{c0}$ ,  $\tilde{\Lambda}_{c1}^{+}$ ,  $\tilde{\Lambda}_{c2}^{+}$  in  $I = 0$  and  $\tilde{\Xi}_{c0}$ ,  $\tilde{\Xi}_{c1}$ ,  $\tilde{\Xi}_{c2}$  in  $I = \frac{1}{2}$ .

<span id="page-5-0"></span>The leading Lagrangian terms describing *P*-wave couplings among the *s*-wave baryons and *S*-wave couplings between the *s*-wave and *p*-wave baryons are

$$
\mathcal{L}_P = \frac{3}{2} i g_1 \epsilon_{\mu\nu\sigma\lambda} \operatorname{Tr} (\bar{S}^{\mu} v^{\nu} A^{\sigma} S^{\lambda})
$$

$$
- \sqrt{3} g_2 \operatorname{Tr} (\bar{B}_{\bar{3}} A^{\mu} S_{\mu} + \bar{S}^{\mu} A_{\mu} B_{\bar{3}}), \qquad (3.8)
$$

<span id="page-5-2"></span>and $3$ 

$$
\mathcal{L}_{S} = h_{2} \{ \epsilon_{ijk} \bar{\mathcal{R}}_{i}^{\mu} v_{\nu} A_{jl}^{\nu} \mathcal{S}_{\mu}^{kl} + \epsilon_{ijk} \bar{\mathcal{S}}_{\mu}^{kl} v_{\nu} A_{lj}^{\nu} \mathcal{R}_{i}^{\mu} \} \n+ h_{3} \operatorname{Tr}(\bar{B}_{\bar{3}} v_{\mu} A^{\mu} U + \bar{U} v^{\mu} A_{\mu} B_{\bar{3}}) \n+ h_{4} \operatorname{Tr} {\tilde{V}}_{\mu} v_{\nu} A^{\nu} \mathcal{S}^{\mu} + \bar{\mathcal{S}}_{\mu} v_{\nu} A^{\nu} \mathcal{V}^{\mu} \} \n+ h_{5} \operatorname{Tr}({\tilde{\mathcal{R}}}_{\mu} v_{\nu} A^{\nu} \mathcal{S}^{\mu} + \bar{\mathcal{S}}^{\mu} v_{\nu} A^{\nu} {\tilde{\mathcal{R}}}_{\mu}) \n+ h_{6} ({\tilde{T}}_{i} v_{\nu} A_{ji}^{\nu} {\tilde{U}}_{j} + {\tilde{U}}_{i} v_{\nu} A_{ji}^{\mu} {\tilde{T}}_{j}) \n+ h_{7} {\{\epsilon_{ijk} {\tilde{\mathcal{V}}}}_{\mu}^{\nu} v_{\nu} A_{ji}^{\nu} \mathcal{S}_{kl}^{\mu} + {\epsilon_{ijk} {\bar{\mathcal{S}}}_{kl}^{\mu} v_{\nu} A_{lj}^{\nu} {\tilde{\mathcal{V}}}_{\mu}^i \}, (3.9)
$$

respectively. The Goldstone bosons couple to the matter fields through the nonlinear axial-vector field  $A_\mu$  defined as

$$
A_{\mu} = \frac{i}{2} (\xi^{\dagger} \partial_{\mu} \xi - \xi \partial_{\mu} \xi^{\dagger})
$$
  
= 
$$
-\frac{1}{f_{\pi}} \partial_{\mu} \phi + \frac{1}{6 f_{\pi}^{3}} [\phi, [\phi, \partial_{\mu} \phi]] + \cdots,
$$
 (3.10)

with  $\xi = \exp(i\phi/f_\pi)$ ,  $\phi = \frac{1}{\sqrt{2}}\pi^a \lambda^a$  and  $f_\pi = 132$  MeV.

The *D*-wave couplings of the *p*-wave baryons to *s*-wave baryons are described by dimension-5 terms in the effective Lagrangian [[8\]](#page-13-4)

<span id="page-5-1"></span>
$$
\mathcal{L}_{D} = i h_{8} \epsilon_{ijk} \tilde{S}_{\mu}^{kl} \Big( \mathcal{D}^{\mu} A^{\nu} + \mathcal{D}^{\nu} A^{\mu} + \frac{2}{3} g^{\mu \nu} (\nu \cdot \mathcal{D}) (\nu \cdot A) \Big)_{lj} \mathcal{R}_{\nu}^{i}
$$
\n
$$
+ i h_{9} \operatorname{Tr} \Big[ \tilde{S}_{\mu} \Big( \mathcal{D}^{\mu} A^{\nu} + \mathcal{D}^{\nu} A^{\mu} + \frac{2}{3} g^{\mu \nu} (\nu \cdot \mathcal{D}) (\nu \cdot A) \Big) \mathcal{V}_{\nu} \Big] + i h_{10} \epsilon_{ijk} \tilde{T}_{i} (\mathcal{D}_{\mu} A_{\nu} + \mathcal{D}_{\nu} A_{\mu})_{jl} \mathcal{X}_{kl}^{\mu \nu}
$$
\n
$$
+ h_{11} \epsilon_{\mu \nu \sigma \lambda} \operatorname{Tr} \Big\{ \bar{S}^{\mu} (\mathcal{D}^{\nu} A_{\alpha} + \mathcal{D}_{\alpha} A^{\nu}) \mathcal{X}^{\alpha \sigma} \Big\} \nu^{\lambda} + i h_{12} \operatorname{Tr} \Big\{ \bar{S}_{\mu} \Big( \mathcal{D}^{\mu} A^{\nu} + \mathcal{D}^{\nu} A^{\mu} + \frac{2}{3} g^{\mu \nu} (\nu \cdot \mathcal{D}) (\nu \cdot A) \Big) \tilde{T}_{\nu} \Big\} + i h_{13} \epsilon_{ijk} \bar{S}_{\mu}^{kl} \Big( \mathcal{D}^{\mu} A^{\nu} + \mathcal{D}^{\nu} A^{\mu} + \frac{2}{3} g^{\mu \nu} (\nu \cdot \mathcal{D}) (\nu \cdot A) \Big)_{lj} \tilde{V}_{\nu}^{i} + i h_{14} \tilde{T}_{i} (\mathcal{D}^{\mu} A^{\nu} + \mathcal{D}^{\nu} A^{\mu})_{ji} \tilde{X}_{\mu \nu}^{j}
$$
\n
$$
+ h_{15} \epsilon_{\mu \nu \sigma \lambda} \epsilon_{ijk} \bar{S}_{kl}^{\mu} (\mathcal{D}^{\nu} A_{\alpha} + \mathcal{D}_{\alpha} A^{\nu})_{lj} \tilde{X}_{i}^{\alpha \sigma} \nu^{\lambda}, \
$$

where the covariant derivative of the axial-vector field  $A_{\mu}$   $V_{\mu} = \frac{1}{2}$ <br>is defined as  $\mathcal{D}_{\mu}A_{\nu} = \partial_{\mu}A_{\nu} + [V_{\mu}, A_{\nu}]$  with is defined as  $\mathcal{D}_{\mu}A_{\nu} = \partial_{\mu}A_{\nu} + [$ 

$$
V_{\mu} = \frac{1}{2} (\xi^{\dagger} \partial_{\mu} \xi + \xi \partial_{\mu} \xi^{\dagger}) = \frac{1}{2f_{\pi}^{2}} [\phi, \partial_{\mu} \phi] + \cdots,
$$
\n(3.12)

and satisfies the relation  $\mathcal{D}_{\mu}A_{\nu} - \mathcal{D}_{\nu}A_{\mu} = 0$ . Note that a pure *D*-wave is described by the configuration

<sup>&</sup>lt;sup>3</sup>The original  $h_1$  term defined in [[12](#page-13-6)] is now the  $g_2$  term in Eq.  $(3.8)$  where we have followed [\[20\]](#page-13-14) for the definition of  $g_1$ and  $g_2$  couplings. The  $h_3, \dots, h_7$  terms were first introduced in [\[8\]](#page-13-4).

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$$
D_{\mu\nu} = (\partial_{\mu} - \nu_{\mu} \nu \cdot \partial)(\partial_{\nu} - \nu_{\nu} \nu \cdot \partial)
$$
  
 
$$
- \frac{1}{3} (g_{\mu\nu} - \nu_{\mu} \nu_{\nu})(\partial - \nu \nu \cdot \partial)^2, \qquad (3.13)
$$

satisfying  $v^{\mu}D_{\mu\nu} = 0$ ,  $D_{\mu}^{\mu} = 0$  and  $D_{\mu\nu} = D_{\nu\mu}$ . It is straightforward to show that the structure  $\mathcal{D}^{\mu}A^{\nu}$  +  $\mathcal{D}^{\nu}A^{\mu} + \frac{2}{3}g^{\mu\nu}(v \cdot \mathcal{D})(v \cdot A)$  appearing in Eq. ([3.11](#page-5-1)) indeed projects out a pure *D*-wave.

