

Lattice study of the masses of singlet 0^{++} mesonsA. Hart,¹ C. McNeile,^{2,*} C. Michael,² and J. Pickavance²¹*SUPA, School of Physics, University of Edinburgh, Edinburgh EH9 3JZ, Scotland*²*Theoretical Physics Division, Dept. of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK*

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We compute the masses of the flavor singlet 0^{++} mesons using ($n_f = 2$) unquenched lattice QCD with the Iwasaki and Wilson gauge actions. Both fermionic and glueball interpolating operators are used to create the states. The mass of the lightest 0^{++} meson is suppressed relative to the mass of the 0^{++} glueball in quenched QCD at an equivalent lattice spacing. The low value for mass of the lightest flavor singlet 0^{++} meson obtained in our calculation could be due to either: that the mass of the meson is much lower than that of the 0^{++} glueball in quenched QCD, or due to the effect of lattice artifacts. We present arguments for the first option, but we are unable to rule out the explanation due to lattice artifacts.

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I. INTRODUCTION

The interpretation of the experimental f_0 mesons in terms of fundamental quark and glue fields is still not settled [1–6]. Quenched lattice QCD predicts that the mass of the scalar ($J^{PC} = 0^{++}$) glueball is around 1.6 GeV [7–10]. Hence, attention has focused on finding evidence for a glueball component (mixing with quark-antiquark) in the physical $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ mesons. The quark model predicts that there are two f_0 mesons in this mass regime, so the existence of three mesons is suggestive of the presence of additional degrees of freedom such as the elusive 0^{++} glueball.

The experimental f_0 spectrum contains more puzzles. The experimental data for the $f_0(1370)$ still seems controversial [2]. There is also a new state $f_0(1790)$ reported by BES [5,11]. It is not clear how this affects the “standard” mixing scenario for the 0^{++} mesons, that considers only three mesons. The interpretation of the $f_0(980)$ meson which is close to the kaon-kaon threshold, in terms of quark and antiquarks, is also uncertain. Even extracting the mass and width of the $f_0(400–1200)$ state is still controversial [2,3], but progress seems to have been made recently [12,13]. It may be difficult for lattice calculations to explore the $f_0(400–1200)$, because it has such a large width. Although there are attempts to study this state [14,15] on the lattice.

The glueball spectrum in pure gauge theory is theoretically well defined, because the glueball operators do not mix with fermionic $\bar{\psi}\psi$ operators. The work of many authors has shown that the lightest 0^{++} state is at 1640(40) MeV (plus errors of around 10% in setting the scale) [7,16] in pure SU(3) Yang-Mills theory. In unquenched calculations there are new complications. The dynamical sea quarks will cause the glueball and flavor singlet fermionic 0^{++} interpolating operators to couple to the same physical states. In fact, it no longer makes sense

to talk about glueball states in dynamical QCD calculations (although there are glueball interpolating operators). There are only flavor singlet 0^{++} mesons that we sometimes denote by FS. Also the 0^{++} states can decay into meson pairs, so the decay width of hadron may play an important role in the dynamics.

Unlike in quenched QCD which has a degenerate isoscalar and isovector scalar meson (made from $\bar{q}q$), the isoscalar state mixes with the glueball, being reduced in mass if it is lighter than the glueball. In detail, this mixing will be a function of the lattice parameters and sea-quark masses. These will have to be extrapolated to their physical values to get the physical mixing.

There are predictions [17,18] that the width for glueball decay to two mesons is large relative to its mass. Although one exploratory lattice calculation [19] found that the decay width of the 0^{++} was 108(29) MeV in the quenched approximation.

Indeed, the standard formalism for determining the masses of hadrons on a lattice may be inappropriate in the presence of open decay channels and special techniques may be needed [20–22]. The MILC collaboration reported problems in extracting the masses of nonsinglet 0^{++} [23] and 1^{-+} [24] mesons, that they attributed to open decay channels. The techniques to study decay widths using lattice QCD have recently been reviewed [25]. All the above discussions indicate that it is essential to study the singlet 0^{++} mass spectrum with dynamical fermions.

