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Three-dimensional nonanticommutative superspace

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We propose two alternative formulations for a three-dimensional nonanticommutative superspace in which some of the fermionic coordinates obey Clifford anticommutation relations. For this superspace, we construct the supersymmetry generators satisfying standard anticommutation relations and the corresponding supercovariant derivatives. We formulate a scalar superfield theory in such a superspace and calculate its propagator. We also suggest a prescription for the introduction of interactions in such theories.

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The concept of noncommutative coordinates has deep motivations originated from fundamental properties of the spacetime and string theory [1]. In this context a recent and very important development was the introduction of a nonanticommutative superspace [2,3] in which the noncommutativity affects not only the usual spacetime coordinates, like in [1], but also the fermionic coordinates. Such an idea was shown to have some foundations in string theory [4].

In the four-dimensional case, the superspace in which the fermionic coordinates are nonanticommutative possesses a very specific form of supersymmetry, called $N = \frac{1}{2}$ supersymmetry [3,5]. This fact generated a great deal of interest in field theories formulated in the four-dimensional nonanticommutative superspace. Among the results obtained in the ensuing investigations, we would like to mention the proof of the renormalizability of the nonanticommutative Wess-Zumino model [6] and the development of the background field method for the super-Yang-Mills theory [7]. However, until this time the nonanticommutative superspace formulation for the three-dimensional spacetime has not been discussed, despite the fact that supersymmetry in three dimensions is very simple (see its detailed description in [8]; interesting examples of noncommutative three-dimensional superfield theories can be found in [9–12]). The main reason for such situation seems to be the following: the relative simplicity of the four-dimensional noncommutative superspace is based on the fact that one can keep untouched half of the supersymmetry generators, which obeys standard anticommutation relations. This possibility arises because the Lorentz group has two independent fundamental spinor representations (the dotted and the undotted ones), and correspondingly there are two sets of supersymmetry generators. In this case, we have the choice of imposing nontrivial anticommutation relations between the fermionic coordinates affecting the algebra of only one of the sets (the dotted ones, let us say), preserving the algebra of the other set (namely, the undotted ones) [13]. In the three-dimensional spacetime, however, the lowest dimensional spinor representation of the Lorentz group is real, so there is only one set of supersymmetry generators. Hence, imposing nontrivial anticommutation relations between the fermionic coordinates could break completely the supersymmetry algebra. Although this possibility does not necessarily lead to inconsistencies, in this work we require that at least some explicit supersymmetry survive. Hence, we have to develop alternative ways to introduce nonanticommutativity in the three-dimensional superspace.

Starting with $N = 1$ supersymmetry we may define modified supersymmetry generators satisfying the standard anticommutation relations but constructing the field theories would be problematic. We will return to this point later. For this reason, in this paper we begin with the $N = 2$ supersymmetry. So the starting point of the three-dimensional nonanticommutative superspace is the following anticommutation relation for the fermionic coordinates $\theta^{i\alpha}$:

$$\{\theta^{i\alpha}, \theta^{j\beta}\} = \Sigma^{ij\alpha\beta}. \qquad (1)$$

Here $\alpha, \beta = 1, 2$ are Lorentz spinor indices, $i, j = 1, 2$ are labels for the fermionic coordinates corresponding to each supercharge, and $\Sigma^{ij\alpha\beta}$ is a constant matrix symmetric under the exchange of $i\alpha$ and $j\beta$. Let us construct the supersymmetry generators for this case. For simplicity and for the sake of clarification of the impact of this relation, we assume that all non(anti)commutativity is concentrated in the fermionic coordinate sector, the bosonic spacetime coordinates being assumed to commute.

In the case of nonanticommutative superspace, the usual supersymmetry generators (here $\partial^i_\alpha = \frac{\partial}{\partial\theta^{i\alpha}}$, we use the notations of [8])

$$Q^i_\alpha = i\partial^i_\alpha + \theta^{i\beta}\partial_{\beta\alpha}, \qquad (2)$$

have the following anticommutation relation (we assume

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that $\{\partial^i_\alpha, \partial^j_\beta\} = 0$):

$$\{Q^i_\alpha, Q^j_\beta\} = 2P_{\alpha\beta}\delta^{ij} - \Sigma^{ij\gamma\delta}P_{\gamma\alpha}P_{\delta\beta}, \tag{3}$$

where $P_{\alpha\beta} = i\partial_{\beta\alpha}$ are the momentum generators. This relation differs in an essential way from the general assumptions of standard supersymmetric theories since it is nonlinear, implying that the anticommutator of two transformations generated by the Q's is not a translation but a more complicated transformation.

