Meson-octet-baryon couplings using light cone QCD sum rules

T. M. Aliev,[1,*] A. Ozpineci,[1,†] S. B. Yakovlev,[2] and V. Zamiralov[2,‡]

[1]*Middle East Technical University, Ankara, Turkey*
[2]*Institute of Nuclear Physics, M. V. Lomonosov MSU, Moscow, Russia*

The coupling constants of K and π mesons with the octet baryons are studied in light cone QCD sum rules taking into account $SU(3)_f$ flavor symmetry breaking effects, but keeping the $SU(2)$ isospin symmetry intact. It is shown that, in the $SU(3)_f$ flavor symmetry breaking case, all of the couplings can be written in terms of four universal functions instead of F and D couplings which exist in the $SU(3)_f$ symmetry case. A comparison of our results of kaon and pion-baryon couplings with existing theoretical and experimental results in the literature is performed.

I. INTRODUCTION

The analysis of baryon-baryon, baryon-meson scattering, and photoproduction experiments requires the knowledge of the hadronic coupling constants involving mesons. Experimentally, only the hadronic coupling constant with pion $g_{NN\pi}$ is determined accurately both from nucleon-nucleon and pion-nucleon scattering. However, the situation for the kaon case is not simple and, to reproduce an experimental result for kaon-nucleon scattering cross section and kaon photoproduction, many phenomenologically undetermined coupling constants are needed (see [1]). Therefore, it seems a formidable task to determine the kaon-baryon and pion-baryon coupling constants. For this reason, a reliable theoretical approach for estimating these coupling constants is needed. Among all nonperturbative methods, QCD sum rules [2] are especially powerful in studying the properties of hadrons. This method is successfully applied to the investigation of a variety of problems in hadron physics, in particular, the calculation of the meson-baryon coupling constants. The calculation of the pion-nucleon coupling constants received a lot of attention (see e.g. [3–11]).

The K meson-baryon coupling constants also were studied in the framework of the sum rules (SR) method in [12–15] and in the light cone SR (LCSR) in [16].

The main result of these works is that the predictions of SR for meson-baryon couplings depend strongly on the choice of the Dirac structure (see e.g. [9,10]). Our numerical calculations show that only the $\not{p}\not{q}\gamma_5$ structure leads to a reliable prediction on the meson-baryon coupling constants.

In this work, we will study pseudoscalar π and K meson-baryon coupling constants in the framework of an alternative approach to the traditional SR, i.e. LCSR using the most general expression for hadronic currents as well

as $SU(3)_f$ flavor symmetry breaking strange quark effects. LCSR is based on the operator product expansion near the light cone, which is an expansion of the time ordered product over the twist rather than the dimension of the operators. The main contribution in this approach comes from the operators having the lowest twist. The main ingredient of LCSR is the wave functions of hadrons, which define the matrix element of nonlocal operators between the vacuum and the one particle hadron state (for more see e.g. [17,18]).

It should be noted that kaon-baryon coupling constants in the $SU(3)_f$ symmetry limit have been studied in [19] based on the known sum rules for the pion-baryon couplings. In the present work, we have taken into account the $SU(3)_f$ breaking effects. It is well known that in the $SU(3)_f$ symmetry limit, the coupling constants of pions and kaons with baryons are described in terms of two universal constants: F and D (see Sec. II). In the absence of this symmetry, it is natural to ask how much of this structure still remains. One of the central problems addressed in this paper is the discussion of this question.

The plan of the paper is as follows. In Sec. II, we demonstrate how kaon-baryon coupling constants and the pion-baryon couplings can be related in the $SU(3)_f$ symmetry breaking case. In Sec. III, the LCSR for the meson-baryon coupling constants using the most general form of the baryon currents is derived. Section IV is devoted to the analysis of the LCSR and comparison of our results with the predictions of the other approaches.

II. RELATIONS BETWEEN K AND π COUPLING CONSTANTS

In this section, we will demonstrate how K and π coupling constants to the baryons can be related.

Let us briefly review the formulas for the coupling constants KNY, $K\Xi Y$, and $\pi\Sigma Y$ $(Y = \Sigma, \Lambda)$ in the $SU(3)_f$ symmetry limit. In this limit, the interaction Lagrangian can be written as

$$\mathcal{L} = \sqrt{2}(D\,\mathrm{Tr}\bar{B}\{P, B\} + F\,\mathrm{Tr}\bar{B}[P, B]), \qquad (1)$$

───────────
*Permanent address: Institute of Physics, Baku, Azerbaijan.
Electronic address: taliev@metu.edu.tr
†Electronic address: ozpineci@metu.edu.tr
‡Electronic address: zamir@depni.sinp.msu.ru

where

$$B_\beta^\alpha = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} \quad (2)$$

and

$$P_\beta^\alpha = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \quad (3)$$

Here B_β^α represent the $\frac{1}{2}^+$ octet baryons and P_β^α represents the 0^- pseudoscalar mesons. From the Lagrangian, the expressions for the K and π couplings can easily be read off as

$$g_{\pi^0 pp} = F + D, \qquad g_{\pi^0 \Sigma^+ \Sigma^+} = 2F,$$

$$g_{\pi^- \Sigma^+ \Sigma^0} = -2F, \qquad g_{\pi^0 \Xi^0 \Xi^0} = F - D,$$

$$g_{\pi^+ \Xi^+ \Xi^0} = -\sqrt{2}(F - D), \qquad g_{\pi^{0,\pm}\Sigma^{0,\pm}\Lambda} = \frac{2}{\sqrt{3}}D,$$

$$g_{K^- p\Lambda} = g_{\eta\Xi\Xi} = -\frac{1}{\sqrt{3}}(3F + D), \qquad g_{K^- p\Sigma^0} = D - F$$

$$g_{K^0 \Xi^0 \Lambda} = g_{\eta NN} = \frac{1}{\sqrt{3}}(3F - D),$$

$$g_{K^0 \Xi^0 \Sigma^0} = -(D + F), \quad \text{etc.} \quad (4)$$

In order to motivate the treatment in the $SU(3)_f$ violating case, let us write the π^0 current as

$$j_{\pi^0} = \sum_{q=u,d,s} g_{\pi^0 qq}\bar{q}\gamma_5 q, \quad (5)$$

where $g_{\pi^0 uu} = -g_{\pi^0 dd} = \frac{1}{\sqrt{2}}$ and $g_{\pi^0 ss} = 0$. Then the coupling of the pion to the $B(qq, q')$ baryon which consists of two identical q-quarks and a third different q'-quark, can be written as

$$\frac{1}{\sqrt{2}}g_{\pi^0 BB} = g_{\pi^0 qq}2F + g_{\pi^0 q'q'}(F - D) \quad (6)$$

or explicitly

$$\frac{1}{\sqrt{2}}g_{pp\pi^0} = g_{\pi^0 uu}2F + g_{\pi^0 dd}(F - D) = \frac{1}{\sqrt{2}}(F + D),$$

$$\frac{1}{\sqrt{2}}g_{\pi^0 \Sigma^+ \Sigma^+} = g_{\pi^0 uu}2F + g_{\pi^0 ss}(F - D) = \sqrt{2}F, \quad (7)$$

$$\frac{1}{\sqrt{2}}g_{\pi^0 \Xi^0 \Xi^0} = g_{\pi^0 ss}2F + g_{\pi^0 uu}(F - D) = \frac{1}{\sqrt{2}}(F - D),$$

etc.

In order to obtain relations between these coupling constants, let us write formally the coupling constant of the π^0 to the Σ^0 as

$$\frac{1}{\sqrt{2}}g_{\pi^0 \Sigma^0 \Sigma^0} = g_{\pi^0 uu}F + g_{\pi^0 dd}F + g_{\pi^0 ss}(F - D). \quad (8)$$

Note that this coupling is exactly equal to zero. Exchanging, first $d \leftrightarrow s$ and then $u \leftrightarrow s$, one obtains two auxiliary quantities:

$$\frac{1}{\sqrt{2}}g_{\pi^0 \tilde{\Sigma}^{0ds} \tilde{\Sigma}^{0ds}} = g_{\pi^0 uu}F + g_{\pi^0 ss}F + g_{\pi^0 dd}(F - D)$$

$$= \frac{1}{\sqrt{2}}D,$$

$$\frac{1}{\sqrt{2}}g_{\pi^0 \tilde{\Sigma}^{0us} \tilde{\Sigma}^{0us}} = g_{\pi^0 ss}F + g_{\pi^0 dd}F + g_{\pi^0 uu}(F - D)$$

$$= -\frac{1}{\sqrt{2}}D. \quad (9)$$

The following relation holds:

$$g_{\pi^0 \tilde{\Sigma}^{0ds} \tilde{\Sigma}^{0ds}} - g_{\pi^0 \tilde{\Sigma}^{0us} \tilde{\Sigma}^{0us}} = 2D = \sqrt{3}g_{\pi^0 \Sigma^0 \Lambda}. \quad (10)$$

Up to now, we have considered only the neutral π^0. To study the couplings of the charged pions and kaons, let us first define the auxiliary "hyperons" $\Lambda_{us,ds}$ and $\Sigma^0_{us,ds}$ obtained from the normal Λ and Σ by the exchanges $u \leftrightarrow s$ and $d \leftrightarrow s$. Using the wave functions of Λ and Σ^0, it can be shown that

$$\begin{cases} 2|\Sigma^0_{ds}\rangle = -|\Sigma^0\rangle - \sqrt{3}|\Lambda\rangle \\ 2|\Lambda_{ds}\rangle = -\sqrt{3}|\Sigma^0\rangle + |\Lambda\rangle, \\ 2|\Sigma^0_{us}\rangle = -|\Sigma^0\rangle + \sqrt{3}|\Lambda\rangle \\ 2|\Lambda_{us}\rangle = \sqrt{3}|\Sigma^0\rangle + |\Lambda\rangle, \end{cases} \quad (11)$$

where $|\Sigma^0_{ds}\rangle$ and $|\Lambda_{ds}\rangle$ are the $V = 1$ and $V = 0$ V-spin states, respectively, and $|\Sigma^0_{us}\rangle$ and $|\Lambda_{us}\rangle$ are the $U = 1$ and $U = 0$ U-spin states.

Now, let us write the formal coupling of the π^- with Σ^+ and the auxiliary Λ_{ds}

$$2g_{\pi^- \Sigma^+ \Lambda_{ds}} = -\sqrt{3}g_{\pi^- \Sigma^+ \Sigma^0} + g_{\pi^- \Sigma^+ \Lambda} = \frac{2}{\sqrt{3}}(3F + D). \quad (12)$$

Performing the exchange $d \leftrightarrow s$, Λ_{ds} becomes the "physical" Λ, π^- becomes K^-, and Σ^+ becomes $-p$. Then we get

$$2(g_{\pi^- \Sigma^+ \Lambda_{ds}})(d \leftrightarrow s) = -2g_{K^- p\Lambda} = \frac{2}{\sqrt{3}}(3F + D) \quad (13)$$

which coincides with the $SU(3)_f$ symmetry prediction.

Similarly, writing the coupling of the π^- to Σ^+ and the auxiliary Λ_{us},

$$2g_{\pi^-\Sigma^+\Lambda_{us}} = \sqrt{3}g_{\pi^-\Sigma^+\Sigma^0} + g_{\pi^-\Sigma^+\Lambda} = -\frac{2}{\sqrt{3}}(3F - D),$$

$$(14)$$

and performing the $u \leftrightarrow s$ exchange, one obtains

$$2(g_{\pi^-\Sigma^+\Lambda_{us}})(u \leftrightarrow s) = 2g_{K^-\Xi^0\Lambda} = -\frac{2}{\sqrt{3}}(3F + D). \quad (15)$$

In a similar way, starting from the formal couplings $g_{\pi^-\Sigma^+\Sigma^0_{ds}}$ and $g_{\pi^-\Sigma^0\Sigma^0_{us}}$, and performing the $d \leftrightarrow s$ and $u \leftrightarrow s$ exchanges, respectively, one can obtain the following relations:

$$-2(g_{\pi^-\Sigma^+\Sigma^0_{ds}})(d \leftrightarrow s) = 2g_{K^-p\Sigma^0} = 2(-F + D),$$
$$-2(g_{\pi^-\Sigma^+\Sigma^0_{us}})(u \leftrightarrow s) = 2g_{K^0\Xi^0\Sigma^0} = -2(F + D). \quad (16)$$

These expression show how we can construct sum rules for the K baryon coupling constants, starting from the corresponding sum rules for the π baryon coupling constants.

III. LIGHT CONE QCD SUM RULES FOR THE MESON-BARYON COUPLINGS

In this section, we will derive light cone sum rules for the meson-baryon couplings. Sum rules for the meson-baryon couplings can be obtained by equating two different representations of a suitably chosen correlator, written in terms of hadrons and quark-gluons. We begin our calculation by constructing the following correlator:

$$\Pi^{B_2 \to B_1 \mathcal{M}} = i \int d^4x e^{ipx} \langle \mathcal{M}(q) | \mathcal{T} \eta_{B_1}(x) \bar{\eta}_{B_2}(0) | 0 \rangle, \quad (17)$$

where $\mathcal{M}$ is either a pion or a kaon, η_B is the interpolating current of the baryon under consideration, $\mathcal{T}$ is the time ordering operator, and q is the momentum of the $\mathcal{M}$-meson. This correlator can be calculated on one side phenomenologically, in terms of the hadron parameters, and on the other side by the operator product expansion (OPE) in the deep Euclidean region $p^2 \to -\infty$, using the quark gluon language. By matching both representations through the dispersion relations, one obtains the sum rules.

Let us first discuss the phenomenological part of the correlator function equation (17). Saturating the correlator function by ground state baryons with quantum numbers of the corresponding baryons, we get

$$\Pi^{B_2 \to B_1 \mathcal{M}}(p_1^2, p_2^2) = \frac{\langle 0|\eta_{B_1}|B_1(p_1)\rangle}{p_1^2 - M_1^2}$$
$$\times \langle B_1(p_1)\mathcal{M}(q)|B_2(p_2)\rangle$$
$$\times \frac{\langle B_2(p_2)|\eta_{B_2}|0\rangle}{p_2^2 - M_2^2} + \cdots, \quad (18)$$

where $p_2 = p_1 + q$, M_i is the mass of the baryon B_i, and $\cdots$ stand for the contributions of the higher states and the continuum.

The matrix elements of the interpolating currents between the vacuum and a single baryon state, B_i, with momentum p and having spin s is defined as

$$\langle 0|\eta_{B_i}|B_i(p, s)\rangle = \lambda_{B_i}u(p, s), \quad (19)$$

where λ_{B_i} is the overlap amplitude and u is the Dirac spinor for the baryon. In order to write down the phenomenological part of the sum rules from Eq. (18), it follows that one also needs the matrix element $\langle B_1(p_1)\mathcal{M}(q)|B_2(p_2)\rangle$. This matrix element is defined as

$$\langle B_1(p_1)K(q)|B_2(p_2)\rangle = g_{B_2B_1\mathcal{M}}\bar{u}(p_1)i\gamma_5 u(p_2). \quad (20)$$

Using Eqs. (19) and (20) and summing over the baryons' spins, we get the following phenomenological representation of the correlator:

$$\Pi^{B_2 \to B_1 \mathcal{M}}(p_1^2, p_2^2) = i\frac{g_{B_2B_1\mathcal{M}}\lambda_{B_1}\lambda_{B_2}}{(p_1^2 - M_1^2)(p_2^2 - M_2^2)}$$
$$\times (-\not{p}\not{q}\gamma_5 - M_1\not{q}\gamma_5$$
$$+ (M_2 - M_1)\not{p}\gamma_5$$
$$+ (M_1M_2 - p^2)\gamma_5) + \cdots, \quad (21)$$

where $\cdots$ stands for the contribution of the higher states and the continuum. From Eq. (21), it is seen that the correlator has numerous structures and in principle any structure can be used for the determination of the meson-baryon coupling constant. But our numerical analysis shows that the sum rules obtained from $\not{q}\gamma_5$ and γ_5 do not converge and hence cannot be used for a reliable determination of the coupling constant. The sum rules obtained from the structure $\not{p}\gamma_5$ do converge. But due to the small factor $M_1 - M_2$ multiplying the structure [this factor goes to zero in the $SU(3)_f$ symmetry limit], any uncertainty in the sum rule gets enhanced, and hence the results are also not reliable. Thus, we are left with the $\not{p}\not{q}\gamma_5$ structure only.

