Discovery potential of radiative neutralino production at the ILC

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We study radiative neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$ at the linear collider with longitudinally polarized beams. We consider the standard model background from radiative neutrino production $e^+e^- \to \nu\bar{\nu}\gamma$, and the supersymmetric radiative production of sneutrinos $e^+e^- \to \tilde{\nu}\tilde{\nu}^*\gamma$, which can be a background for invisible sneutrino decays. We give the complete tree-level formulas for the amplitudes and matrix elements squared. In the minimal supersymmetric standard model, we study the dependence of the cross sections on the beam polarizations, on the parameters of the neutralino sector, and on the selectron masses. We show that for b-inolike neutralinos longitudinal polarized beams enhance the signal and simultaneously reduce the background, such that statistics is significantly enhanced. We point out that there are parameter regions where radiative neutralino production is the only channel to study SUSY particles, since heavier neutralinos, charginos and sleptons are too heavy to be pair-produced in the first stage of the linear collider with $\sqrt{s} = 500$ GeV.

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I. INTRODUCTION

Supersymmetry (SUSY) is an attractive concept for theories beyond the standard model (SM) of particle physics. SUSY models like the minimal supersymmetric standard model (MSSM) [1–3] predict SUSY partners of the SM particles with masses of the order of a few hundred GeV. Their discovery is one of the main goals of present and future colliders in the TeV range. In particular, the international e^+e^- linear collider (ILC) will be an excellent tool to determine the parameters of the SUSY model with high precision [4–8]. Such a machine provides high luminosity $\mathcal{L} = 500$ fb^{-1}, a center-of-mass energy of $\sqrt{s} = 500$ GeV in the first stage, and a polarized electron beam with the option of a polarized positron beam [9].

The neutralinos are the fermionic SUSY partners of the neutral gauge and CP-even Higgs bosons. Since they are among the lightest particles in many SUSY models, they are expected to be also the first states to be observed. At the ILC, they can be directly produced in pairs

$$e^+ + e^- \to \tilde{\chi}_i^0 + \tilde{\chi}_j^0, \qquad (1.1)$$

which proceeds via Z boson and selectron exchange [10,11]. At tree level, the neutralino sector depends only on the four parameters M_1, M_2, μ, and $\tan\beta$, which are the $U(1)_Y$ and $SU(2)_L$ gaugino masses, the Higgsino mass parameter, and the ratio of the vacuum expectation values of the two Higgs fields, respectively. These parameters can be determined by measuring the neutralino production cross sections and decay distributions [7,12–15]. In the MSSM with R-parity (or proton hexality, P_6, [16]) conservation, the lightest neutralino $\tilde{\chi}_1^0$ is typically the lightest SUSY particle (LSP) and as such is stable and a good dark matter candidate [17,18]. In collider experiments the LSP escapes detection such that the direct production of the lightest neutralino pair $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0$ is invisible. Their pair production can only be observed indirectly via radiative production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$, where the photon is radiated off the incoming beams or off the exchanged selectrons. Although this higher order process is suppressed by the square of the additional photon-electron coupling, it might be the lightest state of SUSY particles to be observed at colliders. The signal is a single high energetic photon and missing energy, carried by the neutralinos.

As a unique process to search for the first SUSY signatures at e^+e^- colliders, the radiative production of neutralinos has been intensively studied in the literature [19–35].[1] Early investigations focus on LEP energies and discuss special neutralino mixing scenarios only, in particular, the pure photino case [19–26]. More recent studies assume general neutralino mixing [27–35] and some of them underline the importance of longitudinal [27–30] and even transverse beam polarizations [27,30]. The transition amplitudes are given in a generic factorized form [28], which allows the inclusion of anomalous $WW\gamma$ couplings. Cross sections are calculated with the program COMPHEP [29], or in the helicity formalism [30]. Some of the studies [31–35] however do not include longitudinal beam polarizations, which might be essential for measuring radiative neutralino production at the ILC. Special scenarios are considered, where besides the sneutrinos also the heavier neutralinos [32–34], and even charginos [38–40] decay invisibly or almost invisibly. However, a part of such unconventional signatures are by now ruled out by LEP2 data [32,39,41]. For the ILC, such "effective" LSP scenarios have been analyzed [33], and strategies for detecting invisible decays of neutralinos and charginos have been proposed [38,40]. Moreover, the radiative production of neutralinos can serve as a direct test, whether neutralinos

[1] In addition we found two references, Refs. [36,37], which are however almost identical in wording and layout to Ref. [31].

are dark matter candidates. See, for example, Ref. [42], which presents a model independent calculation for the cross section of radiatively produced dark matter candidates at high-energy colliders, including polarized beams for the ILC.

The signature "photon plus missing energy" has been studied intensively by the LEP collaborations ALEPH [43], DELPHI [44], L3 [45], and OPAL [41,46]. In the SM, $e^+e^- \to \nu\bar{\nu}\gamma$ is the leading process with this signature. Since the cross section depends on the number N_ν of light neutrino generations [47], it has been used to measure N_ν consistent with three. In addition, the LEP collaborations have tested physics beyond the SM, like nonstandard neutrino interactions, extra dimensions, and SUSY particle productions. However, no deviations from SM predictions have been found, and only bounds on SUSY particle masses have been set, e.g. on the gravitino mass [43–46]. This process is also important in determining collider bounds on a very light neutralino [48]. For a combined short review, see, for example, Ref. [49].

Although there are so many theoretical studies on radiative neutralino production in the literature, a thorough analysis of this process is still missing in the light of the ILC with a high center-of-mass energy, high luminosity, and longitudinally polarized beams. As noted above, most of the existing analyses discuss SUSY scenarios with parameters which are ruled out by LEP2 already, or without taking beam polarizations into account. In particular, the question of the role of the positron beam polarization has to be addressed. If both beams are polarized, the discovery potential of the ILC might be significantly extended, especially if other SUSY states like heavier neutralino, chargino or even slepton pairs are too heavy to be produced at the first stage of the ILC at $\sqrt{s} = 500$ GeV. Moreover the SM background photons from radiative neutrino production $e^+e^- \to \nu\bar{\nu}\gamma$ have to be included in an analysis with beam polarizations. Proper beam polarizations could enhance the signal photons and reduce those from the SM background at the same time, which enhances the statistics considerably. In this respect also the MSSM background photons from radiative sneutrino production $e^+e^- \to \tilde{\nu}\tilde{\nu}^*\gamma$ have to be discussed, if sneutrino production is kinematically accessible and if the sneutrino decay is invisible.

Finally the studies which analyze beam polarizations do not give explicit formulas for the squared matrix elements, but only for the transition amplitudes [27,28,30]. Other authors admit sign errors [34] in some interfering amplitudes in precedent works [33], however do not provide the corrected formulas for radiative neutrino and sneutrino production. Additionally, we found inconsistencies and sign errors in the Z exchange terms in some works [27,30], which yield wrong results for scenarios with dominating Z exchange. Thus we will give the complete tree-level amplitudes and the squared matrix elements including longitudinal beam polarizations, such that the formulas can be used for further studies on radiative production of neutralinos, neutrinos and sneutrinos.

In Sec. II, we discuss our signal process, radiative neutralino pair production, as well as the major SM and MSSM background processes. In Sec. III, we define cuts on the photon angle and energy, and define a statistical significance for measuring an excess of photons from radiative neutralino production over the backgrounds. We analyze numerically the dependence of cross sections and significances on the electron and positron beam polarizations, on the parameters of the neutralino sector, and on the selectron masses. We summarize and conclude in Sec. IV. In the Appendix, we define neutralino mixing and couplings, and give the tree-level amplitudes as well as the squared matrix elements with longitudinal beam polarizations for radiative production of neutralinos, neutrinos and sneutrinos. In addition we give details on the parametrization of the phase space.

II. RADIATIVE NEUTRALINO PRODUCTION AND BACKGROUNDS

A. Signal process

Within the MSSM, radiative neutralino production [19–35]

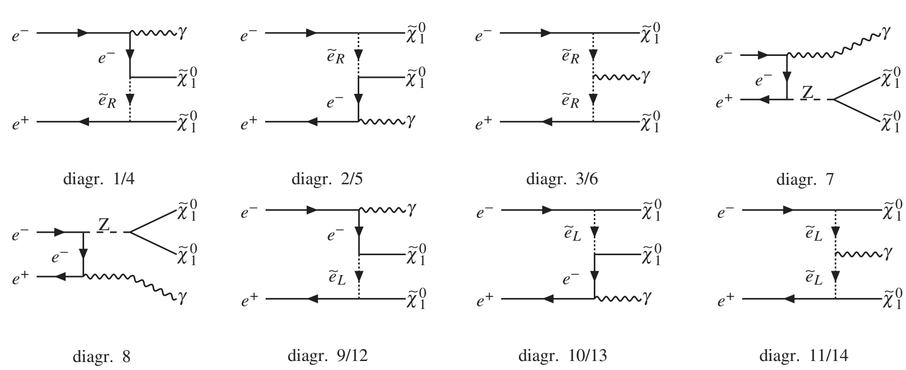

FIG. 1. Diagrams for radiative neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$ [74]. For the calculation in Appendix A, the first number of the diagrams labels t channel, the second one u channel exchange of selectrons, where the neutralinos are crossed.

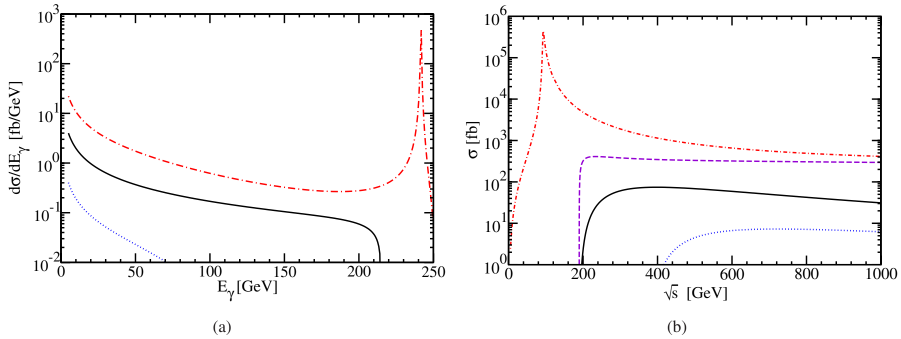

FIG. 2 (color online). (a) Photon energy distributions for $\sqrt{s} = 500$ GeV, and (b) $\sqrt{s}$ dependence of the cross sections σ for radiative neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$ (black, solid), neutrino production $e^+e^- \to \nu\bar{\nu}\gamma$ (violet, dashed) and sneutrino production $e^+e^- \to \tilde{\nu}\tilde{\nu}^*\gamma$ (blue, dotted) for scenario SPS 1a [55,56], see Table I, with $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$. The red dot-dashed line is in (a) the photon energy distribution for radiative neutrino production $e^+e^- \to \nu\bar{\nu}\gamma$, and in (b) the cross section without the upper cut on the photon energy E_γ, see Eq. (3.5).

$$e^+ + e^- \to \tilde{\chi}_1^0 + \tilde{\chi}_1^0 + \gamma \tag{2.1}$$

proceeds at tree level via t- and u-channel exchange of right and left selectrons $\tilde{e}_{R,L}$, as well as Z boson exchange in the s channel. The photon is radiated off the incoming beams or the exchanged selectrons; see the contributing diagrams in Fig. 1. We give the relevant Feynman rules for general neutralino mixing, the tree-level amplitudes, and the complete analytical formulas for the amplitude squared, including longitudinal electron and positron beam polarizations, in Appendix A. We also summarize the details of the neutralino mixing matrix there. For the calculation of cross sections and distributions we use cuts, as defined in Eq. (3.5). An example of the photon energy distribution and the $\sqrt{s}$ dependence of the cross section is shown in Fig. 2.

B. Neutrino background

Radiative neutrino production [33,47,50–52]

$$e^+ + e^- \to \nu_\ell + \bar{\nu}_\ell + \gamma, \qquad \ell = e, \mu, \tau \tag{2.2}$$

is a major SM background. Electron neutrinos ν_e are produced via t-channel W boson exchange, and $\nu_{e,\mu,\tau}$ via s-channel Z boson exchange. We show the corresponding diagrams in Appendix B, where we also give the tree-level amplitudes and matrix elements squared including longitudinal beam polarizations.

C. MSSM backgrounds

Next we consider radiative sneutrino production [33,53,54]

$$e^+ + e^- \to \tilde{\nu}_\ell + \tilde{\nu}_\ell^* + \gamma, \qquad \ell = e, \mu, \tau. \tag{2.3}$$

We present the tree-level Feynman graphs as well as the amplitudes and amplitudes squared, including beam polarizations, in Appendix C. The process has t-channel contributions via virtual charginos for $\tilde{\nu}_e\tilde{\nu}_e^*$ production, as well as s-channel contributions from Z boson exchange for $\tilde{\nu}_{e,\mu,\tau}\tilde{\nu}^*_{e,\mu,\tau}$ production, see Fig. 8. Radiative sneutrino production, Eq. (2.3), can be a major MSSM background to neutralino production, Eq. (2.1), if the sneutrinos decay mainly invisibly, e.g., via $\tilde{\nu} \to \tilde{\chi}_1^0\nu$. This leads to so called "virtual LSP" scenarios [32–34]. However, if kinematically allowed, other visible decay channels like $\tilde{\nu} \to \tilde{\chi}_1^\pm\ell^\mp$ reduce the background rate from radiative sneutrino production. For example in the SPS 1a scenario [55,56] we have $\mathrm{BR}(\tilde{\nu}_e \to \tilde{\chi}_1^0\nu_e) = 85\%$, see Table I.

In principle, also neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ followed by the subsequent radiative neutralino decay [57] $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\gamma$ is a potential background. However, significant branching ratios $\mathrm{BR}(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\gamma) > 10\%$ are only obtained for small values of $\tan\beta < 5$ and/or $M_1 \sim M_2$ [35,58,59]. Thus we neglect this background in the following. For details see Refs. [58–60].

III. NUMERICAL RESULTS

We present numerical results for the tree-level cross section for radiative neutralino production, Eq. (2.1), and the background from radiative neutrino and sneutrino production, Eqs. (2.2) and (2.3), respectively. We define the cuts on the photon energy and angle, and define the statistical significance. We study the dependence of the cross

TABLE I. Parameters and masses for SPS 1a scenario [55,56].

$\tan\beta = 10$	$\mu = 352$ GeV	$M_2 = 193$ GeV	$m_0 = 100$ GeV
$m_{\chi_1^0} = 94$ GeV $m_{\chi_2^0} = 178$ GeV	$m_{\chi_1^\pm} = 178$ GeV $m_{\chi_2^\pm} = 376$ GeV	$m_{\tilde{e}_R} = 143$ GeV $m_{\tilde{e}_L} = 204$ GeV	$m_{\tilde{\nu}_e} = 188$ GeV $\mathrm{BR}(\tilde{\nu}_e \to \tilde{\chi}_1^0\nu_e) = 85\%$

sections and the significance on the beam polarizations P_{e^-} and P_{e^+}, the supersymmetric parameters μ and M_2, and on the selectron masses. In order to reduce the number of parameters, we assume the SUSY GUT relation

$$M_1 = \tfrac{5}{3}\tan^2\theta_w M_2. \tag{3.1}$$

Therefore the mass of the lightest neutralino is $m_{\chi_1^0} \gtrsim 50$ GeV [61]. We also use the approximate renormalization group equations (RGE) for the slepton masses [62–64],

$$m_{\tilde{e}_R}^2 = m_0^2 + 0.23M_2^2 - m_Z^2\cos 2\beta \sin^2\theta_w, \tag{3.2}$$

$$m_{\tilde{e}_L}^2 = m_0^2 + 0.79M_2^2 + m_Z^2\cos 2\beta(-\tfrac{1}{2} + \sin^2\theta_w), \tag{3.3}$$

$$m_{\tilde{\nu}_e}^2 = m_0^2 + 0.79M_2^2 + \tfrac{1}{2}m_Z^2\cos 2\beta, \tag{3.4}$$

with m_0 the common scalar mass parameter. Since in our scenarios the dependence on $\tan\beta$ is rather mild, we fix $\tan\beta = 10$.

A. Cuts on photon angle and energy

To regularize the infrared and collinear divergencies of the tree-level cross sections, we apply cuts on the photon scattering angle θ_γ and on the photon energy E_γ

$$-0.99 \le \cos\theta_\gamma \le 0.99, \qquad 0.02 \le x \le 1 - \frac{m_{\chi_1^0}^2}{E_{\text{beam}}^2}, \qquad x = \frac{E_\gamma}{E_{\text{beam}}}, \tag{3.5}$$

with the beam energy $E_{\text{beam}} = \sqrt{s}/2$. The cut on the scattering angle corresponds to $\theta_\gamma \in [8°, 172°]$, and reduces much of the background from radiative Bhabha scattering, $e^+e^- \to e^+e^-\gamma$, where both leptons escape close to the beam pipe [43,44]. The lower cut on the photon energy is $E_\gamma = 5$ GeV for $\sqrt{s} = 500$ GeV. The upper cut on the photon energy $x^{\max} = 1 - m_{\chi_1^0}^2/E_{\text{beam}}^2$ is the kinematical limit of radiative neutralino production. At $\sqrt{s} = 500$ GeV and for $m_{\chi_1^0} \gtrsim 70$ GeV, this cut reduces much of the on-shell Z boson contribution to radiative neutrino production, see Refs. [29,32,54,65] and Fig. 2(a). We assume that the neutralino mass $m_{\chi_1^0}$ is known from LHC or ILC measurements [7]. If $m_{\chi_1^0}$ is unknown, a fixed cut, e.g., $E_\gamma^{\max} = 175$ GeV at $\sqrt{s} = 500$ GeV, could be used instead [65].

B. Theoretical significance

In order to quantify whether an excess of signal photons from neutralino production, $N_{\text{S}} = \sigma\mathcal{L}$, for a given integrated luminosity $\mathcal{L}$, can be measured over the SM background photons, $N_{\text{B}} = \sigma_{\text{B}}\mathcal{L}$, from radiative neutrino production, we define the theoretical significance

$$S = \frac{N_{\text{S}}}{\sqrt{N_{\text{S}} + N_{\text{B}}}} = \frac{\sigma}{\sqrt{\sigma + \sigma_{\text{B}}}}\sqrt{\mathcal{L}}. \tag{3.6}$$

A theoretical significance of, e.g., $S = 1$ implies that the signal can be measured at the statistical 68% confidence level. Also the signal to background ratio $N_{\text{S}}/N_{\text{B}}$ should be considered to judge the reliability of the analysis. For example, if the background cross section is known experimentally to 1% accuracy, we should have $N_{\text{S}}/N_{\text{B}} > 1/100$.

