Nonlinear spinor field in Bianchi type-I cosmology: Inflation, isotropization, and late time acceleration

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A self-consistent system of interacting nonlinear spinor and scalar fields within the scope of a Bianchi type-I cosmological model filled with perfect fluid is considered. Exact self-consistent solutions to the corresponding field equations are obtained. A role of the spinor field in the evolution of the Universe is studied. It is shown that the spinor field gives rise to an accelerated mode of expansion of the Universe. Early in the evolution, the spinor field nonlinearity generates an accelerated mode of expansion. The rapid growth of the Universe in this case results in the earlier isotropization. At the later evolution the acceleration of the Universe is provided with a spinor mass.

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I. INTRODUCTION

Some recent observations suggest that the universe is spatially flat and undergoing a period of accelerated expansion. In order to explain this accelerated mode of expansion of the present-day Universe, cosmologists introduced different kinds of source fields:

(i) Quintessence [1–4] with $w = p_{\rm DE}/\varepsilon_{\rm DE}$. A special member of this case is the cosmological constant $w = -1$ [5–7].
(ii) Chaplygin gas [8,9] with the equation of state $p_{\rm DE} = A/\varepsilon_{\rm DE}$ with A being some positive constant.
(iii) Phantom dark energy (DE) with $w < -1$.
(iv) Oscillating DE.
(v) models with interaction between DE and dark matter.
(vi) Scalar-tensor DE models, etc.

Recently cosmological models with spinor field have been extensively studied by a number of authors in a series of papers [10–15]. A principal goal of the papers [10–14] was to find out the regular solutions of the corresponding field equations. In some special cases, namely, with a cosmological constant (Λ term) that plays the role of an additional gravitation field, we indeed find singularity-free solutions. It was also found that the introduction of nonlinear spinor field results in a rapid growth of the Universe. This allows us to consider the spinor field as a possible candidate to explain the accelerated mode of expansion. Note that similar attempt is made in a recent paper by Kremer *et al.* [16].

The simplest models of expanding Universe are those which are spatially homogeneous and isotropic. These models were first studied by Friedmann [17], Robertson [18,19] and Walker [20]. Though spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) models are widely considered as good approximation of the present and early stages of the universe, the large scale matter distribution in the observable universe, largely manifested in the form of discrete structures, does not exhibit homogeneity of a higher order. In contrast, the cosmic background radiation, which is significant in the microwave region, is extremely homogeneous, however, recent space investigations detect anisotropy in the cosmic microwave background. The observations from Cosmic Background Explorer's differential radiometer have detected and measured cosmic microwave background anisotropies in different angular scales. These anisotropies are supposed to hide in their fold the entire history of cosmic evolution dating back to the recombination era and are being considered as indicative of the geometry and the content of the universe. More about cosmic microwave background anisotropy is expected to be uncovered by the investigations of microwave anisotropy probe. There is widespread consensus among the cosmologists that cosmic microwave background anisotropies in small angular scales have the key to the formation of discrete structure. The theoretical arguments [21] and recent experimental data that support the existence of an anisotropic phase that approaches an isotropic one leads to consider the models of universe with anisotropic background. The earliest use of anisotropic cosmological models to study a real cosmological problem was the investigation by Lemaitre [22]. The purpose of that study was to verify whether the big-bang singularity which occurs in FRW model was just a consequence of the assumed symmetry. In the late 60's three different facets of astrophysical research directed interest to homogeneous but anisotropic cosmological models [23]: in discussing the possibility of a primordial magnetic field Zel'dovich [24] and Thorne [25] considered anisotropic cosmologies; in studying factors which might affect the amount of primordial helium production in a big-bang cosmology Hawking and Tayler [26] considered anisotropic models; Kristian and Sachs [27] as well as Kantowski and Sachs [28], in studying the degree to which our universe is actually isotropic, considered anisotropic cosmologies. Zel'dovich was first to assume that the early isotropization of cosmological expanding process can take

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place as a result of quantum effect of particle creation near singularity [29]. This assumption was further justified by several authors [30–32]. Interest in studying Klein-Gordon and Dirac equations in anisotropic models has increased since Hu and Parker [32] have shown that the creation of scalar particles in anisotropic backgrounds can dissipate the anisotropy as the Universe expands. Investigation similar to that of Lamaitre was carried out in the 60's. The purpose was to see if the helium abundance could be fitted better by anisotropic cosmologies than by the FRW one. It might happen because anisotropy speeds up the evolution between the time when deuterium can first form and time when neutrons and photons no longer find each other to combine. Pioneering work in this effort was done by Hawking and Tayler [26].