In order to obtain meson-baryon coupling, we need the explicit forms of the interpolating currents for the baryons. It is well known that there is a continuum number of interpolating currents for the octet baryons. In our calculations we will use the following general forms of baryon currents:

$$\eta^{\Sigma^0} = \sqrt{\frac{1}{2}}\epsilon^{abc}[(u^{aT}Cs^b)\gamma_5 d^c + t(u^{aT}C\gamma_5 s^b)d^c$$
$$- (s^{aT}Cd^b)\gamma_5 u^c - t(s^{aT}C\gamma_5 d^b)u^c],$$

$$\eta^{\Sigma^+} = -\frac{1}{\sqrt{2}}\eta^{\Sigma^0}(d \to u),$$

$$\eta^{\Sigma^-} = \frac{1}{\sqrt{2}}\eta^{\Sigma^0}(u \to d),$$

$$\eta^p = \eta^{\Sigma^+}(s \to d),$$

$$\eta^n = \eta^{\Sigma^-}(s \to u), \qquad (22)$$

$$\eta^{\Xi^0} = \eta^n(d \to s),$$

$$\eta^{\Xi^-} = \eta^p(u \to s),$$

$$\eta^{\Lambda} = -\sqrt{\frac{1}{6}}\epsilon^{abc}[2(u^{aT}Cd^b)\gamma_5 s^c + 2t(u^{aT}C\gamma_5 d^b)s^c$$
$$+ (u^{aT}Cs^b)\gamma_5 d^c + t(u^{aT}C\gamma_5 s^b)d^c$$
$$+ (s^{aT}Cd^b)\gamma_5 u^c + t(s^{aT}C\gamma_5 d^b)u^c],$$

where a, b, c are the color indices, and t is an arbitrary parameter, and C is the charge conjugation operator. The Ioffe current corresponds to the choice $t = -1$. Note that all currents except the current of Λ can be obtained from the current of Σ^0 by simple replacements. Recently, it has been shown in [20,21] that it is also possible to obtain the current of Λ from the current of Σ^0 through

$$2\eta_{\Sigma^0}(d \leftrightarrow s) + \eta_{\Sigma^0} = -\sqrt{3}\eta_{\Lambda},$$
$$2\eta_{\Sigma^0}(u \leftrightarrow s) - \eta_{\Sigma^0} = -\sqrt{3}\eta_{\Lambda}. \qquad (23)$$

A. Relations between the correlation functions

Identities presented in Eqs. (22) and (23) allow us to write all the correlation functions for the strong coupling constants of the $\pi^{0,\pm}$ and $K^{0,\pm}$ to the baryon octet, in terms of only four functions. In the $SU(3)_f$ symmetry limit, all these couplings are related using symmetry arguments. The main power of our approach is that our relations do not make use of the exact $SU(3)_f$ symmetry and hence can be used to study various $SU(3)_f$ symmetry violation effects.

Two of these four independent functions can be obtained from a slightly modified form of the correlation function $\Pi^{\Sigma^0 \to \Sigma^0 \pi^0}$. To start the derivation of the relationships between the various correlation functions, define

$$\Pi^{\Sigma^0 \to \Sigma^0 \pi^0} = g_{\pi \bar{u}u}\Pi_1(u, d, s) + g_{\pi \bar{d}d}\Pi_1'(u, d, s)$$
$$+ g_{\pi \bar{s}s}\Pi_2(u, d, s), \qquad (24)$$

where we formally write the quark content of π^0 in the form of Eq. (5). For a real pion, we have $g_{\pi \bar{u}u} = -g_{\pi \bar{d}d} = \frac{1}{\sqrt{2}}$ and $g_{\pi \bar{s}s} = 0$. Hence, essentially $\Pi_1(u, d, s)$,

$\Pi_1'(u, d, s)$, and $\Pi_2(u, d, s)$ is the contribution to the correlation function when the pion is emitted from the u, d, and s quark in Σ^0, respectively.

Note that the current for Σ^0 is symmetric under the exchange of the u and d quark field operators. Hence, the contribution of emission from the d quark can be obtained from the contribution of the emission from the u quark by a simple exchange of the u and d quarks, i.e. $\Pi_1'(u, d, s) = \Pi_1(d, u, s)$. This leaves us with only two independent expressions, i.e. $\Pi_1(u, d, s)$ and $\Pi_2(u, d, s)$. In the following, we will use the formal notation:

$$\Pi_1(u, d, s) = \langle \bar{u}u|\Sigma^0 \bar{\Sigma}^0|0 \rangle,$$
$$\Pi_2(u, d, s) = \langle \bar{s}s|\Sigma^0 \bar{\Sigma}^0|0 \rangle. \qquad (25)$$

In Π_1 substituting d instead of u, and using the fact that $\Sigma^0(d \to u) = \sqrt{2}\Sigma^+$, we obtain

$$4\Pi_1(u, u, s) = 2\langle \bar{u}u|\Sigma^+ \bar{\Sigma}^+|0 \rangle. \qquad (26)$$

[The factor 4 on the left-hand side is introduced due to the fact that, since Σ^+ has two u quark, there are 4 ways that the π^0 can be emitted, but $\Pi_1(u, u, s)$ takes into account only one of these.] Also noting that since Σ^+ does not contain any d quark,

$$\Pi^{\Sigma^+ \to \Sigma^+ \pi^0} = g_{\pi \bar{u}u}\langle \bar{u}u|\Sigma^+ \bar{\Sigma}^+|0 \rangle + g_{\pi \bar{s}s}\langle \bar{s}s|\Sigma^+ \bar{\Sigma}^+|0 \rangle$$
$$= \sqrt{2}\Pi_1(u, u, s). \qquad (27)$$

Similarly, for Σ^-, we obtain

$$\Pi^{\Sigma^- \to \Sigma^- \pi^0} = g_{\pi \bar{d}d}\langle \bar{d}d|\Sigma^- \bar{\Sigma}^-|0 \rangle + g_{\pi \bar{s}s}\langle \bar{s}s|\Sigma^- \bar{\Sigma}^-|0 \rangle$$
$$= -\sqrt{2}\Pi_1'(d, d, s) = -\sqrt{2}\Pi_1(d, d, s) \qquad (28)$$

which concludes derivation of the relations between the couplings of π^0 to the Σ baryons.

In order to derive relations for the coupling of π^0 to the proton and neutron, we need the matrix elements $\langle \bar{u}u|N\bar{N}|0 \rangle$ and $\langle \bar{d}d|N\bar{N}|0 \rangle$. In order to obtain the first matrix element, note that proton current can be obtained from the Σ^+ current by replacing the s quark by the d quark. Hence,

$$\langle \bar{u}u|p\bar{p}|0 \rangle = (\langle \bar{u}u|\Sigma^+ \bar{\Sigma}^+|0 \rangle)(s \to d) = \Pi_1(u, u, d). \qquad (29)$$

In order to obtain $\langle \bar{d}d|p\bar{p}|0 \rangle$, $\Pi_2(u, d, s)$ will be needed. First, replacing the d quark by the u quark, we obtain

$$\Pi_2(u, u, s) = \langle \bar{s}s|\Sigma^+ \bar{\Sigma}^+|0 \rangle, \qquad (30)$$

and next, by replacing the s quark by the d quark, we obtain

$$\Pi_2(u, u, d) = \langle \bar{d}d | p\bar{p} | 0 \rangle. \qquad (31)$$

Hence, we see that

$$\Pi^{p \to p\pi^0} = g_{\pi \bar{u}u} \langle \bar{u}u | p\bar{p} | 0 \rangle + g_{\pi \bar{d}d} \langle \bar{d}d | p\bar{p} | 0 \rangle$$
$$= \sqrt{2}\Pi_1(u, u, d) - \frac{1}{\sqrt{2}} \Pi_2(u, u, d). \qquad (32)$$

Using similar reasoning, one can also derive the following relationships for the coupling of π^0 to the nucleon and Ξ baryons:

$$\Pi^{n \to n\pi^0} = \frac{1}{\sqrt{2}} \Pi_2(d, d, u) - \sqrt{2}\Pi_1(d, d, u),$$

$$\Pi^{\Xi^0 \to \Xi^0 \pi^0} = \frac{1}{\sqrt{2}} \Pi_2(s, s, u), \qquad (33)$$

$$\Pi^{\Xi^- \to \Xi^- \pi^0} = -\frac{1}{\sqrt{2}} \Pi_2(s, s, d)$$

which concludes the derivation of the couplings of π^0 to the baryons in terms of $\Pi_1(u, d, s)$ and $\Pi_2(u, d, s)$.

Relating the analytical expression of the couplings of the neutral pion to the coupling of the charged pion is more subtle. To motivate the reasoning, consider the matrix element $\langle \bar{d}d | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle$. This Σ^0 baryon contains one of each of the u, d, and s quarks. In this matrix element, it is the d quarks which emit the final $\bar{d}d$ and the other u and d quarks act as spectators. Similarly, in the matrix element $\langle \bar{u}d | \Sigma^+ \bar{\Sigma}^0 | 0 \rangle$, the d quark in Σ^0 and one of the u quarks in Σ^+ form the state $\langle \bar{u}d |$ and the other u and s quarks in both the baryons act as spectators. Thus, it is reasonable to expect that $\langle \bar{d}d | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle$ and $\langle \bar{u}d | \Sigma^+ \bar{\Sigma}^0 | 0 \rangle$ are proportional. Indeed an explicit calculation of the correlation functions showed that

$$\Pi^{\Sigma^0 \to \Sigma^+ \pi^-} = \langle \bar{u}d | \Sigma^+ \bar{\Sigma}^0 | 0 \rangle = -\sqrt{2}\langle \bar{d}d | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle$$
$$= -\sqrt{2}\Pi_1'(u, d, s) = -\sqrt{2}\Pi_1(d, u, s) \quad (34)$$

from which, after exchanging u and d quarks, one obtains

$$\Pi^{\Sigma^0 \to \Sigma^- \pi^+} = \langle \bar{d}u | \Sigma^- \bar{\Sigma}^0 | 0 \rangle = \sqrt{2}\langle \bar{u}u | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle$$
$$= \sqrt{2}\Pi_1(u, d, s). \qquad (35)$$

Using a similar reasoning, it is expected that $\langle \bar{u}u | \Xi^0 \bar{\Xi}^0 | 0 \rangle$ should be proportional to $\langle \bar{d}u | \Xi^- \bar{\Xi}^0 | 0 \rangle$. An explicit calculation showed that

$$\Pi^{\Xi^0 \to \Xi^- \pi^+} = \langle \bar{d}u | \Xi^- \bar{\Xi}^0 | 0 \rangle = -\sqrt{2}\langle \bar{u}u | \Xi^0 \bar{\Xi}^0 | 0 \rangle$$
$$= -\Pi_2(s, s, u) \qquad (36)$$

and, exchanging u and d quarks, we obtain

$$\Pi^{\Xi^- \to \Xi^0 \pi^-} = \langle \bar{u}d | \Xi^0 \bar{\Xi}^- | 0 \rangle = -\sqrt{2}\langle \bar{d}d | \Xi^0 \bar{\Xi}^0 | 0 \rangle$$
$$= -\Pi_2(s, s, d). \qquad (37)$$

Using similar arguments, one can show the following relations for the other correlation functions involving the pion and not involving Λ:

$$\Pi^{\Sigma^- \to \Sigma^0 \pi} = \sqrt{2}\Pi_1(u, d, s),$$
$$\Pi^{\Sigma^+ \to \Sigma^0 \pi} = -\sqrt{2}\Pi_1'(u, d, s) = -\sqrt{2}\Pi_1(d, u, s),$$
$$\Pi^{\Sigma^- \to nK^-} = -\Pi_2(d, d, s),$$
$$\Pi^{p \to \Sigma^+ K^0} = -\Pi_2(u, u, d), \qquad (38)$$
$$\Pi^{\Sigma^+ \to pK^0} = -\Pi_2(u, u, s),$$
$$\Pi^{n \to \Sigma^- K^+} = -\Pi_2(d, d, u).$$

In order to obtain the expressions for the correlations involving the Λ baryon, one uses the relations given in Eq. (23). Using those relations, one obtains

$$\Pi^{\Lambda \to \Lambda \pi^0} = \frac{\sqrt{2}}{3}\left[\Pi_1(u, s, d) - \Pi_1(d, s, u) + \Pi_2(s, d, u) - \Pi_2(s, u, d) - \frac{1}{2}\Pi_1(u, d, s) + \frac{1}{2}\Pi_1(d, u, s) \right],$$

$$\Pi^{\Lambda \to \Sigma^0 \pi^0} + \Pi^{\Sigma^0 \to \Lambda \pi^0} = \frac{2}{\sqrt{6}}[\Pi_1(u, s, d) + \Pi_1(d, s, u) - \Pi_2(s, d, u) - \Pi_2(s, u, d)],$$

$$\Pi^{\Xi^- \to \Sigma^0 K} + \sqrt{3}\Pi^{\Xi^- \to \Lambda K} = 2\sqrt{2}\Pi_1(u, s, d), \qquad \Pi^{n \to \Sigma^0 K} - \sqrt{3}\Pi^{n \to \Lambda K} = 2\sqrt{2}\Pi_1(s, d, u), \qquad (39)$$

$$\Pi^{p \to \Sigma^0 K^+} + \sqrt{3}\Pi^{p \to \Lambda K^+} = -2\sqrt{2}\Pi_1(s, u, d), \qquad -\Pi^{\Xi^0 \to \Sigma^0 K^0} + \sqrt{3}\Pi^{\Xi^0 \to \Lambda K^0} = 2\sqrt{2}\Pi_1(d, s, u),$$

$$\Pi^{\Sigma^0 \to pK^-} + \sqrt{3}\Pi^{\Lambda \to pK^-} = -2\sqrt{2}\Pi_1(s, u, d), \qquad \Pi^{\Sigma^0 \to nK^0} - \sqrt{3}\Pi^{\Lambda \to nK^0} = 2\sqrt{2}\Pi_1(s, d, u),$$

$$\Pi^{\Sigma^0 \to \Xi^0 K^0} - \sqrt{3}\Pi^{\Lambda \to \Xi^0 K^0} = -2\sqrt{2}\Pi_1(d, s, u), \qquad \Pi^{\Sigma^0 \to \Xi^- K^+} + \sqrt{3}\Pi^{\Lambda \to \Xi^- K^+} = 2\sqrt{2}\Pi_1(u, s, d).$$

As one can see from the relations in Eq. (39), it is not possible to separate the correlations involving the Λ baryon from expressions involving the Σ^0 baryon using only the functions Π_1 and Π_2. In order to separate these correlation functions, we will choose two more independent functions:

$$\Pi_3(u, d, s) = -\Pi^{\Sigma^0 \to \Xi^- K^+} = -\langle u\bar{s}|\Xi^- \bar{\Sigma}^0|0\rangle \quad (40)$$

and

$$\Pi_4(u, d, s) = -\Pi^{\Xi^- \to \Sigma^0 K^-} = -\langle s\bar{u}|\Sigma^0 \bar{\Xi}^-|0\rangle. \quad (41)$$

The main motivation for choosing these two correlation functions is that one involves Σ^0 as an initial state and the other one involves Σ^0 as a final state. There are many other possible choices and each one would work equally well. For convention, these two are chosen in this work.

Using these two correlations, by suitable replacements or exchanges of the quarks, one obtains the following relations:

$$
\begin{aligned}
\Pi^{\Xi^0 \to \Sigma^+ K^-} &= \Pi^{n \to p\pi^-}(s \leftrightarrow d) = -\sqrt{2}\Pi_3(s, s, u), \\
\Pi^{\Xi^- \to \Sigma^- K^0} &= \Pi^{\Xi^0 \to \Sigma^+ K^-}(u \to d) = -\sqrt{2}\Pi_3(s, s, d), \\
\Pi^{\Sigma^+ \to \Xi^0 K^+} &= \sqrt{2}\Pi^{\Sigma^0 \to \Xi^- K^+}(d \to u) = -\sqrt{2}\Pi_3(u, u, s), \\
\Pi^{p \to n\pi} &= \Pi^{\Sigma^+ \to \Xi^0 K^+}(s \to d) = -\sqrt{2}\Pi_3(u, u, d), \\
\Pi^{n \to p\pi^-} &= \Pi^{p \to n\pi^+}(u \leftrightarrow d) = -\sqrt{2}\Pi_3(d, d, u), \\
\Pi^{\Sigma^0 \to pK^-} - \sqrt{3}\Pi^{\Lambda \to pK^-} &= 2\Pi^{\Sigma^0 \to \Xi^- K^+}(s \leftrightarrow u) = -2\Pi_3(s, d, u), \\
\Pi^{\Sigma^0 \to nK^0} + \sqrt{3}\Pi^{\Lambda \to nK^0} &= -2\Pi^{\Sigma^0 \to \Xi^- K^+}(s \leftrightarrow u, u \leftrightarrow d) = 2\Pi_3(s, u, d), \\
\Pi^{\Sigma^0 \to \Sigma^- \pi^+} + \sqrt{3}\Pi^{\Lambda \to \Sigma^- \pi^+} &= 2\Pi^{\Sigma^0 \to \Xi^- K^+}(s \leftrightarrow d) = -2\Pi_3(u, s, d), \\
\Pi^{\Lambda \to \Sigma^+ \pi^-} &= \Pi^{\Lambda \to \Sigma^- \pi^+}(u \leftrightarrow d)\Pi^{\Sigma^- \to \Xi^- K} = \Pi^{\Sigma^+ \to \Xi^0 K}(u \to d) = -\sqrt{2}\Pi_3(d, d, s); \\
\Pi^{\Sigma^0 \to \Xi^0 K^0} &= -\Pi^{\Sigma^0 \to \Xi^- K^+}(d \leftrightarrow u) = \Pi_3(d, u, s), \\
\Pi^{\Xi^0 \to \Sigma^0 K^0} &= -\Pi^{\Xi^- \to \Sigma^0 K^+}(u \leftrightarrow d) = \Pi_4(d, u, s), \\
\Pi^{p \to \Sigma^0 K^+} - \sqrt{3}\Pi^{p \to \Lambda K^+} &= 2\Pi^{\Xi^- \to \Sigma^0 K^-}(u \leftrightarrow s) = -2\Pi_4(s, d, u), \\
\Pi^{n \to \Sigma^0 K^0} + \sqrt{3}\Pi^{n \to \Lambda K^0} &= -2\Pi^{\Xi^- \to \Sigma^0 K^-}(u \leftrightarrow s, u \leftrightarrow d) = 2\Pi_4(s, u, d), \\
\Pi^{\Sigma^- \to \Sigma^0 \pi^-} + \sqrt{3}\Pi^{\Sigma^- \to \Lambda \pi^-} &= 2\Pi^{\Xi^- \to \Sigma^0 K^-}(d \leftrightarrow s) = -2\Pi_4(u, s, d), \\
\Pi^{\Sigma^+ \to \Lambda \pi^+} &= \Pi^{\Sigma^- \to \Lambda \pi^-}(u \leftrightarrow d).
\end{aligned}
\quad (42)
$$

Hence, we conclude our claim that all 45 strong coupling constants of the pions and the kaon to the octet baryons can be expressed in terms of only 4 independent functions without using any flavor symmetry. In Appendix A, we present each one of the correlation functions in terms of the four functions $\Pi_i(u, d, s)$, $(i = 1, 2, 3, 4)$. In this work, we will work in the exact isosymmetry limit. In this limit, not all of the coupling constants are independent. In Appendix B, we present the relationships between the coupling constants in this limit.

