$\mu - \tau$ symmetry and radiatively generated leptogenesis

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We consider a $\mu - \tau$ symmetry in neutrino sectors realized at the GUT scale in the context of a seesaw model. In our scenario, the exact $\mu - \tau$ symmetry realized in the basis where the charged lepton and heavy Majorana neutrino mass matrices are diagonal leads to vanishing lepton asymmetries. We find that, in the minimal supersymmetric extension of the seesaw model with large $\tan\beta$, the renormalization group (RG) evolution from the GUT scale to seesaw scale can induce a successful leptogenesis even without introducing any symmetry breaking terms by hand, whereas such RG effects lead to tiny deviations of θ_{23} and θ_{13} from $\pi/4$ and zero, respectively. It is shown that the right amount of the baryon asymmetry η_B can be achieved via so-called resonant leptogenesis, which can be realized at rather low seesaw scale with large $\tan\beta$ in our scenario so that the well-known gravitino problem is safely avoided.

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I. INTRODUCTION

Recent precise neutrino experiments appear to show robust evidence for the neutrino oscillation. The present neutrino experimental data [1–3] exhibit that the atmospheric neutrino deficit points toward a maximal mixing between the tau and muon neutrinos. However, the solar neutrino deficit favors a not-so-maximal mixing between the electron and muon neutrinos. In addition, although we do not have yet any firm evidence for the neutrino oscillation arisen from the 1st and 3rd generation flavor mixing, there is a bound on the mixing element U_{e3} from CHOOZ reactor experiment, $|U_{e3}| < 0.2$ [4]. Although neutrinos have gradually revealed their properties in various experiments since the historic Super-Kamiokande confirmation of neutrino oscillations [1], properties related to the leptonic *CP* violation are completely unknown so far. To understand in detail the neutrino mixings observed in various oscillation experiments is one of the most interesting issues in particle physics. The large values of $\theta_{\rm sol}$ and $\theta_{\rm atm}$ may be telling us about some underlying new symmetries of leptons which are not present in the quark sector, and may provide a clue to understanding the nature of quark-lepton complementarity beyond the standard model.

Recently, there have been some attempts to explain the maximal mixing of the atmospheric neutrinos and very tiny value of the 3rd mixing element U_{e3} by introducing some approximate discrete symmetries [5,6] or the mass splitting among the heavy Majorana neutrinos in the seesaw framework [7]. In the basis where charged leptons are mass eigenstates, the $\mu - \tau$ interchange symmetry has become useful in understanding the maximal atmospheric neutrino mixing and the smallness of U_{e3} [8–12]. The mass difference between the muon and the tau leptons, of course, breaks this symmetry in such a basis. So we expect this symmetry to be an approximate one, and thus it must hold only for the neutrino sector. To generate nonvanishing but tiny mixing element U_{e3}, in the literatures [11] the authors introduced $\mu - \tau$ symmetry breaking terms in leptonic mass matrices by hand at tree level. We have also proposed a scheme for breaking of $\mu - \tau$ symmetry through an appropriate *CP* phase in neutrino Dirac-Yukawa matrix so as to achieve both nonvanishing U_{e3} and successful leptogenesis [12]. In our scheme, $\mu - \tau$ symmetry breaking factor associated with the *CP* phase is essential to achieve both nonvanishing U_{e3} and leptogenesis. However, besides the soft breaking terms, the $\mu - \tau$ symmetry is still approximate one in the sense that its breaking effects in the lepton sector can arise via the radiative corrections generated by the charged lepton Yukawa couplings which are not subject to the $\mu - \tau$ symmetry.

In this work we propose that the precise $\mu - \tau$ symmetry, imposed in Ref. [12], exists only at high energy scale such as the GUT scale and a renormalization group (RG) evolution from high scale to low scale gives rise to the breaking of $\mu - \tau$ symmetry in the lepton sector without introducing any *ad hoc* soft symmetry breaking terms. However, it turns out that the RG effects in the standard model (SM) and even its minimal supersymmetric extension are quite meager such that the size of U_{e3} and the deviation of θ_{23} from the maximal mixing are tiny.[1] In this paper, however, we shall show that such small RG effects in supersymmetric seesaw model can lead to successful leptogenesis which is absent in the exact $\mu - \tau$ symmetry, whereas lepton asymmetry generated in the context of the SM is too small to achieve successful leptogenesis. We note that the leptogenesis realized in our scheme is, in

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[1]This is so mainly because our scheme reflects normal hierarchical light neutrino mass spectrum.

fact, a kind of radiatively induced leptogenesis which has been discussed in Refs. [13,14]. As will be shown later, in our scheme both real and imaginary parts of the combination of neutrino Dirac-Yukawa matrix $(Y_\nu Y_\nu^\dagger)_{jk}$, which are needed for leptogenesis, are zero in the limit of the exact $\mu - \tau$ symmetry at tree level. We note that each of them is generated via RG effects proportional to $\tan^2\beta$ at low energy. Thus, the lepton asymmetry generated in our scheme is proportional to $\tan^4\beta$ and it can be enhanced by taking large value of $\tan\beta$. This observation is different from the results in Refs. [13,14], in which only real part of $(Y_\nu Y_\nu^\dagger)_{jk}$ is radiatively generated and thus lepton asymmetry is proportional to $\tan^2\beta$.