<span id="page-6-0"></span>Some of the partial widths derived from the Lagrangians [\(3.8\)](#page-5-0) and [\(3.9\)](#page-5-2) are [[8\]](#page-13-4):

$$
\Gamma(\Sigma_c^* \to \Sigma_c \pi) = \frac{g_1^2}{2\pi f_\pi^2} \frac{m_{\Sigma_c}}{m_{\Sigma_c^*}} p_\pi^3,
$$
  
\n
$$
\Gamma(\Sigma_c \to \Lambda_c \pi) = \frac{g_2^2}{2\pi f_\pi^2} \frac{m_{\Lambda_c}}{m_{\Sigma_c}} p_\pi^3,
$$
  
\n
$$
\Gamma(\Lambda_{c1}(1/2^-) \to \Sigma_c \pi) = \frac{h_2^2}{2\pi f_\pi^2} \frac{m_{\Sigma_c}}{m_{\Lambda_c}} E_\pi^2 p_\pi,
$$
  
\n
$$
\Gamma(\Sigma_{c0}(1/2^-) \to \Lambda_c \pi) = \frac{h_3^2}{2\pi f_\pi^2} \frac{m_{\Lambda_c}}{m_{\Sigma_c}} E_\pi^2 p_\pi,
$$
  
\n
$$
\Gamma(\Sigma_{c1}(1/2^-) \to \Sigma_c \pi) = \frac{h_4^2}{4\pi f_\pi^2} \frac{m_{\Sigma_c}}{m_{\Sigma_c}} E_\pi^2 p_\pi,
$$
  
\n(3.14)

$$
\Gamma(\tilde{\Sigma}_{c1}(1/2^{-}) \to \Sigma_{c}\pi) = \frac{h_{5}^{2}}{4\pi f_{\pi}^{2}} \frac{m_{\Sigma_{c}}}{m_{\tilde{\Sigma}_{c1}}} E_{\pi}^{2} p_{\pi},
$$
  

$$
\Gamma(\tilde{\Xi}_{c0}(1/2^{-}) \to \Xi_{c}\pi) = \frac{h_{6}^{2}}{2\pi f_{\pi}^{2}} \frac{m_{\Xi_{c}}}{m_{\tilde{\Xi}_{c0}}} E_{\pi}^{2} p_{\pi},
$$
  

$$
\Gamma(\tilde{\Lambda}_{c1}(1/2^{-}) \to \Sigma_{c}\pi) = \frac{h_{7}^{2}}{2\pi f_{\pi}^{2}} \frac{m_{\Sigma_{c}}}{m_{\tilde{\Lambda}_{c1}}} E_{\pi}^{2} p_{\pi},
$$

where  $p_{\pi}$  is the c.m. momentum of the pion and  $f_{\pi}$  = 132 MeV. Unfortunately, the decay  $\Sigma_c^* \to \Sigma_c \pi$  is kinematically prohibited since the mass difference between  $\Sigma_c^*$  and  $\Sigma_c$  is only of order 65 MeV. Consequently, the coupling *g*<sup>1</sup> cannot be extracted directly from the strong decays of heavy baryons. Note that since the charge of the final state is not specified in Eq.  $(3.14)$  $(3.14)$ , care must be taken for the neutral pion state. For example, an additional factor of  $\frac{1}{2}$  should be taken for  $\Sigma_c^{*+} \to \Sigma_c^+ \pi^0$  and  $\tilde{E}_{c0}^+ \to \Xi_c^+ \pi^0$ , but not for  $\Sigma_c^+ \to \Lambda_c^+ \pi^0$ .

<span id="page-6-2"></span>In the quark model, various couplings in Eqs.  $(3.9)$  $(3.9)$  $(3.9)$  and [\(3.11](#page-5-1)) are related to each other. The *S*-wave couplings between the *s*-wave and the *p*-wave baryons are related by [[8](#page-13-4)]

$$
\frac{|h_3|}{|h_4|} = \frac{\sqrt{3}}{2}, \qquad \frac{|h_2|}{|h_4|} = \frac{1}{2},
$$
  

$$
\frac{|h_5|}{|h_6|} = \frac{2}{\sqrt{3}}, \qquad \frac{|h_5|}{|h_7|} = 1.
$$
 (3.15)

The *D*-wave couplings satisfy the relations [[8\]](#page-13-4)

<span id="page-6-3"></span>
$$
|h_8| = |h_9| = |h_{10}|, \t \frac{|h_{11}|}{|h_{10}|} = \frac{|h_{15}|}{|h_{14}|} = \sqrt{2},
$$
  

$$
\frac{|h_{12}|}{|h_{13}|} = 2, \t \frac{|h_{14}|}{|h_{13}|} = 1.
$$
 (3.16)

From the dimensional analysis, it is expected that the dimensional *D*-wave couplings  $h_{8, \cdots, 14}$  are of order  $1/\Lambda$ <sub>x</sub> ~ (1.0 – 1.2) × 10<sup>-3</sup> MeV<sup>-1</sup>, where  $\Lambda$ <sub>x</sub> =  $(0.83 \sim 1)$  GeV is the chiral symmetry breaking scale.

As discussed in Sec. II, there exist first positive-parity excited charmed baryon states. Their orbital angular momentum *L* is 2, 1 or 0. The transitions of these excited baryons to the *s*-wave baryons involve *S*-wave, *P*-wave and *F*-wave couplings. However, given the complications with the even-parity excitations, here we will not generalize HHChPT to include them.

In terms of the Wigner 6-j symbol, the decay rate for the baryon decay process  $J \rightarrow J' + \pi$  after spin-averaging over the initial spin and summing over final spins has the expression [\[23,](#page-13-17)[24\]](#page-13-18)

<span id="page-6-4"></span>
$$
\Gamma(J \to J' + \pi) = (2J_{\ell} + 1)(2J' + 1) \left| \begin{bmatrix} L_{\pi} & J'_{\ell} & J_{\ell} \\ S_Q & J & J' \end{bmatrix} \right|^2
$$
  
 
$$
\times p_{\pi}^{2L_{\pi}+1} |M_{L_{\pi}}|^2, \qquad (3.17)
$$

where  $L_{\pi}$  is the orbital angular momentum of the pion,  $S_Q = \frac{1}{2}$  is the heavy quark spin and  $M_{L_{\pi}}$  is the reduced matrix element which is independent of *J* and *J'*. This relation is very useful in relating strong decays into different multiplets for a given partial wave.

#### **A. Strong decays of** *s***-wave charmed baryons**

In the framework of heavy hadron chiral pertrubation theory (HHChPT), one can use some measurements as input to fix the coupling  $g_2$  which, in turn, can be used to predict the rates of other strong decays. We shall use the measured rates of  $\Sigma_c^{++} \to \Lambda_c^+ \pi^+$ ,  $\Sigma_c^{*++} \to \Lambda_c^+ \pi^+$  and  $\Sigma_c^{*0} \rightarrow \Lambda_c^+ \pi^-$  as inputs to obtain

<span id="page-6-1"></span>
$$
|g_2| = 0.605^{+0.039}_{-0.043}, \qquad 0.57 \pm 0.04, \qquad 0.60 \pm 0.04,
$$
\n(3.18)

respectively, where we have neglected the tiny contributions from electromagnetic decays. Note that  $|g_2|$  obtained from  $\Sigma_c^0 \rightarrow \Lambda_c^+ \pi^-$  has the same central value as the first one in Eq.  $(3.18)$  except that the errors are slightly large.<sup>4</sup> Hence, the averaged  $g_2$  is<sup>5</sup>

$$
|g_2| = 0.591 \pm 0.023. \tag{3.19}
$$

<sup>&</sup>lt;sup>4</sup>Historically, based on the nonrelativistic quark model, the prediction  $\Gamma(\Sigma_c^0 \to \Lambda_c^+ \pi^-) = 2.45$  MeV was made long before experiment [[20](#page-13-14)].

For previous efforts of extracting  $g_2$  from experiment using HHChPT, see [[8](#page-13-4),[25](#page-13-19)].

As pointed out in  $[20]$  $[20]$  $[20]$ , within the framework of the nonrelativistic quark model, the couplings  $g_1$  and  $g_2$  can be related to  $g_A^q$ , the axial-vector coupling in a single quark transition of  $u \rightarrow d$ , via

$$
g_1 = \frac{4}{3} g_A^q
$$
,  $g_2 = \sqrt{\frac{2}{3}} g_A^q$ . (3.20)

Using  $g_A^q = 0.75$  which is required to reproduce the correct value of the nucleon axial coupling  $g_A^N = 1.25$ , we obtain

$$
g_1 = 1, \qquad g_2 = 0.61. \tag{3.21}
$$

Hence, the quark model prediction is in good agreement with experiment, while the large- $N_c$  prediction  $|g_2|$  = with experiment, while the large- $N_c$  prediction  $|g_2| = g_A^N/\sqrt{2} = 0.88$  [[26](#page-13-20)] deviates from the data by  $2\sigma$ . Applying  $(3.18)$  $(3.18)$  leads to (see also Table V)

$$
\Gamma(\Xi_c^{*+}) = \Gamma(\Xi_c^{*+} \to \Xi_c^{+} \pi^0, \Xi_c^0 \pi^+)
$$
  
\n
$$
= \frac{g_2^2}{4\pi f_\pi^2} \left( \frac{1}{2} \frac{m_{\Xi_c^+}}{m_{\Xi_c^{*+}}} p_\pi^3 + \frac{m_{\Xi_c^0}}{m_{\Xi_c^{*+}}} p_\pi^3 \right)
$$
  
\n
$$
= (2.7 \pm 0.2) \text{ MeV},
$$
  
\n
$$
\Gamma(\Xi_c^{*0}) = \Gamma(\Xi_c^{*0} \to \Xi_c^{+} \pi^-, \Xi_c^0 \pi^0)
$$
  
\n
$$
= \frac{g_2^2}{4\pi f_\pi^2} \left( \frac{m_{\Xi_c^+}}{m_{\Xi_c^{*0}}} p_\pi^3 + \frac{1}{2} \frac{m_{\Xi_c^0}}{m_{\Xi_c^{*0}}} p_\pi^3 \right)
$$
  
\n
$$
= (2.8 \pm 0.2) \text{ MeV}.
$$
  
\n(3.22)

Note that we have neglected the effect of  $\Xi_c - \Xi_c'$  mixing in calculations (for recent considerations, see [[31](#page-13-21)[,32](#page-13-22)]). Therefore, the predicted total width of  $\Xi_c^{*+}$  is in the vicinity of the current limit  $\Gamma(\Xi_c^{*+})$  < 3.1 MeV [[33\]](#page-13-23).