A Bianchi type-I (BI) universe, being a straightforward generalization of the flat Friedmann-Robertson-Walker (FRW) Universe, is one of the simplest models of an anisotropic Universe that describes a homogenous and spatially flat Universe. Unlike the FRW Universe which has the same scale factor for each of the three spatial directions, the BI universe has a different scale factor in each direction, thereby introducing an anisotropy to the system. It moreover has an agreeable property that near the singularity it behaves like a Kasner Universe, even in the presence of matter, and consequently falls within the general analysis of the singularity given by Belinskii *et al.* [33]. Also in a Universe filled with matter for $p = \zeta\varepsilon$, $\zeta < 1$, it has been shown that any initial anisotropy in the BI universe quickly dies away and a BI universe eventually evolves into a FRW Universe [34]. Since the present-day Universe is surprisingly isotropic, this feature of the BI universe makes it a prime candidate for studying the possible effects of an anisotropy in the early Universe on present-day observations. In light of its importance several authors have studied BI universe from different aspects.

In this paper we study the role of a spinor field in generating an accelerated mode of expansion of the Universe. Note that we consider the late time acceleration as well as the initial inflation, i.e., a period when the Universe might be significantly anisotropic. Taking this into mind as well as the importance and generality of anisotropic models, we consider the gravitational field be given by a BI universe. Beside the accelerated mode of expansion, we also examine the possibilities for singularity-free solutions and isotropization of the initially anisotropic unverse.

II. BASIC EQUATIONS

We consider a system of the nonlinear spinor, scalar, and BI gravitational fields given by the action

$$\mathcal{S}(g; \psi, \bar{\psi}, \varphi) = \int \mathcal{L}\sqrt{-g}d\Omega \tag{2.1}$$

with

$$\mathcal{L} = \mathcal{L}_{\rm g} + \mathcal{L}_{\rm m} + \mathcal{L}_{\rm pf}. \tag{2.2}$$

The gravitational part of the Lagrangian (2.2) $\mathcal{L}_{\rm g}$ is given by a Bianchi type-I metric, whereas the term $\mathcal{L}_{\rm m}$ describe the interacting system of the spinor and the scalar fields. Finally, $\mathcal{L}_{\rm pf}$ describes the perfect fluid.

The interacting system the spinor and the scalar fields is given by the Lagrangian

$$\mathcal{L}_{\rm m} = \frac{i}{2}[\bar{\psi}\gamma^{\mu}\nabla_{\mu}\psi - \nabla_{\mu}\bar{\psi}\gamma^{\mu}\psi] - m\bar{\psi}\psi + F + \frac{1}{2}(1 + \lambda_1 F_1)\varphi_{,\alpha}\varphi^{,\alpha}. \tag{2.3}$$

The term F describes the self-action of a spinor field and $\lambda_1 F_1$ ascribes the interaction between the spinor and the scalar fields with λ_1 being the coupling constant. F and F_1 can be chosen as some arbitrary functions of invariants generated from the real bilinear forms of a spinor field. Thanks to Pauli-Fierz theorem, one may assume $F = F(I, J)$ and $F_1 = F_1(I, J)$ with $I = S^2$, $S = \bar{\psi}\psi$, $J = P^2$ and $P = i\bar{\psi}\gamma^5\psi$, since the three remaining invariants are the functions of these two. In our previous studies [cf. e.g., [13,14]] when $F = F(J)$ and/or $F_1 = F_1(J)$ we considered a massless spinor field since in the nonlinear generalization of the classical field equations, the massive term does not possess the significance that it possesses in the linear one, as it by no means defines the total energy (or mass) of the nonlinear field system. As the spinor mass plays a significant role in the late time acceleration of the Universe, in this paper we consider the case when F and F_1 are the functions of I (or S) only.

The gravitational field is chosen in the form

$$ds^2 = dt^2 - a_1^2 dx_1^2 - a_2^2 dx_2^2 - a_3^2 dx_3^2, \tag{2.4}$$

where a_i are the functions of t only and the speed of light is taken to be unity. We also define

$$\tau = a_1 a_2 a_3. \tag{2.5}$$

Using the variation principle we find the equations for the spinor, scalar and gravitational fields. The nonlinear spinor field equation takes the form

$$i\gamma^{\mu}\nabla_{\mu}\psi - (m - \mathcal{D})\psi = 0, \tag{2.6a}$$

$$i\nabla_{\mu}\bar{\psi}\gamma^{\mu} + (m - \mathcal{D})\bar{\psi} = 0, \tag{2.6b}$$

with ∇_{μ} denoting the covariant derivative of the spinor field and $\mathcal{D} = dF/dS + (\lambda_1/2)\varphi_{,\mu}\varphi^{,\mu}dF_1/dS$. The equation for the massless scalar field reads

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\nu}}[\sqrt{-g}g^{\nu\mu}(1 + \lambda_1 F_1)\varphi_{,\mu}] = 0. \tag{2.7}$$

Einstein's gravitational field equation corresponding to the BI spacetime can be written in the form:

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} = \kappa T_1^1, \tag{2.8a}$$

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} = \kappa T_2^2, \tag{2.8b}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} = \kappa T_3^3, \tag{2.8c}$$

$$\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} + \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} = \kappa T_0^0. \tag{2.8d}$$

Here T_μ^ν is the energy-momentum tensor of the spinor and scalar fields and the perfect fluid.

We consider the spinor and scalar field to be space independent.