This paper is organized as follows. In Sec. II, we present a supersymmetric seesaw model reflecting $\mu - \tau$ symmetry at a high energy scale such as the GUT scale. The discussion for RG evolution from high scale such as the GUT scale to low scale is given in Sec. III. In Sec. IV, we show how successful leptgenesis can be radiatively induced in our scheme. Numerical results and conclusion are given in Sec. V.

II. SUPERSYMMETRIC SEESAW MODEL WITH $\mu - \tau$ SYMMETRY REALIZED AT THE GUT SCALE

Let us begin by considering a supersymmetric version of the seesaw model, which is given as the following leptonic superpotential:

$$W_{\rm lepton} = \hat{l}_L^c \mathbf{Y}_l \hat{L} \cdot \hat{H}_1 + \hat{N}_L^c \mathbf{Y}_\nu \hat{L} \cdot \hat{H}_2 - \tfrac{1}{2}\hat{N}_L^{cT}\mathbf{M}_R \hat{N}_L^c, \quad (1)$$

where the family indices have been omitted and $\hat{L}_j$, $j = e, \mu, \tau \equiv 1, 2, 3$ stand for the chiral supermultiplets of the $SU(2)_L$ doublet lepton fields, $\hat{H}_{1,2}$ are the Higgs doublet fields with hypercharge $\mp 1/2$, $\hat{N}_{jL}^c$ and $\hat{l}_{jL}^c$ are the supermultiplet of the $SU(2)_L$ singlet neutrino and charged lepton field, respectively. In the above superpotential, $\mathbf{M}_R$ is the heavy Majorana neutrino mass matrix, and $\mathbf{Y}_l$ and $\mathbf{Y}_\nu$ are the 3×3 charged lepton and neutrino Dirac-Yukawa matrices, respectively. After spontaneous symmetry breaking, the seesaw mechanism leads to the following effective light neutrino mass term:

$$m_{\rm eff} = -\mathbf{Y}_\nu^T \mathbf{M}_R^{-1} \mathbf{Y}_\nu v_2^2, \quad (2)$$

where v_2 is the vacuum expectation value of the Higgs field with positive hypercharge and denoted as $v_2 = v\sin\beta$ with $v \approx 174$ GeV.

Let us impose the $\mu - \tau$ symmetry for the neutrino sectors in the basis in which both the charged lepton mass and heavy Majorana mass matrices are diagonal, and then the neutrino Dirac-Yukawa matrix and the heavy Majorana neutrino mass matrix are given as

$$\mathbf{Y}_\nu = \begin{pmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} & \mathbf{y}_{12} \\ \mathbf{y}_{12} & \mathbf{y}_{22} & \mathbf{y}_{23} \\ \mathbf{y}_{12} & \mathbf{y}_{23} & \mathbf{y}_{22} \end{pmatrix}, \quad \mathbf{M}_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{pmatrix}, \quad (3)$$

where the elements $\mathbf{y}_{ij}$ of the neutrino Dirac-Yukawa matrix are all complex in general. As is shown in Ref. [12], the $\mu - \tau$ symmetry imposed as above is responsible for the neutrino mixing pattern with $\theta_{23} = 45°$ and $\theta_{13} = 0°$ after seesawing. Here, we assume that the above matrices Eq. (3) reflecting the $\mu - \tau$ symmetry are realized at the GUT scale, $Q_{\rm GUT} = 2 \times 10^{16}$ GeV. As is also shown in [12], the seesaw model based on Eq. (3) leads to only the normal hierarchical light neutrino mass spectrum because we take diagonal form of heavy Majorana neutrino mass matrix [12,15]. Thus, the RG effects on the neutrino mixing matrix $U_{\rm PMNS}$ are expected to be very small even in the supersymmetric case. However, as will be shown later, such small RG effects can trigger leptogenesis which is absent in the case of the exact $\mu - \tau$ symmetry. With those exact $\mu - \tau$ symmetric structures in the neutrino sectors, we shall show that a successful leptogenesis could be achieved solely through the RG running effects between the GUT and the seesaw scales without being in conflict with experimental low energy constraints.

III. RELEVANT RGE'S IN MSSM

In the minimal supersymmetric standard model (MSSM), the radiative behavior of the heavy Majorana neutrinos mass matrix $\mathbf{M}_R$ is dictated by the following RG equation [16,17]:

$$\frac{d}{dt}\mathbf{M}_R = 2[(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger)\mathbf{M}_R + \mathbf{M}_R(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger)^T], \quad (4)$$

where $t = \frac{1}{16\pi^2}\ln(Q/Q_{\rm GUT})$ with an arbitrary renormalization scale Q. The RG equation for the neutrino Dirac-Yukawa matrix can be written as

$$\frac{d\mathbf{Y}_\nu}{dt} = \mathbf{Y}_\nu\Big[\Big(T - 3g_2^2 - \frac{3}{5}g_1^2\Big) + (Y_l^\dagger Y_l + 3\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)\Big], \quad (5)$$

where $T = {\rm Tr}(3Y_u^\dagger Y_u + \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)$, and g_2, g_1 are the $SU(2)_L$ and $U(1)_Y$ gauge coupling constants, respectively.

