Supersymmetric intersecting branes in time-dependent backgrounds

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We construct a family of supersymmetric solutions in time-dependent backgrounds in supergravity theories. One class of the solutions are intersecting brane solutions and another class are brane solutions in pp-wave backgrounds, and their intersection rules are also given. The relation to existing literature is also discussed. An example of D1–D5 with linear null dilaton together with its possible dual theory is briefly discussed.

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I. INTRODUCTION

There has been much interest in time-dependent supersymmetric solutions of supergravities in ten- and 11-dimensional spacetime because of their applications to cosmology and to our understanding of spacelike singularities [1–5]. Among others, solutions with the linear dilaton background in the null direction with $\frac{1}{2}$ supersymmetry are proposed to describe the singular region in the spacetime without difficulty [6], and various extensions have been considered [7–26]. For a detailed review of the big-bang models in string theory see [27]. It has been argued that it is possible to map the theory near the singularity at a very early time to the dual matrix theory which allows us to discuss the behaviors of the solution in perturbative picture, whereas in the far future it can be thought of as a perturbative string theory with weak string coupling. It is the matrix degrees of freedom but not the perturbative string states that describe the physics near the singularities, but later the spacetime picture in terms of the closed strings becomes relevant. Furthermore the celebrated AdS/CFT duality has also been used to argue that the dual field theory is a time-dependent supersymmetric gauge theory on the boundary of the AdS space in other backgrounds. In view of these interesting developments in the study of time-dependent solutions in string theory, it is important to explore further examples of supersymmetric solutions of string and supergravity theories in time-dependent backgrounds.

On the other hand, D-branes can probe the nonperturbative dynamics of the string theory and they have been used to study various duality aspects of string theory. A systematic derivation of the general D-brane solutions in the pp-wave backgrounds has been given in [28]. It is thus interesting to find if we can have such brane solutions in time-dependent backgrounds with time-dependent dilaton. In fact, D3-brane solutions have been found and discussed in [17,19] and other single brane solutions in [24,26], but it is not known if there exist further general intersecting brane solutions of this type in a time-dependent setup.

Motivated by the recent surge of interest in finding out time-dependent solutions in supergravity and speculation on the dual field theory in this setup, in this paper we present a general class of intersecting brane solutions in time-dependent supersymmetric backgrounds with and without pp-wave by using the method developed in Ref. [28,29]. We start with a general ansatz for the metric and solve for the field equations of the supergravity. We also derive the intersection rules for the branes in this background. Some examples of dual field theories are also discussed briefly.

The rest of the paper is organized as follows. In Sec.. II we present the classical solution of supergravity by directly solving the equations of motion. For simplicity, we take the factorized ansatz for the metric functions as the product of usual r-dependent part and the time-dependent part. This way of taking the ansatz has a clear advantage in that the supergravity equations of motion essentially have two different contributions coming from the r-dependent part and a differential equation involving only time derivatives. We then present three kinds of solutions. We further show that special cases of our general solutions reproduce known solutions. We also write explicitly a particular example of intersecting branes of D1–D5 system with a linear dilaton and make some comments about the dual field theory. In Sec. III we present our conclusions and outlook.

II. SUPERGRAVITY SOLUTION

The low-energy effective action for the supergravity system coupled to dilaton and n_A-form field strength is given by

$$I = \frac{1}{16\pi G_D}\int d^D x\sqrt{-g}\Big[R - \frac{1}{2}(\partial\phi)^2 - \sum_{A=1}^{m}\frac{1}{2n_A!}e^{a_A\phi}F_{n_A}^2\Big], \tag{1}$$

where G_D is the Newton constant in D dimensions and g is the determinant of the metric. The last term includes both

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RR and NS-NS field strengths and $a_A = \frac{1}{2}(5 - n_A)$ for RR field strengths and $a_A = -1$ for NS-NS 3-form. We put fermions and other background fields to be zero.