From the spinor field equation we find

$$S = C_0/\tau, \tag{2.9}$$

with C_0 being an integration constant. The components of the spinor field in this case read

$$\psi_{1,2}(t) = (C_{1,2}/\sqrt{\tau})e^{-i\int(m-\mathcal{D})dt}, \quad \psi_{3,4}(t) = (C_{3,4}/\sqrt{\tau})e^{i\int(m-\mathcal{D})dt}, \tag{2.10}$$

with the integration constants obeying $C_0 = C_1^2 + C_2^2 - C_3^2 - C_4^2$.

For the scalar field we find

$$\varphi = C\int\frac{dt}{\tau(1+\lambda_1 F_1)} + C_1, \tag{2.11}$$

where C and C_1 are integration constants.

Before solving the equation for τ, we have to write the components of the energy-momentum tensor of the source fields in details:

$$T_0^0 = mS - F + \frac{1}{2}(1+\lambda_1 F_1)\dot{\varphi}^2 + \varepsilon_{pf},$$
$$T_1^1 = T_2^2 = T_3^3 = \mathcal{D}S - F - \frac{1}{2}(1+\lambda_1 F_1)\dot{\varphi}^2 - p_{pf}. \tag{2.12}$$

In (2.12) ε_{pf} and p_{pf} are the energy density and pressure of the perfect fluid, respectively, and related by the equation of state

$$p_{pf} = \zeta\varepsilon_{pf}, \qquad \zeta \in [0,1]. \tag{2.13}$$

In this case the Bianchi identity $T^\mu_{\nu;\mu} = 0$ gives

$$\dot{\varepsilon} + \frac{\dot{\tau}}{\tau}(\varepsilon + p) = 0. \tag{2.14}$$

In view of (2.13) from (2.14) for the energy density and pressure of the perfect fluid one finds

$$\varepsilon_{pf} = \frac{\varepsilon_0}{\tau^{1+\zeta}}, \qquad p_{pf} = \frac{\zeta\varepsilon_0}{\tau^{1+\zeta}}. \tag{2.15}$$

In account of $T_1^1 = T_2^2 = T_3^3$ from Eqs. (2.8a)–(2.8c) we find

$$a_i(t) = A_i[\tau(t)]^{1/3}\exp\left[X_i\int[\tau(t')]^{-1}dt'\right], \tag{2.16}$$

with the integration constants A_i and X_i obeying the following conditions

$$A_1A_2A_3 = 1, \tag{2.17a}$$

$$X_1 + X_2 + X_3 = 0. \tag{2.17b}$$

As it was mentioned above, in order to give a more realistic description of the early day Universe we need to consider the anisotropic cosmological models such as Bianchi type-I (BI). On the other hand the modern day Universe is wonderfully isotropic. So we have to find out how and when the initially anisotropic spacetime evolves into an isotropic one. There exists a number of isotropization criteria in literature. In [34] Jacobs used anisotropy parameter in order to find the time when anisotropies ceased to be large one. Two common criteria for isotropization are

$$\mathcal{A} = \frac{1}{3}\sum_{i=1}^{3}\frac{H_i^2}{H^2} - 1 \to 0, \tag{2.18a}$$

$$\Sigma^2 = \frac{1}{2}\mathcal{A}H^2 \to 0. \tag{2.18b}$$

Here $\mathcal{A}$ and Σ^2 are the mean anisotropy parameter and shear parameter, respectively. $H_i = \dot{a}_i/a_i$ are the directional Hubble parameters and $H = \dot{a}/a$ is the mean Hubble parameter, with $a(t) = \tau^{1/3}$ being the mean scale factor. In this paper we use the isotropization condition introduced by Bronnikov *et al.* [35]. Isotropization means that at large physical times, when the volume scale τ tends to infinity, the three scale factors $a_i(t)$ grow at the same rate. Therefore, we will say that a model is isotropizing if

$$a_i/a \to \text{const} > 0 \quad \text{as } \tau \to \infty. \tag{2.19}$$

As is seen from (2.16) in our case $a_i/a \to A_i = \text{const}$ as $\tau \to \infty$. Recall that the isotropic FRW model has same scale factor in all three directions, i.e., $a_1(t) = a_2(t) = a_3(t) = a(t)$. So for the BI universe to evolve into a FRW one the constants A_i's are likely to be identical, i.e., $A_1 = A_2 = A_3 = 1$. Note that by rescaling some coordinates we can come to $a_i/a \to 1$ and the metric will become manifestly isotropic at large t. Moreover, the isotropic nature of the present Universe leads to the fact that the three other constants X_i should be close to zero as well, i.e., $|X_i| \ll 1$, ($i = 1, 2, 3$), so that $X_i\int[\tau(t)]^{-1}dt \to 0$ for $t < \infty$ (for $\tau(t) = t^n$ with $n > 1$ the integral tends to zero as $t \to \infty$ for any X_i). The rapid growth of the Universe due to the introduction of the nonlinear spinor field to the system results in the earlier isotropization.