For our convenience, let us reformulate the RG equation, Eq. (4), in the basis where $\mathbf{M}_R$ is diagonal. Since $\mathbf{M}_R$ is symmetric, it can be diagonalized with a unitary matrix V,

$$V^T \mathbf{M}_R V = {\rm diag}(M_1, M_2, M_3). \quad (6)$$

Note that as the structure of mass matrix $\mathbf{M}_R$ changes with the evolution of the scale, that of the unitary matrix V depends on the scale, too. The RG evolution of the unitary matrix $V(t)$ can be written as

$$\frac{d}{dt}V = VA, \tag{7}$$

where matrix A is anti-Hermitian, $A^\dagger = -A$, due to the unitarity of V. Then, differentiating Eq. (6), we obtain

$$\frac{dM_i\delta_{ij}}{dt} = A^T_{ij}M_j + M_iA_{ij} + 2\{V^T[(\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)\mathbf{M}_R + \mathbf{M}_R(\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)^T]V\}_{ij}. \tag{8}$$

It immediately follows from the anti-Hermiticity of A that $A_{ii} = 0$ in Eq. (8). Absorbing the unitary transformation into the neutrino Dirac-Yukawa coupling

$$Y_\nu \equiv V^T\mathbf{Y}_\nu, \tag{9}$$

the real diagonal part of Eq. (8) becomes

$$\frac{dM_i}{dt} = 4M_i(Y_\nu Y_\nu^\dagger)_{ii}. \tag{10}$$

On the other hand, the off-diagonal part of Eq. (8) leads to

$$A_{jk} = 2\frac{M_k + M_j}{M_k - M_j}\,\mathrm{Re}[(Y_\nu Y_\nu^\dagger)_{jk}] + 2i\frac{M_j - M_k}{M_j + M_k}\,\mathrm{Im}[(Y_\nu Y_\nu^\dagger)_{jk}], \qquad (j \neq k). \tag{11}$$

Note that the real part of A_{jk} is singular for the degenerate cases with $M_j = M_k$, and the RG equation for Y_ν in $\mathbf{M}_R$ diagonal basis is written as

$$\frac{dY_\nu}{dt} = Y_\nu\left[\left(T - 3g_2^2 - \frac{3}{5}g_1^2\right) + (Y_l^\dagger Y_l + 3Y_\nu^\dagger Y_\nu)\right] + A^TY_\nu. \tag{12}$$

The singularity in $\mathrm{Re}[A_{jk}]$ can be eliminated with the help of an appropriate rotation between degenerate heavy Majorana neutrino states. Such a rotation does not change any physics and it is equivalent to absorbing the rotation matrix R into the neutrino Dirac-Yukawa matrix Y_ν,

$$Y_\nu \rightarrow \tilde{Y}_\nu = RY_\nu, \tag{13}$$

where the matrix R, particularly rotating 2nd and 3rd generations of heavy Majorana neutrinos, can be parameterized as

$$R(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos x & \sin x \\ 0 & -\sin x & \cos x \end{pmatrix}. \tag{14}$$

Then, the singularity in the real part of A_{jk} is indeed removed when the rotation angle x is taken to fulfill the condition

$$\mathrm{Re}[(\tilde{Y}_\nu\tilde{Y}_\nu^\dagger)_{jk}] = 0, \quad \text{for any pair } j, k \text{ corresponding to } M_j = M_k. \tag{15}$$

For our purpose, let us parameterize $\mathbf{Y}_\nu$ at the GUT scale as follows:

$$Y_\nu = d\begin{pmatrix} \rho e^{i\varphi_{11}} & \omega e^{i\varphi_{12}} & \omega e^{i\varphi_{12}} \\ \omega e^{i\varphi_{12}} & \kappa e^{i\varphi_{22}} & e^{i\varphi_{23}} \\ \omega e^{i\varphi_{12}} & e^{i\varphi_{23}} & \kappa e^{i\varphi_{22}} \end{pmatrix}, \tag{16}$$

where φ_{ij} denote CP phases in Y_ν, and define the following useful Hermitian parameter

$$H \equiv (Y_\nu Y_\nu^\dagger) = d^2\begin{pmatrix} H_{11} & H_{12} & H_{12} \\ H_{12}^* & H_{22} & H_{23} \\ H_{12}^* & H_{23} & H_{22} \end{pmatrix}, \tag{17}$$

where

$$\begin{aligned} H_{11} &= \rho^2 + 2\omega^2, \\ H_{12} &= \rho\omega e^{i(\varphi_{11}-\varphi_{12})} + \omega\kappa e^{i(\varphi_{12}-\varphi_{22})} + \omega e^{i(\varphi_{12}-\varphi_{23})}, \\ H_{22} &= \omega^2 + \kappa^2 + 1, \\ H_{23} &= \omega^2 + 2\kappa\cos(\varphi_{22} - \varphi_{23}). \end{aligned} \tag{18}$$