We will not include additional cuts on the missing mass or on the transverse momentum distributions of the photons [29,65]. Detailed Monte Carlo analyses, including detector simulations and particle identification and reconstruction efficiencies, would be required to predict the significance more accurately, which is however beyond the scope of the present work. Also the effect of beamstrahlung should be included in such an experimental analysis [65–67]. Beamstrahlung distorts the peak of the beam energy spectrum to lower values of $E_{\text{beam}} = \sqrt{s}/2$, and is more significant at colliders with high luminosity. In the processes we consider, the cross sections for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$ and $e^+e^- \to \nu\bar{\nu}\gamma$ depend significantly on the beam energy only near threshold. In most of the parameter space we consider, for $\sqrt{s} = 500$ GeV the cross sections are nearly constant, see, for example, Fig. 2(b), so we expect that the effect of beamstrahlung will be rather small. However, for M_2, $\mu \gtrsim 300$ GeV, $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$ is the only SUSY production, which is kinematically accessible, see Fig. 4. In order to exactly determine the kinematic reach of the ILC beamstrahlung must be taken into account.

C. Energy distribution and $\sqrt{s}$ dependence

In Fig. 2(a) we show the energy distributions of the photon from radiative neutralino production, neutrino production, and sneutrino production for scenario SPS 1a [55,56], see Table I, with $\sqrt{s} = 500$ GeV, beam polarizations $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$, and cuts as defined in Eq. (3.5). The energy distribution of the photon from neutrino production peaks at $E_\gamma = (s - m_Z^2)/(2\sqrt{s}) \approx 242$ GeV due to radiative Z return, which is possible for $\sqrt{s} > m_Z$. Much of this photon background from radiative neutrino production can be reduced by the upper cut on the photon energy $x^{\max} = E_\gamma^{\max}/E_{\text{beam}} = 1 - m_{\chi_1^0}^2/E_{\text{beam}}^2$, see Eq. (3.5), which is the kinematical endpoint $E_\gamma^{\max} \approx 215$ GeV of the energy distribution of the photon from radiative neutralino production, see the solid line in Fig. 2(a). Note that in principle the neutralino mass could be determined by a measurement of this endpoint $E_\gamma^{\max} = E_\gamma^{\max}(m_{\chi_1^0})$

$$m_{\chi_1^0}^2 = \tfrac{1}{4}(s - 2\sqrt{s}E_\gamma^{\max}). \tag{3.7}$$

For this one would need to be able to separate the signal and background processes very well. This might be possible if the neutralino is heavy enough, such that the endpoint is sufficiently removed from the Z^0-peak of the background distribution.

In Fig. 2(b) we show the $\sqrt{s}$ dependence of the cross sections. Without the upper cut on the photon energy $x^{\max}$, see Eq. (3.5), the background cross section from radiative neutrino production $e^+e^- \to \nu\bar{\nu}\gamma$, see the dashed-dotted line in Fig. 2(b), is much larger than the corresponding cross section with the cut, see the dashed line. However with the cut, the signal cross section from radiative neutralino production, see the solid line, is then only about 1 order of magnitude smaller than the background.

D. Beam polarization dependence

In Fig. 3(a) we show the beam polarization dependence of the cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma)$ for the SPS 1a scenario [55,56], where radiative neutralino production proceeds mainly via right selectron $\tilde{e}_R$ exchange. Since the neutralino is mostly b-ino, the coupling to the right selectron is more than twice as large as to the left selectron. Thus the contributions from right selectron exchange to the cross section are about a factor 16 larger than the $\tilde{e}_L$ contributions. In addition the $\tilde{e}_L$ contributions are suppressed compared to the $\tilde{e}_R$ contributions by a factor of about 2 since $m_{\tilde{e}_R} < m_{\tilde{e}_L}$, see Eqs. (3.2) and (3.3). The Z boson exchange is negligible. The background process, radiative neutrino production, mainly proceeds via W boson exchange, see the corresponding diagram in Fig. 7. Thus positive electron beam polarization P_{e^-} and negative positron beam polarization P_{e^+} enhance the signal cross section and reduce the background at the same time, see Figs. 3(a) and 3(c), which was also observed in Refs. [28,65]. The positive electron beam polarization and negative positron beam polarization enhance $\tilde{e}_R$ exchange and suppress $\tilde{e}_L$ exchange, such that it becomes negligible. Opposite polarizations would lead to comparable contributions from both selectrons. In going from unpolarized beams $(P_{e^-}, P_{e^+}) = (0, 0)$ to polarized beams, e.g., $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$, the signal cross section is enhanced by a factor ≈ 3, and the background cross section is reduced by a factor ≈ 10. The signal to background ratio increases from $N_S/N_B \approx 0.007$ to $N_S/N_B \approx 0.2$, such that the statistical significance S, shown in Fig. 3(b), is increased by a factor ≈ 8.5 to $S \approx 77$. If only the electron beam is polarized, $(P_{e^-}, P_{e^+}) = (0.8, 0)$, we still have $N_S/N_B \approx 0.06$ and $S \approx 34$, thus the option of a polarized positron beam at the ILC doubles the significance for radiative neutralino production, but is not needed or essential to observe this process at $\sqrt{s} = 500$ GeV and $\mathcal{L} = 500\ \mathrm{fb}^{-1}$ for the SPS 1a scenario.

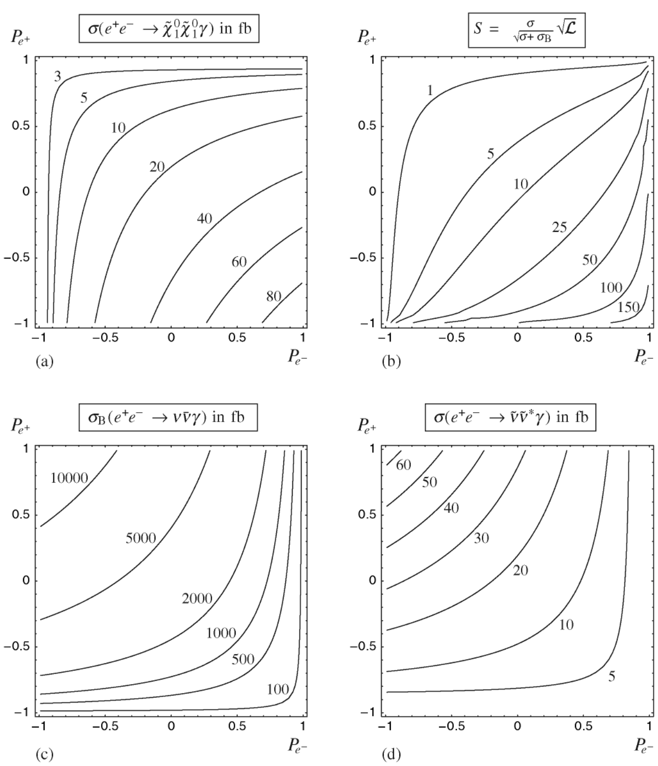

FIG. 3. (a) Contour lines of the cross section and (b) the significance S for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$ at $\sqrt{s} = 500$ GeV and $\mathcal{L} = 500\ \mathrm{fb}^{-1}$ for scenario SPS 1a [55,56], see Table I. The beam polarization dependence of the cross section for radiative neutrino and sneutrino production are shown in (c) and (d), respectively.

The conclusion of Ref. [29] is, however, that an almost pure level of beam polarizations is needed at the ILC to observe this process at all. The authors have used a scenario with $M_1 = M_2$, leading to a lightest neutralino, which is mostly a W-ino. Thus larger couplings to the left selectron than to the right selectron are obtained. In such a scenario, one cannot simultaneously enhance the signal and reduce the background. Moreover their large selectron masses $m_{\tilde{e}_{L,R}} = 500$ GeV lead to an additional suppression of the signal, see also Sec. III F.

Finally we note that positive electron beam polarization and negative positron beam polarization also suppress the cross section of radiative sneutrino production, see Fig. 3(d). Since it is the corresponding SUSY process to radiative neutrino production, we expect such a similar quantitative behavior.

E. μ & M_2 dependence

In Fig. 4(a) we show contour lines of the cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma)$ in fb in the (μ, M_2) plane. For $\mu \gtrsim 300$ GeV the signal and the background cross sections are nearly independent of μ, and consequently also the significance, which is shown in Fig. 4(b). In addition, the dependence of the neutralino mass on μ is fairly weak for $\mu \gtrsim 300$ GeV, as can be seen in Fig. 4(d). Also the couplings have a rather mild μ-dependence in this parameter region.

The cross section $\sigma_B(e^+e^- \to \nu\bar{\nu}\gamma)$ of the SM background process due to radiative neutrino production, shown in Fig. 4(c), can reach more than 340 fb and is considerably reduced due to the upper cut on the photon energy $x^{\max}$, see Eq. (3.5). Without this cut we would have $\sigma_B = 825$ fb.

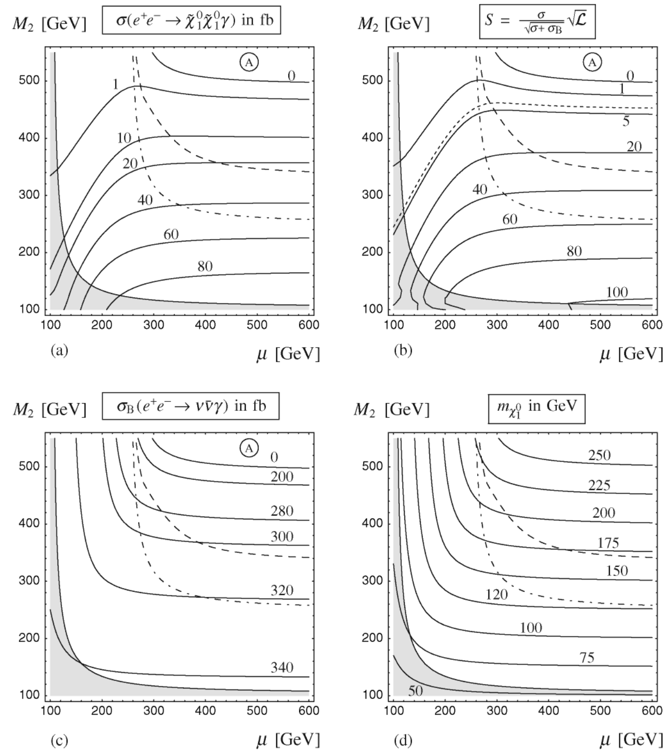

FIG. 4. Contour lines (solid) of (a) the cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma)$, (b) the significance S, (c) the neutrino background $\sigma_B(e^+e^- \to \nu\bar{\nu}\gamma)$, and (d) the neutralino mass $m_{\chi_1^0}$ in the μ-M_2 plane for $\sqrt{s} = 500$ GeV, $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$, $\mathcal{L} = 500\ \text{fb}^{-1}$, with $\tan\beta = 10$, $m_0 = 100$ GeV, and RGEs for the selectron masses, see Eqs. (3.2) and (3.3). The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV. The dashed line indicates the kinematical limit $m_{\chi_1^0} + m_{\chi_2^0} = \sqrt{s}$, and the dashed-dotted line the kinematical limit $2m_{\chi_1^\pm} = \sqrt{s}$. Along the dotted line in (b) the signal to background ratio is $\sigma/\sigma_B = 0.01$. The area A is kinematically forbidden by the cut on the photon energy E_γ, see Eq. (3.5).

Thus the signal can be observed with high statistical significance S, see Fig. 4(b). Because of the large integrated luminosity $\mathcal{L} = 500\ \text{fb}^{-1}$ of the ILC, we have $S \gtrsim 25$ with N_S/N_B up to 1/4 for $M_2 \lesssim 350$ GeV. For $\mu < 0$ we get similar results for the cross sections in shape and size, since the dependence of N_{11} on the sign of μ, see Eq. (A5), is weak due to the large value of $\tan\beta = 10$.

In Fig. 4, we also indicate the kinematical limits of the lightest observable associated neutralino production process, $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ (dashed), and those of the lightest chargino production process, $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ (dashed-dotted). In the region above these lines μ, $M_2 \gtrsim 300$ GeV, heavier neutralinos and charginos are too heavy to be pair-produced at the first stage of the ILC with $\sqrt{s} = 500$ GeV. In this case radiative neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$ will be the only channel to study the gaugino sector. Here significances of $5 < S \lesssim 25$ can be obtained for 350 GeV $\lesssim M_2 \lesssim 450$ GeV, see Fig. 4(b).

Note that the production of right sleptons $e^+e^- \to \tilde{\ell}_R^+\tilde{\ell}_R^-$, $\tilde{\ell} = \tilde{e}, \tilde{\mu}$, and, in particular, the production of the lighter staus $e^+e^- \to \tilde{\tau}_1^+\tilde{\tau}_1^-$, due to mixing in the stau sector [68], are still open channels to study the direct production of SUSY particles for $M_2 \lesssim 500$ GeV in our GUT scenario with $m_0 = 100$ GeV.

F. Dependence on the selectron masses

The cross section for radiative neutralino production $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma)$ depends mainly on selectron $\tilde{e}_{R,L}$ exchange in the t and u channels. Besides the beam polarizations, which enhance $\tilde{e}_R$ or $\tilde{e}_L$ exchange, the cross section is also very sensitive to the selectron masses. In the mSUGRA universal supersymmetry breaking scenario [69], the masses are parametrized by m_0 and M_2, besides $\tan\beta$, which enter the RGEs, see Eqs. (3.2) and (3.3). We show the contour lines of the selectron masses $\tilde{e}_{R,L}$ in the $m_0 - M_2$ plane in Fig. 5(c) and 5(d), respectively. The selectron masses increase with increasing m_0 and M_2.

For the polarizations $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$, the cross section is dominated by $\tilde{e}_R$ exchange, as discussed in Sec. III D. In Fig. 5(a) and 5(b) we show the m_0 and M_2

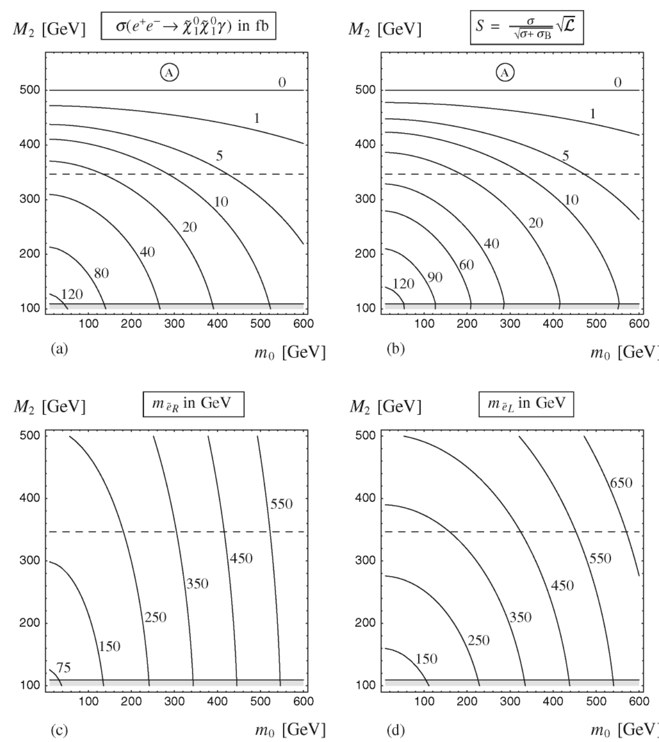

FIG. 5. (a) Contour lines of the cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma)$, (b) the significance S, and (c), (d) the selectron masses $m_{\tilde{e}_R}$, $m_{\tilde{e}_L}$, respectively, in the m_0-M_2 plane for $\sqrt{s} = 500$ GeV, $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$, $\mathcal{L} = 500\ \text{fb}^{-1}$, with $\mu = 500$ GeV, $\tan\beta = 10$, and RGEs for the selectron masses, see Eqs. (3.2) and (3.3). The dashed line indicates the kinematical limit $m_{\chi_1^0} + m_{\chi_2^0} = \sqrt{s}$. The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV, the area A is kinematically forbidden.

dependence of the cross section and the significance S, Eq. (3.6). With increasing m_0 and M_2 the cross section and the significance decrease, due to the increasing mass of $\tilde{e}_R$. In Fig. 4(d) we see that for $\mu \gtrsim 7/10 M_2$, the neutralino mass $m_{\chi_1^0}$ is practically independent of μ and rises with M_2. Thus for increasing M_2, and thereby increasing neutralino mass, the cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma)$ reaches the kinematical limit at $M_2 \approx 500$ GeV for $\sqrt{s} = 500$ GeV. A potential background from radiative sneutrino production is only relevant for $M_2 \lesssim 200$ GeV, $m_0 \lesssim 200$ GeV. For larger values the production is kinematically forbidden.

In Fig. 5 we also indicate the kinematical limit of associated neutralino pair production $m_{\chi_1^0} + m_{\chi_2^0} = \sqrt{s} = 500$ GeV, reached for $M_2 \approx 350$ GeV. If in addition $m_0 > 200$ GeV, also selectron and smuon pairs cannot be produced at $\sqrt{s} = 500$ GeV due to $m_{\tilde{\ell}_R} > 250$ GeV. Thus, in this parameter range $M_2 > 350$ GeV and $m_0 > 200$ GeV, radiative production of neutralinos will be the *only* possible production process of SUSY particles, if we neglect stau mixing. A statistical significance of $S > 1$ can be obtained for selectron masses not larger than $m_{\tilde{e}_R} \approx 500$ GeV, corresponding to $m_0 \lesssim 500$ GeV and $M_2 \lesssim 450$ GeV. Thus radiative neutralino production extends the discovery potential of the ILC in the parameter range $m_0 \in [200, 500]$ GeV and $M_2 \in [350, 450]$ GeV. Here, the beam polarizations will be essential, see Fig. 6. We show contour lines of the statistical significance S for three different sets of (P_{e^-}, P_{e^+}). The first set has both beams polarized, $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$, the second one has only electron beam polarization, $(P_{e^-}, P_{e^+}) = (0.8, 0)$, and the third has zero beam polarizations $(P_{e^-}, P_{e^+}) = (0, 0)$. We observe that beam polarizations significantly enhance the discovery potential of the ILC. At least electron polarization $P_{e^-} = 0.8$ is needed to extend an exploration of the $m_0 - M_2$ parameter space.

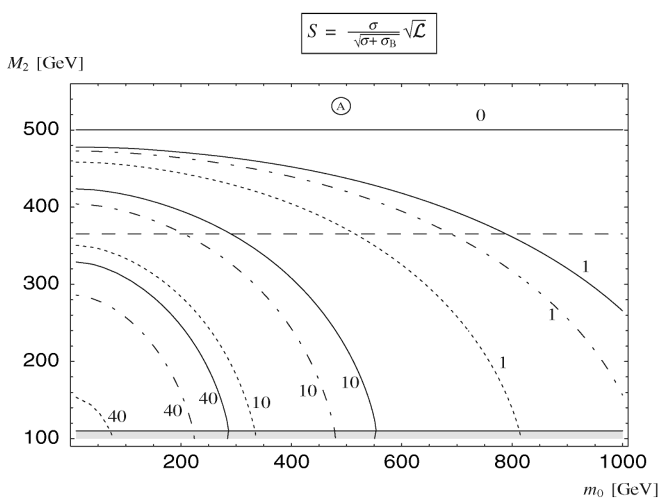

FIG. 6. Contour lines of the significance S in the m_0-M_2 plane for different beam polarizations $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$ (solid), $(P_{e^-}, P_{e^+}) = (0.8, 0)$ (dashed-dotted), and $(P_{e^-}, P_{e^+}) = (0, 0)$ (dotted), for $\sqrt{s} = 500$ GeV, $\mathcal{L} = 500$ fb^{-1}, $\mu = 500$ GeV, $\tan\beta = 10$, and RGEs for the selectron masses, see Eqs. (3.2) and (3.3). The dashed line indicates the kinematical limit $m_{\chi_1^0} + m_{\chi_2^0} = \sqrt{s}$. The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV, the area A is kinematically forbidden.