From the action (1), one can derive the field equations/Bianchi identities

$$R_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \sum_A \frac{1}{2n_A!} e^{a_A\phi}\left[n_A(F^2_{n_A})_{\mu\nu} - \frac{n_A - 1}{D-2}F^2_{n_A} g_{\mu\nu}\right], \tag{2}$$

$$\Box\phi = \sum_A \frac{a_A}{2n_A!} e^{a_A\phi} F^2_{n_A}, \tag{3}$$

$$\partial_{\mu_1}(\sqrt{-g}e^{a_A\phi}F^{\mu_1\cdots\mu_{n_A}}) = 0, \tag{4}$$

$$\partial_{[\mu}F_{\mu_1\cdots\mu_{n_A}]} = 0. \tag{5}$$

In this paper we consider the case in which the metric functions depend on u and r. Specifically we take the metric ansatz

$$ds^2_D = e^{2(u_0+v_0)}[-2dudv + K(u, y_\alpha, z_i)du^2] + \sum_{\alpha=1}^{d-2} e^{2(u_\alpha+v_\alpha)}dy^2_\alpha + e^{2(B+C)}[dr^2 + r^2 d\Omega^2_{\tilde{d}+1}], \tag{6}$$

where $D = d + \tilde{d} + 2$, the coordinates u, v and y_α, ($\alpha = 1, \ldots, d-2$) parameterize the d-dimensional world-volume directions and the remaining $\tilde{d} + 2$ coordinates r and angles are transverse to the brane world-volume (sometimes denoted also by the orthogonal coordinates z_i), $d\Omega^2_{\tilde{d}+1}$ is the line element of the $(\tilde{d}+1)$-dimensional sphere. Here u_0, u_α and B are assumed to be functions of r only, and v_0, v_α and C are those of u only. The dilaton ϕ is also taken as a sum of u-dependent and r-dependent terms: $\phi = \phi_r + \phi_u$. Our ansatz includes more general solutions than those in [17,19].

For the field strength backgrounds, we take

$$F_{n_A} = e^{2f_A(u)}E'_A(r)du \wedge dv \wedge dy^{\alpha_1} \wedge \cdots \wedge dy^{\alpha_{q_A-1}} \wedge dr, \tag{7}$$

where $n_A = q_A + 2$. Throughout this paper, the dot and prime denote derivatives with respect to u and r, respectively. Equation (7) is an electric background and we could also consider magnetic background, but that is basically the same as the electric case with the replacement

$$g_{\mu\nu} \to g_{\mu\nu}, \qquad F_n \to e^{a\phi} * F_n, \qquad \phi \to -\phi. \tag{8}$$

This is due to the S-duality symmetry of the original system (1). So we do not have to consider it separately.

The NS-NS 3-form responsible for the off-diagonal component of the metric is separately written as

$$H_{uij} = e^{2g(u)}\partial_{[i}b_{j]}, \tag{9}$$

such that it satisfies the Bianchi identity. Here the indices i, j denote the directions transverse to the branes (z_i, z_j or r and angles). The field equation for this NS-NS 3-form leads to

$$\partial_{[i}b_{j]} = e^{-U+2(u_0+v_0+B+C)+\phi}\mu_{ij}, \tag{10}$$

where μ_{ij} is constant and U is defined by

$$U = 2u_0 + \sum_{\alpha=1}^{d-2} u_\alpha + \tilde{d}B. \tag{11}$$

There is no restriction on the u-dependent part of b_j from the field equation, but we have chosen it like Eq. (10) as our convention.

With our ansatz, the Einstein equations (2) reduce to

$$u''_0 + u'_0\left\{U' + \frac{\tilde{d}+1}{r}\right\} = \sum_A \frac{D - q_A - 3}{2(D-2)} S_A T_A (E'_A)^2, \tag{12}$$

