PHYSICAL REVIEW D **74,** 121502(R) (2006)

Thermodynamics via creation from nothing: Limiting the cosmological constant landscape

A. O. Barvinsky

Theory Department, Lebedev Physics Institute, Leninsky Prospect 53, 119991 Moscow, Russia

A. Yu. Kamenshchik

Dipartimento di Fisica and INFN, Via Irnerio 46, 40126 Bologna, Italy
and L. D. Landau Institute for Theoretical Physics, Kosygin str. 2, 119334 Moscow, Russia

(Received 27 June 2006; published 19 December 2006)

The creation of a quantum Universe is described by a *density matrix* which yields an ensemble of universes with the cosmological constant limited to a bounded range $\Lambda_{\rm min} \leq \Lambda \leq \Lambda_{\rm max}$. The domain $\Lambda < \Lambda_{\rm min}$ is ruled out by a cosmological bootstrap requirement (the self-consistent back reaction of hot matter). The upper cutoff results from the quantum effects of vacuum energy and the conformal anomaly mediated by a special ghost-avoidance renormalization. The cutoff $\Lambda_{\rm max}$ establishes a new quantum scale—the accumulation point of an infinite sequence of garland-type instantons. The dependence of the cosmological constant range on particle phenomenology suggests a possible dynamical selection mechanism for the landscape of string vacua.

DOI: 10.1103/PhysRevD.74.121502 PACS numbers: 04.60.Gw, 04.62.+v, 98.80.Bp, 98.80.Qc

Quantum cosmology [1,2] and Euclidean quantum gravity [3] might effectively restrict the landscape of string vacua. This landscape is too big [4] to predict either the observed particle phenomenology or large-scale structure formation within string theory itself. Other methods have to be invoked, at least some of them based on the cosmological wavefunction [5–7]. This is dominated by the exponentiated Euclidean action, $\exp(-S_{\rm E})$, calculated on the gravitational instanton which is a saddle point of an underlying path integral over Euclidean 4-geometries. This instanton gives rise to Lorentzian signature spacetime by analytic continuation across minimal hypersurfaces. The continuation can be interpreted either as quantum tunneling or as the creation of the Universe from "nothing". Thus, the most probable vacua of the landscape become weighted by the minima of $S_{\rm E}$. This might serve as a method of selecting a vacuum from the enormously big string landscape.

An immediate difficulty with this program arises from the infrared catastrophe of small cosmological constant Λ. The Hartle-Hawking wave function [3], which describes nucleation of the de Sitter Universe from the Euclidean 4-dimensional hemisphere, has the form

$$\Psi_{\rm HH} \sim \exp(-S_{\rm E}) = \exp(3\pi/2G\Lambda). \tag{1}$$

This diverges for $\Lambda \to 0$ because of unboundedness of the Euclidean gravitational action. Despite some early attempts to interpret it as the origin of a zero value of Λ [8], this result remains both controversial and anti-intuitive because it disfavors inflation and prefers creation of infinitely large universes. Apart from the tunneling proposals of [9] which employ an opposite sign in the exponential of (1) and thus open the possibility for opposite conclusions [10], no convincing resolution of this problem has thus far been suggested.

In this Letter we show that Euclidean path integration framework naturally avoids this infrared catastrophe. We attain this result by: (i) extending the notion of Hartle-Hawking *pure* state to a density matrix which describes a *mixed* quantum state of the Universe and (ii) incorporating the nonperturbative back reaction of hot quantum matter on the instanton background [11]. These extensions seem natural because whether the initial state of the Universe is pure or mixed is a dynamical question rather than a postulate. We address this question below by embedding both types of states into a unified framework of a density matrix.

A density matrix $\rho[\varphi, \varphi']$ is represented in Euclidean quantum gravity [12] by an instanton having two disjoint boundaries Σ and Σ' associated with its two entries φ and φ' (collecting both gravity and matter observables). The instanton interpolates between these, thus establishing mixing correlations, see Fig. 1. In contrast, for the density matrix of the pure Hartle-Hawking state the bridge between Σ and Σ' is broken, so that the instanton is a union of two disjoint hemispheres which smoothly close up at their poles (Fig. 2)—a picture illustrating the factorization of $\hat{\rho} = |\Psi_{\rm HH}\rangle\langle\Psi_{\rm HH}|$.