G. Note on LEP2

We have also calculated the unpolarized cross sections and the significances for radiative neutralino production at LEP2 energies $\sqrt{s} = 200$ GeV, for a luminosity of $\mathcal{L} = 100$ pb^{-1}. We have used the cuts $|\cos\theta_\gamma| \le 0.95$ and $0.2 \le x \le 1 - m_{\chi_1^0}^2/E_{\rm beam}^2$, cf. Eq. (3.5). Even for rather small selectron masses $m_{\tilde{e}_{R,L}} = 80$ GeV, the cross sections are not larger than 100 fb. If we alter the GUT relation, Eq. (3.1), to $M_1 = rM_2$, and vary r, we only obtain statistical significances of $S < 0.2$. These values have also been reported by other theoretical studies at LEP2 energies, see, for example, Ref. [35].

If we drop the GUT relation, M_1 is a free parameter. For

$$M_1 = \frac{M_2 m_Z^2 \sin(2\beta)\sin^2\theta_w}{\mu M_2 - m_Z^2 \sin(2\beta)\cos^2\theta_w} \tag{3.8}$$

the neutralino is massless [70] at tree level and is apparently experimentally allowed [48]. A massless neutralino should enlarge the cross section for radiative neutralino production due to the larger phase space, although the coupling is also modified to almost pure b-ino. However, we still find $S = \mathcal{O}(10^{-1})$ at most. This is in accordance with the experimental SUSY searches in photon events with missing energy at LEP [41,43–46], where no evidence of SUSY particles was found.

IV. SUMMARY AND CONCLUSIONS

We have studied radiative neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$ at the linear collider with polarized beams. We have considered the standard model background process $e^+e^- \to \nu\bar{\nu}\gamma$ and the SUSY background $e^+e^- \to \tilde{\nu}\tilde{\nu}^*\gamma$, which also has the signature of a high energetic photon and missing energy, if the sneutrinos decay invisibly. For these processes we have given the complete tree-level amplitudes and the full squared matrix elements including longitudinal polarizations from the electron and positron beam. In the MSSM, we have studied the dependence of the cross sections on the beam polarizations, on the gaugino and Higgsino mass parameters M_2 and μ, as well as the dependence on the selectron masses. Finally, in order to quantify whether an excess of signal photons, N_S, can be measured over the background photons, N_B, from radiative neutrino production, we have analyzed the theoretical statistical significance $S = N_S/\sqrt{N_S + N_B}$ and the signal to background ratio N_S/N_B. Our results can be summarized as follows.

(i) The cross section for $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$ reaches up to 100 fb in the μ-M_2 and the m_0-M_2 plane at $\sqrt{s} =$ 500 GeV. The significance can be as large as 120, for a luminosity of $\mathcal{L} = 500\ \text{fb}^{-1}$, such that radiative neutralino production should be well accessible at the ILC.

(ii) At the ILC, electron and positron beam polarizations can be used to significantly enhance the signal and suppress the background simultaneously. We have shown that the significance can then be increased almost by an order of magnitude, e.g., with $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$ compared to $(P_{e^-}, P_{e^+}) = (0, 0)$. In the SPS 1a scenario the cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma)$ increases from 25 fb to 70 fb with polarized beams, whereas the background $\sigma(e^+e^- \rightarrow \nu\bar{\nu}\gamma)$ is reduced from 3600 fb to 330 fb. Although a polarized positron beam is not essential to study radiative neutralino production at the ILC, it will help to increase statistics.

(iii) We note that charginos and heavier neutralinos could be too heavy to be pair-produced at the ILC in the first stage at $\sqrt{s} = 500$ GeV. If only slepton pairs are accessible, the radiative production of the lightest neutralino might be the only SUSY process to study the neutralino sector. Even in the regions of the parameter space near the kinematical limits of $\tilde{\chi}_1^0 - \tilde{\chi}_2^0$ pair production we find a cross section of about 20 fb and corresponding significances up to 20.

(iv) Finally we want to remark that our given values for the statistical significance S can only be seen as rough estimates, since we do not include a detector simulation. However, since we have obtained large values up to $S \approx 120$, we hope that our results encourage further experimental studies, including detailed Monte Carlo simulations.

ACKNOWLEDGMENTS

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APPENDIX A: RADIATIVE NEUTRALINO PRODUCTION

1. Neutralino mixing matrix

In the b-ino, W-ino, Higgsino basis $(\tilde{B}, \tilde{W}_3^0, \tilde{H}_u, \tilde{H}_d)$, the neutralino mass matrix is given by [2,71]

$$M = \begin{pmatrix} M_1 & 0 & -m_Z\sin\theta_w\cos\beta & m_Z\sin\theta_w\sin\beta \\ 0 & M_2 & m_Z\cos\theta_w\cos\beta & -m_Z\cos\theta_w\sin\beta \\ -m_Z\sin\theta_w\cos\beta & m_Z\cos\theta_w\cos\beta & 0 & -\mu \\ m_Z\sin\theta_w\sin\beta & -m_Z\cos\theta_w\sin\beta & -\mu & 0 \end{pmatrix}, \tag{A1}$$

with M_1 and M_2 the $U(1)_Y$ and the $SU(2)_w$ gaugino mass parameters, respectively, μ the Higgsino mass parameter, $\tan\beta = \frac{v_2}{v_1}$ the ratio of the two vacuum expectation values of the Higgs fields, m_Z the Z boson mass, and $\tan\theta_w$ the weak mixing angle. In the CP conserving case, M is real symmetric and can be diagonalized by an orthogonal matrix. Since at least one eigenvalue of M is negative, we can use a unitary matrix N to get a positive semidefinite diagonal matrix with the neutralino masses $m_{\chi_i^0}$ [1]

$$\text{diag}\left(m_{\chi_1^0}, \quad m_{\chi_2^0}, \quad m_{\chi_3^0}, \quad m_{\chi_4^0} \right) = N^* M N^{-1}. \tag{A2}$$

Note that the transformation Eq. (A2) is only a similarity transformation if N is real.

2. Lagrangian and couplings

For radiative neutralino production

$$e^-(p_1) + e^+(p_2) \rightarrow \tilde{\chi}_1^0(k_1) + \tilde{\chi}_1^0(k_2) + \gamma(q), \tag{A3}$$

the SUSY Lagrangian is given by [1]

$$\mathcal{L} = \sqrt{2}ea\bar{f}_e P_L \tilde{\chi}_1^0 \tilde{e}_R + \sqrt{2}eb\bar{f}_e P_R \tilde{\chi}_1^0 \tilde{e}_L + \tfrac{1}{2}eZ_\mu \bar{\tilde{\chi}}_1^0 \gamma^\mu [gP_L + fP_R]\tilde{\chi}_1^0 + \text{H.c.}, \tag{A4}$$

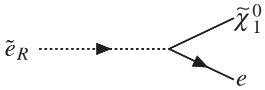
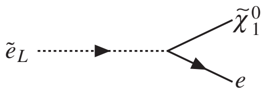
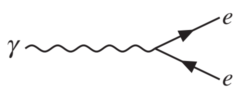
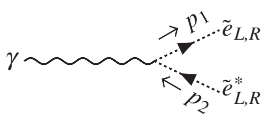
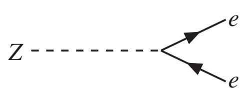
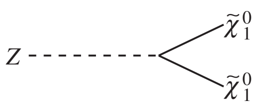
with the electron, selectron, neutralino, and Z boson fields f_e, $\tilde{e}_{L,R}$, $\tilde{\chi}_1^0$, and Z_μ, respectively, and $P_{L,R} = (1 \mp \gamma^5)/2$. The couplings are

$$a = -\frac{1}{\cos\theta_w} N_{11}^*, \qquad b = \frac{1}{2\sin\theta_w}(N_{12} + \tan\theta_w N_{11}),$$
$$g = -\frac{1}{2\sin\theta_w\cos\theta_w}(|N_{13}|^2 - |N_{14}|^2), \qquad f = -g, \tag{A5}$$

see the Feynman rules in Table II. The $Z - e - e$ couplings are

$$c = \frac{1}{\sin\theta_w\cos\theta_w}\left(\frac{1}{2} - \sin^2\theta_w\right), \qquad d = -\tan\theta_w. \tag{A6}$$

TABLE II. Vertex factors with parameters a, b, c, d, f, and g defined in Eqs. (A5) and (A6), with $e > 0$.

Vertex	Factor
	$ie\sqrt{2}aP_L$
	$ie\sqrt{2}bP_R$
	$ie\gamma^\mu$
	$ie(p_1 + p_2)^\mu$
	$ie\gamma^\mu(cP_L + dP_R)$
	$\frac{ie}{2}\gamma^\mu(gP_L + fP_R)$

3. Amplitudes for radiative neutralino production

We define the selectron and Z boson propagators as

$$\Delta_{\tilde{e}_{L,R}}(p_i, k_j) \equiv \frac{1}{m^2_{\tilde{e}_{L,R}} - m^2_{\chi_1^0} + 2p_i \cdot k_j}, \tag{A7}$$

$$\Delta_Z(k_1, k_2) \equiv \frac{1}{m_Z^2 - 2m^2_{\chi_1^0} - 2k_1 \cdot k_2 - i\Gamma_Z m_Z}. \tag{A8}$$

The tree-level amplitudes for right selectron exchange in the t channel, see the diagrams 1–3 in Fig. 1, are

$$\mathcal{M}_1 = 2ie^3|a|^2\left[\bar{u}(k_1)P_R\frac{(\not{p}_1 - \not{q})}{2p_1 \cdot q}\not{\epsilon}^* u(p_1)\right] \times [\bar{v}(p_2)P_L v(k_2)]\Delta_{\tilde{e}_R}(p_2, k_2), \tag{A9}$$

$$\mathcal{M}_2 = 2ie^3|a|^2[\bar{u}(k_1)P_R u(p_1)] \times \left[\bar{v}(p_2)\not{\epsilon}^*\frac{(\not{q} - \not{p}_2)}{2p_2 \cdot q}P_L v(k_2)\right]\Delta_{\tilde{e}_R}(p_1, k_1), \tag{A10}$$

$$\mathcal{M}_3 = 2ie^3|a|^2[\bar{u}(k_1)P_R u(p_1)][\bar{v}(p_2)P_L v(k_2)] \times (2p_1 - 2k_1 - q) \cdot \epsilon^* \Delta_{\tilde{e}_R}(p_1, k_1)\Delta_{\tilde{e}_R}(p_2, k_2). \tag{A11}$$

The amplitudes for u-channel $\tilde{e}_R$ exchange, see the diagrams 4–6 in Fig. 1, are

$$\mathcal{M}_4 = -2ie^3|a|^2\left[\bar{u}(k_2)P_R\frac{(\not{p}_1 - \not{q})}{2p_1 \cdot q}\not{\epsilon}^* u(p_1)\right] \times [\bar{v}(p_2)P_L v(k_1)]\Delta_{\tilde{e}_R}(p_2, k_1), \tag{A12}$$

$$\mathcal{M}_5 = -2ie^3|a|^2[\bar{u}(k_2)P_R u(p_1)] \times \left[\bar{v}(p_2)\not{\epsilon}^*\frac{(\not{q} - \not{p}_2)}{2p_2 \cdot q}P_L v(k_1)\right]\Delta_{\tilde{e}_R}(p_1, k_2), \tag{A13}$$

$$\mathcal{M}_6 = -2ie^3|a|^2[\bar{u}(k_2)P_R u(p_1)][\bar{v}(p_2)P_L v(k_1)] \times (2p_1 - 2k_2 - q) \cdot \epsilon^* \Delta_{\tilde{e}_R}(p_1, k_2)\Delta_{\tilde{e}_R}(p_2, k_1). \tag{A14}$$

In the photino limit, our amplitudes $\mathcal{M}_1 - \mathcal{M}_6$, Eqs. (A9)–(A14), agree with those given in Ref. [21].

The amplitudes for Z boson exchange, see the diagrams 7 and 8 in Fig. 1, are

$$\mathcal{M}_7 = ie^3\left[\bar{v}(p_2)\gamma^\mu(cP_L + dP_R)\frac{(\not{p}_1 - \not{q})}{2p_1 \cdot q}\not{\epsilon}^* u(p_1)\right] \times [\bar{u}(k_1)\gamma_\mu(gP_L + fP_R)v(k_2)]\Delta_Z(k_1, k_2), \tag{A15}$$

$$\mathcal{M}_8 = ie^3\left[\bar{v}(p_2)\not{\epsilon}^*\frac{(\not{q} - \not{p}_2)}{2p_2 \cdot q}\gamma^\mu(cP_L + dP_R)u(p_1)\right] \times [\bar{u}(k_1)\gamma_\mu(gP_L + fP_R)v(k_2)]\Delta_Z(k_1, k_2). \tag{A16}$$

Note that additional sign factors appear in the amplitudes $\mathcal{M}_4 - \mathcal{M}_6$ and $\mathcal{M}_7 - \mathcal{M}_8$, compared to $\mathcal{M}_1 - \mathcal{M}_3$. They stem from the reordering of fermionic operators in the Wick expansion of the S matrix. For radiative neutralino production $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$, such sign factors appear since the two external neutralinos are fermions.[2] For details see Refs. [10,31]. In addition an extra factor 2 is obtained in the Wick expansion of the S matrix, since the Majorana field $\tilde{\chi}_1^0$ contains both creation and annihilation operators. In our conventions we follow here closely Ref. [10]. Other authors take care of this factor by multiplying the $Z\tilde{\chi}_1^0\tilde{\chi}_1^0$ vertex by a factor 2 already [1]. For more details of this subtlety, see Ref. [1].

The amplitudes $\mathcal{M}_9 - \mathcal{M}_{14}$ for left selectron exchange, see the diagrams 9–14 in Fig. 1, are obtained from the $\tilde{e}_R$ amplitudes by substituting

$$a \to b, \qquad P_L \to P_R, \qquad P_R \to P_L, \qquad \Delta_{\tilde{e}_R} \to \Delta_{\tilde{e}_L}. \tag{A17}$$

[2]Note that in Ref. [27] the relative sign in the amplitudes for Z boson and t-channel $\tilde{e}_R$ exchange is missing.

For $\tilde{e}_L$ exchange in the t channel they read

$$\mathcal{M}_9 = 2ie^3|b|^2\left[\bar{u}(k_1)P_L\frac{(\not{p}_1-\not{q})}{2p_1\cdot q}\not{\epsilon}^* u(p_1)\right]\times[\bar{v}(p_2)P_R v(k_2)]\Delta_{\tilde{e}_L}(p_2,k_2), \quad \text{(A18)}$$

$$\mathcal{M}_{10} = 2ie^3|b|^2[\bar{u}(k_1)P_L u(p_1)]\times\left[\bar{v}(p_2)\not{\epsilon}^*\frac{(\not{q}-\not{p}_2)}{2p_2\cdot q}P_R v(k_2)\right]\Delta_{\tilde{e}_L}(p_1,k_1), \quad \text{(A19)}$$

$$\mathcal{M}_{11} = 2ie^3|b|^2[\bar{u}(k_1)P_L u(p_1)][\bar{v}(p_2)P_R v(k_2)]\times(2p_1-2k_1-q)\cdot\epsilon^*\Delta_{\tilde{e}_L}(p_1,k_1)\Delta_{\tilde{e}_L}(p_2,k_2). \quad \text{(A20)}$$

The u-channel $\tilde{e}_L$ exchange amplitudes are

$$\mathcal{M}_{12} = -2ie^3|b|^2\left[\bar{u}(k_2)P_L\frac{(\not{p}_1-\not{q})}{2p_1\cdot q}\not{\epsilon}^* u(p_1)\right]\times[\bar{v}(p_2)P_R v(k_1)]\Delta_{\tilde{e}_L}(p_2,k_1), \quad \text{(A21)}$$

$$\mathcal{M}_{13} = -2ie^3|b|^2[\bar{u}(k_2)P_L u(p_1)]\times\left[\bar{v}(p_2)\not{\epsilon}^*\frac{(\not{q}-\not{p}_2)}{2p_2\cdot q}P_R v(k_1)\right]\Delta_{\tilde{e}_L}(p_1,k_2), \quad \text{(A22)}$$

$$\mathcal{M}_{14} = -2ie^3|b|^2[\bar{u}(k_2)P_L u(p_1)][\bar{v}(p_2)P_R v(k_1)]\times(2p_1-2k_2-q)\cdot\epsilon^*\Delta_{\tilde{e}_L}(p_1,k_1)\Delta_{\tilde{e}_L}(p_2,k_2). \quad \text{(A23)}$$

Our amplitudes $\mathcal{M}_1 - \mathcal{M}_{14}$ agree with those given in Ref. [30,31], however there is an obvious misprint in amplitude T_5 of Ref. [30]. In addition we have checked that the amplitudes $\mathcal{M}_i = \epsilon_\mu \mathcal{M}_i^\mu$ for $i = 1, \ldots, 14$ fulfill the Ward identity $q_\mu(\sum_i \mathcal{M}_i^\mu) = 0$, as done in Refs. [27,30]. We find $q_\mu(\mathcal{M}_1^\mu + \mathcal{M}_2^\mu + \mathcal{M}_3^\mu) = 0$ for t-channel $\tilde{e}_R$ exchange, $q_\mu(\mathcal{M}_4^\mu + \mathcal{M}_5^\mu + \mathcal{M}_6^\mu) = 0$ for u-channel $\tilde{e}_R$ exchange, $q_\mu(\mathcal{M}_7^\mu + \mathcal{M}_8^\mu) = 0$ for Z boson exchange, and analog relations for the $\tilde{e}_L$ exchange amplitudes.