The basic formalism for the mixing of pure glue and $\bar{q}q$ states was described in previous work [26] from UKQCD. That paper obtained surprisingly light values for the flavor-singlet scalar meson, but at relatively coarse lattice spacing. Here we explore this issue using a smaller lattice spacing. We also determine the spectrum using lattices with a different gauge action since the lattice artifacts (for instance big order a^2 corrections to the scalar mass) will then be different.

In detail, we use dynamical gauge configurations generated by UKQCD using fully nonperturbatively improved clover fermions [27]. The lattice spacing is finer, $a \sim 0.1$

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fm, than in our previous study [26]. Another technical improvement is that we use fermionic scalar mesons at nonzero momentum. This helps to get a better signal for the larger volumes. This was regularly used for pure glue operators [28], but is not routinely used for fermionic operators. The glueball spectrum using only glue operators has been presented in [27,28]. The inclusion of $\bar{q}q$ operators in this study allows us to address mixing issues. UKQCD has recently published a lattice study of the non-singlet 0^{++} mesons [29].

We also use gauge configurations from the CP-PACS collaboration [30]. These configurations were generated with an improved gauge action. This can help to address some of the potential issues with using the clover action in combination with the Wilson pure gauge action [31,32]. In particular the lattice spacing dependence of the mass of the 0^{++} states is known to be large with the Wilson pure gauge action [8,9,33,34]. There have also been claims that the unquenched calculations that use the clover fermion action, with the Wilson single plaquette action, are affected by bulk phase transition in the adjoint plane [31,32,34–37]. One conjectured consequence of the phase transition was the suppression of the 0^{++} glueball masses obtained by Hart and Teper [28] from $N_f = 2$ unquenched QCD. The JLQCD collaboration [35] found the effect of the adjoint phase transition was reduced by the use of improved gauge actions, such as the Iwasaki action. Hence the CP-PACS data will be an important cross check on results.

II. PARAMETERS OF THE LATTICE CALCULATION

We use gauge configurations from the UKQCD [27] and CP-PACS [30] collaborations. For the gauge configurations from UKQCD, the nonperturbatively improved clover action was used to generate the unquenched gauge configurations, with a clover coefficient $c_{SW} = 2.0171$. The β value was 5.2 and the lattice volume was $16^3 \times 32$. The sea quark κ values of 0.1350 and 0.1355 were used for the singlet correlators. The hadron spectrum, potential, and some glueball estimates from this data set have been presented in [27,28].

We use two sets of data with $N_f = 2$ from the CP-PACS collaboration at $\beta = 1.95$ [30]. The tadpole improved clover action with clover coefficient of $c_{SW} = 1.53$ and the Iwasaki renormalized group improved gauge action were used. The lattice size is $16^3 \times 32$ and we use valence quark masses equal to those of the sea quarks. The lattice details are summarized in Table I.

The methods we use to compute the disconnected diagrams have been described in earlier publications [26,38]. Essentially we use random Z_2 volume sources to estimate the bubble diagrams, with the variance reduction technique described in [26]. We have also published data for the singlet pseudoscalar channel [38] from some of this data

TABLE I. Simulation details for CP-PACS and UKQCD data sets.

Code	N_{gauge}	κ	r_0/a	am_{PS}
C390	648	0.1390	2.651	0.729
C410	490	0.1410	3.014	0.427
U350	144	0.1350	4.75	0.405
U355	416	0.1355	5.04	0.294

set. We have presented preliminary results for a subset of the data used in this paper in [39].