$$\sum_{\alpha=1}^{d-2}\ddot{v}_\alpha + (\tilde{d}+2)\ddot{C} + \sum_{\alpha=1}^{d-2}\dot{v}^2_\alpha + (\tilde{d}+2)\dot{C}^2 - 2\dot{v}_0\left\{\sum_{\alpha=1}^{d-2}\dot{v}_\alpha + (\tilde{d}+2)\dot{C}\right\} + e^{2(u_0+v_0-B-C)}K\left[u''_0 + \frac{\tilde{d}+1}{r}u'_0 + \frac{1}{2}K^{-1}\Box^{(\tilde{d}+2)}K + \partial_i\left(u_0 + \frac{1}{2}\ln K\right)\partial^i U\right] + \frac{1}{2}\sum_{\alpha=1}^{d-2} e^{2(u_0+v_0-u_\alpha-v_\alpha)}\partial^2_\alpha K$$
$$= -\sum_A \frac{D-q_A-3}{2(D-2)} e^{2(u_0+v_0-B-C)}KS_A T_A (E'_A)^2 + \frac{1}{4}e^{-2U-(4B+4C+\phi)+4g(u)}(\partial_{[i}b_{j]})^2 - \frac{1}{2}(\dot{\phi})^2, \tag{13}$$

$$u''_\alpha + u'_\alpha\left(U' + \frac{\tilde{d}+1}{r}\right) = \sum_A \frac{\delta^{(\alpha)}_A}{2(D-2)} S_A T_A (E'_A)^2, \tag{14}$$

$$U'' + B'' - B'\left(2u'_0 + \sum_{\alpha=1}^{d-2} u'_\alpha - \frac{\tilde{d}+1}{r}\right) + 2(u'_0)^2 + \sum_{\alpha=1}^{d-2}(u'_\alpha)^2 = -\frac{1}{2}(\phi')^2 + \sum_A \frac{D-q_A-3}{2(D-2)} S_A T_A (E'_A)^2, \tag{15}$$

$$B'' - \frac{1}{r^2} + \left(B' + \frac{1}{r}\right)\left(U' + \frac{\tilde{d}+1}{r}\right) - \frac{\tilde{d}}{r^2} = -\sum_A \frac{q_A+1}{2(D-2)} S_A T_A (E_A')^2, \tag{16}$$

where S_A, T_A and $\delta_A^{(\alpha)}$ are defined by

$$S_A = \exp\left(\epsilon_A a_A \phi_r - 2\sum_{\alpha\in q_A} u_\alpha\right), \qquad T_A = \exp\left(\epsilon_A a_A \phi_u - 2\sum_{\alpha\in q_A} v_\alpha + 4f_A\right), \tag{17}$$

$$\delta_A^{(\alpha)} = \begin{cases} D - q_A - 3 \\ -(q_A+1) \end{cases} \quad \text{for} \begin{cases} y_\alpha \text{ belonging to} q_A\text{-brane} \\ \text{otherwise} \end{cases}, \tag{18}$$

respectively, the sum of α runs over the world-volume of the q_A-brane (u, v and $(q_A-1)y^\alpha$ coordinates, and so $\sum_{\alpha\in q_A} u_\alpha = 2u_0 + \sum_{\alpha=1}^{q_A-1} u_\alpha$ for example), and $\epsilon_A = +1(-1)$ is for electric (magnetic) backgrounds. The Eqs. (12)–(16) are the uv, uu, $\alpha\beta$, rr and ab components of the Einstein equations (2), respectively. The dilaton equation (3) and the remaining Eqs. (4) and/or (5) yield

$$e^{-U}(e^U\phi')' = -\frac{1}{2}\sum_A \epsilon_A a_A S_A T_A (E_A')^2, \tag{19}$$

$$(r^{\tilde{d}+1} S_A E_A')' = 0, \tag{20}$$

where the dilaton field is written as a sum of r- and u-dependent terms: $\phi = \phi_r + \phi_u$.

The field equations (12)–(16), (19), and (20) are simplified considerably by imposing the condition

$$U = 0. \tag{21}$$

It is known that under this condition, all the supersymmetric intersecting brane solutions can be derived [29].

Now the dilaton equation (19) gives

$$\phi'' + \frac{(\tilde{d}+1)}{r}\phi' = -\frac{1}{2}\sum_A \epsilon_A a_A S_A T_A (E_A')^2, \tag{22}$$

In order to cancel the u-dependence in Eq. (22), we should set

$$4f_A = -\epsilon_A a_A \phi_u + 2\sum_{\alpha\in q_A} v_\alpha, \tag{23}$$

namely $T_A = 1$. Then the field equation for the dilaton reduces to the one discussed in [28].

