PHYSICAL REVIEW D **74,** 121701(R) (2006)

Effective nonintercommutation of local cosmic strings at high collision speeds

Ana Achúcarro*

Instituut-Lorentz for Theoretical Physics, Leiden, The Netherlands and Department of Theoretical Physics, The University of the Basque Country UPV-EHU, 38940 Bilbao, Spain

Roland de Putter†

Instituut-Lorentz for Theoretical Physics, Leiden, The Netherlands

(Received 8 May 2006; published 13 December 2006)

We present evidence that Abrikosov-Nielsen-Olesen (ANO) strings pass through each other for very high speeds of approach due to a double intercommutation. In near-perpendicular collisions numerical simulations give threshold speeds bounded above by $\sim 0.98c$ for type I, and by $\sim 0.88c$ for deep type II strings. The second intercommutation occurs because at ultrahigh collision speeds, the connecting segments formed by the first intercommutation are nearly static and almost antiparallel, which gives them time to interact and annihilate. A simple model explains the rough features of the threshold velocity dependence with the incidence angle. For deep type II strings and large incidence angles a second effect becomes dominant, the formation of a loop that catches up with the interpolating segments. The loop is related to the observed vortex-antivortex reemergence in two dimensions. In this case the critical value for double intercommutation can become much lower.

DOI: 10.1103/PhysRevD.74.121701 PACS numbers: 11.27.+d, 98.80.Cq

Cosmic strings were intensely studied in the eighties as a possible explanation of the small deviations from homogeneity that are necessary to seed structure formation in the early Universe. Since then, this picture has been abandoned in favor of the inflationary scenario, mainly because of observations of the cosmic microwave background radiation [1]. The recent revival of interest in cosmic strings is largely motivated by fundamental theory. First, the formation of cosmic strings in the early Universe appears to be a fairly generic prediction of grand unified theories of particle physics [2]. Second, some brane inflation models from superstring theory predict the formation of cosmic (super)-string networks as well [3–6]. It should be possible in the near future, for example using B modes of the CMB polarization [7], to detect the effects of cosmic strings with a sensitivity one or two orders of magnitude higher than at present, which makes strings an excellent probe of physics at ultrahigh energies that are otherwise extremely hard to probe (even the absence of strings can discriminate between particle physics models).

An important property of cosmic strings is their behavior when two string segments collide. In principle, there are four possible outcomes (we refer the reader to [8] for general background and references): they can (1) simply pass through each other, (2) exchange ends and reconnect, (3) form a Y-junction with a bridge between the original strings, or (4) do neither and get tangled up, which happens if the Y-junction is kinematically forbidden [9]. Outcomes (3) and (4) apply to non-Abelian gauge theory strings, where there is a topological obstruction that forbids the first two outcomes, and also to (p, q) strings, which are bound states of fundamental superstrings and $D1$-branes. Another example of outcome (3) are the zipper bound states that can be formed between multiply-winding type I Abrikosov-Nielsen-Olesen (ANO) vortices when they collide at low speeds and incidence angles, due to their attractive interaction [10].

The second outcome is usually called *intercommutation* (or reconnection) and is extremely important in cosmological scenarios as it provides a mechanism for a string network to lose energy and reach a scaling regime in which the energy density in strings remains a constant fraction of the dominant form of energy density in the Universe (matter or radiation). Intercommutation leads to loops and small scale structure that decay efficiently into particles and radiation. The question of the precise effect of intercommutation on scaling (see for example [11–19]) has taken center stage recently after the realization that for cosmic superstrings the probability of intercommutation p is very low ($p \sim 10^{-3}$ to $\sim 10^{-1}$ depending on the type of string [20]). While all previous studies agree that the network will reach a scaling solution if $p = 1$, the situation is less clear for lower intercommutation probabilities. Since full field theory simulations of string networks on cosmological scales are computationally too demanding, numerical studies of such networks [21,22] typically use the effective Nambu-Goto action [23], which treats strings as infinitely thin objects. However, this action cannot be used to describe what happens when two strings intersect and therefore the intercommutation behavior needs to be studied using the full field theory.

