PHYSICAL REVIEW D **75,** 014002 (2007)

Improved method for CKM constraints in charmless three-body B and B_s decays

Michael Gronau,[1,2] Dan Pirjol,[3] Amarjit Soni,[4] and Jure Zupan[5,6]

[1]*Physics Department, Technion-Israel Institute of Technology, 32000 Haifa, Israel*
[2]*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309, USA*
[3]*Center for Theoretical Physics, Massachusetts Institute for Technology, Cambridge, Massachusetts 02139, USA*
[4]*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*
[5]*Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA*
[6]*J. Stefan Institute, Jamova 39, P.O. Box 3000, 1001 Ljubljana, Slovenia*

(Received 28 August 2006; published 2 January 2007)

Recently Ciuchini, Pierini, and Silvestrini proposed a method for constraining CKM parameters in $B \to K\pi\pi$ and $B_s \to K\pi\pi$ through phase measurements of amplitudes involving $I = 3/2$ $K^*\pi$ final states. We show that complementary information on CKM parameters may be obtained by studying the phases of $\Delta I = 1$ $B \to (K^*\pi)_{I=1/2}$, $B_s \to (K^*\bar{K})_{I=1}$ and $B_s \to (\bar{K}^*K)_{I=1}$ amplitudes. Hadronic uncertainties in these constraints from electroweak penguin operators O_9 and O_{10}, studied using flavor SU(3), are shown to be very small in $B \to K\pi\pi$ and $B_s \to K\pi\pi$ and somewhat larger in $B_s \to K\bar{K}\pi$. The first processes imply a precise linear relation between $\bar{\rho}$ and $\bar{\eta}$, with a measurable slope and an intercept at $\bar{\eta} = 0$ involving a theoretical error of 0.03. The decays $B_s \to K\pi\pi$ permit a measurement of γ involving a theoretical error below a degree. We note that while time-dependence is required when studying B^0 decays at the $\Upsilon(4S)$, it may not be needed when studying B_s decays at hadronic colliders.

DOI: 10.1103/PhysRevD.75.014002 PACS numbers: 13.20.He

I. INTRODUCTION

Recently a method has been proposed by Ciuchini, Pierini, and Silvestrini [1,2] for determining Cabibbo-Kobayashi-Maskawa (CKM) parameters in three body $B \to K\pi\pi$ and $B_s \to K\pi\pi$ decays. The proposed method is reminiscent of early suggestions for determining γ using rates and asymmetries in two-body decays $B \to K\pi$ [3–6] and $B_s \to K\pi$ [7]. A unique feature of the new method is being able to measure through interference in the Dalitz plot relative phases between quasi two-body decay amplitudes for $B_{(s)} \to K^*\pi$ and $\bar{B}_{(s)} \to \bar{K}^*\pi$. This is similar to a proposal for measuring relative phases among $B \to \rho\pi$ amplitudes by studying the Dalitz plot for $B^0 \to \pi^+\pi^-\pi^0$ [8]. When neglecting electroweak penguin (EWP) contributions, the relative phase between a combination of decay amplitudes describing $B_{(s)} \to (K^*\pi)_{I=3/2}$ and a corresponding combination of $\bar{B}_{(s)}$ amplitudes determines the weak phase γ. A small hadronic uncertainty caused by EWP amplitudes was estimated, based on factorization and assuming certain input values for B-to-light-mesons form factors [1].

In the present paper we propose extending the method to $\Delta I = 1$, $I(K^*\pi) = 1/2$ amplitudes in the above decays and to $I = 1$ amplitudes in $B_s \to K^*\bar{K}$ and $B_s \to \bar{K}^*K$. We use flavor SU(3) to study theoretical uncertainties caused by EWP contributions, suggesting a way for reducing these uncertainties. The resulting theoretical precision in determining CKM parameters in $B \to K\pi\pi$ and $B_s \to K\pi\pi$ is shown to be very high, essentially at the level of isospin breaking corrections. This happens because the method is based primarily on isospin symmetry considerations, while flavor SU(3) is used only to estimate uncertainties from a subset of small EWP contributions.

In Sec. II we analyze $B \to K^*\pi$, $B_s \to K^*\pi$, and $B_s \to K^*\bar{K}(\bar{K}^*K)$ decays in terms of isospin amplitudes, selecting several ratios of $\bar{B}_{(s)}$ and $B_{(s)}$ isospin amplitudes which can be used to determine γ in the absence of EWP contributions. Section III studies the effects of EWP amplitudes, turning the determination of γ into a generic constraint on CKM parameters. The constraint involves an uncertainty from a ratio of two hadronic matrix elements of $(V - A)$ current-current operators. Flavor SU(3) calculations show that this ratio is small for judiciously chosen combinations of isospin amplitudes in $B \to K^*\pi$, vanishes approximately in $B_s \to K^*\pi$ in the isospin symmetry limit, and is larger in $B_s \to K^*\bar{K}(\bar{K}^*K)$. This implies precise constraints on CKM parameters from knowledge of amplitudes and their relative phases for $B \to K^*\pi$ and $B_s \to K^*\pi$.

Section IV discusses measurements of these quasi two-body decay amplitudes and of $B \to K^*\bar{K}(\bar{K}^*K)$ in three classes of three-body decays, $B \to K\pi\pi$, $B_s \to K\pi\pi$ and $B_s \to K\bar{K}\pi$ decays, respectively. We point out that measuring a relative phase between the amplitudes for $B^0 \to K^{*+}\pi^-$ and $\bar{B}^0 \to K^{*-}\pi^+$ in $B^0 \to K_S\pi^+\pi^-$ produced in e^+e^- collisions at the $\Upsilon(4S)$ requires time-dependence. In contrast, no time-dependence may be needed for a similar measurement in $B_s \to K_S\pi^+\pi^-$ performed at hadronic colliders if a width difference in the B_s system is measured. In order to obtain a most precise determination of CKM parameters, we propose applying amplitude analyses to the entire $B \to K\pi\pi$ class, using isospin amplitudes as variables. Section V concludes with several remarks about the implementation of this method and its sensitivity to physics beyond the standard model, comparing it with two other methods for determining γ in B and B_s decays.

II. ISOSPIN DECOMPOSITIONS AND γ WITHOUT ELECTROWEAK PENGUIN TERMS

The cleanest method for extracting the weak phase α or γ in $\Delta S = 0$ and $\Delta S = 1$ charmless hadronic B decays stems from applying isospin symmetry to these decays, eliminating the effect of QCD penguin amplitudes which transform in these processes as $\Delta I = 1/2$ and $\Delta I = 0$, respectively [9]. We will now discuss separately the four cases, $B \to K^*\pi$, $B_s \to K^*\pi$, $B_s \to K^*\bar{K}$ and $B_s \to \bar{K}^*K$ in terms of their isospin amplitudes.

A. $B \to K^*\pi$

In strangeness changing decays of the type $B \to K\pi$ or $B \to K^*\pi$, where K^* denotes any kaon resonance, $K^*(892)$, $K_0^*(1430)$, $K_2^*(1430), \ldots$, the four physical amplitudes for B^0 and B^+ are decomposed into three isospin-invariant amplitudes [3],

$$\begin{aligned} -A(K^{*+}\pi^-) &= B_{1/2} - A_{1/2} - A_{3/2}, \\ A(K^{*0}\pi^+) &= B_{1/2} + A_{1/2} + A_{3/2}, \\ -\sqrt{2}A(K^{*+}\pi^0) &= B_{1/2} + A_{1/2} - 2A_{3/2}, \\ \sqrt{2}A(K^{*0}\pi^0) &= B_{1/2} - A_{1/2} + 2A_{3/2}. \end{aligned} \tag{1}$$

Here we use a phase convention [10] in which a minus sign is associated with a $\bar{u}$ quark in a meson. The amplitudes B and A correspond to $\Delta I = 0$ and $\Delta I = 1$ parts of $\mathcal{H}_{\rm eff}$, respectively. Their subscripts denote the isospin of the final $K^*\pi$ state. Here and elsewhere we will denote by B isospin amplitudes obtaining contributions from QCD penguin operators, and by A other amplitudes. Our study will focus on the latter.

The amplitude quadrangle relation

$$\begin{aligned} 3A_{3/2} &= A(K^{*+}\pi^-) + \sqrt{2}A(K^{*0}\pi^0) \\ &= A(K^{*0}\pi^+) + \sqrt{2}A(K^{*+}\pi^0) \end{aligned} \tag{2}$$

defines $A_{3/2}$ as one diagonal of the quadrangle, while $A_{1/2}$ is given by

$$\begin{aligned} 6A_{1/2} &= A(K^{*+}\pi^-) + 3A(K^{*0}\pi^+) - 2\sqrt{2}A(K^{*0}\pi^0) \\ &= 3A(K^{*+}\pi^-) + A(K^{*0}\pi^+) - 2\sqrt{2}A(K^{*+}\pi^0). \end{aligned} \tag{3}$$

The two $\Delta I = 1$ amplitudes, $A_{3/2}$ and $A_{1/2}$, do not contain a QCD penguin contribution and would carry a single weak phase γ if EWP contributions could be neglected. Here we will proceed under this assumption, postponing a discussion of the effects of EWP amplitudes to the next section. Denoting amplitudes for charge-conjugate initial and final states by $\bar{A}$, and defining two ratios of amplitudes,

$$R_I \equiv \frac{\bar{A}_I}{A_I}, \qquad I = 1/2, 3/2, \tag{4}$$

the phase γ is determined by

$$\Phi_I \equiv -\tfrac{1}{2}\arg(R_I) = \gamma. \tag{5}$$

Note that although the ratios R_I do not depend on the magnitudes of A_I in the limit of vanishing EWP contributions, an extraction of γ requires measuring both the magnitudes and the relative phases of physical $B \to K^*\pi$ amplitudes and their charge conjugate.

