Suppressing proton decay by separating quarks and leptons

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Arkani-Hamed and Schmaltz have shown that proton stability need not originate from symmetries in a high energy theory. Instead the proton decay rate is suppressed if quarks and leptons are spatially separated in a compact extra dimension. This separation may be achieved by coupling five-dimensional fermions to a bulk scalar field with a nontrivial vacuum profile and requires relationships between the associated quark and lepton Yukawa couplings. We hypothesize that these relationships are the manifestation of an underlying symmetry. We further show that the Arkani-Hamed and Schmaltz proposal may suggest that proton stability *is* the result of an underlying symmetry, though not necessarily the traditional baryon number symmetry.

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In recent years it has been proposed that the fundamental scale of nature may be much less than the Planck scale [1–3]. By introducing large extra dimensions one is able to reframe the hierarchy problem and remove the need to explain the disparity between the Planck scale and the electroweak scale. In the standard model (SM) it is known that proton decay proceeds at the nonrenormalizable level via the dimension six operator Q^3L/Λ^2, where Q (L) generically denotes a quark (lepton) field operator and Λ is the SM cutoff. The stringent lower bound of 1.6×10^{33} yr on the decay mode $p \rightarrow e^+\pi$ leads to the bound $\Lambda \gtrsim 10^{16}$ GeV [4]. In models with large extra dimensions the fundamental gravitational scale may be reduced to TeV energies, removing the order 10^{16} GeV cutoff required to suppress the proton decay rate.

Arkani-Hamed and Schmaltz (AS) [5] have suggested that proton longevity need not imply a conserved symmetry in the more fundamental theory [6]. They have shown that proton decay can be suppressed in models with a low fundamental scale if quarks and leptons are localized at different four-dimensional slices of a five-dimensional spacetime. If the fifth dimension forms an S^1/Z_2 orbifold, maximal suppression of the proton decay rate results when quarks and leptons are localized at different fixed points. It is known that the zero mode of a five-dimensional fermion, which may be identified with a SM fermion, can be localized at an S^1/Z_2 orbifold fixed point by coupling the fermion to a bulk scalar field with a nontrivial vacuum profile [7]. The sign of the associated Yukawa coupling determines the fixed point at which the fermion zero mode is localized [7,8]. Thus proton decay may be suppressed by arbitrarily choosing different sign Yukawa couplings for quarks and leptons with a bulk scalar (see e.g. [9]).

It is interesting to speculate that an underlying theory may possess symmetries which fix the relative bulk scalar Yukawa coupling signs between quarks and leptons. In this work we ask if the separation of quarks and leptons required to achieve the AS proposal may itself be the manifestation of an underlying symmetry. Thus proton stability would result from the symmetries of an underlying theory, though not necessarily the traditional baryon number symmetry. Indeed, it was suggested in [10] that it may be possible to understand the separation of quarks and leptons in a five-dimensional $SO(10)$ model, through fermion couplings to a symmetry breaking vacuum expectation value (VEV) in the B–L direction. However, this does not ensure that the theory will separate quarks and leptons. This may be seen as follows. Consider a five-dimensional spacetime with the fifth dimension forming an S^1/Z_2 orbifold. Take Ψ as a bulk field in the **16** of $SO(10)$, containing a family of SM fermions, and H as a bulk scalar in the adjoint representation of $SO(10)$. The Yukawa Lagrangian for H is

$$\mathcal{L}_H = \sum_i g_i \bar{\Psi}_i \Psi_i H, \qquad (1)$$

where $i = 1, 2, 3$ labels the different generations. If H develops a kink profiled VEV in the B–L direction, chiral zero mode fermions will be localized at one of the orbifold fixed points, with the point of localization determined by the sign of the Yukawa coupling between the given fermion and the B–L direction scalar H_{B-L}. Observe that the quarks of a given generation will couple to H_{B-L} with a different Yukawa coupling sign than the leptons of the same generation. However, quarks and leptons of different generations may still couple to H_{B-L} with the same sign. The mixing observed in the quark sector requires all quarks to be separated from the light leptons in order to suppress the proton decay rate. This will not occur unless one arbitrarily chooses $g_i > 0$ or $g_i < 0$ for all i. Thus breaking $SO(10)$ in the B–L direction by a bulk scalar does not guarantee suppression of the proton decay rate in a higher-dimensional theory.