B. Expressions for the functions Π_i

In the previous subsection, we have shown that all the correlation functions can be expressed in terms of only four analytical functions which can be obtained from the corre-lation functions for the transitions $\Sigma^0 \to \Sigma^0 \pi^0$, $\Sigma^0 \to \Xi^0 K^0$, and $\Xi^0 \to \Sigma^0 K^0$. Hence, it is enough to evaluate only the correlation functions for these transitions only.

In the large Euclidean momentum $-p_1^2 \to \infty$ and $-p_2^2 \to \infty$ region, the correlators can be calculated using the OPE. For this calculation, the propagators of the light quarks and matrix elements of the form $\langle\mathcal{M}|\bar{q}(x_1)\Gamma q'(x_2)|0\rangle$, where $\mathcal{M} = K^0$ or π^0, and Γ is a member of the Dirac basis of the gamma matrices are needed. In order to study the $SU(3)_f$ violation effects, we have expressed the light quark propagator up to linear order in m_q and then for numerical analysis set $m_u = m_d = 0$ and $m_s \neq 0$. The matrix elements $\langle\mathcal{M}|\bar{q}(x_1)\Gamma q'(x_2)|0\rangle$ can be written in terms of the meson light cone distribution amplitudes. The explicit forms of these matrix elements are given in [22,23]:

$$\langle \mathcal{M}(p)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle = -if_\mathcal{M} p_\mu \int_0^1 du\, e^{i\bar{u}px}\left(\varphi_\mathcal{M}(u) + \frac{1}{16}m_\mathcal{M}^2 x^2 \mathbb{A}(u)\right) - \frac{i}{2}f_\mathcal{M} m_\mathcal{M}^2 \frac{x_\mu}{px}\int_0^1 du\, e^{i\bar{u}px}\mathbb{B}(u),$$

$$\langle \mathcal{M}(p)|\bar{q}(x)i\gamma_5 q(0)|0\rangle = \mu_\mathcal{M}\int_0^1 du\, e^{i\bar{u}px}\varphi_P(u), \langle \mathcal{M}(p)|\bar{q}(x)\sigma_{\alpha\beta}\gamma_5 q(0)|0\rangle$$

$$= \frac{i}{6}\mu_\mathcal{M}(1-\tilde{\mu}_\mathcal{M}^2)(p_\alpha x_\beta - p_\beta x_\alpha)\int_0^1 du\, e^{i\bar{u}px}\varphi_\sigma(u),$$

$$\langle \mathcal{M}(p)|\bar{q}(x)\sigma_{\mu\nu}\gamma_5 g_s G_{\alpha\beta}(vx)q(0)|0\rangle = i\mu_\mathcal{M}\left[p_\alpha p_\mu\left(g_{\nu\beta} - \frac{1}{px}(p_\nu x_\beta + p_\beta x_\nu)\right) - p_\alpha p_\nu\left(g_{\mu\beta} - \frac{1}{px}(p_\mu x_\beta + p_\beta x_\mu)\right)\right.$$

$$\left. - p_\beta p_\mu\left(g_{\nu\alpha} - \frac{1}{px}(p_\nu x_\alpha + p_\alpha x_\nu)\right) + p_\beta p_\nu\left(g_{\mu\alpha} - \frac{1}{px}(p_\mu x_\alpha + p_\alpha x_\mu)\right)\right]$$

$$\times \int \mathcal{D}\alpha\, e^{i(\alpha_{\bar{q}}+v\alpha_g)px}\mathcal{T}(\alpha_i), \langle \mathcal{M}(p)|\bar{q}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)q(0)|0\rangle$$

$$= p_\mu(p_\alpha x_\beta - p_\beta x_\alpha)\frac{1}{px}f_\mathcal{M} m_\mathcal{M}^2\int \mathcal{D}\alpha\, e^{i(\alpha_{\bar{q}}+v\alpha_g)px}\mathcal{A}_\parallel(\alpha_i)$$

$$+\left[p_\beta\left(g_{\mu\alpha} - \frac{1}{px}(p_\mu x_\alpha + p_\alpha x_\mu)\right) - p_\alpha\left(g_{\mu\beta} - \frac{1}{px}(p_\mu x_\beta + p_\beta x_\mu)\right)\right]f_\mathcal{M} m_\mathcal{M}^2$$

$$\times \int \mathcal{D}\alpha\, e^{i(\alpha_{\bar{q}}+v\alpha_g)px}\mathcal{A}_\perp(\alpha_i),$$

$$\langle \mathcal{M}(p)|\bar{q}(x)\gamma_\mu i g_s G_{\alpha\beta}(vx)q(0)|0\rangle = p_\mu(p_\alpha x_\beta - p_\beta x_\alpha)\frac{1}{px}f_\mathcal{M} m_\mathcal{M}^2\int \mathcal{D}\alpha\, e^{i(\alpha_{\bar{q}}+v\alpha_g)px}\mathcal{V}_\parallel(\alpha_i)$$

$$+\left[p_\beta\left(g_{\mu\alpha} - \frac{1}{px}(p_\mu x_\alpha + p_\alpha x_\mu)\right) - p_\alpha\left(g_{\mu\beta} - \frac{1}{px}(p_\mu x_\beta + p_\beta x_\mu)\right)\right]f_\mathcal{M} m_\mathcal{M}^2$$

$$\times \int \mathcal{D}\alpha\, e^{i(\alpha_{\bar{q}}+v\alpha_g)px}\mathcal{V}_\perp(\alpha_i), \tag{43}$$

where $\mu_\mathcal{M} = f_\mathcal{M}\frac{m_\mathcal{M}^2}{m_{q_1}+m_{q_2}}$, $\tilde{\mu}_\mathcal{M} = \frac{m_\mathcal{M}}{m_{q_1}+m_{q_2}}$, where q_1 and q_2 are the quarks in the meson $\mathcal{M}$, $\mathcal{D}\alpha = d\alpha_{\bar{q}}d\alpha_q d\alpha_g\delta(1-\alpha_{\bar{q}}-\alpha_q-\alpha_g)$, and the functions $\varphi_\mathcal{M}(u)$, $\mathbb{A}(u)$, $\mathbb{B}(u)$, $\varphi_P(u)$, $\varphi_\sigma(u)$, $\mathcal{T}(\alpha_i)$, $\mathcal{A}_\perp(\alpha_i)$, $\mathcal{A}_\parallel(\alpha_i)$, $\mathcal{V}_\perp(\alpha_i)$, and $\mathcal{V}_\parallel(\alpha_i)$ are functions of definite twist and their expressions will be given in Sec. IV.

For the explicit form of the light quark propagator, we have used the expression

$$S_q(x) = \frac{i\slashed{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle\bar{q}q\rangle}{12}\left(1 - i\frac{m_q}{4}\slashed{x}\right) - \frac{x^2}{192}m_0^2\langle\bar{q}q\rangle\left(1 - i\frac{m_q}{6}\slashed{x}\right)$$

$$- ig_s\int_0^1 du\left[\frac{\slashed{x}}{16\pi^2 x^2}G_{\mu\nu}(ux)\sigma_{\mu\nu} - ux^\mu G_{\mu\nu}(ux)\gamma^\nu\frac{i}{4\pi^2 x^2} - i\frac{m_q}{32\pi^2}G_{\mu\nu}\sigma^{\mu\nu}\left(\ln\left(\frac{-x^2\Lambda^2}{4}\right) + 2\gamma_E\right)\right], \tag{44}$$

where γ_E is the Euler constant, $\gamma_E \simeq 0.577$ and Λ is a scale that separates the long and short distance physics. For numerical analysis, we used the value $\Lambda = 300$ MeV.

Using the explicit expressions of the full propagator of the light quark, Eq. (44), and the meson wave functions, and separating the coefficient of the structure $\slashed{p}\slashed{q}\gamma_5$, one can obtain the theoretical expression for the correlation function in terms of a few condensates, distribution amplitudes of the mesons, and the QCD parameter.

Having the explicit expression for the correlator from QCD, QCD sum rules are obtained by applying Borel transformations on the variables p_1^2 and $p_2^2 = (p_1 + q)^2$ in order to suppress the contribution of the higher states and the continuum (for details, see e.g. [24–26]). Then equating the final results from phenomenological and corresponding parts, we arrive at the sum rules for the corresponding kaon-baryon couplings. Our final results for the four analytical functions Π_i, $i = 1, 2, 3$, and 4 (for the structure $\slashed{p}\slashed{q}\gamma_5$) are

$$\Pi_1(u, d, s) = \frac{f_{\mathcal{M}}}{64\pi^2} M^4(m_s(1 - t)^2 - m_d(1 - t^2))i_2(\phi_{\mathcal{M}}) - \frac{\mu_{\mathcal{M}}}{64\pi^2} M^4(1 - \tilde{\mu}_{\mathcal{M}}^2)(1 - t^2)i_2(\phi_\sigma)$$

$$- \frac{f_{\mathcal{M}}}{32\pi^2} m_{\mathcal{M}}^2 M^2(m_s(1 - t)^2 + 3m_d(1 - t^2))i_1(\mathcal{V}, 1)\left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right)$$

$$+ \frac{f_{\mathcal{M}}}{16\pi^2} m_{\mathcal{M}}^2 M^2(m_s(1 - t)^2 - m_d(1 - t^2))i_1(\mathcal{V}_\perp, 1)\left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right) + \frac{f_{\mathcal{M}}}{768\pi^2}\langle g_s^2 GG\rangle(m_s(1 - t)^2$$

$$- m_d(1 - t^2))i_2(\phi_{\mathcal{M}})\left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right) - \frac{m_0^2 + 2M^2}{3456M^6}\mu_{\mathcal{M}}(1 - \tilde{\mu}_{\mathcal{M}}^2)\langle g_s^2 GG\rangle(m_d\langle \bar{s}s\rangle$$

$$+ m_s\langle \bar{d}d\rangle)(3 + 2t + 3t^2)i_2(\phi_\sigma) - \frac{m_0^2}{648M^2}(1 - \tilde{\mu}_{\mathcal{M}}^2)\mu_{\mathcal{M}}(m_s\langle \bar{d}d\rangle + m_d\langle \bar{s}s\rangle)(5 + 4t + 5t^2)i_2(\phi_\sigma)$$

$$- \frac{f_{\mathcal{M}}}{3072\pi^2 M^2}\langle g_s^2 GG\rangle m_{\mathcal{M}}^2(m_s(1 - t)^2 - m_d(1 - t^2))(4i_1(\mathcal{A}_\parallel, 1 - 2v) - i_2(\mathbb{A}))$$

$$+ \frac{f_{\mathcal{M}}}{768\pi^2 M^2}\langle g_s^2 GG\rangle m_{\mathcal{M}}^2(m_s(1 - t)^2 + m_d(1 - t^2))i_1(\mathcal{V}_\parallel, 1) - \frac{f_{\mathcal{M}}}{1152\pi^2}\langle g_s^2 GG\rangle(m_s(1 - t)^2$$

$$- m_d(1 - t^2))i_2(\varphi_{\mathcal{M}}) - \frac{f_{\mathcal{M}}}{128\pi^2} m_{\mathcal{M}}^2 M^2(m_s(1 - t)^2 - m_d(1 - t^2))i_2(\mathbb{A}) - \frac{f_{\mathcal{M}}}{24}M^2(\langle \bar{s}s\rangle(1 - t)^2$$

$$- \langle \bar{d}d\rangle(1 - t^2))i_1(\varphi_{\mathcal{M}}) + \frac{f_{\mathcal{M}}}{32\pi^2} m_{\mathcal{M}}^2 M^2(m_s(1 - t)^2 - m_d(1 - t^2))i_1(\mathcal{A}_\parallel, 1 - 2v)$$

$$- \frac{f_{\mathcal{M}}}{16\pi^2} m_{\mathcal{M}}^2 M^2(m_s(1 - t)^2 + 2m_d(1 - t^2))i_1(\mathcal{V}_\parallel, 1) + \frac{f_{\mathcal{M}}}{16\pi^2} m_{\mathcal{M}}^2 M^2(m_s(1 - t)^2 - m_d(1 - t^2))i_1(\mathcal{V}_\perp, 1)$$

$$+ \frac{\mu_{\mathcal{M}}}{144}(1 - \tilde{\mu}_{\mathcal{M}}^2)[\langle \bar{d}d\rangle(-3m_d(1 - t^2) - 2m_s(3 + 2t + 3t^2)) - \langle \bar{s}s\rangle(3m_s(1 - t^2) + m_d(6 + 4t + 6t^2))]$$

$$\times i_2(\phi_\sigma) + \frac{f_{\mathcal{M}}}{432} m_0^2(3\langle \bar{s}s\rangle(1 - t)^2 - 2\langle \bar{d}d\rangle(1 - t)^2)i_2(\varphi_{\mathcal{M}}) + \frac{f_{\mathcal{M}}}{96} m_{\mathcal{M}}^2(\langle \bar{s}s\rangle(1 - t)^2 - \langle \bar{d}d\rangle(1 - t^2))i_2(\mathbb{A})$$

$$- \frac{\mu_{\mathcal{M}}}{48}(1 - t^2)(\langle \bar{d}d\rangle m_d - \langle \bar{s}s\rangle m_s)i_1'(\mathcal{T}, 1 - 2v) - \frac{f_{\mathcal{M}}}{24} m_{\mathcal{M}}^2(\langle \bar{s}s\rangle(1 - t)^2 - \langle \bar{d}d\rangle(1 - t^2))i_1(\mathcal{A}_\parallel, 1 - 2v)$$

$$+ \frac{f_{\mathcal{M}}}{24} m_{\mathcal{M}}^2(\langle \bar{d}d\rangle(1 - t^2) + \langle \bar{s}s\rangle(1 - t)^2)i_1(\mathcal{V}_\parallel, 1), \tag{45}$$

$$\Pi_2(u, d, s) = -\frac{M^4}{192\pi^2}\mu_{\mathcal{M}}(1 - \tilde{\mu}_{\mathcal{M}})(1 - t)^2 i_2(\phi_\sigma) + \frac{f_{\mathcal{M}}}{64\pi^2} M^4(m_u + m_d)(1 - t^2)i_2(\varphi_{\mathcal{M}})$$

$$- \frac{M^4}{32\pi^2}\mu_{\mathcal{M}}(1 - t)^2 i_1'(\mathcal{T}, 1 - 2v) + \frac{f_{\mathcal{M}}}{768\pi^2}(m_u + m_d)(1 - t^2)\langle g_s^2 GG\rangle\left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right)i_2(\varphi_{\mathcal{M}})$$

$$- \frac{f_{\mathcal{M}}}{32\pi^2} m_{\mathcal{M}}^2(m_u + m_d)M^2(1 - t^2)\left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right)i_1(3\mathcal{V}_\parallel - 2\mathcal{V}_\perp, 1)$$

$$+ \frac{\mu_{\mathcal{M}}}{3456M^6}(1 - \tilde{\mu}^2)(m_0^2 + 2M^2)\langle g_s^2 GG\rangle(1 + t)^2(\langle \bar{d}d\rangle m_u + m_d\langle \bar{u}u\rangle)i_2(\phi_\sigma)$$

$$- \frac{m_0^2}{1296M^2}\mu_{\mathcal{M}}(1 - \tilde{\mu}_{\mathcal{M}}^2)(1 + t)^2(\langle \bar{d}d\rangle m_u + m_d\langle \bar{u}u\rangle)i_2(\phi_\sigma)$$

$$- \frac{f_{\mathcal{M}}}{3072\pi^2 M^2} m_{\mathcal{M}}^2\langle g_s^2 GG\rangle(m_u + m_d)(1 - t^2)(4i_1(\mathcal{A}_\parallel, 1 - 2v) + 4i_1(\mathcal{V}_\parallel, 1) - i_2(\mathbb{A}))$$

$$- \frac{f_{\mathcal{M}}}{24}M^2(1 - t^2)(\langle \bar{d}d\rangle + \langle \bar{u}u\rangle)i_2(\phi_{\mathcal{M}}) + \frac{f_{\mathcal{M}}}{128\pi^2} m_{\mathcal{M}}^2(m_u + m_d)M^2(1 - t^2)(4i_1(\mathcal{A}_\parallel, 1 - 2v)$$