It is clear from Table V that the strong decay width of  $\Sigma_c$ is smaller than that of  $\Sigma_c^*$  by a factor of  $\sim$ 7, although they will become the same in the limit of heavy quark symmetry. This is ascribed to the fact that the c.m. momentum of the pion is around 90 MeV in the decay  $\Sigma_c \rightarrow \Lambda_c \pi$  while it is 2 times bigger in  $\Sigma_c^* \to \Lambda_c \pi$ . Since  $\Sigma_c$  states are significantly narrower than their spin- $\frac{3}{2}$  counterparts, this explains why the measurement of their widths came out much later. Instead of using the data to fix the coupling constants in a model-independent manner, there exist some calculations of couplings in various models such as the relativistic light-front model [[27](#page-13-24)], the relativistic three-quark model [\[28\]](#page-13-25) and light-cone sum rules [[29](#page-13-26)[,34](#page-13-27)]. The calculated results are summarized in Table V.

It is worth remarking that although the coupling  $g_1$ cannot be determined directly from the strong decay such as  $\Sigma_c^* \to \Sigma_c \pi$ , some information of  $g_1$  can be learned from the radiative decay  $\Xi_c^{*0} \to \Xi_c^0 \gamma$ , which is prohibited at tree level by SU(3) symmetry but can be induced by chiral loops. A measurement of  $\Gamma(\Xi_c^{*0} \to \Xi_c^0 \gamma)$  will yield two possible solutions for  $g_1$ . Assuming the validity of the quark model relations among different coupling constants, the experimental value of  $g_2$  implies  $|g_1| = 0.93 \pm 0.16$ [\[25\]](#page-13-19).

#### **B. Strong decays of** *p***-wave charmed baryons**

Some of the *S*-wave and *D*-wave couplings of *p*-wave baryons to *s*-wave baryons can be determined. In principle, the coupling  $h_2$  is readily extracted from  $\Lambda_c(2593)^+$   $\rightarrow$  $\Sigma_c^0 \pi^+$  with  $\Lambda_c$ (2593) being identified as  $\Lambda_{c1}(\frac{1}{2}^-)$ . However, since  $\Lambda_c(2593)^+ \to \Sigma_c \pi$  is kinematically barely allowed, the finite width effects of the intermediate resonant states could become important [[35](#page-13-28)]. If these effects are neglected, then from Eq.  $(3.14)$  $(3.14)$  $(3.14)$  and the measured decay rates of  $\Lambda_c(2593)^+ \to \Sigma_c^0 \pi^+$  and  $\Lambda_c(2593)^+ \to \Sigma_c^{++} \pi^-,$ we find

$$
|h_2| = 0.41 \pm 0.11. \tag{3.23}
$$

<span id="page-7-0"></span>Before proceeding to a more precise determination of *h*2, we make several remarks on the partial widths of  $\Lambda_c(2593)^+$  decays. (i) PDG [[7](#page-13-0)] has assumed the isospin relation, namely,  $\Gamma(\Lambda_c^+\pi^+\pi^-) = 2\Gamma(\Lambda_c^+\pi^0\pi^0)$  to extract the branching ratios for  $\Sigma_c \pi$  modes. However, the decay  $\Lambda_c(2593) \rightarrow \Lambda_c \pi \pi$  occurs very close to the threshold as  $m_{\Lambda_c(2593)} - m_{\Lambda_c} = 308.9 \pm 0.6$  MeV. Hence, the phase space is very sensitive to the small isospin-violating mass differences between members of pions and charmed Sigma

TABLE V. Decay widths (in units of MeV) of *s*-wave charmed baryons. Theoretical predictions of [[27](#page-13-24)] are taken from Table IV of [\[28\]](#page-13-25).

Decay	Expt. $[7]$	This work <b>HHChPT</b>	Tawfiq <i>et al.</i> $[27]$	Ivanov <i>et al.</i> [28]	Huang <i>et al.</i> [29]	Albertus <i>et al.</i> [30]
$\Sigma_c^{++} \to \Lambda_c^+ \pi^+$	$2.23 \pm 0.30$	input	$1.51 \pm 0.17$	$2.85 \pm 0.19$	2.5	$2.41 \pm 0.07$
$\Sigma_c^+ \rightarrow \Lambda_c^+ \pi^0$	$<$ 4.6	$2.5 \pm 0.2$	$1.56 \pm 0.17$	$3.63 \pm 0.27$	3.2	$2.79 \pm 0.08$
$\Sigma_c^0 \rightarrow \Lambda_c^+ \pi^-$	$2.2 \pm 0.4$	input	$1.44 \pm 0.16$	$2.65 \pm 0.19$	2.4	$2.37 \pm 0.07$
$\Sigma_c (2520)^{++} \to \Lambda_c^+ \pi^+$	$14.9 \pm 1.9$	input	$11.77 \pm 1.27$	$21.99 \pm 0.87$	8.2	$17.52 \pm 0.75$
$\Sigma_c(2520)^+ \rightarrow \Lambda_c^+ \pi^0$	<17	$16.6 \pm 1.3$			8.6	$17.31 \pm 0.74$
$\Sigma_c(2520)^0 \rightarrow \Lambda_c^+ \pi^-$	$16.1 \pm 2.1$	input	$11.37 \pm 1.22$	$21.21 \pm 0.81$	8.2	$16.90 \pm 0.72$
$\Xi_c(2645)^+ \to \Xi_c^{0,+} \pi^{+,0}$	$<$ 3.1	$2.7 \pm 0.2$	$1.76 \pm 0.14$	$3.04 \pm 0.37$		$3.18 \pm 0.10$
$\Xi_c(2645)^0 \to \Xi_c^{+,0} \pi^{-,0}$	< 5.5	$2.8 \pm 0.2$	$1.83 \pm 0.06$	$3.12 \pm 0.33$		$3.03 \pm 0.10$

baryon multiplets. Since the neutral pion is slightly lighter than the charged one, it turns out that both  $\Lambda_c^+ \pi^+ \pi^-$  and  $\Lambda_c^+ \pi^0 \pi^0$  have very similar rates [see Eq. [\(3.35\)](#page-9-0) below]. (ii) Taking  $\mathcal{B}(\Lambda_c(2593)^+ \to \Lambda_c^+ \pi^+ \pi^-) \approx 0.5$  and using the measured ratios [[7](#page-13-0)]

$$
\frac{\Gamma(\Lambda_c(2593)^+ \to \Sigma_c^{++} \pi^-)}{\Gamma(\Lambda_c(2593)^+ \to \Lambda_c^+ \pi^+ \pi^-)} = 0.36 \pm 0.10,
$$
\n
$$
\frac{\Gamma(\Lambda_c(2593)^+ \to \Sigma_c^0 \pi^+)}{\Gamma(\Lambda_c(2593)^+ \to \Lambda_c^+ \pi^+ \pi^-)} = 0.37 \pm 0.10,
$$
\n(3.24)

we obtain

$$
\mathcal{B}(\Lambda_c(2593)^+ \to \Sigma_c^{++} \pi^-) = 0.18 \pm 0.05,
$$
  

$$
\mathcal{B}(\Lambda_c(2593)^+ \to \Sigma_c^0 \pi^+) = 0.19 \pm 0.05,
$$
 (3.25)

and

$$
\Gamma(\Lambda_c(2593)^+ \to \Sigma_c^{++} \pi^-) = 0.65^{+0.41}_{-0.31} \text{ MeV},
$$
  

$$
\Gamma(\Lambda_c(2593)^+ \to \Sigma_c^0 \pi^+) = 0.67^{+0.41}_{-0.31} \text{ MeV}.
$$
 (3.26)

(iii) The nonresonant or direct three-body decay mode  $\Lambda_c^+ \pi^+ \pi^-$  has a branching ratio of  $0.14 \pm 0.08$  [[7\]](#page-13-0). Assuming the same for  $\Lambda_c^+\pi^0\pi^0$ , then the fractions for resonant and nonresonant  $\Lambda_c^+ \pi \pi$  are  $0.73 \pm 0.15$ and  $0.27 \pm 0.15$ , respectively, where  $\Lambda_c^+ \pi \pi =$  $\Lambda_c^+ \pi^+ \pi^- + \Lambda_c^+ \pi^0 \pi^0$ . From the measured total width of  $\Lambda_c(2593)^+$ ,  $3.6^{+2.0}_{-1.3}$  MeV [\[7\]](#page-13-0), we are led to

$$
\Gamma(\Lambda_c(2593)^+ \to \Lambda_c^+ \pi \pi)_R = (2.63^{+1.56}_{-1.09}) \text{ MeV},
$$
  
\n
$$
\Gamma(\Lambda_c(2593)^+ \to \Lambda_c^+ \pi \pi)_{NR} = (0.97^{+0.76}_{-0.64}) \text{ MeV}.
$$
\n(3.27)

