

Can the $X(3872)$ be a 1^{++} four-quark state?

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We use QCD spectral sum rules to test the nature of the meson $X(3872)$, assumed to be an exotic four-quark ($c\bar{c}q\bar{q}$) state with $J^{PC} = 1^{++}$. For definiteness, we work with the diquark-antidiquark current proposed recently, at leading order in α_s , consider the contributions of higher dimension condensates and keep terms which are linear in the light quark mass m_q . We find $M_X = (3925 \pm 127)$ MeV which is compatible, within the errors, with the experimental candidate $X(3872)$, while the SU(3) breaking-terms lead to an unusual mass-splitting $M_{X_s} - M_X = -(61 \pm 30)$ MeV. The mass-difference between the neutral states due to isospin violation is about $(2.6 \sim 3.9)$ MeV. For the b -quark, we predict $M_{X_b} = (10144 \pm 106)$ MeV for the $X_b(b\bar{b}q\bar{q})$, which is much below the $\bar{B}B^*$ threshold, and for the $X_b^s(b\bar{b}s\bar{s})$, a mass-splitting $M_{X_b^s} - M_{X_b} = -(121 \pm 182)$ MeV. Our analysis also indicates that the mass-splitting between the ground state and the radial excitation of about $(225 \sim 250)$ MeV is much smaller than in the case of ordinary mesons and is (within the errors) flavor-independent. We also extract the decay constants, analogous to f_π , of such mesons, which are useful for further studies of their leptonic and hadronic decay widths. The uncertainties of our estimates are mainly due to the ones from the c and b quark masses.

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I. INTRODUCTION

In August 2003, Belle reported evidence for a new narrow state in the decay $B^+ \rightarrow X(3872)K^+ \rightarrow J/\psi\pi^+\pi^-K^+$ [1], which has been confirmed by three other experiments [2]. The $X(3872)$ is the best studied of the new $c\bar{c}$ -associated states, $X(3872)$, $X(3940)$, $Y(4260)$, etc. [3]. It has a mass of 3872 MeV and a very narrow width $\Gamma < 2.3$ MeV at 95%. Upon discovery, $X(3872)$ seemed a likely candidate for $\psi_2(^3D_2)$ or $\psi_3(^3D_3)$ [4], but the expected radiative transitions to χ_c states have never been seen. The $\pi\pi$ mass spectrum favors high dipion masses, suggesting a $J/\psi\rho$ decay that is incompatible with the identification of $X(3872) \rightarrow \pi^+\pi^-J/\psi$ as the strong decay of a pure isoscalar state. Belle's observation of the decay $X(3872) \rightarrow J/\psi\gamma$ [5] determines $C = +$, opposite to the charge-conjugation of the leading charmonium candidates. The same paper [5] also reports the observation of the X decaying to $J/\psi\pi^+\pi^-\pi^0$, with a rate which is comparable to that of the $J/\psi\pi^+\pi^-$ mode. This decay suggests an appreciable transition rate to $J/\psi\omega$ and establishes sizeable isospin violating effects. Finally, an analysis of angular distributions supports the assignment $J^{PC} = 1^{++}$, but the mass of $X(3872)$ is too low to be gracefully identified with the 2^3P_1 charmonium state. More recently, the Belle collaboration reported a peak in $D^0\bar{D}^0\pi^0$ which can be interpreted as the dominant decay mode of the X [6].

The anomalous nature of the X has led to many speculations: tetraquark [7,8], cusp [9], hybrid [10], or glueball

[11]. Another explanation is that the $X(3872)$ is a $D\bar{D}^*$ bound state [12–16], as predicted before its discovery.

In this work we use QCD spectral sum rules (QSSR) (the Borel/Laplace sum rules (LSR) [17–19] and finite energy sum rules (FESR) [19–21]), to study the two-point functions of the axial-vector meson, $X(3872)$, assumed to be a four-quark state. In previous calculations, the sum rule (SR) approach was used to study the light scalar mesons [22–25] and the $D_{sJ}^+(2317)$ meson [26,27], considered as four-quark states and a good agreement with the experimental masses was obtained. However, the tests were not decisive as the usual quark-antiquark assignments also provide predictions consistent with data and more importantly with chiral symmetry expectations [19,23,28,29]. In the four-quark scenario, scalar mesons can be considered as S -wave bound states of diquark-antidiquark pairs, where the diquark was taken to be a spin zero color antitriplet. Here we follow Ref. [7], and consider the $X(3872)$ as the $J^{PC} = 1^{++}$ state with the symmetric spin distribution: $[cq]_{S=1}[\bar{c}\bar{q}]_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}]_{S=1}$. Therefore, the corresponding lowest-dimension interpolating operator for describing X_q is given by

$$j_\mu = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}} [(q_a^T C \gamma_5 c_b)(\bar{q}_d \gamma_\mu C \bar{c}_e^T) + (q_a^T C \gamma_\mu c_b) \times (\bar{q}_d \gamma_5 C \bar{c}_e^T)], \quad (1)$$

where a, b, c, \dots are color indices, C is the charge-conjugation matrix and q denotes a u or d quark.

In general, one should consider all possible combinations of different 1^{++} four-quark operators, similar to e.g. done in [30] for the 0^{++} light mesons and consider their mixing under renormalizations [31] from which one can

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form renormalization group invariant (RGI) physical currents. However, we might expect that, working with a particular choice of current given above will provide a general feature of the four-quark model predictions for the $X(3872)$, provided that we can work with quantities less affected by radiative corrections and where the OPE converges quite well¹ As pointed out in [7], isospin forbidden decays are possible if X is not a pure isospin state. Pure isospin states are

$$X(I=0) = \frac{X_u + X_d}{\sqrt{2}}, \quad \text{and} \quad X(I=1) = \frac{X_u - X_d}{\sqrt{2}}. \quad (2)$$

If the physical states are just the mass eigenstates X_u or X_d , maximal isospin violations are possible. Deviations from these two ideal situations are described by a mixing angle between X_u and X_d [7]

$$X_l = X_u \cos\theta + X_d \sin\theta, \quad X_h = -X_u \sin\theta + X_d \cos\theta. \quad (3)$$

In Ref. [7], by considering the X decays into two and three pions, a mixing angle $\theta \sim 20^\circ$ is deduced and a mass difference

$$m(X_h) - m(X_l) = (8 \pm 3) \text{ MeV}. \quad (4)$$

In this work, we want to test in which conditions the results of the sum rules are compatible with the above predictions.

II. THE QCD EXPRESSION OF THE TWO-POINT CORRELATOR

The SR are constructed from the two-point correlation function

$$\begin{aligned} \Pi_{\mu\nu}(q) &= i \int d^4x e^{iq \cdot x} \langle 0 | T [j_\mu(x) j_\nu^\dagger(0)] | 0 \rangle \\ &= -\Pi_1(q^2) \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \Pi_0(q^2) \frac{q_\mu q_\nu}{q^2}. \end{aligned} \quad (5)$$

Since the axial-vector current is not conserved, the two functions, Π_1 and Π_0 , appearing in Eq. (5) are independent and have, respectively, the quantum numbers of the spin 1 and 0 mesons.