The main effect that we advocate here is that thermal fluctuations and quantum conformal anomaly destroy the Hartle-Hawking instanton and replace it with one filled by

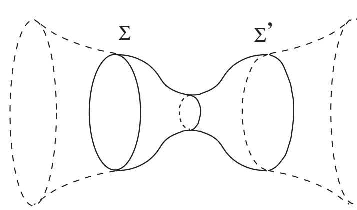

FIG. 1. Density matrix instanton. Dashed lines depict the Lorentzian Universe nucleating at minimal surfaces Σ and Σ'.

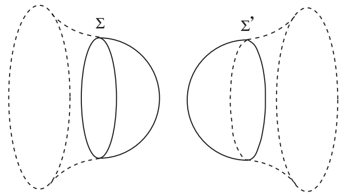

FIG. 2. Density matrix of the pure Hartle-Hawking state represented by the union of two vacuum instantons.

radiation. This is already manifest in classical theory, specifically in the Euclidean Friedmann equation for a scale factor $a(\tau)$, $\dot{a}^2/a^2 = 1/a^2 - H^2 - C/a^4$ (we use spatially closed FRW metric $ds^2 = N^2(\tau)d\tau^2 + a^2(\tau)d^2\Omega^{(3)}$ in the gauge $N = 1$ and express $\Lambda = 3H^2$ in terms of the Hubble constant). The radiation density C/a^4 prevents the half-instantons from closing and allows a to vary between two turning points [13,14] $a_\pm = (1/\sqrt{2}H) \times (1 \pm \sqrt{1-4CH^2})^{1/2}$. This forces a tubular structure on the instanton which spans (at least) one period of oscillation between $a_\pm$, provided the constant C characterizing the amount of radiation satisfies the bound $4H^2C \leq 1$.

The existence of radiation naturally follows from the partition function of this state associated with the toroidal instanton obtained by identifying Σ' and Σ. The radiation back reaction supports the instanton geometry in which it exists. Remarkably, when the vacuum energy and conformal anomaly are taken into account this bootstrap yields a set of instantons—a landscape—only in the bounded range of Λ,

$$\Lambda_{\rm min} < \Lambda < \Lambda_{\rm max}. \tag{2}$$

All values $\Lambda < \Lambda_{\rm min}$ are completely eliminated either because of the absence of instanton solutions or because of their *infinitely large positive* action. A similar situation holds for $\Lambda > \Lambda_{\rm max}$—no instantons exist there, and the Lorentzian configurations in this overbarrier domain (if any) are exponentially suppressed relative to those of (2).

To quantify the above picture consider the density matrix given by the Euclidean path integral [12]

$$\rho[\varphi, \varphi'] = e^{\Gamma} \int D[g, \phi] \exp(-S_{\rm E}[g, \phi]), \tag{3}$$

where $S_{\rm E}[g, \phi]$ is the classical action, and the integration runs over gravitational g and matter ϕ fields interpolating between φ and φ' at Σ and Σ'. The statistical sum $\exp(-\Gamma)$ is given by a similar path integral over periodic fields on the torus with identified boundaries Σ and Σ'.

The back reaction follows from decomposing $[g, \phi]$ into a minisuperspace $g_0(\tau) = (a(\tau), N(\tau))$, and the "matter" sector which includes also inhomogeneous metric perturbations on minisuperspace background $\Phi(x) = (\phi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), \ldots)$. With a relevant decomposition of the measure $D[g, \phi] = Dg_0(\tau) \times D\Phi(x)$, the integral for Γ expresses in terms of the effective action $\Gamma[g_0(\tau)]$ of quantized matter on the background $g_0(\tau)$, $\exp(-\Gamma[g_0]) = \int D\Phi(x) \exp(-S_{\rm E}[g_0, \Phi(x)])$, as

$$e^{-\Gamma} = \int Dg_0(\tau) \exp(-\Gamma[g_0(\tau)]). \tag{4}$$

Our approximation will be to consider $\Gamma[g_0(\tau)]$ in the one-loop order, $\Gamma[g_0] = S_{\rm E}[g_0] + \Gamma_{\text{1-loop}}[g_0]$, and handle (4) at the tree level, which is equivalent to solving the *effective equations* for $\Gamma[g_0]$.