III. FIT METHODS

To extract a good signal from lattice QCD calculations, it has been found to be essential to use a variational basis of correlators

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)^\dagger \mathcal{O}_j(0) | 0 \rangle. \quad (1)$$

In terms of the path integral the matrix C_{ij} is

$$C_{ij}(t) = \frac{1}{Z} \int dU \int d\psi \int d\bar{\psi} e^{-S_F - S_G} \mathcal{O}_i(t)^\dagger \mathcal{O}_j(0), \quad (2)$$

where S_F is the clover fermion action (lattice approximation to the Dirac Lagrangian) and S_G is Wilson gauge action (lattice approximation to the gauge action). The full details of the Lagrangians are described in our earlier work on the light hadron spectrum [27]. In the fermion sector we use fuzzed and local operators as basis states ($\mathcal{O}_i(t)$). The fuzzing method is described in the spectrum study [27] of UKQCD. In the pure glue sector, two types of smeared glueball operators [28] are included.

We use factorising (or variational) fits to extract the masses and amplitudes

$$C_{ij}(t) = \sum_{m=0}^M c_i^m c_j^m e^{-E_m t}. \quad (3)$$

The scalar channel (0^{++}) at zero momentum has a vacuum contribution which we account for either by fixing $E_0 = 0$ [40] or by evaluating vacuum subtracted correlations. M is the number of states in the fit which can be 1 to 3. For our multistate fits, we quote the masses for all states. However we regard the highest state as a “truncation error”, and this mass may not correspond to a physical state.

Any state with the same quantum numbers as the operator \mathcal{O}_j will couple to that channel. If the amplitude is small then it will be difficult to extract the mass of the state. We feel it is better to use as many different interpolating operators in constructing the correlator matrix. If the singlet 0^{++} states couple to both pure glue and fermionic operators, then it must be better to use additional basis states, provided they are not too noisy and are sufficiently independent of the other states. This can help to stabilize the multistate fits. Hence our best results are from fits from

order 4 smearing matrices that include both glueball and fermionic operators, each with two spatial sizes.

The Iwasaki action has ghost states that contribute to the correlators at small time distances [34,41]. This issue has recently been studied by Necco [34] in quenched QCD for a variety of improved gauge actions. Here we shall restrict our fits to $t > a$ which should reduce these potential problems.

IV. RESULTS

In Fig. 1 we show an effective mass plot for the U355 data at zero momentum together with a fit. This illustrates the quality of the data and also the advantage of having many different correlations to fit simultaneously. In Figs. 2 and 3 we show effective mass plots for the C410 and C390 data sets, respectively. With our statistics there is no sign of any qualitative difference between the effective mass plots in Fig. 1 and in the two Figs. 2 and 3, this gives us confidence that we are not affected by ghost states caused by the nonlocality of the Iwasaki gauge action. This is consistent with the perturbative calculation by Necco [34], who estimated that the ghost state would not contribute for times $t \gg a$, using the formalism of Luscher and Weisz [41]. The starting time for our fits is time slice 2 (see Table II).

In Table II, we collect our fit results for the flavor-singlet scalar. Results for the flavor nonsinglet were presented previously [29]. These results use the 4 by 4 matrix of correlators that includes pure glue and pure fermionic operators in the same fit. For connected correlators we use the conventional method with sources at the origin