$$- 8i_1(\mathcal{V}_\parallel - \mathcal{V}_\perp, 1) - i_2(\mathbb{A})) + \frac{\mu_{\mathcal{M}}}{144}(1 - \tilde{\mu}_{\mathcal{M}}^2)[\langle \bar{d}d\rangle(m_d(1 - t^2) + 2m_u(1 + t)^2)$$

$$+ \langle \bar{u}u\rangle(m_u(1 - t^2) - 2m_d(1 + t)^2)]i_2(\varphi_\sigma) + \frac{f_{\mathcal{M}}}{864}(1 - t^2)(\langle \bar{d}d\rangle + \langle \bar{u}u\rangle)(9m_{\mathcal{M}}^2 i_2(\mathbb{A}) + 10m_0^2 i_2(\varphi_{\mathcal{M}}))$$

$$- \frac{f_{\mathcal{M}}}{1152\pi^2}\langle g_s^2 GG\rangle(m_u + m_d)(1 - t^2)i_2(\varphi_{\mathcal{M}}) - \frac{f_{\mathcal{M}}}{24} m_{\mathcal{M}}^2(1 - t^2)(\langle \bar{d}d\rangle + \langle \bar{u}u\rangle)(i_1(\mathcal{A}_\parallel, 1 - 2v)$$

$$+ i_1(\mathcal{V}_\parallel, 1)) - \frac{\mu_{\mathcal{M}}}{48}(1 - t)^2(\langle \bar{d}d\rangle m_d + \langle \bar{u}u\rangle m_u)i_1'(\mathcal{T}, 1 - 2v), \tag{46}$$

$$\sqrt{2}\Pi_3(u,d,s) = \frac{f_{\mathcal{M}}}{64\pi^2}M^4(m_d+m_s)(1+2t-3t^2) + \frac{1}{192\pi^2}(1-\tilde{\mu}_{\mathcal{M}}^2)M^4(5+2t-7t^2)$$

$$- \frac{1}{32\pi^2}\mu_{\mathcal{M}}(1-t)^2 i_1'(\mathcal{T},1-2v) + \frac{f_{\mathcal{M}}}{768\pi^2}\langle g_s^2 GG\rangle(m_d+m_s)(1+2t-3t^2)i_2(\varphi_{\mathcal{M}})$$

$$\times \left(\gamma_E - \ln\frac{M^2}{\Lambda^2} - \frac{2}{3}\right) - \frac{f_{\mathcal{M}}}{32\pi^2}m_{\mathcal{M}}^2 M^2(1-t)^2((m_d-m_s)i_1(\mathcal{A}_\|,1) - (m_d+m_s)i_1(\mathcal{V}_\|,1))\left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right)$$

$$- \frac{f_{\mathcal{M}}}{16\pi^2}m_{\mathcal{M}}^2 M^2(1+2t-3t^2)((m_d-m_s)i_1(\mathcal{A}_\perp,1) - (m_d+m_s)i_1(\mathcal{V}_\perp,1))\left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right)$$

$$+ \frac{\mu_{\mathcal{M}}}{3456 M^6}(1-\tilde{\mu}_{\mathcal{M}}^2)(m_0^2+2M^2)\langle g_s^2 GG\rangle(7+6t+7t^2)(\langle\bar{d}d\rangle m_s + \langle\bar{s}s\rangle m_d)i_2(\varphi_\sigma)$$

$$+ \frac{f_{\mathcal{M}}}{3072 M^2\pi^2}\langle g_s^2 GG\rangle m_{\mathcal{M}}^2(m_d+m_s)(1+2t-3t^2)i_2(\mathbb{A}) - \frac{f_{\mathcal{M}}}{768 M^2\pi^2}\langle g_s^2 GG\rangle m_{\mathcal{M}}^2(m_d+m_s)(1-t)$$

$$\times ((1+3t)i_1(\mathcal{A}_\|,1-2v) + (3+t)i_1(\mathcal{V}_\|,1)) + \frac{f_{\mathcal{M}}}{384 M^2\pi^2}m_{\mathcal{M}}^2\langle g_s^2 GG\rangle(m_d-m_s)(1-t)$$

$$\times ((1+3t)i_1(\mathcal{A}_\perp,1) + (3+t)i_1(\mathcal{V}_\perp,1-2v)) + \frac{m_0^2}{1296 M^2}(1-\tilde{\mu}_{\mathcal{M}}^2)\mu_{\mathcal{M}}(\langle\bar{d}d\rangle m_s + \langle\bar{s}s\rangle m_d)$$

$$\times (19+14t+19t^2)i_2(\phi_\sigma) - \frac{f_{\mathcal{M}}}{384\pi^2}(1+2t-3t^2)(3m_{\mathcal{M}}^2(m_d+m_s)i_2(\mathbb{A})$$

$$+ 16\pi^2(\langle\bar{d}d\rangle + \langle\bar{s}s\rangle)i_2(\varphi_{\mathcal{M}})) - \frac{f_{\mathcal{M}}}{16\pi^2}m_{\mathcal{M}}^2(m_d+m_s)M^2(1-t)((1+3t)i_1(\mathcal{A}_\|,v) - 2i_1(\mathcal{V}_\|,1))$$

$$+ \frac{f_{\mathcal{M}}}{16\pi^2}m_{\mathcal{M}}^2 M^2(1-t)((m_s(1+t) + 2m_d t)i_1(\mathcal{A}_\|,1) + 2(2m_s(1+t) - m_d(1-t))i_1(\mathcal{V}_\perp,1))$$

$$- \frac{f_{\mathcal{M}}}{8\pi^2}m_{\mathcal{M}}^2(m_d-m_s)M^2(1-t)((1+3t)i_1(\mathcal{A}_\perp,1) - (3+t)i_1(\mathcal{V}_\perp,v))$$

$$- \frac{\mu_{\mathcal{M}}}{144}(1-\tilde{\mu}_{\mathcal{M}}^2)(\langle\bar{d}d\rangle m_d + \langle\bar{s}s\rangle m_s)(-5-2t+7t^2)i_2(\varphi_\sigma)$$

$$+ \frac{\mu_{\mathcal{M}}}{72}(1-\tilde{\mu}_{\mathcal{M}}^2)(\langle\bar{d}d\rangle m_s + \langle\bar{s}s\rangle m_d)(7+6t+7t^2)i_2(\varphi_\sigma) + \frac{f_{\mathcal{M}}}{864}(\langle\bar{d}d\rangle + \langle\bar{s}s\rangle)$$

$$\times (9m_{\mathcal{M}}^2(1+2t-3t^2)i_2(\mathbb{A}) + 4m_0^2(2+3t-5t^2)i_2(\varphi_{\mathcal{M}})) - \frac{\mu_{\mathcal{M}}}{24}(1-t)(-2m_s\langle\bar{s}s\rangle t$$

$$+ \langle\bar{d}d\rangle m_d(1+t))i_1'(\mathcal{T},1) + \frac{\mu_{\mathcal{M}}}{24}(\langle\bar{d}d\rangle m_d + \langle\bar{s}s\rangle m_s)(1-t)^2 i_1'(\mathcal{T},v)$$

$$- \frac{f_{\mathcal{M}}}{24}m_{\mathcal{M}}^2(\langle\bar{d}d\rangle + \langle\bar{s}s\rangle)(1-t)[(1+3t)i_1(\mathcal{A}_\|,1-2v) + (3+t)i_1(\mathcal{V}_\|,v)]$$

$$+ \frac{f_{\mathcal{M}}}{12}m_{\mathcal{M}}^2(\langle\bar{d}d\rangle - \langle\bar{s}s\rangle)(1-t)[(1+3t)i_1(\mathcal{A}_\perp) + (3+t)i_1(\mathcal{V}_\perp,1-2v)], \tag{47}$$

$$\sqrt{2}\Pi_4(u,d,s) = \sqrt{2}\Pi_3(u,d,s) - \frac{f_{\mathcal{M}}}{16\pi^2}m_{\mathcal{M}}^2(m_d-m_s)M^2(1-t)[(-1+t)i_1(\mathcal{A}_\|,1) - 2(1+3t)i_1(\mathcal{A}_\perp,1)]\left(\gamma_E - \ln\frac{M^2}{\Lambda^2}\right)$$

$$- \frac{f_{\mathcal{M}}}{192 M^2\pi^2}m_{\mathcal{M}}^2\langle g_s^2 GG\rangle(m_d-m_s)(1-t)[(1+3t)i_1(\mathcal{A}_\perp,1) + (3+t)i_1(\mathcal{V}_\perp,1-2v)]$$

$$- \frac{f_{\mathcal{M}}}{16\pi^2}m_{\mathcal{M}}^2(m_d-m_s)M^2(1-t)[(-1+t)i_1(\mathcal{A}_\|,1) - 4(1+3t)i_1(\mathcal{A}_\perp,1)]$$

$$- \frac{f_{\mathcal{M}}}{8\pi^2}m_{\mathcal{M}}^2(m_d-m_s)M^2(-3+2t+t^2)i_1(\mathcal{V}_\perp,1-2v)$$

$$- \frac{\mu_{\mathcal{M}}}{24}(\langle\bar{d}d\rangle m_d - \langle\bar{s}s\rangle m_s)(-1-2t+3t^2)i_1'(\mathcal{T},1) - \frac{f_{\mathcal{M}}}{6}m_{\mathcal{M}}^2(\langle\bar{d}d\rangle - \langle\bar{s}s\rangle)(1-t)$$

$$\times [(1+3t)i_1(\mathcal{A}_\perp,1) + (3+t)i_1(\mathcal{V}_\perp,1-2v)], \tag{48}$$

where $\mathcal{M} = \pi$ or K.

The functions i_n, i'_n, $\tilde{i}_n$, $\tilde{\tilde{i}}_n$ are defined as

$$
\begin{aligned}
i_1(\varphi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \varphi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta(k - \bar{u}_0), \\
i'_1(\varphi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \varphi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta'(k - \bar{u}_0), \\
\tilde{i}_1(\varphi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \int_0^{\alpha_{\bar{q}} + v\alpha_g} dk \varphi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta(k - \bar{u}_0), \qquad i_2(f) = f(u_0),
\end{aligned}
\tag{49}
$$

where $k = \alpha_{\bar{q}} + v\alpha_g$ when there is no integration over k, $u_0 = M_1^2/(M_1^2 + M_2^2)$.

For obtaining the meson-baryon couplings, we also need to know the overlap amplitude of the hadrons. The overlap amplitudes can be obtained from the mass sum rules and their expressions can be found in [24,25]. As we noted that the other currents can be obtained from Σ^0 current, therefore we present the result for the residue of this current only:

$$
\begin{aligned}
\lambda_{\Sigma^0}^2 e^{-(m_\Sigma^2/M^2)} =&\; \frac{M^6}{1024\pi^2}(5 + 2t + 5t^2) - \frac{m_0^2}{96M^2}(-1 + t)^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle - \frac{m_0^2}{16M^2}(-1 + t^2)\langle\bar{s}s\rangle(\langle\bar{d}d\rangle + \langle\bar{u}u\rangle) \\
&+ \frac{3}{128m_0^2}m_0^2(-1 + t^2)[m_s(\langle\bar{d}d\rangle + \langle\bar{u}u\rangle) + \langle\bar{s}s\rangle(m_u + m_d)] - \frac{1}{64\pi^2}(-1 + t)^2(\langle\bar{d}d\rangle m_u + \langle\bar{u}u\rangle m_d)M^2 \\
&- \frac{3}{64\pi^2}(-1 + t^2)(m_s(\langle\bar{d}d\rangle + \langle\bar{u}u\rangle) + \langle\bar{s}s\rangle(m_u + m_d))M^2 \\
&+ \frac{1}{128\pi^2}(5 + 2t + 5t^2)(\langle\bar{u}u\rangle m_u + \langle\bar{d}d\rangle m_d + \langle\bar{s}s\rangle m_s) + \frac{1}{24}[3\langle\bar{s}s\rangle(\langle\bar{d}d\rangle + \langle\bar{u}u\rangle)(-1 + t^2) \\
&+ (-1 + t)^2\langle\bar{u}u\rangle\langle\bar{d}d\rangle] + \frac{m_0^2}{256\pi^2}(-1 + t)^2(m_d\langle\bar{u}u\rangle + m_u\langle\bar{d}d\rangle) \\
&+ \frac{m_0^2}{256\pi^2}(-1 + t^2)[13m_s(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle) + 11\langle\bar{s}s\rangle(m_d + m_u)] \\
&- \frac{m_0^2}{192\pi^2}(1 + t + t^2)(\langle\bar{u}u\rangle m_u + \langle\bar{d}d\rangle m_d - 2m_s\langle\bar{s}s\rangle).
\end{aligned}
\tag{50}
$$

Note that, from the mass sum rules, one can only extract the square of the residue, and not the sign. We have chosen our sign convention in defining the currents such that, in the $SU(3)_f$ symmetry limit, the signs correctly reproduce the F and D expressions (see, for example, [27]).

The contribution of the continuum to the sum rules obtained from Eqs. (45)–(48) and to the mass sum rules equation (50) are subtracted using the replacements

$$
M^{2n} \rightarrow \frac{1}{\Gamma(n)} \int_{q^2}^{s_0} ds\, s^{n-1} e^{-(s/M^2)}, \qquad n > 0
$$
$$
M^{2n} \ln\frac{M^2}{\Lambda^2} \rightarrow \frac{1}{\Gamma(n)} \int_{q^2}^{s_0} ds\, s^{n-1}\left(\ln\frac{s}{\Lambda^2} - \psi(n)\right),
\tag{51}
$$

where $q^2 = 0$ in the mass sum rules and $q^2 = m_{\mathcal{M}}^2$ in the sum rules for the coupling constants since the meson is real, $\psi(n)$ is the digamma function:

$$
\psi(x) = \frac{d}{dx} \ln\Gamma[x].
\tag{52}
$$

The subtraction scheme, Eq. (51), corresponds to taking a triangular domain in the double dispersion relation for the coupling constant outside of which we use the quark hadron duality to subtract the contributions of the higher states and continuum.

IV. NUMERICAL ANALYSIS

In this section we present our numerical results for the sum rules obtained in the previous section for the meson-baryon coupling constants. The meson-baryon coupling constants are physically measurable quantities, they should be independent on the auxiliary Borel parameter M^2, the continuum threshold s_0, and the parameter t. Therefore we need to find regions of these parameters where meson-baryon couplings are independent of them.

From the sum rules, one sees that the main input parameters are the meson wave functions. In our calculations, we will use the following forms of the meson wave functions [22,23]:

$$\phi_{\mathcal{M}}(u) = 6u\bar{u}(1 + a_1^{\mathcal{M}} C_1(2u-1) + a_2^{\mathcal{M}} C_2^{3/2}(2u-1)),$$

$$\mathcal{T}(\alpha_i) = 360\eta_3 \alpha_{\bar{q}}\alpha_q \alpha_g^2 \left(1 + w_3 \frac{1}{2}(7\alpha_g - 3)\right),$$

$$\phi_P(u) = 1 + \left(30\eta_3 - \frac{5}{2}\frac{1}{\mu_{\mathcal{M}}^2}\right)C_2^{1/2}(2u-1) + \left(-3\eta_3 w_3 - \frac{27}{20}\frac{1}{\mu_{\mathcal{M}}^2} - \frac{81}{10}\frac{1}{\mu_{\mathcal{M}}^2}a_2^{\mathcal{M}}\right)C_4^{1/2}(2u-1), \phi_\sigma(u)$$

$$= 6u\bar{u}\left[1 + \left(5\eta_3 - \frac{1}{2}\eta_3 w_3 - \frac{7}{20}\mu_{\mathcal{M}}^2 - \frac{3}{5}\mu_{\mathcal{M}}^2 a_2^{\mathcal{M}}\right)C_2^{3/2}(2u-1)\right],$$

$$\mathcal{V}_\parallel(\alpha_i) = 120\alpha_q \alpha_{\bar{q}}\alpha_g(v_{00} + v_{10}(3\alpha_g - 1)),$$

$$\mathcal{A}_\parallel(\alpha_i) = 120\alpha_q \alpha_{\bar{q}}\alpha_g(0 + a_{10}(\alpha_q - \alpha_{\bar{q}})),$$

$$\mathcal{V}_\perp(\alpha_i) = -30\alpha_g^2\left[h_{00}(1 - \alpha_g) + h_{01}(\alpha_g(1 - \alpha_g) - 6\alpha_q \alpha_{\bar{q}}) + h_{10}\left(\alpha_g(1 - \alpha_g) - \frac{3}{2}(\alpha_{\bar{q}}^2 + \alpha_q^2)\right)\right],$$

$$\mathcal{A}_\perp(\alpha_i) = 30\alpha_g^2(\alpha_{\bar{q}} - \alpha_q)\left[h_{00} + h_{01}\alpha_g + \frac{1}{2}h_{10}(5\alpha_g - 3)\right],$$

$$B(u) = g_{\mathcal{M}}(u) - \phi_{\mathcal{M}}(u),$$

$$g_{\mathcal{M}}(u) = g_0 C_0^{1/2}(2u-1) + g_2 C_2^{1/2}(2u-1) + g_4 C_4^{1/2}(2u-1),$$

$$\mathbb{A}(u) = 6u\bar{u}\left[\frac{16}{15} + \frac{24}{35}a_2^{\mathcal{M}} + 20\eta_3 + \frac{20}{9}\eta_4 + \left(-\frac{1}{15} + \frac{1}{16} - \frac{7}{27}\eta_3 w_3 - \frac{10}{27}\eta_4\right)C_2^{3/2}(2u-1)\right.$$

$$\left. + \left(-\frac{11}{210}a_2^{\mathcal{M}} - \frac{4}{135}\eta_3 w_3\right)C_4^{3/2}(2u-1)\right] + \left(-\frac{18}{5}a_2^{\mathcal{M}} + 21\eta_4 w_4\right)$$

$$\times [2u^3(10 - 15u + 6u^2)\ln u + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln\bar{u} + u\bar{u}(2 + 13u\bar{u})], \tag{53}$$

where $C_n^k(x)$ are the Gegenbauer polynomials,

$$h_{00} = v_{00} = -\frac{1}{3}\eta_4,$$

$$a_{10} = \frac{21}{8}\eta_4 w_4 - \frac{9}{20}a_2^{\mathcal{M}},$$

$$v_{10} = \frac{21}{8}\eta_4 w_4,$$

$$h_{01} = \frac{7}{4}\eta_4 w_4 - \frac{3}{20}a_2^{\mathcal{M}},$$

$$h_{10} = \frac{7}{4}\eta_4 w_4 + \frac{3}{20}a_2^{\mathcal{M}}, \tag{54}$$

$$g_0 = 1,$$

$$g_2 = 1 + \frac{18}{7}a_2^{\mathcal{M}} + 60\eta_3 + \frac{20}{3}\eta_4,$$

$$g_4 = -\frac{9}{28}a_2^{\mathcal{M}} - 6\eta_3 w_3,$$

and the parameters entering Eqs. (53) and (54) are given in Table I for the pion and the kaon.