In this paper we focus on the intercommutation behavior of ANO strings in the Abelian Higgs model—the relativistic Landau-Ginzburg model—but our results should apply to other Abelian local (gauged) strings provided there

*Electronic address: achucar@lorentz.leidenuniv.nl
†Electronic address: rdeputter@lorentz.leidenuniv.nl

are no topological obstructions. Since intercommutation is a local effect we work in flat space-time. In units $c = \hbar = 1$, the Lagrangian is

$$\mathcal{L} = (\partial_\mu + ieA_\mu)\phi(\partial^\mu - ieA^\mu)\phi^\dagger - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\lambda}{4}(|\phi|^2 - \eta^2)^2. \quad (1)$$

After a rescaling of units this model can be characterized by a single parameter, the ratio of the Higgs mass to the gauge boson mass $\beta := (m_\phi/m_A)^2 = \lambda/2e^2$. Following [24,25], we place a superposition of two (boosted) ANO strings on a three-dimensional lattice and evolve this configuration in the Hamiltonian formalism using a leapfrog algorithm. The initial configuration is characterized by only two parameters [see Fig. 1(a)]: the center of mass speed v of the strings when they are far apart and the angle α between them.

The main conclusion from previous simulations (e.g. [24]) of the interaction of ANO strings with unit winding is that intercommutation takes place in all cases. This can be understood by looking at the field configuration in certain two-dimensional slices through the point, say (x_0, y_0, z_0), in which the strings come to intersect [8]. In the $z = z_0$ plane (see Fig. 1), the string interaction looks like the collision and annihilation of a vortex and an antivortex, whereas in the $y = y_0$ plane it looks like vortex-vortex scattering at 90° [26]. Hence, large-scale network simulations of ANO strings use the Nambu-Goto approximation and assume that strings *always* exchange ends when they collide. Here we argue that this picture has to be modified for sufficiently high collision speeds. We find evidence of a threshold speed v_t above which strings exchange ends *twice* and thus effectively pass through each other. Note that this is a different effect than the threshold speed suggested elsewhere for the first intercommutation [25,27,28], of which we find no evidence. We will now first describe some of the more technical aspects of our simulations and then explain our results in some detail.

For each simulation, the lattice spacing a is determined as follows. The vortex configuration [29,30] is $\phi = \eta X(r)e^{i\theta}$ in cylindrical polar coordinates, with $X(0) = 0$ and $X(\infty) = 1$, such that $(X'(0))^{-1}$ gives a characteristic scale for the (Higgs) core. For $\beta = 1/8$, 1, we always take the lattice spacing to be $a \approx (5\gamma(v)X'(0))^{-1}$, where $\gamma(v) = 1/\sqrt{1 - v^2}$, while for $\beta = 8$, 32, we sometimes have to settle for slightly less resolution (but a is never larger than twice the size given above). The typical time step size is $\Delta t \approx a/2$, so the Courant condition (here $\Delta t \leq a/\sqrt{3}$) holds, and most simulations are performed on a 400^4 grid. The initial string separation is taken to be $\Delta x \approx 2R/\gamma(v)$, where R is the radius of a stationary string/vortex outside of which both the Higgs and the gauge fields are within 5% of their vacuum values (this means that v is not exactly what it would be at infinite separation, but the difference is negligible). We use the same boundary conditions as [24]: after each round the fields inside the box are updated using the equations of motion, and the fields on the boundaries are calculated assuming the strings move unperturbed and at constant speeds at the boundaries.

