

Braneworld gravitational collapse from a radiative bulk

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We study the fate of a collapsing star on the brane in a generalized braneworld gravity with bulk matter. Specifically, we investigate the possibility of having a static exterior for a collapsing star in the radiative bulk scenario. Here, the nonlocal correction due to bulk matter is manifest in an induced mass that adds up to the physical mass of the star resulting in an effective mass. A Schwarzschild solution for the exterior in terms of this effective mass is obtained, which reveals that, even if the star exchanges energy with the bulk, the exterior may appear to be static to a braneworld observer located outside the collapsing region. The possible explanation of the situation from the discussion on the role of bulk matter is provided. The nature of bulk matter and the corresponding bulk geometry have also been obtained and analyzed, which gives a complete picture of both brane and bulk viewpoints.

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I. INTRODUCTION

The search for a consistent description of black hole physics and gravitational collapse in braneworlds [1] has been a challenge to theoretical physics. As a matter of fact, black holes and gravitational collapse are not yet well understood in the braneworld scenario. The idea of an extended singularity in higher dimensions leads to a black string solution [2] which is neither stable nor localized on the brane. The first solution for localized black holes on the brane came out to be Reissner-Nördstrom type with a “tidal charge” contribution arising from the bulk Weyl tensor [3]. Later on, attempts were made to include a Schwarzschild metric with a nonvacuum brane and the black hole intersecting the bulk [4,5]. Solutions for the charged rotating black holes on the brane were also obtained [6] (see also [7] for a review and [8] for some very recent results). Based on this tidal charge scenario, an Oppenheimer-Snyder type [9] gravitational collapse of spherically symmetric objects was studied in [10], which led to an interesting conclusion. This was formulated by a no-go theorem that indicated a nonstatic exterior for the collapsing sphere on the brane. Subsequently, it was shown that the exterior for this radiative sphere can be described by a Vaidya metric that envelops the collapsing region [11]. Possible generalizations of the nonstatic nature for induced gravity with or without the Gauss-Bonnet term also exist [12]. However, it was demonstrated in [13] that a static exterior can be obtained by relaxing the idea of dust inside the star, thereby introducing a nonvanishing surface pressure, and by ignoring the tidal effect.

Out of simplicity, these descriptions are based on the assumption that the bulk is empty, comprised of a negative cosmological constant. No doubt, in the simplest case, they provide important insight into the bulk-brane interplay. But

a careful look at the scenario reveals that the source of the above conclusions may, in fact, lie in this very assumption. Nevertheless, an empty bulk might be only a special situation to deal with. For a more realistic description of the physics on the brane, it is instructive to take into account the effects of bulk fields as well. There are many reasons why one should consider bulk fields. The braneworld models are motivated by the p -brane solutions of M theory [14], which are obtained by considering scalars (e.g., dilatons), 3-form fields (e.g., the Kalb-Ramond field), etc. in the bulk. It is expected that these properties of p -branes will be reflected in braneworld physics too. Besides phenomenological motivations [15], the urge to consider bulk fields comes from the so-called radion stabilization problem, which was solved by the Goldberger-Wise mechanism [16]. From the geometric point of view, a bulk field can also be thought of as an outcome of global topological defects which supply a nontrivial stress-energy tensor outside the brane [17]. The motivations from the gravitational sector include addressing the age-old problems like the cosmological constant problem [18]. Also, a Schwarzschild black hole on the brane will have a regular anti de-Sitter (AdS) horizon only if the bulk contains exotic matter [19]. In fact, in a realistic brane cosmological scenario, the bulk gravitons produced by the fluctuations on brane matter act like an effective field residing in the bulk [20]. Thus the need for considering matter fields in the bulk is apparently unavoidable. With this motivation, we intend to analyze gravitational collapse for a nonempty bulk.