In this brief note we assume that (a) the hierarchy problem tells us that the SM cutoff must be low (order ~10 TeV) and that (b) the longevity of the proton results

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from the separation of quarks and leptons in an extra dimension. Desiring simplicity we further assume that (c) the separation of quarks and leptons results from the simplest Yukawa Lagrangian which naturally localizes quarks and leptons at opposite boundaries of an S^1/Z_2 orbifold. We identify a minimal set of symmetries required to preserve the Yukawa coupling relationships this Lagrangian contains. In order to construct a complete theory possessing this minimal set of symmetries one is required to extend the SM gauge group. We identify candidate extensions. Let us emphasize that our main point is that *proton stability may be the manifestation of an underlying symmetry in the AS proposal*. We illustrate this with a concrete example. Although alternative symmetries which achieve quark-lepton separation may exist, our observation holds independent of the specific construct. We note that neutrino mixing has recently been investigated using symmetrical configurations of bulk fermions [11].

We assume a five-dimensional product spacetime $M^4 \times S^1/Z_2$, where M^4 denotes a four-dimensional Minkowski spacetime. The action of the Z_2 transformation is defined by $y \to -y$, where y labels the extra dimension. We include the following five-dimensional fermions:

$$U, D, N, E, U^c, D^c, E^c, N^c, \tag{2}$$

where the zero modes of the fields U, D, N, E form the usual SM $SU_L(2)$ doublets and the zero modes of U^c, D^c, and E^c will be identified with the SM $SU_L(2)$ singlet fields. We have included a SM gauge singlet field N^c for reasons which will become evident. The zero mode of this field is the charge conjugate of the usual right-chiral neutrino. We also include a gauge singlet bulk scalar Σ. The action of the orbifold discrete symmetry Z_2 on the fields is

$$\begin{aligned} \Sigma(x^\mu, y) &\to \Sigma(x^\mu, -y) = -\Sigma(x^\mu, y), \\ F(x^\mu, y) &\to F(x^\mu, -y) = \gamma^5 F(x^\mu, y), \end{aligned} \tag{3}$$

where x^μ labels M^4, F generically labels the fermions (2) and γ^5 is the usual product of Dirac matrices. The scalar potential is given by

$$V(\Sigma) = \frac{\lambda}{4\Lambda}(\Sigma^2 - u^2)^2, \tag{4}$$

where λ is dimensionless and Λ is the cutoff. The combination of the scalar's orbifold parity and the potential V result in the VEV profile [8]

$$\langle\Sigma\rangle(y) \approx u \tanh[\beta y] \tanh[\beta(L/2 - y)], \tag{5}$$

where $\beta^2 = \lambda L u^2/4$ and the orbifold fixed points are located at $y = 0$ and $y = L/2$. The fermion orbifold parities (3) permit only the (four-dimensional) left-chiral component of a given fermion F to possess a zero mode (call it $f_L^{(0)}$). The sign of the Yukawa coupling constant between the field F and Σ then determines the fixed point at which this zero mode $f_L^{(0)}$ is localized [7].