The ratio $R_{3/2}$ was studied in [1] (where it was denoted by $R^0 = R^{\pm}$) while the ratio $R_{1/2}$ studied here provides independent information on CKM parameters.

B. $B_s \to K^*\pi$

The isospin decomposition of the two $\Delta S = 0$ $B_s \to K^*\pi$ decay amplitudes is

$$\begin{aligned} A_s(K^{*+}\pi^-) &= A^s_{3/2} - \sqrt{2}B^s_{1/2}, \\ A_s(K^{*0}\pi^0) &= \sqrt{2}A^s_{3/2} + B^s_{1/2}, \end{aligned} \tag{6}$$

where the superscript s denotes B_s instead of B^0 and subscripts denote the isospin of both the transition operator and the final $K^*\pi$ state. Since in $\Delta S = 0$ decays the QCD penguin operator behaves as $\Delta I = 1/2$ it is contained only in $B^s_{1/2}$. On the other hand, the amplitude

$$3A^s_{3/2} = A_s(K^{*+}\pi^-) + \sqrt{2}A_s(K^{*0}\pi^0) \tag{7}$$

is pure tree when neglecting EWP contributions, thus providing information on γ. Defining a ratio of $\bar{B}_s$ and B_s amplitudes (denoted R_d in [2]),

$$R^s_{3/2} \equiv \frac{\bar{A}^s_{3/2}}{A^s_{3/2}}, \tag{8}$$

one now has

$$\Phi^s_{3/2} \equiv -\tfrac{1}{2}\arg(R^s_{3/2}) = \gamma. \tag{9}$$

C. $B_s \to K^*\bar{K}$ and $B_s \to \bar{K}^*K$

These $\Delta S = 1$ decays involve two independent pairs of isospin amplitudes,

$$A_s(K^{*+}K^-) = A^s_1 + B^s_0, \qquad A_s(K^{*0}\bar{K}^0) = A^s_1 - B^s_0, \tag{10}$$

and

$$A_s(K^{*-}K^+) = A'^s_1 + B'^s_0, \qquad A_s(\bar{K}^{*0}K^0) = A'^s_1 - B'^s_0. \tag{11}$$

Thus, one has

$$\begin{aligned} 2A^s_1 &= A_s(K^{*+}K^-) + A_s(K^{*0}\bar{K}^0), \\ 2A'^s_1 &= A_s(K^{*-}K^+) + A_s(\bar{K}^{*0}K^0). \end{aligned} \tag{12}$$

Defining for each of these processes a ratio of $\bar{B}_s$ and B_s amplitudes

$$R_1^s \equiv \frac{\bar{A}_1^s}{A_1^s}, \qquad R_1'^s \equiv \frac{\bar{A}_1'^s}{A_1'^s}, \tag{13}$$

one obtains two new independent equations for γ,

$$\Phi_1^s \equiv -\tfrac{1}{2}\arg(R_1^s) = \gamma, \qquad \Phi_1'^s \equiv -\tfrac{1}{2}\arg(R_1'^s) = \gamma. \tag{14}$$

III. CKM CONSTRAINTS INCLUDING ELECTROWEAK PENGUIN AMPLITUDES

In section II we have neglected $\Delta S = 1$, $\Delta I = 1$ and $\Delta S = 0$, $\Delta I = 3/2$ EWP contributions. In this limit measurements of the ratios $R_I^{(s)}$ in (5), (9), and (14), determine γ. The inclusion of EWP operators modifies these relations since these operators involve different weak phase than the tree operators. These effects are important in the $\Delta S = 1$ relations (5) and (14), where EWP contributions are CKM-enhanced, and are negligible in the $\Delta S = 0$ relation (9). We will first obtain a general constraint in the $(\bar{\rho}, \bar{\eta})$ plane [11] following from fixed values of $\Phi_I^{(s)} \equiv -\frac{1}{2}\arg(R_I^{(s)})$.

Let us study the effect of EWP operators on obtaining CKM constraints in $\Delta S = 1$ decays. The dominant $(V - A)$ EWP operators, O_9^s and O_{10}^s, in the $\Delta S = 1$ effective Hamiltonian [12] are related to current-current operators, $O_1^s \equiv [\bar{s}b]_{V-A}[\bar{u}u]_{V-A}$ and $O_2^s \equiv [\bar{u}b]_{V-A}[\bar{s}u]_{V-A}$, through operator relations

$$O_{9,10}^s = \tfrac{3}{2}O_{1,2}^s + [\text{operators with } \Delta I = 0]. \tag{15}$$

Neglecting EWP operators, O_7^s and O_8^s, involving small Wilson coefficients, the $\Delta I = 1$ part of the $\Delta S = 1$ weak Hamiltonian can be rewritten as

$$\begin{aligned} H_{\Delta I=1}^s = {}& (\lambda_u^s C_+ - \tfrac{3}{2}\lambda_t^s C_+^{\rm EWP}) O_+^{\Delta I=1} \\ & + (\lambda_u^s C_- - \tfrac{3}{2}\lambda_t^s C_-^{\rm EWP}) O_-^{\Delta I=1}, \end{aligned} \tag{16}$$

where $\lambda_{u(t)}^s \equiv V_{u(t)b}^* V_{u(t)s}$, $O_\pm^{\Delta I=1} \equiv \frac{1}{2}(O_1^s \pm O_2^s)$, and $C_\pm \equiv C_1 \pm C_2$, $C_\pm^{\rm EWP} \equiv C_9 \pm C_{10}$ are sums and differences of Wilson coefficients.

Terms in (16) involving $C_\pm^{\rm EWP}$ introduce in $\Delta I = 1$ amplitudes a weak phase different from γ, with coefficients depending on hadronic matrix elements for $O_-^{\Delta I=1}$ and $O_+^{\Delta I=1}$. Using a relation between Wilson coefficients which holds up to 1% corrections [12],

$$\frac{C_+^{\rm EWP}}{C_+} = -\frac{C_-^{\rm EWP}}{C_-}, \tag{17}$$

one obtains a generic expression for the four ratios $R_{1/2}$, $R_{3/2}$, R_1^s, and $R_1'^s$ in Eqs. (4) and (13),

$$R_I^{(s)} = e^{-2i[\gamma + \arg(1+\kappa)]}\frac{1 + c_\kappa^* r_I^{(s)}}{1 + c_\kappa r_I^{(s)}}. \tag{18}$$

Here we define

$$c_\kappa \equiv \frac{1-\kappa}{1+\kappa}, \qquad \kappa \equiv -\frac{3}{2}\frac{C_+^{\rm EWP}}{C_+}\frac{\lambda_t^s}{\lambda_u^s}, \tag{19}$$

$$r_I^{(s)} \equiv \frac{\langle f_I | C_- O_-^{\Delta I=1} | B_{(s)}\rangle}{\langle f_I | C_+ O_+^{\Delta I=1} | B_{(s)}\rangle}. \tag{20}$$

The parameter κ depends only on calculable Wilson coefficients and on CKM parameters. In order to illustrate the sizable shift in $\Phi_I^{(s)} \equiv -\frac{1}{2}\arg(R_I^{(s)})$ caused by this parameter alone, we use the central values for CKM parameters [13] and next to leading order (NLO) values for Wilson coefficients at $\mu = m_b = 4.8$ GeV, $C_1(m_b) = -0.178$, $C_2(m_b) = 1.079$, $C_9(m_b) = -0.0102$, $C_{10} = 0.0017$. We find

$$\kappa = \frac{\lambda_t^s}{\lambda_u^s}(1.404 \pm 0.038) \times 10^{-2} = -0.35 + 0.56i, \tag{21}$$

where the error in the brackets corresponds to varying the scale μ in the NLO Wilson coefficients in the range $m_b/2 \le \mu \le m_b$. On the right-hand side we give the result for central values of Wilson coefficients and CKM elements. This value of κ translates into $\arg(1+\kappa) = 41°$.