4. Spin formalism and squared matrix elements

We include the longitudinal beam polarizations of electron, P_{e^-}, and positron, P_{e^+}, with $-1 \le P_{e^\pm} \le +1$ in their density matrices

$$\rho^-_{\lambda_-\lambda'_-} = \tfrac{1}{2}(\delta_{\lambda_-\lambda'_-} + P_{e^-}\sigma^3_{\lambda_-\lambda'_-}), \quad \text{(A24)}$$

$$\rho^+_{\lambda_+\lambda'_+} = \tfrac{1}{2}(\delta_{\lambda_+\lambda'_+} + P_{e^+}\sigma^3_{\lambda_+\lambda'_+}), \quad \text{(A25)}$$

where σ^3 is the third Pauli matrix and λ_-, λ'_-, and λ_+, λ'_+ are the helicity indices of electron and positron, respectively. The squared matrix elements are then obtained by

$$T_{ii} = \sum_{\lambda_-,\lambda'_-,\lambda_+,\lambda'_+}\rho^-_{\lambda_-\lambda'_-}\rho^+_{\lambda_+\lambda'_+}\mathcal{M}_i^{\lambda_-\lambda_+}\mathcal{M}_i^{*\lambda'_-\lambda'_+}, \quad \text{(A26)}$$

$$T_{ij} = 2\Re\mathrm{e}\left(\sum_{\lambda_-,\lambda'_-,\lambda_+,\lambda'_+}\rho^-_{\lambda_-\lambda'_-}\rho^+_{\lambda_+\lambda'_+}\mathcal{M}_i^{\lambda_-\lambda_+}\mathcal{M}_j^{*\lambda'_-\lambda'_+}\right), \quad i \neq j, \quad \text{(A27)}$$

$$|\mathcal{M}|^2 = \sum_{i\le j} T_{ij}, \quad \text{(A28)}$$

where an internal sum over the helicities of the outgoing neutralinos, as well as a sum over the polarizations of the photon is included. Note that we suppress the electron and positron helicity indices of the amplitudes $\mathcal{M}_i^{\lambda_-\lambda_+}$ in the formulas (A9)–(A23). The product of the amplitudes in Eqs. (A26) and (A27) contains the projectors

$$u(p,\lambda_-)\bar{u}(p,\lambda'_-) = \tfrac{1}{2}(\delta_{\lambda_-\lambda'_-} + \gamma^5\sigma^3_{\lambda_-\lambda'_-})\not{p}, \quad \text{(A29)}$$

$$v(p,\lambda'_+)\bar{v}(p,\lambda_+) = \tfrac{1}{2}(\delta_{\lambda_+\lambda'_+} + \gamma^5\sigma^3_{\lambda_+\lambda'_+})\not{p}. \quad \text{(A30)}$$

The contraction with the density matrices of the electron and positron beams leads finally to

$$\sum_{\lambda_-,\lambda'_-}\rho^-_{\lambda_-\lambda'_-}u(p,\lambda_-)\bar{u}(p,\lambda'_-) = \left(\frac{1-P_{e^-}}{2}P_L + \frac{1+P_{e^-}}{2}P_R\right)\not{p}, \quad \text{(A31)}$$

$$\sum_{\lambda_+,\lambda'_+}\rho^+_{\lambda_+\lambda'_+}v(p,\lambda'_+)\bar{v}(p,\lambda_+) = \left(\frac{1+P_{e^+}}{2}P_L + \frac{1-P_{e^+}}{2}P_R\right)\not{p}. \quad \text{(A32)}$$

In the squared amplitudes, we include the electron and positron beam polarizations in the coefficients

$$C_R = (1+P_{e^-})(1-P_{e^+}), \qquad C_L = (1-P_{e^-})(1+P_{e^+}). \quad \text{(A33)}$$

In the following we give the squared amplitudes T_{ij}, as defined in Eqs. (A26) and (A27), for $\tilde{e}_R$ and Z exchange. To obtain the corresponding squared amplitudes for $\tilde{e}_L$ exchange, one has to substitute

$$a \to b, \quad d \leftrightarrow c, \quad f \leftrightarrow g, \qquad C_R \to C_L, \quad \Delta_{\tilde{e}_R} \to \Delta_{\tilde{e}_L}. \quad \text{(A34)}$$

There is no interference between diagrams with $\tilde{e}_R$ and $\tilde{e}_L$ exchange, since we assume the high-energy limit where ingoing particles are considered massless.

$$T_{11} = 4e^6 C_R |a|^4 \Delta^2_{\tilde{e}_R}(p_2, k_2) \frac{p_2 \cdot k_2 q \cdot k_1}{q \cdot p_1}, \tag{A35}$$

$$T_{22} = 4e^6 C_R |a|^4 \Delta^2_{\tilde{e}_R}(p_1, k_1) \frac{p_1 \cdot k_1 q \cdot k_2}{q \cdot p_2}, \tag{A36}$$

$$T_{33} = 4e^6 C_R |a|^4 \Delta^2_{\tilde{e}_R}(p_1, k_1) \Delta^2_{\tilde{e}_R}(p_2, k_2) p_1 \cdot k_1 p_2 \cdot k_2 \times [-4m^2_{\chi^0_1} + 8p_1 \cdot k_1 + 4q \cdot p_1 - 4q \cdot k_1], \tag{A37}$$

$$T_{44} = 4e^6 C_R |a|^4 \Delta^2_{\tilde{e}_R}(p_2, k_1) \frac{p_2 \cdot k_1 q \cdot k_2}{q \cdot p_1}, \tag{A38}$$

$$T_{55} = 4e^6 C_R |a|^4 \Delta^2_{\tilde{e}_R}(p_1, k_2) \frac{p_1 \cdot k_2 q \cdot k_1}{q \cdot p_2}, \tag{A39}$$

$$T_{66} = 4e^6 C_R |a|^4 \Delta^2_{\tilde{e}_R}(p_1, k_2) \Delta^2_{\tilde{e}_R}(p_2, k_1) p_1 \cdot k_2 p_2 \cdot k_1 \times [-4m^2_{\chi^0_1} + 8p_1 \cdot k_2 + 4q \cdot p_1 - 4q \cdot k_2], \tag{A40}$$

$$T_{77} = \frac{4e^6}{q \cdot p_1} |\Delta_Z(k_1, k_2)|^2 [(C_R d^2 f^2 + C_L c^2 g^2) \times p_2 \cdot k_1 q \cdot k_2 + (C_R d^2 g^2 + C_L c^2 f^2) p_2 \cdot k_2 q \cdot k_1 + fg(C_L c^2 + C_R d^2) m^2_{\chi^0_1} q \cdot p_2], \tag{A41}$$

$$T_{88} = \frac{4e^6}{q \cdot p_2} |\Delta_Z(k_1, k_2)|^2 [(C_R d^2 f^2 + C_L c^2 g^2) p_1 \cdot k_2 q \cdot k_1 + (C_R d^2 g^2 + C_L c^2 f^2) p_1 \cdot k_1 q \cdot k_2 + fg(C_L c^2 + C_R d^2) m^2_{\chi^0_1} q \cdot p_1], \tag{A42}$$

$$T_{12} = -4e^6 C_R |a|^4 \Delta_{\tilde{e}R}(p_1, k_1) \Delta_{\tilde{e}R}(p_2, k_2) \frac{1}{q \cdot p_1 q \cdot p_2} \times [q \cdot k_2 p_1 \cdot k_1 p_1 \cdot p_2 - p_1 \cdot k_1 q \cdot p_2 p_1 \cdot k_2 + p_1 \cdot k_1 p_2 \cdot k_2 q \cdot p_1 + p_1 \cdot p_2 q \cdot k_1 p_2 \cdot k_2 - q \cdot p_1 p_2 \cdot k_2 p_2 \cdot k_1 + p_1 \cdot k_1 p_2 \cdot k_2 q \cdot p_2 - 2p_1 \cdot p_2 p_1 \cdot k_1 p_2 \cdot k_2], \tag{A43}$$

$$T_{13} = 8e^6 C_R |a|^4 \Delta_{\tilde{e}_R}(p_1, k_1) \Delta^2_{\tilde{e}_R}(p_2, k_2) \frac{p_2 \cdot k_2}{q \cdot p_1} \times [m^2_{\chi^0_1} q \cdot p_1 + 2(p_1 \cdot k_1)^2 + p_1 \cdot k_1 q \cdot p_1 - 2p_1 \cdot k_1 q \cdot k_1], \tag{A44}$$

$$T_{14} = -4e^6 C_R |a|^4 \Delta_{\tilde{e}_R}(p_2, k_1) \Delta_{\tilde{e}_R}(p_2, k_2) \frac{m^2_{\chi^0_1} q \cdot p_2}{q \cdot p_1}, \tag{A45}$$

$$T_{15} = 4e^6 C_R |a|^4 \Delta_{\tilde{e}_R}(p_1, k_2) \Delta_{\tilde{e}_R}(p_2, k_2) \frac{m^2_{\chi^0_1} p_1 \cdot p_2}{q \cdot p_1 q \cdot p_2} \times [q \cdot p_1 - p_1 \cdot p_2 + q \cdot p_2], \tag{A46}$$

$$T_{16} = 4e^6 C_R |a|^4 \Delta_{\tilde{e}_R}(p_1, k_2) \Delta_{\tilde{e}_R}(p_2, k_1) \Delta_{\tilde{e}_R}(p_2, k_2) \frac{m^2_{\chi^0_1}}{q \cdot p_1} \times [-2p_1 \cdot k_2 p_1 \cdot p_2 - q \cdot p_1 p_1 \cdot p_2 + q \cdot k_2 p_1 \cdot p_2 - q \cdot p_1 p_2 \cdot k_2 + q \cdot p_2 p_1 \cdot k_2], \tag{A47}$$

$$T_{17} = 4e^6 |a|^2 C_R d \frac{1}{q \cdot p_1} \Delta_{\tilde{e}_R}(p_2, k_2) \Re\mathrm{e}\{\Delta_Z(k_1, k_2)\} \times [2g p_2 \cdot k_2 q \cdot k_1 + f m^2_{\chi^0_1} q \cdot p_2], \tag{A48}$$

$$T_{18} = -4e^6 C_R |a|^2 d \frac{1}{q \cdot p_1 q \cdot p_2} \Delta_{\tilde{e}_R}(p_2, k_2) \times \Re\mathrm{e}\{\Delta_Z(k_1, k_2)\} [g(-2p_1 \cdot p_2 p_2 \cdot k_2 p_1 \cdot k_1 + p_2 \cdot k_2 (q \cdot k_1 p_1 \cdot p_2 - p_2 \cdot k_1 q \cdot p_1 + p_1 \cdot k_1 q \cdot p_2) + p_1 \cdot k_1 (q \cdot p_1 p_2 \cdot k_2 + q \cdot k_2 p_1 \cdot p_2 - q \cdot p_2 p_1 \cdot k_2)) - f m^2_{\chi^0_1} p_1 \cdot p_2 (p_1 \cdot p_2 - q \cdot p_2 - q \cdot p_1)], \tag{A49}$$

$$T_{23} = 8e^6 C_R |a|^4 \frac{p_1 \cdot k_1}{q \cdot p_2} \Delta^2_{\tilde{e}_R}(p_1, k_1) \Delta_{\tilde{e}_R}(p_2, k_2) \times [m^2_{\chi^0_1} q \cdot p_2 + 2(p_2 \cdot k_2)^2 + p_2 \cdot k_2 q \cdot p_2 - 2p_2 \cdot k_2 q \cdot k_2], \tag{A50}$$

$$T_{24} = 4e^6 C_R |a|^4 \Delta_{\tilde{e}_R}(p_1, k_1) \Delta_{\tilde{e}_R}(p_2, k_1) \frac{m^2_{\chi^0_1} p_1 \cdot p_2}{q \cdot p_1 q \cdot p_2} \times (q \cdot p_1 - p_1 \cdot p_2 + q \cdot p_2), \tag{A51}$$

$$T_{25} = 4e^6 C_R |a|^4 \Delta_{\tilde{e}_R}(p_1, k_1) \Delta_{\tilde{e}_R}(p_1, k_2) \frac{m^2_{\chi^0_1} q \cdot p_1}{q \cdot p_2}, \tag{A52}$$

$$T_{26} = 4e^6 C_R |a|^4 \Delta_{\tilde{e}_R}(p_1, k_2) \Delta_{\tilde{e}_R}(p_2, k_1) \Delta_{\tilde{e}_R}(p_1, k_1) \frac{m^2_{\chi^0_1}}{q \cdot p_2} \times [-2p_2 \cdot k_1 p_1 \cdot p_2 - q \cdot p_2 p_1 \cdot p_2 + q \cdot k_1 p_1 \cdot p_2 - q \cdot p_2 p_1 \cdot k_1 + q \cdot p_1 p_2 \cdot k_1], \tag{A53}$$

$$T_{27} = \frac{4e^6 C_R |a|^2 d}{q\cdot p_1 q\cdot p_2} \Delta_{\tilde{e}_R}(p_1,k_1)\Re\{\Delta_Z(k_1,k_2)\} \times [g(2p_1\cdot p_2 p_2\cdot k_2 p_1\cdot k_1 + p_2\cdot k_2(-q\cdot k_1 p_1\cdot p_2 + p_2\cdot k_1 q\cdot p_1 - p_1\cdot k_1 q\cdot p_2) + p_1\cdot k_1(-q\cdot p_1 p_2\cdot k_2 - q\cdot k_2 p_1\cdot p_2 + q\cdot p_2 p_1\cdot k_2)) + f m^2_{\chi^0_1} p_1\cdot p_2(p_1\cdot p_2 - q\cdot p_2 - q\cdot p_1)], \quad \text{(A54)}$$

$$T_{28} = \frac{4e^6 C_R |a|^2 d}{q\cdot p_2} \Delta_{\tilde{e}_R}(p_1,k_1)\Re\{\Delta_Z(k_1,k_2)\} \times [2g p_1\cdot k_1 q\cdot k_2 + f m^2_{\chi^0_1} q\cdot p_1], \quad \text{(A55)}$$

$$T_{34} = -4e^6 C_R |a|^4 \frac{m^2_{\chi^0_1}}{q\cdot p_1} \Delta_{\tilde{e}_R}(p_1,k_1)\Delta_{\tilde{e}_R}(p_2,k_1)\Delta_{\tilde{e}_R}(p_2,k_2) \times [2p_1\cdot p_2 p_1\cdot k_1 + p_1\cdot p_2 q\cdot p_1 - p_1\cdot k_1 q\cdot p_2 + p_2\cdot k_1 q\cdot p_1 - p_1\cdot p_2 q\cdot k_1], \quad \text{(A56)}$$

$$T_{35} = -4e^6 C_R |a|^4 \frac{m^2_{\chi^0_1}}{q\cdot p_2} \Delta_{\tilde{e}_R}(p_1,k_1)\Delta_{\tilde{e}_R}(p_1,k_2)\Delta_{\tilde{e}_R}(p_2,k_2) \times [2p_1\cdot p_2 p_2\cdot k_2 - p_1\cdot p_2 q\cdot k_2 + p_1\cdot p_2 q\cdot p_2 - p_2\cdot k_2 q\cdot p_1 + p_1\cdot k_2 q\cdot p_2], \quad \text{(A57)}$$

$$T_{36} = 8e^6 C_R |a|^4 \Delta_{\tilde{e}_R}(p_1,k_1)\Delta_{\tilde{e}_R}(p_1,k_2)\Delta_{\tilde{e}_R}(p_2,k_1) \times \Delta_{\tilde{e}_R}(p_2,k_2) m^2_{\chi^0_1} p_1\cdot p_2[-2p_1\cdot k_1 - 2q\cdot p_1 - 2p_1\cdot k_2 + 2k_1\cdot k_2 + q\cdot k_2 + q\cdot k_1], \quad \text{(A58)}$$

$$T_{37} = \frac{4e^6 C_R |a|^2 d}{q\cdot p_1} \Delta_{\tilde{e}_R}(p_1,k_1)\Delta_{\tilde{e}_R}(p_2,k_2)\Re\{\Delta_Z(k_1,k_2)\} \times [2g p_2\cdot k_2(q\cdot p_1 p_1\cdot k_1 - 2p_1\cdot k_1 q\cdot k_1 + 2(p_1\cdot k_1)^2 + m^2_{\chi^0_1} q\cdot p_1) + f m^2_{\chi^0_1}(2p_1\cdot p_2 p_1\cdot k_1 + p_1\cdot p_2 q\cdot p_1 - p_1\cdot p_2 q\cdot k_1 - p_1\cdot k_1 q\cdot p_2 + q\cdot p_1 p_2\cdot k_1)], \quad \text{(A59)}$$

$$T_{38} = \frac{4e^6 C_R |a|^2 d}{q\cdot p_2} \Delta_{\tilde{e}_R}(p_1,k_1)\Delta_{\tilde{e}_R}(p_2,k_2)\Re\{\Delta_Z(k_1,k_2)\} \times [2g p_1\cdot k_1(2(p_2\cdot k_2)^2 + p_2\cdot k_2 q\cdot p_2 - 2p_2\cdot k_2 q\cdot k_2 + m^2_{\chi^0_1} q\cdot p_2) + f m^2_{\chi^0_1}(2p_1\cdot p_2 p_2\cdot k_2 + p_1\cdot p_2 q\cdot p_2 - p_1\cdot p_2 q\cdot k_2 + q\cdot p_2 p_1\cdot k_2 - q\cdot p_1 p_2\cdot k_2)], \quad \text{(A60)}$$

$$T_{45} = -\frac{4e^6 C_R |a|^4}{q\cdot p_1 q\cdot p_2} \Delta_{\tilde{e}_R}(p_1,k_2)\Delta_{\tilde{e}_R}(p_2,k_1) \times [q\cdot k_1 p_1\cdot k_2 p_1\cdot p_2 - p_1\cdot k_2 q\cdot p_2 p_1\cdot k_1 + p_1\cdot k_2 p_2\cdot k_1 q\cdot p_1 + p_1\cdot p_2 q\cdot k_2 p_2\cdot k_1 - q\cdot p_1 p_2\cdot k_1 p_2\cdot k_2 + p_1\cdot k_2 p_2\cdot k_1 q\cdot p_2 - 2p_1\cdot p_2 p_1\cdot k_2 p_2\cdot k_1], \quad \text{(A61)}$$

$$T_{46} = 8e^6 C_R |a|^4 \frac{p_2\cdot k_1}{q\cdot p_1} \Delta_{\tilde{e}_R}(p_1,k_2)\Delta^2_{\tilde{e}_R}(p_2,k_1) \times [m^2_{\chi^0_1} q\cdot p_1 + 2(p_1\cdot k_2)^2 + p_1\cdot k_2 q\cdot p_1 - 2p_1\cdot k_2 q\cdot k_2], \quad \text{(A62)}$$

$$T_{47} = -\frac{4e^6 C_R |a|^2 d}{q\cdot p_1} \Delta_{\tilde{e}_R}(p_2,k_1)\Re\{\Delta_Z(k_1,k_2)\} \times [2g p_2\cdot k_1 q\cdot k_2 + f m^2_{\chi^0_1} q\cdot p_2], \quad \text{(A63)}$$

$$T_{48} = \frac{-4e^6 C_R |a|^2 d}{q\cdot p_1 q\cdot p_2} \Delta_{\tilde{e}_R}(p_2,k_1)\Re\{\Delta_Z(k_1,k_2)\} \times [g(2p_1\cdot p_2 p_1\cdot k_2 p_2\cdot k_1 + p_2\cdot k_1(-q\cdot k_2 p_1\cdot p_2 - p_1\cdot k_2 q\cdot p_2 + p_2\cdot k_2 q\cdot p_1) + p_1\cdot k_2(-q\cdot p_1 p_2\cdot k_1 + q\cdot p_2 p_1\cdot k_1 - q\cdot k_1 p_1\cdot p_2)) + f m^2_{\chi^0_1} p_1\cdot p_2(p_1\cdot p_2 - q\cdot p_2 - q\cdot p_1)], \quad \text{(A64)}$$

$$T_{56} = 8e^6 C_R |a|^4 \frac{p_1\cdot k_2}{q\cdot p_2} \Delta^2_{\tilde{e}_R}(p_1,k_2)\Delta_{\tilde{e}_R}(p_2,k_1) \times [p_2\cdot k_1 q\cdot p_2 - 2p_2\cdot k_1 q\cdot k_1 + 2(p_2\cdot k_1)^2 + m^2_{\chi^0_1} q\cdot p_2], \quad \text{(A65)}$$