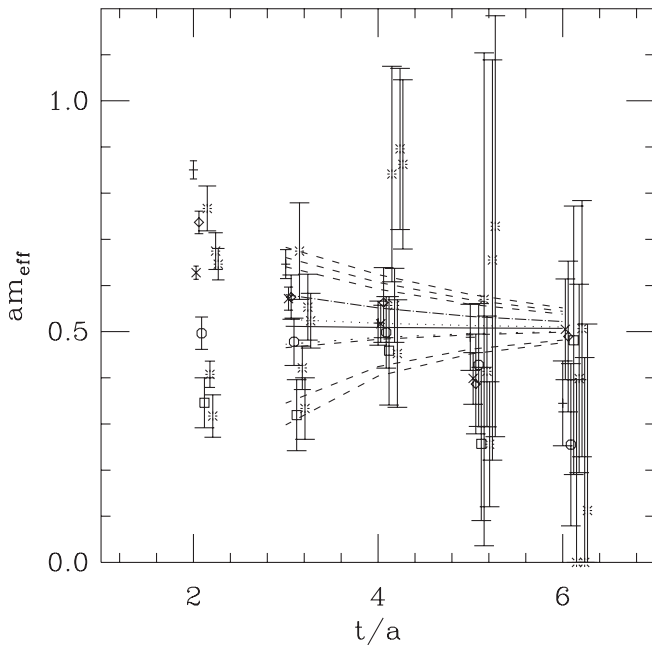


FIG. 1. Singlet scalar meson effective mass plot from fermionic and gluonic operators from U355 with momentum zero.

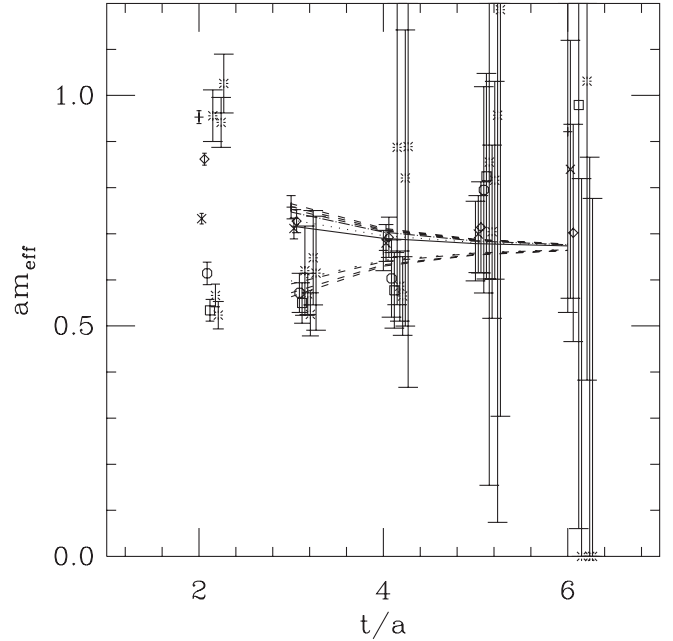


FIG. 2. Singlet scalar meson effective mass plot from fermionic and gluonic operators from C410 with momentum zero.

(with 4 time sources for U355). For disconnected fermionic loops we use the method of [26] with 100 stochastic samples. For “glueball” operators we used square fuzzed Wilson loops of differing sizes, selected to have good overlap with the scalar glueball [28]. We measure glueball operators every 10 gauge configurations and fermionic disconnected operators every 10 (40 for U350 and 20 for

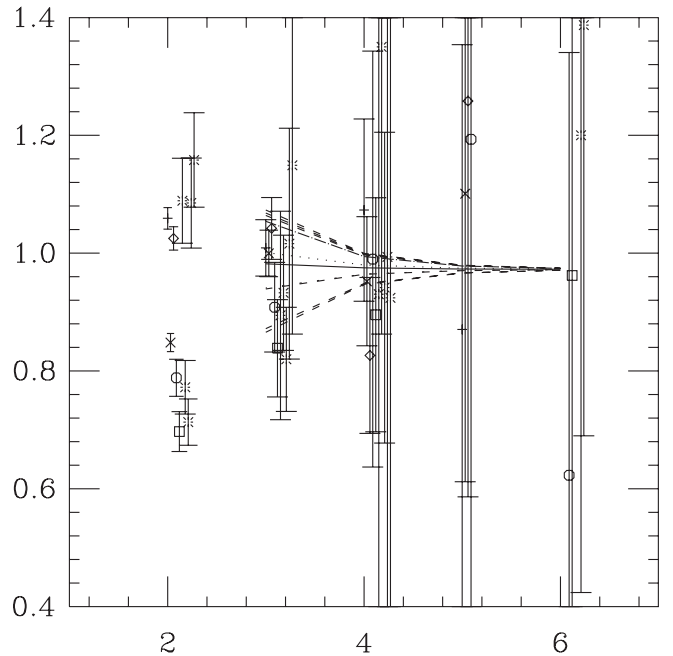


FIG. 3. Singlet scalar meson effective mass plot from fermionic and gluonic operators from C390 with momentum zero.

