Mass splittings in Σ_b and Σ_b^*

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The charged Σ_b and Σ_b^* states have recently been reported by the CDF Collaboration. The relation of their reported charge-averaged masses to expectations based on the quark model is reviewed briefly. A relation is proved among the $\Delta I = 1$ electromagnetic mass differences $\Sigma_1 \equiv M(\Sigma^+) - M(\Sigma^-)$, $\Sigma_1^* \equiv M(\Sigma^{*+}) - M(\Sigma^{*-})$, $\Sigma_{b1} \equiv M(\Sigma_b^+) - M(\Sigma_b^-)$, and $\Sigma_{b1}^* \equiv M(\Sigma_b^{*+}) - M(\Sigma_b^{*-})$. The relation is $\Sigma_{b1}^* - \Sigma_{b1} = (m_s/m_b)(\Sigma_1^* - \Sigma_1)$, leading to the expectation $\Sigma_{b1}^* - \Sigma_{b1} = 0.40 \pm 0.07$ MeV.

I. INTRODUCTION

The Collider Detector Facility (CDF) Collaboration at Fermilab has recently announced the observation of four new candidates for $\Sigma_b^{\pm}$ and $\Sigma_b^{*\pm}$ [1], with masses very close to those expected in theory. Ref. [2] uses a double expansion in $1/N_c$ and $1/m_Q$, where N_c is the number of quark colors and m_Q is the heavy quark mass, while Ref. [3] uses the quark model. A recent relativistic calculation and comparison with some earlier predictions [4] may be found in Ref. [5].

The Σ_b and Σ_b^* states are illustrated in Fig. 1. They would have quark content buu, bdd with total spins $J(\Sigma_b^{\pm}) = 1/2$ and $J(\Sigma_b^{*\pm}) = 3/2$.

The analysis of Ref. [1] studies the spectra of $\Lambda_b \pi^{\pm}$ states, finding peaks at the values of $Q^{(*)\pm} \equiv M(\Sigma^{(*)\pm}) - M(\pi^{\pm}) - M(\Lambda_b)$ shown in Table I. These may be combined with the newly reported CDF value $M(\Lambda_b) = 5619.7 \pm 1.7 \pm 1.7$ MeV [6] to obtain masses of the $\Sigma_b^{(*)\pm}$ states. Here Q and Q^* denote the averages of $Q^{\pm}$ and $Q^{*\pm}$, respectively. In this analysis it was assumed that $Q^{*+} - Q^{*-} = Q^+ - Q^-$. The main point of the present paper is to examine the validity of this assumption.

We begin by discussing the nonelectromagnetic mass splittings briefly. Here the basic physics is the same as that in Ref. [7], which may be consulted for earlier references. The charge-averaged hyperfine splitting between the $J = 1/2$ and $J = 3/2$ states may be predicted from that for charmed particles:

$$\frac{M(\Sigma_b^*) - M(\Sigma_b)}{M(\Sigma_c^*) - M(\Sigma_c)} = \frac{m_c}{m_b} = \frac{1.5 \text{ GeV}}{4.9 \text{ GeV}} = 0.31, \quad (1)$$

where we use "constituent" quark masses from Ref. [8]. Using isospin-averaged differences $M(\Sigma_c) - M(\Lambda_c) = (167.09 \pm 0.13)$ MeV and $M(\Sigma_c^*) - M(\Lambda_c) = (231.5 \pm 0.8)$ MeV based on Ref. [9], we find this ratio to be 0.33 ± 0.03. The first of Refs. [2] finds $M(\Sigma_b^*) - M(\Sigma_b) = 23.8$ MeV, the second finds 15.8 MeV, and Ref. [5] finds 29 MeV.

As for the splitting of the spin-weighted average $[2M(\Sigma_b^*) + M(\Sigma_b)]/3$ from the Λ_b, it is expected to be the same as the corresponding values for hyperons con-

taining strange or charmed quarks. The experimental values are [1]

$$\frac{M(\Sigma_b) + 2M(\Sigma_b^*)}{3} - M(\Lambda_b) = (205.9 \pm 1.8) \text{ MeV}, \quad (2)$$

where we have used the averages of the differences for $\Sigma_b^{(*)\pm}$ as no information is available on $M(\Sigma_b^{(*)0})$. This is to be compared with

$$\frac{M(\Sigma_c) + 2M(\Sigma_c^*)}{3} - M(\Lambda_c) = (210.0 \pm 0.5) \text{ MeV}, \quad (3)$$

where we have used the differences with respect to $M(\Lambda_c)$ mentioned above, and

$$\frac{M(\Sigma) + 2M(\Sigma^*)}{3} - M(\Lambda) = (205.1 \pm 0.3) \text{ MeV}, \quad (4)$$