Since the mass of the initial and final baryons are close to each other, we can set $M_1^2 = M_2^2 = 2M^2$. Then $u_0 = \frac{1}{2}$. For this reason we need the value of the wave functions only at $u = \frac{1}{2}$. The values of the other input parameters appearing in the sum rules are: $\langle\bar{q}q\rangle = -(0.243\text{ GeV})^3$, $m_0^2 = (0.8 \pm 0.2)\text{ GeV}^2$ [24], $f_K = 0.160\text{ GeV}$, and $f_\pi = 0.131\text{ GeV}$ [22]

An upper bound for the Borel parameter M^2 is obtained by requiring that the contribution of the continuum to the correlation function is less than 50% of the value of the correlation function. A lower bound is obtained by requiring that the contribution of the term with the highest power of $\frac{1}{M^2}$ is less than 20%. Using these constraints, one can find a working region for the Borel parameter M^2. The continuum threshold is varied in the range between $s_0 = (m_B + 0.5)^2$ and $s_0 = (m_B + 0.7)^2$.

In Fig. 1, we present the dependence of $g_{\Lambda nK}$ on M^2 at three fixed values of the parameter t and two fixed values of the continuum threshold, s_0. From this figure, we see that the results are rather stable with respect to variations of M^2 in the working region of M^2. It is also seen that the result depends on the value of t. Our next task is to find a region of t where the results are independent of the value of t. For this aim, in Fig. 2, we depict the dependence of $g_{\Lambda nK}$

TABLE I. Parameters of the wave function calculated at the renormalization scale $\mu = 1\text{ GeV}^2$.

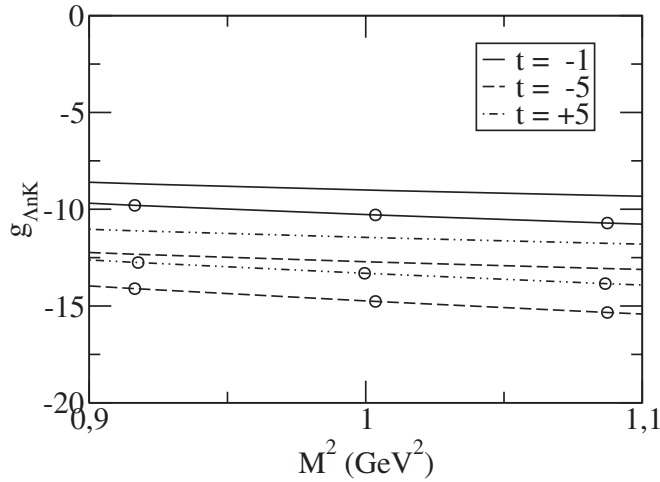|	π	K
$a_1^{\mathcal{M}}$	0	0.050
$a_2^{\mathcal{M}}$	0.44	0.16
η_3	0.015	0.015
η_4	10	0.6
w_3	-3	-3
w_4	0.2	0.2

FIG. 1.   The dependence of the coupling constant $g_{\Lambda nK}$ on the Borel parameter M^2 for the values of the arbitrary parameter $t = -1$, ± 5 and the continuum threshold $s_0 = 2.25$ GeV^2 (the curves without any symbols) and $s_0 = 2.75$ GeV (the curves with circles).

on $\cos\theta$, where θ is defined through $t = \tan\theta$. We see that when $\cos\theta$ varies in between $-0.5 < \cos\theta < 0.25$, the coupling constant $g_{\Lambda nK}$ is practically independent of the unphysical parameter t. And we find that $g_{\Lambda nK} = -13 \pm 3$.

A similar analysis of the stability of the other coupling constants with respect to the variation of the Borel mass M^2, the parameter t, and the continuum threshold s_0 is performed (see Figs. 3–36). The results on the kaon and pion couplings are presented in Table II under the column "General current." In the second column, labeled "Ioffe current," are listed our predictions if we set the arbitrary parameter $t = -1$. In the third column labeled "SU(3)$_f$,"
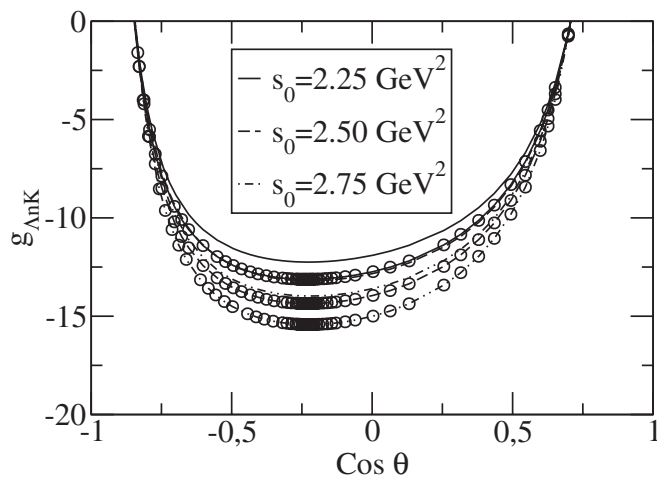
FIG. 2.   The dependence of the coupling constant $g_{\Lambda nK}$ on $\cos\theta$ for $s_0 = 2.50 \pm 0.25$ GeV^2 and for the Borel parameter $M^2 = 0.9$ GeV^2 (the curves without any symbols) and $M^2 = 1.1$ GeV^2 (the curves with circles on them).
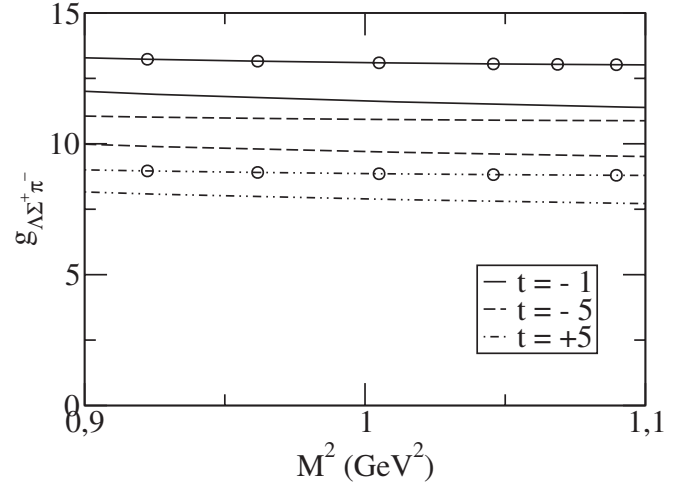
FIG. 3.   Same as Fig. 1 but for the $\Lambda \rightarrow \Sigma^+ \pi^-$ transition.
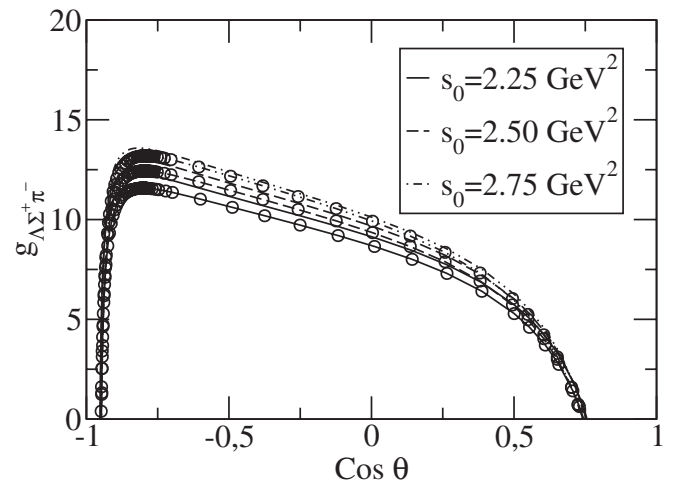
FIG. 4.   Same as Fig. 2 but for the $\Lambda \rightarrow \Sigma^+ \pi^-$ transition.
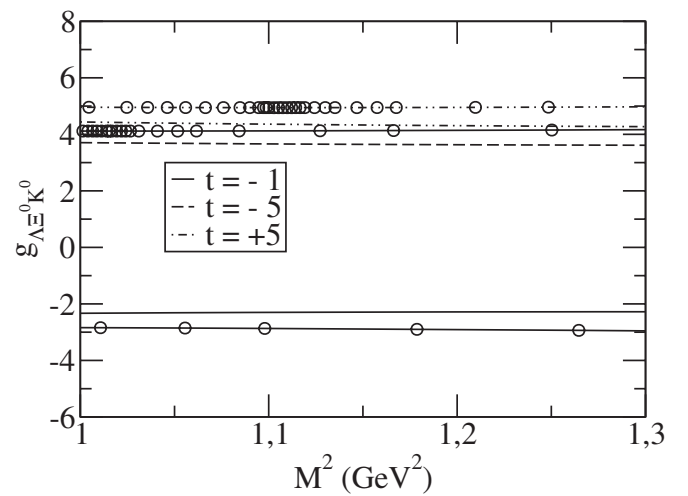
FIG. 5.   Same as Fig. 1 but for the $\Lambda \rightarrow \Xi^0 K^0$ transition and for the threshold values $s_0 = 2.75$ (the curves without any symbols) and $s_0 = 3.25$ (curves with circles).
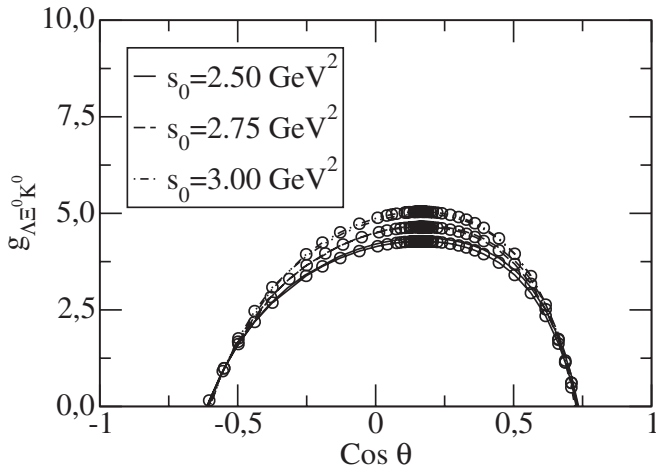
FIG. 6. Same as Fig. 2 but for the $\Lambda \to \Xi^0 K^0$ transition and for the values of the threshold $s_0 = 2.75 \pm 0.25$ GeV^2 and the Borel mass $M^2 = 1.1$ (curves without any circles), and $M^2 = 1.3$ (curves with circles).
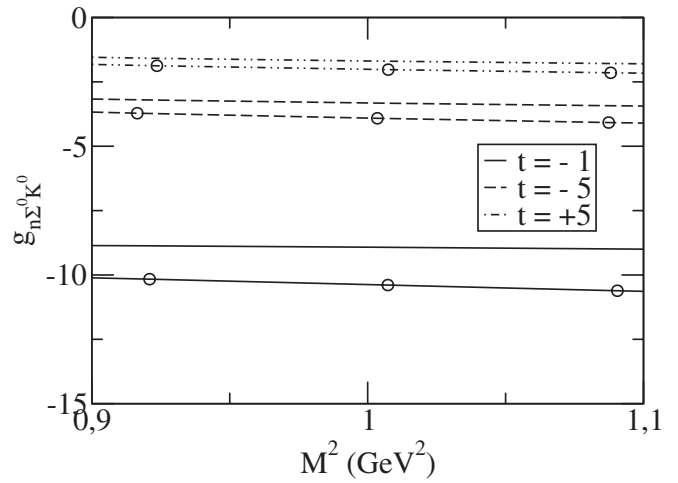
FIG. 9. The same as Fig. 1 but for the $n \to \Sigma^0 K^0$ transition.
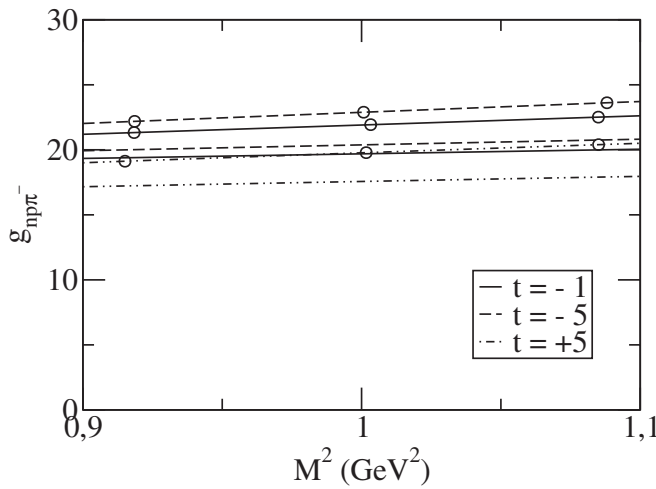
FIG. 7. The same as Fig. 1 but for the $n \to p\pi^-$ transition.
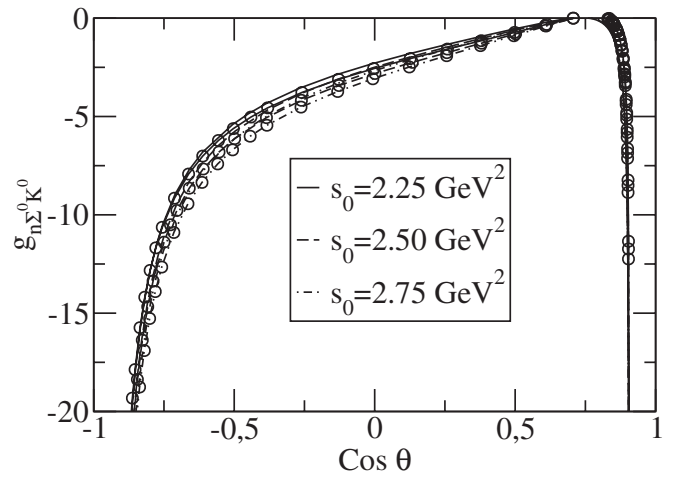
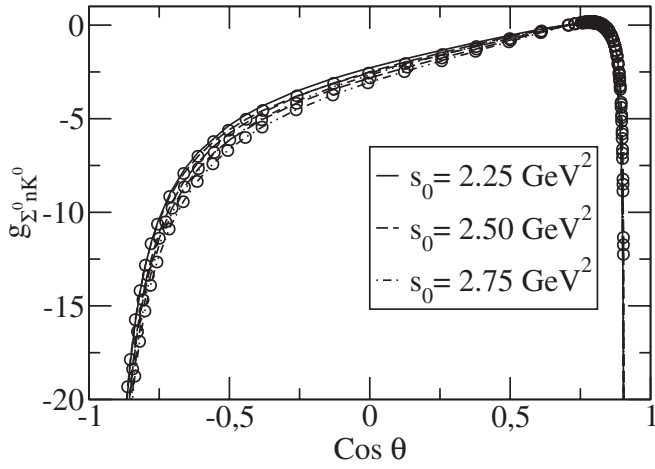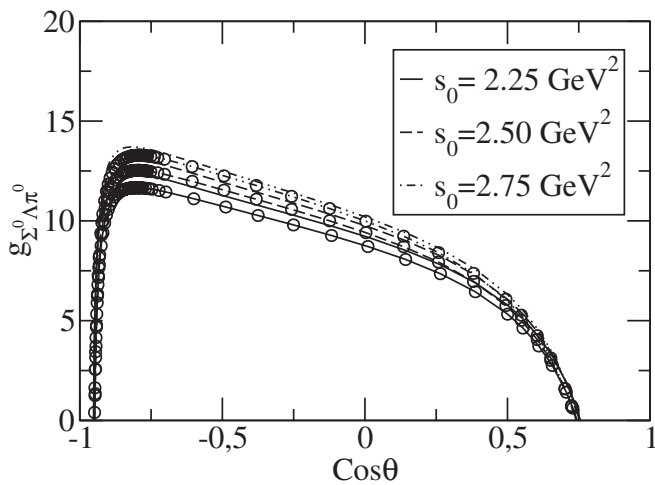FIG. 10. The same as Fig. 2 but for the $n \to \Sigma^0 K^0$ transition.
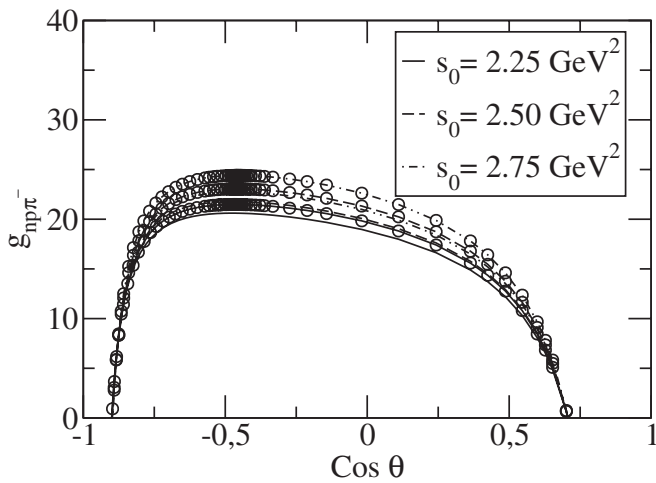
FIG. 8. The same as Fig. 2 but for the $n \to p\pi^-$ transition.
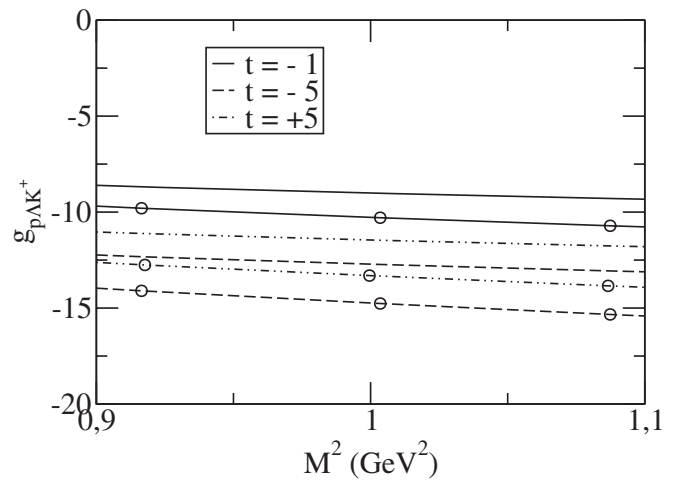
FIG. 11. The same as Fig. 1 but for the $p \to \Lambda K^+$ transition.
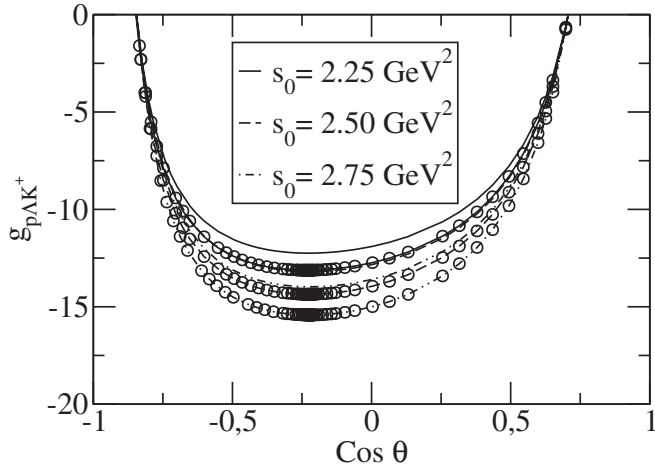
FIG. 12. The same as Fig. 2 but for the $p \to \Lambda K^+$ transition.
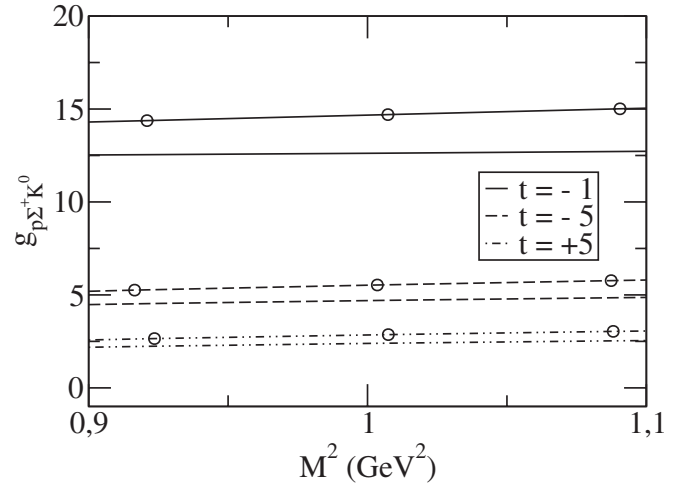
FIG. 15. The same as Fig. 1 but for the $p \to \Sigma^+ K^0$ transition.
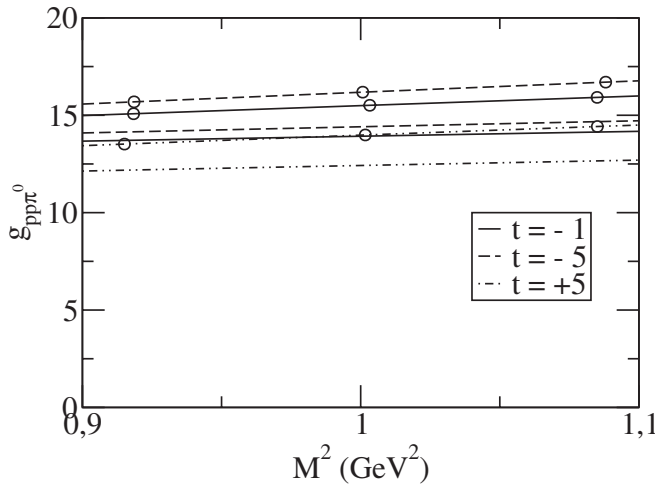
FIG. 13. The same as Fig. 1 but for the $p \to p\pi^0$ transition.
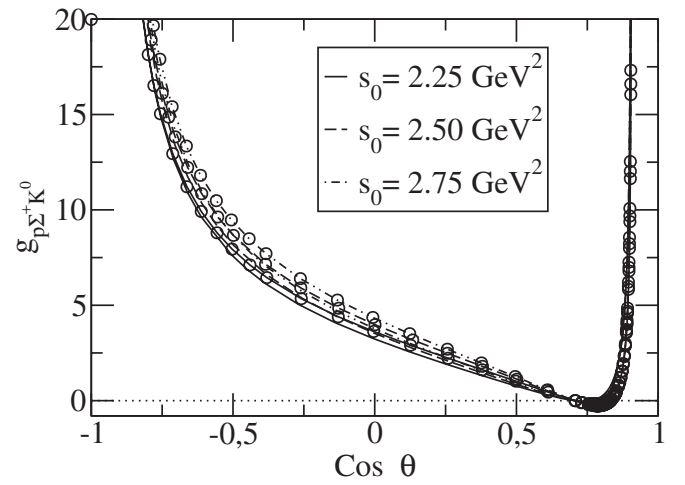
FIG. 16. The same as Fig. 2 but for the $p \to \Sigma^+ K^0$ transition.
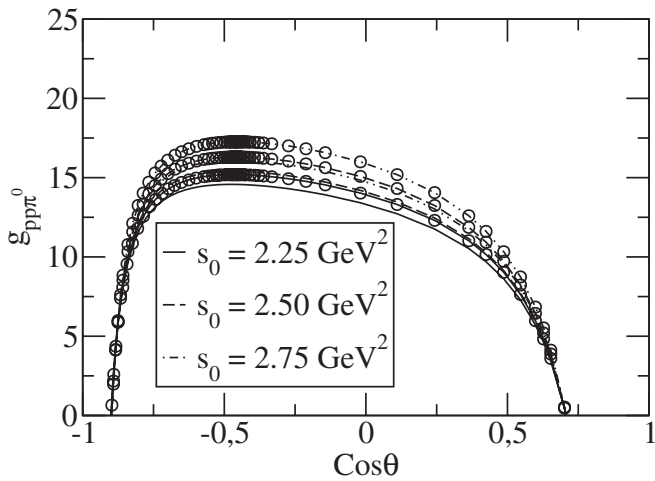
FIG. 14. The same as Fig. 2 but for the $p \to p\pi^0$ transition.
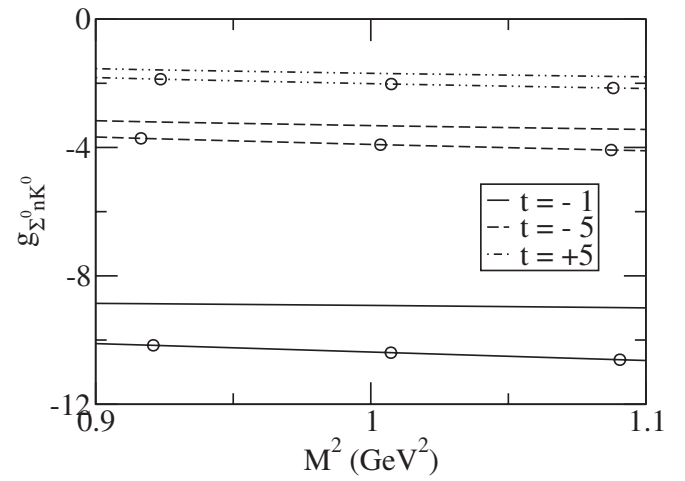
FIG. 17. The same as Fig. 1 but for the $\Sigma^0 \to nK^0$ transition.