A nonzero value of the parameter $r_I^{(s)}$ leads to an additional shift in $\Phi_I^{(s)}$, given by $-\frac{1}{2}\arg[(1 + c_\kappa^* r_I^{(s)})/(1 + c_\kappa r_I^{(s)}]$. A given value of the observable $\Phi_I^{(s)}$ can be shown to imply the following constraint in the $(\bar{\rho}, \bar{\eta})$ plane (we use $\lambda = 0.227$):

$$\frac{\bar{\eta} + (\bar{\rho} + C)t}{(\bar{\rho} + C) - \bar{\eta}t} = \tan\Phi_I^{(s)}. \tag{22}$$

Here we define

$$C \equiv \frac{3}{2}\frac{C_+^{\rm EWP}}{C_+}\frac{1 - \lambda^2/2}{\lambda^2} = -0.27, \tag{23}$$

$$t \equiv \frac{1 + t_+ t_- - \sqrt{(1 + t_+^2)(1 + t_-^2)}}{t_+ - t_-}, \tag{24}$$

$$t_\pm \equiv \frac{(\bar{\rho}^2 + \bar{\eta}^2 - C^2)\,{\rm Im}(r_I^{(s)}) \mp 2C\bar{\eta}\,{\rm Re}(r_I^{(s)})}{(\bar{\rho} + C)^2 + \bar{\eta}^2 + (\bar{\rho}^2 + \bar{\eta}^2 - C^2)\,{\rm Re}(r_I^{(s)}) \pm 2C\bar{\eta}\,{\rm Im}(r_I^{(s)})}. \tag{25}$$

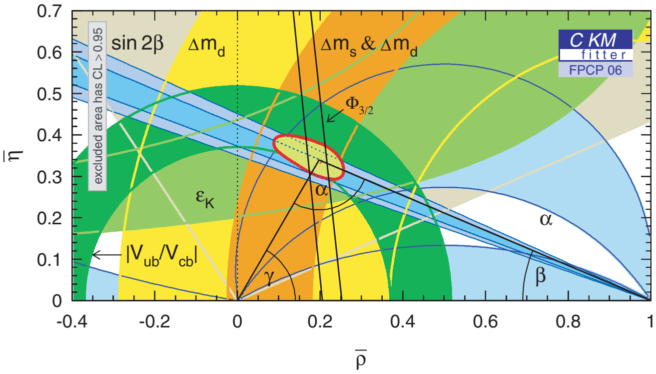

FIG. 1 (color online). 1σ constraint in the $\bar{\rho}$-$\bar{\eta}$ plane (two almost vertical parallel black lines) from a precisely measured $\Phi_{3/2}$ in $B \to K\pi\pi$, taking values for $r_{3/2}$ as in (30). All other constraints are taken from Ref. [13].

For a small value of $r_I^{(s)}$ and for a given value of the observable $\Phi_I^{(s)}$, one obtains the following constraint,

$$\bar{\eta} = \tan\Phi_I^{(s)}[\bar{\rho} + C(1 - 2\,\mathrm{Re}(r_I^{(s)})) + \mathcal{O}(r_I^{(s)2})]. \qquad (26)$$

This describes a straight line in the $(\bar{\rho}, \bar{\eta})$ plane (cf. Figure 1), with a slope $\tan\Phi_I^{(s)}$ and an intercept $\bar{\rho}_0 = -C[1 - 2\,\mathrm{Re}(r_I^{(s)})]$ at $\bar{\eta} = 0$. A theoretical error $\delta r_I^{(s)}$ in $r_I^{(s)}$ translates into an uncertainty of $\pm 2|C|\delta r_I^{(s)}$ in the intercept $\bar{\rho}_0$ but no uncertainty in the slope $\tan\Phi_I^{(s)}$ which is measured through the ratio $R_I^{(s)}$. Assuming for illustration a negligible value of $r_I^{(s)}$, one may estimate the slope required in the standard model by choosing central values of $(\bar{\rho}, \bar{\eta})$ from a CKM fit [13], $\bar{\rho} = 0.20$, $\bar{\eta} = 0.34$. This implies a slope $\tan\Phi_I^{(s)} = -5.0$, which is quite sensitive to the value of $r_I^{(s)}$.

A similar treatment of EWP contributions can be applied to $\Delta S = 0$ processes involving CKM factors $\lambda_{u(t)}^d$. In the isospin symmetry limit the $\Delta S = 0$ part of O_- is pure $\Delta I = 1/2$, hence $O_-^{\Delta I=3/2} = 0$. Consequently, in $B_s \to (K^*\pi)_{I=3/2}$ one has $r_{3/2}^s = 0$.[1] This implies that there is no hadronic uncertainty in the CKM constraint from the ratio $R_{3/2}^s$ in $B_s \to K^*\pi$, aside from tiny corrections from the operators O_7 and O_8 which we have neglected. The parameter κ' in $\Delta S = 0$ decays, whose complex phase is related to γ, is of order a few percent [see Eqs. (19) and (21)],

$$|\kappa'| = -\frac{3C_+^{\mathrm{EWP}}}{2C_+}\frac{|\lambda_t^d|}{|\lambda_u^d|} = (1.40 \pm 0.04) \times 10^{-2}\frac{\sin\gamma}{\sin\beta}. \qquad (27)$$

Thus, the dependence of the shift $\arg(1 + \kappa')$ on γ is very small and calculable in terms of γ.

[1] This observation has been overlooked in Ref. [2].

Another case where $r = 0$ holds in a symmetry limit is $B \to (K\pi)_{I=3/2}$, where the K and π mesons are in an S-wave and must be in a symmetric SU(3) state [6,14]. This SU(3) argument does not hold in $B \to (K^*\pi)_{I=1/2,3/2}$ [15], nor does it hold in $B_s \to (K^*\bar{K})_{I=1}$ and $B_s \to (\bar{K}^*K)_{I=1}$. We will study now the values of $r_I^{(s)}$ for these decays within flavor SU(3). Theoretical errors in these values lead to uncertainties in the resulting CKM constraints.

A. Ratios r_I and CKM constraints in $B \to K^*\pi$

The two ratios R_I in Eq. (4), providing independent pieces of information on γ, are given by Eq. (18) with $r = r_I$ given by

$$r_I \equiv \frac{\langle (K^*\pi)_I | C_- O_-^{\Delta I=1} | B\rangle}{\langle (K^*\pi)_I | C_+ O_+^{\Delta I=1} | B\rangle}, \qquad I = 1/2, 3/2. \qquad (28)$$

The ratio $r_{3/2}$ was estimated in [1], based on factorization and assuming certain input values for B-to-light-mesons form factors. Here we wish to present a different approach based on flavor SU(3) for calculating both $r_{3/2}$ and $r_{1/2}$. Using flavor SU(3) r_I may be calculated from tree-dominated strangeness-conserving B decays, which are CKM-enhanced relative to tree amplitudes in $B \to K^*\pi$, and which have already been measured. Furthermore, one may apply Eqs. (18) and (28) to $|(K^*\pi)_X\rangle$, an arbitrary superposition of $I = 1/2$ and $I = 3/2$ $K^*\pi$ states. The corresponding ratio of hadronic matrix elements will be denoted r_X. One is searching for a linear superposition of isospin states which leads to a small value of r_X in order to obtain a small uncertainty in CKM parameters.

The operators $O_+^{\Delta I=1}$ and $O_-^{\Delta I=1}$ transform as $\mathbf{\overline{15}}$ and $\mathbf{6}$ representations of SU(3), respectively, while a general $K^*\pi$ state is a combination of $\mathbf{8_S}$, $\mathbf{8_A}$, $\mathbf{10}$, $\mathbf{\overline{10}}$, and $\mathbf{27}$ [16,17]. Thus, the numerator in r_X involves in general a linear combination of three reduced matrix elements, $\langle \mathbf{8_S}|\mathbf{6}|\mathbf{3}\rangle$, $\langle \mathbf{8_A}|\mathbf{6}|\mathbf{3}\rangle$, $\langle \mathbf{10}|\mathbf{6}|\mathbf{3}\rangle$, and the denominator involves a combination of four matrix elements, $\langle \mathbf{8_S}|\mathbf{\overline{15}}|\mathbf{3}\rangle$, $\langle \mathbf{8_A}|\mathbf{\overline{15}}|\mathbf{3}\rangle$, $\langle \mathbf{\overline{10}}|\mathbf{\overline{15}}|\mathbf{3}\rangle$, $\langle \mathbf{27}|\mathbf{\overline{15}}|\mathbf{3}\rangle$. The same reduced matrix elements occur in $\Delta S = 0$ amplitudes. One is seeking two sums of $\Delta S = 0$ amplitudes which are given by the same two combinations in the numerator and denominator of r_X.