Remarkably, $\Gamma[g_0]$ is *exactly* calculable for conformally-invariant fields by a conformal transformation [15] relating generic FRW metric $ds^2 = a^2 d\bar{s}^2$ to that of a static universe of a unit size, $d\bar{s}^2 = d\eta^2 + d^2\Omega^{(3)}$ (these metrics are denoted below as g and $\bar{g}$, while η is the conformal time). The total action reads

$$\begin{aligned}\Gamma[a(\tau), N(\tau)] &= 2\int_{\tau_-}^{\tau_+} d\tau\left(-\frac{a\dot{a}^2}{N} - Na + NH^2a^3\right)\\ &\quad + 2B\int_{\tau_-}^{\tau_+} d\tau\left(\frac{\dot{a}^2}{Na} - \frac{1}{6}\frac{\dot{a}^4}{N^3a}\right)\\ &\quad + B\int_{\tau_-}^{\tau_+} d\tau N/a + F\left(2\int_{\tau_-}^{\tau_+} d\tau N/a\right).\end{aligned} \tag{5}$$

We work in units of $m_P = \sqrt{3\pi/4G}$, and the integration runs between two turning points at $\tau_\pm$. The first line is the classical part, the second line is the conformal contribution and the last line is the one-loop action on the static instanton of the metric $\bar{g}$.

The conformal contribution $\Gamma_{\text{1-loop}}[g] - \Gamma_{\text{1-loop}}[\bar{g}]$ is determined by the coefficients of $\Box R$, the Gauss-Bonnet invariant $E = R^2_{\mu\nu\alpha\gamma} - 4R^2_{\mu\nu} + R^2$ and Weyl tensor term in the conformal anomaly $g_{\mu\nu}\delta\Gamma_{1-\text{loop}}/\delta g_{\mu\nu} = g^{1/2}(\alpha\Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta})/4(4\pi)^2$. Specifically this contribution can be obtained by the technique of [16]; it contains higher-derivative terms $\sim\ddot{a}^2$ which produce ghost instabilities in solutions of effective equations. However, such terms are proportional to the coefficient α which can be put to zero by adding the following finite *local* counterterm

$$\Gamma_R[g] = \Gamma_{\text{1-loop}}[g] + \frac{1}{2(4\pi)^2}\frac{\alpha}{12}\int d^4x g^{1/2} R^2(g). \tag{6}$$

This ghost-avoidance renormalization is justified by the requirement of consistency of the theory at the quantum level. The contribution $\Gamma_R[g] - \Gamma_R[\bar{g}]$ to the *renormalized* action then gives the second line of (5) with $B = 3\beta/4$.

The static instanton with a period η_0 playing the role of inverse temperature contributes $\Gamma_{\text{1-loop}}[\bar{g}] = E_0\eta_0 + F(\eta_0)$, where the vacuum energy E_0 and free energy $F(\eta_0)$ are the typical boson and fermion sums over field oscillators with energies ω on a unit 3-sphere $E_0 = \pm\sum_\omega \omega/2$, $F(\eta_0) = \pm\sum_\omega \ln(1 \mp e^{-\omega\eta_0})$. The renormalization (6) which should be applied also to $\Gamma_{\text{1-loop}}[\bar{g}]$

modifies E_0, so that $\Gamma_R[\bar{g}] = C_0\eta_0 + F(\eta_0)$, $C_0 \equiv E_0 + 3\alpha/16$. This gives the third line of Eq. (5) with $C_0 = B/2$. This universal relation between C_0 and $B = 3\beta/4$ follows from the known anomaly coefficients [17] and the UV-renormalized Casimir energy in a static universe [18] for scalar, Weyl spinor and vector fields, respectively, having:

$$\alpha = \frac{1}{90} \times \begin{cases} -1 \\ -3 \\ 18 \end{cases}, \qquad \beta = \frac{1}{360} \times \begin{cases} 2 \\ 11 \\ 124 \end{cases}, \qquad E_0 = \frac{1}{960} \times \begin{cases} 4 \\ 17 \\ 88 \end{cases}. \tag{7}$$

It is important that for conformally-invariant fields the nonlocal action (5) is exact, and contains no other terms of higher order in the curvature.