It has been shown extensively in numerous papers [20–23] that, in the presence of bulk matter, the bulk metric for which a Friedmann-Robertson-Walker (FRW) geometry on the brane is recovered is given by a higher dimensional generalization of the Vaidya AdS (VAdS₅) spacetime [24], with the radiation flowing from the brane to the bulk (incoming radiation). This black hole, being radiative,

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exchanges energy with the brane, which is manifest through a *brane matter nonconservation equation*, so that the matter on the brane is no longer strictly conserved [25]. However, the total matter energy of the bulk-brane system still remains conserved, confirming no global violation of matter conservation. Of late, the idea of the VAdS₅ radiative bulk scenario has been generalized for both incoming and outgoing radiation [26]. This generalized idea has opened up new avenues of visualizing the brane phenomena from the point of view of bulk-brane energy exchange. Subsequently, different possibilities have been investigated. For example, Ref. [27] discusses the scenario for asymmetric embedding and Ref. [28] accounts for nonradial emission of bulk gravitons. Several other models with radiating black holes in the bulk have also been brought forth [29].

Our aim in this article is to utilize the generic feature of the VAdS₅ bulk discussed in [26] in order to study gravitational collapse of spherically symmetric objects. In our model, the bulk energy-momentum tensor is given by a phantom (ghost) radiation field having negative energy density. The phantom field plays an important role in gravitational physics. In four dimensions, it raises a fair possibility of explaining observed accelerated expansion of the universe [30] and unifying dark matter and dark energy [31]. Additionally, the radiative behavior of a phantom null dust has been employed to obtain wormhole solutions [32]. In the braneworld context too, a phantom field in the bulk has shown much promise. It has been shown in [33] that a bulk phantom field makes it possible to localize both massless and massive fermions on the brane and it results in the localization of massless modes of all the standard model fields as well as gravity on the same brane for six dimensional models [34]. Nevertheless, since such exotic matter fields can indeed be present in the bulk, it is interesting to investigate the possible consequences of these fields on the physics of the four dimensional world. This serves as the basic motivation for considering phantom radiation in the bulk and for studying gravitational collapse on the brane. We show, with the help of the modified Einstein equation and the nonconservation equation, that the collapsing star on the brane can exchange energy with the bulk even if the exterior may appear to be static. The unique feature of this radiative bulk scenario is that a braneworld (local) observer located near the surface of the star cannot *feel* this effect so that the star would appear to be static even though there is energy flow between the brane and the bulk. In this way, we generalize the analysis of gravitational collapse of spherical objects on the brane and, at the same time, reestablish the possibility of having a static exterior for a collapsing sphere.

Throughout the article, we use μ, ν, \dots for the brane indices and M, N, \dots for those in the bulk. Specifically, we choose the following notations for the coordinates: $(\tau, r, \theta, \phi) \equiv$ brane coordinates for the interior of the col-

lapsing sphere; $(T, R, \Theta, \Phi) \equiv$ brane coordinates for the exterior; and $(t, \mathcal{R}, x, y, z) \equiv$ bulk coordinates $\equiv (v, \mathcal{R}, x, y, z)$ in terms of the null coordinates.

II. BRANE VIEWPOINT WITH A RADIATIVE BULK

In the presence of a bulk field exchanging energy with the brane, the effective Einstein equation on the brane [35] is generalized to [21]

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa^2 T_{\mu\nu} + \kappa_5^4 \mathcal{S}_{\mu\nu} - \mathcal{E}_{\mu\nu} + \mathcal{F}_{\mu\nu}, \quad (2.1)$$

where μ, ν, \dots are the brane (4D) indices; $\mathcal{S}_{\mu\nu}$, $\mathcal{E}_{\mu\nu}$, and $\mathcal{F}_{\mu\nu}$ are, respectively, the quadratic brane energy-momentum tensor, the projected bulk Weyl tensor, and the bulk energy-momentum tensor projected on the brane, and $\kappa_5^4 = 6\kappa^2/\lambda$ the 5D coupling constant. For a general VAdS₅ bulk with the possibilities of both incoming and outgoing radiation, the matter conservation equation on the brane is modified to [26]

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -2\epsilon\psi, \quad (2.2)$$

where $\epsilon\psi \propto d\mathcal{M}/dv$, with $\mathcal{M}(v)$ the sum total of the masses of the bulk black hole and the radiation field. A crucial term in the above expression is ϵ which can take the values $\epsilon = \pm 1$ and results in the bulk-brane energy exchange. Since, for a phantom radiative field, $\psi < 0$, it appears that $\epsilon < 0 \Rightarrow d\mathcal{M}/dv > 0$ so that the bulk gains energy, whereas $\epsilon > 0 \Rightarrow d\mathcal{M}/dv < 0$, implying that the bulk loses energy. A negative signature for ϵ will thus indicate that an object on the brane releases phantom radiation to the bulk, whereas a positive ϵ will mean that it absorbs phantom radiation from the bulk. To a braneworld observer, ψ is the quantitative estimate of the brane projection of the bulk energy density, reflected by the equation