TABLE II. Fit results for flavor-singlet scalar mesons. These are fits to four by four matrices of correlations to the t -range shown with momentum ($p = 2\pi n/L$). Fits to momentum zero are to the vacuum subtracted correlators here. For these fits involving disconnected contributions, we expect a χ^2/dof close to 1.0 since the absolute error stays constant with increasing t .

Code	n	Region	χ^2/dof	E_0	E_1	E_2
U350	000	2–6	24/40	0.64(3)	1.17(25)	...
U350	100	2–6	24/35	0.69(4)	0.86(7)	1.23(29)
U355	000	2–6	24/40	0.510(14)	1.06(12)	...
U355	100	2–6	34/35	0.62(3)	1.01(5)	1.47(10)
U355	110	2–6	36/35	0.75(2)	1.05(3)	1.63(9)
C390	000	2–6	45/45	0.98(4)
C410	000	2–6	19/40	0.67(2)	1.56(40)	...

U355). The data are binned into groups of 40 trajectories to avoid the impact of any autocorrelation on the error analysis.

We use correlated fits [42] to ascertain the minimum t -value that gives an acceptable fit. We then quote results from an uncorrelated fit to that range. We find similar results for the two lightest singlet states at nonzero momentum from 3 state fits to t ranges of 2–6 to 2–10 and from 2 state fits for t ranges of 3–6 to 3–10. Thus the systematic error on the ground state energy from varying the fits seems to be smaller than the statistical error which is quoted in the table.

For the FS channel, the nonzero momentum results are more stable since they do not involve a vacuum subtraction and also they involve additional statistics from constructing the correlator from momenta in each spatial direction. We expect that

$$E^2 = m^2 + p^2, \quad (4)$$

where $p = 2n\pi/L$ with integer vector n where we explore $n.n = 0, 1, 2$ here. As shown in Fig. 4, we find consistency (U355 is plotted since for that case we have 3 momentum values) within our statistical errors. The mass value is then the intercept and is consistent with the results in Table II.

Using glueball operators at zero and nonzero momentum, UKQCD [27,28] previously found $aM^{0^{++}} = 0.628(30)$ and $aM^{0^{++}} = 0.626(41)$ at $\kappa = 0.1355$ and at $\kappa = 0.1350$ respectively. The masses were estimated using effective masses after finding the optimal basis using a variational technique [28]. This previous analysis claimed that the 0^{++} state was lighter in $N_f = 2$ with the mass ratio from dynamical to quenched lattices of 0.85(3).

Using Eq. (4) with the data in Table II, we obtain $am_1(0^{++}) = 0.51(2)$ for U355 and 0.60(4) for U350. Hence, doing a combined fit to glueball and $\bar{q}q$ 0^{++} operators has produced additional suppression of the 0^{++} mass with the inclusion of dynamical fermions. It is only for U355 that we have a reliable estimate of the first excited state, obtaining $am_2(0^{++}) = 0.92(6)$. For U355, we find that the fit coefficients of our two lightest states have the