The results of our simulations are presented in Figs. 2 and 3. We investigate values of $\beta = 1/8, 1, 8, 32$ and $\alpha = 30°, 45°, 60°, 90°, 120°, 135°, 150°$ and center of mass speeds up to $v = 0.98$ and find threshold speeds in most cases. We suspect that if we could probe even higher initial speeds, we would find a threshold speed in all cases. For $v < v_t$, the strings intercommute and then move away from each other without interacting again. However, for $v > v_t$ the interaction of the strings after intercommutation is such that they exchange ends a second time. The exact mechanism through which the strings reconnect the second time depends on the value of β and α. In most cases (always for $\beta = 1/8$ and $\beta = 1$), the process unfolds roughly as in Fig. 4: the strings attract after the first

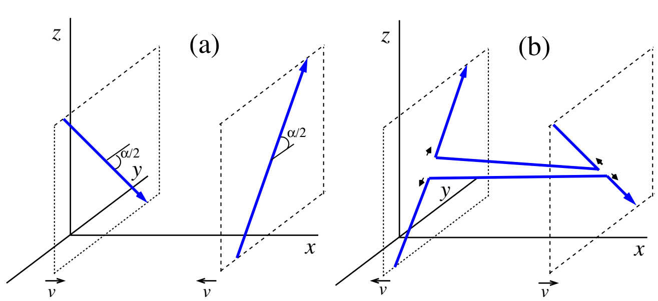

FIG. 1 (color online). (a) Initial positions and orientations of the strings in the center of mass frame. The strings lie in $x =$ const planes and approach each other with speed v. The arrows indicate the orientations of the strings, which form an angle α. (b) The configuration after one intercommutation. If $v \sim c$, the kinks' motion along the strings is negligible and the connecting horizontal segments are practically antiparallel and immobile, making a second interaction possible.

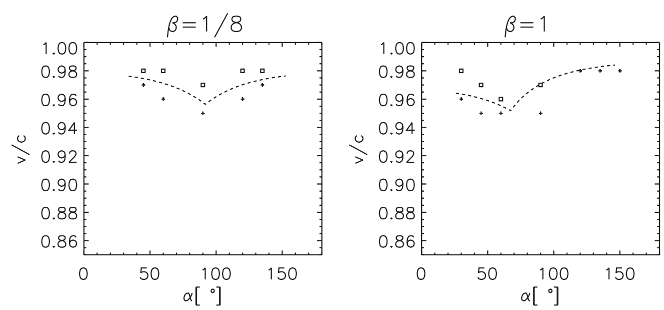

FIG. 2. Threshold speed as a function of incidence angle for $\beta = 1/8, 1$. For each α, a dot gives the *highest* approach speed v (in our simulations) for which strings only exchange ends once, and a square gives the *lowest* speed for which strings reconnect twice (so the threshold speed v_t is in between). Dashed lines are based on a simple theoretical model (see text; the plots shown have $\delta_t = 156°$, 150° and $w_t = 0.18$, 0.17 for $\beta = 1/8$, 1 respectively).

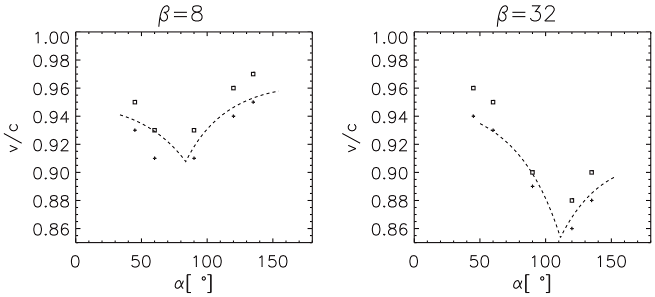

FIG. 3. Threshold speeds for type II strings. For very large β and α, a different mechanism governs the second intercommutation (interaction with an emerging string loop) and the threshold speeds are considerably smaller than for lower β. However, the results still agree well with our model. We simply need a higher critical speed w_t for $\beta = 32$. Fits use $\delta_t = 142°$, $142°$ and $w_t = 0.28$, 0.43 for $\beta = 8$, 32, respectively.

intercommutation, come to intersect again in the center of the box, and then intercommute a second time. Afterwards, the strings move on as if they had simply passed through each other, except that the parts of the strings that have been involved in the interaction lag behind the rest of the strings a little. For $\beta \gg 1$ (deep type II, e.g. $\beta = 32$) and large initial angle $\alpha = 135°$, the second exchange of ends proceeds differently. In this case, a string loop is formed (the three-dimensional manifestation of vortex-antivortex reemergence, see also [24,26]) after the first intercommutation. The loop starts at the intersection point and grows in the $y = y_0$ plane (see Fig. 1) to eventually catch up with the two original strings. When this happens, the strings reconnect again through the loop and move on as if they have passed through each other. For certain values of $v < v_t$, we also find loop formation but the loop does not grow large enough to catch up with the strings. Note that our results agree with the conventional picture of string interactions in the sense that, initially, the strings *always* intercommute.