$$\mathcal{F}_{\mu\nu} = \frac{2}{3}\kappa_5^2\psi h_{\mu\nu}, \quad (2.3)$$

where $h_{\mu\nu}$ is the induced metric on the brane. Consequently, the Bianchi identity on the brane leads to the equation governing the evolution of the so-called Weyl term,

$$\dot{\rho}^* + 4\frac{\dot{a}}{a}\rho^* = 2\epsilon\psi - \frac{2\kappa_5^2}{3\kappa^2}\left[\dot{\psi} + 3\frac{\dot{a}}{a}\psi\right]. \quad (2.4)$$

Equations (2.2) and (2.4) show that, in general, there is a coupling [36] between the bulk energy-momentum tensor and its brane counterpart, which is responsible for the bulk-brane energy exchange. However, as already pointed out in [36], since the bulk matter is not determined *a priori*, the ansatz for the coupling term \mathcal{Q} (which is precisely the right-hand side of the above equation) can be arbitrarily chosen, so far as it is physically acceptable. Several possible ansatz have been considered in [36–40] and the

consequences have been investigated. Brane cosmological dynamics of bulk scalar fields have been studied in detail in [20,41,42]. What follows is that we can take an ansatz for the coupling term of the form $\mathcal{Q} = H\rho^*$. With this ansatz, Eq. (2.4) reveals that the Weyl term behaves as

$$\rho^* = \frac{C(\tau)}{a^4}, \quad C(\tau) = C^*a(\tau), \quad (2.5)$$

where $C(\tau)$ is the scaled on-brane mass function $\mathcal{M}(\tau)$, τ being the proper time on the brane. Hence, for this type of ansatz, the Weyl term supplies an additional matterlike effect to the brane. We shall show *a posteriori* what type of bulk matter can give rise to this matterlike nature of the Weyl term.

III. GRAVITATIONAL COLLAPSE ON THE BRANE

Let us now analyze gravitational collapse for spherically symmetric objects using the effective Einstein equation with bulk fields discussed above. For a sphere undergoing Oppenheimer-Snyder collapse, the collapsing region can be conveniently expressed by a Robertson-Walker metric,

$$ds_{\text{int}}^2 = -d\tau^2 + a^2(\tau)(1 + \frac{1}{4}kr^2)^{-2}[dr^2 + r^2d\Omega^2] \quad (3.1)$$

with (τ, r, θ, ϕ) as the brane coordinates for the interior. This metric has to be a solution of the generalized brane Friedmann equation, which, with the help of Eq. (2.5) and the Randall-Sundrum (RS) fine-tuning ($\Lambda = 0$), turns out to be

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{C^*}{a^3} - \frac{k}{a^2}. \quad (3.2)$$

Also, the Raychaudhuri equation for geodesic focusing can be written as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho\left(1 + \frac{\rho}{2\lambda}\right) + 3p\left(1 + \frac{\rho}{\lambda}\right)\right] - \frac{C^*}{a^3} - \frac{\kappa_5^2}{3}\psi. \quad (3.3)$$

Now, expressing it in terms of the proper radius $R(\tau) = ra(\tau)/(1 + \frac{1}{4}kr^2)$, the generalized Friedmann equation (3.2) for the collapsing region reads

$$\dot{R}^2 = \frac{2GM}{R} + \frac{3GM^2}{4\pi\lambda R^4} + \frac{2GM^*}{R} + E \quad (3.4)$$

with

$$M = \frac{4}{3}\pi R_0^3, \quad M^* = \frac{C^*}{2G}R_0^3, \quad E = -k\left(\frac{R_0}{a_0}\right)^2, \quad (3.5)$$

where $R_0 = r_0a_0/(1 + \frac{1}{4}kr_0^2)$. Here, the constant E is the energy per unit physical mass. So, unlike the vacuum bulk scenario, a radiative bulk with matter gives rise to two mass terms for the brane black hole: one (M) is the physical mass, i.e., the total energy per proper star volume, and another (M^*) is the ‘‘induced mass’’ of the star that encodes

the bulk information on the brane. Precisely, the role of bulk matter is to provide an additional *mass* to the collapsing star, resulting in an effective mass $M^{\text{eff}} = M + M^*$. It should be noted that this induced mass correction is quite small due to the presence of G . That the physical mass of a braneworld object is modified in the presence of bulk matter has been shown in [42] and has been depicted as the comoving mass in order to study its cosmological consequences.