From (2.10), (2.11), and (2.16) one finds that the spinor, scalar and metric functions are in some functional dependence of τ. It could be shown that the other physical quantities such as spin-current, charge etc. and invariant of spacetime are too expressed via τ [13,14]. In fact all these quantities are inverse proportional to τ^n. Thus we see that at any spacetime points where $\tau = 0$ the spinor, scalar and gravitational fields become infinity, hence the spacetime becomes singular at this point [14]. So it is very important to study the equation for τ (which can be viewed as master equation) in details, exactly what we shall do in the section to follow. In doing so, we analyze the role of spinor field in the character of evolution of the Universe.

III. EVOLUTION OF BI UNIVERSE AND ROLE OF SPINOR FIELD

In this section we study the role of spinor field in the evolution of the Universe. But first of all let me qualitatively show the differences that occur at the later stage of expansion depending on how the sources of the gravitational field were introduced in the system. After a little manipulation from (2.8) one finds the equation for τ which is indeed the acceleration equation and has the following general form:

$$\frac{\ddot{\tau}}{\tau} = \frac{3}{2}\kappa(T_1^1 + T_0^0). \quad (3.1)$$

On the other hand, from the Bianchi identity $G^{\nu}_{\mu;\nu} = 0$ we have

$$\dot{T}_0^0 = -\frac{\dot{\tau}}{\tau}(T_0^0 - T_1^1). \quad (3.2)$$

After a little manipulation from (3.1) and (3.2) one finds the following expression for T_0^0:

$$\kappa T_0^0 = 3H^2 - C_{00}/\tau^2, \quad (3.3)$$

where in analogy with FRW model we define the generalized Hubble constant:

$$3H = \frac{\dot{\tau}}{\tau} = \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} = H_1 + H_2 + H_3. \quad (3.4)$$

Taking into account that even in case of $H = 0$ the energy density should be nonnegative, the integration constant C_{00} in (3.3) should be nonpositive.

Note that the Einstein equations for the FRW model read

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = \kappa T_1^1, \quad (3.5a)$$

$$3\left(\frac{\dot{a}}{a}\right)^2 = \kappa T_0^0. \quad (3.5b)$$

From (3.5) one finds

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{6}(T_0^0 - 3T_1^1). \quad (3.6)$$

The Eq. (3.6) is known as the acceleration equation. On the other hand, for the BI Universe from (2.8) we obtain

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} = -\frac{\kappa}{2}(T_0^0 - 3T_1^1). \quad (3.7)$$

As is seen, setting $a_1 = a_2 = a_3$ we come to Eq. (3.6), hence one may argue to define (3.7) as acceleration equation for BI model. Nevertheless, as acceleration equation for BI universe we consider Eq. (3.1), since when we talk about the expansion of the Universe, we mean the growth rate of the volume as a whole, not a particular scale factor.

Keeping this in mind let us now define the deceleration parameter for the BI universe. Recall that in FRW cosmology the deceleration parameter has the form

$$d_{\rm frw} = -\frac{a\ddot{a}}{\dot{a}^2} = -\left[1 + \frac{\dot{H}_{\rm frw}}{H_{\rm frw}}\right] = \frac{d}{dt}\left(\frac{1}{H_{\rm frw}}\right) - 1, \quad (3.8)$$

where $H_{\rm frw} = \dot{a}/a$ is the Hubble parameter for FRW model. In analogy we can define a deceleration parameter as well. If we define the generalized deceleration parameter as:

$$d = -\left[1 + \frac{\dot{H}_1 + \dot{H}_2 + \dot{H}_3}{H_1^2 + H_2^2 + H_3^2}\right], \quad (3.9)$$

where $H_i = \dot{a}_i/a_i$, then the standard deceleration parameter is recovered at $a_1 = a_2 = a_3$. But in this case the definition for acceleration adopted here is no longer valid. So we switch to the second choice and following Belinchon and Harko *et al.* [36,37] define the generalized deceleration parameter as

$$d = \frac{d}{dt}\left(\frac{1}{3H}\right) - 1 = -\frac{\tau\ddot{\tau}}{\dot{\tau}^2}. \quad (3.10)$$

Thus we have defined all the basic quantities needed to verify the character of evolution. Let us now study the master Eq. (3.1) in detail. In doing so we first choose the nonlinear spinor terms explicitly. Assume that $F = \lambda S^q$ and $F_1 = S^r$ where λ is the self-coupling constant. Further for simplicity we set $\varepsilon_0 = 1$ and $C_0 = 1$. Then on account of (2.9) and (2.15) for the energy density and the pressure from (2.12) we find

$$T_0^0 = \frac{m}{\tau} - \frac{\lambda}{\tau^q} + \frac{\tau^{r-2}}{2(\lambda_1 + \tau^r)} + \frac{1}{\tau^{1+\zeta}} \equiv \varepsilon$$
$$T_1^1 = \frac{(q-1)\lambda}{\tau^q} - \frac{[(1-r)\lambda_1 + \tau^r]\tau^{r-2}}{2(\lambda_1 + \tau^r)^2} - \frac{\zeta}{\tau^{1+\zeta}} \equiv p. \quad (3.11)$$