FIG. 18. The same as Fig. 2 but for the $\Sigma^0 \rightarrow nK^0$ transition.
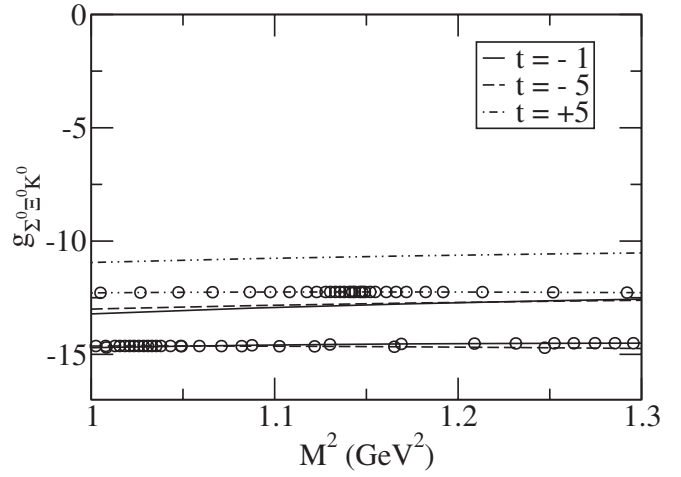
FIG. 21. The same as Fig. 5 but for the $\Sigma^0 \rightarrow \Xi^0 K^0$ transition.
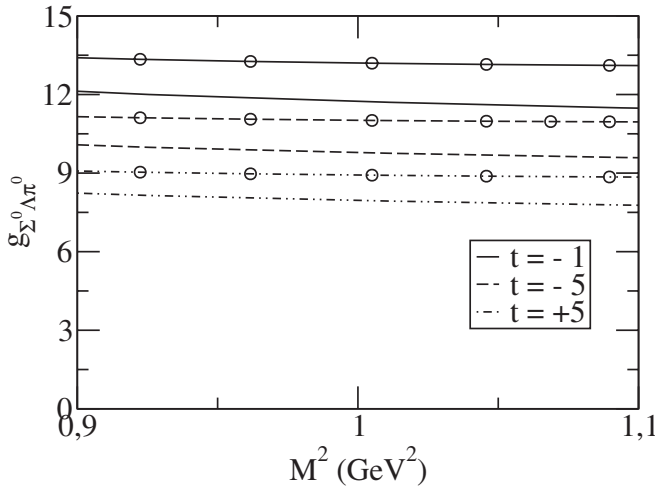
FIG. 19. The same as Fig. 1 but for the $\Sigma^0 \rightarrow \Lambda \pi^0$ transition.
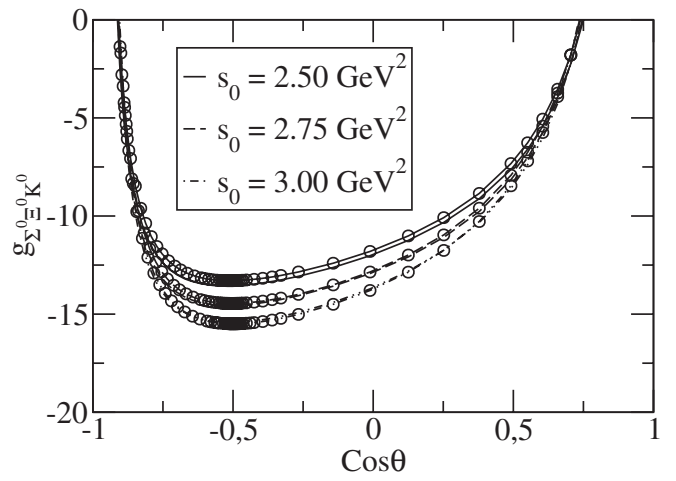
FIG. 22. The same as Fig. 6 but for the $\Sigma^0 \rightarrow \Xi^0 K^0$ transition.

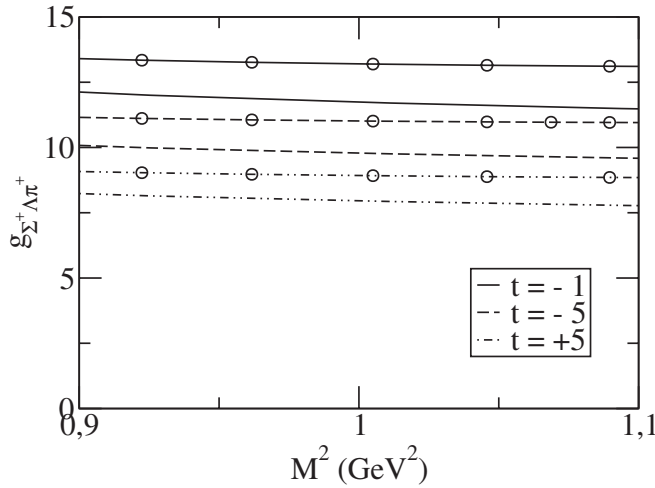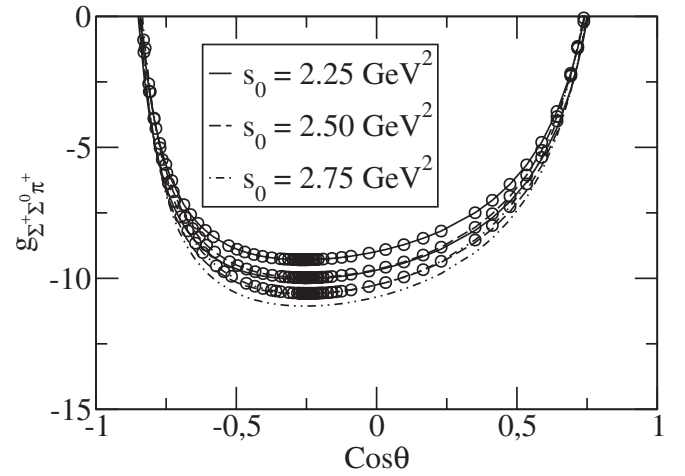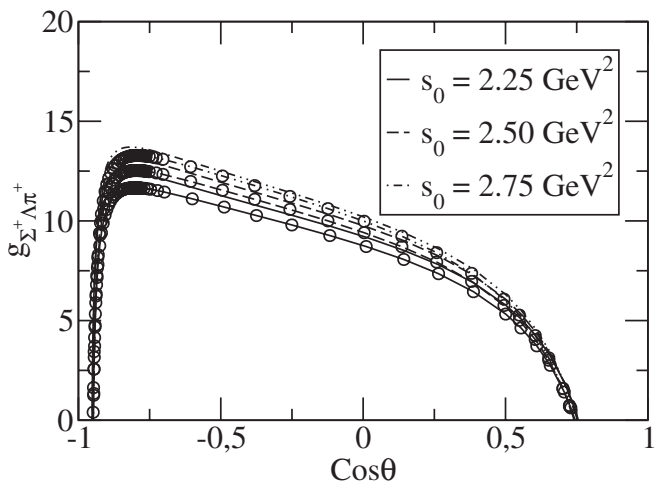FIG. 20. The same as Fig. 2 but for the $\Sigma^0 \rightarrow \Lambda \pi^0$ transition.
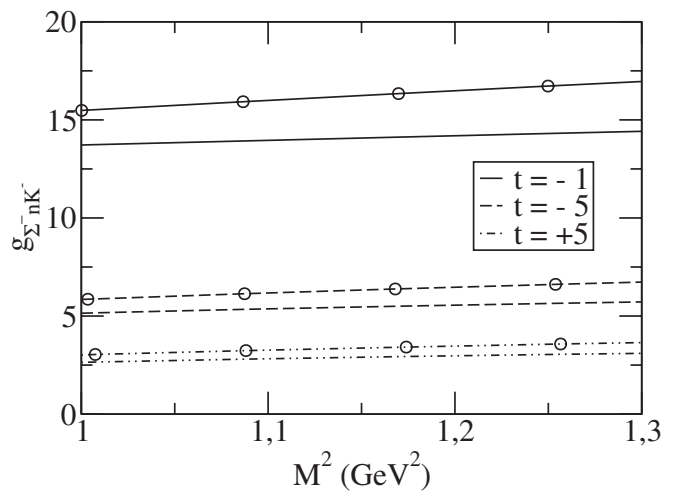
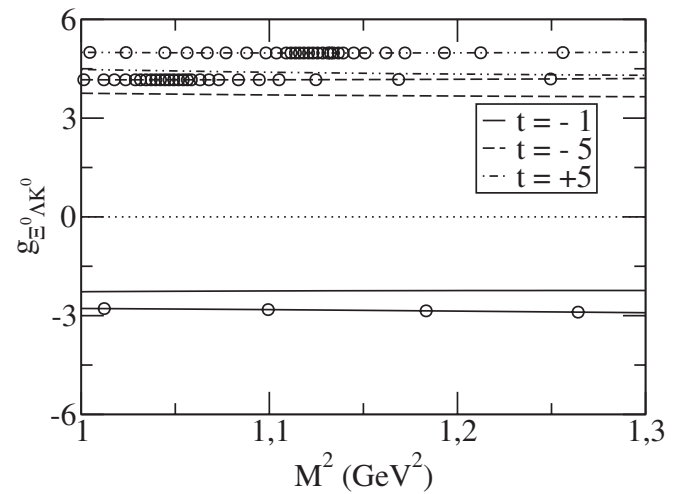