where the masses are taken directly from Ref. [9], and an average over the Σ isospin multiplet is taken. In each case the dominant source of error is the mass of the $I_3 = 0$, $J = 3/2$ state, Σ_c^{*+} or Σ^{*0}.
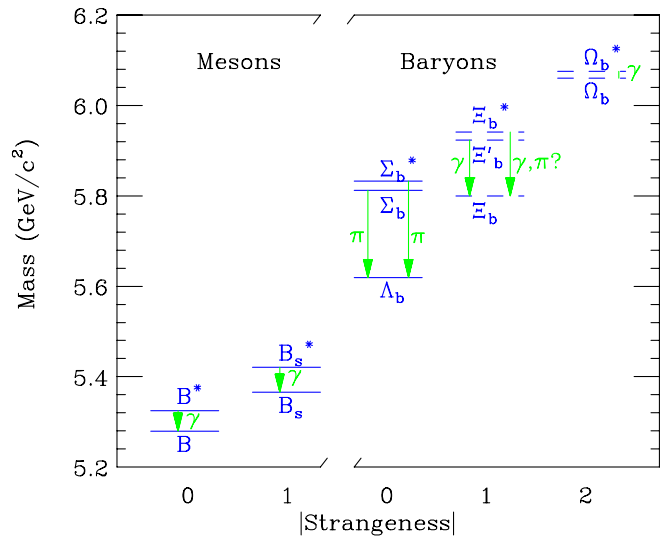

FIG. 1 (color online). Hadrons containing a single beauty quark. The Σ_b and Σ_b^* have recently been reported by CDF [1]. Dashed lines denote states not yet observed.

TABLE I. Values of $Q^{(*)\pm} \equiv M(\Sigma_b^{(*)\pm}) - M(\pi^\pm) - M(\Lambda_b)$ and $M(\Sigma^{(*)\pm})$ reported by the CDF Collaboration [1].

Quantity	Value (MeV)
Q^+	$48.4^{+2.0}_{-2.3} \pm 0.1$
Q^-	$55.9 \pm 1.0 \pm 0.1$
$Q^* - Q$	$21.3^{+2.0+0.4}_{-1.9-0.2}$

Ref. [2] also predicts the equality of these mass splittings, and estimates that the b and c quantities should be equal to about ± 5 MeV. Ref. [3] uses quark-model arguments to estimate the Σ_b mass but eliminates reference to actual quark masses by using other hadron mass splittings.

We now turn to electromagnetic mass splittings. The discussion will be conducted in a quark model in which there are several sources of baryon electromagnetic mass differences [10]. Most of these cancel out when one takes the $\Delta I = 1$ mass differences

$$\Sigma_1 \equiv M(\Sigma^+) - M(\Sigma^-), \qquad \Sigma_1^* \equiv M(\Sigma^{*+}) - M(\Sigma^{*-}),$$
(5)

$$\Sigma_{b1} \equiv M(\Sigma_b^+) - M(\Sigma_b^-),$$
$$\Sigma_{b1}^* \equiv M(\Sigma_b^{*+}) - M(\Sigma_b^{*-}).$$
(6)

However, we review briefly all sources of isospin violation in baryon masses.

II. INTRINSIC QUARK MASSES

The u and d quarks have intrinsic masses which differ by a couple of MeV [9]. Corresponding estimates for the strange quark mass are in the vicinity of 100 MeV. However, quarks in hadrons are more suitably described by the constituent values (see, e.g., Refs. [7,11]) $m_u, m_d = \mathcal{O}(350)$ MeV, $m_s = \mathcal{O}(500)$ MeV, with $m_d - m_u$ of order a few MeV but quite uncertain. The quarks' kinetic energies may also depend on their masses. Without detailed knowledge of dynamics, it is difficult to anticipate this dependence. One may simply parametrize kinetic energies with labels K_q for those contributions which act as one-body operators and $K_{q_i q_j}$ for those contributions which depend on interactions with each individual other quark.

III. COULOMB INTERACTIONS BETWEEN QUARKS

Each quark pair in a hadron has a Coulomb interaction energy

$$\Delta E_{ij\text{em}} = \alpha Q_i Q_j \left\langle \frac{1}{r_{ij}} \right\rangle,$$
(7)

where α is the electromagnetic fine structure constant, Q_i is the charge of quark i in units of the proton charge, and $\langle 1/r_{ij} \rangle$ is the expectation value of the inverse distance

between the members of the pair. In the flavor-SU(3) limit $\langle 1/r_{ij} \rangle$ will be universal throughout a multiplet. In this limit, we parametrize the interaction energy $\Delta E_{ij\text{em}} = aQ_iQ_j$, where a is some universal constant.