In general, the Yukawa Lagrangian for Σ takes the form

$$-\mathcal{L}_{\rm Yuk} = \sum_{F,i} h_{F_i} \bar{F}_i F_i \Sigma, \tag{6}$$

where the sum is over all fermion fields (2) and all generations $i = 1, 2, 3$. The Yukawa constants h_F are in general independent and the separation of quarks and leptons required to ensure proton longevity demands that one enforce the relationships

$$\mathrm{sign}\,(h_q) = -\mathrm{sign}(h_l), \tag{7}$$

where the subscript q (l) labels quarks (leptons). We shall not employ the most general Yukawa Lagrangian (6). Instead we assume the simplest Yukawa Lagrangian which naturally separates quarks and leptons, namely,

$$\begin{aligned} -\mathcal{L}_{\rm Min} = h\Big\{ &\sum_i (U_i^2 + D_i^2 + U_i^{c2} + D_i^{c2}) \\ &- \sum_i (N_i^2 + E_i^2 + N_i^{c2} + E_i^{c2})\Big\}\Sigma, \end{aligned} \tag{8}$$

where h denotes a common Yukawa coupling constant and we ignore quark color for the moment. We employ an obvious notation with $F^2 = \bar{F}F$. The sign of h determines the fixed point at which, e.g., quarks are localized, with leptons automatically localized at the opposite boundary of the compact extra dimension.

The simplicity of (8) motivates us to identify five-dimensional extensions of the SM which naturally produce this Yukawa Lagrangian. To this end we note that $\mathcal{L}_{\rm Min}$ possesses the symmetry [12]

$$\mathcal{G} = U(3)_f \otimes U(4)_g \otimes Z_2^{\rm QL}. \tag{9}$$

Here $U(3)_f$ [$U(4)_g$] is the group of unitary rotations of three (four) objects and $Z_2^{\rm QL}$ is a discrete symmetry interchanging two objects. The sets of fermions

$$\{U_i, D_i, U_i^c, D_i^c\} \quad \text{and} \quad \{N_i, E_i, N_i^c, E_i^c\} \tag{10}$$

each form (3, 4) representations of $U(3)_f \otimes U(4)_g$, with $U(3)_f$ mixing the three families and $U(4)_g$ mixing the four states within a family. The symmetry $Z_2^{\rm QL}$ is defined by the interchange of the two sets (10). Note that Σ transforms trivially under $U(3)_f \otimes U(4)_g$ and is necessarily odd under $Z_2^{\rm QL}$.

Clearly the group $\mathcal{G}$ is not a symmetry of the SM and naïvely we may expect any extension of the SM which reproduces the Lagrangian $\mathcal{L}_{\rm Min}$ to exhibit this rather restrictive symmetry. There are, however, more transparent subgroups of $\mathcal{G}$ which, when enforced upon an extended SM, ensure the Yukawa coupling relationships in $\mathcal{L}_{\rm Min}$. First observe that

$$U(4)_g \supset Z_2^{\mathrm{LR}}, \tag{11}$$

where the action of Z_2^{LR} is defined by

$$U_i \leftrightarrow U_i^c, \qquad D_i \leftrightarrow D_i^c, \qquad N_i \leftrightarrow N_i^c, \qquad E_i \leftrightarrow E_i^c. \tag{12}$$

Also note that

$$U(3)_f \supset Z_3^f, \tag{13}$$

where Z_3^f is a cyclic symmetry group acting on the three generations of fermions. The symmetry group

$$\mathcal{G}_{\mathrm{Min}} = Z_3^f \otimes Z_2^{\mathrm{LR}} \otimes Z_2^{\mathrm{QL}}, \tag{14}$$

is in fact a minimal symmetry set required to preserve the Yukawa coupling relations in $\mathcal{L}_{\mathrm{Min}}$.