As shown in Ref. [12], the Hermitian parameter H in the limit of the exact $\mu - \tau$ symmetry leads to vanishing lepton asymmetry which is disastrous for successful leptogenesis. To generate nonvanishing lepton asymmetry, we need to break the exact degeneracy of the masses of 2nd and 3rd heavy Majorana neutrinos and the $\mu - \tau$ symmetric texture of Y_ν proposed in Eq. (16). In our scenario, as will be shown later, only the RG evolution, without including any *ad hoc* soft breaking terms, is responsible for such a breaking required for successful leptogenesis.

For our $\mu - \tau$ symmetric Y_ν given in Eq. (16), the angle satisfying the condition Eq. (15) is $x = \pm\pi/4$. Without a loss of generality, taking $x = \pi/4$, we obtain

$$\tilde{H} \equiv (\tilde{Y}_\nu\tilde{Y}_\nu^\dagger) = RHR^T = \begin{pmatrix} H_{11} & \sqrt{2}H_{12} & 0 \\ \sqrt{2}H_{12}^* & H_{23} + H_{22} & 0 \\ 0 & 0 & -H_{23} + H_{22} \end{pmatrix}. \tag{19}$$

It is obvious from Eq. (19) that $\mathrm{Re}[\tilde{H}_{23(32)}] = 0$ and thus the singularity in $A_{23(32)}$ does not appear. We also note that $\mathrm{Im}[\tilde{H}_{23(32)}] = 0$, which is due to the $\mu - \tau$ symmetric structure of $\tilde{H}$.[2] As will be shown later, since the CP asymmetry required for leptogenesis is proportional to $\mathrm{Im}[\tilde{H}_{23}^2] = 2\,\mathrm{Re}[\tilde{H}_{23}]\,\mathrm{Im}[\tilde{H}_{23}]$, both real and imaginary parts of $\tilde{H}_{23}$ should be nonzero for successful leptogenesis. In this work, we shall show that nonvanishing values of them can be generated through the RG evolution.

Now, let us consider RG effects which may play an important role in successful leptogenesis. First, the parameter $\delta_N = 1 - M_3/M_2$ reflecting the mass splitting of the degenerate heavy Majorana neutrinos is governed by the following RGE which can be derived from Eq. (10),

[2]In Ref. [13,14], the authors considered the radiatively induced leptogenesis based on arbitrary textures of neutrino Dirac-Yukawa matrix for which $\mathrm{Im}[\tilde{H}_{23}]$ needs not to be zero in general.

$$\frac{d\delta_N}{dt} = 4(1-\delta_N)[\tilde{H}_{22} - \tilde{H}_{33}] \simeq 8\,\mathrm{Re}[H_{23}]. \quad (20)$$

The solution of the RGE (20) is approximately given by

$$\delta_N = 8d^2\{\omega^2 + 2\kappa\cos(\varphi_{22} - \varphi_{23})\}\cdot t, \quad (21)$$

where we used Eq. (18). Note that the radiative splitting of degenerate heavy Majorana neutrinos masses depends particularly on the phase difference, $\varphi_{22} - \varphi_{23}$.

RGE of the parameter $\tilde{H}$ is written as

$$\frac{d\tilde{H}}{dt} = 2\bigg[\bigg(T - 3g_2^2 - \frac{3}{5}g_1^2\bigg)\tilde{H} + \tilde{Y}_\nu(Y_l^\dagger Y_l)\tilde{Y}_\nu^\dagger + 3\tilde{H}^2\bigg] + A^T\tilde{H} + \tilde{H}A^*. \quad (22)$$

Considering the structure of $\tilde{H}$ in Eq. (19), up to nonzero leading contributions in the right side of Eq. (22), RGE of $\tilde{H}_{23}$ is given by

$$\frac{d\,\mathrm{Re}[\tilde{H}_{23}]}{dt} \simeq y_\tau^2\,\mathrm{Re}[(\tilde{Y}_{\nu 23}\tilde{Y}^*_{\nu 33})],$$
$$\frac{d\mathrm{Im}[\tilde{H}_{23}]}{dt} \simeq 2\,\mathrm{Im}[(\tilde{Y}_\nu Y_l^\dagger Y_l \tilde{Y}_\nu^\dagger)_{23}] \simeq 2y_\tau^2\,\mathrm{Im}[(\tilde{Y}_{\nu 23}\tilde{Y}^*_{\nu 33})]. \quad (23)$$

In terms of the parameters in Eq. (18), the radiatively generated $\tilde{H}_{23}$ is given approximately by

$$\mathrm{Re}[\tilde{H}_{23}] \simeq y_\tau^2 d^2\frac{\kappa^2 - 1}{2}\cdot t,$$
$$\mathrm{Im}[\tilde{H}_{23}] \simeq 2y_\tau^2 d^2\kappa\sin(\varphi_{23} - \varphi_{22})\cdot t. \quad (24)$$

Interestingly enough, the radiatively generated $\mathrm{Im}[\tilde{H}_{23}]$ is proportional to $\sin(\varphi_{23} - \varphi_{22})$.