$$T_{57} = -\frac{4e^6 C_R |a|^2 d}{q\cdot p_2 q\cdot p_1} \Delta_{\tilde{e}_R}(p_1,k_2)\Re\{\Delta_Z(k_1,k_2)\} \times [g(2p_1\cdot p_2 p_1\cdot k_2 p_2\cdot k_1 + p_1\cdot k_2(-p_1\cdot p_2 q\cdot k_1 + p_1\cdot k_1 q\cdot p_2 - q\cdot p_1 p_2\cdot k_1) + p_2\cdot k_1(-p_1\cdot p_2 q\cdot k_2 - p_1\cdot k_2 q\cdot p_2 + q\cdot p_1 p_2\cdot k_2)) + f m^2_{\chi^0_1} p_1\cdot p_2(p_1\cdot p_2 - q\cdot p_2 - q\cdot p_1)], \quad \text{(A66)}$$

$$T_{58} = -\frac{4e^6 C_R |a|^2 d}{q \cdot p_2} \Delta_{\tilde{e}_R}(p_1, k_2) \Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[2g p_1 \cdot k_2 q \cdot k_1 + f m^2_{\chi^0_1} q \cdot p_1], \tag{A67}$$

$$T_{67} = -\frac{4e^6 C_R |a|^2 d}{q \cdot p_1} \Delta_{\tilde{e}_R}(p_1, k_2)\Delta_{\tilde{e}_R}(p_2, k_1) \Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[2g p_2 \cdot k_1 (p_1 \cdot k_2 q \cdot p_1 - 2q \cdot k_2 p_1 \cdot k_2 + 2(p_1 \cdot k_2)^2 + m^2_{\chi^0_1} q \cdot p_1) + f m^2_{\chi^0_1}(2p_1 \cdot k_2 p_1 \cdot p_2 + q \cdot p_1 p_1 \cdot p_2 - q \cdot k_2 p_1 \cdot p_2 - q \cdot p_2 p_1 \cdot k_2 + q \cdot p_1 p_2 \cdot k_2)], \tag{A68}$$

$$T_{68} = -\frac{4e^6 C_R |a|^2 d}{q \cdot p_2} \Delta_{\tilde{e}_R}(p_1, k_2)\Delta_{\tilde{e}_R}(p_2, k_1) \Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[2g p_1 \cdot k_2 (2(p_2 \cdot k_1)^2 + q \cdot p_2 p_2 \cdot k_1 - 2p_2 \cdot k_1 q \cdot k_1 + m^2_{\chi^0_1} q \cdot p_2) + f m^2_{\chi^0_1}(2p_1 \cdot p_2 p_2 \cdot k_1 + p_1 \cdot p_2 q \cdot p_2 - p_2 \cdot k_1 q \cdot p_1 + p_1 \cdot k_1 q \cdot p_2 - p_1 \cdot p_2 q \cdot k_1)], \tag{A69}$$

$$T_{78} = \frac{4e^6}{q \cdot p_2 q \cdot p_1} |\Delta_Z(k_1, k_2)|^2 [(C_R g^2 d^2 + C_L f^2 c^2)(2p_1 \cdot p_2 p_1 \cdot k_1 p_2 \cdot k_2 + p_1 \cdot k_1 (p_1 \cdot k_2 q \cdot p_2 - p_1 \cdot p_2 q \cdot k_2 - p_2 \cdot k_2 q \cdot p_2)) + p_2 \cdot k_2 (p_2 \cdot k_1 q \cdot p_1 - p_1 \cdot p_2 q \cdot k_1 - p_1 \cdot k_1 q \cdot p_1)) + (C_L g^2 c^2 + C_R f^2 d^2)(2p_1 \cdot p_2 p_1 \cdot k_2 p_2 \cdot k_1 + p_1 \cdot k_2 (p_1 \cdot k_1 q \cdot p_2 - p_1 \cdot p_2 q \cdot k_1 - p_2 \cdot k_1 q \cdot p_2)) + p_2 \cdot k_1 (p_2 \cdot k_2 q \cdot p_1 - p_1 \cdot p_2 q \cdot k_2 - p_1 \cdot k_2 q \cdot p_1)) + 2gf(C_L c^2 + C_R d^2) m^2_{\chi^0_1} p_1 \cdot p_2 (p_1 \cdot p_2 - q \cdot p_2 - q \cdot p_1)]. \tag{A70}$$

We have calculated the squared amplitudes with FEYNCALC [72]. When integrating the squared amplitude over the phase space, see Appendix D, the $s-t$-interference terms cancel the $s-u$-interference terms due to a symmetry in these channels, caused by the Majorana properties of the neutralinos [28]. Note that in principle also terms proportional to $\epsilon_{\kappa\lambda\mu\nu} k_1^\kappa p_1^\lambda p_2^\mu q^\nu \Im\mathrm{m}\{\Delta_Z\}$ would appear in the squared amplitudes T_{ij}, due to the inclusion of the Z width to regularize the pole of the propagator Δ_Z. However, since this is a higher order effect which is small far off the Z resonance, we neglect such terms. In addition they would vanish after performing a complete phase space integration.

APPENDIX B: AMPLITUDES FOR RADIATIVE NEUTRINO PRODUCTION

For radiative neutrino production

$$e^-(p_1) + e^+(p_2) \to \nu(k_1) + \bar{\nu}(k_2) + \gamma(q), \tag{B1}$$

we define the W and Z boson propagators as

$$\Delta_W(p_i, k_j) \equiv \frac{1}{m_W^2 + 2p_i \cdot k_j}, \tag{B2}$$

$$\Delta_Z(k_1, k_2) \equiv \frac{1}{m_Z^2 - 2k_1 \cdot k_2 - \mathrm{i}\Gamma_Z m_Z}. \tag{B3}$$

The tree-level amplitudes for W boson exchange, see the diagrams 1–3 in Fig. 7, are then

$$\mathcal{M}_1 = \frac{\mathrm{i}e^3 a^2}{4q \cdot p_1} \Delta_W(p_2, k_2)[\bar{v}(p_2)\gamma^\mu P_L v(k_2)] \times [\bar{u}(k_1)\gamma_\mu P_L(\not{q} - \not{p}_1)\not{\epsilon}^* u(p_1)], \tag{B4}$$

$$\mathcal{M}_2 = \frac{\mathrm{i}e^3 a^2}{4q \cdot p_2} \Delta_W(p_1, k_1)[\bar{u}(k_1)\gamma^\mu P_L u(p_1)] \times [\bar{v}(p_2)\not{\epsilon}^*(\not{p}_2 - \not{q})\gamma_\mu P_L v(k_2)], \tag{B5}$$

$$\mathcal{M}_3 = \frac{1}{2}\mathrm{i}e^3 a^2 \Delta_W(p_1, k_1)\Delta_W(p_2, k_2)[\bar{u}(k_1)\gamma^\beta P_L u(p_1)] \times [\bar{v}(p_2)\gamma^\alpha P_L v(k_2)]((2k_1 - 2p_1 + q)_\mu g_{\alpha\beta} + (p_1 - k_1 - 2q)_\beta g_{\mu\alpha} + (p_1 - k_1 + q)_\alpha g_{\beta\mu})(\epsilon^\mu)^*, \tag{B6}$$

with the parameter

$$a = \frac{1}{\sin\theta_w}, \tag{B7}$$

see the Feynman rules listed in Table III. The amplitudes for Z boson exchange, see diagrams 4 and 5 in Fig. 7, are

$$\mathcal{M}_4 = \frac{\mathrm{i}e^3 f}{4q \cdot p_1} \Delta_Z(k_1, k_2)[\bar{u}(k_1)\gamma^\nu P_L v(k_2)] \times [\bar{v}(p_2)\gamma_\nu(cP_L + dP_R)(\not{q} - \not{p}_1)\not{\epsilon}^* u(p_1)], \tag{B8}$$

$$\mathcal{M}_5 = \frac{\mathrm{i}e^3 f}{4q \cdot p_2} \Delta_Z(k_1, k_2)[\bar{u}(k_1)\gamma^\nu P_L v(k_2)] \times [\bar{v}(p_2)\not{\epsilon}^*(\not{p}_2 - \not{q})\gamma_\nu(cP_L + dP_R)u(p_1)], \tag{B9}$$

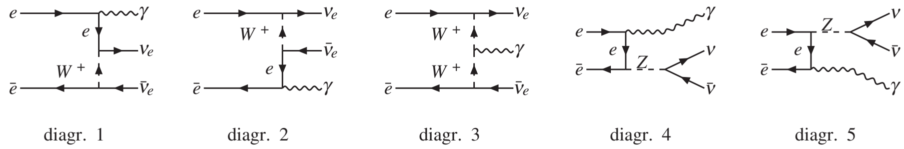


FIG. 7. Contributing diagrams to $e^+e^- \to \nu\bar{\nu}\gamma$ [74].

with the parameters

$$c = \frac{1}{\sin\theta_w \cos\theta_w}\left(\frac{1}{2} - \sin^2\theta_w\right), \qquad d = -\tan\theta_w, \qquad f = \frac{1}{\sin\theta_w \cos\theta_w}. \tag{B10}$$

We have checked that the amplitudes $\mathcal{M}_i = \epsilon_\mu \mathcal{M}_i^\mu$ for $i = 1, \ldots, 5$ fulfill the Ward identity $q_\mu(\sum_i \mathcal{M}_i^\mu) = 0$. We find $q_\mu(\mathcal{M}_1^\mu + \mathcal{M}_2^\mu + \mathcal{M}_3^\mu) = 0$ for W exchange and $q_\mu(\mathcal{M}_4^\mu + \mathcal{M}_5^\mu) = 0$ for Z exchange.

We obtain the squared amplitudes T_{ii} and T_{ij} as defined in Eqs. (A26) and (A27):

$$T_{11} = \frac{e^6 C_L a^4}{q \cdot p_1}\Delta_W^2(p_2, k_2) p_2 \cdot k_1 q \cdot k_2, \tag{B11}$$

$$T_{22} = \frac{e^6 C_L a^4}{q \cdot p_2}\Delta_W^2(p_1, k_1) p_1 \cdot k_2 q \cdot k_1, \tag{B12}$$

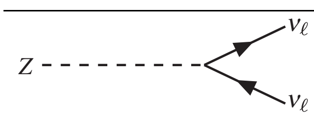
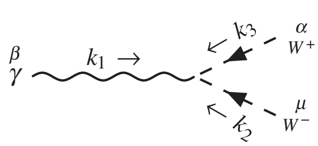
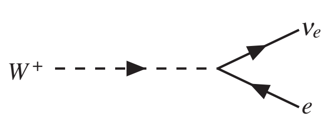
$$\begin{aligned} T_{33} = {}& e^6 C_L a^4 \Delta_W^2(p_2, k_2)\Delta_W^2(p_1, k_1)[p_2 \cdot k_2 p_1 \cdot k_1 p_1 \cdot k_1 + (p_2 \cdot k_1(7p_1 \cdot k_2 - 6q \cdot k_2) + p_2 \cdot k_2(q \cdot k_1 - q \cdot p_1) \\ & - q \cdot k_2(p_1 \cdot p_2 + 2q \cdot p_2) + p_1 \cdot k_2(p_1 \cdot p_2 + 6q \cdot p_2))p_1 \cdot k_1 + p_2 \cdot k_1 q \cdot k_1 p_1 \cdot k_2 - 3q \cdot k_1 p_1 \cdot k_2 p_1 \cdot p_2 \\ & + q \cdot k_1 q \cdot k_2 p_1 \cdot p_2 - p_2 \cdot k_1 p_1 \cdot k_2 q \cdot p_1 + q \cdot k_1 p_2 \cdot k_2 q \cdot p_1 + 2p_2 \cdot k_1 q \cdot k_2 q \cdot p_1 + 2q \cdot k_1 p_1 \cdot k_2 q \cdot p_2 \\ & + k_1 \cdot k_2(-2q \cdot k_1 p_1 \cdot p_2 + p_1 \cdot k_1(p_2 \cdot k_1 - p_1 \cdot p_2 + q \cdot p_2) + q \cdot p_1(3p_2 \cdot k_1 + 2p_1 \cdot p_2 + q \cdot p_2))], \end{aligned} \tag{B13}$$

$$T_{44} = 3\frac{e^6 f^2}{q \cdot p_1}|\Delta_Z(k_1, k_2)|^2(C_L c^2 p_2 \cdot k_1 q \cdot k_2 + C_R d^2 p_2 \cdot k_2 q \cdot k_1), \tag{B14}$$

$$T_{55} = 3\frac{e^6 f^2}{q \cdot p_2}|\Delta_Z(k_1, k_2)|^2(C_L c^2 p_1 \cdot k_2 q \cdot k_1 + C_R d^2 p_1 \cdot k_1 q \cdot k_2), \tag{B15}$$

$$\begin{aligned} T_{12} = {}& \frac{e^6 C_L a^4}{q \cdot p_1 q \cdot p_2}\Delta_W(p_1, k_1)\Delta_W(p_2, k_2)[2p_2 \cdot k_1 p_1 \cdot k_2 p_1 \cdot p_2 - q \cdot k_1 p_1 \cdot k_2 p_1 \cdot p_2 - p_2 \cdot k_1 q \cdot k_2 p_1 \cdot p_2 \\ & - p_2 \cdot k_1 p_1 \cdot k_2 q \cdot p_1 + p_2 \cdot k_1 p_2 \cdot k_2 q \cdot p_1 + p_1 \cdot k_1 p_1 \cdot k_2 q \cdot p_2 - p_2 \cdot k_1 p_1 \cdot k_2 q \cdot p_2], \end{aligned} \tag{B16}$$

TABLE III. Vertex factors with the parameters a, c, d, and f defined in Eq. (B7) and (B10).

Vertex	Factor
	$-\frac{ie}{2} f \gamma^\mu P_L$, $\ell = e, \mu, \tau$
	$-ie[(k_1 - k_2)_\alpha g_{\beta\mu} + (k_2 - k_3)_\beta g_{\mu\alpha} + (k_3 - k_1)_\mu g_{\alpha\beta}]$
	$-\frac{1}{\sqrt{2}} iea\gamma_\mu P_L$

$$T_{13} = \frac{e^6 C_L a^4}{q\cdot p_1} \Delta_W^2(p_2, k_2)\Delta_W(p_1, k_1)[4p_1\cdot k_1 p_2\cdot k_1 p_1\cdot k_2 - p_2\cdot k_1 q\cdot k_1 p_1\cdot k_2 - 3q\cdot k_1 p_1\cdot p_2 p_1\cdot k_2 + 3p_1\cdot k_1 q\cdot p_2 p_1\cdot k_2 - 3p_1\cdot k_1 p_2\cdot k_1 q\cdot k_2 + q\cdot k_1 q\cdot k_2 p_1\cdot p_2 + k_1\cdot k_2 p_2\cdot k_1 q\cdot p_1 - p_1\cdot k_1 p_2\cdot k_2 q\cdot p_1 + 3p_2\cdot k_1 q\cdot k_2 q\cdot p_1 + k_1\cdot k_2 p_1\cdot p_2 q\cdot p_1 - p_1\cdot k_1 q\cdot k_2 q\cdot p_2], \tag{B17}$$

$$T_{14} = -\frac{2e^6 C_L c f a^2}{q\cdot p_1}\Delta_W(p_2, k_2)\,\Re\mathrm{e}\{\Delta_Z(k_1, k_2)\} p_2\cdot k_1 q\cdot k_2, \tag{B18}$$

$$T_{15} = -\frac{e^6 C_L c f a^2}{q\cdot p_1 q\cdot p_2}\Delta_W(p_2, k_2)\,\Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[2p_2\cdot k_1 p_1\cdot k_2 p_1\cdot p_2 - q\cdot k_1 p_1\cdot k_2 p_1\cdot p_2 - p_2\cdot k_1 q\cdot k_2 p_1\cdot p_2 - p_2\cdot k_1 p_1\cdot k_2 q\cdot p_1 + p_2\cdot k_1 p_2\cdot k_2 q\cdot p_1 + p_1\cdot k_1 p_1\cdot k_2 q\cdot p_2 - p_2\cdot k_1 p_1\cdot k_2 q\cdot p_2], \tag{B19}$$

$$T_{23} = \frac{e^6 C_L a^4}{q\cdot p_2}\Delta_W^2(p_1, k_1)\Delta_W(p_2, k_2)[-3p_1\cdot k_2 p_2\cdot k_1 p_2\cdot k_1 + 3q\cdot k_1 p_1\cdot k_2 p_2\cdot k_1 - p_1\cdot k_1 p_2\cdot k_1 p_2\cdot k_2 + k_1\cdot k_2 p_1\cdot p_2 p_2\cdot k_1 + 2p_1\cdot k_2 p_1\cdot p_2 p_2\cdot k_1 - q\cdot k_2 p_1\cdot p_2 p_2\cdot k_1 - 2p_1\cdot k_2 q\cdot p_1 p_2\cdot k_1 + p_2\cdot k_2 q\cdot p_1 p_2\cdot k_1 - 3p_1\cdot k_2 q\cdot p_2 p_2\cdot k_1 + p_1\cdot k_1 q\cdot k_1 p_2\cdot k_2 - k_1\cdot k_2 q\cdot k_1 p_1\cdot p_2 - q\cdot k_1 q\cdot k_2 p_1\cdot p_2 + q\cdot k_1 p_2\cdot k_2 q\cdot p_1 + 2p_1\cdot k_1 p_1\cdot k_2 q\cdot p_2 + 3q\cdot k_1 p_1\cdot k_2 q\cdot p_2], \tag{B20}$$

$$T_{24} = -\frac{e^6 C_L c f a^2}{q\cdot p_1 q\cdot p_2}\Delta_W(p_1, k_1)\,\Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[2p_2\cdot k_1 p_1\cdot k_2 p_1\cdot p_2 - q\cdot k_1 p_1\cdot k_2 p_1\cdot p_2 - p_2\cdot k_1 q\cdot k_2 p_1\cdot p_2 - p_2\cdot k_1 p_1\cdot k_2 q\cdot p_1 + p_2\cdot k_1 p_2\cdot k_2 q\cdot p_1 + p_1\cdot k_1 p_1\cdot k_2 q\cdot p_2 - p_2\cdot k_1 p_1\cdot k_2 q\cdot p_2], \tag{B21}$$

$$T_{25} = -\frac{2e^6 C_L c f a^2}{q\cdot p_2}\Delta_W(p_1, k_1)\,\Re\mathrm{e}\{\Delta_Z(k_1, k_2)\} p_1\cdot k_2 q\cdot k_1, \tag{B22}$$

$$T_{34} = -\frac{e^6 C_L c f a^2}{q\cdot p_1}\Delta_W(p_1, k_1)\Delta_W(p_2, k_2)\,\Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[4p_1\cdot k_1 p_2\cdot k_1 p_1\cdot k_2 - p_2\cdot k_1 q\cdot k_1 p_1\cdot k_2 - 3q\cdot k_1 p_1\cdot p_2 p_1\cdot k_2 + 3p_1\cdot k_1 q\cdot p_2 p_1\cdot k_2 - 3p_1\cdot k_1 p_2\cdot k_1 q\cdot k_2 + q\cdot k_1 q\cdot k_2 p_1\cdot p_2 + k_1\cdot k_2 p_2\cdot k_1 q\cdot p_1 - p_1\cdot k_1 p_2\cdot k_2 q\cdot p_1 + 3p_2\cdot k_1 q\cdot k_2 q\cdot p_1 + k_1\cdot k_2 p_1\cdot p_2 q\cdot p_1 - p_1\cdot k_1 q\cdot k_2 q\cdot p_2], \tag{B23}$$