Taking into account that T_0^0 and T_1^1 are the functions of τ only, the Eq. (3.1) can now be presented as

$$\ddot{\tau} = \mathcal{F}(q_1, \tau), \quad (3.12)$$

where we define

$$\mathcal{F}(q_1, \tau) = (3/2)\kappa(m + \lambda(q-2)\tau^{1-q} + \lambda_1 r\tau^{r-1}/2(\lambda_1 + \tau^r)^2 + (1-\zeta)/\tau^{\zeta}), \quad (3.13)$$

where $q_1 = \{\kappa, m, \lambda, \lambda_1, q, r, \zeta\}$ is the set of problem parameters. Equation (3.12) allows the following first integral:

$$\dot{\tau} = \sqrt{2[E - \mathcal{U}(q_1, \tau)]} \tag{3.14}$$

where we denote

$$\mathcal{U}(q_1, \tau) = -\frac{3}{2}[\kappa(m\tau - \lambda/\tau^{q-2} - \lambda_1/2(\lambda_1 + \tau^r) + \tau^{1-\zeta})]. \tag{3.15}$$

From a mechanical point of view Eq. (3.12) can be interpreted as an equation of motion of a single particle with unit mass under the force $\mathcal{F}(q_1, \tau)$. In (3.14) E is the integration constant which can be treated as energy level, and $\mathcal{U}(q_1, \tau)$ is the potential of the force $\mathcal{F}(q_1, \tau)$. We solve Eq. (3.12) numerically using Runge-Kutta method. The initial value of τ is taken to be a reasonably small one, while the corresponding first derivative $\dot{\tau}$ is evaluated from (3.14) for a given E.

Before presenting numerical results let us first study Eqs. (3.12), (3.13), (3.14), and (3.15) qualitatively. In view of (3.13) from (3.12) one finds $\ddot{\tau} \to (3/2)\kappa m > 0$ as $\tau \to \infty$, i.e., if $\ddot{\tau}$ is considered to be the acceleration of the BI Universe, then the massive spinor field essentially can be viewed as a source for everlasting acceleration. As far as initial stage of expansion is concerned (here we are exclusively dealing with an expanding Universe), the positivity of the radical imposes some restriction on the value of τ, namely, in case of $\lambda > 0$ and $q \geq 2$ the value of τ cannot be too close to zero at any spacetime point. In this case there exists an infinitely high potential wall as $\tau \to 0$ making it impossible for any classical system to reach the point where $\tau = 0$ [cf. Figure 1]. Thus we conclude that for some special choice of problem parameters the introduction of nonlinear spinor field given by a self-action provides singularity-free solutions. As it was shown in [13] the regular solution is obtained only at the expense of broken dominant-energy condition in the Hawking-Penrose theorem [38] which in case of BI unverse can be written in the form:

$$T_0^0 \geq T_1^1 a_1^2 + T_2^2 a_2^2 + T_3^3 a_3^2, \tag{3.16a}$$
$$T_0^0 \geq T_1^1 a_1^2, \tag{3.16b}$$
$$T_0^0 \geq T_2^2 a_2^2, \tag{3.16c}$$
$$T_0^0 \geq T_3^3 a_3^2. \tag{3.16d}$$

Let us now consider the case when λ is negative. From (3.15) one sees, in the vicinity of $\tau = 0$ there exists a bottomless potential hole [cf Fig. 2]. If the initial value of τ is too close to zero and the constant E is less than $\mathcal{U}_{\max}$ (the maximum value of the potential in presence of a self-action), the Universe will never come out of the hole.

Let us now solve the (3.12) numerically. In doing this we choose the problem parameters as follows: Einstein's gravitational constant $\kappa = 1$, spinor mass $m = 1$, the power of nonlinearity $q = 4$, $r = 4$ and $\zeta = 1/3$ that corresponds to a radiation. We also set $C_{00} = -0.001$ and $E = 10$. The initial value of τ is taken to be $\tau_0 = 0.4$. The coupling constant is chosen to be $\lambda_1 = 0.5$, while the self-coupling constant is taken to be either $\lambda = 0.5$ or $\lambda = -0.5$. Here, in the figures we use the following notations:

(1) corresponds to the case with self-action and interaction;

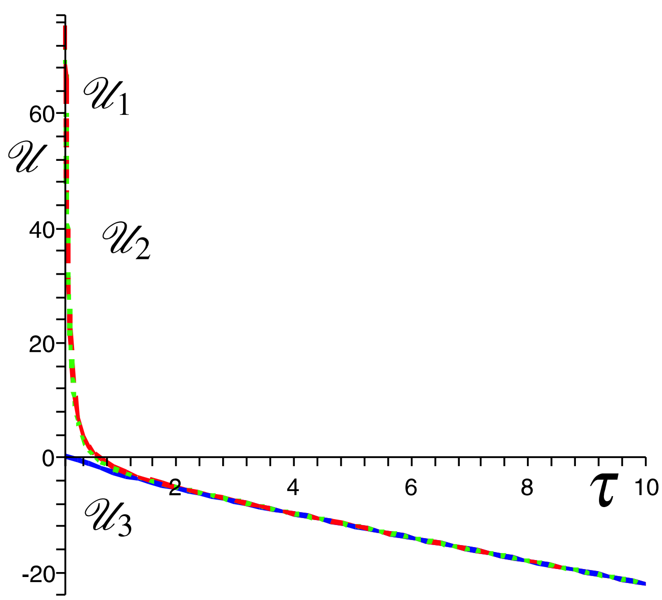

FIG. 1 (color online). View of the potential $\mathcal{U}(\tau)$ for $\lambda > 0$.