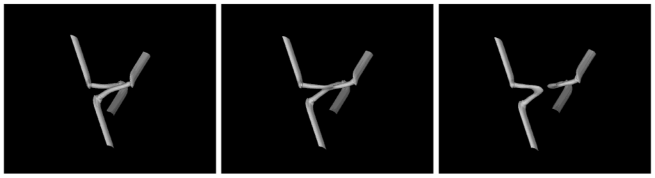

FIG. 4. Double intercommutation of strings with $\beta = 1$, $\alpha = 60°$ and $v = 0.96$. The strings effectively pass through each other, with some distortion. We use a box size $35 \times 35 \times 36.4$. Within the shaded surfaces, the energy density is above $\sim$10% of the peak energy density of a *static* ANO string/vortex. Fast moving segments appear to be thicker. The strings first intercommute ($T \sim 1.0$, not shown) and separate (top image, $T = 7.2$). Next, they attract, come to intersect again (center, $T = 9.0$), intercommute a second time, and move away from each other (bottom, $T = 10.8$). We use length and time units which take $e = \eta = 1$. Total energy is conserved to better than 0.7% up until the last figure shown.

The existence of a threshold speed and its angle dependence is easily explained in the Nambu-Goto approximation. Immediately after the first intercommutation each string has two kinks separating the parts unaffected by the interaction from the straight horizontal segments created in between [see Fig. 1(b)]. These kinks have to move at the speed of light, and their vertical motion along the strings pulls the horizontal segments apart. But for high collision speeds $v \sim c$, the kinks' *horizontal* velocity is almost luminal and so the vertical component negligible; moreover the horizontal segments are almost antiparallel (in the unphysical limit $v = c$, the kinks do not move up at all and the segments are exactly antiparallel and lying on top of each other; so we would expect the second reconnection with probability one).

More precisely, the angle between the horizontal segments is $\cos(\delta/2) = \frac{\cos(\alpha/2)/(v\gamma(v))}{\sqrt{1+(\cos(\alpha/2)/(v\gamma(v)))^2}}$ (antiparallel for $\delta = \pi$) and they move apart with velocity $w = \sin(\alpha/2)/\gamma(v)$, the vertical velocity of the kinks. If we assume that strings intercommute a second time only for δ above a critical angle δ_t and w below a critical speed w_t, we can get a surprisingly good fit to the data. First, for $\delta_t \sim 150$ and $w_t \sim 0.2$ (or significantly higher $w_t \sim 0.43$ for $\beta = 32$ where the second exchange of ends occurs through a loop for large α) we get a threshold speed of about the right magnitude that does not depend strongly on α. Second, when we do look at the α dependence, the model nicely explains the minimum of v_t somewhere around $\alpha = 90°$. The heuristic picture is that if the attractive interaction energy of the horizontal segments exceeds their kinetic energy they will come together and annihilate again. What the Nambu-Goto approximation misses is the interaction energy of the bridging segments.

In conclusion, we find that ANO strings effectively pass through each other at high speeds of approach. The result is consistent with a simple kinematic argument so we expect it to apply to any other local (gauged) string as long as there is no topological obstruction to intercommutation. In particular we have preliminary evidence that multiple-winding type I strings also pass through each other at these high collision speeds. An interesting open question is whether strings carrying zero modes or bound states will also pass through each other at very high collision speeds. In the particular case of semilocal strings and Skyrmions in the Bogomol'nyi limit it has recently been found [31] that at the location of the first intercommutation the strings revert to ANO strings, so we expect the result to hold there as well.