The case of $r_{3/2}$ is particularly simple since the state $(K^*\pi)_{I=3/2}$ contains only two pieces transforming as $\mathbf{10}$ and $\mathbf{27}$ of SU(3). The transformation properties of the operators imply that only the $\mathbf{10}$ and $\mathbf{27}$ pieces contribute to the numerator and denominator, respectively. Thus, $r_{3/2}$ is proportional to a ratio of corresponding reduced matrix elements, $\langle \mathbf{10}|\mathbf{6}|\mathbf{3}\rangle / \langle \mathbf{27}|\mathbf{\overline{15}}|\mathbf{3}\rangle$. The numerical coefficient multiplying this ratio can be read off SU(3) Clebsch-Gordan tables in Ref. [17] (after translating into our phase convention). These tables can also be used to express $\langle \mathbf{10}|\mathbf{6}|\mathbf{3}\rangle$ and $\langle \mathbf{27}|\mathbf{\overline{15}}|\mathbf{3}\rangle$ in terms of $\Delta S = 0$ amplitudes,

$$r_{3/2} = \frac{[A(\rho^+\pi^0) - A(\rho^0\pi^+)] - \sqrt{2}[A(K^{*+}\bar{K}^0) - A(\bar{K}^{*0}K^+)]}{A(\rho^+\pi^0) + A(\rho^0\pi^+)}. \tag{29}$$

This expression can be simplified by neglecting the $\Delta S = 0$ QCD penguin amplitude given by the second term in the numerator, and by assuming that the strong phase difference between the two amplitudes in the remaining term is small, as this phase is expected to be suppressed by $1/m_b$ and $\alpha_s(m_b)$ [18–20]. This is supported by studies of QCD penguin amplitudes (including charming penguins) in $B \to \rho\pi$ which have been found to be small, with a penguin-to-tree ratio of about 0.2 [21]. Signs of color-allowed amplitudes are assumed to be given by factorization. Using branching ratios given in Table I, one finds

$$r_{3/2} = \frac{|\sqrt{\mathcal{B}(\rho^+\pi^0)} - \sqrt{\mathcal{B}(\rho^0\pi^+)}|}{\sqrt{\mathcal{B}(\rho^+\pi^0)} + \sqrt{\mathcal{B}(\rho^0\pi^+)}} = 0.054 \pm 0.045 \pm 0.023. \tag{30}$$

The first error is caused by experimental errors in $B \to \rho\pi$ branching ratios. The second error, due to SU(3) breaking, is calculated by allowing an uncertainty of 30% in each of the reduced matrix elements entering the physical amplitudes.

The value (30), obtained by applying SU(3) to $B \to \rho\pi$ branching ratios, may be compared with an estimate based on naive factorization [1] in which we include a color factor,

$$r_{3/2} = \frac{C_-}{C_+}\frac{(1 - 1/N_c)}{(1 + 1/N_c)}\frac{(f_\pi A_0^{BK^*} - f_{K^*}F_0^{B\pi})}{(f_\pi A_0^{BK^*} + f_{K^*}F_0^{B\pi})} = 0.012 \pm 0.083. \tag{31}$$

We used the following values for decay constants and form factors [19], $f_\pi = 131$ MeV, $f_{K^*} = 218 \pm 4$ MeV, $F_0^{B\pi} = 0.28 \pm 0.05$, $A_0^{BK^*} = 0.45 \pm 0.07$. Note that naive factorization may be a reasonable approximation because the ratio $r_{3/2}$ defined in (28) does not involve QCD penguin contributions.

TABLE I. Branching ratios for $B \to \rho\pi$ and $B \to K^*K$ decays, in units of 10^{-6}, taken from Ref. [22] unless quoted otherwise.

	Mode	Branching ratio
$B^+ \to$	$\rho^0\pi^+$	$8.7^{+1.0}_{-1.1}$
	$\rho^+\pi^0$	$10.8^{+1.4}_{-1.5}$
	$\overline{K^{*0}}K^+$	<5.3
$B^0 \to$	$\rho^\pm\pi^\mp$	24.0 ± 2.5
	$\rho^+\pi^-$	16.0 ± 2.0[a]
	$\rho^-\pi^+$	7.6 ± 1.3[b]
	$\rho^0\pi^0$	$1.83^{+0.56}_{-0.55}$
	$K^{*0}\overline{K^0}$	<1.9

[a]We take an average of 19.5 ± 5.0 [23] and 15.3 ± 2.2 [24].
[b]We take an average of 9.6 ± 3.4 [23] and 7.3 ± 1.4 [24].

A bound on the error in $r_{3/2}$ caused by neglecting a difference of two $\Delta S = 0$ QCD penguin amplitudes in (29) can be obtained in terms of upper bounds on branching fractions for $B^+ \to K^{*+}\bar{K}^0$ and $B^+ \to \bar{K}^{*0}K^+$. Although current upper bounds are not very useful (see Table I), we expect the bounds to improve in the future, such that the error caused by neglecting these terms will be at most at the level of SU(3) breaking.

The error in $r_{3/2}$ affects the CKM constraint (26) through the term involving $\mathrm{Re}(r_{3/2})$. The error from neglecting a strong phase difference between $A_{\rho^+\pi^0}$ and $A_{\rho^0\pi^+}$ is expected to be very small, since $\mathrm{Re}(r_{3/2})$ depends quadratically on this phase. This is gratifying since the size of $1/m_b$ suppressed strong phases cannot be reliably calculated [18,25]. Information on the above phase is provided by the isospin pentagon relation [3]

$$A(\rho^+\pi^0) + A(\rho^0\pi^+) = \frac{1}{\sqrt{2}}(A(\rho^+\pi^-) + A(\rho^-\pi^+)) + \sqrt{2}A(\rho^0\pi^0). \tag{32}$$

Relative phases between amplitudes on the right-hand-side can be measured through a Dalitz plot analysis of $B^0 \to \pi^+\pi^-\pi^0$ [8,26]. Assuming that phases between $B \to \rho\pi$ amplitudes can be neglected, and using branching ratios from Table I and a lifetime ratio [22], $\tau_+/\tau_0 = 1.076 \pm 0.008$, Eq. (32) reads $6.01 \pm 0.27 = 6.69 \pm 0.38$. This agreement shows that relative phases between $B \to \rho\pi$ amplitudes are not large. Assuming in contrast a negative sign for the color-suppressed amplitude $A(\rho^0\pi^0)$, for which factorization does not hold, would imply that $6.01 \pm 0.27 = 2.86 \pm 0.38$ which is badly broken.

The value of $r_{3/2}$ in (30) can now be substituted in Eq. (26). The resulting linear constraint in $\bar{\rho}$-$\bar{\eta}$ plane is shown in Fig. 1, assuming a precisely measured value for $\Phi_{3/2}$. The current error in $r_{3/2}$ translates into a very small error of 0.03 in the intercept where $\bar{\eta} = 0$, $\bar{\rho}_0 = 0.24 \pm 0.03$, but no theoretical error in the slope which is given by a value measured for $\tan\Phi_{3/2}$. The small error in the intercept, partly from SU(3) breaking in $r_{3/2}$, is linear in the uncertainty in $r_{3/2}$, and may be reduced only slightly by measuring more precisely $B \to \rho\pi$ branching ratios.

The calculation of $r_{1/2}$ proceeds in a similar manner to the calculation of $r_{3/2}$, leading to a larger value of order one. Instead, one may search for a superposition of $I = 1/2$ and $I = 3/2$ $K^*\pi$ states for which r_X is small. Using

$$|(K^*\pi)_X\rangle = \frac{1}{\sqrt{5}}(|I = 1/2\rangle - 2|I = 3/2\rangle), \tag{33}$$

we find

$$r_X = \frac{A(\rho^+\pi^0) - 2A(\rho^0\pi^+) + \sqrt{2}A(\rho^0\pi^0) + N_X(K^*K)}{A(\rho^+\pi^0) + \sqrt{2}A(\rho^-\pi^+) + \sqrt{2}A(\rho^0\pi^0) + D_X(K^*K)}. \tag{34}$$

$\Delta S = 0$ QCD penguin and annihilation amplitudes in the numerator and denominator,

$$\begin{aligned} N_X(K^*K) &\equiv [2A(\bar{K}^{*0}K^+) - 3A(K^{*+}\bar{K}^0) - A(K^{*0}\bar{K}^0) \\ &\quad + A(K^{*+}K^-)]/\sqrt{2}, \\ D_X(K^*K) &\equiv [A(K^{*0}\bar{K}^0) - A(K^{*+}\bar{K}^0) + A(K^{*+}K^-)]/\sqrt{2}, \end{aligned} \tag{35}$$

are expected to be no larger than SU(3) breaking corrections and will be neglected. Assuming small strong phase differences between $B \to \rho\pi$ amplitudes, and using measured branching ratios in Table I, we find

$$r_X = -0.068 \pm 0.057 \pm 0.044. \tag{36}$$

The first error originates in experimental errors in $B \to \rho\pi$ branching ratios, while the second error is calculated assuming 30% SU(3) breaking in reduced matrix elements. The central value of r_X and its error are comparable to $r_{3/2}$. As in the latter case, this translates to a very small error in the intercept, but no error in the slope of the linear relation (26) between $\bar{\rho}$ and $\bar{\eta}$ provided by a measurement of Φ_X. We define R_X and Φ_X as in Eqs. (4) and (5) using the $K^*\pi$ state defined in (33):

$$R_X \equiv \frac{\bar{A}_{1/2} - 2\bar{A}_{3/2}}{A_{1/2} - 2A_{3/2}}, \qquad \Phi_X \equiv -\frac{1}{2}\arg(R_X). \tag{37}$$

The result (36) may be compared with an estimate based on naive factorization,

$$\begin{aligned} r_X &= \frac{C_-}{C_+}\frac{(1 - 1/N_c)}{(1 + 1/N_c)}\frac{(2f_\pi A_0^{BK^*} - f_{K^*}F_0^{B\pi})}{(2f_\pi A_0^{BK^*} + f_{K^*}F_0^{B\pi})} \\ &= -0.22 \pm 0.07, \end{aligned} \tag{38}$$

where the error reflects only errors on the assumed form factors. Our result (36) using flavor SU(3) agrees within uncertainties with this more crude approximation which gives a somewhat larger central value.