$$T_{35} = -\frac{e^6 C_L c f a^2}{q\cdot p_2}\Delta_W(p_1, k_1)\Delta_W(p_2, k_2)\,\Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[-3p_1\cdot k_2 p_2\cdot k_1 p_2\cdot k_1 + 3q\cdot k_1 p_1\cdot k_2 p_2\cdot k_1 - p_1\cdot k_1 p_2\cdot k_2 p_2\cdot k_1 + k_1\cdot k_2 p_1\cdot p_2 p_2\cdot k_1 + 2p_1\cdot k_2 p_1\cdot p_2 p_2\cdot k_1 - q\cdot k_2 p_1\cdot p_2 p_2\cdot k_1 - 2p_1\cdot k_2 q\cdot p_1 p_2\cdot k_1 + p_2\cdot k_2 q\cdot p_1 p_2\cdot k_1 - 3p_1\cdot k_2 q\cdot p_2 p_2\cdot k_1 + p_1\cdot k_1 q\cdot k_1 p_2\cdot k_2 - k_1\cdot k_2 p_1\cdot p_2 q\cdot k_1 - q\cdot k_1 q\cdot k_2 p_1\cdot p_2 + q\cdot k_1 p_2\cdot k_2 q\cdot p_1 + 2p_1\cdot k_1 p_1\cdot k_2 q\cdot p_2 + 3q\cdot k_1 p_1\cdot k_2 q\cdot p_2], \tag{B24}$$

$$T_{45} = \frac{3e^6 f^2}{q\cdot p_1 q\cdot p_2}|\Delta_Z(k_1, k_2)|^2[C_L c^2(2p_2\cdot k_1 p_1\cdot k_2 p_1\cdot p_2 - q\cdot k_1 p_1\cdot k_2 p_1\cdot p_2 - p_2\cdot k_1 q\cdot k_2 p_1\cdot p_2 - p_2\cdot k_1 p_1\cdot k_2 q\cdot p_1 + p_2\cdot k_1 p_2\cdot k_2 q\cdot p_1 + p_1\cdot k_1 p_1\cdot k_2 q\cdot p_2 - p_2\cdot k_1 p_1\cdot k_2 q\cdot p_2) + C_R d^2(2p_1\cdot k_1 p_2\cdot k_2 p_1\cdot p_2 - q\cdot k_1 p_2\cdot k_2 p_1\cdot p_2 - p_1\cdot k_1 q\cdot k_2 p_1\cdot p_2 - p_1\cdot k_1 p_2\cdot k_2 q\cdot p_1 + p_2\cdot k_1 p_2\cdot k_2 q\cdot p_1 + p_1\cdot k_1 p_1\cdot k_2 q\cdot p_2 - p_1\cdot k_1 p_2\cdot k_2 q\cdot p_2)]. \tag{B25}$$

We have calculated the squared amplitudes with FEYNCALC [72]. We neglect terms proportional to $\epsilon_{\kappa\lambda\mu\nu}k_1^\kappa p_1^\lambda p_2^\mu q^\nu \Im\mathrm{m}\{\Delta_Z\}$, see the discussion at the end of Appendix A.

APPENDIX C: AMPLITUDES FOR RADIATIVE SNEUTRINO PRODUCTION

For radiative sneutrino production

$$e^-(p_1) + e^+(p_2) \to \tilde{\nu}(k_1) + \tilde{\nu}^*(k_2) + \gamma(q) \qquad \text{(C1)}$$

we define the chargino and Z boson propagators as

$$\Delta_{\chi^+_{1,2}}(p_i, k_j) \equiv \frac{1}{m^2_{\chi^+_{1,2}} - m^2_{\tilde{\nu}} + 2p_i \cdot k_j}, \qquad \text{(C2)}$$

$$\Delta_Z(k_1, k_2) \equiv \frac{1}{m_Z^2 - 2m^2_{\tilde{\nu}} - 2k_1 \cdot k_2} - \mathrm{i}\Gamma_Z m_Z. \qquad \text{(C3)}$$

The tree-level amplitudes for chargino $\tilde{\chi}^\pm_1$ exchange, see the contributing diagrams 1–3 in Fig. 8, are

$$\mathcal{M}_1 = \frac{\mathrm{i}e^3 a^2 |V_{11}|^2}{2q \cdot p_1} \Delta_{\chi^+_1}(p_2, k_2)[\bar{v}(p_2) P_R(\not{p}_2 - \not{k}_2 - m_{\chi^+_1}) \times P_L(\not{p}_1 - \not{q})\not{\epsilon}^* u(p_1)], \qquad \text{(C4)}$$

$$\mathcal{M}_2 = -\frac{\mathrm{i}e^3 a^2 |V_{11}|^2}{2q \cdot p_2} \Delta_{\chi^+_1}(p_1, k_1)[\bar{v}(p_2)\not{\epsilon}^*(\not{p}_2 - \not{q}) \times P_R(\not{k}_1 - \not{p}_1 - m_{\chi^+_1}) P_L u(p_1)], \qquad \text{(C5)}$$

$$\mathcal{M}_3 = -\mathrm{i}e^3 a^2 |V_{11}|^2 \Delta_{\chi^+_1}(p_1, k_1)\Delta_{\chi^+_1}(p_2, k_2) \times [\bar{v}(p_2) P_R(\not{p}_2 - \not{k}_2 - m_{\chi^+_1})\not{\epsilon}^*(\not{k}_1 - \not{p}_1 - m_{\chi^+_1}) \times P_L u(p_1)], \qquad \text{(C6)}$$

with the parameter a defined in Eq. (B7). The 2×2 matrices U and V diagonalize the chargino mass matrix X [1]

$$U^* X V^{-1} = \mathrm{diag}\big(m_{\chi^+_1}, \quad m_{\chi^+_2} \big). \qquad \text{(C7)}$$

The amplitudes for chargino $\tilde{\chi}^\pm_2$ exchange, see the contributing diagrams 4–6 in Fig. 8, are

$$\mathcal{M}_4 = \frac{\mathrm{i}e^3 a^2 |V_{21}|^2}{2q \cdot p_1} \Delta_{\chi^+_2}(p_2, k_2)[\bar{v}(p_2) P_R(\not{p}_2 - \not{k}_2 - m_{\chi^+_2}) \times P_L(\not{p}_1 - \not{q})\not{\epsilon}^* u(p_1)], \qquad \text{(C8)}$$

$$\mathcal{M}_5 = -\frac{\mathrm{i}e^3 a^2 |V_{21}|^2}{2q \cdot p_1} \Delta_{\chi^+_2}(p_1, k_1)[\bar{v}(p_2)\not{\epsilon}^*(\not{p}_2 - \not{q}) \times P_R(\not{k}_1 - \not{p}_1 - m_{\chi^+_2}) P_L u(p_1)], \qquad \text{(C9)}$$

$$\mathcal{M}_6 = -\mathrm{i}e^3 a^2 |V_{21}|^2 \Delta_{\chi^+_2}(p_1, k_1)\Delta_{\chi^+_2}(p_2, k_2) \times [\bar{v}(p_2) P_R(\not{p}_2 - \not{k}_2 - m_{\chi^+_2})\not{\epsilon}^*(\not{k}_1 - \not{p}_1 - m_{\chi^+_2}) \times P_L u(p_1)]. \qquad \text{(C10)}$$

The amplitudes for Z boson exchange, see the diagrams 7 and 8 in Fig. 8, read

$$\mathcal{M}_7 = \frac{\mathrm{i}e^3 f}{4q \cdot p_1} \Delta_Z(k_1, k_2)[\bar{v}(p_2)(\not{k}_1 - \not{k}_2) \times (cP_L + dP_R)(\not{p}_1 - \not{q})\not{\epsilon}^* u(p_1)], \qquad \text{(C11)}$$

$$\mathcal{M}_8 = \frac{\mathrm{i}e^3 f}{4q \cdot p_2} \Delta_Z(k_1, k_2)[\bar{v}(p_2)\not{\epsilon}^*(\not{q} - \not{p}_2)(\not{k}_1 - \not{k}_2) \times (cP_L + dP_R) u(p_1)], \qquad \text{(C12)}$$

with the parameters c, d, and f defined in Eq. (B10), and the Feynman rules listed in Table IV. We have checked that the amplitudes $\mathcal{M}_i = \epsilon_\mu \mathcal{M}_i^\mu$, $i = 1, \ldots, 8$, fulfill the Ward identity $q_\mu(\sum_i \mathcal{M}_i^\mu) = 0$, as done in Ref. [53]. We find $q_\mu(\mathcal{M}_1^\mu + \mathcal{M}_2^\mu + \mathcal{M}_3^\mu) = 0$ for $\tilde{\chi}^\pm_1$ exchange, $q_\mu(\mathcal{M}_4^\mu + \mathcal{M}_5^\mu + \mathcal{M}_6^\mu) = 0$ for $\tilde{\chi}^\pm_2$ exchange, and $q_\mu(\mathcal{M}_7^\mu + \mathcal{M}_8^\mu) = 0$ for Z boson exchange. Our amplitudes for chargino and Z boson exchange agree with those given in Refs. [53,54], and in the limit of vanishing chargino mixing with those of Ref. [25]. However, there are obvious misprints in the amplitudes M_2 and M_4 of

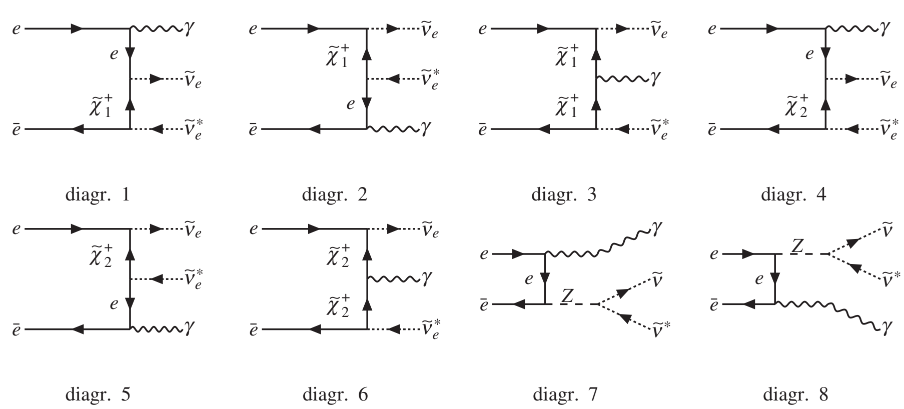
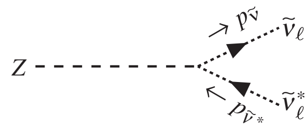
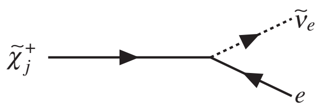
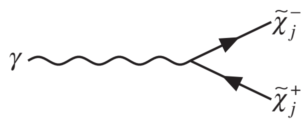


FIG. 8. Contributing diagrams to $e^+e^- \to \tilde{\nu}\tilde{\nu}^*\gamma$ [74].

TABLE IV. Vertex factors with parameters a, f defined in Eqs. (B7) and (B10), and C the charge conjugation operator.

Vertex	Factor
	$-\frac{1}{2}\mathrm{i}ef(p_{\tilde{\nu}} + p_{\tilde{\nu}^*})_\mu$, $\quad \ell = e, \mu, \tau$
	$-\mathrm{i}eaV_{j1}P_RC$, $\tilde{\chi}_j^+$ transposed
	$-\mathrm{i}e\gamma_\mu$

Ref. [54], see their Eqs. (7) and (9), respectively, and in the amplitude T_5 of Ref. [25], see their Eq. (F.3).

We then obtain the squared amplitudes T_{ii} and T_{ij} as defined in Eqs. (A26) and (A27):

$$T_{11} = \frac{e^6 C_L a^4 |V_{11}|^4}{2q \cdot p_1} \Delta^2_{\chi_1^+}(p_2, k_2)(2p_2 \cdot k_2 q \cdot k_2 - m^2_{\tilde{\nu}} q \cdot p_2), \tag{C13}$$

$$T_{22} = \frac{e^6 C_L a^4 |V_{11}|^4}{2q \cdot p_2} \Delta^2_{\chi_1^+}(p_1, k_1)(2p_1 \cdot k_1 q \cdot k_1 - m^2_{\tilde{\nu}} q \cdot p_1), \tag{C14}$$

$$\begin{aligned} T_{33} = {} & e^6 C_L a^4 |V_{11}|^4 \Delta^2_{\chi_1^+}(p_1, k_1) \Delta^2_{\chi_1^+}(p_2, k_2) \\ & \times [m^4_{\chi_1^+} p_1 \cdot p_2 + 4m^2_{\chi_1^+} p_1 \cdot k_1 p_2 \cdot k_2 \\ & - 2m^2_{\tilde{\nu}} p_1 \cdot k_1 p_2 \cdot k_1 + 4k_1 \cdot k_2 p_1 \cdot k_1 p_2 \cdot k_2 \\ & - 2m^2_{\tilde{\nu}} p_1 \cdot k_2 p_2 \cdot k_2 + m^4_{\tilde{\nu}} p_1 \cdot p_2], \end{aligned} \tag{C15}$$

$$T_{44} = \frac{e^6 C_L a^4 |V_{21}|^4}{2q \cdot p_1} \Delta^2_{\chi_2^+}(p_2, k_2)(2p_2 \cdot k_2 q \cdot k_2 - m^2_{\tilde{\nu}} q \cdot p_2), \tag{C16}$$

$$T_{55} = \frac{e^6 C_L a^4 |V_{21}|^4}{2q \cdot p_2} \Delta^2_{\chi_2^+}(p_1, k_1)(2p_1 \cdot k_1 q \cdot k_1 - m^2_{\tilde{\nu}} q \cdot p_1), \tag{C17}$$

$$\begin{aligned} T_{66} = {} & e^6 C_L a^4 |V_{21}|^4 \Delta^2_{\chi_2^+}(p_1, k_1) \Delta^2_{\chi_2^+}(p_2, k_2) \\ & \times [m^4_{\chi_2^+} p_1 \cdot p_2 + 4m^2_{\chi_2^+} p_1 \cdot k_1 p_2 \cdot k_2 \\ & - 2m^2_{\tilde{\nu}} p_1 \cdot k_1 p_2 \cdot k_1 + 4k_1 \cdot k_2 p_1 \cdot k_1 p_2 \cdot k_2 \\ & - 2m^2_{\tilde{\nu}} p_1 \cdot k_2 p_2 \cdot k_2 + m^4_{\tilde{\nu}} p_1 \cdot p_2], \end{aligned} \tag{C18}$$

$$\begin{aligned} T_{77} = {} & 3\frac{e^6 f^2 (C_L c^2 + C_R d^2)}{4q \cdot p_1} |\Delta_Z(k_1, k_2)|^2 \\ & \times [p_2 \cdot k_1 q \cdot k_1 - p_2 \cdot k_2 q \cdot k_1 - p_2 \cdot k_1 q \cdot k_2 \\ & + p_2 \cdot k_2 q \cdot k_2 - m^2_{\tilde{\nu}} q \cdot p_2 + k_1 \cdot k_2 q \cdot p_2], \end{aligned} \tag{C19}$$

$$\begin{aligned} T_{88} = {} & 3\frac{e^6 f^2 (C_L c^2 + C_R d^2)}{4q \cdot p_2} |\Delta_Z(k_1, k_2)|^2 \\ & \times [p_1 \cdot k_1 q \cdot k_1 - p_1 \cdot k_2 q \cdot k_1 - p_1 \cdot k_1 q \cdot k_2 \\ & + p_1 \cdot k_2 q \cdot k_2 - m^2_{\tilde{\nu}} q \cdot p_1 + k_1 \cdot k_2 q \cdot p_1], \end{aligned} \tag{C20}$$

$$\begin{aligned} T_{12} = {} & -\frac{e^6 C_L a^4 |V_{11}|^4}{q \cdot p_1 q \cdot p_2} \Delta_{\chi_1^+}(p_1, k_1) \Delta_{\chi_1^+}(p_2, k_2)[-k_1 \cdot k_2 p_1 \cdot p_2 p_1 \cdot p_2 + p_2 \cdot k_1 p_1 \cdot k_2 p_1 \cdot p_2 + p_1 \cdot k_1 p_2 \cdot k_2 p_1 \cdot p_2 \\ & - q \cdot k_1 p_2 \cdot k_2 p_1 \cdot p_2 - p_1 \cdot k_1 q \cdot k_2 p_1 \cdot p_2 + k_1 \cdot k_2 q \cdot p_1 p_1 \cdot p_2 + k_1 \cdot k_2 q \cdot p_2 p_1 \cdot p_2 \\ & - p_2 \cdot k_1 p_1 \cdot k_2 q \cdot p_1 + p_2 \cdot k_1 p_2 \cdot k_2 q \cdot p_1 + p_1 \cdot k_1 p_1 \cdot k_2 q \cdot p_2 - p_2 \cdot k_1 p_1 \cdot k_2 q \cdot p_2], \end{aligned} \tag{C21}$$

$$\begin{aligned} T_{13} = {} & -\frac{e^2 C_L a^4 |V_{11}|^4}{q \cdot p_1} \Delta_{\chi_1^+}(p_1, k_1) \Delta^2_{\chi_1^+}(p_2, k_2)[m^2_{\chi_1^+} q \cdot k_2 p_1 \cdot p_2 + m^2_{\chi_1^+} q \cdot p_1 p_2 \cdot k_2 - m^2_{\chi_1^+} q \cdot p_2 p_1 \cdot k_2 \\ & - 4p_1 \cdot k_1 p_1 \cdot k_2 p_2 \cdot k_2 + 4p_1 \cdot k_1 p_2 \cdot k_2 q \cdot k_2 + 2m^2_{\tilde{\nu}} p_1 \cdot k_1 p_1 \cdot p_2 - 2m^2_{\tilde{\nu}} p_1 \cdot k_1 q \cdot p_2], \end{aligned} \tag{C22}$$

$$T_{14} = \frac{e^6 C_L a^4 |V_{11}|^2 |V_{21}|^2}{q \cdot p_1} \Delta_{\chi_1^+}(p_2, k_2) \Delta_{\chi_2^+}(p_2, k_2)[2p_2 \cdot k_2 q \cdot k_2 - m_{\tilde{\nu}}^2 q \cdot p_2], \tag{C23}$$

$$\begin{aligned} T_{15} = &-\frac{e^6 C_L a^4 |V_{11}|^2 |V_{21}|^2}{q \cdot p_1 q \cdot p_2} \Delta_{\chi_2^+}(p_1, k_1) \Delta_{\chi_1^+}(p_2, k_2)[-k_1 \cdot k_2 p_1 \cdot p_2 p_1 \cdot p_2 + p_2 \cdot k_1 p_1 \cdot k_2 p_1 \cdot p_2 \\ &+ p_1 \cdot k_1 p_2 \cdot k_2 p_1 \cdot p_2 - q \cdot k_1 p_2 \cdot k_2 p_1 \cdot p_2 - p_1 \cdot k_1 q \cdot k_2 p_1 \cdot p_2 + k_1 \cdot k_2 q \cdot p_1 p_1 \cdot p_2 + k_1 \cdot k_2 q \cdot p_2 p_1 \cdot p_2 \\ &- p_2 \cdot k_1 p_1 \cdot k_2 q \cdot p_1 + p_2 \cdot k_1 p_2 \cdot k_2 q \cdot p_1 + p_1 \cdot k_1 p_1 \cdot k_2 q \cdot p_2 - p_2 \cdot k_1 p_1 \cdot k_2 q \cdot p_2], \end{aligned} \tag{C24}$$