The fundamental assumption of the sum rules approach is the principle of duality. Specifically, we assume that there is an interval over which the correlation function

¹In the well-known case of baryon sum rules, a simplest choice of operator [32] and a more general choice [33] have been given in the literature. Though technically apparently different, mainly for the region of convergence of the OPE, the two choices of interpolating currents have provided the same predictions for the proton mass and mixed condensate but only differs for values of higher dimension four-quark condensates.

may be equivalently described at both the quark and the hadron levels. Therefore, on the one hand, we calculate the correlation function at the quark level in terms of quark and gluon fields. On the other hand, the correlation function is calculated at the hadronic level introducing hadron characteristics such as masses and coupling constants. At the quark level, the complex structure of the QCD vacuum leads us to employ the Wilson's operator product expansion (OPE). The calculation of the phenomenological side proceeds by inserting intermediate states for the meson X . Parametrizing the coupling of the axial-vector meson 1^{++} , X , to the current, j_μ , in Eq. (1) in terms of the meson decay constant f_X as

$$\langle 0 | j_\mu | X \rangle = \sqrt{2} f_X M_X^4 \epsilon_\mu, \quad (6)$$

the phenomenological side of Eq. (5) can be written as

$$\Pi_{\mu\nu}^{\text{phen}}(q^2) = \frac{2f_X^2 M_X^8}{M_X^2 - q^2} \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_X^2} \right) + \dots, \quad (7)$$

where the Lorentz structure projects out the 1^{++} state. The dots denote higher axial-vector resonance contributions that will be parametrized, as usual, through the introduction of a continuum threshold parameter s_0 .

In the OPE side, we work at leading order in α_s and consider the contributions of condensates up to dimension eight. We keep the term which is linear in the light-quark mass m_q , in order to estimate the mass difference in Eq. (4). Keeping the charm-quark mass finite, we use the momentum-space expression for the charm-quark propagator. The light-quark part of the correlation function is calculated in the coordinate-space, and then Fourier transformed to the momentum space in D dimensions. The resulting light-quark part is combined with the charm-quark part before it is dimensionally regularized at $D = 4$.

The correlation function, Π_1 , in the OPE side can be written as a dispersion relation

$$\Pi_1^{\text{OPE}}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho(s)}{s - q^2}, \quad (8)$$

where the spectral density is given by the imaginary part of the correlation function: $\pi\rho(s) = \text{Im}[\Pi_1^{\text{OPE}}(s)]$. After making an inverse-Laplace (or Borel) transform of both sides, and transferring the continuum contribution to the OPE side, the sum rule for the axial-vector meson X up to dimension-eight condensates can be written as²

²We have not included the effects of a dimension 2 term induced by the UV renormalon, [34,35], which we expect to be numerically negligible like in the other channels [36], though this result needs to be checked. Instanton-like contributions which appear as a high-dimension operators will also be neglected like some other higher dimension condensate effects.

$$2f_X^2 M_X^8 e^{-M_X^2/M^2} = \int_{4m_c^2}^{s_0} ds e^{-s/M^2} \rho(s) + \Pi_1^{\text{mix}\langle\bar{q}q\rangle}(M^2), \quad (9)$$

where

$$\rho(s) = \rho^{\text{pert}}(s) + \rho^{m_q}(s) + \rho^{\langle\bar{q}q\rangle}(s) + \rho^{\langle G^2\rangle}(s) + \rho^{\text{mix}}(s) + \rho^{\langle\bar{q}q\rangle^2}(s), \quad (10)$$

with

$$\begin{aligned} \rho^{\text{pert}}(s) &= \frac{1}{2^{10}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta)(1+\alpha+\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]^4, \\ \rho^{m_q}(s) &= -\frac{m_q}{2^3\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha} \left\{ -\frac{\langle\bar{q}q\rangle}{2^2} \frac{[m_c^2 - \alpha(1-\alpha)s]^2}{(1-\alpha)} + \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} [(\alpha+\beta)m_c^2 - \alpha\beta s] \right. \\ &\quad \left. \times \left[-m_c^2\langle\bar{q}q\rangle + \frac{\langle\bar{q}q\rangle}{2^2} [(\alpha+\beta)m_c^2 - \alpha\beta s] + \frac{m_c}{2^5\pi^2\alpha\beta^2} (3+\alpha+\beta)(1-\alpha-\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]^2 \right] \right\}, \\ \rho^{\langle\bar{q}q\rangle}(s) &= -\frac{m_c\langle\bar{q}q\rangle}{2^5\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} (1+\alpha+\beta)[(\alpha+\beta)m_c^2 - \alpha\beta s]^2, \\ \rho^{\langle G^2\rangle}(s) &= \frac{\langle g^2 G^2\rangle}{2^9 3\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} [(\alpha+\beta)m_c^2 - \alpha\beta s] \left[\frac{m_c^2(1-(\alpha+\beta)^2)}{\beta} - \frac{(1-2\alpha-2\beta)}{2\alpha} [(\alpha+\beta)m_c^2 - \alpha\beta s] \right]. \end{aligned} \quad (11)$$

where the integration limits are given by $\alpha_{\min} = (1 - \sqrt{1 - 4m_c^2/s})/2$, $\alpha_{\max} = (1 + \sqrt{1 - 4m_c^2/s})/2$ and $(\beta_{\min} = \alpha m_c^2)/(s\alpha - m_c^2)$. We have also included the dominant contributions from the dimension-five condensates

$$\rho^{\text{mix}}(s) = \frac{m_c\langle\bar{q}g\sigma \cdot Gq\rangle}{2^6\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left[-\frac{2}{\alpha} (m_c^2 - \alpha(1-\alpha)s) + \int_{\beta_{\min}}^{1-\alpha} d\beta [(\alpha+\beta)m_c^2 - \alpha\beta s] \left(\frac{1}{\alpha} + \frac{\alpha+\beta}{\beta^2} \right) \right], \quad (12)$$

where the contribution of dimension-six condensates $\langle g^3 G^3\rangle$ is neglected, since assumed to be suppressed by the loop factor $1/16\pi^2$. The usual estimate $\langle g^3 G^3\rangle \simeq 1 \text{ GeV}^2 \langle \alpha_s G^2\rangle$ [19] would deserve to be checked in more detail. We have included the contribution of the dimension-six four-quark condensate

$$\rho^{\langle\bar{q}q\rangle^2}(s) = \frac{m_c^2\langle\bar{q}q\rangle^2}{12\pi^2} \sqrt{\frac{s-4m_c^2}{s}}, \quad (13)$$

and (for completeness) a part of the dimension-8 condensate contributions³

³We should note that a complete evaluation of these contributions require more involved analysis including a nontrivial choice of the factorization assumption basis [37]. We wish that we can perform this analysis in the future.

$$\begin{aligned} \Pi_1^{\text{mix}\langle\bar{q}q\rangle}(M^2) &= -\frac{m_c^2\langle\bar{q}g\sigma \cdot Gq\rangle\langle\bar{q}q\rangle}{24\pi^2} \\ &\quad \times \int_0^1 d\alpha \left[1 + \frac{m_c^2}{\alpha(1-\alpha)M^2} - \frac{1}{2(1-\alpha)} \right] \\ &\quad \times \exp\left[-\frac{m_c^2}{\alpha(1-\alpha)M^2} \right]. \end{aligned} \quad (14)$$

III. LSR PREDICTIONS OF M_X

In order to extract the mass M_X without worrying about the value of the decay constant f_X , we take the derivative of Eq. (9) with respect to $1/M^2$, divide the result by Eq. (9) and obtain

$$M_X^2 = \frac{\int_{4m_c^2}^{s_0} ds e^{-s/M^2} s \rho(s)}{\int_{4m_c^2}^{s_0} ds e^{-s/M^2} \rho(s)}. \quad (15)$$

This quantity has the advantage to be less sensitive to the perturbative radiative corrections than the individual

moments. Therefore, we expect that our results obtained to leading order in α_s will be quite accurate.