Pole contributions to the decays  $\Lambda_c(2593)^+$ ,  $\Lambda_c(2625)^+ \to \Lambda_c^+ \pi \pi$  have been considered in [\[8](#page-13-4)[,12](#page-13-6)[,29\]](#page-13-26) with the finite width effects included. The intermediate states of interest are  $\Sigma_c$  and  $\Sigma_c^*$  poles. The resonant contribution arises from the  $\Sigma_c$  pole, while the nonresonant term receives a contribution from the  $\Sigma_c^*$  pole. (Since  $\Lambda_c(2593)^+$ ,  $\Lambda_c(2625)^+$   $\to \Sigma_c^* \pi$  are not kinematically allowed, the  $\Sigma_c^*$  pole is not a resonant contribution.) The decay rates thus depend on two coupling constants  $h_2$  and

*h*<sub>8</sub>. The decay rate for the process  $\Lambda_{c_1}^+(2593) \to \Lambda_c^+ \pi^+ \pi^$ can be calculated in the framework of heavy hadron chiral perturbation theory [[8\]](#page-13-4)

$$
\frac{d^2\Gamma(\Lambda_{c1}^+(2593) \to \Lambda_c^+\pi^+(E_1)\pi^-(E_2))}{dE_1dE_2}
$$
\n
$$
= \frac{g_2^2}{16\pi^3 f_\pi^4} m_{\Lambda_c^+} {\mathbf{p}_2^2 |A|^2 + \mathbf{p}_1^2 |B|^2 + 2\mathbf{p}_1 \cdot \mathbf{p}_2 \operatorname{Re}(AB^*)},
$$
\n(3.28)

with

$$
A(E_1, E_2) = \frac{h_2 E_1}{\Delta_R - \Delta_{\Sigma_c^0} - E_1 + i\Gamma_{\Sigma_c^0}/2} -\frac{\frac{2}{3} h_8 \mathbf{p}_2^2}{\Delta_R - \Delta_{\Sigma_c^{*0}} - E_1 + i\Gamma_{\Sigma_c^{*0}}/2} +\frac{2h_8 \mathbf{p}_1 \cdot \mathbf{p}_2}{\Delta_R - \Delta_{\Sigma_c^{*++}} - E_2 + i\Gamma_{\Sigma_c^{*++}}/2},
$$
 (3.29)

$$
B(E_1, E_2; \Delta_{\Sigma_c^{(*)0}}, \Delta_{\Sigma_c^{(*)++}}) = A(E_2, E_1; \Delta_{\Sigma_c^{(*)++}}, \Delta_{\Sigma_c^{(*)0}}),
$$
\n(3.30)

where  $\Delta_R = m_{\Lambda_c(2593)} - m_{\Lambda_c}$  and  $\Delta_{\Sigma_c^{(*)}} = m_{\Sigma_c^{(*)}} - m_{\Lambda_c}$ .

For the spin- $\frac{3}{2}$  state  $\Lambda_c(2625)$ , its decay is dominated by the three-body channel  $\Lambda_c^+ \pi \pi$  as the major two-body decay  $\Sigma_c \pi$  is a *D*-wave one. As for the decay  $\Lambda_{c1}^+(2625) \rightarrow$  $\Lambda_c^+ \pi^+ \pi^-$ , its rate is given by [\[8](#page-13-4)]

$$
\frac{d^2\Gamma(\Lambda_{c1}^+(2625) \to \Lambda_c^+ \pi^+(E_1)\pi^-(E_2))}{dE_1 dE_2}
$$
\n
$$
= \frac{g_2^2}{16\pi^3 f_\pi^4} M_{\Lambda_c^+} {\mathbf{p}_1^2 |C|^2 + \mathbf{p}_2^2 |E|^2}
$$
\n
$$
+ 2\mathbf{p}_1 \cdot \mathbf{p}_2 \operatorname{Re}(CE^*) + [\mathbf{p}_1^2 \mathbf{p}_2^2 - (\mathbf{p}_1 \cdot \mathbf{p}_2)^2]
$$
\n
$$
\times [\mathbf{p}_1^2 |D|^2 + \mathbf{p}_2^2 |F|^2 - \operatorname{Re}(CF^*) + \operatorname{Re}(DE^*)
$$
\n
$$
+ 2\mathbf{p}_1 \cdot \mathbf{p}_2 \operatorname{Re}(DF^*)], \tag{3.31}
$$

with

$$
C(E_1, E_2) = \left(h_2 E_2 - \frac{2}{3} h_8 \mathbf{p}_2^2\right) \frac{1}{\Delta_{R^*} - \Delta_{\Sigma_c^{*++}} - E_2 + i\Gamma_{\Sigma_c^{*++}}/2} + \frac{2}{3} h_8 \mathbf{p}_1 \cdot \mathbf{p}_2 \left(\frac{1}{\Delta_{R^*} - \Delta_{\Sigma_c^0} - E_1 + i\Gamma_{\Sigma_c^0}/2} + \frac{2}{\Delta_{R^*} - \Delta_{\Sigma_c^{*0}} - E_1 + i\Gamma_{\Sigma_c^{*0}}/2}\right),
$$
  
\n
$$
D(E_1, E_2) = \frac{2}{3} h_8 \left(-\frac{1}{\Delta_{R^*} - \Delta_{\Sigma_c^0} - E_1 + i\Gamma_{\Sigma_c^0}/2} + \frac{1}{\Delta_{R^*} - \Delta_{\Sigma_c^{*0}} - E_1 + i\Gamma_{\Sigma_c^{*0}}/2}\right),
$$
  
\n
$$
E(E_1, E_2; \Delta_{\Sigma_c^{(*)0}}, \Delta_{\Sigma_c^{(*)++}}) = C(E_2, E_1; \Delta_{\Sigma_c^{(*)++}}, \Delta_{\Sigma_c^{(*)0}}),
$$
  
\n
$$
F(E_1, E_2; \Delta_{\Sigma_c^{(*)0}}, \Delta_{\Sigma_c^{(*)++}}) = -D(E_2, E_1; \Delta_{\Sigma_c^{(*)++}}, \Delta_{\Sigma_c^{(*)0}}),
$$
  
\n(3.32)

<span id="page-8-0"></span>where  $\Delta_{R^*} = m_{\Lambda_c(2625)} - m_{\Lambda_c}$ .

The total widths of the  $\Lambda_c(2593)$  and  $\Lambda_c(2625)$  states obtained after integrating out the variables  $E_1$  and  $E_2$  and including the  $\pi^{0}\pi^{0}$  channel are

TABLE VI. Same as Table V except for *p*-wave charmed baryons.

Decay	Expt. $[7]$	This work <b>HHChPT</b>	Tawfiq <i>et al.</i> [27]	Ivanov <i>et al.</i> [28]	Huang <i>et al.</i> [29]	Zhu $\left[34\right]$
$\Lambda_c(2593)^+ \to (\Lambda_c^+\pi\pi)_R$	$2.63_{-1.09}^{+1.56}$	input			2.5	
$\Lambda_c(2593)^+\to\Sigma_c^{++}\pi^-$	$0.65^{+0.41}_{-0.31}$	$0.72^{+0.43}_{-0.30}$	$1.47 \pm 0.57$	$0.79 \pm 0.09$	$0.55^{+1.3}_{-0.55}$	0.64
$\Lambda_c(2593)^+\to\Sigma_c^0\pi^+$	$0.67^{+0.41}_{-0.31}$	$0.77^{+0.46}_{-0.32}$	$1.78 \pm 0.70$	$0.83 \pm 0.09$	$0.89 \pm 0.86$	0.86
$\Lambda_c(2593)^+ \to \Sigma_c^+ \pi^0$		$1.57^{+0.93}_{-0.65}$	$1.18 \pm 0.46$	$0.98 \pm 0.12$	$1.7 \pm 0.49$	1.2
$\Lambda_c(2625)^+ \to \Sigma_c^{++} \pi^-$	< 0.10	$\leq 0.029$	$0.44 \pm 0.23$	$0.076 \pm 0.009$	0.013	0.011
$\Lambda_c(2625)^+\to\Sigma_c^0\pi^+$	< 0.09	$\leq 0.029$	$0.47 \pm 0.25$	$0.080 \pm 0.009$	0.013	0.011
$\Lambda_c(2625)^+ \rightarrow \Sigma_c^+ \pi^0$		$\leq 0.041$	$0.42 \pm 0.22$	$0.095 \pm 0.012$	0.013	0.011
$\Lambda_c(2625)^+ \to \Lambda_c^+ \pi \pi$	< 1.9	$\leq 0.21$			0.11	
$\Sigma_c(2800)^{++} \rightarrow \Lambda_c \pi$ , $\Sigma_c^{(*)} \pi$	$75^{+22}_{-17}$	input				
$\Sigma_c(2800)^+ \rightarrow \Lambda_c \pi$ , $\Sigma_c^{(*)} \pi$	$62^{+60}_{-40}$	input				
$\Sigma_c(2800)^0 \rightarrow \Lambda_c \pi$ , $\Sigma_c^{(*)} \pi$	$61^{+28}_{-18}$	input				
$\Xi_c(2790)^+ \rightarrow \Xi_c^{\prime 0,+} \pi^{+,0}$	<15	$8.0^{+4.7}_{-3.3}$				
$\Xi_c(2790)^0 \to \Xi_c^{l+,0} \pi^{-,0}$	<12	$8.5^{+5.0}_{-3.5}$				
$\Xi_c(2815)^+ \to \Xi_c^{*+,0} \pi^{0,+}$	< 3.5	$3.4^{+2.0}_{-1.4}$	$2.35 \pm 0.93$	$0.70 \pm 0.04$		
$\Xi_c(2815)^0 \to \Xi_c^{*+,0} \pi^{-,0}$	<6.5	$3.6^{+2.1}_{-1.5}$				

$$
\Gamma(\Lambda_c(2593)^+ \to \Lambda_c^+ \pi \pi) = 13.82h_2^2 + 26.28h_8^2 - 2.97h_2h_8,
$$
  