We shall now investigate whether there is a static vacuum exterior that can match the interior metric (3.1) at the boundary of the collapsing star. The field for the exterior of the collapsing star is a solution of the modified Einstein equation (2.1) for vacuum ($T_{\mu\nu} = 0 = \mathcal{S}_{\mu\nu}$). By employing the RS fine-tuning, the equation reads

$$R_{\mu\nu} = -\mathcal{E}_{\mu\nu} + \mathcal{F}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{F}, \quad (3.6)$$

where \mathcal{F} is the trace of $\mathcal{F}_{\mu\nu}$. We express the static, spherically symmetric metric for the vacuum exterior by

$$ds_{\text{ext}}^2 = -F^2\left[1 - \frac{2Gm}{R}\right]dT^2 + \frac{dR^2}{1 - \frac{2Gm}{R}} + R^2d\Omega^2, \quad (3.7)$$

where $F = F(R)$ and $m = m(R)$ are, in general, radial functions characterizing the exterior spacetime given by the coordinates (T, R, Θ, Φ) . The matching conditions across the boundary, assumed to satisfy 4D Israel junction conditions, are (i) continuity of the metric and (ii) continuity of the extrinsic curvature, implying continuity of \dot{R} . The two conditions are simultaneously satisfied by expressing the metrics (3.1) and (3.7) in terms of null coordinates by adapting the method of [10], which results in the following conclusions: $F(R) = 1$ by rescaling, and

$$m(R) = M + M^* + \frac{3M^2}{8\pi\lambda R^3}. \quad (3.8)$$

In terms of the effective mass, the expression for $m(R)$ can be rewritten as

$$m(R) = M^{\text{eff}} + \frac{3(M^{\text{eff}})^2}{8\pi\lambda R^3}. \quad (3.9)$$

In obtaining Eq. (3.9) from Eq. (3.8), we have neglected higher order terms involving M^* , since the correction due to the induced mass is sufficiently small so far as the low energy condition $\lambda \gg M/R^3$ holds well. The most striking feature of the above equation is that it gives nothing but the Schwarzschild solution for the exterior, with the correction term arising from perturbative braneworld gravity. However, there is some intriguing differences between the above solution and the standard 4D Schwarzschild solution. Here the Schwarzschild mass of the star is not its physical mass M but an effective mass M^{eff} that incorporates the effects of the radiative bulk. In the low energy regime ($\lambda \gg M/R^3$), this Schwarzschild metric essentially

means a static exterior. Thus, we find that the exterior may appear to be static even if there is energy exchange between the collapsing star and the bulk. In this sense, the exterior is manifestly static.

It is worthwhile to make some comments on the gravitational potential of the collapsing sphere. In braneworlds, the law of gravitation gets modified from the standard inverse square law, though the correction term becomes negligible at low energy [43]. Equation (3.9) reveals that this radiative bulk scenario is in accordance with the high energy correction of the Newtonian potential in braneworld gravity. The only point is that the correction to the potential is further suppressed due to the presence of the bulk matter.

How does the bulk matter play such a crucial role? The answer lies in the calculation of the brane Ricci scalar, once by using Eqs. (3.7) and (3.8), that gives

$$R^\mu{}_\mu = \frac{9GM^2}{2\pi\lambda R^6} \quad (3.10)$$

and once more by utilizing the trace-free property of $\mathcal{E}_{\mu\nu}$ in the vacuum field equation (3.6). Using the expression for $\mathcal{F}_{\mu\nu}$ from Eq. (2.3), it results in