In the numerical analysis of the sum rules, the values used for the quark masses and condensates are (see e.g. [19,38–40])⁴

$$\begin{aligned}
 m_c(m_c) &= (1.23 \pm 0.05) \text{ GeV}, & m_b(m_b) &= (4.24 \pm 0.06) \text{ GeV}, \\
 m_u &= 2.3 \text{ MeV}, & m_d &= 6.4 \text{ MeV}, \\
 m_q &\equiv (m_u + m_d)/2 = 4.3 \text{ MeV}, & \langle \bar{q}q \rangle &= -(0.23 \pm 0.03)^3 \text{ GeV}^3, \\
 m_s &= 100 \text{ MeV}, & \langle \bar{s}s \rangle / \langle \bar{q}q \rangle &= 0.8 \pm 0.2, \\
 \langle \bar{q}g\sigma \cdot Gq \rangle &= m_0^2 \langle \bar{q}q \rangle & \text{with} & m_0^2 = 0.8 \text{ GeV}^2, \\
 \langle g^2 G^2 \rangle &= 0.88 \text{ GeV}^4.
 \end{aligned} \tag{16}$$

We evaluate the sum rules in the range $2.0 \leq M^2 \leq 2.8$ for two values of s_0 : $s_0^{1/2} = 4.1 \text{ GeV}$, $s_0^{1/2} = 4.2 \text{ GeV}$.

Comparing the relative contribution of each term in Eqs. (11)–(14), to the right hand side of Eq. (9) we obtain a quite good OPE convergence for $M^2 > 1.9 \text{ GeV}^2$, as can be seen in Fig. 1. This analysis allows us to determine the lower limit constraint for M^2 in the sum rules window. This figure also shows that, although there is a change of sign between dimension-six and dimension-eight condensates contributions, the contribution of the latter being smaller, where, we have assumed, in Fig. 1–4, the validity of the vacuum saturation for these condensates. The relatively small contribution of the dimension-eight condensates may justify the validity of our approximation, unlike in the case of the 5-quark current correlator, as noticed in [41]. However, the partial compensation of these two terms indicate the sensitivity of the central value of the mass prediction on the way the OPE is truncated.

In Fig. 2 are shown the contributions of the individual condensates to M_X obtained from Eq. (15). From Fig. 2, it appears that the results oscillates around the perturbative result, and that the results obtained up to dimension-5 are very close to the ones obtained up to dimension-8. For definiteness, the value of M_X obtained by including the

dimension-5 mixed condensate will be considered as the final prediction from the LSR, and the effects of the higher condensates as the error due to the truncation of the OPE.

We get an upper limit constraint for M^2 by imposing the rigorous constraint that the QCD continuum contribution should be smaller than the pole contribution.⁵ The maximum value of M^2 for which this constraint is satisfied depends on the value of s_0 . The comparison between pole and continuum contributions for $s_0^{1/2} = 4.2 \text{ GeV}$ is shown in Fig. 3. The same analysis for the other value of the continuum threshold gives $M^2 < 2.2 \text{ GeV}^2$ for $s_0^{1/2} = 4.1 \text{ GeV}$.

In Fig. 4, we show the X meson mass obtained from Eq. (15), in the relevant sum rules window, with the upper and lower validity limits indicated. From Fig. 4 we see that the results are reasonably stable as a function of M^2 . In our numerical analysis, we shall then consider the range of M^2 values from 2.0 GeV^2 until the one allowed by the sum rule window criteria as can be deduced from Fig. 4 for each value the s_0 -range of values.

Using the QCD parameters in Eq. (16), we obtain the LSR predictions for different values of s_0 and including the dimension-5 condensates

$$\begin{aligned}
 M_X &= (3908 \pm 26 \pm 13 \pm 100 \pm 46) \text{ MeV} & \text{for } s_0^{1/2} = 4.1 \text{ GeV}, \\
 &(3943 \pm 30 \pm 10 \pm 80 \pm 48) \text{ MeV} & \text{for } s_0^{1/2} = 4.2 \text{ GeV}.
 \end{aligned} \tag{17}$$

The errors are due, respectively, to M^2 , $\langle \bar{q}q \rangle$, m_c and the truncation of the OPE. We have estimated the absolute value of the last error by varying the dimension-six and eight condensates from their vacuum saturation values to the ones where a violation of the factorization assumption

by a factor two is assumed. The errors due to other parameters are negligible. One can notice that the main error comes from the uncertainties in the determination of the charm-quark mass, which plays a crucial role in the analy-

⁴To leading order approximation in α_s , at which we are working, we do not consistently consider the running scale dependence of these parameters. We shall use here the values of the quark masses obtained within the same QCD spectral sum rules methods compiled in [19]. They are defined in the \overline{MS} -scheme, and have obtained within the same truncation of the QCD series from different channels and by different authors.

⁵More restrictive conditions are sometimes imposed in the literature, where, for example, it is required that the continuum contribution is smaller than 30% of the total contribution. In this case no sum rule-window is allowed. In our analysis, we use a less restrictive criterion, having in mind that the role of the continuum is expected to be larger for high-dimensional current operators than in the usual ρ -meson channel, as indicated by different sum rules analyses in the existing literature.

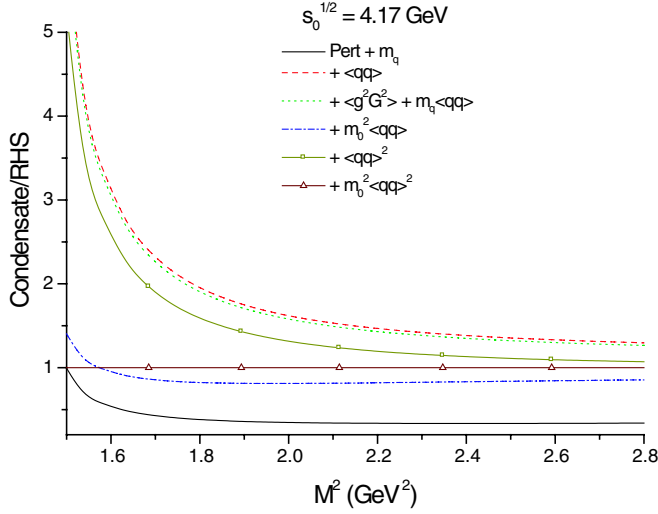


FIG. 1 (color online). The OPE convergence in the region $1.6 \leq M^2 \leq 2.8 \text{ GeV}^2$ for $s_0^{1/2} = 4.17 \text{ GeV}$. We start with the perturbative contribution (plus a very small m_q contribution) and each subsequent line represents the addition of one extra condensate dimension in the expansion.