IV. RADIATIVELY INDUCED RESONANT LEPTOGENESIS

When two lightest heavy Majorana neutrinos are nearly degenerate, the *CP* asymmetry through their decays gets dominant contributions from self-energy diagrams and can be written as [18–21]

$$\varepsilon_i = \frac{\Gamma(N_i \to l\varphi) - \Gamma(N_i \to \bar{l}\varphi^\dagger)}{\Gamma(N_i \to l\varphi) + \Gamma(N_i \to \bar{l}\varphi^\dagger)} \simeq \sum_{k\neq i}\frac{\mathrm{Im}[(Y_\nu Y_\nu^\dagger)^2_{ik}]}{16\pi(Y_\nu Y_\nu^\dagger)_{ii}\delta_{N,ik}}\bigg(1 + \frac{\Gamma_k^2}{4M_i^2\delta^2_{N,ik}}\bigg)^{-1}, \quad (25)$$

where Γ_k is the tree-level decay width of the k-th right-handed neutrino,

$$\Gamma_k = \frac{(Y_\nu Y_\nu^\dagger)_{kk}M_k}{8\pi}, \quad (26)$$

and $\delta_{N,ik}$ is a parameter which denotes the degree of the mass splitting between two degenerate heavy Majorana neutrinos,

$$\delta_{N,ik} \equiv 1 - \frac{M_k}{M_i}. \quad (27)$$

As shown in Ref. [12], the neutrino Dirac-Yukawa matrix and the heavy Majorana neutrino mass matrix given in the forms of Eq. (3) are consistent with neutrino oscillation data only when $M_1 \gg M_2 \simeq M_3$. Here we note that it is rather difficult to realize naturally such an inverted hierarchy of the heavy Majorana neutrino mass spectrum in GUT models. For the mass hierarchy $M_1 \gg M_2 \simeq M_3$, the decay of N_1 takes place in thermal equilibrium and thus the lepton asymmetry required for successful leptogenesis will be accomplished by $\varepsilon_{2(3)}$ given as follows:

$$\varepsilon_{2(3)} = \frac{\mathrm{Im}[(Y_\nu Y_\nu^\dagger)^2_{23}]}{16\pi(Y_\nu Y_\nu^\dagger)_{22(33)}\delta_N}\bigg(1 + \frac{\Gamma^2_{3(2)}}{4M^2_{2(3)}\delta_N^2}\bigg)^{-1}, \quad (28)$$

where $\delta_N = \delta_{N,23}$. From Eqs. (19), (21), and (24), the lepton asymmetry is given by

$$\varepsilon_{2(3)} \simeq \frac{\mathrm{Im}[(\tilde{H}_{23})^2]}{16\pi\tilde{H}_{22(33)}\delta_N} \simeq \frac{y_\tau^4\kappa(\kappa^2 - 1)\sin(\Delta\varphi)\cdot t}{64\pi\{\omega^2 + 2\kappa\cos(\Delta\varphi)\}\cdot h_{2(3)}}, \quad (29)$$

where $\Delta\varphi \equiv \varphi_{23} - \varphi_{22}$, and two parameters $h_{2(3)}$ are defined as

$$h_2 = \tilde{H}_{22}/d^2 = 1 + \kappa^2 + 2\omega^2 + 2\kappa\cos(\Delta\varphi),$$
$$h_3 = \tilde{H}_{33}/d^2 = 1 + \kappa^2 - 2\kappa\cos(\Delta\varphi). \quad (30)$$

In Eq. (29) we neglected the term containing the decay width since it turns out to be very small in our scenario. Note that due to the opposite sign of the term $2\kappa\cos(\Delta\varphi)$ in h_2 and h_3, either h_2 or h_3 becomes larger depending on the sign of $\cos(\Delta\varphi)$. This implies that either of the two degenerate heavy Majorana neutrinos, N_2 or N_3, would dominantly contribute to the leptogenesis over two distinct regions of $\Delta\varphi$. More specifically, for $\Delta\varphi < 90°$ or $\Delta\varphi > 270°$, ϵ_2 is dominant over ϵ_3 because of $h_2 \ll h_3$. Otherwise, ϵ_3 is dominant over ϵ_2. However, in our scenario as shown in Fig. 1, only the former case ($\epsilon_2 \gg \epsilon_3$) is allowed, mainly because of the experimental constraint $\Delta m^2_{\mathrm{sol}}/m^2_{\mathrm{atm}} \ll 1$, as will be shown later in detail.