At first sight one could try to overcome this difficulty by finding new supersymmetry generators, $\tilde{Q}^i_\alpha$, which would satisfy the standard anticommutation relation

$$\{\tilde{Q}^i_\alpha, \tilde{Q}^j_\beta\} = 2\delta^{ij}P_{\alpha\beta}. \tag{4}$$

It is easily verified that this relation is satisfied if

$$\tilde{Q}\,^i_\alpha = Q^i_\alpha + \frac{i}{2}\Sigma^{ij\beta\gamma}\partial^j_\beta P_{\gamma\alpha}. \tag{5}$$

However, in the next step, to fix a (new) supercovariant derivative $\tilde{D}^i_\alpha$ this approach meets a stumbling block. In fact, the condition of supersymmetric covariance demands

$$\{\tilde{D}^i_\alpha, \tilde{Q}^j_\beta\} = 0. \tag{6}$$

Taking into account the basic relation (1) and the expression (5) for the supersymmetry generator, a natural candidate would be

$$\tilde{D}\,^i_\alpha = D^i_\alpha + \frac{1}{2}\Sigma^{ij\beta\gamma}\partial^j_\beta P_{\gamma\alpha}, \tag{7}$$

where $D^i_\alpha = \partial^i_\alpha + i\theta^{i\beta}\partial_{\beta\alpha}$ is the standard supercovariant derivative. Nevertheless, these operators involve second derivatives and do not satisfy Leibnitz's rule, so one cannot use integration by parts, which turns cumbersome the D-algebra transformations [14]. Had we insisted in the formulation of a nonanticommutative superspace beginning with $N = 1$ supersymmetry, we would arrive at the same difficulty. This is the problem mentioned earlier.

However, since we started with $N = 2$, there are alternative ways to introduce the nonanticommutativity. We will describe two of them. Our first proposal is similar to the conception of the $N = (1, 1)$ superspace [15] in four-dimensional superspace. We break half of supersymmetries, that is, those corresponding to the generators Q^2_α: we choose $\Sigma^{11\alpha\beta} = \Sigma^{21\alpha\beta} = \Sigma^{12\alpha\beta} = 0$, so that the only nontrivial anticommutation relation for the fermionic coordinates is

$$\{\theta^{2\alpha}, \theta^{2\beta}\} = \Sigma^{\alpha\beta}, \tag{8}$$

hence only the unbroken generators Q^1_α satisfy the standard anticommutation relation

$$\{Q^1_\alpha, Q^1_\beta\} = 2i\partial_{\alpha\beta}. \tag{9}$$

The supercovariant derivatives defined to anticommute with these generators are [16]

$$D^i_\alpha = \partial^i_\alpha + i\theta^{i\beta}\partial_{\beta\alpha}, \tag{10}$$

and one can verify that

$$\{D^1_\alpha, D^1_\beta\} = 2i\partial_{\alpha\beta}, \qquad \{D^1_\alpha, D^2_\beta\} = 0,$$
$$\{D^2_\alpha, D^2_\beta\} = 2i\partial_{\alpha\beta} - \Sigma^{\gamma\delta}\partial_{\gamma\alpha}\partial_{\delta\beta}. \tag{11}$$

The most natural definition of the Moyal product compatible with the condition of supersymmetric covariance is

$$\Phi_1(z) * \Phi_2(z) = \exp\Big[-\frac{1}{2}\Sigma^{\alpha\beta}D^2_\alpha(z_1)D^2_\beta(z_2)\Big] \times \Phi_1(z_1)\Phi_2(z_2)\Big|_{z_1=z_2=z}. \tag{12}$$