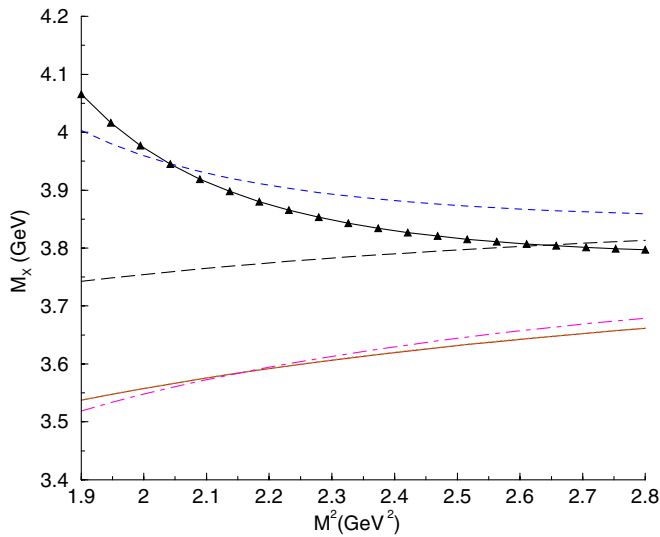


FIG. 2 (color online). The OPE convergence for M_X in the region $1.9 \leq M^2 \leq 2.8 \text{ GeV}^2$ for $s_0^{1/2} = 4.1 \text{ GeV}$. We start with the perturbative contribution plus a very small m_q contribution (long-dashed line) and each subsequent line represents the addition of one extra condensate dimension in the expansion: $+\langle\bar{q}q\rangle$ (solid line), $+\langle g^2 G^2\rangle + m_q \langle\bar{q}q\rangle$ (dotted-line in top of the solid line), $+m_0^2 \langle\bar{q}q\rangle$ (dashed line), $+\langle\bar{q}q\rangle^2$ (dot-dashed line), $+m_0^2 \langle\bar{q}q\rangle^2$ (solid line with triangles).

sis like in the one of other heavy quark systems. One can also notice that the central value of the mass prediction increases with s_0 . Apart the intuitive observation from an extrapolation of the known mass splittings from ordinary mesons which may not be applied for the multi-quark states (see e.g. [42]), s_0 remains a free parameter. We shall try to fix its value using FESR.

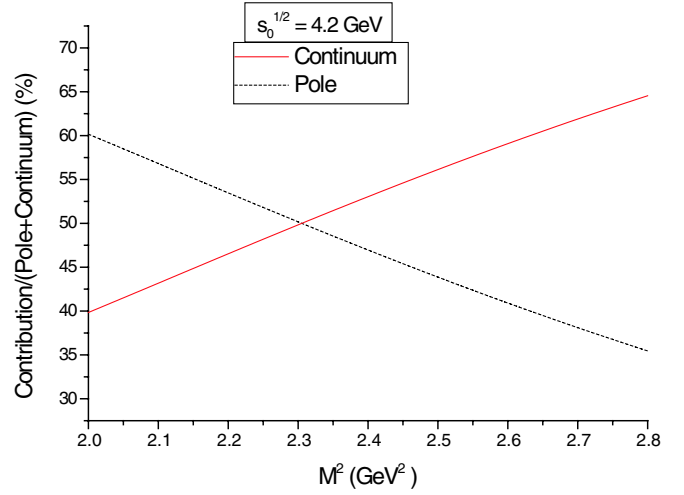


FIG. 3 (color online). The dashed line shows the relative pole contribution (the pole contribution divided by the total, pole plus continuum, contribution) and the solid line shows the relative continuum contribution.

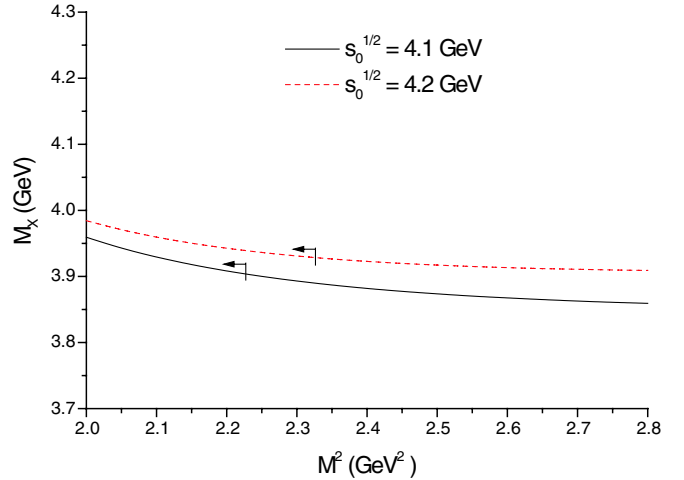


FIG. 4 (color online). The X meson mass as a function of the sum rule parameter (M^2) for different values of the continuum threshold: $s_0^{1/2} = 4.1 \text{ GeV}$ (solid line) and $s_0^{1/2} = 4.2 \text{ GeV}$ (dashed line). The arrows indicate the region allowed for the sum rules: the lower limit (cut below 2.0 GeV^2) is given by OPE convergence requirement and the upper limit by the dominance of the QCD pole contribution.

IV. FESR PREDICTION FOR M_X

As an alternative, we use the FESR, which can be obtained from Eq. (9) by taking the limit $1/M^2 \rightarrow 0$ and equating the same power in $1/M^2$ in the two sides of the sum rules. We get up to dimension-six condensates

$$2f_X^2 M_X^8 \sum_n (-M_X^2)^n \left(\frac{1}{M^2}\right)^n = \sum_n \int_{4m_c^2}^{s_0} ds (-s)^n \left(\frac{1}{M^2}\right)^n \rho(s), \quad n = 0, 1, 2, \dots \quad (18)$$

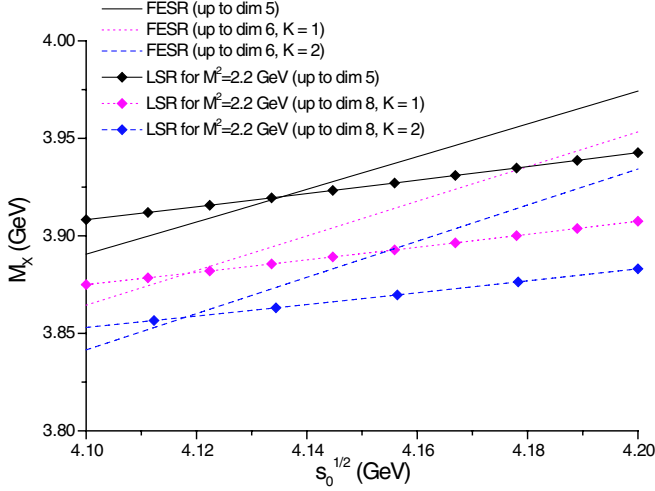


FIG. 5 (color online). The FESR results in Eq. (20) for M_X as a function of s_0 for $n = 0$ including condensates up to different dimensions. The LSR results have been also inserted for easy comparison.