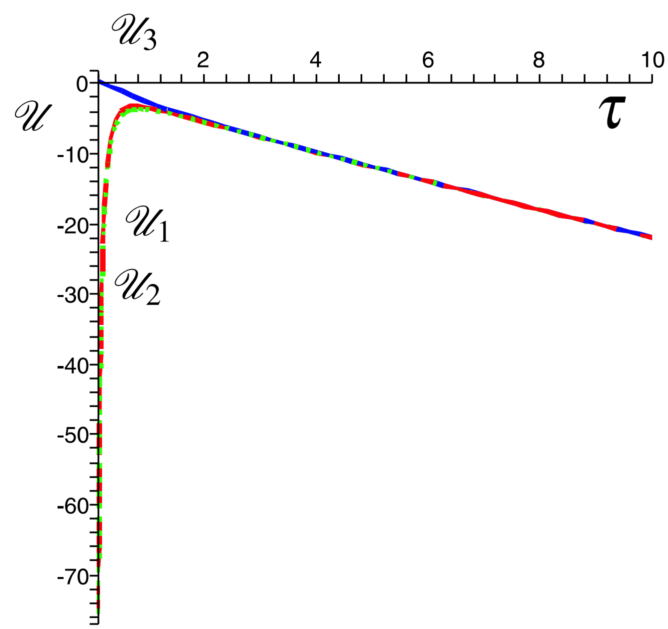

FIG. 2 (color online). View of the potential $\mathcal{U}(\tau)$ for a negative λ.

(2) corresponds to the case with self-action only;
(3) corresponds to the case with interaction only.

As one sees from Fig. 1, in presence of a self-action of the spinor field with a positive λ, there occurs an infinitely high barrier as $\tau \to 0$, it means that in the case considered here τ cannot be trivial [if treated classically, the Universe cannot approach to a point unless it stays at an infinitely high energy level]. Thus, the nonlinearity of the spinor field provided by the self-action generates singularity-free evolution of the Universe. But this regularity can be achieved only at the expense of dominant-energy condition in Hawking-Penrose theorem. It is also clear that if the nonlinearity is induced by a scalar field, τ may be trivial as well, thus giving rise to spacetime singularity [14]. We would like to note that the singularity-free evolution of the Universe can be achieved by introducing a Λ term into the system. The system in question is thoroughly studied in [13,14]. It was shown that introduction of a positive Λ that corresponds to a repulsive force and can be viewed as a form of dark energy accelerates the speed of expansion, whereas, a negative Λ corresponding to an additional gravitational force, depending of the choice of E, generates oscillatory or nonperiodic mode of evolution. Note also that the regular solution obtained by means of a negative Λ in case of interaction does not result in broken dominant-energy condition [14].

In Figs. 3 and 4 we plot the corresponding energy density and pressure. In case of a positive λ the energy density is initially negative while the pressure is positive. In this case though the solution is singularity-free, the violation of dominant energy takes place. In case of negative λ the pressure is always negative.

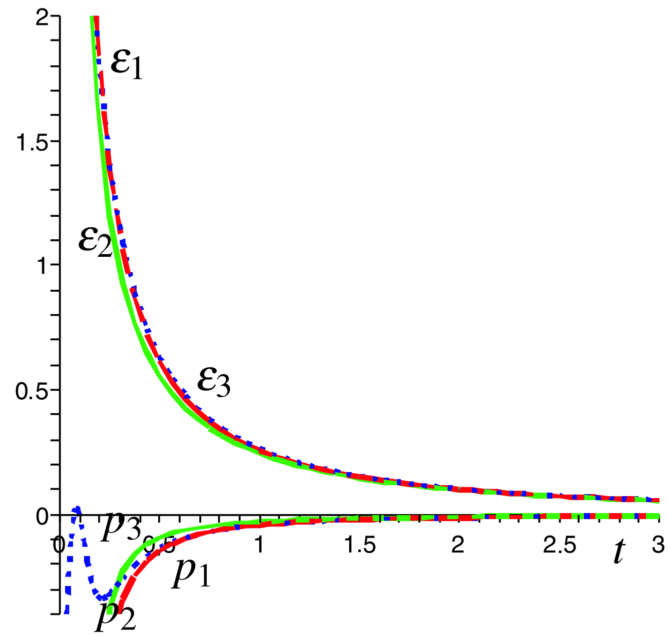

FIG. 4 (color online). Energy density and pressure in case of a negative λ.