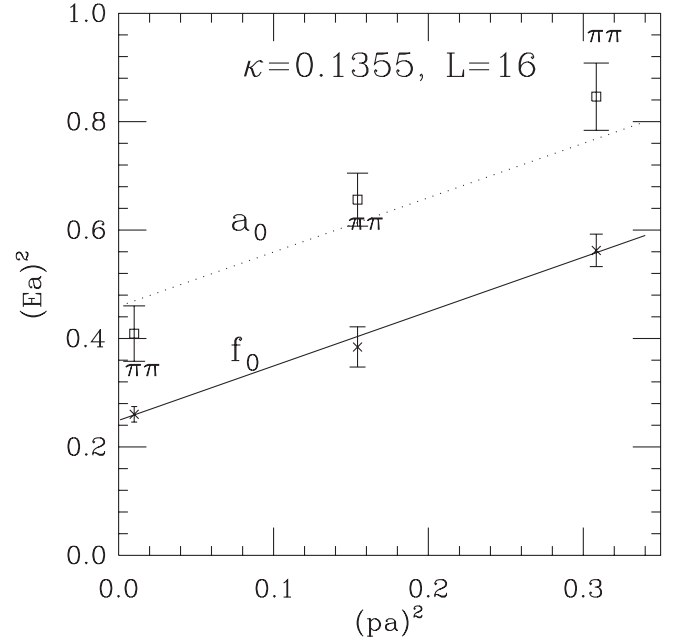


FIG. 4. Scalar meson masses (for U355) versus momentum for both flavor-singlet (f_0) and nonsinglet (a_0). The straight lines show the expected slope for a relativistic mass-energy relationship. The lowest $\pi\pi$ thresholds are also illustrated.

structure expected from a maximal mixing of a gluonic state and a fermionic state. The previous analysis [27,28] using only gluonic operators appears to have been getting close to the weighted mean of the first and second masses which is consistent with the maximal mixing we find here.

In Fig. 5 the 0^{++} masses (from the four basis fit) are plotted with the continuum limit of the quenched data (with standard Wilson glue). The masses are in units of r_0 , that is convenient unit for lattice studies. To convert to MeV the reader can use $1/r_0 \sim 400$ MeV.

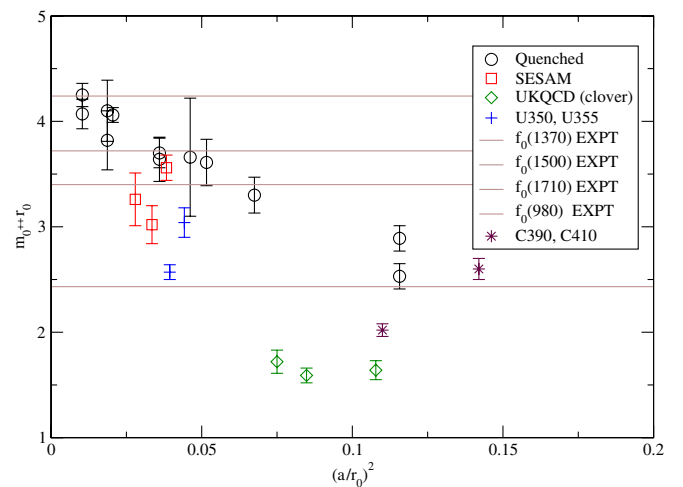


FIG. 5 (color online). Compilation of FS 0^{++} masses versus a^2 . The quenched data are from lattice calculations that use the Wilson gauge action [7,61–64].

Our study using the CP-PACS lattices with Iwasaki glue has revealed somewhat similar results to those obtained previously using Wilson glue [26] at similar lattice spacings. Our results (U355 and U350) with finer lattice spacings also illustrate the suppression of the mass of the FS 0^{++} state relative to the quenched 0^{++} glueball mass at a similar lattice spacing. Thus the feature found previously [26] of surprisingly light f_0 mass is consistent with the new results at a finer lattice spacing and with an improved gauge action at the coarse action. We discuss this further in Sec. VI.

We note that in this study the lightest singlet scalar (f_0) is not substantially lighter than the nonsinglet (a_0), as shown in Fig. 4 for instance. In our previous work at $\beta = 5.2$ using the clover action with $c_{SW} = 1.76$ [26], the singlet mass was around 50% lighter than the nonsinglet scalar mass.