Equating the coefficients of the polynomial in $1/M^2$ in both sides of Eq. (18) gives n equations:

$$2f_X^2 M_X^8 M_X^{2n} = \int_{4m_c^2}^{s_0} ds s^n \rho(s), \quad n = 0, 1, 2, \dots \quad (19)$$

Finally, dividing two subsequent equations (with n and $n + 1$), we can obtain the mass M_X for any chosen value of n (which, formally, is expected to be the same for any n)

$$M_X^2 = \frac{\int_{4m_c^2}^{s_0} ds s^{n+1} \rho(s)}{\int_{4m_c^2}^{s_0} ds s^n \rho(s)}, \quad n = 0, 1, 2, \dots \quad (20)$$

In contrast to the previous method, the FESR have the advantage of giving correlations between the mass and the continuum threshold s_0 , which can be used to avoid inconsistencies in the determination of these parameters. Ideally, one looks at a minimum in the function $M_X(s_0)$, which would provide a good criteria for fixing both s_0 and M_X . The results for different values of n are very similar, therefore, in Fig. 5, we only show the result for $n = 0$. One can see in Fig. 5 that there is no stability in s_0 , which can indicate the important role of the QCD continuum in the analysis.

One can also notice, from Fig. 5, that the FESR converges faster than the LSR due mainly to the fact that here we use an expansion in $1/s_0$ where $\sqrt{s_0} \sim 4.1$ GeV, while in the LSR, the expansion is done in $1/M^2$, where $M \sim 1.4$ GeV is much smaller.

V. FINAL QSSR PREDICTIONS FOR M_X AND f_X

In order to exploit the complementary role of the LSR and FESR, we also show in Fig. 5, the result obtained from the Laplace sum rule using $M^2 = 2.2$ GeV². The factor K was introduced to account for deviations of the factoriza-

tion hypothesis for the $D = 6$ [20,33,40,43–45] and 8 [20,37,40,45]—condensates; $K = 1(2)$ refers to an assumption that respects (violates by a factor 2) vacuum saturation.

The intersection point fixes the range of values of s_0 to be

$$s_0^{1/2} = (4.15 \pm 0.03) \text{ GeV}, \quad (21)$$

which is smaller than intuitively expected. This small value of the continuum threshold relative to the value of the resonance mass signals again the important role of the QCD continuum in the analysis, which is expected for correlators described by high-dimension current operators. Using this range of values of s_0 , one can, definitely, fix the X mass to be

$$M_X = (3925 \pm 20 \pm 46 \pm 117) \text{ MeV}, \quad (22)$$

in remarkable agreement (within the errors) with the experimental candidate $X(3872)$ and with an estimate from relativistic quark model [46].⁶ The first error comes from s_0 , the second from the truncation of the OPE, and the third from the QCD inputs, such as m_c and $\langle \bar{q}q \rangle$. Despite this large dependence of our results on the value of the continuum threshold, the error induced by s_0 is comparable with the ones from other sources. The errors due to s_0 could be reduced by using a more involved parametrization based on some effective Lagrangian or eventually, alternatives forms of the sum rules, such as those used in [47] for describing the hadronic τ decay or some other sum rules [48].

Assuming that the mass of the first radial excitation is given by $\sqrt{s_0}$, one can deduce a crude estimate of the splitting

$$X' - X \approx 225 \text{ MeV}, \quad (23)$$

which is expected to be valid if the local quark-hadron duality is at work.⁷ Within this assumption, one can notice that the mass-splitting is much smaller than the naive extrapolation from the ordinary meson spectrum. Such a situation has been also encountered in the analysis of the pentaquark sum rule [42] and, in general, in the analysis of correlators described by high-dimension operators such as hybrids and gluonia [19]. This result can indicate the existence of higher states near the lowest ground state mass, which can manifest as large continuum in the data analysis.

One can also deduce to leading order in α_s , from the individual lowest moments, the decay constant defined in

⁶However, due to the large error of our result, it can also be compatible with the $X(3940)$, $Y(3940)$ and $Z(3930)$ if some of them are found to be a 1^{++} state. We plan to study carefully the splitting of the different states of a given spin in a future work.

⁷If one uses similar assumption for the $D_s'(0^{++})$, one can identify $\sqrt{s_0}$ with the radial excitation predicted in [49] using some other approaches.

Eq. (6)

$$f_X = (4.66 \pm 0.16 \pm 0.29 \pm 0.68) \times 10^{-5} \text{ GeV}, \quad (24)$$

which can be more affected by the radiative corrections than M_X . The first error comes from s_0 , the second from the truncation of the OPE, and the third from the QCD inputs, such as m_c and $\langle \bar{q}q \rangle$. f_X is useful for the estimate of its hadronic width using vertex sum rules. As X is an axial-vector meson, its decay constant can measure its weak transition into $l\nu$ via a W -exchange, which might be difficult to measure experimentally. It would be useful to have a measurement of this decay constant from some other methods, like e.g. lattice calculations.

VI. SU(3) BREAKINGS AND MASS OF THE X^s

It is straightforward to extend the previous analysis to the case of the strange quark by using the QCD parameters given in Eq. (16). One can e.g. work with the ratios given in Eqs. (15) and (20). However, as the errors in the determination of M_X are relatively large, it will be difficult, to extract the SU(3) splitting from these individual ratio of moments.

For extracting this relatively small mass-splitting, it is appropriate to use the double ratio of moments [19,50]

$$d_c^s \equiv \frac{M_{X^s}^2}{M_X^2} \quad (25)$$

for the LSR and for the FESR, which suppress different systematic errors (m_c, \dots) and the dependence on the sum rule parameters (s_0, M^2). The results of the analysis from LSR are given in Fig. 6 from which one can deduce, with a good accuracy

$$\sqrt{d_c^s} = 0.984 \pm 0.002 \pm 0.007, \quad (26)$$

where the first error comes from the QCD and sum rules

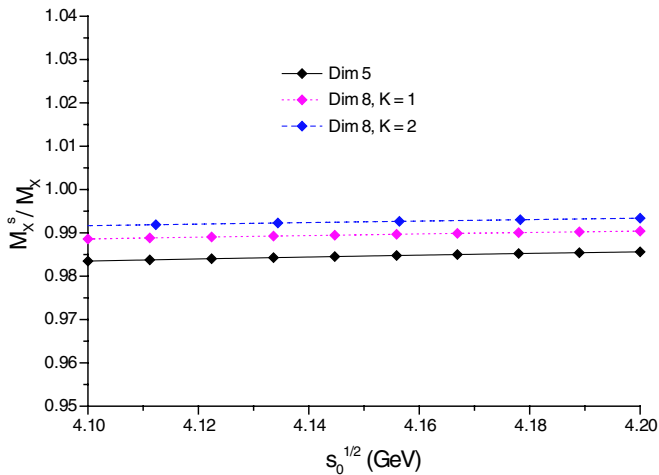


FIG. 6 (color online). The ratio M_{X^s}/M_X as a function of s_0 , obtained from the LSR results up to different dimensions in the OPE.

parameters including the SU(3) breaking of the quark condensates. The second error from an estimate of the truncation of the OPE. This leads to the mass splitting

$$M_{X^s} - M_X \simeq -(61 \pm 30) \text{ MeV}. \quad (27)$$

Similar methods used in [19,50] have predicted successfully the values of $M_{D_s^*}/M_{D^*}$ and $M_{B_s^*}/M_{B^*}$, which is not quite surprising, as in the double ratios, all irrelevant sum rules systematics cancel out.