Why is the group $\mathcal{G}_{\mathrm{Min}}$ more transparent than $\mathcal{G}$? The groups Z_2^{LR} and Z_2^{QL} are the familiar discrete symmetries found in the left-right (LR) symmetric model and the quark-lepton (QL) symmetric model, respectively. These models are well known and have been studied in both four-dimensional [13–21] and higher-dimensional [22–26] frameworks. The LR model arises when one postulates that the fundamental theory describing nature possesses a left-right interchange symmetry. One is required to introduce the additional gauge symmetry $SU_R(2)$ to permit Z_2^{LR}. The QL model results from the assumption that nature displays a quark-lepton interchange symmetry at a fundamental level. When quark color is introduced in (8) one must also introduce leptonic color $SU_l(3)$ to permit the QL symmetry. The so called quark-lepton left-right symmetric model, which contains the discrete symmetry group $Z_2^{\mathrm{LR}} \otimes Z_2^{\mathrm{QL}}$, also has been studied [27]. Each of these models would allow one to achieve some degree of quark-lepton separation in 5D, but would not naturally separate all quarks and leptons.

An existing model of greater interest to us is that based on the gauge group [28,29]

$$\mathcal{H} \equiv [SU(3)]^2 \otimes [SU(2)]^2 \otimes [U_X(1)]^3. \tag{15}$$

Here the $SU(3)$ factors are the color groups of the QL symmetric model, namely, the usual color group $SU_c(3)$ and the leptonic color group $SU_l(3)$ (required to construct a QL symmetric Lagrangian). The $SU(2)$ factors are the familiar chiral groups of the LR model, while the fermion quantum numbers under $[U_X(1)]^3$ distinguish the different generations. Of importance is the fact that the model admits the discrete symmetry group

$$Z_3^f \otimes Z_2^{\mathrm{LR}} \otimes Z_2^{\mathrm{QL}}, \tag{16}$$

namely, $\mathcal{G}_{\mathrm{Min}}$. Thus models based on the gauge group $\mathcal{H}$ automatically admit the Yukawa Lagrangian $\mathcal{L}_{\mathrm{Min}}$ in 5D and thus naturally separate quarks and leptons. Having made this identification, a few comments are in order.

First note that the group $\mathcal{H}$ requires the introduction of leptonic color $SU_l(3)$ in order to admit the discrete symmetry Z_2^{QL}. It is known however that leptonic color does not emerge from many of the popular grand unified theory gauge groups like $SU(5)$, $SO(10)$, and E_6. Nonetheless, there exist alternative approaches to gauge unification. Quartification models are based on the gauge group $[SU(3)]^4$ [30–33] and admit the discrete symmetry $Z_2^{\mathrm{LR}} \otimes Z_2^{\mathrm{QL}}$. A 5D quartification model would allow one to obtain quark-lepton separation with three independent bulk scalar Yukawa coupling constants (one per generation). In this sense the longevity of the proton may be as natural in 5D quartification models as it is in the proposal of [10].

The fact that $Z_3^f \subset U(3)_f$ suggests that a unified model containing a horizontal gauge symmetry would naturally explain the longevity of the proton via quark-lepton separation. Quintification models offer an alternative unified framework and employ the gauge group $[SU(3)]^5 = [SU(3)]^4 \otimes SU_H(3)$ [34], where the subscript H labels a horizontal symmetry. Thus it would be possible to realize $\mathcal{L}_{\mathrm{Min}}$ within a five-dimensional quintification model, demonstrating that our approach may be compatible with the notion of unification.

In conclusion, we have investigated symmetries which allow one to naturally separate quarks and leptons in five-dimensional models. This separation is important in that it allows one to understand the long lifetime of the proton in models with a low fundamental scale. We have shown that higher-dimensional extensions of the SM with the gauge group $[SU(3)]^2 \otimes [SU(2)]^2 \otimes [U_X(1)]^3$ admit the discrete symmetry $Z_3^f \otimes Z_2^{\mathrm{LR}} \otimes Z_2^{\mathrm{QL}}$ and allow one to achieve quark-lepton separation in both a natural and minimal fashion.

It is intriguing that in this approach proton longevity remains a manifestation of underlying symmetries in the high energy theory, though not necessarily baryon number as traditional approaches would suggest. Irrespective of the specific example we have constructed, the observation that proton stability within the AS proposal may imply underlying symmetries remains of interest.

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