The effective equation $\delta\Gamma/\delta N(\tau) = 0$ has the form of the classical equation modified by the quantum B-term

$$\frac{\dot{a}^2}{a^2} + B\left(\frac{1}{2}\frac{\dot{a}^4}{a^4} - \frac{\dot{a}^2}{a^4}\right) = \frac{1}{a^2} - H^2 - \frac{C}{a^4}, \tag{8}$$

$$C = B/2 + F'(\eta_0), \qquad \eta_0 = 2\int_{\tau_-}^{\tau_+} d\tau/a(\tau). \tag{9}$$

Remarkably, the contribution of the nonlocal $F(\eta_0)$ in (5) reduces to the radiation *constant* C as a *nonlocal functional* of $a(\tau)$, determined by the *bootstrap* Eq. (9). Here $F'(\eta_0) \equiv dF(\eta_0)/d\eta_0 > 0$ is the energy of a hot gas of particles, which adds to their vacuum energy $B/2$.

Periodic instanton solutions of Eqs. (8) and (9) exist only inside the curvilinear wedge of (H^2, C)-plane between bold segments of the upper hyperbolic boundary and the lower straight line boundary of Fig. 3,

$$4CH^2 \leq 1, \qquad C \geq B - B^2H^2, \qquad BH^2 \leq 1/2. \tag{10}$$

Below this domain the solutions are either complex and aperiodic or suppressed by *infinite positive* Euclidean action. Above this domain only Lorentzian (overbarrier) configurations exist, but they are again exponentially damped relative to instantons in (10).

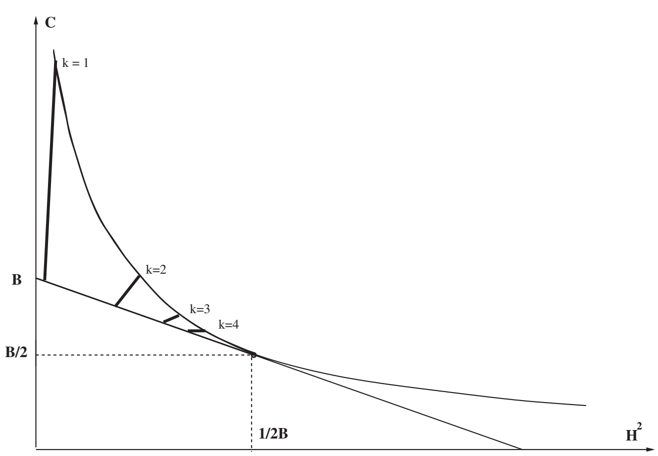

FIG. 3. Instanton domain in the (H^2, C)-plane. Garland families are shown for $k = 1, 2, 3, 4$. Their sequence accumulates at the critical point $(1/2B, B/2)$.

These properties are based on the fact that the turning points of (8) exactly coincide with classical $a_\pm$, but a_- exists only when $a_-^2 \geq B$, which gives rise to (10). Otherwise, $a(\tau)$ at the contraction phase becomes complex or runs to zero which violates instanton periodicity. In the latter case a smooth Hartle-Hawking instanton with $a_- = 0$ forms and yields $\eta_0 \to \infty$ in view of (9), so that $F(\eta_0) \sim F'(\eta_0) \to 0$. Therefore, its on-shell action

$$\Gamma_0 = F(\eta_0) - \eta_0 F'(\eta_0) + 4\int_{a_-}^{a_+} \frac{da\dot{a}}{a}\left(B - a^2 - \frac{B\dot{a}^2}{3}\right) \tag{11}$$

due to $B > 0$ diverges to $+\infty$ at $a_- = 0$ and completely rules out pure-state instantons [11].

Moreover, inside the range (10) our bootstrap eliminates the infrared catastrophe of $\Lambda \to 0$. Indeed $\eta_0 \to \infty$ as $H^2 \to 0$, so that due to (9) $C \to B/2$, but this is impossible because in view of (10) $C \geq B$ at $H^2 = 0$. Thus, instanton family never hits the C-axes of $H^2 = 0$ and can only interpolate between the points on the boundaries of the domain (10). For a conformal scalar field the numerical analysis gives such a family [11] starting at $H^2 \approx 2.00$, $C \approx 0.004$, $\Gamma_0 \approx -0.16$, and terminating at $H^2 \approx 13.0$, $C \approx 0.02$, $\Gamma_0 \approx -0.09$. The upper point describes the static universe filled by a hot radiation with the temperature $T = H/\pi\sqrt{1 - 2BH^2}$, whereas the lower point establishes the lower bound of the Λ-range.

The upper bound of the landscape follows from the existence of *garlands* that can be obtained by gluing together into a torus k copies of a simple instanton [13,19]; see Fig. 4. Their formalism is the same as above except that the conformal time in (9) and the integral term of (11) should be multiplied by k.