FIG. 23. The same as Fig. 5 but for the $\Sigma^- \rightarrow nK^-$ transition.
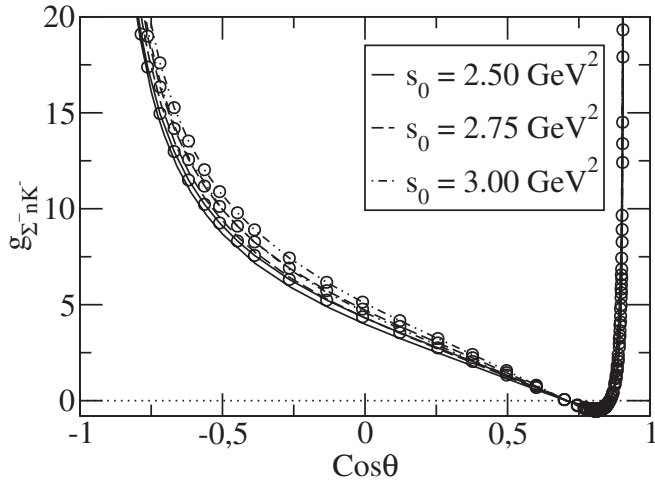
FIG. 24. The same as Fig. 6 but for the $\Sigma^- \to nK^-$ transition.

FIG. 25. The same as Fig. 1 but for the $\Sigma^+ \to \Lambda\pi^+$ transition.

FIG. 26. The same as Fig. 2 but for the $\Sigma^+ \to \Lambda\pi^+$ transition.
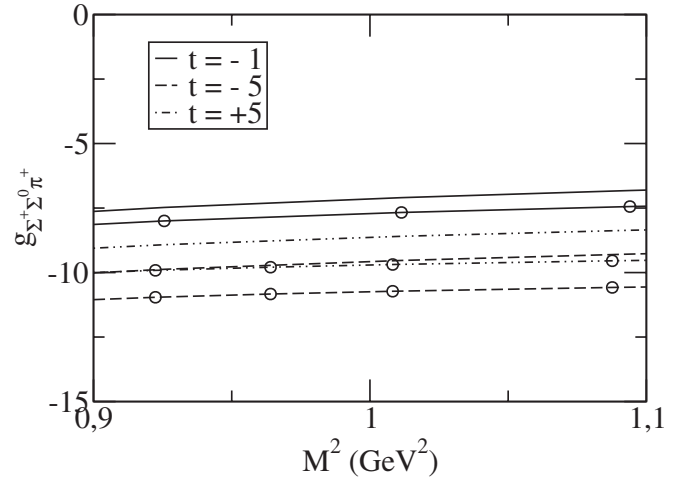
FIG. 27. The same as Fig. 1 but for the $\Sigma^+ \to \Sigma^0\pi^+$ transition.

FIG. 28. The same as Fig. 2 but for the $\Sigma^+ \to \Sigma^0\pi^+$ transition.

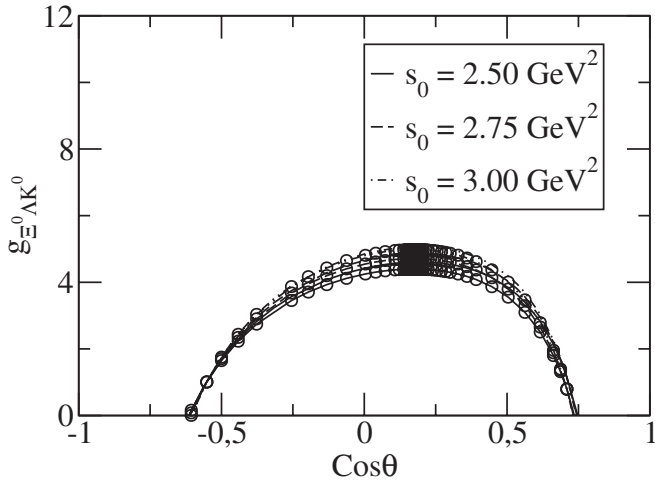FIG. 29. The same as Fig. 5 but for the $\Xi^0 \to \Lambda K^0$ transition.

FIG. 30.   The same as Fig. 6 but for the $\Xi^0 \to \Lambda K^0$ transition.

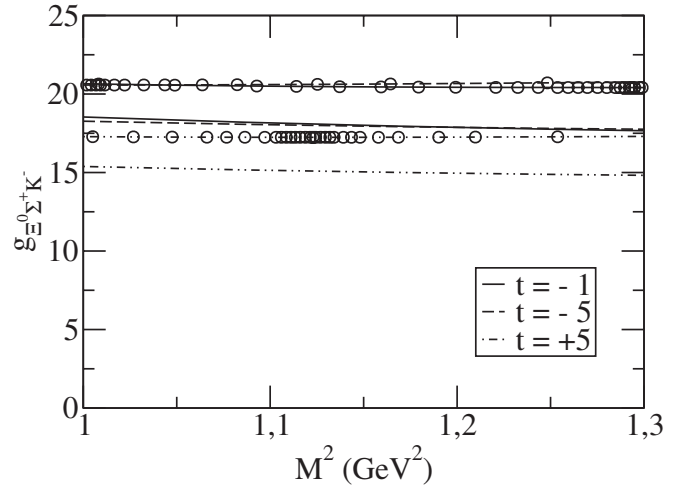FIG. 33.   The same as Fig. 5 but for the $\Xi^0 \to \Sigma^+ K^-$ transition.
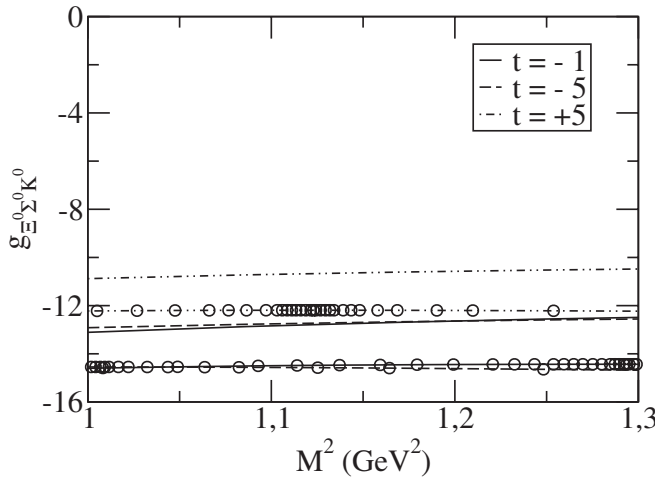

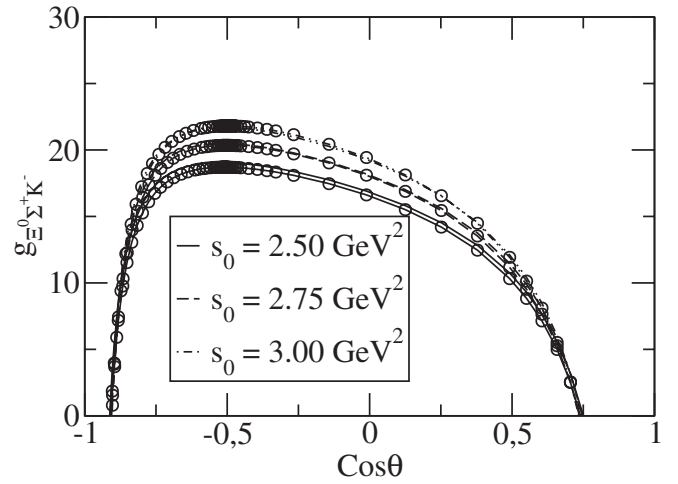FIG. 31.   The same as Fig. 5 but for the $\Xi^0 \to \Sigma^0 K^0$ transition.

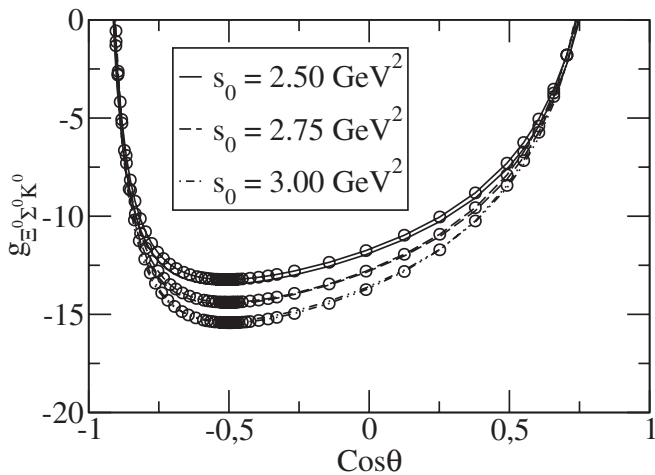FIG. 34.   The same as Fig. 6 but for the $\Xi^0 \to \Sigma^+ K^-$ transition.

FIG. 32.   The same as Fig. 6 but for the $\Xi^0 \to \Sigma^0 K^0$ transition.
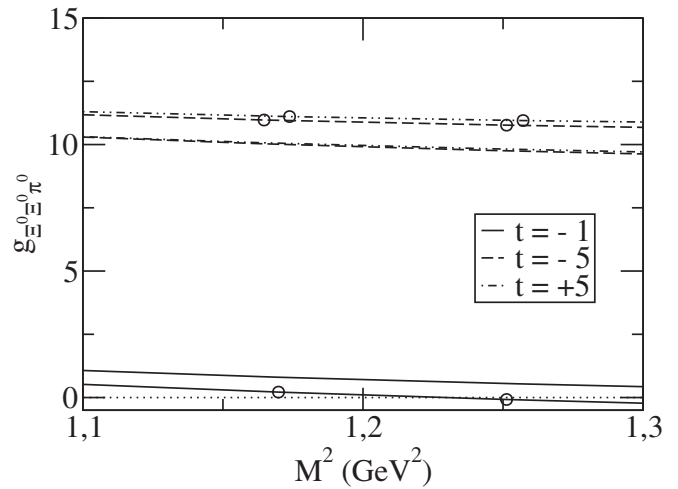

FIG. 35.   The same as Fig. 1 but for the $\Xi^0 \to \Xi^0 \pi^0$ transition and the threshold values $s_0 = 2.75$ (curves without any symbols), and $s_0 = 3.25$ (curves with symbols).
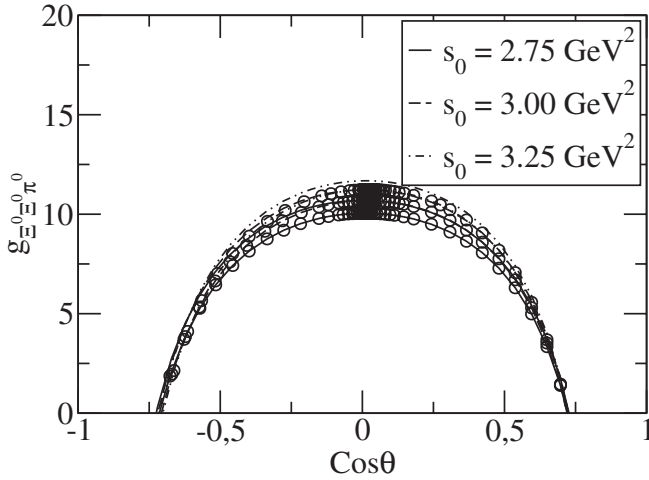
FIG. 36. Same as Fig. 2 but for the $\Xi^0 \to \Xi^0 \pi^0$ transition and for the values of the threshold $s_0 = 3.00 \pm 0.25$ GeV^2 and the Borel mass $M^2 = 1.1$ (curves without any circles), and $M^2 = 1.3$ (curves with circles).

we present the predictions of SU(3)$_f$ if one uses the central values of the prediction of the general current for the transitions $p \to p\pi^0$ and $\Sigma^+ \to \Lambda \pi^+$ (marked as "Input" in the table) to determine the F and D values. In the same table, we also present the existing theoretical and experimental results in the literature (columns 5, 6, and 7) for the same coupling constants.

Comparing the results in Table II, we obtain the following main conclusions:

(i) There are many cases in which the predictions of our analysis using the most general current differ considerably from the prediction of the Ioffe current, e.g. for the $\Lambda \to \Xi^0 K$ and $\Xi^0 \to \Lambda K^0$ coupling constants, the magnitudes as well as the signs are different. Also for the $p \to \Sigma^+ K^0$, $\Sigma^- \to nK^-$, and $n \to \Sigma^0 K^0$ cases, the magnitudes differ by at least a factor of 3. This difference is mainly due to the fact that, in these decays, the predictions for the coupling constants depend strongly on the exact value of t around $t = -1$. Hence $t = -1$ does not fall in the stability region of the sum rules.

(ii) Our results are also different from the results obtained in [15,28,31] but are closer to the experimental results [29,30].

(iii) In all cases except the $\Xi^0 \to \Xi^0 \pi^0$ transition, the prediction of the general current is consistent with the SU(3)$_f$ symmetry, whereas the Ioffe current predicts a large violation of SU(3)$_f$ symmetry. The connection between the SU(3)$_f$ symmetry and the usability of Ioffe current and the reason why there is a large violation of SU(3)$_f$ flavor symmetry in $\Xi^0 \to \Xi^0 \pi^0$ transition needs further study and is beyond the scope of this work. A plausibility argument can be that, since Ξ baryons contain two strange quarks, it is reasonable to expect that the SU(3)$_f$ violation will be more pronounced in this channel.

In conclusion, the coupling constants of the pseudoscalar K and π with the octet baryons are studied within the framework of light cone QCD sum rules. In numerical

TABLE II. The strong coupling constants for various channels both for the general current and for the Ioffe current. The first three columns are the results of this work. QSR*(†) are the predictions of the QCD sum rules using the $\sigma_{\mu\nu}\gamma_5 p^\mu q^\nu$ ($i\not{q}\gamma_5$) structure.

Channel	General current	Ioffe current ($t = -1$)	SU(3)$_f$	QSR*	QSR† [28]	Experiment
$\Lambda \to nK$	-13 ± 3	-9.5 ± 1	-14.3	-2.37 ± 0.09 [15]	-2.49 ± 1.25	-13.5 [29]
$\Lambda \to \Sigma^+ \pi^-$	10 ± 3	12 ± 1	10.0			
$\Lambda \to \Xi^0 K^0$	4.5 ± 2	-2.5 ± 0.5	4.25			
$n \to p\pi^-$	21 ± 4	20 ± 2	19.8			21.2 [30]
$n \to \Sigma^0 K^0$	-3.2 ± 2.2	-9.5 ± 0.5	-3.3	-0.025 ± 0.015 [15]	-0.40 ± 0.38	-4.25 [29]
$p \to \Lambda K^+$	-13 ± 3	-10 ± 1	-14.25	-2.37 ± 0.09 [15]	-2.49 ± 1.25	-13.5 [29]
$p \to p\pi^0$	14 ± 4	15 ± 1	Input	13.5 ± 0.5 [10]		14.9 [30]
$p \to \Sigma^+ K^0$	4 ± 3	14 ± 1	5.75			
$\Sigma^0 \to nK^0$	-4 ± 3	-9.5 ± 1	-3.32	-0.025 ± 0.015 [15]	-0.40 ± 0.38	-4.25 [29]
$\Sigma^0 \to \Lambda \pi^0$	11 ± 3	12 ± 1.5	10.0	6.9 ± 1 [31]		
$\Sigma^0 \to \Xi^0 K^0$	-13 ± 3	-13.5 ± 1	-14			
$\Sigma^- \to nK^-$	5 ± 3	15 ± 2	4.7			
$\Sigma^+ \to \Lambda \pi^+$	10 ± 3.5	12.5 ± 1	Input			
$\Sigma^+ \to \Sigma^0 \pi^+$	-9 ± 2	-7.5 ± 0.7	-10.7	-11.9 ± 0.4 [10]		
$\Xi^0 \to \Lambda K^0$	4.5 ± 1	-2.6 ± 0.3	4.25			
$\Xi^0 \to \Sigma^0 K^0$	-12.5 ± 3	-13.5 ± 1	-14			
$\Xi^0 \to \Sigma^+ K^-$	18 ± 4	19 ± 2	19.8			
$\Xi^0 \to \Xi^0 \pi^0$	10 ± 2	0.3 ± 0.6	-3.32	-1.60 ± 0.05 [10]		

analysis, we studied the $SU(3)_f$ flavor symmetry breaking effects due to $m_s \neq m_u = m_d = 0$ and $\langle \bar{s}s \rangle \neq \langle \bar{u}u \rangle = \langle \bar{d}d \rangle$. It is found that, without assuming any symmetry between the light quarks, all the coupling constants of the octet baryons to K and π can be written in terms of only four analytical functions, which reduce to the well known F and D functions in the $SU(3)_f$ symmetry case. We also perform comparison of our results with the existing theoretical and experimental results in the literature.