We remark that the radiatively induced lepton asymmetry ϵ_i is proportional to $y_\tau^4 = y^4_{\tau,SM}(1 + \tan^2\beta)^2$, and thus for large $\tan\beta$ it can be highly enhanced and proportional to $\tan^4\beta$. Furthermore, it has an explicit dependence of the evolution scale t. These two points are different from what was obtained in Refs. [13,14], where the neutrino Dirac - Yukawa matrix has been arbitrary chosen so that $\mathrm{Im}[\tilde{H}_{23}]$ could be initially nonzero and the lepton asymmetry became proportional to y_τ^2 at leading order and, at the same time, the scale dependence was cancelled out.

The resulting baryon-to-photon ratio is estimated in the context of MSSM to be

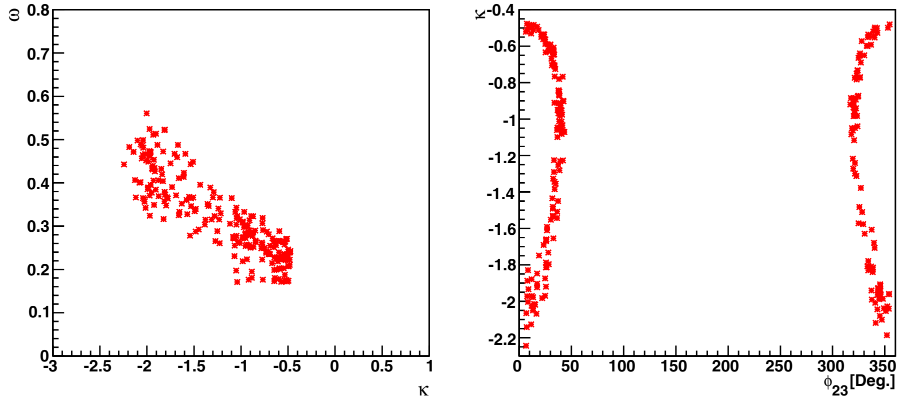

FIG. 1 (color online). Parameter regions allowed by the 3σ experimental constraints in Eq. (37) for $M_1 = 10^{13}$ GeV, $M_2 = 10^6$ GeV, and $\tan\beta = 25$.

$$\eta_B \simeq -1.67 \times 10^{-2} \sum_i \varepsilon_i \cdot \kappa_i, \tag{31}$$

where the efficiency factor κ_i describes the washout of the produced lepton asymmetry ε_i. The efficiency in generating the resultant baryon asymmetry is usually controlled by the parameter defined as

$$K_i \equiv \frac{\Gamma_i}{H} = \frac{\tilde{m}_i}{m_*}, \tag{32}$$

where H is the Hubble constant, and $\tilde{m}_i$, the so-called effective neutrino mass, is given by

$$\tilde{m}_i = \frac{[m_D m_D^\dagger]_{ii}}{M_i}, \tag{33}$$

and m_* is defined as

$$m_* = \frac{16\pi^{\frac{5}{2}}}{3\sqrt{5}} g_*^{\frac{1}{2}} \frac{v^2}{M_{\rm Planck}} \simeq 1.08 \times 10^{-3}\ {\rm eV}, \tag{34}$$

where we adopted $M_{\rm Planck} = 1.22 \times 10^{19}$ GeV and the effective number of degrees of freedom $g_* \simeq g_{*\rm SM} = 106.75$, $g_{*\rm MSSM} \simeq 2g_{*\rm SM}$. Although most analyses on baryogenesis via leptogenesis conservatively consider $K_i < 1$, much larger values of K_i, even larger than 10^3, can be tolerated [21].

From the actual numerical calculations, we find that our scenario resides in the so-called *strong washout regime* with

$$K_2 \gtrsim 1, \qquad K_3 \gtrsim 10. \tag{35}$$

Thus, for our numerical calculations, we will adopt approximate expressions of the efficiency factor given for large K_i by [22],

$$\begin{aligned} \kappa_i &\approx \frac{1}{2\sqrt{K_i^2 + 9}} \quad \text{for } 0 \lesssim K_i \lesssim 10, \\ \kappa_i &\approx \frac{0.3}{K_i(\ln K_i)^{0.6}} \quad \text{for } 10 \lesssim K_i \lesssim 10^6. \end{aligned} \tag{36}$$

V. NUMERICAL ANALYSIS AND DISCUSSIONS

As can be seen in the approximate expression in Eq. (29), the lepton asymmetry ϵ_i depends dominantly on one phase difference, $\Delta\varphi = \varphi_{23} - \varphi_{22}$, among the phases assigned in the neutrino Dirac-Yukawa matrix given in the form of Eq. (16). Therefore, we focus on the phase difference $\Delta\varphi$ and study how the prediction of η_B varies with the choice of the input values of $\Delta\varphi$ at the GUT scale. In order to estimate the RG evolutions of neutrino Dirac-Yukawa matrix and heavy Majorana neutrino masses from the GUT scale to the seesaw scale, we numerically solve all the relevant RGE's presented in Ref. [23].