\n
$$
\Gamma(\Lambda_c(2625)^+ \to \Lambda_c^+ \pi \pi) = 0.617h_2^2 + 0.136 \times 10^6h_8^2 - 27h_2h_8,
$$
\n(3.33)

where use of  $(3.18)$  for  $g_2$  has been made. It is clear that the experimental limit on  $\Gamma(\Lambda_c(2625))$  gives an upper bound on  $h_8$  of order 10<sup>-3</sup> (in units of MeV<sup>-1</sup>), whereas the decay width of  $\Lambda_c(2593)$  is entirely governed by the coupling  $h_2$ . This indicates that the direct nonresonant  $\Lambda_c^+ \pi \pi$ contribution cannot be described by the  $\Sigma_c^*$  pole alone. Some other mechanisms are needed to account for the nonresonant contributions. Identifying the calculated  $\Gamma(\Lambda_c(2593)^+ \to \Lambda_c^+ \pi \pi)$  with the resonant one, we find

<span id="page-9-1"></span>
$$
|h_2| = 0.437^{+0.114}_{-0.102}, \qquad |h_8| < 3.65 \times 10^{-3}. \tag{3.34}
$$

Comparing  $(3.34)$  with  $(3.23)$  $(3.23)$  we see that the magnitude of  $h<sub>2</sub>$  is enhanced slightly by finite width effects.

Assuming that the total width of  $\Lambda_c(2593)^+$  is saturated by the resonant  $\Lambda_c^+ \pi \pi$  3-body decays, Pirjol and Yan obtained  $|h_2| = 0.572^{+0.322}_{-0.197}$  and  $|h_8| \le (3.50 - 3.68) \times$  $10^{-3}$  MeV<sup>-1</sup> [[8\]](#page-13-4). Using the updated hadron masses and  $\Gamma(\Lambda_c(2593))$ , we find  $|h_2| = 0.499_{-0.100}^{+0.134}$ . Taking into account the fact that the  $\Sigma_c$  and  $\Sigma_c^*$  poles only describe the resonant contributions to the total width of  $\Lambda_c(2593)$ , we finally reach at the value of  $h_2$  given by [\(3.34\)](#page-9-1). Using this result for  $h_2$ , the two-body  $\Lambda_c(2593) \to \Sigma_c \pi$  rates are shown in Table VI. The three-body partial rates are found to be

<span id="page-9-0"></span>
$$
\Gamma(\Lambda_c(2593)^+ \to \Lambda_c^+ \pi^+ \pi^-) = 1.29 \text{ MeV}, \n\Gamma(\Lambda_c(2593)^+ \to \Lambda_c^+ \pi^0 \pi^0) = 1.34 \text{ MeV}.
$$
\n(3.35)

Therefore, isospin violation is manifested in the relations  $\Gamma(\Sigma_c^{\dagger} \pi^0) \approx 2\Gamma(\Sigma_c^{++} \pi^-) \sim 2\Gamma(\Sigma_c^0 \pi^+$  and  $\Gamma(\Lambda_c^+\pi^0\pi^0) \approx \Gamma(\Lambda_c^+\pi^+\pi^-)$  in  $\Lambda_c$ (2593) decays.

The  $\Xi_c(2790)$  and  $\Xi_c(2815)$  baryons form a doublet  $E_{c1}(\frac{1}{2}^{-}, \frac{3}{2}^{-})$ .  $E_{c}(2790)$  decays to  $E'_{c}\pi$ , while  $E_{c}(2815)$ decays to  $\Xi_c \pi \pi$ , resonating through  $\Xi_c^*$ , i.e.  $\Xi_c$  (2645). Using the coupling  $h_2$  obtained from ([3.34\)](#page-9-1) and the experimental observation that the  $\Xi_c \pi \pi$  mode in  $\Xi_c$  (2815) decays is consistent with being entirely via  $\Xi_c^* \pi$  [[37](#page-13-30)], the predicted  $E_c(2790)$  and  $E_c(2815)$  widths are shown in Table VI, where uses have been made of

$$
\Gamma(\Xi_{c1}(1/2)^{+}) \approx \Gamma(\Xi_{c1}(1/2)^{+} \rightarrow \Xi_{c}^{\prime+} \pi^{0}, \Xi_{c}^{\prime 0} \pi^{+})
$$
\n
$$
= \frac{h_{2}^{2}}{4\pi f_{\pi}^{2}} \left(\frac{1}{2} \frac{m_{\Xi_{c}^{\prime+}}}{m_{\Xi_{c1}(1/2)}} E_{\pi}^{2} p_{\pi} + \frac{m_{\Xi_{c}^{\prime 0}}}{m_{\Xi_{c1}(1/2)}} E_{\pi}^{2} p_{\pi}\right),
$$
\n
$$
\Gamma(\Xi_{c1}(3/2)^{+}) \approx \Gamma(\Xi_{c1}(3/2)^{+} \rightarrow \Xi_{c}^{*+} \pi^{0}, \Xi_{c}^{*0} \pi^{+})
$$
\n
$$
= \frac{h_{2}^{2}}{4\pi f_{\pi}^{2}} \left(\frac{1}{2} \frac{m_{\Xi_{c}^{*+}}}{m_{\Xi_{c1}(3/2)}} E_{\pi}^{2} p_{\pi} + \frac{m_{\Xi_{c}^{\prime 0}}}{m_{\Xi_{c1}(3/2)}} E_{\pi}^{2} p_{\pi}\right),
$$
\n(3.36)

and similar expressions for the neutral  $\Xi_{c1}(\frac{1}{2}^{-}, \frac{3}{2}^{-})$  states.

<sup>&</sup>lt;sup>6</sup>The CLEO result  $\Gamma(\Lambda_c(2593)) = 3.9^{+2.4}_{-1.6}$  MeV [[36](#page-13-31)] is used in [\[8\]](#page-13-4) to fix  $h_2$ .

The predictions are consistent with the current experimental limits.

Some information on the coupling  $h_{10}$  can be inferred from the strong decays of  $\Sigma_c(2800)$ . As noticed in passing, the states  $\Sigma_c(2800)^{+,+,+,0}$  which are observed in the  $\Lambda_c^+\pi$ spectrum are most likely to be  $\Sigma_{c2}(\frac{3}{2})$ . From Table III we see that there are three low-lying *p*-wave  $\Sigma_c$  multiplets:  $\Sigma_{c0}$ ,  $\Sigma_{c1}$  and  $\Sigma_{c2}$ . Both  $\Sigma_{c0}$  and  $\Sigma_{c2}$  decay to  $\Lambda_c \pi$  in an *S*-wave and a *D*-wave, respectively, while  $\Sigma_{c1}$  decays mainly to the two-pion system  $\Lambda_c \pi \pi$  in a *P*-wave. From Eqs. ([3.14](#page-6-0)), [\(3.15\)](#page-6-2), and ([3.34](#page-9-1)) we find  $\Gamma(\Sigma_{c0} \to \Lambda_c \pi) \approx$ 406 MeV. Hence, it is too broad to be observable. Therefore,  $\Sigma_c(2800)^{++,+0}$  are likely to be  $\Sigma_{c2}(\frac{3}{2})^{-}$ . Assuming their widths are dominated by the two-body *D*-wave modes  $\Lambda_c \pi$ ,  $\Sigma_c \pi$  and  $\Sigma_c^* \pi$ , we have [\[8](#page-13-4)]

$$
\Gamma(\Sigma_{c2}(3/2)^{++}) \approx \Gamma(\Sigma_{c2}(3/2)^{++} \to \Lambda_c^+ \pi^+)
$$
  
+ 
$$
\Gamma(\Sigma_{c2}(3/2)^{++} \to \Sigma_c^+ \pi^+)
$$
  
+ 
$$
\Gamma(\Sigma_{c2}(3/2)^{++} \to \Sigma_c^{*+} \pi^+)
$$
  
+ 
$$
\Gamma(\Sigma_{c2}(3/2)^{++} \to \Sigma_c^{++} \pi^0)
$$
  
+ 
$$
\Gamma(\Sigma_{c2}(3/2)^{++} \to \Sigma_c^{*++} \pi^0), \quad (3.37)
$$

and similar expressions for  $\Sigma_{c2}(\frac{3}{2})^+$  and  $\Sigma_{c2}(\frac{3}{2})^0$ . Using<sup>7</sup>

$$
\Gamma(\Sigma_{c2}(3/2^-) \to \Lambda_c \pi) = \frac{4h_{10}^2}{15\pi f_\pi^2} \frac{m_{\Lambda_c}}{m_{\Sigma_{c2}}} p_\pi^5,
$$
\n
$$
\Gamma(\Sigma_{c2}(3/2^-) \to \Sigma_c^{(*)} \pi) = \frac{h_{11}^2}{10\pi f_\pi^2} \frac{m_{\Sigma_c^{(*)}}}{m_{\Sigma_{c2}}} p_\pi^5,
$$
\n(3.38)

and the quark model relation  $n_{11}^2 = 2h_{10}^2$ [cf. Equation  $(3.16)$  $(3.16)$  $(3.16)$ ] and the measured widths of  $\Sigma_c(2800)^{+,+,+,0}$  (Table II), we obtain

$$
|h_{10}| = (0.86^{+0.08}_{-0.10}) \times 10^{-3} \text{ MeV}^{-1}.
$$
 (3.39)