IV. STRONG HYPERFINE INTERACTIONS

Quarks in hadrons experience a spin-dependent force due to gluon exchange which acts dominantly on pairs in an S-wave state. For quark pairs in a baryon, one has a strong hyperfine interaction energy

$$\Delta E_{ij\text{HFs}} = \text{const} \frac{|\Psi_{ij}(0)|^2 \langle \sigma_i \cdot \sigma_j \rangle}{m_i m_j},$$
(8)

where $|\Psi_{ij}(0)|^2$ is the square of the S-wave wave function of two quarks at zero relative separation, and the constant is universal for all quark pairs in a baryon. We shall assume that $|\Psi_{ij}(0)|^2$ is universal for all quark pairs in S-wave baryons. We then find a contribution to the hyperfine energy $\Delta E_{ij\text{HFs}} = \beta \langle \sigma_i \cdot \sigma_j \rangle / (m_i m_j)$. The calculation of strong hyperfine splittings in baryons, requiring evaluation of $\langle \sigma_i \cdot \sigma_j \rangle$ for each quark pair, is described in more detail in Ref. [10].

V. ELECTROMAGNETIC HYPERFINE INTERACTIONS

The electromagnetic interaction between quarks in a baryon has a hyperfine contribution

$$\Delta E_{ij\text{HFe}} = -\frac{2\pi\alpha Q_i Q_j |\Psi(0)_{ij}|^2 \langle \sigma_i \cdot \sigma_j \rangle}{3m_i m_j}.$$
(9)

Assuming universality of the wave functions, we parametrize this effect as $\Delta E_{ij\text{HFe}} = \gamma Q_i Q_j \langle \sigma_i \cdot \sigma_j \rangle / (m_i m_j)$.

We now form the *differences* of $\Delta I = 1$ mass differences for $\Sigma^\pm$ and $\Sigma^{*\pm}$ states. We find

$$\Sigma_1^* - \Sigma_1 = \beta \left(\frac{6}{m_u m_s} - \frac{6}{m_d m_s} \right) - \frac{\gamma}{9} \left(\frac{6}{m_d m_s} + \frac{12}{m_u m_s} \right)$$
$$= -2\sqrt{3} M_{\Lambda\Sigma^0},$$
(10)

where $M_{\Lambda\Sigma^0}$ is an isospin-violating term mixing the Λ and Σ^0. Corresponding relations may be written for Σ_b and Σ_b^* with the substitution $s \to b$:

$$\Sigma_{b1}^* - \Sigma_{b1} = \beta \left(\frac{6}{m_u m_b} - \frac{6}{m_d m_b} \right) - \frac{\gamma}{9} \left(\frac{6}{m_d m_b} + \frac{12}{m_u m_b} \right)$$
$$= -2\sqrt{3} M_{\Lambda_b \Sigma_b^0}.$$
(11)

The crucial point is that these differences are of order $1/m_s$ and $1/m_b$, respectively. They are thus related by

$$\Sigma_{b1}^* - \Sigma_{b1} = (m_s/m_b)(\Sigma_1^* - \Sigma_1).$$
(12)

This relation is implicit in many previous treatments (see, e.g., Table I in the first of Refs. [2]), as any such hyperfine

differences are expected to scale as the inverse of the heavy quark mass and should be the same for quarks of the same charge (s and b in the present case).

We now use the experimental averages [9]

$$\Sigma_1 = -8.08 \pm 0.08 \text{ MeV}, \qquad \Sigma_1^* = -4.4 \pm 0.64 \text{ MeV}, \tag{13}$$

and the constituent-quark masses $m_s = 538$ MeV [7] and $m_b = 4.9$ GeV [8] to predict

$$\Sigma_{b1}^* - \Sigma_{b1} = 0.40 \pm 0.07 \text{ MeV}. \tag{14}$$

While the analysis of Ref. [1] assumed $\Sigma_{b1}^* - \Sigma_{b1} = 0$ (which would be accurate in the limit of $m_b \rightarrow \infty$), the relatively small value in Eq. (14) is not likely to lead to a substantial change in masses obtained from experiment. Earlier discussions of electromagnetic mass splittings in

heavy baryons also find small values of $\Sigma_{b1}^* - \Sigma_{b1}$, without explicitly noting the relation (12): 0.6 MeV in Ref. [12] and 0.2 MeV in Ref. [13].

A corresponding relation cannot be obtained for the charmed baryon mass differences $\Sigma_{c1} \equiv M(\Sigma_c^{++}) - M(\Sigma_c^0)$ and $\Sigma_{c1}^* \equiv M(\Sigma_c^{*++}) - M(\Sigma_c^{*0})$. All that can be said is that in the limit of $m_c \rightarrow \infty$, one would have $\Sigma_{c1}^* - \Sigma_{c1} \rightarrow 0$. Present data give $\Sigma_{c1} = (0.27 \pm 0.11)$ MeV and $\Sigma_{c1}^* = (0.3 \pm 0.6)$ MeV [9].

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