The purpose of plotting the energy density and pressure is to show that the energy density of the source field indeed decreases with the increase of the Universe. This also shows that there exists an interval where the energy density of the system with spinor field nonlinearity generated by the self-action is negative. Moreover, we see the pressure of the source field gets negative in course of evolution (in case of self-action with a positive λ pressure is initially

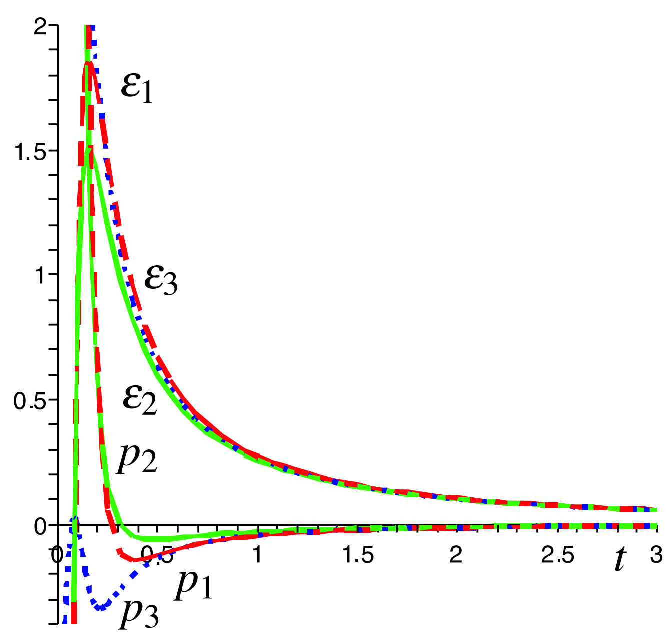

FIG. 3 (color online). Energy density and pressure corresponding to a positive λ.

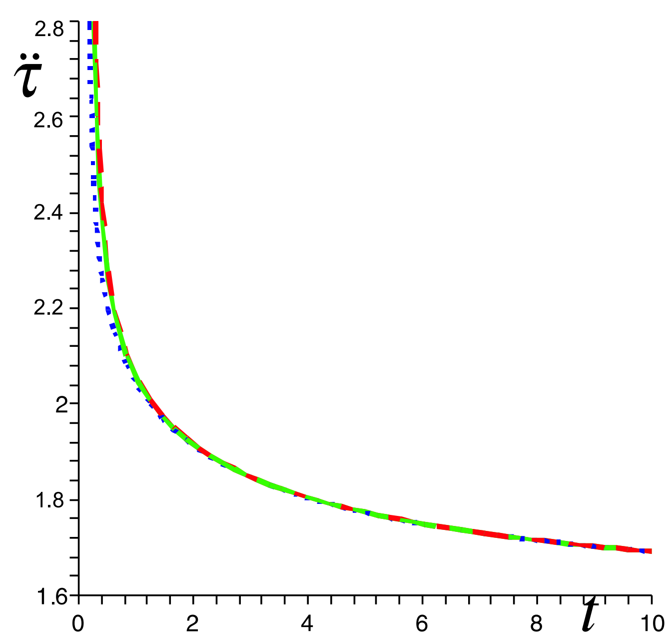

FIG. 5 (color online). Acceleration of the Universe corresponding to a positive λ.

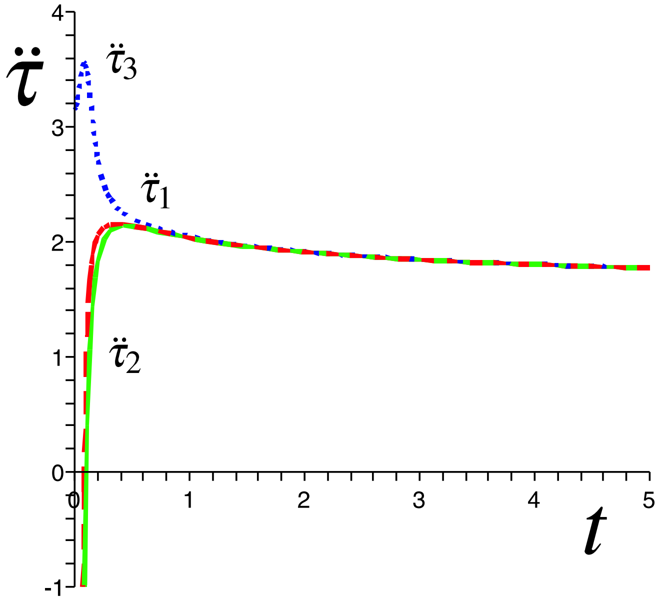

FIG. 6 (color online). Acceleration of the Universe in case of a negative λ.