It is difficult at this point to make a reliable estimate of whether double intercommutation has a significant effect on the network's evolution and scaling properties; this may have to be determined in large numerical simulations. Obviously v_t is found to be high, so the probability of two very fast moving segments colliding is extremely low. On the other hand, nonintercommutation affects the highest energy components of the network, and moreover these fast string segments are expected to have higher collision rates simply because they cross larger distances, so the question is definitely worth a second look.

We thank Jon Urrestilla for advice on computational issues and for very helpful discussions, and also Kepa Sousa, Petja Salmi, and Tanmay Vachaspati. Our work was partly supported by the Netherlands Organization for Scientific Research (N.W.O) under the VICI programme. We acknowledge the generous allocation of computational resources by the Dutch National Computer Facility Foundation (NCF).

[1] L. Pogosian, M. Wyman, and I. Wasserman, J. Cosmol. Astropart. Phys. 09 (2004) 008.
[2] R. Jeannerot, J. Rocher, and M. Sakellariadou, Phys. Rev. D **68**, 103514 (2003).
[3] S. Sarangi and S. H. H. Tye, Phys. Lett. B **536**, 185 (2002).
[4] M. Majumdar and A. C. Davis, J. High Energy Phys. 03 (2002) 056.
[5] E. J. Copeland, R. C. Myers, and J. Polchinski, J. High Energy Phys. 06 (2004) 013.
[6] G. Dvali and A. Vilenkin, J. Cosmol. Astropart. Phys. 03 (2004) 010.
[7] U. Seljak and A. Slosar, Phys. Rev. D **74**, 063523 (2006).
[8] A. Vilenkin and E. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, England, 1994), 1st ed.
[9] E. J. Copeland, T. W. B. Kibble, and D. A. Steer, Phys. Rev. Lett. **97**, 021602 (2006).
[10] L. Bettencourt, P. Laguna, and R. Matzner, Phys. Rev. Lett. **78**, 2066 (1997).
[11] G. Vincent, N. D. Antunes, and M. Hindmarsh, Phys. Rev. Lett. **80**, 2277 (1998).
[12] J. Moore, E. Shellard, and C. Martins, Phys. Rev. D **65**, 023503 (2001).
[13] C. Martins, E. Shellard, and J. Moore, Phys. Rev. Lett. **92**, 251601 (2004).
[14] N. T. Jones, H. Stoica, and S. H. H. Tye, Phys. Lett. B **563**, 6 (2003).
[15] T. Damour and A. Vilenkin, Phys. Rev. D **71**, 063510 (2005).
[16] M. Sakellariadou, J. Cosmol. Astropart. Phys. 04 (2005) 003.
[17] A. Avgoustidis and E. P. S. Shellard, Phys. Rev. D **73**, 041301 (2006).
[18] V. Vanchurin, K. D. Olum, and A. Vilenkin, Phys. Rev. D **74**, 063527 (2006).
[19] C. Ringeval, M. Sakellariadou, and F. Bouchet, astro-ph/0511646.
[20] M. G. Jackson, N. T. Jones, and J. Polchinski, J. High Energy Phys. 10 (2005) 013.
[21] A. Albrecht and N. Turok, Phys. Rev. Lett. **54**, 1868 (1985).
[22] D. Bennett, Phys. Rev. D **33**, 872 (1986).
[23] M. Anderson, F. Bonjour, R. Gregory, and J. Stewart, Phys. Rev. D **56**, 8014 (1997).
[24] R. Matzner, Comput. Phys. **2**, 51 (1988).
[25] E. Shellard, Nucl. Phys. B **B283**, 624 (1987).
[26] E. Myers, C. Rebbi, and R. Strilka, Phys. Rev. D **45**, 1355 (1992).
[27] E. Copeland and N. Turok, Fermilab Report No. 86/127-A, 1986 (unpublished).
[28] A. Hanany and K. Hashimoto, J. High Energy Phys. 06 (2005) 021.
[29] A. Abrikosov, Zh. Eksp. Teor. Fiz. **32**, 1442 (1957).
[30] H. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).
[31] P. Laguna, V. Natchu, R. A. Matzner, and T. Vachaspati, hep-th/0604177.