$$\begin{aligned} T_{16} = &-\frac{e^6 C_L a^4 |V_{11}|^2 |V_{21}|^2}{q \cdot p_1} \Delta_{\chi_2^+}(p_1, k_1) \Delta_{\chi_1^+}(p_2, k_2) \Delta_{\chi_2^+}(p_2, k_2)[m_{\chi_2^+}^2 q \cdot k_2 p_1 \cdot p_2 + m_{\chi_2^+}^2 q \cdot p_1 p_2 \cdot k_2 \\ &- m_{\chi_2^+}^2 q \cdot p_2 p_1 \cdot k_2 - 4p_1 \cdot k_1 p_1 \cdot k_2 p_2 \cdot k_2 + 4p_1 \cdot k_1 p_2 \cdot k_2 q \cdot k_2 + 2m_{\tilde{\nu}}^2 p_1 \cdot k_1 p_1 \cdot p_2 - 2m_{\tilde{\nu}}^2 p_1 \cdot k_1 q \cdot p_2], \end{aligned} \tag{C25}$$

$$\begin{aligned} T_{17} = &-\frac{e^6 C_L a^2 cf |V_{11}|^2}{2q \cdot p_1} \Delta_{\chi_1^+}(p_2, k_2) \Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[q \cdot k_1 p_2 \cdot k_2 - 2q \cdot k_2 p_2 \cdot k_2 + p_2 \cdot k_1 q \cdot k_2 + m_{\tilde{\nu}}^2 q \cdot p_2 \\ &- k_1 \cdot k_2 q \cdot p_2], \end{aligned} \tag{C26}$$

$$\begin{aligned} T_{18} = &-\frac{e^6 C_L a^2 cf |V_{11}|^2}{2q \cdot p_1 q \cdot p_2} \Delta_{\chi_1^+}(p_2, k_2) \Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[-q \cdot p_2 p_1 \cdot k_2 p_1 \cdot k_2 + p_2 \cdot k_1 p_1 \cdot p_2 p_1 \cdot k_2 \\ &- 2p_2 \cdot k_2 p_1 \cdot p_2 p_1 \cdot k_2 + q \cdot k_2 p_1 \cdot p_2 p_1 \cdot k_2 - p_2 \cdot k_1 q \cdot p_1 p_1 \cdot k_2 + p_2 \cdot k_2 q \cdot p_1 p_1 \cdot k_2 + p_1 \cdot k_1 q \cdot p_2 p_1 \cdot k_2 \\ &- p_2 \cdot k_1 q \cdot p_2 p_1 \cdot k_2 + p_2 \cdot k_2 q \cdot p_2 p_1 \cdot k_2 + m_{\tilde{\nu}}^2 p_1 \cdot p_2 p_1 \cdot p_2 - k_1 \cdot k_2 p_1 \cdot p_2 p_1 \cdot p_2 + p_1 \cdot k_1 p_2 \cdot k_2 p_1 \cdot p_2 \\ &- q \cdot k_1 p_2 \cdot k_2 p_1 \cdot p_2 - p_1 \cdot k_1 q \cdot k_2 p_1 \cdot p_2 + p_2 \cdot k_2 q \cdot k_2 p_1 \cdot p_2 - p_2 \cdot k_2 p_2 \cdot k_2 q \cdot p_1 + p_2 \cdot k_1 p_2 \cdot k_2 q \cdot p_1 \\ &- m_{\tilde{\nu}}^2 p_1 \cdot p_2 q \cdot p_1 + k_1 \cdot k_2 p_1 \cdot p_2 q \cdot p_1 - m_{\tilde{\nu}}^2 p_1 \cdot p_2 q \cdot p_2 + k_1 \cdot k_2 p_1 \cdot p_2 q \cdot p_2], \end{aligned} \tag{C27}$$

$$\begin{aligned} T_{23} = &-\frac{e^6 C_L a^4 |V_{11}|^4}{q \cdot p_2} \Delta_{\chi_1^+}^2(p_1, k_1) \Delta_{\chi_1^+}(p_2, k_2)[m_{\chi_1^+}^2 q \cdot k_1 p_1 \cdot p_2 - m_{\chi_1^+}^2 p_2 \cdot k_1 q \cdot p_1 + m_{\chi_1^+}^2 p_1 \cdot k_1 q \cdot p_2 \\ &- 4p_1 \cdot k_1 p_2 \cdot k_1 p_2 \cdot k_2 + 4p_1 \cdot k_1 q \cdot k_1 p_2 \cdot k_2 + 2m_{\tilde{\nu}}^2 p_2 \cdot k_2 p_1 \cdot p_2 - 2m_{\tilde{\nu}}^2 p_2 \cdot k_2 q \cdot p_1], \end{aligned} \tag{C28}$$

$$\begin{aligned} T_{24} = &-\frac{e^6 C_L a^4 |V_{11}|^2 |V_{21}|^2}{q \cdot p_1 q \cdot p_2} \Delta_{\chi_1^+}(p_1, k_1) \Delta_{\chi_2^+}(p_2, k_2)[-k_1 \cdot k_2 p_1 \cdot p_2 p_1 \cdot p_2 + p_2 \cdot k_1 p_1 \cdot k_2 p_1 \cdot p_2 \\ &+ p_1 \cdot k_1 p_2 \cdot k_2 p_1 \cdot p_2 - q \cdot k_1 p_2 \cdot k_2 p_1 \cdot p_2 - p_1 \cdot k_1 q \cdot k_2 p_1 \cdot p_2 + k_1 \cdot k_2 q \cdot p_1 p_1 \cdot p_2 + k_1 \cdot k_2 q \cdot p_2 p_1 \cdot p_2 \\ &- p_2 \cdot k_1 p_1 \cdot k_2 q \cdot p_1 + p_2 \cdot k_1 p_2 \cdot k_2 q \cdot p_1 + p_1 \cdot k_1 p_1 \cdot k_2 q \cdot p_2 - p_2 \cdot k_1 p_1 \cdot k_2 q \cdot p_2], \end{aligned} \tag{C29}$$

$$T_{25} = \frac{e^6 C_L a^4 |V_{11}|^2 |V_{21}|^2}{q \cdot p_2} \Delta_{\chi_1^+}(p_1, k_1) \Delta_{\chi_2^+}(p_1, k_1)[2p_1 \cdot k_1 q \cdot k_1 - m_{\tilde{\nu}}^2 q \cdot p_1], \tag{C30}$$

$$\begin{aligned} T_{26} = &-\frac{e^6 C_L a^4 |V_{11}|^2 |V_{21}|^2}{q \cdot p_2} \Delta_{\chi_2^+}(p_1, k_1) \Delta_{\chi_1^+}(p_1, k_1) \Delta_{\chi_2^+}(p_2, k_2)[m_{\chi_2^+}^2 q \cdot k_1 p_1 \cdot p_2 - m_{\chi_2^+}^2 q \cdot p_1 p_2 \cdot k_1 + m_{\chi_2^+}^2 q \cdot p_2 p_1 \cdot k_1 \\ &- 4p_1 \cdot k_1 p_2 \cdot k_1 p_2 \cdot k_2 + 4p_1 \cdot k_1 p_2 \cdot k_2 q \cdot k_1 + 2m_{\tilde{\nu}}^2 p_2 \cdot k_2 p_1 \cdot p_2 - 2m_{\tilde{\nu}}^2 p_2 \cdot k_2 q \cdot p_1], \end{aligned} \tag{C31}$$

$$\begin{aligned} T_{27} = &\frac{e^6 C_L a^2 cf |V_{11}|^2}{2q \cdot p_1 q \cdot p_2} \Delta_{\chi_1^+}(p_1, k_1) \Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[q \cdot p_2 p_1 \cdot k_1 p_1 \cdot k_1 + 2p_2 \cdot k_1 p_1 \cdot p_2 p_1 \cdot k_1 - q \cdot k_1 p_1 \cdot p_2 p_1 \cdot k_1 \\ &- p_2 \cdot k_2 p_1 \cdot p_2 p_1 \cdot k_1 + q \cdot k_2 p_1 \cdot p_2 p_1 \cdot k_1 - p_2 \cdot k_1 q \cdot p_1 p_1 \cdot k_1 - p_2 \cdot k_1 q \cdot p_2 p_1 \cdot k_1 - p_1 \cdot k_2 q \cdot p_2 p_1 \cdot k_1 \\ &- m_{\tilde{\nu}}^2 p_1 \cdot p_2 p_1 \cdot p_2 + k_1 \cdot k_2 p_1 \cdot p_2 p_1 \cdot p_2 - p_2 \cdot k_1 q \cdot k_1 p_1 \cdot p_2 - p_2 \cdot k_1 p_1 \cdot k_2 p_1 \cdot p_2 + q \cdot k_1 p_2 \cdot k_2 p_1 \cdot p_2 \\ &+ p_2 \cdot k_1 p_2 \cdot k_1 q \cdot p_1 + p_2 \cdot k_1 p_1 \cdot k_2 q \cdot p_1 - p_2 \cdot k_1 p_2 \cdot k_2 q \cdot p_1 + m_{\tilde{\nu}}^2 p_1 \cdot p_2 q \cdot p_1 - k_1 \cdot k_2 p_1 \cdot p_2 q \cdot p_1 \\ &+ p_2 \cdot k_1 q \cdot p_2 p_1 \cdot k_2 + m_{\tilde{\nu}}^2 p_1 \cdot p_2 q \cdot p_2 - k_1 \cdot k_2 p_1 \cdot p_2 q \cdot p_2], \end{aligned} \tag{C32}$$

$$T_{28} = \frac{e^6 C_L a^2 cf|V_{11}|^2}{2q \cdot p_2} \Delta_{\chi_1^+}(p_1, k_1) \Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[-q \cdot k_1 p_1 \cdot k_2 + 2q \cdot k_1 p_1 \cdot k_1 - p_1 \cdot k_1 q \cdot k_2 - m_{\tilde{\nu}}^2 q \cdot p_1 + k_1 \cdot k_2 q \cdot p_1], \tag{C33}$$

$$T_{34} = -\frac{e^6 C_L a^4 |V_{11}|^2 |V_{21}|^2}{q \cdot p_1} \Delta_{\chi_1^+}(p_1, k_1)\Delta_{\chi_1^+}(p_2, k_2)\Delta_{\chi_2^+}(p_2, k_2)[m_{\chi_1^+}^2(q \cdot k_2 p_1 \cdot p_2 + p_2 \cdot k_2 q \cdot p_1 - p_1 \cdot k_2 q \cdot p_2) - 4p_1 \cdot k_1 p_2 \cdot k_2(p_1 \cdot k_2 - q \cdot k_2) + 2m_{\tilde{\nu}}^2 p_1 \cdot k_1(p_1 \cdot p_2 - q \cdot p_2)], \tag{C34}$$

$$T_{35} = -\frac{e^6 C_L a^4 |V_{11}|^2 |V_{21}|^2}{q \cdot p_2} \Delta_{\chi_1^+}(p_1, k_1)\Delta_{\chi_1^+}(p_2, k_2)\Delta_{\chi_2^+}(p_1, k_1)[m_{\chi_1^+}^2(q \cdot k_1 p_1 \cdot p_2 + p_1 \cdot k_1 q \cdot p_2 - p_2 \cdot k_1 q \cdot p_1) - 4p_1 \cdot k_1 p_2 \cdot k_2(p_2 \cdot k_1 - q \cdot k_1) + 2m_{\tilde{\nu}}^2 p_2 \cdot k_2(p_1 \cdot p_2 - q \cdot p_1)], \tag{C35}$$

$$T_{36} = 2e^6 C_L a^4 |V_{11}|^2 |V_{21}|^2 \Delta_{\chi_1^+}(p_1, k_1)\Delta_{\chi_1^+}(p_2, k_2)\Delta_{\chi_2^+}(p_1, k_1)\Delta_{\chi_2^+}(p_2, k_2)[2p_1 \cdot k_1 p_2 \cdot k_2(m_{\chi_1^+}^2 + m_{\chi_2^+}^2 + 2k_1 \cdot k_2) + m_{\chi_1^+}^2 m_{\chi_2^+}^2 p_1 \cdot p_2 - 2m_{\tilde{\nu}}^2(p_1 \cdot k_1 p_2 \cdot k_1 + p_1 \cdot k_2 p_2 \cdot k_2) + m_{\tilde{\nu}}^4 p_1 \cdot p_2], \tag{C36}$$

$$T_{37} = \frac{e^6 C_L a^2 cf|V_{11}|^2}{2q \cdot p_1} \Delta_{\chi_1^+}(p_1, k_1)\Delta_{\chi_1^+}(p_2, k_2) \Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[m_{\chi_1^+}^2(q \cdot k_1 p_1 \cdot p_2 - q \cdot k_2 p_1 \cdot p_2 + p_2 \cdot k_1 q \cdot p_1 - p_2 \cdot k_2 q \cdot p_1 - p_1 \cdot k_1 q \cdot p_2 + p_1 \cdot k_2 q \cdot p_2) - 2p_1 \cdot k_1 p_2 \cdot k_1 p_1 \cdot k_2 - 2p_1 \cdot k_1 p_1 \cdot k_1 p_2 \cdot k_2 + 2p_1 \cdot k_1 p_2 \cdot k_2 q \cdot k_1 + 4p_1 \cdot k_1 p_2 \cdot k_2 p_1 \cdot k_2 + 2p_1 \cdot k_1 p_2 \cdot k_1 q \cdot k_2 - 4p_1 \cdot k_1 p_2 \cdot k_2 q \cdot k_2 - 2m_{\tilde{\nu}}^2 p_1 \cdot p_2 p_1 \cdot k_1 + 2k_1 \cdot k_2 p_1 \cdot k_1 p_1 \cdot p_2 + 2m_{\tilde{\nu}}^2 p_1 \cdot k_1 q \cdot p_2 - 2k_1 \cdot k_2 p_1 \cdot k_1 q \cdot p_2], \tag{C37}$$

$$T_{38} = -\frac{e^6 C_L a^2 cf|V_{11}|^2}{2q \cdot p_2} \Delta_{\chi_1^+}(p_1, k_1)\Delta_{\chi_1^+}(p_2, k_2) \Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[m_{\chi_1^+}^2(q \cdot k_1 p_1 \cdot p_2 - q \cdot k_2 p_1 \cdot p_2 - p_2 \cdot k_1 q \cdot p_1 + p_2 \cdot k_2 q \cdot p_1 + p_1 \cdot k_1 q \cdot p_2 - p_1 \cdot k_2 q \cdot p_2) + 2p_1 \cdot k_1 p_2 \cdot k_2 p_2 \cdot k_2 - 4p_1 \cdot k_1 p_2 \cdot k_1 p_2 \cdot k_2 + 4p_1 \cdot k_1 p_2 \cdot k_2 q \cdot k_1 + 2p_2 \cdot k_1 p_1 \cdot k_2 p_2 \cdot k_2 - 2p_1 \cdot k_2 p_2 \cdot k_2 q \cdot k_1 - 2p_1 \cdot k_1 p_2 \cdot k_2 q \cdot k_2 + 2m_{\tilde{\nu}}^2 p_1 \cdot p_2 p_2 \cdot k_2 - 2k_1 \cdot k_2 p_2 \cdot k_2 p_1 \cdot p_2 - 2m_{\tilde{\nu}}^2 p_2 \cdot k_2 q \cdot p_1 + 2k_1 \cdot k_2 p_2 \cdot k_2 q \cdot p_1], \tag{C38}$$

$$T_{45} = -\frac{e^6 C_L a^4 |V_{21}|^4}{q \cdot p_1 q \cdot p_2} \Delta_{\chi_2^+}(p_1, k_1)\Delta_{\chi_2^+}(p_2, k_2)[-k_1 \cdot k_2 p_1 \cdot p_2 p_1 \cdot p_2 + p_1 \cdot k_2 p_2 \cdot k_1 p_1 \cdot p_2 + p_1 \cdot k_1 p_2 \cdot k_2 p_1 \cdot p_2 - q \cdot k_1 p_2 \cdot k_2 p_1 \cdot p_2 - p_1 \cdot k_1 q \cdot k_2 p_1 \cdot p_2 + k_1 \cdot k_2 q \cdot p_1 p_1 \cdot p_2 + k_1 \cdot k_2 q \cdot p_2 p_1 \cdot p_2 - p_2 \cdot k_1 p_1 \cdot k_2 q \cdot p_1 + p_2 \cdot k_1 p_2 \cdot k_2 q \cdot p_1 + p_1 \cdot k_1 p_1 \cdot k_2 q \cdot p_2 - p_2 \cdot k_1 p_1 \cdot k_2 q \cdot p_2], \tag{C39}$$

$$T_{46} = -\frac{e^6 C_L a^4 |V_{21}|^4}{q \cdot p_1} \Delta_{\chi_2^+}(p_1, k_1)\Delta_{\chi_2^+}^2(p_2, k_2)[m_{\chi_2^+}^2 q \cdot k_2 p_1 \cdot p_2 + m_{\chi_2^+}^2 q \cdot p_1 p_2 \cdot k_2 - m_{\chi_2^+}^2 q \cdot p_2 p_1 \cdot k_2 - 4p_1 \cdot k_1 p_1 \cdot k_2 p_2 \cdot k_2 + 4p_1 \cdot k_1 p_2 \cdot k_2 q \cdot k_2 + 2m_{\tilde{\nu}}^2 p_1 \cdot k_1 p_1 \cdot p_2 - 2m_{\tilde{\nu}}^2 p_1 \cdot k_1 q \cdot p_2], \tag{C40}$$

$$T_{47} = -\frac{e^6 C_L a^2 cf|V_{21}|^2}{2q \cdot p_1} \Delta_{\chi_2^+}(p_2, k_2) \Re\mathrm{e}\{\Delta_Z(k_1, k_2)\}[q \cdot k_1 p_2 \cdot k_2 - 2q \cdot k_2 p_2 \cdot k_2 + p_2 \cdot k_1 q \cdot k_2 + m_{\tilde{\nu}}^2 q \cdot p_2 - k_1 \cdot k_2 q \cdot p_2], \tag{C41}$$