This is consistent with the naive expectation that  $h_{10} \sim$  $1/\Lambda_{\chi}$ . Since the state  $\Sigma_{c1}(\frac{3}{2})$  is broader, even a small mixing of  $\Sigma_{c2}(\frac{3}{2})$  with  $\Sigma_{c1}(\frac{3}{2})$  could enhance the decay width of the former [[8\]](#page-13-4). Moreover, the nonresonant threebody mode  $\Lambda_c^+ \pi \pi$  may have contributions to the width of  $\Sigma_{c2}$ , the above value for  $h_{10}$  should be regarded as an upper limit of  $|h_{10}|$ . Using the quark model relation  $|h_8| = |h_{10}|$ [Eq.  $(3.16)$ ], we then have

$$
|h_8| \lesssim (0.86^{+0.08}_{-0.10}) \times 10^{-3} \text{ MeV}^{-1}, \tag{3.40}
$$

which improves the previous limit  $(3.34)$  $(3.34)$  $(3.34)$  by a factor of 4.

Using the above value of  $h_8$ , the rates of  $\Lambda_c(2625)$ decays to  $\Lambda_c \pi \pi$  and  $\Sigma_c \pi$  are presented in Table VI, where we have used Eq.  $(3.33)$  $(3.33)$  and

$$
\Gamma(\Lambda_{c1}(3/2^-) \to \Sigma_c \pi) = \frac{2h_8^2}{9\pi f_\pi^2} \frac{m_{\Sigma_c}}{m_{\Lambda_{c1}(3/2)}} p_\pi^5. \tag{3.41}
$$

## **C. Strong decays of first positive-parity excited charmed baryons**

Besides the *p*-wave charmed baryons discussed in the previous subsection, some of the higher orbitally excited charmed baryons listed in Table I are likely to be the first positive-parity excitations. For example, the recent Belle studies favor the  $J^P$  quantum numbers of  $\Lambda_c(2880)^+$  to be  $\frac{5}{2}$  + [[4](#page-12-2)]. The quantum numbers for the first positive-parity excited charmed baryons are listed in Table IV. Those states have  $L_K + L_k = 2$  and hence the orbital angular momentum  $L_{\ell}$  can be 2, 1 or 0. Besides  $\Lambda_c(2880)^+$ , the states  $\Lambda_c(2765)^+$ ,  $\Lambda_c(2940)^+$ ,  $\Xi_c(2980)^{+,0}$  and  $\Xi_c(3077)^{+,0}$  are also likely to be the first positive-parity excitations of charmed baryons as we are going to discuss.

As noticed in Sec. II, Belle has studied the experimental constraint on the  $J^P$  quantum numbers of  $\Lambda_c(2880)^+$  and found that the assignment of  $J = \frac{5}{2}$  is favored over  $J = \frac{1}{2}$  or  $\frac{3}{2}$  by the angular analysis of  $\Lambda_c(2880)^+ \to \Sigma_c^{0,++} \pi^{\pm}$  [[4\]](#page-12-2). The measurement of the ratio of  $\Lambda_c(2880)$  partial widths [\[4\]](#page-12-2)

<span id="page-10-0"></span>
$$
R = \frac{\Gamma(\Lambda_c(2880) \to \Sigma_c^* \pi^{\pm})}{\Gamma(\Lambda_c(2880) \to \Sigma_c \pi^{\pm})} = (24.1 \pm 6.4^{+1.1}_{-4.5})\% \quad (3.42)
$$

can be used to determine the parity assignment. From Tables III and IV we see that the candidates for the spin- $\frac{5}{2}$  state are  $\tilde{\Lambda}_{c2}(\frac{5}{2}^{-})$ ,  $\Lambda_{c2}(\frac{5}{2}^{+})$ ,  $\tilde{\Lambda}'_{c2}(\frac{5}{2}^{+})$ ,  $\tilde{\Lambda}''_{c2}(\frac{5}{2}^{+})$ and  $\bar{\Lambda}_{c3}''(\frac{5}{2}^+)$ . For  $J^P = \frac{5}{2}^-$ ,  $\Lambda_c(2880)$  decays to  $\bar{\Sigma}_c^* \pi$ and  $\Sigma_c \pi$  in a *D* wave. From Eq. [\(3.17](#page-6-4)) we obtain

$$
\frac{\Gamma(\tilde{\Lambda}_{c2}(5/2^{-}) \to [\Sigma_c^* \pi]_D)}{\Gamma(\tilde{\Lambda}_{c2}(5/2^{-}) \to [\Sigma_c \pi]_D)} = \frac{7}{2} \frac{p_\pi^5(\Lambda_c(2880) \to \Sigma_c^* \pi)}{p_\pi^5(\Lambda_c(2880) \to \Sigma_c \pi)} = \frac{7}{2} \times 0.42 = 1.45. \tag{3.43}
$$

Hence, the assignment of  $J^P = \frac{5}{2}$  for  $\Lambda_c(2880)$  is disfavored. For  $J^P = \frac{5}{2}^+$ ,  $\Lambda_{c2}$ ,  $\tilde{\Lambda}'_{c2}$  and  $\tilde{\Lambda}''_{c2}$  with  $J_\ell = 2$  decay to  $\Sigma_c \pi$  in a *F* wave and  $\Sigma_c^* \pi$  in *F* and *P* waves. Neglecting the *P*-wave contribution for the moment,

$$
\frac{\Gamma(\Lambda_{c2}(5/2^+) \to [\Sigma_c^* \pi]_F)}{\Gamma(\Lambda_{c2}(5/2^+) \to [\Sigma_c \pi]_F)} = \frac{4}{5} \frac{p_\pi^7(\Lambda_c(2880) \to \Sigma_c^* \pi)}{p_\pi^7(\Lambda_c(2880) \to \Sigma_c \pi)} \n= \frac{4}{5} \times 0.29 = 0.23.
$$
\n(3.44)

At first glance, it appears that this is in good agreement with experiment. However, the  $\Sigma_c^* \pi$  channel is available via a *P*-wave and is enhanced by a factor of  $1/p_{\pi}^4$  (or more precisely,  $(\Lambda_{\chi}/p_{\pi})^4$ ) relative to the *F*-wave one. Unfortunately, we cannot apply Eq.  $(3.17)$  $(3.17)$  to calculate the contribution of the  $[\Sigma_c^* \pi]_F$  channel to the ratio *R* as the reduced matrix elements are different for *P*-wave and

<sup>&</sup>lt;sup>7</sup>It is useful to apply Eq.  $(3.17)$  $(3.17)$  to check the consistency of the partial decay rate formulas.

*F*-wave modes. In any event, the  $\Sigma_c^* \pi$  mode produced in  $\Lambda_c(2880)$  is *a priori* not necessarily suppressed relative to  $[\Sigma_c \pi]_F$ . Therefore, if  $\Lambda_c(2880)^+$  is one of the states  $\Lambda_{c2}$ ,  $\tilde{\Lambda}'_{c2}$  and  $\tilde{\Lambda}''_{c2}$ , the prediction  $R = 0.23$  is not robust as it can be easily upset by the contribution from the *P*-wave  $\Sigma_c^* \pi$ .

As for  $\tilde{\Lambda}_{c3}^{\prime\prime}(\frac{5}{2}^+)$ , it decays to  $\Sigma_c^*\pi$ ,  $\Sigma_c\pi$  and  $\Lambda_c\pi$  all in *F* waves. Since  $J_{\ell} = 3$ ,  $L_{\ell} = 2$ , it turns out that

$$
\frac{\Gamma(\Lambda_{c3}''(5/2^+) \to [\Sigma_c^* \pi]_F)}{\Gamma(\Lambda_{c3}''(5/2^+) \to [\Sigma_c \pi]_F)} = \frac{5}{4} \frac{p_\pi^7(\Lambda_c(2880) \to \Sigma_c^* \pi)}{p_\pi^7(\Lambda_c(2880) \to \Sigma_c \pi)} \n= \frac{5}{4} \times 0.29 = 0.36. \quad (3.45)
$$

Although this deviates from the experimental measurement  $(3.42)$  $(3.42)$  by  $1\sigma$ , it is a robust prediction. However, there are

two issues with this assignment. First,  $\Lambda_{c3}''(\frac{5}{2}^+)$  can decay to a *F*-wave  $\Lambda_c \pi$  and this has not been seen by *BABAR* and Belle. Second, the quark model indicates a  $\Lambda_{c2}(\frac{5}{2}^+)$  state around 2910 MeV which is close to the mass of  $\Lambda_c(2880)$ , while the mass of  $\Lambda_{c3}''(\frac{5}{2}^+)$  is higher [[9](#page-13-3)]. Therefore, we conjecture that the first positive-parity excited charmed baryon  $\Lambda_c(2880)^+$  could be an admixture of  $\Lambda_{c2}(\frac{5}{2}^+)$ and  $\Lambda''_{c3}(\frac{5}{2}^+)$ .