ACKNOWLEDGMENTS

APPENDIX A: EXPRESSIONS OF THE CORRELATION FUNCTIONS

Correlation functions for the couplings involving π^0:

$$\Pi^{\Sigma^0 \to \Sigma^0 \pi^0} = \frac{1}{\sqrt{2}}(\Pi_1(u, d, s) - \Pi_1(d, u, s)),$$

$$\Pi^{\Sigma^+ \to \Sigma^+ \pi^0} = \sqrt{2}\Pi_1(u, u, s),$$

$$\Pi^{\Sigma^- \to \Sigma^- \pi^0} = -\sqrt{2}\Pi_1(d, d, s),$$

$$\Pi^{p \to p \pi^0} = \sqrt{2}\Pi_1(u, u, d) - \frac{1}{\sqrt{2}}\Pi_2(u, u, d),$$

$$\Pi^{n \to n \pi^0} = -\sqrt{2}\Pi_1(d, d, u) + \frac{1}{\sqrt{2}}\Pi_2(d, d, u),$$

$$\Pi^{\Xi^0 \to \Xi^0 \pi^0} = \frac{1}{\sqrt{2}}\Pi_2(s, s, u),$$

$$\Pi^{\Xi^- \to \Xi^- \pi^0} = -\frac{1}{\sqrt{2}}\Pi_2(s, s, d),$$

$$\Pi^{\Sigma^0 \to \Lambda \pi^0} + \Pi^{\Lambda \to \Sigma^0 \pi^0} = \frac{2}{\sqrt{6}}[\Pi_1(u, s, d) + \Pi_1(d, s, u) - \Pi_2(s, d, u) - \Pi_2(s, u, d)],$$

$$\Pi^{\Lambda \to \Lambda \pi^0} = \frac{\sqrt{2}}{3}\Big[\Pi_1(u, s, d) - \Pi_1(d, s, u) + \Pi_2(s, d, u) - \Pi_2(s, u, d) - \frac{1}{2}\Pi_1(u, d, s) + \frac{1}{2}\Pi_1(d, u, s)\Big].$$

$$(A1)$$

Correlation functions for the couplings involving π^-:

$$\Pi^{\Sigma^0 \to \Sigma^+ \pi^-} = -\sqrt{2}\Pi_1(d, u, s),$$

$$\Pi^{\Sigma^- \to \Sigma^0 \pi^-} = \sqrt{2}\Pi_1(u, d, s),$$

$$\Pi^{\Xi^- \to \Xi^0 \pi^-} = -\Pi_2(s, s, d),$$

$$\Pi^{\Lambda \to \Sigma^+ \pi^-} = -\frac{1}{\sqrt{3}}[2\Pi_3(d, s, u) + \sqrt{2}\Pi_1(d, u, s)],$$

$$\Pi^{\Sigma^- \to \Lambda \pi^-} = -\frac{1}{\sqrt{3}}[2\Pi_4(u, s, d) + \sqrt{2}\Pi_1(u, d, s)],$$

$$\Pi^{n \to p \pi^-} = -\sqrt{2}\Pi_3(d, d, u).$$

$$(A2)$$

Correlation functions for the couplings involving π^+:

$$\Pi^{\Sigma^+ \to \Sigma^0 \pi^+} = -\sqrt{2}\Pi_1(d, u, s),$$

$$\Pi^{\Sigma^0 \to \Sigma^- \pi^+} = \sqrt{2}\Pi_1(u, d, s),$$

$$\Pi^{\Xi^0 \to \Xi^- \pi^+} = -\Pi_2(s, s, u),$$

$$\Pi^{\Sigma^+ \to \Lambda \pi^+} = -\frac{1}{\sqrt{3}}[2\Pi_4(d, s, u) + \sqrt{2}\Pi_1(d, u, s)],$$

$$\Pi^{\Lambda \to \Sigma^- \pi^+} = -\frac{1}{\sqrt{3}}[2\Pi_3(u, s, d) + \sqrt{2}\Pi_1(u, d, s)],$$

$$\Pi^{p \to n \pi^+} = -\sqrt{2}\Pi_3(u, u, d),$$

$$(A3)$$

Correlation functions for the couplings involving K^+:

$$\Pi^{p \to \Sigma^0 K} = -\sqrt{2}\Pi_1(s, u, d) - \Pi_4(s, d, u),$$

$$\Pi^{p \to \Lambda K} = -\frac{1}{\sqrt{3}}[\sqrt{2}\Pi_1(s, u, d) - \Pi_4(s, d, u)],$$

$$\Pi^{n \to \Sigma^- K} = -\Pi_2(d, d, u),$$

$$\Pi^{\Sigma^+ \to \Xi^0 K} = -\sqrt{2}\Pi_3(u, u, s),$$

$$\Pi^{\Sigma^0 \to \Xi^- K} = -\Pi_3(u, d, s),$$

$$\Pi^{\Lambda \to \Xi^- K} = \frac{1}{\sqrt{3}}[2\sqrt{2}\Pi_1(u, s, d) + \Pi_3(u, d, s)].$$

$$(A4)$$

Correlation functions for the couplings involving K^-:

$$\Pi^{\Sigma^0 \to p K} = -\sqrt{2}\Pi_1(s, u, d) - \Pi_3(s, d, u),$$

$$\Pi^{\Lambda \to p K} = -\frac{1}{\sqrt{3}}[\sqrt{2}\Pi_1(s, u, d) - \Pi_3(s, d, u)],$$

$$\Pi^{\Sigma^- \to n K} = -\Pi_2(d, d, s),$$

$$\Pi^{\Xi^0 \to \Sigma^+ K} = -\sqrt{2}\Pi_3(s, s, u),$$

$$\Pi^{\Xi^- \to \Sigma^0 K} = -\Pi_4(u, d, s),$$

$$\Pi^{\Xi^- \to \Lambda K} = \frac{1}{\sqrt{3}}[2\sqrt{2}\Pi_1(u, s, d) + \Pi_4(u, d, s)].$$

$$(A5)$$

Correlation functions for the couplings involving $K^0(s\bar{d})$:

$$\Pi^{\Xi^0 \to \Sigma^0 K} = \Pi_4(d, u, s), \qquad \Pi^{\Xi^0 \to \Lambda K} = \frac{1}{\sqrt{3}}[2\sqrt{2}\Pi_1(d, s, u) + \Pi_4(d, u, s)], \qquad \Pi^{\Xi^- \to \Sigma^- K} = -\sqrt{2}\Pi_3(s, s, d),$$

$$\Pi^{\Sigma^0 \to nK} = \Pi_3(s, u, d) + \sqrt{2}\Pi_1(s, d, u), \qquad \Pi^{\Lambda \to nK} = \frac{1}{\sqrt{3}}[\Pi_3(s, u, d) - \sqrt{2}\Pi_1(s, d, u)], \qquad \Pi^{\Sigma^+ \to pK} = -\Pi_2(u, u, s).$$

$$(A6)$$

Correlation functions for the couplings involving $\bar{K}^0(d\bar{s})$:

$$\Pi^{\Sigma^0 \to \Xi^0 K} = \Pi_3(d, u, s), \qquad \Pi^{\Lambda \to \Xi^0 K} = \frac{1}{\sqrt{3}}[2\sqrt{2}\Pi_1(d, s, u) + \Pi_3(d, u, s)], \qquad \Pi^{\Sigma^- \to \Xi^- K} = -\sqrt{2}\Pi_3(d, d, s),$$

$$\Pi^{n \to \Sigma^0 K} = \Pi_4(s, u, d) + \sqrt{2}\Pi_1(s, d, u), \qquad \Pi^{n \to \Lambda K} = \frac{1}{\sqrt{3}}[\Pi_4(s, u, d) - \sqrt{2}\Pi_1(s, d, u)], \qquad \Pi^{p \to \Sigma^+ K} = -\Pi_2(u, u, d).$$

$$(A7)$$

APPENDIX B: RELATIONS IN THE SU(2) LIMIT

Correlation functions involving the pion:

$$\Pi^{\Sigma^0 \to \Sigma^0 \pi} = \Pi^{\Lambda \to \Lambda \pi^0} = 0,$$

$$\sqrt{2}\Pi_1(q, q, s) = \Pi^{\Sigma^+ \to \Sigma^+ \pi} = -\Pi^{\Sigma^- \to \Sigma^- \pi} = -\Pi^{\Sigma^0 \to \Sigma^+ \pi} = \Pi^{\Sigma^- \to \Sigma^0 \pi} = -\Pi^{\Sigma^+ \to \Sigma^0 \pi} = \Pi^{\Sigma^0 \to \Sigma^- \pi},$$

$$\Pi^{\Xi^0 \to \Xi^0 \pi} = \frac{1}{\sqrt{2}}\Pi_2(s, s, q) = -\Pi^{\Xi^- \to \Xi^- \pi} = -\frac{1}{\sqrt{2}}\Pi^{\Xi^- \to \Xi^0 \pi} = -\frac{1}{\sqrt{2}}\Pi^{\Xi^0 \to \Xi^- \pi},$$

$$\Pi^{p \to p\pi} = -\Pi^{n \to n\pi} = \sqrt{2}\Pi_1(q, q, q) - \frac{1}{\sqrt{2}}\Pi_2(q, q, q),$$

$$\Pi^{\Lambda \to \Sigma^+ \pi} = \Pi^{\Lambda \to \Sigma^- \pi} = -\frac{1}{\sqrt{3}}[2\Pi_3(q, s, q) + \sqrt{2}\Pi_1(q, q, s)],$$

$$\Pi^{\Sigma^+ \to \Lambda \pi} = \Pi^{\Sigma^- \to \Lambda \pi} = -\frac{1}{\sqrt{3}}[2\Pi_4(q, s, q) + \sqrt{2}\Pi_1(q, q, s)],$$

$$\Pi^{n \to p\pi} = \Pi^{p \to n\pi} = -\sqrt{2}\Pi_3(q, q, q),$$

$$\Pi^{\Sigma^0 \to \Lambda \pi} + \Pi^{\Lambda \to \Sigma^0 \pi^0} = \frac{4}{\sqrt{6}}[\Pi_1(q, s, q) - \Pi_2(s, q, q)].$$

$$(B1)$$

Correlations involving the kaons:

$$\Pi^{n\to\Sigma^0 K} = -\Pi^{p\to\Sigma^0 K} = \Pi_4(s,q,q) + \sqrt{2}\Pi_1(s,q,q),$$

$$\Pi^{p\to\Lambda K} = \Pi^{n\to\Lambda K} = -\frac{1}{\sqrt{3}}[\sqrt{2}\Pi_1(s,q,q) - \Pi_4(s,q,q)],$$

$$\Pi^{p\to\Sigma^+ K} = \Pi^{n\to\Sigma^- K} = -\Pi_2(q,q,q),$$

$$-\Pi^{\Sigma^0\to\Xi^- K} = -\frac{1}{\sqrt{2}}\Pi^{\Sigma^+\to\Xi^0 K} = \Pi^{\Sigma^0\to\Xi^0 K} = -\frac{1}{\sqrt{2}}\Pi^{\Sigma^-\to\Xi^- K} = \Pi_3(q,q,s),$$

$$\Pi^{\Lambda\to\Xi^0 K} = \Pi^{\Lambda\to\Xi^- K} = \frac{1}{\sqrt{3}}[2\sqrt{2}\Pi_1(q,s,q) + \Pi_3(q,q,s)],$$

$$\Pi^{\Sigma^0\to n K} = -\Pi^{\Sigma^0\to p K} = \sqrt{2}\Pi_1(s,q,q) + \Pi_3(s,q,q),$$

$$-\Pi^{\Lambda\to p K} = -\Pi^{\Lambda\to n K} = \frac{1}{\sqrt{3}}[\sqrt{2}\Pi_1(s,q,q) - \Pi_3(s,q,q)],$$

$$\Pi^{\Sigma^-\to n K} = \Pi^{\Sigma^+\to p K} = -\Pi_2(q,q,s),$$

$$-\Pi^{\Xi^0\to\Sigma^+ K} = -\Pi^{\Xi^-\to\Sigma^- K} = \sqrt{2}\Pi_3(s,s,q),$$

$$-\Pi^{\Xi^-\to\Sigma^0 K} = \Pi^{\Xi^0\to\Sigma^0 K} = \Pi_4(q,q,s),$$

$$\Pi^{\Xi^-\to\Lambda K} = \Pi^{\Xi^0\to\Lambda K} = \frac{1}{\sqrt{3}}[2\sqrt{2}\Pi_1(q,s,q) + \Pi_4(q,q,s)].$$

(B2)

[1] M. K. Cheoun, B. S. Han, B. G. Yu, and I. T. Cheon, Phys. Rev. C **54**, 1811 (1996).
[2] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).
[3] H. Shiomi and T. Hatsuda, Nucl. Phys. **A594**, 294 (1998).
[4] M. C. Birse and B. Krippa, Phys. Lett. B **373**, 9 (1996).
[5] T. Doi, H. Kim, and M. Oka, Phys. Rev. C **62**, 055202 (2000).
[6] T. M. Aliev, A. Ozpineci, and M. Savci, Phys. Rev. D **64**, 034001 (2001); T. M. Aliev and M. Savci, Phys. Rev. D **61**, 016008 (1999).
[7] H. Kim, S. H. Lee, and M. Oka, Phys. Rev. D **60**, 034007 (1999).
[8] H.-C. Kim, Eur. Phys. J. A **7**, 121 (2000).
[9] H. Kim, T. Doi, M. Oka, and S. H. Lee, Nucl. Phys. **A662**, 371 (2000).
[10] H. Kim, T. Doi, M. Oka, and S. H. Lee, Nucl. Phys. **A678**, 295 (2000).
[11] V. M. Braun and I. E. Filyanov, Z. Phys. C **44**, 157 (1989).
[12] S. Choe, M. K. Cheoun, and S. H. Lee, Phys. Rev. C **53**, 1363 (1996).
[13] S. Choe, Phys. Rev. C **57**, 2061 (1998).
[14] B. Krippa, Phys. Lett. B **420**, 13 (1998).
[15] M. E. Bracco, F. S. Navarra, and M. Nielsen, Phys. Lett. B **454**, 346 (1999).
[16] T. M. Aliev and M. Savci, Phys. Rev. C **61**, 045201 (2000).
[17] V. M. Braun, hep-ph/9801222.
[18] P. Colangelo and A. Khodjamirian, *At Frontier of Particle Physics/Handbook of QCD*, edited by M. Shifman (World Scientific, Singapore), Vol. 3, p. 1495.
[19] T. M. Aliev, A. Ozpineci, S. B. Yakovlev, and V. S. Zamiralov, AIP Conf. Proc. **806**, 40 (2006).
[20] A. Ozpineci, S. B. Yakovlev, and V. S. Zamiralov, Phys. At. Nucl. **68**, 279 (2005).
[21] A. Ozpineci, S. B. Yakovlev, and V. S. Zamiralov, Mod. Phys. Lett. A **20**, 243 (2005).
[22] P. Ball, J. High Energy Phys. 01, 010 (1999); P. Ball, V. M. Braun, and A. Lenz, J. High Energy Phys. 05 (2006) 004.
[23] P. Ball and R. Zwicky, Phys. Rev. D **71**, 014015 (2005).
[24] V. M. Belyaev and B. L. Ioffe, Sov. Phys. JETP **57**, 716 (1983); Zh. Eksp. Teor. Fiz. **84**, 1236 (1983).
[25] T. M. Aliev and A. Ozpineci, Nucl. Phys. **B732**, 291 (2006).
[26] T. M. Aliev, A. Ozpineci, and M. Savci, Phys. Rev. D **64**, 034001 (2001).
[27] J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).
[28] S. Choe, Phys. Rev. C **62**, 025204 (2000).
[29] R. Lawall *et al.*, Eur. Phys. J. A **24**, 275 (2005).
[30] R. A. Arndt, Z.-J. Li, L. D. Roper, and R. L. Workman, Phys. Rev. Lett. **65**, 157 (1990).
[31] T. Doi, Y. Kondo, and M. Oka, Phys. Rep. **398**, 253 (2004).