This form is similar to the Moyal product introduced in [3]. We note the presence of supercovariant derivatives in the exponent instead of ordinary derivatives ∂_α. This is so because ordinary spinor derivatives do not anticommute with the supersymmetry generators Q^1_α and their use in (12) would lead to more complicated transformation rules. We note also that the sign of the exponent is fixed by the condition that the (Moyal) anticommutator $\{\theta^{2\alpha}, \theta^{2\beta}\}_*$ must reproduce the relation (8). Expanding the exponential in (12) we obtain,

$$\begin{aligned}
&\exp\Big[-\frac{1}{2}\Sigma^{\alpha\beta}D^2_\alpha(z_1)D^2_\beta(z_2)\Big]\Phi_1(z_1) * \Phi_2(z_2)\Big|_{z_1=z_2=z} \\
&\quad\equiv\Big[1 - \frac{1}{2}\Sigma^{\alpha\beta}D^2_\alpha(z_1)D^2_\beta(z_2) \\
&\qquad + \frac{1}{8}\Sigma^{\alpha\beta}D^2_\alpha(z_1)D^2_\beta(z_2)\Sigma^{\gamma\delta}D^2_\gamma(z_1)D^2_\delta(z_2) + \ldots\Big] \\
&\qquad \times \Phi_1(z_1)\Phi_2(z_2)\Big|_{z_1=z_2=z} \\
&\quad= \Phi_1(z)\Phi_2(z) - \frac{1}{2}\Sigma^{\alpha\beta}D^2_\alpha\Phi_1(z)D^2_\beta\Phi_2(z) \\
&\qquad - \frac{1}{8}\Sigma^{\alpha\beta}\Sigma^{\gamma\delta}D^2_\alpha D^2_\gamma\Phi_1(z)D^2_\beta D^2_\delta\Phi_2(z) + \ldots
\end{aligned} \tag{13}$$

A distinct feature of (13) is that, unlike the four-dimensional case, it involves all orders in D^2_α, since there is no power of the supercovariant derivative in three dimensions which identically vanishes. For the Moyal product of three or more superfields this definition can be straightforwardly generalized.

We are now in a position to define the propagators for field theories in the nonanticommutative superspace. We consider the simplest example, i.e., the scalar superfield theory. The most natural form of its free action is

$$S = -\frac{1}{4}\int d^7z\Phi * \Delta\Phi, \tag{14}$$

where $d^7z = d^3xd^2\theta^1d^2\theta^2$ and Δ is some operator which does not depend on θ^2 so that it is not affected by the Moyal product.

The reader should not be misled by the apparent simplicity of this action. Indeed, in contrast with the four-dimensional case, the above expression nontrivially involves the noncommutativity matrix $\Sigma^{\alpha\beta}$! In other words, the superspace integral of the Moyal product of two fields essentially differs from the usual case. Really, it follows from (13) that

$$S = \frac{1}{2}\int d^7z\Big(\Phi(z)\Delta\Phi(z) - \frac{1}{2}\Sigma^{\alpha\beta}D^2_\alpha\Phi(z)D^2_\beta\Delta\Phi(z) - \frac{1}{8}\Sigma^{\alpha\beta}\Sigma^{\gamma\delta}D^2_\alpha D^2_\gamma\Phi(z)D^2_\beta D^2_\delta\Delta\Phi(z) + \ldots\Big). \tag{15}$$

By moving all derivatives to the field affected by Δ by means of integration by parts, we get

$$S = \frac{1}{2}\int d^7z\Big(\Phi(z)\Delta\Phi(z) + \frac{1}{2}\Sigma^{\alpha\beta}\Phi(z)D^2_\alpha D^2_\beta\Delta\Phi(z) + \frac{1}{8}\Sigma^{\alpha\beta}\Sigma^{\gamma\delta}\Phi(z)D^2_\gamma D^2_\alpha D^2_\beta D^2_\delta\Delta\Phi(z) + \ldots\Big). \tag{16}$$

In general, the nth order in Σ term has the form

$$A_n = \frac{1}{2^{n+1}n!}\int d^7z\Phi(z)\Sigma^{\alpha_n\beta_n}\ldots \times \Sigma^{\alpha_2\beta_2}\Sigma^{\alpha_1\beta_1}D^2_{\alpha_n}D^2_{\alpha_{n-1}}\ldots D^2_{\alpha_1}D^2_{\beta_1}\ldots D^2_{\beta_n}\Delta\Phi(z). \tag{17}$$