The quark potential model predicts a  $\frac{5}{2}$   $\Lambda_c$  state at 2900 MeV and a  $\frac{3}{2}$  +  $\Lambda_c$  state at 2910 MeV [[9\]](#page-13-3). Given the uncertainty of order 50 MeV for the quark model calculation, this suggests that the possible allowed  $J<sup>P</sup>$  numbers of the highest  $\Lambda_c(2940)^+$  are  $\frac{5}{2}$  and  $\frac{3}{2}$  +. Hence, the potential candidates are  $\tilde{\Lambda}_{c2}(\frac{5}{2}^{-})$ ,  $\Lambda_{c2}(\frac{3}{2}^{+})$ ,  $\tilde{\Lambda}'_{c1}(\frac{3}{2}^{+})$ ,

TABLE VII. Possible strong decays of the first positive-parity excitations of the  $\Xi_c$ , where *L* denotes the orbital angular momentum of the light meson(s). The final state  $\Sigma_c^* K$  is kinematically allowed for  $\Xi_c(3077)$ .

$J^P$	Diquark transition	L	Decay channel	Final states
$rac{1}{2}$ +	$0^+ \rightarrow 1^+ + 0^-$	$\mathbf{1}$	$\frac{1}{2}$ + $\rightarrow$ $\{\frac{1}{2}$ + $\frac{3}{2}$ + $\}$ + 0 <sup>-</sup>	$\Xi_c^{\prime}\pi$ , $\Xi_c^*\pi$ , $\Sigma_c^{(*)}K$
	$0^+ \rightarrow 0^+ + 0^- + 0^-$	$\boldsymbol{0}$	$\frac{1}{2}$ + $\rightarrow$ $\frac{1}{2}$ + $+$ 0 <sup>-</sup> + 0 <sup>-</sup>	$\Xi_c \pi \pi$ , $\Lambda_c K \pi$
	$1^+ \rightarrow 0^+ + 0^-$	$\mathbf{1}$	$\frac{1}{2}$ + $\rightarrow$ $\frac{1}{2}$ + $\rightarrow$ 0 <sup>-</sup>	$\Xi_c \pi$ , $\Lambda_c K$ , $D \Lambda$
	$1^+ \rightarrow 1^+ + 0^-$	$\mathbf{1}$	$\frac{1}{2}$ + $\rightarrow$ $\{\frac{1}{2}$ + $\frac{3}{2}$ + $\}$ + 0 <sup>-</sup>	$\Xi_c^{\prime}\pi$ , $\Xi_c^*\pi$ , $\Sigma_c^{(*)}K$
	$1^+ \rightarrow 1^- + 0^-$	$\boldsymbol{0}$	$\frac{1}{2}$ – $\rightarrow$ $\frac{1}{2}$ + + 0–	$\Xi_c(2790)\pi$ , $\Lambda_c(2593)K$
	$1^+ \rightarrow 1^- + 0^-$	$\overline{c}$	$\frac{1}{2}$ – $\rightarrow$ $\frac{3}{2}$ + + 0 <sup>-</sup>	$\Xi_c(2815)\pi$ , $\Lambda_c(2625)K$
	$1^+ \rightarrow 0^+ + 0^- + 0^-$	$\overline{c}$	$\frac{1}{2}$ + $\rightarrow$ $\frac{1}{2}$ + $+$ 0 <sup>-</sup> + 0 <sup>-</sup>	$\Xi_c \pi \pi$ , $\Lambda_c K \pi$
	$1^+ \rightarrow 1^+ + 0^- + 0^-$	$\overline{0}$	$\frac{1}{2}$ + $\rightarrow$ $\frac{1}{2}$ + $+$ 0 <sup>-</sup> + 0 <sup>-</sup>	$\Xi_c^{\prime} \pi \pi$
	$1^+ \rightarrow 1^+ + 0^- + 0^-$	2	$\frac{1}{2}$ + $\rightarrow$ $\frac{3}{2}$ + + 0 <sup>-</sup> + 0 <sup>-</sup>	$\Xi_c^*\pi\pi$
$rac{3}{2}$ +	$1^+ \rightarrow 0^+ + 0^-$	$\mathbf{1}$	$\frac{3}{2}$ + $\rightarrow$ $\frac{1}{2}$ + $\rightarrow$ 0 <sup>-</sup>	$\Xi_c \pi$ , $\Lambda_c K$ , $D \Lambda$
	$1^+ \rightarrow 1^+ + 0^-$	$\mathbf{1}$	$\frac{3}{2}$ + $\rightarrow$ $\{\frac{1}{2}$ + $\frac{3}{2}$ + $\}$ + 0 <sup>-</sup>	$\Xi_c^{\prime}\pi$ , $\Xi_c^*\pi$ , $\Sigma_c^{(*)}$ K
	$1^+ \rightarrow 1^- + 0^-$	$\boldsymbol{0}$	$\frac{3}{2}$ – $\rightarrow$ $\frac{3}{2}$ + + 0 <sup>-</sup>	$\Xi_c(2815)\pi$ , $\Lambda_c(2625)K$
	$1^+ \rightarrow 1^- + 0^-$	$\overline{c}$	$\frac{3}{2}$ – $\rightarrow \{\frac{1}{2}$ – $\frac{3}{2}$ + } + 0 –	$\Xi_c(2790, 2815)\pi$ , $\Lambda_c(2593, 2625)K$
	$1^+ \rightarrow 0^+ + 0^- + 0^-$	$\overline{c}$	$\frac{3}{2}$ + $\rightarrow$ $\frac{1}{2}$ + + 0 <sup>-</sup> + 0 <sup>-</sup>	$\Xi_c \pi \pi$ , $\Lambda_c K \pi$
	$1^+ \rightarrow 1^+ + 0^- + 0^-$	$\boldsymbol{0}$	$\frac{3}{2}$ + $\rightarrow$ $\frac{3}{2}$ + + 0 <sup>-</sup> + 0 <sup>-</sup>	$\Xi_c^*\pi\pi$
	$1^+ \rightarrow 1^+ + 0^- + 0^-$	2	$\frac{3}{2}$ + $\rightarrow$ $\{\frac{1}{2}$ + $\frac{3}{2}$ + $\}$ + 0 <sup>-</sup> + 0 <sup>-</sup>	$\Xi_c^{\prime} \pi \pi$ , $\Xi_c^* \pi \pi$
	$2^+ \rightarrow 1^+ + 0^-$	$\mathbf{1}$	$\frac{3}{2}$ + $\rightarrow$ $\{\frac{1}{2}$ + $\frac{3}{2}$ + $\}$ + 0 <sup>-</sup>	$\Xi_c^{\prime}\pi$ , $\Xi_c^*\pi$ , $\Sigma_c^{(*)}$ K
	$2^+ \rightarrow 1^- + 0^-$	$\overline{2}$	$\frac{3}{2}$ – $\rightarrow$ $\{\frac{1}{2}$ – $\frac{3}{2}$ + $\}$ + 0 –	$\Xi_c(2790, 2815)\pi$ , $\Lambda_c(2593, 2625)K$
	$2^+ \rightarrow 0^+ + 0^- + 0^-$	$\overline{2}$	$\frac{3}{2}$ + $\rightarrow$ $\frac{1}{2}$ + $+$ 0 <sup>-</sup> + 0 <sup>-</sup>	$\Xi_c \pi \pi$ , $\Lambda_c K \pi$
	$2^+ \rightarrow 1^+ + 0^- + 0^-$	$\overline{2}$	$\frac{3}{2}$ + $\rightarrow$ $\{\frac{1}{2}$ + $\frac{3}{2}$ + $\}$ + 0 <sup>-</sup> + 0 <sup>-</sup>	$\Xi_c^{\prime} \pi \pi$ , $\Xi_c^* \pi \pi$
$rac{5}{2}$ +	$2^+ \rightarrow 1^+ + 0^-$	1	$\frac{5}{2}$ + $\rightarrow$ $\frac{3}{2}$ + $+$ 0 <sup>-</sup>	$\Xi_c^* \pi$ , $\Sigma_c^* K$
	$2^+ \rightarrow 1^+ + 0^-$	3	$\frac{5}{2}$ + $\rightarrow$ $\{\frac{1}{2}$ + $\frac{3}{2}$ + $\}$ + 0 <sup>-</sup>	$\Xi_c^{\prime}\pi$ , $\Xi_c^*\pi$ , $\Sigma_c^{(*)}K$
	$2^+ \rightarrow 1^- + 0^-$	$\overline{2}$	$\frac{5}{2}$ $^ \longrightarrow$ $\{\frac{1}{2}$ $^-$ , $\frac{3}{2}$ $^+\}$ + $0^-$	$\Xi_c(2790, 2815)\pi$ , $\Lambda_c(2593, 2625)K$
	$2^+ \rightarrow 0^+ + 0^- + 0^-$	$\overline{c}$	$\frac{5}{2}$ + $\rightarrow$ $\frac{1}{2}$ + $+$ 0 <sup>-</sup> + 0 <sup>-</sup>	$\Xi_c \pi \pi$ , $\Lambda_c K \pi$
	$2^+ \rightarrow 1^+ + 0^- + 0^-$	$\overline{c}$	$\frac{5}{2}$ + $\rightarrow$ $\{\frac{1}{2}$ + $\frac{3}{2}$ + $\}$ + 0 <sup>-</sup> + 0 <sup>-</sup>	$\Xi_c^{\prime} \pi \pi$ , $\Xi_c^* \pi \pi$
	$3^+ \rightarrow 0^+ + 0^-$	3	$\frac{5}{2}$ + $\rightarrow \frac{1}{2}$ + + 0 <sup>-</sup>	$\Xi_c \pi$ , $\Lambda_c K$ , $D \Lambda$
	$3^+ \rightarrow 1^+ + 0^-$	$\overline{\mathbf{3}}$	$\frac{5}{2}$ + $\rightarrow$ $\{\frac{1}{2}$ + $\frac{3}{2}$ + $\}$ + 0 <sup>-</sup>	$\Xi_c^{\prime}\pi$ , $\Xi_c^*\pi$ , $\Sigma_c^{(*)}K$
	$3^+ \rightarrow 1^- + 0^-$	$\overline{2}$	$\frac{5}{2}$ – $\rightarrow$ $\{\frac{1}{2}$ – $\frac{3}{2}$ + $\}$ + 0 –	$\Xi_c(2790, 2815)\pi$ , $\Lambda_c(2593, 2625)K$
	$3^+ \rightarrow 1^+ + 0^- + 0^-$	$\overline{c}$	$\frac{5}{2}$ + $\rightarrow$ $\{\frac{1}{2}$ + $\text{,}$ $\frac{3}{2}$ + } + 0^ + 0^	$\Xi_c^{\prime} \pi \pi$ , $\Xi_c^* \pi \pi$