In our numerical calculation of the RG running effects, we first fix the values of two masses of heavy Majorana neutrinos with hierarchy $M_1 \gg M_2$ and $\tan\beta$, then we solve the RGE's by varying input values of all the parameter space $\{d, \kappa, \omega, \rho, \Delta\varphi\}$ given at the GUT scale. Then finally we determine the parameter space allowed by low energy neutrino experimental data. At present, we have five experimental data, which are taken as low energy constraints in our numerical analysis, given at 3σ by [24],

$$\begin{gathered} 29.3° < \theta_{12} < 39.2°, \qquad 35.7° < \theta_{23} < 55.6°, \\ 0° < \theta_{13} < 11.5°, \qquad 7.1 < \Delta m_{21}^2 [10^{-5}\ {\rm eV}^2] < 8.9, \\ 2.0 < \Delta m_{31}^2 [10^{-3}\ {\rm eV}^2] < 3.2. \end{gathered} \tag{37}$$

Using the results of the RG evolutions, we estimate the lepton asymmetry for the allowed parameter space from these low energy experimental constraints. In Fig. 1, we show the parameter regions constrained by the experimental data given in Eq. (37). The two figures exhibit how the parameters κ and ω are correlated and how κ depends on

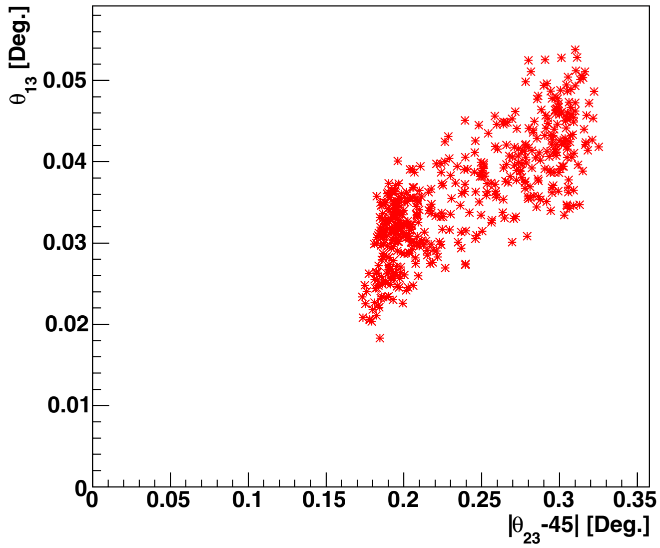

FIG. 2 (color online). Radiatively generated deviations of θ_{13} and θ_{23} from the $\mu - \tau$ symmetric prediction, ($\theta_{13} = 0$ and $\theta_{23} = 45°$), for the same parameter space used in Fig. 1.

the phase difference $\Delta\varphi$, respectively. Here we adopted $M_1 = 10^{13}$ GeV, $M_2 = 10^6$ GeV and $\tan\beta = 25$ as inputs, so that the gravitino could not be overproduced in early Universe.[3]

We find that due to the mass hierarchy of the heavy Majorana neutrinos $M_1 \gg M_2$, the RG running effects depend very weakly on the parameter ρ in our analysis.

As mentioned earlier, since our scenario allows only the normal hierarchical spectrum of light neutrino masses, there are very tiny deviations of the mixing angles arising from the RG evolutions, even in the supersymmetric case with large $\tan\beta$. In Fig. 2, we show the deviations of θ_{13} and θ_{23} from their $\mu - \tau$ symmetric initial values at the GUT scale, i.e. $\theta_{13} = 0$ and $\theta_{23} = 45°$, resulting from the RG evolution. It turned out from our numerical estimate that the radiatively generated deviations of θ_{13} and θ_{23} from the initial angles are at most 0.2° and 1.2°, respectively, even for $\tan\beta \sim 50$. In addition, there can exist radiative corrections associated with low energy supersymmetric threshold effects, which might be important in some cases [25]. The typical size of the flavor diagonal threshold corrections denoted by $I^{TH}_{\alpha} \sim g_2^2/(32\pi^2) f_{\alpha}$ ($\alpha = e, \mu, \tau$) with a loop function f_{α} [25] is of order 10^{-3} and corresponding additional deviations of mixing angles are at most less than 0.1°. However, in the case that either I^{TH}_{τ} or I^{TH}_{μ} is dominant and its size reaches maximally allowed value 0.03 [26], the additional deviations of the mixing angles can be $\delta\theta_{13} \sim 0.2°$ and $\delta\theta_{23} \sim 1°$.