Using FESR and taking the SU(3) breaking correction for the continuum threshold, which is the most important effect in the FESR analysis, we confirm the previous LSR result. It is interesting to notice that we predict a X^s mass slightly lighter than the X , which is quite unusual. This is due to the fact that, in the sum rules expression of $M_{X^s}^2$, the linear quark mass term tends to decrease the X^s -mass, which is partly compensated by the effect of the quark condensates. Such a small and negative mass-splitting is rather striking and needs to be checked using alternative methods. Note, however, that a partial restoration of SU(3) symmetry is already observed in the neighborhood of heavy quarks, illustrated by the almost equal hyperfine splittings $D_s^* - D_s$ and $D^* - D$. In potential models, the mass splitting $M_{X^s} - M_X$ is certainly larger than the value in Eq. (27), but smaller than $2(m_s - m_q)$ as the increase of the constituent mass from m_q to m_s is partially cancelled by the deeper binding of the strange quarks. The existence of the X^s , which can be experimentally checked, can serve for a further test of the four-quark model for the X . The (almost) degenerate value of the X and of the X^s masses may suggest that the physically observed X state can result from a mixing between the $c\bar{c}q\bar{q}$ and $c\bar{c}s\bar{s}$ bare states, which may be dominated by its $c\bar{c}q\bar{q}$ component. However, we expect that a careful and perfect analysis of the $c\bar{c}s\bar{s}$ sector should feel the X in the spectrum, though with a small coupling. One should also notice that these $c\bar{c}q\bar{q}$ and $c\bar{c}s\bar{s}$ components can be comparable if the X is a SU(3) singlet state.

Using the ratio of the s - over the q -quark sum rules, one can predict also the ratio of decay constants

$$\frac{f_{X^s}}{f_X} \simeq 1.025 \pm 0.010 \quad (28)$$

where, in the individual sum rules, the m_s corrections act positively implying that this ratio is larger than 1. We also expect the reliability of a such result advocating the previous arguments for the ratio of mass. Similar sum rule leads to $f_{B^*}/f_B = 1.16 \pm 0.03$ [51], which has been confirmed later on by different lattice calculations.

However, despite the different successful predictions of the ratio of moments for the B -meson parameters, we expect that the method will be less predictive for the four-quark state. This can be signaled by the large error in the previous prediction of the mass-difference. The inclusion of radiative or some other higher dimension

condensates corrections or some other effects not accounted for in this paper will be useful for confirming or disproving the previous results.

VII. TEST OF THE ISOSPIN VIOLATION

We attempt to use of the sum rule, for a rough estimate of the small mass difference $M(X_h) - M(X_l)$ defined in Eq. (4). Using Eq. (15), we get

$$M^2(X_h) - M^2(X_l) = \frac{\int_{4m_c^2}^{s_0} ds e^{-s/M^2} s [\rho_h(s) - \rho_l(s)]}{\int_{4m_c^2}^{s_0} ds e^{-s/M^2} \rho(s)}, \quad (29)$$

where

$$\begin{aligned} \rho_l(s) &= \cos^2\theta \rho_u(s) + \sin^2\theta \rho_d(s) \quad \text{and} \\ \rho_h(s) &= \sin^2\theta \rho_u(s) + \cos^2\theta \rho_d(s). \end{aligned} \quad (30)$$

Here, $\rho_u(s)$ and $\rho_d(s)$ are simply the spectral density $\rho(s)$ defined before with the flavor of the light quark chosen as u and d , respectively.

Clearly the only terms depending on the light quark flavor will contribute to the numerator of Eq. (29). In fact the expression $\rho_h(s) - \rho_l(s)$ can be written in terms of the isospin breaking quantities: $\langle \bar{u}u \rangle - \langle \bar{d}d \rangle = -\gamma \langle \bar{q}q \rangle$, $m_u - m_d$, $m_u \langle \bar{u}u \rangle - m_d \langle \bar{d}d \rangle = -\gamma \langle \bar{q}q \rangle m_q + \langle \bar{q}q \rangle (m_u - m_d)$, and $\langle \bar{u}u \rangle^2 - \langle \bar{d}d \rangle^2 = -2\gamma \langle \bar{q}q \rangle^2$, where $\langle \bar{q}q \rangle = (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle)/2$ and $\gamma = (\langle 0 | \bar{d}d - \bar{u}u | 0 \rangle) / \langle 0 | \bar{u}u | 0 \rangle$.

The value of γ has been estimated in a variety of approaches with results varying over almost 1 order of magnitude: $-1 \times 10^{-2} \leq \gamma \leq -2 \times 10^{-3}$ [52]. However, studies based on chiral perturbation theory [53], LSR [19] and FESR [54] analysis of the (pseudo)scalar channels, analysis of the neutron-proton mass difference [52] and of the heavy meson decay widths [55] leads to the value

$$\gamma \simeq -(1 \pm 0.5) \times 10^{-2} \quad (31)$$

which we shall consider in our analysis. The results for the mass difference using $s_0^{1/2} = 4.2$ GeV can be seen in Fig. 7, where we have considered two values for the mixing angle: $\theta = 0^\circ$, corresponding to maximal isospin violations, and $\theta = 20^\circ$ which was the value determined in [7].

One can notice from Fig. 7 that the sign of the mass difference is reversed when one includes the dimension-six condensates, while the effect of the (partial) dimension-8 contribution is relatively small, indicating that the OPE starts to behave quite well. However, one needs a more complete evaluation of the dimension-8 contributions for a more precise determination of the mass splitting. For a more conservative estimate, we consider the range of the absolute value of the mass difference, which is not strongly affected by the truncation of the OPE. In this way, one obtains from Fig. 7

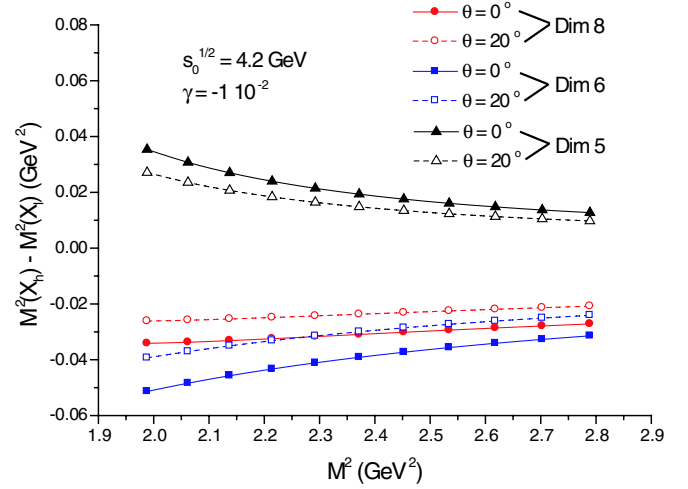


FIG. 7 (color online). $M^2(X_h) - M^2(X_l)$ as a function of the sum rule parameter (M^2) for $\gamma = -1 \times 10^{-2}$. The solid line is for $\theta = 0^\circ$ and the dashed one is for $\theta = 20^\circ$.

$$|M(X_h) - M(X_l)| \simeq (2.6-3.9) \text{ MeV}, \quad (32)$$

which is smaller than the (8 ± 3) MeV value given in [7], but larger than the decay width of the $X(3872)$, which is less than 2.3 MeV. However, one can notice from Fig. 7 that the sum rule cannot fix with a good precision the sign of the mass splitting, though it is tempting to conclude that the sign is negative, in disagreement with the result of [7].