$$T_{48} = -\frac{e^6 C_L a^2 c f |V_{21}|^2}{2q\cdot p_1 q\cdot p_2}\Delta_{\chi_2^+}(p_2,k_2)\Re\{\Delta_Z(k_1,k_2)\}[-q\cdot p_2 p_1\cdot k_2 p_1\cdot k_2 + p_2\cdot k_1 p_1\cdot p_2 p_1\cdot k_2 - 2p_2\cdot k_2 p_1\cdot p_2 p_1\cdot k_2 + q\cdot k_2 p_1\cdot p_2 p_1\cdot k_2 - p_2\cdot k_1 q\cdot p_1 p_1\cdot k_2 + p_2\cdot k_2 q\cdot p_1 p_1\cdot k_2 + p_1\cdot k_1 q\cdot p_2 p_1\cdot k_2 - p_2\cdot k_1 q\cdot p_2 p_1\cdot k_2 + p_2\cdot k_2 q\cdot p_2 p_1\cdot k_2 + m_{\tilde\nu}^2 p_1\cdot p_2 p_1\cdot p_2 - k_1\cdot k_2 p_1\cdot p_2 p_1\cdot p_2 + p_1\cdot k_1 p_2\cdot k_2 p_1\cdot p_2 - q\cdot k_1 p_2\cdot k_2 p_1\cdot p_2 - p_1\cdot k_1 q\cdot k_2 p_1\cdot p_2 + p_2\cdot k_2 q\cdot k_2 p_1\cdot p_2 - p_2\cdot k_2 p_2\cdot k_2 q\cdot p_1 + p_2\cdot k_1 p_2\cdot k_2 q\cdot p_1 - m_{\tilde\nu}^2 p_1\cdot p_2 q\cdot p_1 + k_1\cdot k_2 p_1\cdot p_2 q\cdot p_1 - m_{\tilde\nu}^2 p_1\cdot p_2 q\cdot p_2 + k_1\cdot k_2 p_1\cdot p_2 q\cdot p_2], \quad \text{(C42)}$$

$$T_{56} = -\frac{e^6 C_L a^4 |V_{21}|^4}{q\cdot p_2}\Delta^2_{\chi_2^+}(p_1,k_1)\Delta_{\chi_2^+}(p_2,k_2)[m^2_{\chi_2^+} q\cdot k_1 p_1\cdot p_2 - m^2_{\chi_2^+} p_2\cdot k_1 q\cdot p_1 + m^2_{\chi_2^+} p_1\cdot k_1 q\cdot p_2 - 4p_1\cdot k_1 p_2\cdot k_1 p_2\cdot k_2 + 4p_1\cdot k_1 q\cdot k_1 p_2\cdot k_2 + 2m_{\tilde\nu}^2 p_2\cdot k_2 p_1\cdot p_2 - 2m_{\tilde\nu}^2 p_2\cdot k_2 q\cdot p_1], \quad \text{(C43)}$$

$$T_{57} = \frac{e^6 C_L a^2 c f |V_{21}|^2}{2q\cdot p_1 q\cdot p_2}\Delta_{\chi_2^+}(p_1,k_1)\Re\{\Delta_Z(k_1,k_2)\}[q\cdot p_2 p_1\cdot k_1 p_1\cdot k_1 + 2p_2\cdot k_1 p_1\cdot p_2 p_1\cdot k_1 - q\cdot k_1 p_1\cdot p_2 p_1\cdot k_1 - p_2\cdot k_2 p_1\cdot p_2 p_1\cdot k_1 + q\cdot k_2 p_1\cdot p_2 p_1\cdot k_1 - p_2\cdot k_1 q\cdot p_1 p_1\cdot k_1 - p_2\cdot k_1 q\cdot p_2 p_1\cdot k_1 - p_1\cdot k_2 q\cdot p_2 p_1\cdot k_1 - m_{\tilde\nu}^2 p_1\cdot p_2 p_1\cdot p_2 + k_1\cdot k_2 p_1\cdot p_2 p_1\cdot p_2 - p_2\cdot k_1 q\cdot k_1 p_1\cdot p_2 - p_2\cdot k_1 p_1\cdot k_2 p_1\cdot p_2 + q\cdot k_1 p_2\cdot k_2 p_1\cdot p_2 + p_2\cdot k_1 p_2\cdot k_1 q\cdot p_1 + p_2\cdot k_1 p_1\cdot k_2 q\cdot p_1 - p_2\cdot k_1 p_2\cdot k_2 q\cdot p_1 + m_{\tilde\nu}^2 p_1\cdot p_2 q\cdot p_1 - k_1\cdot k_2 p_1\cdot p_2 q\cdot p_1 + p_2\cdot k_1 q\cdot p_2 p_1\cdot k_2 + m_{\tilde\nu}^2 p_1\cdot p_2 q\cdot p_2 - k_1\cdot k_2 p_1\cdot p_2 q\cdot p_2], \quad \text{(C44)}$$

$$T_{58} = \frac{e^6 C_L a^2 c f |V_{21}|^2}{2q\cdot p_2}\Delta_{\chi_2^+}(p_1,k_1)\Re\{\Delta_Z(k_1,k_2)\}[2q\cdot k_1 p_1\cdot k_1 - q\cdot k_1 p_1\cdot k_2 - p_1\cdot k_1 q\cdot k_2 - m_{\tilde\nu}^2 q\cdot p_1 + k_1\cdot k_2 q\cdot p_1], \quad \text{(C45)}$$

$$T_{67} = \frac{e^6 C_L a^2 c f |V_{21}|^2}{2q\cdot p_1}\Delta_{\chi_2^+}(p_1,k_1)\Delta_{\chi_2^+}(p_2,k_2)\Re\{\Delta_Z(k_1,k_2)\}[m^2_{\chi_2^+}(q\cdot k_1 p_1\cdot p_2 - q\cdot k_2 p_1\cdot p_2 + p_2\cdot k_1 q\cdot p_1 - p_2\cdot k_2 q\cdot p_1 - p_1\cdot k_1 q\cdot p_2 + p_1\cdot k_2 q\cdot p_2) - 2p_1\cdot k_1 p_2\cdot k_1 p_1\cdot k_2 - 2p_1\cdot k_1 p_1\cdot k_1 p_2\cdot k_2 + 2p_1\cdot k_1 p_2\cdot k_2 q\cdot k_1 + 4p_1\cdot k_1 p_2\cdot k_2 p_1\cdot k_2 + 2p_1\cdot k_1 p_2\cdot k_1 q\cdot k_2 - 4p_1\cdot k_1 p_2\cdot k_2 q\cdot k_2 - 2m_{\tilde\nu}^2 p_1\cdot p_2 p_1\cdot k_1 + 2k_1\cdot k_2 p_1\cdot k_1 p_1\cdot p_2 + 2m_{\tilde\nu}^2 p_1\cdot k_1 q\cdot p_2 - 2k_1\cdot k_2 p_1\cdot k_1 q\cdot p_2], \quad \text{(C46)}$$

$$T_{68} = -\frac{e^6 C_L a^2 c f |V_{21}|^2}{2q\cdot p_2}\Delta_{\chi_2^+}(p_1,k_1)\Delta_{\chi_2^+}(p_2,k_2)\Re\{\Delta_Z(k_1,k_2)\}[m^2_{\chi_2^+}(q\cdot k_1 p_1\cdot p_2 - q\cdot k_2 p_1\cdot p_2 - p_2\cdot k_1 q\cdot p_1 + p_2\cdot k_2 q\cdot p_1 + p_1\cdot k_1 q\cdot p_2 - p_1\cdot k_2 q\cdot p_2) + 2p_1\cdot k_1 p_2\cdot k_2 p_2\cdot k_2 - 4p_1\cdot k_1 p_2\cdot k_1 p_2\cdot k_2 + 4p_1\cdot k_1 p_2\cdot k_2 q\cdot k_1 + 2p_2\cdot k_1 p_1\cdot k_2 p_2\cdot k_2 - 2p_1\cdot k_2 p_2\cdot k_2 q\cdot k_1 - 2p_1\cdot k_1 p_2\cdot k_2 q\cdot k_2 + 2m_{\tilde\nu}^2 p_1\cdot p_2 p_2\cdot k_2 - 2k_1\cdot k_2 p_2\cdot k_2 p_1\cdot p_2 - 2m_{\tilde\nu}^2 p_2\cdot k_2 q\cdot p_1 + 2k_1\cdot k_2 p_2\cdot k_2 q\cdot p_1], \quad \text{(C47)}$$

$$T_{78} = 3\frac{e^6 f^2(C_L c^2 + C_R d^2)}{4q\cdot p_1 q\cdot p_2}|\Delta_Z(k_1,k_2)|^2[p_1\cdot k_1(p_1\cdot k_1 q\cdot p_2 + 2p_2\cdot k_1 p_1\cdot p_2 - q\cdot k_1 p_1\cdot p_2 - 2p_2\cdot k_2 p_1\cdot p_2 + q\cdot k_2 p_1\cdot p_2 - p_2\cdot k_1 q\cdot p_1 + p_2\cdot k_2 q\cdot p_1 - p_2\cdot k_1 q\cdot p_2 - 2p_1\cdot k_2 q\cdot p_2 + p_2\cdot k_2 q\cdot p_2) + p_1\cdot p_2(-2m_{\tilde\nu}^2 p_1\cdot p_2 + 2k_1\cdot k_2 p_1\cdot p_2 - p_2\cdot k_1 q\cdot k_1 - 2p_2\cdot k_1 p_1\cdot k_2 + q\cdot k_1 p_1\cdot k_2 + q\cdot k_1 p_2\cdot k_2 + 2p_1\cdot k_2 p_2\cdot k_2 + p_2\cdot k_1 q\cdot k_2 - p_1\cdot k_2 q\cdot k_2 - p_2\cdot k_2 q\cdot k_2) + q\cdot p_1(p_2\cdot k_1 p_2\cdot k_1 + p_2\cdot k_2 p_2\cdot k_2 + p_2\cdot k_1 p_1\cdot k_2 - 2p_2\cdot k_1 p_2\cdot k_2 - p_1\cdot k_2 p_2\cdot k_2 + 2m_{\tilde\nu}^2 p_1\cdot p_2 - 2k_1\cdot k_2 p_1\cdot p_2) + q\cdot p_2(p_1\cdot k_2 p_1\cdot k_2 + p_2\cdot k_1 p_1\cdot k_2 - p_1\cdot k_2 p_2\cdot k_2 + 2m_{\tilde\nu}^2 p_1\cdot p_2 - 2k_1\cdot k_2 p_1\cdot p_2))]. \quad \text{(C48)}$$

Formulas for the squared amplitudes for radiative sneutrino production can also be found in Refs. [53,54] for longitudinal and transverse beam polarizations. Here, we give however our calculated amplitudes for completeness. We have calculated the squared amplitudes with FEYNCALC [72]. We neglect terms proportional to $\epsilon_{\kappa\lambda\mu\nu}k_1^\kappa p_1^\lambda p_2^\mu q^\nu \Im\mathrm{m}\{\Delta_Z\}$, see the discussion at the end of Appendix A.

APPENDIX D: DEFINITION OF THE DIFFERENTIAL CROSS SECTION AND PHASE SPACE

We present some details of the phase space calculation for radiative neutralino production

$$e^-(p_1) + e^+(p_2) \to \tilde{\chi}_1^0(k_1) + \tilde{\chi}_1^0(k_2) + \gamma(q). \quad \text{(D1)}$$

The differential cross section for (D1) is given by [73]

$$d\sigma = \frac{1}{2}\frac{(2\pi)^4}{2s}\prod_f \frac{d^3\mathbf{p}_f}{(2\pi)^3 2E_f} \times \delta^{(4)}(p_1 + p_2 - k_1 - k_2 - q)|\mathcal{M}|^2, \quad \text{(D2)}$$

where $\mathbf{p}_f$ and E_f denote the final three-momenta and the final energies of the neutralinos and the photon. The squared matrix element $|\mathcal{M}|^2$ is given in Appendix A.

1. Parametrization of momenta and phase space in the neutralino system

We parametrize the four-momenta in the center-of-mass (cms) system of the incoming particles, which we call the laboratory (lab) system. The beam momenta are then parametrized as

$$p_1 = \tfrac{1}{2}(\sqrt{s}, 0, 0, \sqrt{s}), \qquad p_2 = \tfrac{1}{2}(\sqrt{s}, 0, 0, -\sqrt{s}). \quad \text{(D3)}$$

For the outgoing neutralinos and the photon we consider in a first step the local center-of-mass system of the two neutralinos. The photon shall escape along this x_3 axis. We start with general momentum-vectors for the two neutralinos, boost them along their x_3 axis and rotate them around the $x_1$axis to reach the lab system. Note that the three-momenta of the outgoing particles lie in a plane whose normal vector is inclined by an angle θ towards the beam axis. We parametrize the neutralino momenta in their cms frame [21]

$$k_1^* = \begin{pmatrix} \frac{1}{2}\sqrt{s^*} \\ k^*\sin\theta^*\cos\phi^* \\ k^*\sin\theta^*\sin\phi^* \\ k^*\cos\theta^* \end{pmatrix}, \qquad k_2^* = \begin{pmatrix} \frac{1}{2}\sqrt{s^*} \\ -k^*\sin\theta^*\cos\phi^* \\ -k^*\sin\theta^*\sin\phi^* \\ -k^*\cos\theta^* \end{pmatrix}, \quad \text{(D4)}$$

with the local cms energy s^* of the two neutralinos

$$s^* = (k_1 + k_2)^2 = 2m_{\chi_1^0}^2 + 2k_1\cdot k_2, \quad \text{(D5)}$$

the polar angle θ^*, the azimuthal angle ϕ^* and the absolute value of the neutralino three-momenta k^* in their cms frame. These momenta are boosted to the lab system with the Lorentz transformation

$$L(\beta) = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix}, \quad \text{(D6)}$$

with $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ and $\beta = \frac{|\mathbf{k_1}+\mathbf{k_2}|}{(k_1)^0+(k_2)^0}|_{\text{cms beam}}$ the boost velocity from the cms to the lab system

$$\beta = \frac{|\mathbf{q}|}{\sqrt{s} - E_\gamma} = \frac{s - s^*}{s + s^*}. \quad \text{(D7)}$$

Boosting the momenta k_1^* and k_2^*, see Eq. (D4), with the Lorentz transformation Eq. (D6) and then rotating with θ yields the neutralino and photon momenta in the lab system [21]

$$k_1 = \begin{pmatrix} \gamma E^* + \beta\gamma k^*\cos\theta^* \\ k^*\sin\theta^*\cos\phi^* \\ k^*\sin\theta^*\sin\phi^*\cos\theta + (\beta\gamma E^* + \gamma k^*\cos\theta^*)\sin\theta \\ -k^*\sin\theta^*\sin\phi^*\sin\theta + (\beta\gamma E^* + \gamma k^*\cos\theta^*)\cos\theta \end{pmatrix}, \quad \text{(D8)}$$

$$k_2 = \begin{pmatrix} \gamma E^* - \beta\gamma k^*\cos\theta^* \\ -k^*\sin\theta^*\cos\phi^* \\ -k^*\sin\theta^*\sin\phi^*\cos\theta + (\beta\gamma E^* - \gamma k^*\cos\theta^*)\sin\theta \\ k^*\sin\theta^*\sin\phi^*\sin\theta + (\beta\gamma E^* - \gamma k^*\cos\theta^*)\cos\theta \end{pmatrix}, \quad \text{(D9)}$$

$$q = \begin{pmatrix} \frac{s-s^*}{2\sqrt{s}} \\ 0 \\ -\frac{s-s^*}{2\sqrt{s}}\sin\theta \\ -\frac{s-s^*}{2\sqrt{s}}\cos\theta \end{pmatrix}, \quad \text{(D10)}$$

with

$$k^* = \frac{1}{2}\sqrt{s^* - 4m_{\chi_1^0}^2}, \quad \text{(D11)}$$

$$E^* = \frac{\sqrt{s^*}}{2}, \quad \text{(D12)}$$

$$\beta\gamma = \frac{s - s^*}{2\sqrt{ss^*}}. \quad \text{(D13)}$$

The differential cross section for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$ now reads [21]

$$d\sigma = \frac{1}{4096\pi^4 s}\left(1 - \frac{s^*}{s}\right) \times \sqrt{1 - \frac{4m^2_{\chi^0_1}}{s^*}} |\mathcal{M}|^2 d\cos\theta d\cos\theta^* d\phi^* ds^*, \quad \text{(D14)}$$

where the integration variables run over

$$\begin{aligned} &0 \leq \phi^* \leq 2\pi, \\ &-1 \leq \cos\theta^* \leq 1, \\ &4m^2_{\chi^0_1} \leq s^* \leq (1-x)s, \qquad x = \frac{E_\gamma}{E_{\rm beam}}, \\ &-0.99 \leq \cos\theta \leq 0.99. \end{aligned} \quad \text{(D15)}$$

2. Alternative parametrization in the center-of-mass system

For the radiative production of neutralinos $e^+e^- \to \tilde{\chi}^0_i \tilde{\chi}^0_j \gamma$, we choose a coordinate frame in the center-of-mass system, such that the momentum of the photon $\mathbf{p}_\gamma$ points in the z-direction. The scattering angle is $\theta_\gamma \angle(\mathbf{p}_{e^-}, \mathbf{p}_\gamma)$, whereas the azimuthal angle ϕ_γ can be chosen zero. The four-momenta are

$$\begin{aligned} p^\mu_{e^-} &= E_b(1, \sin\theta_\gamma, 0, \cos\theta_\gamma), \\ p^\mu_{e^+} &= E_b(1, -\sin\theta_\gamma, 0, -\cos\theta_\gamma), \end{aligned} \quad \text{(D16)}$$

$$\begin{aligned} p^\mu_\gamma &= E_\gamma(1, 0, 0, 1), \\ p^\mu_{\chi_i} &= |\mathbf{p}_i|(E_i/|\mathbf{p}_i|, \sin\theta_i\cos\phi_i, \sin\theta_i\sin\phi_i, \cos\theta_i), \end{aligned} \quad \text{(D17)}$$

$$\begin{aligned} p^\mu_{\chi_j} &= (E_j, \mathbf{p}_j), \qquad E_j = 2E_b - E_\gamma - E_i, \\ \mathbf{p}_j &= -\mathbf{p}_\gamma - \mathbf{p}_i, \end{aligned} \quad \text{(D18)}$$

with the beam energy $E_b = \sqrt{s}/2$, and the neutralino angle $\theta_i \angle(\mathbf{p}_\gamma, \mathbf{p}_i)$ fixed due to momentum conservation (D18)

$$\cos\theta_i = \frac{m_i^2 - m_j^2 + E_j^2 - E_i^2 - E_\gamma^2}{2E_\gamma\sqrt{E_i^2 - m_i^2}}. \quad \text{(D19)}$$

The phase space integration for the cross section (D2) can then be written as [27,31,53,54]

$$\sigma = \left(1 - \frac{1}{2}\delta_{ij}\right)\frac{1}{16s(2\pi)^4} \times \int \sin\theta_\gamma d\theta_\gamma dE_\gamma \int d\phi_i dE_i |\mathcal{M}|^2. \quad \text{(D20)}$$

For $m_i = m_j$ the integration bounds of the photon and neutralino energy are [27,31]

$$E_\gamma^{\rm max} = E_b\left(1 - \frac{m_i^2}{E_b^2}\right), \quad \text{(D21)}$$

$$E_i^{\rm min,max} = E_b - \frac{E_\gamma}{2}\left[1 \pm \sqrt{1 - \frac{m_i^2}{E_b(E_b - E_\gamma)}}\right], \quad \text{(D22)}$$

whereas the scattering angle θ_γ and the minimal photon energy $E_\gamma^{\rm min}$ have to be cut to regularize infrared and collinear divergencies, see Eq. (3.5). Similar formulas are obtained for the radiative production of neutrinos (2.2) and sneutrinos (2.3).

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