Thus, for our ansatz that the exponents of metric functions, the dilaton and other backgrounds are simply sums of terms dependent on u and r, we see that the Einstein equations and other field equations separate into terms dependent on u and r. The metric corresponds to the "warped form" of supersymmetric time-dependent solutions and static branes, giving a generalization of D3-branes in time-dependent backgrounds in Refs. [17,19]. So these parts in the field equations should be separately satisfied. The r-dependent equations are simply those of Ref. [28] for static branes in the pp-wave backgrounds except for Eq. (13). These r-dependent equations are already solved in Ref. [28].

For Eq. (13), the u-dependent part should balance with each other:

$$-\sum_{\alpha=1}^{d-2} \ddot{v}_\alpha - (\tilde{d}+2)\ddot{C} - \sum_{\alpha=1}^{d-2}\dot{v}_\alpha^2 - (\tilde{d}+2)\dot{C}^2 + 2\dot{v}_0\left\{\sum_{\alpha=1}^{d-2}\dot{v}_\alpha + (\tilde{d}+2)\dot{C}\right\} = \frac{1}{2}(\dot{\phi}_u)^2, \tag{24}$$

which is the only constraint that the u-dependent terms should satisfy.

Here we note that there are three classes of solutions depending on whether we have the plane-wave function K or not and which coordinate dependence it has. We now discuss these solutions separately.

A. Brane solutions in time-dependent backgrounds

If we do not have the function K, its source should be zero:

$$b_i = 0, \tag{25}$$

which is enough to eliminate the u-dependence from Eq. (13), and the only condition that v_α should satisfy is Eq. (24). The resulting equations for r-dependent factors are the same as the static branes [29], and we find the solutions are simply given by

$$\begin{aligned} ds_D^2 &= \prod_A H_A^{2(q_A+1)/\Delta_A}\Bigg[-e^{2v_0(u)}\prod_A H_A^{-2(D-2/\Delta_A)} 2dudv \\ &\quad + \sum_{\alpha=1}^{d-2}\prod_A H_A^{-2\gamma_\alpha^{(A)}/\Delta_A} e^{2v_\alpha(u)} dy_\alpha^2 \\ &\quad + e^{2C(u)}(dr^2 + r^2 d\Omega_{\tilde{d}+1}^2)\Bigg], \\ E_A &= \sqrt{\frac{2(D-2)}{\Delta_A}} H_A^{-1}, \\ \phi &= \sum_A \epsilon_A a_A \frac{D-2}{\Delta_A}\ln H_A + \phi_u, \end{aligned} \tag{26}$$

where Δ_A and $\gamma_A^{(\alpha)}$ are defined by

$$\Delta_A = (q_A + 1)(D - q_A - 3) + \frac{1}{2}a_A^2(D-2), \tag{27}$$
$$\gamma_A^{(\alpha)} = \begin{cases} D-2 \\ 0 \end{cases} \quad \text{for} \begin{cases} y_\alpha \text{ belonging to } q_A\text{-brane} \\ \text{otherwise} \end{cases},$$

respectively, and ϕ_u should be determined by the relation (24). Here and in what follows, $H_A = 1 + \frac{Q_A}{r^{\tilde{d}}}$ represent harmonic functions in $\tilde{d}+2$ dimensions. These give the generalization of the class of solutions discussed in Ref. [19], in which D3-brane solutions are given with $C = 0$. Our solutions generalize these to orthogonally intersecting branes with nonvanishing C.

B. Branes in plane-wave backgrounds

If K does not depend on y_α, $\partial_\alpha^2 K$ term in Eq. (13) is absent. Considering Eq. (10), we find that the condition

$$4g(u) = 2(v_0 + C) + \phi_u, \tag{28}$$

is enough to eliminate u-dependence from Eq. (13). It is then easy to give the solutions, following Ref. [28].”