VIII. SUM RULE PREDICTIONS FOR X_b AND X_b^s

Using the same interpolating field of Eq. (1) with the charm quark replaced by the bottom one, the analysis done for $X(3872)$ in the previous sections can be repeated for X_b , where X_b stands for a $(b\bar{b}q\bar{q})$ tetraquark axial meson. Using consistently the perturbative \overline{MS} -mass $m_b(m_b) = 4.24$ GeV, and working with the LSR, we find a good OPE convergence for $M^2 > 5$ GeV². We also find that, for $s_0 < (10.2 \text{ GeV})^2$, the continuum contribution is always bigger than the pole contribution for all values of $M^2 > 5$ GeV².

In Fig. 8 we show the X_b meson mass obtained from Eq. (15), in the relevant sum rules window, with the upper and lower validity limits indicated. Although we get a good OPE convergence for $M^2 > 5$ GeV², we have now a more restricted lower limit given by $M_{X_b} < \sqrt{s_0}$. Therefore, the lower limit indicated in Fig. 8 is given by this condition.

From Fig. 8 we see that the results are very stable as a function of M^2 in the allowed region. However, the LSR prediction increases with s_0 . Taking into account the variation of M^2 and choosing (*a priori*) some range of s_0 , we arrive at the predictions

$$10.06 \text{ GeV} \leq M_{X_b} \leq 10.50 \text{ GeV}, \quad (33)$$

for $10.2 \text{ GeV} \leq s_0^{1/2} \leq 10.8 \text{ GeV}$ and $5.0 \leq M^2 \leq 8.5 \text{ GeV}^2$.

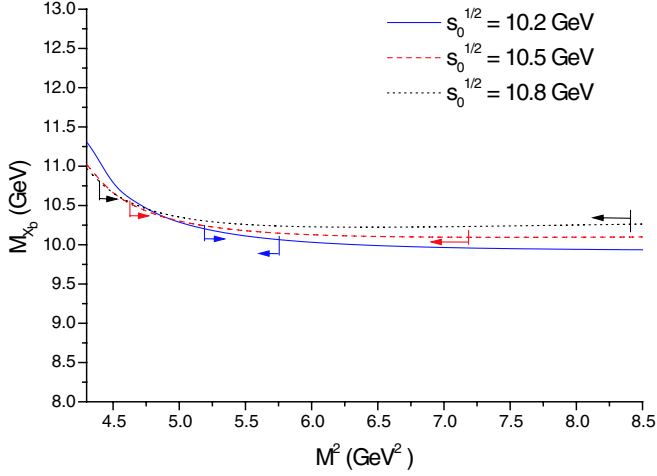


FIG. 8 (color online). The X_b meson mass as a function of the sum rule parameter (M^2) for different values of the continuum threshold: $\sqrt{s_0} = 10.2$ GeV (solid line), $\sqrt{s_0} = 10.5$ GeV (dashed line) and $\sqrt{s_0} = 10.8$ GeV (dotted line). The arrows delimit the region allowed for the LSR sum rules.

The FESR analysis can also be repeated in the case of the b -quarks for improving the LSR results. The results are shown in Fig. 9. As in the case of $X(3872)$ the curves for $n = 0$ and $n = 1$ are quite similar, and again there is no stability in s_0 . We also show in the same figure the LSR results for $M^2 = 5.6$ GeV², as a function of $s_0^{1/2}$. A common solution is obtained for

$$s_0^{1/2} = (10400 \pm 20) \text{ MeV}, \quad (34)$$

to which corresponds the improved final prediction

$$M_{X_b} = (10144 \pm 21 \pm 104) \text{ MeV}. \quad (35)$$

The second error comes again from the QCD inputs. The

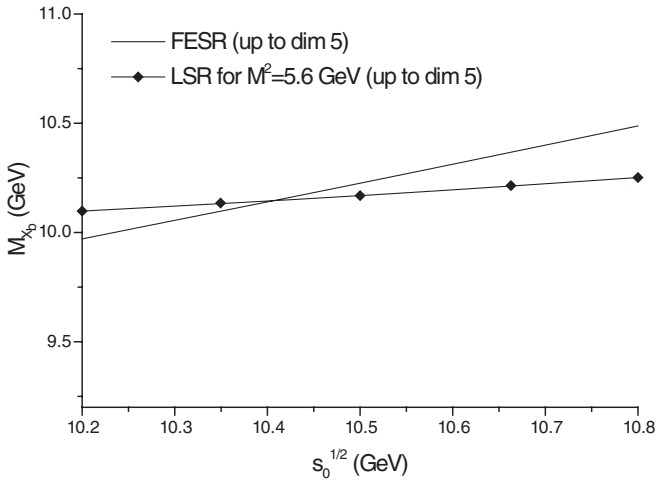


FIG. 9. The FESR results in Eq. (20) for m_{X_b} as a function of $s_0^{1/2}$ for $n = 0$. The LSR result with $M^2 = 5.6$ GeV² has been also inserted for easy comparison.

central value in Eq. (33) is close to the mass of $Y(3S)$, and appreciably below the $B^* \bar{B}$ threshold at about 10.6 GeV. For comparison, the molecular model predicts for X_b a mass which is about 50–60 MeV below this threshold [3], while a relativistic quark model without explicit $(b\bar{b})$ clustering predicts a value of about 133 MeV below this threshold [46]. It would also be interesting to have the (unquenched) lattice results for this state in order to test our QCD-based results. A future discovery of this state, e.g. at LHCb, will certainly test the different theoretical models on this state and clarify, in the same time, the nature of the $X(3872)$.

One can also notice, by assuming that the mass of first radial excitation is about the value of $\sqrt{s_0}$

$$X' - X \approx X'_b - X_b \approx (225 \sim 250) \text{ MeV}. \quad (36)$$

For completeness, we predict the corresponding useful value of the decay constant to leading order in α_s

$$f_{X_b} = (6.9 \sim 7.1) \times 10^{-6} \text{ GeV}. \quad (37)$$

Our previous results will be useful inputs for studying more precisely the phenomenology of the X_b outlined in [56].