As in the case of $k = 1$, garland families interpolate between the lower and upper boundaries of (10). They exist for all k, $1 \leq k \leq \infty$, and their infinite sequence accumulates at the critical point $C = B/2$, $H^2 = 1/2B$, where these boundaries merge. Within the $1/k^2$-accuracy the upper and lower points of each family coincide and read

$$H_{(k)}^2 \simeq \frac{1}{2B}\left(1 - \frac{\ln^2 k^2}{2k^2\pi^2}\right), \qquad \Gamma_0^{(k)} \simeq -B\frac{\ln^3 k^2}{4k^2\pi^2}. \tag{12}$$

With a growing k, garlands become more and more static

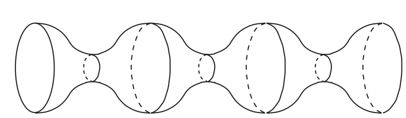

FIG. 4. The garland segment consisting of three folds of a simple instanton glued at surfaces of a maximal scale factor.

and cold with $T_{(k)} \simeq 1/(\sqrt{B}\ln k^2) \to 0$. Contrary to [19] the garland action is not additive in k, so that as $k \to \infty$ garlands do not dominate the ensemble. Their sequence converges to the instanton with $H^2_{\max} = 1/2B$, which gives the upper bound of the range (2).

Thus, our Universe is created in a hot mixed state, but its evolution does not contradict the large-scale structure formation. After nucleation from the instanton the Universe expands; its radiation dilutes, so that Λ starts dominating and generates inflation under an assumption that everywhere above Λ is a composite field (like an inflaton) decaying at the exit by a standard slow-roll scenario.

The ensemble of universes belongs to a bounded range (2) of $\Lambda = 3H^2$. Its infrared cutoff is provided by the radiation back reaction and survives even in the classical limit as $B \to 0$. In contrast, the high-energy cutoff at

$$\Lambda_{\max} = 3m_P^2/2B, \qquad m_P^2 \equiv 3\pi/4G, \tag{13}$$

is the quantum effect of vacuum energy and the conformal anomaly; this generates a new scale in gravity theory.

We have considered only conformal fields which make our model exactly solvable and provide critically important positivity of the constant $B = 3\beta/4$, cf. (7). Moreover, conformal invariance together with the ghost-avoidance renormalization renders a particular value of the vacuum energy $B/2$ in (9) which yields the upper boundary of (2) exactly at the critical point $(1/2B, B/2)$ of Fig. 3. Even if nonconformal fields qualitatively preserve the whole picture, they are likely to break this relation. Then if $C_0 < B/2$ all garlands survive, though they saturate at $\Lambda_{\max}$ with a finite temperature. If $B > C_0 > B/2$, their sequence is truncated at some k. Finally, if $C_0 > B$ the infrared catastrophe occurs again—the $k = 1$ instanton family hits the C-axes at C_0.

Conformal invariance can be justified as a good approximation when conformal particles outnumber nonconformal ones. Moreover, their large number N justifies a semiclassical expansion by scaling down the range (2). Indeed, for a single scalar field the latter is determined by Planckian values, $\Lambda_{\min} \approx 8.99m_P^2$, $\Lambda_{\max} = 360m_P^2$ which, however, decrease as $1/N$ in view of the simple scaling $C \to NC$, $B \to NB$, $F(\eta_0) \to NF(\eta_0)$ and $H^2 \to H^2/N$. Semiclassical expansion can also be justified for large $B = 3\beta/4$ growing with spin, cf. (7), because the domain (2) with (13) shrinks to a narrow subplanckian range when ascending the particle hierarchy.

Though motivated by the string landscape, all the above results hold outside of the string theory context and, as a feedback, suggest a long-sought selection mechanism for the plethora of string vacua. Modulo the details of a relevant $4D$-compactification, this might work as follows. For B growing with N and spin, the upper scale (13) decreases towards the increasing phenomenology scale, and approaches the latter at the string scale m_s^2 where a positive Λ might be generated by the mechanism of [20]. Our conjecture is that at this scale our bootstrap becomes perturbatively consistent, provided $m_P^2/B \simeq m_s^2 \ll m_P^2$, and selects from the string landscape a small subset compatible with observations.