B. Ratio $r_{3/2}^s$ and determining γ in $B_s \to K^*\pi$

As we have shown using isospin symmetry alone, the parameter r in $B_s \to K^*\pi$ vanishes, $r_{3/2}^s = 0$, because the $\Delta S = 0$ part of O_- is pure $\Delta I = 1/2$. The small parameter κ' introduces a small shift $\arg(1 + \kappa')$ in $\Phi_{3/2}^s$ away from γ. Since the shift is calculable in terms of γ [see Eq. (27)], the theoretical error in determining γ using these processes is below 1°.

Note that measuring γ in these processes, and using $B \to K\pi\pi$ for constraining the point $(\bar{\rho}, \bar{\eta})$ to lie on a straight line with measured slope and intercept, fixes the apex of the unitarity triangle as the point where the two straight lines intersect. Thus, in principle, $B \to K\pi\pi$ and $B_s \to K\pi\pi$ alone determine the shape of the unitarity triangle.

C. Ratios r_1^s, $r_1^{\prime s}$ and CKM constraints in $B_s \to K^*\bar{K}$, $\bar{K}^*K$

In the presence of EWP contributions the two ratios R_1^s and $R_1^{\prime s}$ defined in (13) are given by Eq. (18) with

$$\begin{aligned} r_1^s &\equiv \frac{\langle K^*\bar{K}|C_-O_-^{\Delta I=1}|B_s\rangle}{\langle K^*\bar{K}|C_+O_+^{\Delta I=1}|B_s\rangle}, \\ r_1^{\prime s} &\equiv \frac{\langle \bar{K}^*K|C_-O_-^{\Delta I=1}|B_s\rangle}{\langle \bar{K}^*K|C_+O_+^{\Delta I=1}|B_s\rangle}. \end{aligned} \tag{39}$$

We use SU(3) tables in Ref. [17] to express these ratios in terms of $\Delta S = 0$ decay amplitudes for nonstrange B mesons,

$$\begin{aligned} r_1^s &= \frac{A(\rho^+\pi^-) + A(\rho^-\pi^+) - \sqrt{2}A(\rho^0\pi^+) + N_1(K^*K)}{A(\rho^+\pi^-) - A(\rho^-\pi^+) + \sqrt{2}A(\rho^0\pi^+) + D_1(K^*K)}, \\ r_1^{\prime s} &= \frac{A(\rho^-\pi^+) + A(\rho^+\pi^-) - \sqrt{2}A(\rho^+\pi^0) + N_1'(K^*K)}{A(\rho^-\pi^+) - A(\rho^+\pi^-) + \sqrt{2}A(\rho^+\pi^0) + D_1'(K^*K)}. \end{aligned} \tag{40}$$

Penguin and annihilation terms in the numerators and denominators,

$$\begin{aligned} N_1(D_1) &\equiv \pm A(\bar{K}^{*0}K^+) \mp A(K^{*-}K^+) + A(\bar{K}^{*0}K^0), \\ N_1'(D_1') &\equiv \pm A(K^{*+}\bar{K}^0) \mp A(K^{*+}K^-) + A(K^{*0}\bar{K}^0), \end{aligned} \tag{41}$$

will be assumed to be smaller than SU(3) breaking corrections.

Disregarding phase differences between $B \to \rho\pi$ amplitudes which have a very small effect on r_1^s and $r_1^{\prime s}$ [as we argued in obtaining (30)], using measured branching ratios in Table I, and estimating errors from SU(3) breaking as explained above, we have

$$r_1^s = 0.52 \pm 0.10 \pm 0.18, \qquad r_1^{\prime s} = 0.70 \pm 0.21 \pm 0.41. \tag{42}$$

Comparing these values with an estimate based on naive factorization, we find agreement again:

$$r_1^s = r_1^{\prime s} = -\frac{C_-}{C_+}\frac{(1-1/N_c)}{(1+1/N_c)} = 0.70. \quad (43)$$

We do not expect the errors in (42) to improve by reducing errors in $\mathcal{B}(B \to \rho\pi)$, as SU(3) breaking introduces a comparable uncertainty. The values of r_1^s and $r_1^{\prime s}$ can be substituted in Eqs. (22)–(25) to obtain constraints in the $(\bar{\rho}, \bar{\eta})$ plane, for measured values of Φ_1^s and $\Phi_1^{\prime s}$. The larger errors in (42) in comparison with those in (30) and (36) imply larger uncertainties in these constraints than in those following from $\Phi_{3/2}$ and Φ_X.

IV. MEASURING MAGNITUDES AND PHASES FOR QUASI TWO-BODY DECAY AMPLITUDES

As shown in the previous two sections, new constraints in the $(\bar{\rho}, \bar{\eta})$ plane can be obtained within each of the three classes of quasi two-body decay processes, $B \to K^*\pi$, $B_s \to K^*\pi$, and $B_s \to K^*\bar{K}$, $\bar{K}^*K$ and their charge-conjugates. This requires measuring both the magnitudes of the amplitudes in a given class and their relative phases. This can be achieved through amplitude analyses of charmless three-body decays which we discuss now.

A three-body B (or B_s) decay amplitude into a final state f, which is a function of two Dalitz variables, s_{12}, s_{13}, is expressed as a sum of several Breit-Wigner resonant contributions and a nonresonant term. Resonant contributions are given by complex constant amplitudes A_i multiplying Breit-Wigner functions $f_i^{\rm BW}(s_{12}, s_{13})$, while the nonresonant amplitude $A_{\rm NR}$ may vary in the s_{12}, s_{13} plane,

$$A(s_{12}, s_{13}) = A_{\rm NR}(s_{12}, s_{13}) + \sum_i A_i f_i^{BW}(s_{12}, s_{13}). \quad (44)$$

The corresponding amplitude $\bar{A}(s_{12}, s_{23})$, for three-body $\bar{B}$ (or $\bar{B}_s$) decays into a charge-conjugate state $\bar{f}$, is given in terms of an amplitude $\bar{A}_{\rm NR}$ and a set $\bar{A}_i$ corresponding to charge-conjugate resonances. In general, one has $\bar{A}_{\rm NR} \neq A_{\rm NR}$, $\bar{A}_i \neq A_i$ as each amplitude may involve two weak phases and two different strong phases. Direct CP violation in a particular resonant or nonresonant channel would be implied by $|\bar{A}_i| \neq |A_i|$ or $|\bar{A}_{\rm NR}| \neq |A_{\rm NR}|$.

Fitting the event distribution for three body B (or B_s) decays to the squared amplitude (44) permits determining the magnitudes of A_i and their relative phases. A relative phase between two resonant amplitudes is directly measurable when the two resonances overlap in the Dalitz plot. This relative phase can also be measured when there is no overlap between the two resonances, but each of the two resonances overlaps with a third resonance. Alternatively, a phase between two resonance amplitudes can be measured through their interference with the nonresonant amplitude $A_{\rm NR}$.

We will be interested primarily in relative phases between amplitudes associated with K meson resonant states. Charmless three-body decays involving $\pi^+\pi^-$ or K^+K^- obtain also contributions from $c\bar{c}$ resonant states, which involve relatively small rates and are expected to lead to sizable CP asymmetries [27–29].

A. $B \to K\pi\pi$

We start this discussion with the decays $B \to K\pi\pi$ which are currently the most feasible ones among the three classes studied in this paper. Amplitude analyses of $B \to K\pi\pi$, for both charged and neutral B mesons, have been performed by the Belle and *BABAR* collaborations. Decays $B^+ \to K^+\pi^+\pi^-$ have been studied by both Belle [30] and *BABAR* [31]. An amplitude analysis was made by *BABAR* [32] for $B^0 \to K^+\pi^-\pi^0$ [33], and by Belle [34] for $B^0 \to K_S\pi^+\pi^-$. The first two processes are self-tagging whereas the third decay involves final state which is not flavor specific. These measurements have already provided some useful information which is relevant to our proposed study. We note that these studies have averaged over the above processes and their CP conjugates. The proposed study requires separate amplitude analyses for B and $\bar{B}$ decays.

The process $B^+ \to K^+\pi^+\pi^-$ gave information about the magnitudes of amplitudes for $B^+ \to K^{*0}(892)\pi^+$ and $B^+ \to K_0^{*0}(1430)\pi^+$ and their relative phase [30,31]. The statistical error in the measured relative phase is at a level of 10° which is encouraging. However, this three-body decay provides no information on a relative phase between two $B \to K^*(892)\pi$ amplitudes where pairs of K^* and π have different charges.