Necco [34] has recently compared the continuum limit for various quantities, including the mass of the 0^{++} glueball state, in quenched QCD using a number of improved gauge actions. In Fig. 6 we plot the masses obtained from the C390 and C410 data sets with the quenched 0^{++} glueball masses from Necco [34]. This shows that a lower mass is obtained with dynamical quarks for this improved gauge action also. The large errors on the data from Necco [34] are caused by the problems in seeing a plateau in her variational analysis (that also does not include $\bar{\psi}\psi$ operators as we do here).

The plot of the 0^{++} mass as a function of the square of the lattice spacing (Fig. 5) is slightly misleading for unquenched QCD, because it does not show the quark mass dependence. In Fig. 7 we plot the FS 0^{++} mass as a function of the pion mass squared. We did not attempt a chiral extrapolation.

We have not included in the summary Fig. 5, the recent results from [43]. This calculation does not see a deviation of the unquenched FS 0^{++} masses (using glueball inter-

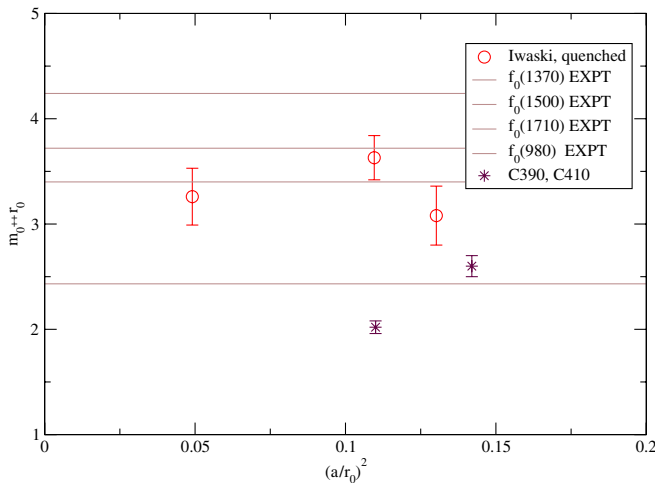


FIG. 6 (color online). FS 0^{++} mass from this calculation compared to the quenched 0^{++} glueball mass from Necco [34].

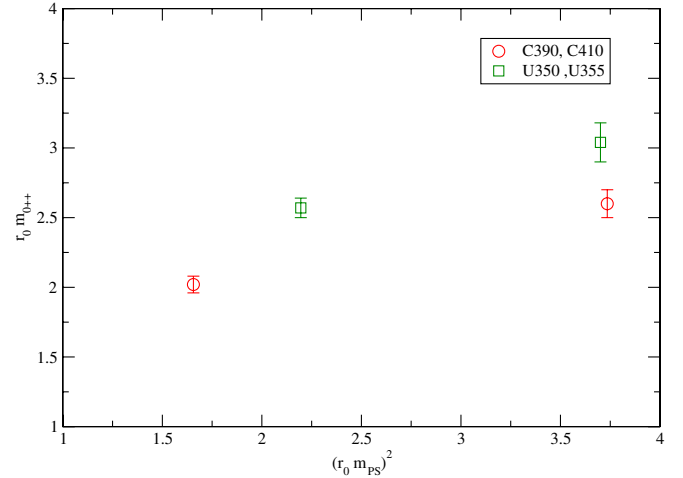


FIG. 7 (color online). FS 0^{++} mass as a function of pion mass squared.

polating operators) from the quenched 0^{++} glueball masses, but did not include simultaneous fits to the glue and fermionic interpolating operators.

V. DECAY

The dominant hadronic decay of the flavor-singlet 0^{++} meson is to two pions. In Fig. 8 we show the relevance of the decay threshold. The proximity of this threshold may have significant impact on the spectrum, also it allows an investigation of the decay transition strength itself.