$$R^\mu{}_\mu = -\frac{8}{3}\kappa_5^2\psi. \quad (3.11)$$

The difference between the above result and the matter-free bulk scenario is noteworthy. For the matter-free bulk, with $\mathcal{F}_{\mu\nu} = 0$, the Ricci scalar in Eq. (3.11) had to vanish, which was in direct contradiction with Eq. (3.10), resulting in the no-go theorem. In light of the generalized bulk scenario, this no-go theorem can now be restated as follows: *In a braneworld with vacuum bulk, a collapsing dust sphere cannot have a static exterior.* On the contrary, for a general nonempty bulk, Eq. (3.11) contains a term comprised of the bulk matter projected onto the brane. A comparison between Eqs. (3.10) and (3.11) reveals that a static exterior for a collapsing star on the brane is indeed possible if the following relation holds well:

$$\psi = -\left(\frac{3M}{8\pi}\right)^2 \sqrt{\frac{3\pi G}{\lambda}} \frac{1}{R^6}, \quad (3.12)$$

where we have used the relations $\kappa_5^4 = 6\kappa^2/\lambda$ and $\kappa^2 = 8\pi G$. This is exactly what is expected from phantom radiation in the bulk, for which $\psi < 0$. Thus, the possibility of having a static exterior is quite consistent with the phantom nature of the bulk field. In this way, we arrive at the radically novel conclusion: *For a general, nonvacuum bulk, a collapsing star on the brane can exchange energy with the bulk even if the exterior may appear to be static to a braneworld observer.*

Equation (3.12) in turn gives an idea about the evolution of the scale factor. The solution for ψ as obtained from this equation should satisfy the evolution equation (2.4) for ρ^* , which means

$$2\epsilon\psi - \frac{2\kappa_5^2}{3\kappa^2} \left[\dot{\psi} + 3\frac{\dot{a}}{a}\psi \right] = C^* \frac{\dot{a}}{a^4}. \quad (3.13)$$

Together with the expression for ψ , this shows the dependence of the comoving radius on the brane proper time as

$$C_1 R^3 + C_2 \ln R = C - C_3 \tau, \quad (3.14)$$

where the coefficients are given by

$$C_1 = \frac{M^*}{3} \left(\frac{R_0}{a_0}\right)^3, \quad C_2 = \left(\frac{3M}{2\pi}\right)^2 \frac{3G}{2\lambda}, \quad (3.15)$$

$$C_3 = \epsilon \left(\frac{3M}{2\pi}\right)^2 \sqrt{\frac{3\pi G}{\lambda}}$$

and C is a constant related to a_0 , the scale factor when the star starts collapsing. The term C_2 being too small, it immediately reveals that the scale factor for the collapsing star approximately evolves as

$$a^3(\tau) \approx a_0^3 - B\tau, \quad (3.16)$$

where the constant B is defined as

$$B = \frac{6\epsilon}{M^*} \left(\frac{3Ma_0^3}{8\pi R_0^3}\right)^2 \sqrt{\frac{3\pi G}{\lambda}}. \quad (3.17)$$

Equation (3.16) represents the scale factor of a collapsing star if the constant B is positive, implying $\epsilon > 0$. The positive signature for ϵ reveals that the star, in fact, receives phantom radiation from the bulk. This provides a possible explanation from the bulk viewpoint. The energy loss by the bulk is manifest on the brane via the induced mass. It is obvious that, after a sufficiently long time $\tau \approx a_0^3/B$, the scale factor becomes practically zero, so that, from the point of view of a local observer near the star's surface, the collapse is completed. Further, the scenario leads to a naked singularity since the effective mass becomes negative after this time is reached. However, this will happen only if the pressure inside the star remains negligible throughout the process. In this context, it may be noted that a star with dust in a radiative bulk collapses at a slower rate than a star with surface pressure in an empty bulk [13].

IV. BULK MATTER AND BULK METRIC NEAR THE BRANE

The collapsing nature of the star on the brane is manifested by its motion in the bulk. To a bulk-based observer, the contraction of the star is identical to its motion along the radial direction of the bulk black hole, with its scale factor being identified with the radial trajectory $\mathcal{R}(\tau)$ at the brane location, with the brane proper time τ chosen as the parameter [20,38,44,45].

It should be mentioned here that the global bulk geometry is difficult to obtain because of the problems associated with embedding simultaneously a FRW brane governing

the interior metric and a static brane region for the exterior of the star, which will result in different extrinsic curvatures for the two regions. Consequently, the global bulk metric may not be strictly VAdS₅. A possible extension is to introduce a dynamic Swiss-cheese-like structure consisting of black cigars penetrating a FRW brane [46]. A detailed calculation of this topic is required in order to find out the global bulk metric. However, if we focus on the physics near the collapsing region of the brane, then the bulk metric can be well approximated to be VAdS₅. Thus, one can put aside the detailed analysis and can safely consider the bulk to be VAdS₅ locally, so far as the vicinity of the collapsing region of the brane is concerned.