The decay $B^0 \to K^+\pi^-\pi^0$ is more interesting in our context, since it measures the magnitudes of $A(B^0 \to K^{*+}\pi^-)$ and $A(B^0 \to K^{*0}\pi^0)$, for both $K^*(892)$ and $K_0^*(1430)$, as well as the three relative phases among these amplitudes. Errors in the measured phases are at a level of 40° [32]. It would be useful to understand the origin of this large error in order to reduce it in future studies of this process, and to perform these measurements separately for B^0 and $\bar{B}^0$. A study of $B^0 \to K^+\pi^-\pi^0$ permits a measurement of the magnitude of $R_{3/2}$ but not its phase. Equation (18) implies that $|R_{3/2}| - 1$ is proportional to $\mathrm{Im}(r_{3/2})$ and vanishes if $r_{3/2}$ is real.

The study of $B^0 \to K_S\pi^+\pi^-$, which is not flavor specific, is more challenging. In order to measure the relative phase between $A(K^{*+}\pi^-)$ and $\bar{A}(K^{*-}\pi^+)$, as required by Eqs. (4) and (5), these amplitudes must interfere through $B^0 - \bar{B}^0$ mixing leading to a common $K_S\pi^+\pi^-$ state. Observing this interference in e^+e^- collisions at the $\Upsilon(4S)$ requires a time-dependent measurements using initially tagged B^0 or $\bar{B}^0$ mesons. The recent time-integrated analysis by Belle [34] assumed no direct CP asymmetry in $B^0 \to K^{*+}\pi^-$, summing over initial B^0 and $\bar{B}^0$. We note that, in fact, an untagged amplitude analysis does not have to make this assumption, permitting separate measurements for the magnitudes of $A(K^{*+}\pi^-)$ and $\bar{A}(K^{*-}\pi^+)$. However, measuring the relative phase between these amplitudes requires a time-dependent measurement.

A fourth process in this class, $B^+ \to K_S \pi^+ \pi^0$, which has not yet been measured, determines the magnitudes of the four amplitudes, $A(K^{*0}\pi^+)$, $A(K^{*+}\pi^0)$, $A(K_0^{*0}\pi^+)$, $A(K_0^{*+}\pi^0)$, and their relative phases. Finally, a very difficult mode which is not needed is $B^0 \to K_S \pi^0 \pi^0$, where measuring the phase difference between $A(K^{*0}\pi^0)$ and $\bar{A}(\bar{K}^{*0}\pi^0)$ would require time-dependence.

In order to apply Eq. (26), the linear constraint in the $(\bar{\rho}, \bar{\eta})$ plane, where $r_{3/2}$ is given in (30), it is sufficient to perform amplitude analyses for merely two processes involving neutral B decays, $B^0 \to K^+ \pi^- \pi^0$ and $B^0 \to K_S \pi^+ \pi^-$. Time-dependence in the second process is crucial. The first process measures the magnitudes of $A(K^{*+}\pi^-)$ and $A(K^{*0}\pi^0)$, their relative phase, and the corresponding CP-conjugate quantities, but not the phase difference between B^0 and $\bar{B}^0$ decays. [Here and below K^* denotes both $K^*(892)$ and $K_0^*(1430)$]. The second process measures the magnitude of $A(K^{*+}\pi^-)$ and its CP conjugate, and the relative phase between these two amplitudes. This set of measurements determining the complex ratio $R_{3/2}$ defined in Eqs. (2) and (4), is over-complete since $|A(K^{*+}\pi^-)|$ and its CP-conjugate are measured both in $B^0 \to K^+ \pi^- \pi^0$ and in $B^0 \to K_S \pi^+ \pi^-$.

Charged B decays, $B^\pm \to K^\pm \pi^\pm \pi^\mp$ and $B^\pm \to K_S \pi^\pm \pi^0$ provide further constraints on CKM parameters using the measurable ratio R_X (37) of $B \to K^* \pi$ and $\bar{B} \to K^* \pi$ decay amplitudes. A measurement of the phase Φ_X is expected to involve a larger experimental error than $\Phi_{3/2}$ since R_X depends on a larger number of amplitudes than $R_{3/2}$, including both neutral and charged B mesons. The measurement of R_X, together with the constraint from $R_{3/2}$, leads to a highly constraining set of measurements for $\bar{\rho}$ and $\bar{\eta}$. Since the four physical $B \to K^* \pi$ amplitudes are not mutually independent [see the quadrangle relation Eq. (2)], we propose studying $B \to K^* \pi$ amplitudes in terms of the isospin amplitudes $B_{1/2}$, $A_{3/2}$ and A_X, where X corresponds to the state defined in (33). In order to demonstrate the extent to which these CKM constraints are over-deterministic, thereby permitting a precise constraint on the point $(\bar{\rho}, \bar{\eta})$, we now count the number of parameters and observables.

We have a total of eight parameters, the magnitudes of $B_{1/2}$ and its CP conjugate $\bar{B}_{1/2}$, the magnitudes of $A_{3/2}$ and A_X, the three relative phases among these four amplitudes, and a CKM ratio $\bar{\eta}/(\bar{\rho} + C)$. [The CP conjugates $\bar{A}_{3/2}$ and $\bar{A}_X$ are not independent parameters and are given by Eqs. (4), (5), and (26).] These eight parameters can be used to fit 17 observables consisting of $|A(K^{*0}\pi^\pm|$ obtained from $B^+ \to K^+ \pi^+ \pi^-$, magnitudes of $A(K^{*0}\pi^\pm)$ and $A(K^{*\pm}\pi^0)$ and their relative phases obtained from $B^\pm \to K_S \pi^\pm \pi^0$, magnitudes of $A(K^{*+}\pi^-), A(K^{*0}\pi^0)$, their CP conjugates and their relative phases obtained from $B^0 \to K^\pm \pi^\mp \pi^0$, and magnitudes and relative phase for $A(B^0 \to K^{*+}\pi^-)$ and $A(\bar{B}^0 \to K^{*-}\pi^+)$ obtained from time-dependent $B^0 \to K_S \pi^+ \pi^-$. We have not included in this counting the decay $B^0 \to K_S \pi^0 \pi^0$ which is most challenging.

B. $B_s \to K\pi\pi$

The weak phase γ can be determined using Dalitz plot analyses for $B_s \to K^\pm \pi^\mp \pi^0$ and $B_s \to K_S \pi^\pm \pi^\mp$. These studies permit extracting the magnitudes $A_s(K^{*+}\pi^-)$, $A_s(K^{*0}\pi^0)$, their CP conjugates and relative phases between these amplitudes. This leads through Eqs. (7)–(9) to a measurement of the phase $\Phi_{3/2}^s$, which gives γ with high theoretical precision, as has been discussed in Sec. III B.

In contrast to the case of $B^0 \to K_S \pi^+ \pi^-$ produced at the $\Upsilon(4S)$, the above measurements can be performed with $B_s \to K_S \pi^+ \pi^-$ produced at hadron colliders without the need for flavor tagging and time-dependence. Because of the lack of quantum coherence between B_s and $\bar{B}_s$ produced in pairs, the charge-averaged time-integrated decay rate for decays into a common state $f \equiv K_S \pi^+ \pi^-$ involves an interference term proportional to the width difference $\Delta\Gamma_s$ in the B_s system, for which one expects $y_s \equiv \Delta\Gamma_s/2\Gamma_s = 0.12 \pm 0.05$ [35].

The untagged integrated decay distribution is given by

$$\frac{d^2\Gamma(B_s \to f)}{ds_{12}ds_{13}} + \frac{d^2\Gamma(\bar{B}_s \to f)}{ds_{12}ds_{13}} = \frac{1}{\Gamma(1-y_s^2)}\left[(|A|^2 + |\bar{A}|^2) - 2y_s \mathcal{R}e\left(\frac{q}{p}\bar{A}A^*\right)\right], \quad (45)$$

where $A \equiv A(B_s \to f)$, $\bar{A} \equiv A(\bar{B}_s \to f)$, $q/p \simeq 1$. Assuming that a reasonably accurate measurement for y_s will exist by the time an amplitude analysis will be performed for this decay, the relative phase between $A_s(K^{*+}\pi^-)$ and $\bar{A}_s(K^{*-}\pi^+)$ can be measured through the interference term involving y_s. Otherwise, a time-dependent measurements of this decay will be required.

In order to show that the above relative phase is measurable using untagged B_s, consider the contributions of $A_s(K^{*+}\pi^-)$ and $\bar{A}_s(K^{*-}\pi^+)$ to A and $\bar{A}$ in (45). Using the dependence of the Breit-Wigner functions $f_{K^{*+}}^{BW}$ and $f_{K^{*-}}^{BW}$ on s_{12} and s_{13}, the untagged decay distribution (45) provides four observables (the real part of the interference term provides two observables) which determine the magnitudes of $A_s(K^{*+}\pi^-)$ and $\bar{A}_s(K^{*-}\pi^+)$ and their relative phase. While in reality this relative phase may be affected by interference with other resonant or nonresonant terms in the amplitude, this proves that, once y_s is given, this phase can be measured through an untagged amplitude analysis of $B_s \to K_S \pi^+ \pi^-$.