The crucial step here is a very specific grouping of derivatives which can be efficiently simplified. Indeed, as $D^2_{\alpha_1}$ and $D^2_{\beta_1}$ in this expression are adjacent and are contracted with the symmetric matrix $\Sigma^{\alpha_1\beta_1}$, we can use (11) to replace $D^2_{\alpha_1}D^2_{\beta_1}$ by $\frac{1}{2}\{D^2_{\alpha 1}, D^2_{\beta 1}\} = P_{\alpha_1\beta_1} + \frac{1}{2}\Sigma^{\gamma\delta}P_{\gamma\alpha_1}P_{\delta\beta_1}$. We call the combination $\Sigma^{\alpha_1\beta_1}D^2_{\alpha_1}D^2_{\beta_1}$ as

$$R(P) \equiv \Sigma^{\alpha\beta}\Big(P_{\alpha\beta} + \frac{1}{2}\Sigma^{\gamma\delta}P_{\gamma\alpha}P_{\delta\beta}\Big) = 2\Sigma\cdot P + 2(\Sigma\cdot P)^2 - \Sigma^2P^2, \tag{18}$$

where we have introduced the vector $\Sigma^n = \frac{1}{2}(\gamma^n)^{\alpha\beta}\Sigma_{\alpha\beta}$, and $\Sigma\cdot P = \Sigma^m P_m$. Then, since $R(P)$ commutes with supercovariant derivatives, we can factor it out from (17) and consider the next pair of derivatives, contracted as $\Sigma^{\alpha_2\beta_2}D_{\alpha_2}D_{\beta_2}$. As a result we obtain another factor of $R(P)$. After repeating this procedure n times, we eventually arrive at

$$A_n = \frac{1}{2n!}\int d^7z\Phi(z)\Big[\frac{R(P)}{2}\Big]^n\Delta\Phi(z). \tag{19}$$

Using this result, the quadratic action (14) takes the form,

$$S = \frac{1}{2}\int d^7z\Phi(z)e^{R(P)/2}\Delta\Phi(z). \tag{20}$$

Notice that, differently from the spacetime noncommutativity, the above quadratic action is deeply affected by the Moyal product. The corresponding momentum space propagator reads

$$\langle\Phi(-p,\theta_1)\Phi(p,\theta_2)\rangle = ie^{-R(P)/2}\Delta^{-1}(p)\delta^4_{12}. \tag{21}$$

One possibility to avoid the blow up of the propagator with the growth of the momentum is to choose the Σ^m vector to be lightlike, $\Sigma^2 = 0$, which gives $\langle\Phi(-p,\theta_1)\Phi(p,\theta_2)\rangle = ie^{-\Sigma\cdot p-(\Sigma\cdot p)^2}\Delta^{-1}(p)\delta^4_{12}$, thus implying in a fast (exponential) decrease with the momentum p along certain directions. A more interesting possibility is to choose Σ^m to be timelike, $\Sigma^m = (\Sigma_0, 0, 0)$, which gives $\langle\Phi(-p,\theta_1)\Phi(p,\theta_2)\rangle = ie^{\Sigma_0p_0-\Sigma_0^2(p_0^2+\vec{p}^2)}\Delta^{-1}(p)\delta^4_{12}$ with exponential decrease along all directions; in the case of spacelike Σ^m the theory is unstable. In all cases the propagator reflects the fact that the quadratic action is highly nonlocal.

The interaction vertices are defined by a direct generalization of (12) for arbitrary number of fields,

$$\Phi_1(z)*\Phi_2(z)*\ldots*\Phi_n(z) = \exp\Big[-\frac{1}{2}\sum_{i\le j\le n}\Sigma^{\alpha\beta}D^2_\alpha(z_i)D^2_\beta(z_j)\Big] \times \Phi_1(z_1)\Phi_2(z_2)\ldots\Phi_n(z_n)\Big|_{z_1=z_2=\ldots z_n=z}. \tag{22}$$

The perturbative series should be ordered by powers of the Σ that appears in the interaction terms (the propagator must not be expanded). This procedure, in the case of the timelike Σ^m guarantees the order by order finiteness of this expansion.