In Fig. 3, we present the predictions for η_B as a function of the phase difference $\Delta\varphi$ imposed initially at the GUT

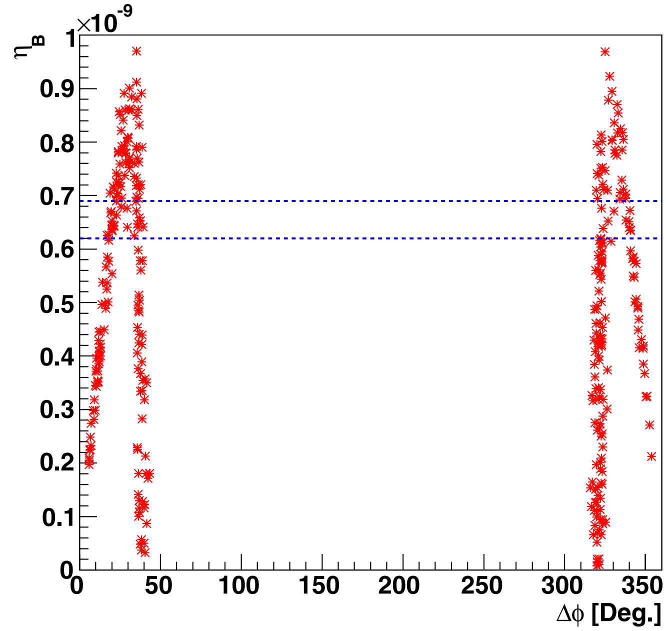

FIG. 3 (color online). Predictions for the baryon asymmetry η_B for the same parameter space as in Fig. 1. The horizontal lines are the current bounds from the CMB observations.

scale. The horizontal lines correspond to the current bounds from the CMB observations [27]:

$$\eta_B^{\text{CMB}} = (6.5^{+0.4}_{-0.3}) \times 10^{-10} (1\sigma). \tag{38}$$

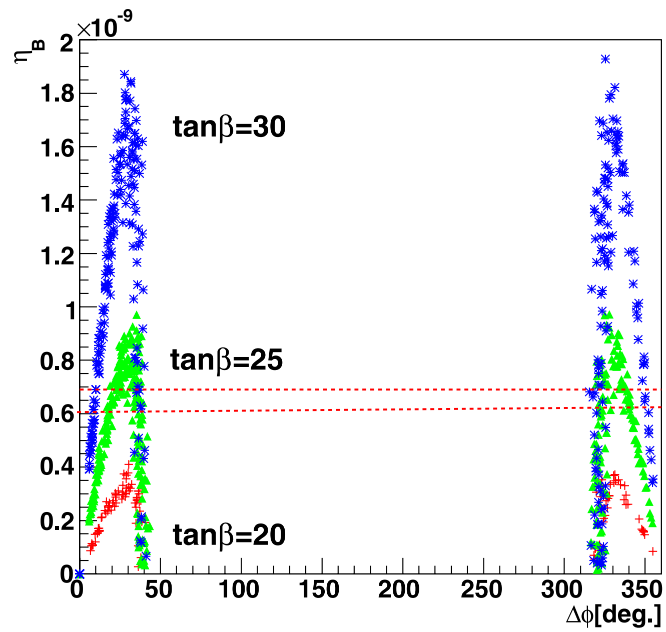

FIG. 4 (color online). Predictions for the baryon asymmetry η_B for the same parameter space as in Fig. 1. The different colors stand for the cases with $\tan\beta = 20$ (red cross), 25 (green triangle), 30 (blue star). The horizontal lines are the current bounds from the CMB observations.

[3]We note that the mass of M_2 can be as light as 10^3 GeV in our scenario.

In Fig. 4, we show how $\tan\beta$ dependence of the baryon asymmetry η_B varies with the phase difference $\Delta\varphi$. The different colored points stand for the results for $\tan\beta = 20$ (red cross), 25 (green triangle), 30 (blue star). We see that the predictions of η_B get smaller as $\tan\beta$ decreases. Thus, we can extract lower limit of $\tan\beta$ from the current observation for η_B given in Eq. (38), which is parameterized as follows:

$$\tan^4\beta \gtrsim 2\times 10^4\left[\frac{(\omega^2 + 2\kappa\cos\Delta\varphi)h_2}{\kappa(\kappa^2-1)\sin\Delta\varphi\cdot t}\right]. \qquad (39)$$

Numerically, the maximum peaks correspond to $\Delta\varphi \simeq 30°$ and $330°$ as can be seen in Fig. 4, and the successful leptogenesis can be achieved in our scenario only for the value of $\tan\beta$ satisfying

$$\tan\beta \gtrsim 23. \qquad (40)$$

As a summary, we have considered an exact $\mu - \tau$ symmetry in neutrino sectors realized at the GUT scale in the context of a seesaw model. The exact $\mu - \tau$ symmetry, which is realized in the basis where the charged lepton and heavy Majorana neutrino mass matrices are diagonal, leads to vanishing lepton asymmetries. We have shown that, in the minimal supersymmetric extensions of the seesaw model with large $\tan\beta$, the RG evolution from the GUT scale to the seesaw scale can induce a successful leptogenesis without introducing any symmetry breaking terms by hand, whereas such small RG effects lead to tiny deviations of θ_{23} and θ_{13} from their initial values at the GUT scale, i.e. $\theta_{23} = \pi/4$ and $\theta_{13} = 0$, respectively. The right amount of the baryon asymmetry η_B has been achieved via so-called resonant leptogenesis. In our scenario the seesaw scale can be lowered down to as much as 10^3 GeV for $\tan\beta = 25$ and so the well-known gravitino problem is safely avoided.

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