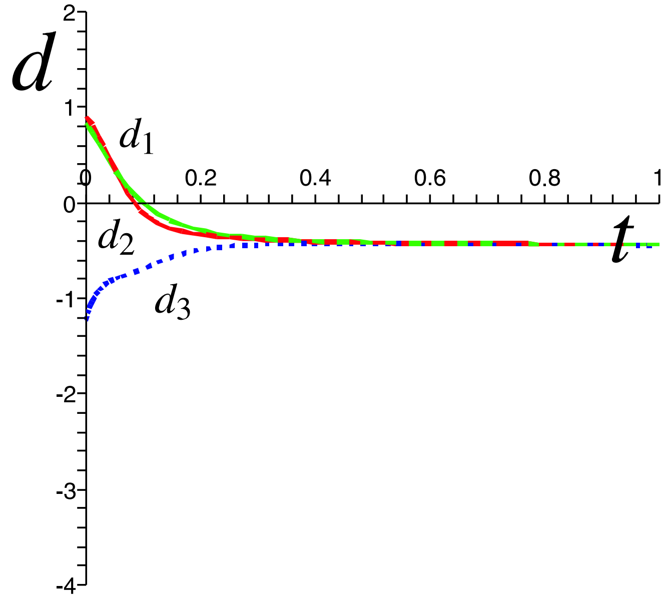

FIG. 8 (color online). Deceleration parameter in case of a negative λ.

positive, but with the expansion of the Universe it gets negative, whereas, in case of of negative λ as well as in case of interacting fields the pressure is always negative). Recall that the dark energy (e.g. quintessence, Chaplygin gas), modeled to explain the late time acceleration of the Universe, has a negative pressure. So we argue that the models with nonlinear spinor field and interacting spinor and scalar fields to some extent can be considered as an alternative to dark energy which is able to explain the late time acceleration of the Universe.

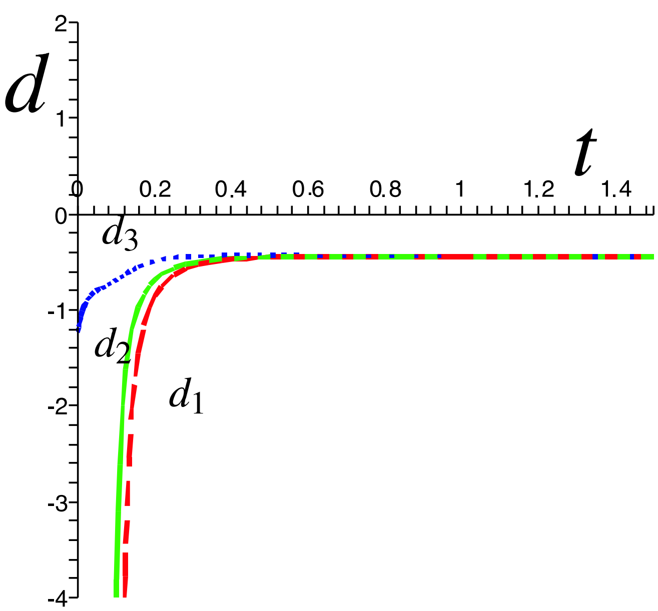

FIG. 7 (color online). Deceleration parameter corresponding to a positive λ.

In Figs. 5 and 6 we illustrate the acceleration of the Universe for positive and negative λ, respectively. As one sees, in both cases we have the decreasing acceleration that tends to $(3/2)\kappa m$ as $\tau \to \infty$.

To cement our claim that the nonlinear spinor field can give rise to a late time acceleration, we also plot deceleration parameters for both positive and negative λ [cf. Figures 7 and 8].

The Figs. 5–8 show the accelerated mode of the expansion of the Universe. As one sees, the acceleration decreases with time. Depending of the choice of nonlinearity it undergoes an initial deceleration phase. It is also seen that the nonlinear term plays proactive role at the initial stage while at the later stage spinor mass is crucial for the accelerated mode of expansion.

IV. CONCLUSION

We considered a system of interacting nonlinear spinor and scalar fields within the scope of a BI cosmological model filled with perfect fluid. It has been shown that for some suitable choice of problem parameters the spinor field nonlinearity gives rise to an effective negative pressure in the course of evolution. Comparing the effective pressure of the nonlinear spinor field with that of a dark energy given by a quintessence or Chaplygin gas we conclude that the spinor field can be seen as an alternative to the dark energy able to explain the acceleration of the Universe. It has been shown that the nonlinear spinor term is proactive at the early stage of the evolution and essentially accelerates the process of evolution, while at the later stage of evolution the spinor mass holds the key. Given the fact that *neutrino* is described by the spinor field equation and it too possesses mass (though too small but nonzero), the presence of huge number of neutrino in the Universe can be seen as one of the possible factor of the late time acceleration of the Universe. It was also shown that for some specific choice of parameters it is possible to construct singularity-free model of the Universe, but this regularity results in the broken dominant-energy condition of the Hawking-Penrose theorem. It should be noted that to get the inflation or late time acceleration introduction of anisotropy is not a must condition. For example, accelerated regimes in the evolution of the Universe by means of a spinor field can be obtained starting with a FRW model as well, exactly which was done in [16]. The problem of the stability of the solutions obtained is not considered here. I plan to turn to this problem sometimes in near future.

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