 $\tilde{\Lambda}''_{c1}(\frac{3}{2}^+)$  and  $\tilde{\Lambda}''_{c2}(\frac{3}{2}^+)$ . Ratios of  $\Lambda_c(2940)$  partial widths are expected in HHChPT to be

$$
\frac{\Gamma(\tilde{\Lambda}_{c2}(5/2^{-}) \to [\Sigma_c^* \pi]_D)}{\Gamma(\tilde{\Lambda}_{c2}(5/2^{-}) \to [\Sigma_c \pi]_D)} = \frac{7}{2} \frac{p_{\pi}^5(\Lambda_c(2940) \to \Sigma_c^* \pi)}{p_{\pi}^5(\Lambda_c(2940) \to \Sigma_c \pi)}
$$
  
\n
$$
= \frac{7}{2} \times 0.48 = 1.68,
$$
  
\n
$$
\frac{\Gamma(\Lambda_{c2}(3/2^{+}) \to [\Sigma_c^* \pi]_P)}{\Gamma(\Lambda_{c2}(3/2^{+}) \to [\Sigma_c \pi]_P)} = \frac{1}{5} \frac{p_{\pi}^3(\Lambda_c(2940) \to \Sigma_c^* \pi)}{p_{\pi}^3(\Lambda_c(2940) \to \Sigma_c \pi)}
$$
  
\n
$$
= \frac{1}{5} \times 0.65 = 0.13,
$$
  
\n
$$
\frac{\Gamma(\tilde{\Lambda}_{c1}'(3/2^{+}) \to [\Sigma_c^* \pi]_P)}{\Gamma(\tilde{\Lambda}_{c1}'(3/2^{+}) \to [\Sigma_c \pi]_P)} = 5 \frac{p_{\pi}^3(\Lambda_c(2940) \to \Sigma_c^* \pi)}{p_{\pi}^3(\Lambda_c(2940) \to \Sigma_c \pi)}
$$
  
\n
$$
= 5 \times 0.65 = 3.25.
$$
 (3.46)

Since the predicted ratios differ significantly for different  $J<sup>P</sup>$  quantum numbers, the measurements of the ratio of  $\sum_{c}^{*} \overline{\pi}/\sum_{c} \pi$  will enable us to discriminate the  $J^{P}$  assignments for  $\Lambda_c(2940)$ .

For the charmed states  $E_c(2980)$  and  $E_c(3077)$ , they could be the first positive-parity excitations of  $\Xi_c$  in viewing of their large masses. Since the mass difference between the antitriplets  $\Lambda_c$  and  $\Xi_c$  for  $J^P = \frac{1}{2}^+, \frac{1}{2}^-, \frac{3}{2}^-$  is of order 180  $\sim$  200 MeV, it is conceivable that  $\Xi_c(2980)$ and  $E_c(3077)$  are the counterparts of  $\Lambda_c(2765)$  and  $\Lambda_c(2880)$ , respectively, in the strange charmed baryon sector. As noted in passing, the state  $\Lambda_c(2765)^+$  could be an even-parity excitation as the quark model [[9\]](#page-13-3) and the Skyrme model [\[13](#page-13-7)] suggest a  $J^P = \frac{1}{2} + \text{state with a mass}$ 2742 and 2775 MeV, respectively. It is thus tempting to assign  $J^P = \frac{1}{2}$  + for  $\Xi_c(2980)$  and  $\frac{5}{2}$  + for  $\Xi_c(3077)$ . Of course, the assignment of  $J^P = \frac{3}{2} + \frac{1}{2}$  is also possible. The possible strong decays of the first positive-parity excitations of the  $\Xi_c$  states are summarized in Table VII. Since the two-body modes  $\Xi_c \pi$ ,  $\Lambda_c K$ ,  $\Xi_c' \pi$  and  $\Sigma_c K$  are in *P* (*F*) waves and the three-body modes  $\Xi_c \pi \pi$  and  $\Lambda_c K \pi$  are in *S* (*D*) waves in the decays of  $\frac{1}{2}$ <sup>+</sup> ( $\frac{5}{2}$ <sup>+</sup>), this explains why  $E_c(2980)$  is broader than  $E_c(3077)$ . Since both  $E_c(2980)$ and  $E_c(3077)$  are above the *D* $\Lambda$  threshold, it is important to search for them in the  $D\Lambda$  spectrum as well.

#### **IV. CONCLUSIONS**

Strong decays of charmed baryons are analyzed in the framework of heavy hadron chiral perturbation theory in which heavy quark symmetry and chiral symmetry are synthesized. Our main conclusions are the following:

- (i) For *s*-wave charmed baryons, we use the channel  $\Sigma_c^{++} \to \Lambda_c^+ \pi^+$  to fix the coupling constant *g*<sub>2</sub>. The value of  $|g_2| = 0.591 \pm 0.023$  are in good agreement with the quark model expectation. The predictions for the strong decays  $\Sigma_c^* \to \Lambda \pi$  and  $\Xi_c^* \rightarrow \Xi_c \pi$  are in excellent agreement with experiment.
- (ii) For  $L = 1$  orbitally excited baryons, two of the unknown couplings, namely,  $h_2$  and  $h_{10}$ , are determined from the resonant  $\Sigma_c^+ \pi \pi$  mode produced in the  $\Lambda_c(2593)$  decay and the width of  $\Sigma_c(2800)$ , respectively. The results are  $|h_2| = 0.437^{+0.114}_{-0.102}$  and  $|h_{10}| \le (0.86^{+0.08}_{-0.10}) \times 10^{-3}$  MeV<sup>-1</sup>. Since the twopion system  $\Lambda_c^+ \pi \pi$  in  $\Lambda_c (2593)^+$  decays receives nonresonant contributions, our value for  $h_2$  is smaller than the previous estimates. Applying the quark model relation  $h_8^2 = h_{10}^2$ , predictions for the strong decays of other *p*-wave charmed baryons such as  $\Lambda_c(2625) \to \Sigma_c \pi$ ,  $\Lambda_c \pi \pi$ ,  $\Xi_c(2790) \to$  $\Xi_c' \pi$  and  $\Xi_c(2815) \rightarrow \Xi_c^* \pi$  are presented in Table VI. Since the decays  $\Lambda_c(2593) \rightarrow \Lambda_c \pi \pi$ and  $\Lambda_c(2593) \to \Sigma_c \pi$  occur very close to the threshold, they are very sensitive to the pion's mass and hence isospin symmetry is violated, for example,  $\Gamma(\Sigma_c^+\pi^0) \approx 2\Gamma(\Sigma_c^{++}\pi^-)$  and  $\Gamma(\Lambda_c^+\pi^0\pi^0) \approx \Gamma(\Lambda_c^+\pi^+\pi^-)$  in  $\Lambda_c$ (2593) decays.
- (iii) We have examined the first positive-parity excited charmed baryons. We conjecture that the state  $\Lambda_c(2880)$  with  $J^P = \frac{5}{2}$  is an admixture of  $\Lambda_{c2}(\frac{5}{2}^+)$  with an  $\tilde{\Lambda}''_{c3}(\frac{5}{2}^+)$ ; both are  $L = 2$  orbitally excited states. Potential models suggest the possible allowed  $J^P$  numbers of the  $\Lambda_c(2940)^+$  to be 5  $\frac{2}{2}$  and  $\frac{2}{2}$ . We have demonstrated that the measurements of the ratio of  $\sum_{c}^{*} \pi / \sum_{c} \pi$  will enable us  $\frac{3}{2}$  + . We have demonstrated that the meato discriminate the  $J^P$  assignments for  $\Lambda_c(2940)$ . We advocate that the  $J<sup>P</sup>$  quantum numbers of  $\Xi_c(2980)$  and  $\Xi_c(3077)$  are  $\frac{1}{2}$  + and  $\frac{5}{2}$  +, respectively. Under this  $J<sup>P</sup>$  assignment, it is easy to understand why  $\Xi_c(2980)$  is broader than  $\Xi_c(3077)$ .

## **ACKNOWLEDGMENTS**

We are grateful to Roman Mizuk for valuable discussions. This research was supported in part by the National Science Council of R.O.C. under Grant Nos. NSC95-2112- M-001-013 and NSC95-2112-M-033-013.

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