The result is

$$ds_D^2 = \prod_A H_A^{2(q_A+1/\Delta_A)}\Big[e^{2v_0(u)}\prod_A H_A^{-2(D-2)/\Delta_A}\{-2dudv + Kdu^2\} + \sum_{\alpha=1}^{d-2}\prod_A H_A^{-2\gamma_A^{(\alpha)}/\Delta_A}e^{2v_\alpha}dy_\alpha^2 + e^{2C(u)}(dr^2 + r^2 d\Omega_{\tilde{d}+1}^2)\Big], \tag{29}$$
$$E_A = \sqrt{\frac{2(D-2)}{\Delta_A}}H_A^{-1},$$
$$\phi = \sum_A \epsilon_A a_A \frac{D-2}{\Delta_A}\ln H_A + \phi_u,$$

where $\gamma_A^{(\alpha)}$ is the same as in Eq. (27) and the function K is defined by

$$\Box^{(\tilde{d}+2)}K = -\frac{1}{2}(\mu_{ij})^2\prod_A H_A^{l_A}, \tag{30}$$

where

$$l_A = \frac{4(q_A+1) + (\epsilon_A a_A - 2)(D-2)}{\Delta_A},$$

and ϕ_u should be determined by the relation (24).

In Ref. [19], D3-brane solutions with $\mu_{ij} = 0$ and $C = 0$ are given. There K is taken to depend only on u, in which case we see that Eq. (30) is trivially satisfied and K can be an arbitrary function of u, in agreement with Ref. [19]. The same solutions in different coordinate system are also given in [17]. Our solutions, when restricted to single branes, are still more general than those.

C. Branes in more general wave backgrounds

If we have the general function K describing waves, we must have

$$4g(u) = 2(v_0 + C) + \phi_u, \qquad v_\alpha = C, \tag{31}$$

in order to get rid of u-dependence from Eq. (13). Then we see that Eq. (24) reduces to

$$-(D-2)(\ddot{C} + \dot{C}^2 - 2\dot{v}_0\dot{C}) = \tfrac{1}{2}(\dot{\phi}_u)^2. \tag{32}$$

and the remaining condition from Eq. (13) precisely gives the corresponding one in Ref. [28].

The determination of the remaining functions are essentially the same as [28], so we do not repeat the details, but simply present the final result:

$$ds_D^2 = \prod_A H_A^{2(q_A+1)/\Delta_A}\Big[e^{2v_0(u)}\prod_A H_A^{-2(D-2)/\Delta_A}\{-2dudv + Kdu^2\} + e^{2C(u)}\sum_{\alpha=1}^{d-2}\prod_A H_A^{-2\gamma_A^{(\alpha)}/\Delta_A}dy_\alpha^2 + e^{2C(u)}(dr^2 + r^2 d\Omega_{\tilde{d}+1}^2)\Big], \tag{33}$$
$$E_A = \sqrt{\frac{2(D-2)}{\Delta_A}}H_A^{-1},$$
$$\phi = \sum_A \epsilon_A a_A \frac{D-2}{\Delta_A}\ln H_A + \phi_u,$$

where $\gamma_A^{(\alpha)}$ is the same as in Eq. (27) and the function K is defined by

$$\Big(\Box^{(\tilde{d}+2)} + \sum_{\alpha=1}^{d-2}\prod_A H_A^{2(\gamma_A^{(\alpha)}/\Delta_A)}\partial_\alpha\Big)K = -\frac{1}{2}(\mu_{ij})^2\prod_A H_A^{l_A}, \tag{34}$$

respectively, and ϕ_u should be determined by the relation (24). The function K is essentially the same as given in Ref. [28]. For a single Dq_A-brane, Eq. (34) admits a solution of the form

$$K = c + \frac{Q}{r^{\tilde{d}}} - \frac{(\mu_{ij})^2}{32}\Big(r^2 + \sum_\alpha y_\alpha^2 + \frac{(q_A-1)}{(\tilde{d}-2)}\frac{Q_A}{r^{\tilde{d}-2}}\Big), \quad (\text{for } \tilde{d} \neq 2) \tag{35}$$

and

$$K = c + \frac{Q}{r^{\tilde{d}}} - \frac{(\mu_{ij})^2}{32}\Big(r^2 + \sum_\alpha y_\alpha^2 - (q_A-1)Q_A\ln r\Big), \quad (\text{for } \tilde{d} = 2). \tag{36}$$

We also have the intersection rules for the branes [28]. If q_A-brane and q_B-brane intersect over $\bar{q}(\le q_A, q_B)$ dimensions, this gives

$$\bar{q} = \frac{(q_A + 1)(q_B + 1)}{D - 2} - 1 - \frac{1}{2}\epsilon_A a_A \epsilon_B a_B. \qquad (37)$$

For D-branes $\epsilon_A a_A = \frac{3-q_A}{2}$, and we get

$$\bar{q} = \frac{q_A + q_B}{2} - 2. \qquad (38)$$

The results presented here are the generalization of the intersection rules already discussed in the literature [29] to the supersymmetric intersecting branes in time-dependent backgrounds.