A natural question which could arise in connection with our construction concerns the role played by the supersymmetry generated by the Q^1 supercharge. To clarify further this point let us consider the following example. The $N = 2$ spinor superfield $\Psi_\alpha(z)$ can be expanded in terms of the $N = 1$ superfields as:

$$\Psi_\alpha(z) = \lambda_\alpha(x,\theta^1) + \theta^2_\alpha\Phi(x,\theta^1) + \theta^{2\beta}A_{\alpha\beta}(x,\theta^1) + (\theta^2)^2F_\alpha(x,\theta^1), \tag{23}$$

where $\Phi(x,\theta^1)$, $\lambda_\alpha(x,\theta^1)$, $A_{\alpha\beta}(x,\theta^1)$, $F_\alpha(x,\theta^1)$ are $N = 1$ superfield components of the $N = 2$ supermultiplet, and $A_{\alpha\beta}(x,\theta^1)$ is a symmetric bispinor. If we introduce the action for such a superfield as being

$$S = -\frac{1}{4}\int d^7z\Psi^\alpha(z)*(D^1)^2\Psi_\alpha(z), \tag{24}$$

which is a special case of the Eq. (14), we can apply the arguments given above to show that this action can be rewritten as

$$-\frac{1}{4}\int d^7z\Psi^\alpha(z)e^{R(P)/2}(D^1)^2\Psi_\alpha(z). \tag{25}$$

Substituting here the expansion (23), we arrive at the following action:

$$-\frac{1}{2}\int d^7z(\theta^2)^2[\Phi(x,\theta^1)e^{R(P)/2}(D^1)^2\Phi(x,\theta^1) + 2\lambda^\alpha(x,\theta^1)e^{R(P)/2}(D^1)^2F_\alpha(x,\theta^1) + A^{\alpha\beta}(x,\theta^1)e^{R(P)/2}(D^1)^2A_{\alpha\beta}(x,\theta^1)], \quad (26)$$

so the integral over $d^2\theta^2$ is factorized out. Hence, after integration over θ^2 the action takes the form

$$\frac{1}{2}\int d^5z[\Phi(x,\theta^1)e^{R(P)/2}(D^1)^2\Phi(x,\theta^1) + 2\lambda^\alpha(x,\theta^1)e^{R(P)/2}(D^1)^2F_\alpha(x,\theta^1) + A^{\alpha\beta}(x,\theta^1)e^{R(P)/2}(D^1)^2A_{\alpha\beta}(x,\theta^1)], \quad (27)$$

and, as a consequence, we have generated the exponential factor for each $N = 1$ superfield component of the $N = 2$ supermultiplet. We conclude that, despite the second supersymmetry is being explicitly broken, its nontrivial impact persists even after integration over the corresponding spinor coordinates. In other words, the $N = 1$ theory still "remembers" the second supersymmetry.

Our second proposal to introduce nonanticommutativity for fermionic coordinates is the following: we first introduce the $N = 2$ supersymmetry with the nontrivially mixed generators:

$$Q^i_\alpha = i\partial^i_\alpha + (\sigma_1)^{ij}\theta^{j\beta}\partial_{\beta\alpha}. \quad (28)$$

To mix two supersymmetries in a nontrivial way, we have introduced the symmetric object $(\sigma_1)^{ij}$, where

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

is a Pauli matrix. The explicit form of the new generators is

$$Q^1_\alpha = i\partial^1_\alpha + \theta^{2\beta}\partial_{\beta\alpha}, \qquad Q^2_\alpha = i\partial^2_\alpha + \theta^{1\beta}\partial_{\beta\alpha}. \quad (29)$$

In the commutative case, their anticommutation relation is

$$\{Q^i_\alpha, Q^j_\beta\} = 2i(\sigma_1)^{ij}\partial_{\alpha\beta}. \quad (30)$$

The covariant derivatives D^i_α which anticommute with these generators, $\{D^i_\alpha, Q^j_\beta\} = 0$, are

$$D^i_\alpha = \partial^i_\alpha + i(\sigma_1)^{ij}\theta^{j\beta}\partial_{\beta\alpha}. \quad (31)$$

Now let us partially break this supersymmetry by imposing the following nontrivial anticommutator for spinor coordinates:

$$\{\theta^2_\alpha, \theta^2_\beta\} = \Sigma_{\alpha\beta}. \quad (32)$$

As a result, $\{Q^1_\alpha, Q^1_\beta\} = \Sigma^{\gamma\delta}\partial_{\gamma\alpha}\partial_{\delta\beta}$ is deformed while other anticommutators of the generators will remain unchanged. It is easy to see that the D^i_α derivatives, both for $i = 1, 2$ still anticommute with the unbroken generators Q^2_α. We note that in this case we cannot factorize two supersymmetries.