Our results hold within the Euclidean path integral (3) which automatically excludes Lorentzian configurations possibly existing above the upper boundary of (10), $4CH^2 > 1$. However, one can imagine an extended formulation of quantum gravity generalizing (3) to a wider path integration domain. Our conclusions nevertheless remain true. Indeed the effective action scales as $\Gamma_0 \sim -\sqrt{B}$, $B \gg 1$, and because it is *negative* our landscape at the scale m_s is weighted by $\exp(\#\sqrt{B}) \simeq \exp(\#m_P/m_s) \gg 1$. Therefore it strongly dominates over Lorentzian configurations, the amplitudes of the latter being $O(1)$ in view of their pure phase nature. Thus, our results look robust against possible generalizations of Euclidean quantum gravity.

A. B. thanks H. Tye, C. Deffayet, A. Gorsky, J.Khoury, R. Woodard, S. Solodukhin, T. Osborn and especially Jim Hartle for thought provoking discussions and helpful suggestions. A. B. was supported by the RFBR grant No. 05-02-17661 and the grant No. LSS 4401.2006.2. A. K. was partially supported by the RFBR grant No. 05-02-17450 and the grant No. LSS 1757.2006.2.

[1] B. S. DeWitt, Phys. Rev. **160**, 1113 (1967).
[2] A. Vilenkin, Phys. Lett. B **117**, 25 (1982); Phys. Rev. D **27**, 2848 (1983).
[3] J. B. Hartle and S. W. Hawking, Phys. Rev. D **28**, 2960 (1983); S. W. Hawking, Nucl. Phys. **B239**, 257 (1984).
[4] R. Bousso and J. Polchinsky, J. High Energy Phys. 06 (2000) 006; M. R. Douglas, J. High Energy Phys. 05 (2003) 046.
[5] H. Ooguri *et al.*, Lett. Math. Phys. **74**, 311 (2005).
[6] H. Firouzjahi *et al.*, J. High Energy Phys. 0409 (2004) 060; S. Sarangi and S.-H. H. Tye, hep-th/0505104; hep-th/0603237.
[7] R. Brustein and S. P. de Alwis, Phys. Rev. D **73**, 046009 (2006).
[8] S. W. Hawking, Phys. Lett. B **134**, 403 (1984); S. R. Coleman, Nucl. Phys. **B310**, 643 (1988).
[9] A. D. Linde, Lett Nuovo Cim. **39**, 401 (1984); A. Vilenkin, Phys. Rev. D **30**, 509 (1984).
[10] A. O. Barvinsky and A. Yu. Kamenshchik, Class. Quant. Grav. **7**, L181 (1990); Phys. Lett. B **332**, 270 (1994).
[11] A. O. Barvinsky and A. Yu. Kamenshchik, J. Cosmol. Astropart. Phys. 09 (2006) 014.

[12] D. N. Page, Phys. Rev. D **34**, 2267 (1986).
[13] J. J. Halliwell and R. C. Myers, Phys. Rev. D **40**, 4011 (1989).
[14] G. V. Lavrelashvili *et al.*, Nucl. Phys. **B329**, 98 (1990); A. Yu. Kamenshchik *et al.*, Phys. Lett. B **357**, 36 (1995).
[15] M. V. Fischetti, J. B. Hartle, and B. L. Hu, Phys. Rev. D **20**, 1757 (1979).
[16] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. B **134**, 187 (1984); A. O. Barvinsky *et al.*, gr-qc/9510037; P. O. Mazur and E. Mottola, Phys. Rev. D **64**, 104022 (2001).
[17] J. S. Dowker and R. Critchley, Phys. Rev. D **16**, 3390 (1977); S. M. Christensen, Phys. Rev. D **17**, 946 (1978).
[18] J. S. Dowker and R. Critchley, J. Phys. A **9**, 535 (1976); S. G. Mamaev, *et al.* Sov. Phys. JETP **43**, 823 (1976); L. H. Ford, Phys. Rev. D **14**, 3304 (1976); P. Candelas and J. S. Dowker, Phys. Rev. D **19**, 2902 (1979).
[19] W. Fischler, D. Morgan, and J. Polchinsky, Phys. Rev. D **42**, 4042 (1990).
[20] S. Kachru *et al.*, Phys. Rev. D **68**, 046005 (2003); S. Kachru *et al.*, J. Cosmol. Astropart. Phys. 10 (2003) 013.