C. $B_s \to K\bar{K}\pi$

As noted above, the CKM constraints following from amplitude analyses of $B_s \to K\bar{K}\pi$ decays are less precise than those following from studies of $B \to K\pi\pi$ and $B_s \to K\pi\pi$. This is due to theoretical errors in the hadronic electroweak penguin parameters, r_1^s and $r_1^{\prime s}$ [Eq. (42)], which are larger than in $r_{3/2}$ [Eq. (30)], r_X [Eq. (36)] and $r^s_{3/2}$ [see discussion in Sec. III B].

In order to obtain a CKM constraint related to the phase Φ^s_1, for instance, one must measure the amplitudes in (12), for $B_s \to K^{*+}K^-$ and $B_s \to K^{*0}\bar{K}^0$, their charge-conjugates, and the three relative phases between these amplitudes. This can be achieved by amplitude analyses for a pair of processes belonging to this class. For instance, using $B_s \to K^+K^-\pi^0$ one can measure the magnitudes of $A(B_s \to K^{*+}K^-)$, $A(B_s \to K^{*-}K^+)$, their charge-conjugates and the relative phases between these amplitudes. A study of $B_s \to K^+K_S\pi^-$ permits measurements of the magnitudes of $A(B_s \to K^{*0}\bar{K}^0)$, $A(B_s \to K^{*-}K^+)$, $A(\bar{B}_s \to K^{*0}\bar{K}^0)$, $A(\bar{B}_s \to K^{*-}K^+)$, and the respective relative phases. This information suffices for fixing Φ^s_1.

Decay distributions in $B_s \to K\bar{K}\pi$ involve twice as many relevant quasi two-body amplitudes as in $B \to K\pi\pi$ and $B_s \to K\pi\pi$, because B_s and $\bar{B}_s$ can decay to a common nonflavor $K^*\bar{K}$ state. The large number of amplitudes and relative phases which must be determined in $B_s \to K\bar{K}\pi$ requires at least as many observables. While in principle possible, this seems to pose a serious challenge to applying this method to $B_s \to K\bar{K}\pi$ decays.

V. CONCLUSION

We have studied in great detail a method proposed in Ref. [1,2] for obtaining new constraints on CKM parameters using $B_{(s)} \to (K^*\pi)_{I=3/2}$ amplitudes, extending the method to $B \to (K^*\pi)_{I=1/2}$ and to $B_s \to K^*\bar{K}(\bar{K}^*K)$ amplitudes measured in $B \to K\pi\pi$ and $B_s \to K\bar{K}\pi$, respectively. Two judiciously chosen isospin amplitudes in $B \to K^*\pi$ have been shown to be over-constrained by several $B \to K\pi\pi$ amplitude analyses, providing a precise linear constraint between the CKM parameters $\bar{\rho}$ and $\bar{\eta}$. The slope of the linear relation is a measurable quantity, while the intercept $\bar{\rho}_0$ where $\bar{\eta} = 0$ is a calculable quantity involving a theoretical error of 0.03. A study of $B_s \to K^*\pi$ amplitudes in $B_s \to K\pi\pi$ leads to a very accurate extraction of the weak phase γ with a theoretical uncertainty below 1°.

The resulting theoretical precision in determining CKM parameters in $B \to K\pi\pi$ and $B_s \to K\pi\pi$ has been shown to be essentially at the level of isospin breaking corrections since the method is based on isospin symmetry considerations, while flavor SU(3) has been used to estimate uncertainties from a subset of small EWP contributions. A larger hadronic uncertainty from EWP contributions is found in a CKM constraint obtained by studying $B \to K^*\bar{K}$ and $B \to \bar{K}^*K$ amplitudes contributing to $B_s \to K\bar{K}\pi$.

There is one crucial theoretical difference between applying this method to $\Delta S = 1$ $B \to K\pi\pi$ and $B_s \to K\bar{K}\pi$ and applying it to $\Delta S = 0$ $B_s \to K\pi\pi$. The first two classes of processes are dominated by $\Delta I = 0$ QCD penguin amplitudes which are eliminated in the relevant isospin amplitudes. In the standard model this implies a delicate cancellation between physical amplitudes defining the numerators and denominators of the $\Delta I = 1$ observables R_I and $R^s_1(R'^s_1)$ on which the method relies. In contrast, in $B_s \to K\pi\pi$ decays the method relies on measuring the $\Delta I = 3/2$ isospin amplitudes which involves dominant tree contributions. This would seem like a disadvantage of using $B \to K\pi\pi$ and $B_s \to K\bar{K}\pi$ relative to $B_s \to K\pi\pi$ for extracting CKM parameters. However, this cancellation in the standard model turns into an advantage when one is searching for new physics in $\Delta I = 1$ operators.

While applications of the method to B_s decays can be foreseen in future experiments at hadron colliders, data for $B \to K\pi\pi$ are already available from e^+e^- collisions at the $\Upsilon(4S)$, and should be analyzed in the manner proposed here. Amplitude analyses of a few processes in the class $B \to K\pi\pi$ have already been performed, measuring amplitudes and relative phases for $B \to K^*(892)\pi$ and $B \to K^*_0(1430)\pi$ [30–32,34]. Since the method is based on $\Delta I = 1$ amplitudes, a first important step toward its full implementation is observing a violation of $\Delta I = 0$ QCD penguin dominance in these quasi two-body decays.

This question has been studied recently [36]. It was shown that in all cases but one $\Delta I = 0$ holds well within current experimental errors. For instance, $\Delta I = 0$ dominance implies

$$2\mathcal{B}(B^0 \to K^{*0}_0\pi^0) = \mathcal{B}(B^0 \to K^{*+}_0\pi^-), \qquad (46)$$

which holds experimentally within large errors, in units of 10^{-6} [22],

$$51.0 \pm 19.8 = 46.6^{+5.6}_{-6.6}. \qquad (47)$$

The exceptional case where $\Delta I = 0$ seems to be violated is the equality,

$$2\mathcal{B}(B^0 \to K^{*0}\pi^0) = \mathcal{B}(B^0 \to K^{*+}\pi^-), \qquad (48)$$

where current experimental values [22]

$$3.4 \pm 1.6 = 9.8 \pm 1.1, \qquad (49)$$

show a discrepancy of 3.3σ. One would have to watch carefully whether this discrepancy holds in future measurements.

This method requires performing amplitude analyses of $B \to K\pi\pi$ separately for B and $\bar{B}$, as one must measure the ratio of $\bar{B} \to \bar{K}^*\pi$ and $B \to K^*\pi$ amplitudes. The method does not require observing a direct CP asymmetry in Dalitz plots for $B \to K\pi\pi$ or an asymmetry in $B \to K^*\pi$ decay rates. We recall that no time-dependence is needed in order to observe direct CP violation in the Dalitz plot of $B^0 \to K_S\pi^+\pi^-$ through an asymmetry with respect to exchanging π^+ and π^- [37]. We have stressed the importance of performing a time-dependent Dalitz plot analysis of $B^0 \to K_S\pi^+\pi^-$, which is required in order to measure separately amplitudes for $B^0 \to K^{*+}\pi^-$ and $\bar{B}^0 \to K^{*-}\pi^+$ and their relative phase.

The method presented here for obtaining a linear relation between $\bar{\rho}$ and $\bar{\eta}$ in $B \to K\pi\pi$ may be compared with a study of γ in $B \to DK$ [38]. The latter method involves an extremely small theoretical uncertainty from $D^0 - \bar{D}^0$ mixing [39] when studying CP-eigenstates and flavor states in D decays. Applying this method to non-CP and nonflavor three-body D decays such as $D^0 \to K_S\pi^+\pi^-$ introduces a theoretical error in γ caused by modeling the three-body decay amplitude in terms of a sum of resonant and nonresonant contributions. Model-dependence in amplitude analyses for $B \to K\pi\pi$ is expected to be larger than in $D^0 \to K_S\pi^+\pi^-$ because the former processes involve lower statistics and higher combinatorial backgrounds. Fortunately, the uncertainty of modeling $B \to K\pi\pi$ is mainly in nonresonant amplitudes [30–32,34], which spread over the entire phase space, but less in $K^*\pi$ amplitudes which are used in the proposed study.