This bulk VAdS₅ metric is a solution of the 5D field equation with the energy-momentum tensor for a radiation field,

$$T_{MN}^{\text{bulk}} = \psi q_M q_N, \quad (4.1)$$

where M, N, \dots are the bulk indices and q_M are the outgoing null vectors indicating the energy flow from the bulk to the brane. Further, for phantom radiation in the bulk, in which we are interested, the quantity ψ is negative. With the help of Eq. (3.12), we find that the bulk matter behaves as

$$T_{MN}^{\text{bulk}} = -\left(\frac{3M}{8\pi}\right)^2 \sqrt{\frac{3\pi G}{\lambda}} \left(\frac{a_0}{R_0}\right)^6 \frac{1}{a^6(\tau)} q_M q_N. \quad (4.2)$$

Thus, the bulk energy-momentum tensor turns out to be

$$T_{MN}^{\text{bulk}} = -\left(\frac{3M}{8\pi}\right)^2 \sqrt{\frac{3\pi G}{\lambda}} \left(\frac{a_0}{R_0}\right)^6 \frac{1}{\mathcal{R}^6} q_M q_N. \quad (4.3)$$

The above equation now gives a purely bulk quantity. The negative signature guarantees that T_{MN}^{bulk} represents the energy-momentum tensor of a phantom radiation field. In this way, we find the nature of the bulk field responsible for the scenario.

One can also find out the bulk metric in the vicinity of the collapsing region of the brane by the perturbative brane-based approach formulated in [36]. In terms of the null coordinate $v = t + \int d\mathcal{R}/f$, the VAdS₅ metric, for both incoming and outgoing radiation, can be written as

$$dS_5^2 = -f(\mathcal{R}, v) dv^2 - 2d\mathcal{R} dv + \mathcal{R}^2 d\Sigma_3^2 \quad (4.4)$$

with $(v, \mathcal{R}, x, y, z)$ representing the bulk coordinates, and Σ_3 is the 3-space. The function $f(\mathcal{R}, v)$ is defined as

$$f(\mathcal{R}, v) = \frac{\mathcal{R}^2}{l^2} - \frac{\mathcal{M}(v)}{\mathcal{R}^2} \quad (4.5)$$

with the length scale l related to the bulk (negative) cosmological constant by $\Lambda_5 = -6/l^2$, and the mass function $\mathcal{M}(v)$, the resultant of the black hole mass $m_1(v)$ and the mass of the radiation field $q(v)$, is given by

$$\mathcal{M}(v) = 2m_1(v) - \frac{q^2(v)}{\mathcal{R}^2}. \quad (4.6)$$

Now, the on-brane mass function $\mathcal{M}(\tau)$ is related to the scale factor [36] via

$$\mathcal{M}(\tau) = \frac{\kappa^2}{3} C(\tau) \propto a(\tau). \quad (4.7)$$

By redefinition of parameters involved in the scale factor of Eq. (3.16), this function turns out to be

$$\mathcal{M}(\tau) = \mathcal{M}_0(\tau_0 - \tau)^{1/3}, \quad (4.8)$$

where $\mathcal{M}_0 = C_0 B \kappa^2 / 3$ is its value when the star starts collapsing. However, this on-brane function will not suffice in describing the bulk geometry relevant to a braneworld observer. We have to further find out the off-brane mass function $\mathcal{M}(v)$. This can be substantiated by keeping note of the fact that the function $f(\mathcal{R}, v)$ at the brane location reduces to

$$f(\mathcal{R}, v)|_{\text{brane}} = \frac{\mathcal{R}^2}{l^2} - C_0 \frac{\mathcal{R}(\tau)|_{\text{brane}}}{\mathcal{R}^2} = \frac{\mathcal{R}^2}{l^2} - C_0 \frac{a(\tau)}{\mathcal{R}^2}. \quad (4.9)$$