The derivatives D^i_α satisfy the following anticommutation relations:

$$\{D^2_\alpha, D^2_\beta\} = 0, \qquad \{D^1_\alpha, D^2_\beta\} = -2i\partial_{\alpha\beta}, \qquad \{D^1_\alpha, D^1_\beta\} = -\Sigma^{\gamma\delta}\partial_{\alpha\gamma}\partial_{\beta\delta}. \quad (33)$$

In this case, the most natural definition of the Moyal product compatible with the condition of supersymmetric covariance is

$$\Phi_1(z) * \Phi_2(z) = \exp\left[-\frac{1}{2}\Sigma^{\alpha\beta}D^2_\alpha(z_1)D^2_\beta(z_2)\right] \times \Phi_1(z_1)\Phi_2(z_2)\Big|_{z_1=z_2=z}. \quad (34)$$

This form is similar to the Moyal product introduced in [3]. For exact the same reason as in (12), only supercovariant derivatives are allowed in the exponent, and the sign of the exponent is fixed for Eq. (34) to be compatible with (8). Similarly to [3], in this case the Moyal product is a finite power series because third and higher degrees of D^2_β are equal to zero due to (33). Also, we can verify that the quadratic action of any nonanticommutative theory will coincide with its anticommutative analog, that is, $\int d^7z\Phi_1 * \Phi_2 = \int d^7z\theta\Phi_1\Phi_2$. Hence, in this case, the propagators will not be changed after the nonanticommutative deformation, only vertices will be affected by the nonanticommutativity, which implies in the arising of a finite number of extra terms. For example, the nonanticommutative analog of the Φ^3 vertex will take the form

$$\int d^7z\Phi * \Phi * \Phi = \int d^7z\{\Phi^3 - (\det\Sigma)\Phi[(D^1)^2\Phi]^2\}. \quad (35)$$

Other field theories in this nonanticommutative superspace can be analogously defined.

Let us now summarize our results. We proposed two alternative formulations for the three-dimensional nonanticommutative superspace. This required the construction of a new representation of the supersymmetry algebra, which was realized by starting with a $N = 2$ algebra, which was explicitly broken down to $N = 1$ in two different ways. By using the supercovariant derivatives, we constructed a Moyal product compatible with the remaining supersymmetry. Unlike the four-dimensional case, in the first formulation, the Moyal product is an infinite power series in the noncommutativity matrix $\Sigma^{\alpha\beta}$. Furthermore, the quadratic action is affected by the noncommutativity. As a result, the propagator has a very unusual form characterized, for timelike Σ^m, by its exponential decay with the momentum, yielding very good convergence properties for the Feynman integrals. In particular, it implies that the problem of UV/IR mixing [17], crucial in the usual noncommutative field theories, is absent in the theories with the fermionic nonanticommutativity, treated as we suggested. In the second formulation, the Moyal product is polynomial just as in the four-dimensional case, its impact consisting in the appearance of extra vertices, with the UV/IR mixing again absent. Our next step consists in a more detailed study of quantum corrections. Another interesting problem is the study of unitarity and causality properties in these new field theories.

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[16] Notice that the supercovariant derivative D^2_α anticommutes with the following extension of the generator Q^2_β: $\tilde{Q}^2_\beta = Q^2_\beta - \Sigma^{\mu\rho}\partial^2_\rho\partial_{\mu\beta}$, with $\{\tilde{Q}^2_\alpha, \tilde{Q}^2_\beta\} = 2P_{\alpha\beta} + \Sigma^{\mu\nu}P_{\mu\alpha}P_{\nu\beta}$.

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