D. Supersymmetry

In the time-dependent background without the NS-NS flux, the amount of unbroken supersymmetry depends on the condition $\gamma_u \epsilon = 0$, and the brane supersymmetry condition. Usually these two conditions are two independent conditions, and one needs to impose them both on the gravitino and the dilatino variations. They are compatible with each other, but the unbroken supersymmetry is $1/2$ compared with the usual static brane solutions. So if $\mu_{ij} = 0$, we have $1/2$ supersymmetry for no branes, $1/4$ supersymmetry for single branes as discussed in Refs. [17,19], $1/8$ supersymmetry for 2 orthogonally intersecting branes and so on.

In the presence of the NS-NS flux H_{uij}, the supersymmetry variations normally restrict the form of the flux for getting a solution of the killing spinor equation of motion. As it was argued in [28], this choice further breaks $1/2$ of the remaining supersymmetry.

To get an idea how the solutions look like, let us consider intersecting D1–D5 system without μ_{ij} (the case in subsection II B). The supergravity solution is given by

$$\begin{aligned} ds^2 &= H_1^{-(3/4)} H_5^{-(1/4)} e^{2v_0(u)}(-2dudv + K(u)du^2) \\ &\quad + \left(\frac{H_1}{H_5}\right)^{1/4} \sum_{\alpha=1}^{4} e^{v_\alpha(u)} dy_\alpha^2 \\ &\quad + H_1^{1/4} H_5^{3/4} e^{2C(u)}(dr^2 + r^2 d\Omega_3^2), \\ \phi &= \ln\left(\frac{H_1}{H_5}\right)^{1/2} + \phi_u. \end{aligned} \qquad (39)$$

We see that the solution resembles the ordinary intersecting branes, and the difference is the presence of time-dependent factors multiplying the metric and terms appearing in the dilaton and forms such as (7). In the present intersecting branes in a wave background with $\mu_{ij} = 0$, the amount of unbroken supersymmetry is four, as this preserves $1/8$ of the full type IIB supersymmetry, corresponding to $N = 2$ supersymmetry in 2 dimensions. If the metric depending on the light-cone coordinate u are finite, the near horizon geometry is $AdS_3 \times S^3 \times M^4$. As argued in Ref. [30], the corresponding dual field theory would be two-dimensional conformal field theory, now in time-dependent backgrounds.

The D1–D5 system has a singularity (big bang) at $v_0(u) \to -\infty$, as some components of the metric vanish and we can easily see that the curvature components also blow up. For example, let us choose the dilaton and metric functions linear in the coordinate u: $\phi = -au(a > 0)$, $v_0(u) = v_\alpha(u) = bu$, $C(u) = cu$. The condition (24) tells us that $a^2 + 16c^2 = 8(b + c)^2$. For the simple choice $c = K = 0$ and $b = a/2\sqrt{2}$, there appears a singularity in the infinite past in the solution (39).

III. CONCLUDING REMARKS

Motivated by the recent interest in supersymmetric time-dependent solutions in supergravity with their possible application to the singularities in our spacetime, we have derived a rather general class of solutions with and without pp-wave. Our solutions reproduce many of the known time-dependent solutions but are more general than those already known. These solutions have time-dependent null dilaton which is related to the string coupling. So the string theory has time-dependent coupling.

According to the AdS/CFT correspondence, it is expected that these solutions have dual description in terms of super Yang-Mills theories. It would be very interesting to further explore the nature of these solutions, especially their singularity structures, and their field theory duals. An interesting question is whether and how the field theory dual gives well-defined description of the behavior of the solutions close to the singularities. It would also be important to find more cosmological applications of these solutions and try to understand the nature of the spacelike singularities.

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