While measuring γ from an interference of tree amplitudes in $B \to DK$ is most likely to receive only small corrections from new physics [39,40], the extraction of a linear constraint between $\bar{\rho}$ and $\bar{\eta}$ in $B \to K\pi\pi$ may be affected more significantly by such effects. Thus, values for CKM parameters obtained in the two methods may differ, indicating short distance $b \to s\bar{q}q$ operators beyond the standard model. The study of $B \to K\pi\pi$ is insensitive to new $\Delta I = 0$ QCD penguinlike operators which cancel in the ratios R_I, but is affected by new $\Delta I = 1$ operators. Such operators are often referred to in the literature as anomalous electroweak (or Trojan) penguin operators [41]. The sensitivity to such contributions is high because in the standard model $\Delta I = 1$ terms in $B \to K^*\pi$ are suppressed relative to $\Delta I = 0$ contributions. Other tests for such $\Delta I = 1$ operators have been proposed in terms of isospin sum rules for rates [42] and CP asymmetries in $B \to K\pi$ [43].

A somewhat similar situation occurs in B_s decays when comparing the theoretically precise measurement of γ in charmless $B_s \to K\pi\pi$ discussed here with the potentially accurate measurement of this phase in $B_s \to D_s^- K^+$ [44]. Both methods require $B_s - \bar{B}_s$ mixing, but no time-dependent measurement is required in $B_s \to K\pi\pi$ due to additional phase information coming from Dalitz plot interferences. In $B_s \to K\pi\pi$ the measurement of γ follows from studying $\Delta I = 3/2$ $\bar{b} \to \bar{u}u\bar{d}$ tree amplitudes, while in $B_s \to D_s^- K^+$ the phase occurs in the interference of $\Delta I = 1/2$ $\bar{b} \to \bar{c}u\bar{s}$ and $\bar{b} \to \bar{u}c\bar{s}$ tree amplitudes. Whereas new physics operators in the latter case are possible in principle, their effects on the determination of γ are less common and are expected to be much smaller than the effects of potentially new $\Delta I = 3/2$ operators contributing in $B_s \to K\pi\pi$. Such $\Delta S = 0$ operators are usually expected in the same class of models where anomalous $\Delta S = 1$ electroweak penguin operators occur.

ACKNOWLEDGMENTS

M. G. is grateful to the SLAC Theory Group for its kind hospitality. We wish to thank Denis Dujmic and Roy Briere for helpful discussions. This work was supported in part by the United States Department of Energy through Grants No. DOE-ER-40682-143, DEAC02-6CH03000, and DE-AC02-98CH10886, by the Israel Science Foundation under Grant No. 1052/04 and by the German-Israeli Foundation under Grant No. I-781-55.14/2003. Research supported in part by the US Department of Energy, Contract No. DE-AC02-76SF00515.

[1] M. Ciuchini, M. Pierini, and L. Silvestrini, Phys. Rev. D **74**, 051301(R) (2006).
[2] M. Ciuchini, M. Pierini, and L. Silvestrini, hep-ph/0602207.
[3] Y. Nir and H. R. Quinn, Phys. Rev. Lett. **67**, 541 (1991); H. J. Lipkin, Y. Nir, H. R. Quinn, and A. Snyder, Phys. Rev. D **44**, 1454 (1991); M. Gronau, Phys. Lett. B **265**, 389 (1991).
[4] M. Gronau, J. L. Rosner, and D. London, Phys. Rev. Lett. **73**, 21 (1994); M. Gronau and J. L. Rosner, Phys. Rev. Lett. **76**, 1200 (1996); Phys. Rev. D **53**, 2516 (1996).
[5] R. Fleischer and T. Mannel, Phys. Rev. D **57**, 2752 (1998); M. Gronau and J. L. Rosner, Phys. Rev. D **57**, 6843 (1998); A. J. Buras and R. Fleischer, Eur. Phys. J. C **11**, 93 (1999).
[6] M. Neubert and J. L. Rosner, Phys. Lett. B **441**, 403 (1998); Phys. Rev. Lett.**81**, 5076 (1998); M. Neubert, J. High Energy Phys. 02 (1999) 014.
[7] M. Gronau and J. L. Rosner, Phys. Lett. B **482**, 71 (2000).
[8] A. E. Snyder and H. R. Quinn, Phys. Rev. D **48**, 2139 (1993).
[9] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).
[10] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Phys. Rev. D **50**, 4529 (1994).
[11] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).
[12] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
[13] J. Charles *et al.* (CKMfitter Group), Eur. Phys. J. C **41**, 1 (2005); updated in www.slac.stanford.edu/xorg/ckmfitter.
[14] M. Gronau, D. Pirjol, and T. M. Yan, Phys. Rev. D **60**, 034021 (1999); **69**, 119901(E) (2004).
[15] M. Gronau, Phys. Rev. Lett. **91**, 139101 (2003).
[16] M. Gronau, Phys. Rev. D **62**, 014031 (2000).
[17] G. Paz, hep-ph/0206312; Y. Grossman, Z. Ligeti, Y. Nir, and H. Quinn, Phys. Rev. D **68**, 015004 (2003).

[18] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. **B591**, 313 (2000); **B606**, 245 (2001).

[19] M. Beneke and M. Neubert, Nucl. Phys. **B675**, 333 (2003).

[20] C. W. Bauer, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D **70**, 054015 (2004); hep-ph/0510241; C. W. Bauer and D. Pirjol, Phys. Lett. B **604**, 183 (2004); A. R. Williamson and J. Zupan, Phys. Rev. D **74**, 014003 (2006).

[21] M. Gronau and J. Zupan, Phys. Rev. D **70**, 074031 (2004).

[22] E. Barberio *et al.* (Heavy Flavor Averaging Group), hep-ex/0603003; updated in www.slac.stanford.edu/xorg/hfag.

[23] C. C. Wang *et al.* (Belle Collaboration) Phys. Rev. Lett. **94**, 121801 (2005).

[24] B. Aubert (*BABAR* Collaboration), hep-ex/0608002.

[25] H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D **71**, 014030 (2005).

[26] B. Aubert *et al.* (*BABAR* Collaboration), hep-ex/0408099.

[27] G. Eilam, M. Gronau, and R. R. Mendel, Phys. Rev. Lett. **74**, 4984 (1995); N. G. Deshpande, G. Eilam, X. G. He, and J. Trampetic, Phys. Rev. D **52**, 5354 (1995).

[28] I. Bediaga, R. E. Blanco, C. Gobel, and R. Mendez-Galain, Phys. Rev. Lett. **81**, 4067 (1998).

[29] S. Fajfer, R. J. Oakes, and T. N. Pham, Phys. Lett. B **539**, 67 (2002); S. Fajfer, T. N. Pham, and A. Prapotnik, Phys. Rev. D **70**, 034033 (2004).

[30] A. Garmash *et al.* (BELLE Collaboration), Phys. Rev. D **71**, 092003 (2005).

[31] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **72**, 072003 (2005).

[32] B. Aubert *et al.* (*BABAR* Collaboration), hep-ex/0408073.

[33] See also P. Chang *et al.* (Belle Collaboration), Phys. Lett. B **599**, 148 (2004).

[34] K. Abe *et al.*, hep-ex/0509047.

[35] A. Lenz, hep-ph/0412007; in the Proceedings of Flavor Physics and *CP* Violation (FPCP 2004), Daegu, Korea, 2004 (unpublished).

[36] M. Gronau and J. L. Rosner, Phys. Rev. D **72**, 094031 (2005).

[37] G. Burdman and J. F. Donoghue, Phys. Rev. D **45**, 187 (1992); S. Gardner, Phys. Lett. B **553**, 261 (2003); S. Gardner and J. Tandean, Phys. Rev. D **69**, 034011 (2004).

[38] M. Gronau and D. Wyler, Phys. Lett. B **265**, 172 (1991); M. Gronau and D. London, Phys. Lett. B **253**, 483 (1991); D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997); M. Gronau, Phys. Rev. D **58**, 037301 (1998); D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. D **63**, 036005 (2001); A. Giri, Y. Grossman, A. Soffer, and J. Zupan, Phys. Rev. D **68**, 054018 (2003); D. Atwood and A. Soni, Phys. Rev. D **71**, 013007 (2005).

[39] Y. Grossman, A. Soffer, and J. Zupan, Phys. Rev. D **72**, 031501 (2005).

[40] S. Khalil, hep-ph/0604118; G. H. Wu and A. Soni, Phys. Rev. D **62**, 056005 (2000).

[41] Y. Grossman, M. Neubert, and A. L. Kagan, J. High Energy Phys. 10 (1999) 029; T. Yoshikawa, Phys. Rev. D **68**, 054023 (2003); M. Gronau and J. L. Rosner, Phys. Lett. B **572**, 43 (2003); A. J. Buras, R. Fleischer, S. Recksiegel, and F. Schwab, Eur. Phys. J. C **32**, 45 (2003).

[42] M. Gronau and J. L. Rosner, Phys. Rev. D **59**, 113002 (1999); H. J. Lipkin, Phys. Lett. B **445**, 403 (1999).

[43] D. Atwood and A. Soni, Phys. Rev. D **58**, 036005 (1998); M. Gronau and J. L. Rosner, Phys. Rev. D **71**, 074019 (2005); M. Gronau, Phys. Lett. B **627**, 82 (2005).

[44] R. Aleksan, I. Dunietz, and B. Kayser, Z. Phys. C **54**, 653 (1992).