The time function for the bulk is, in general, a function of the brane proper time $t = t(\tau)$. With a suitable gauge choice similar to [45], the bulk time is identical to the brane proper time, barring an insignificant constant. By utilizing the identity of the bulk time to the brane proper time, the null coordinate v turns out to be

$$v = t + \frac{l}{2\sqrt{\mathcal{M}_0}(t_0 - t)^{1/6}} \left[\frac{1}{2} \ln \left(\frac{\mathcal{R}/l - \sqrt{\mathcal{M}_0}(t_0 - t)^{1/6}}{\mathcal{R}/l + \sqrt{\mathcal{M}_0}(t_0 - t)^{1/6}} \right) + \tan^{-1} \frac{\mathcal{R}/l}{\sqrt{\mathcal{M}_0}(t_0 - t)^{1/6}} \right]. \quad (4.10)$$

The off-brane mass $\mathcal{M}(v)$ at the vicinity of the brane can be found out by expanding $\mathcal{M}(v)$ in Taylor series around its on-brane value $\mathcal{M}(t)$ as

$$\mathcal{M}(v) = \mathcal{M}(t) + \left[\frac{\partial \mathcal{M}}{\partial t_1} \right]_{t_1=t} \int \frac{d\mathcal{R}}{f} + \frac{1}{2} \left[\frac{\partial^2 \mathcal{M}}{\partial t_1^2} \right]_{t_1=t} \left(\int \frac{d\mathcal{R}}{f} \right)^2 + \dots \quad (4.11)$$

Hence, the off-brane mass function $\mathcal{M}(v)$ is approximately given by

$$\begin{aligned} \mathcal{M}(v) \approx & \mathcal{M}_0(t_0 - t)^{1/3} + \frac{l}{3} \sqrt{\mathcal{M}_0}(t_0 - t)^{-2/3} \\ & \times \left[\frac{1}{2} \ln \left(\frac{\mathcal{R}/l - \sqrt{\mathcal{M}_0}(t_0 - t)^{1/6}}{\mathcal{R}/l + \sqrt{\mathcal{M}_0}(t_0 - t)^{1/6}} \right) \right. \\ & \left. + \tan^{-1} \frac{\mathcal{R}/l}{\sqrt{\mathcal{M}_0}(t_0 - t)^{1/6}} \right]. \end{aligned} \quad (4.12)$$

Thus, by finding out the null coordinate ν and the function $\mathcal{M}(\nu)$, we have been able to obtain the bulk geometry near the collapsing region of the brane, which is sufficient from the point of view of a braneworld observer.

V. SUMMARY AND OPEN ISSUES

In this article, we have shown that a generalized non-empty bulk may lead to a manifestly static exterior for a collapsing spherical star on the brane. In this scenario, the bulk geometry is described by a radiative Vaidya black hole. We have demonstrated that a suitable choice of bulk matter can result in the energy flow from the bulk to the collapsing star in such a way that a braneworld observer located at the exterior of the star cannot feel the energy exchange. We have shown, with the help of the brane Ricci scalar, that a nontrivial contribution from the bulk field is responsible for the scenario. The scale factor for such a collapsing sphere has also been obtained. We have also found the bulk matter and derived the bulk geometry near the brane by a brane-based perturbative analysis. In this way, the article gives a complete picture of both the brane and bulk viewpoints for the situation to be described.

Here we see that the star absorbs energy during the course of its collapse. It may so happen that, after a finite time, the star will collect a sufficient amount of energy to acquire pressure. Then its evolution will be governed by the Raychaudhuri equation (3.3) with pressure, which

might be a generalization of the work reported in [13] for the nonempty bulk. There is another possibility that the star will anti-evaporate. This situation is somewhat comparable to [47]. In either case, we need a better understanding of the bulk-brane dynamics in order to comment further on the ultimate fate of the collapsing star.

Finally, it is natural to ask the following question: How do we account for black holes in this radiative bulk scenario? We recall that black holes are not well understood even in the empty bulk scenario. Given the situation of the modified conservation equation for ρ^* , the effective conservation equation for a braneworld black hole in an empty bulk described in [3] will now be modified by a nontrivial contribution from bulk matter. We expect that the bulk matter contribution may lead to a solution for black holes, consistent with the bulk-brane dynamics. Even the possibility of having a Schwarzschild solution with an effective mass discussed in the present article may be investigated. However, an extensive study in this field is required for a conclusive remark.

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