We extend the analysis to the $X_b^s(b\bar{b}s\bar{s})$. We show in Fig. 10 the LSR prediction for the mass ratio, from which we deduce, by truncating the OPE at $D = 5$:

$$\sqrt{a_b^s} \equiv \frac{M_{X_b^s}}{M_{X_b}} = 0.988 \pm 0.002 \pm 0.018, \quad (38)$$

where the first error comes from s_0 and the QCD parameters, while the second one from the truncation of the OPE.

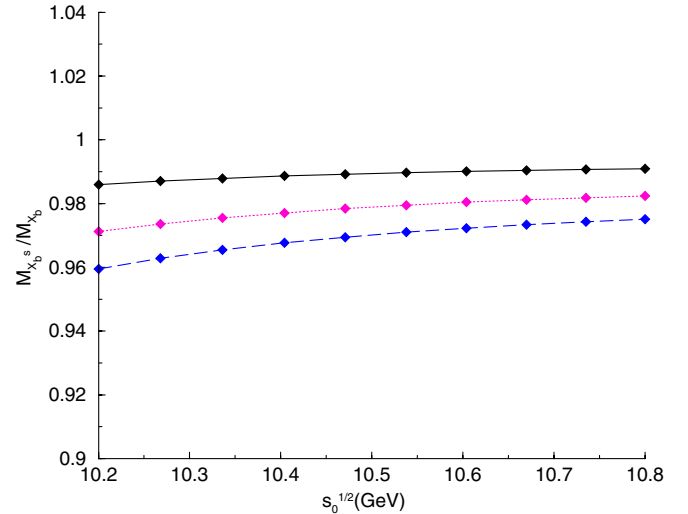


FIG. 10 (color online). The ratio $M_{X_b^s}/M_{X_b}$ as a function of s_0 , obtained from the LSR results up to different dimensions in the OPE. The continuous curve corresponds to the $D = 5$ condensate contributions. The dotted and dashed curves correspond, respectively, to the vacuum saturation ($K = 1$) and violation of the vacuum saturation ($K = 2$) by a factor 2 of the dimension-six- and (partial) eight-contributions.

This leads to:

$$M_{X_b^s} - M_{X_b} = -(123 \pm 182) \text{ MeV} \quad \text{and} \quad (39)$$

$$\frac{f_{X_b^s}}{f_{X_b}} \simeq 1.12 \pm 0.03,$$

where the error due the truncation of the OPE is larger than in the case of the c -quark, which is mainly due to the terms of the form m_b^2/M^2 in the OPE. We expect that the X_b -family will show up at LHCb in the near future, which will serve as a test of our previous predictions.

IX. CONCLUSIONS

We have presented a QSSR analysis of the two-point functions of the $X(3872)$ meson considered as a four-quark state. We find that the sum rules result in Eq. (22) is compatible with experimental data. An improvement of this result needs an accurate determination of running mass m_c of the \overline{MS} -scheme and the inclusion of radiative corrections.

We have extended the analysis for studying the mass splitting between the X^s and X due to SU(3) breaking. Our result in Eq. (27) indicates an unusual ordering which deserves further independent checks from other QCD-based approaches, especially lattice calculations. However, our small mass-splitting suggests that perhaps the observed X has a $c\bar{c}s\bar{s}$ component, though with a small coupling, with a size which depends on the SU(3) assignment of the X .

Allowing possible isospin violations, we have also studied the mass splitting between the states X_l and X_h . Using the common values from different approaches of the leading SU(2) breaking parameter γ of the light quark condensates defined in Eq. (31), we obtain the splitting in Eq. (32) which is smaller than the value 8 MeV predicted in [7].

There are limits [57] on the production of charged partners of the $X(3872)$, but based on the weak decay of B mesons. It cannot be excluded that B decay favors neutral heavy mesons, if it proceeds first via an excited ($c\bar{c}$) state recoiling against a cluster of light quarks and antiquarks. Hence the search for charged partners should be extended to other production mechanisms.

Extending our analysis to the b -quark meson, we found the values of the X_b and X_b^s masses in Eqs. (35) and (39), which are appreciably below the $B^*\bar{B}$ threshold at about 10.6 GeV. This is a common feature of all quark models with proper account for the correlation between the heavy quark and the heavy antiquark that the X_b is more deeply bound with respect to $B\bar{B}^*$ than X_c with respect to $D\bar{D}^*$, for the same reason why the $(b\bar{b})$ family has more narrow states than $(c\bar{c})$. In contrast, the molecular model, in which X_b is a meson-meson system bound by nuclear forces predicts this state rather close below the $B\bar{B}^*$ threshold. Our analysis also indicates that the mass-splitting between

the ground state and the first radial excitation is about (225 ~ 250) MeV, which is much smaller than the one expected from ordinary mesons, and which are (within the errors) flavor independent.

We present in Eqs. (24) and (37) predictions of the decay constants of the X and X_b , and in Eqs. (28) and (39) the ratio of the strange over the non strange decay constants. These are useful quantities for studying the leptonic and hadronic decay widths of such mesons, and which can be checked from (unquenched) lattice calculations or from some other models.

A future discovery at B factories or LHCb of the different states which we have predicted as a consequence of the 1^{++} four-quark nature assumption of the $X(3872)$ will certainly test the different theoretical models proposed for this state and clarify, in the same time, the nature of the $X(3872)$.

Different choices of the four-quark operators have been systematically presented for the 0^{++} light mesons in [30], which should mix under renormalizations [31] from which one can deduce a ‘‘physical’’ renormalization group invariant current (RGI) which can describe the observed state. Though some combinations can provide a faster convergence of the OPE, we do not expect that the choice of the operators will affect much our results, where, in our analysis, the OPE has a good convergence while the renormalization mixing is a higher order effect in α_s . Another choice of operator not included in the previous analysis is, for instance, given in Ref. [8], where it was shown that a simple chromomagnetic model suggests that the color octet-octet in the $[c\bar{c}]_{S=1}[q\bar{q}]_{S=1}$ basis is the most natural candidate for describing the $X(3872)$. However, this choice would correspond to an operator of higher dimension than the one analyzed in Eq. (1), which would therefore induce relatively small corrections to the present analysis. Though done with a particular choice of current [7], we expect that the results given in this paper will reproduce (within the errors of the approach) the general features of the four-quark model for the $X(3872)$. We plan to come back to these issues in a future publication.

Once the mass of the $X(3872)$ is understood, it remains to explain why it is so narrow. There are presumably many multiquark states, but most of them are very broad and cannot be singled out from the continuum. In a recent study [58], based on the same interpolating field as the one used here, it was shown that, in order to explain the small width of the $X(3872)$, one has to choose a particular set of diagrams contributing to its decay. However, it will be desirable if this investigation can be checked from alternative approaches, such as lattice calculations. If confirmed, this method can be straightforwardly repeated to a variety of currents for understanding the width and the internal structure of the $X(3872)$.

If the $X(3872)$ is a four-quark state, as our analysis suggests in answer to the question raised in the title, a

four-quark structure probably holds for the states seen near 3940 MeV and 4260 MeV, on which more experimental information is still needed. It is our intention to extend the present analysis to other J^{PC} configurations which are likely to host multi-quark resonances.

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