The f_0 meson has an allowed S-wave decay to $\pi\pi$ and this has a low threshold so will have a significant impact on the analysis. In a finite spatial volume, as on a lattice, the two-body states are discrete and one can concentrate on the lightest state, with the two pions at rest. A study of the FS sector should then involve gluonic operators (glueball-like); fermionic ($\bar{q}q$) and two meson ($\pi\pi$) to give the fullest coverage. We have studied the spectrum produced

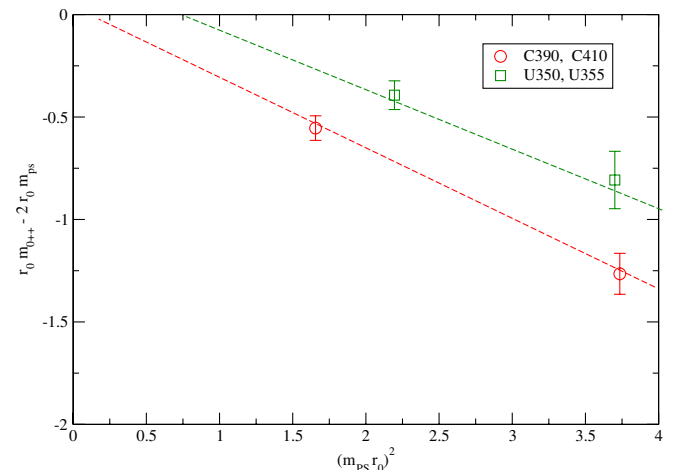


FIG. 8 (color online). Two pion threshold for the FS scalar meson as a function of pion mass squared.

by the first two types of operator above. Here we discuss the prospects for including the two pion channel.

Historically the two pion channel was included in a quenched study of the scalar glueball by Weingarten and Sexton [19] although their results were very preliminary. More recently two-body channels have been studied [29,44] in connection with mesonic decays, such as $\rho \rightarrow \pi\pi$ and $a_0 \rightarrow KK$.

In Fig. 8 the threshold for scalar decay to two lattice pions is plotted. The U355 data has lattice f_0 states at 0.51(2) and 0.92(6) (in lattice units) compared to the $\pi\pi$ threshold on that lattice at 0.59. Thus the heavier f_0 is unstable. This implies that the impact of the two pion channel could potentially be strong.

The fact that the f_0 and two pion states are close means that we may be able to compute decay widths, as was attempted by the GF11 group [19], using the formulation developed for the decay of the heavy 1^{-+} hybrid [45] and ρ meson [25,44].

Of course the theoretical problem of dealing with unstable particles has been solved in principle by Lüscher over ten years ago [46] (see also [47,48]). In his formalism, the scattering phase shift can be extracted from the volume dependence of two-body energy levels. The mass and width of the resonance can then be extracted from the phase shift. Sufficient precision has not been available to pursue this, so far. Thus the more qualitative treatment of hadronic transitions, described above, has been used, instead.

The lack of progress in the study of f_0 meson decay to $\pi\pi$ is due to the presence of disconnected diagrams in the decay—see Fig. 9. Following the methods used for ρ decay, one can estimate the relative importance of this disconnected contribution Q compared to the connected triangle diagram T . The triangle contribution is relatively easy to evaluate unlike the disconnected contribution. In fact results for the transition T of a scalar to two pseudo-scalar mesons have been presented [29] elsewhere.

Using time slice stochastic sources (as used by Ref. [44]) from 100 gauge configurations of U355, and combining these with the volume stochastic source results used above, we can evaluate these contributions: see Fig. 10. This illustrates that even though we have measured the disconnected contribution Q from every time-value and space-value on each of 100 gauge configurations, the resulting error is very large.

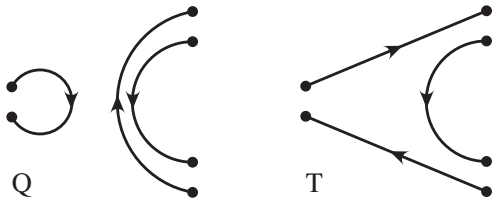


FIG. 9. The quark structure of the disconnected contribution (Q) and the triangle contribution (T) to the transition $f_0 \rightarrow \pi\pi$.

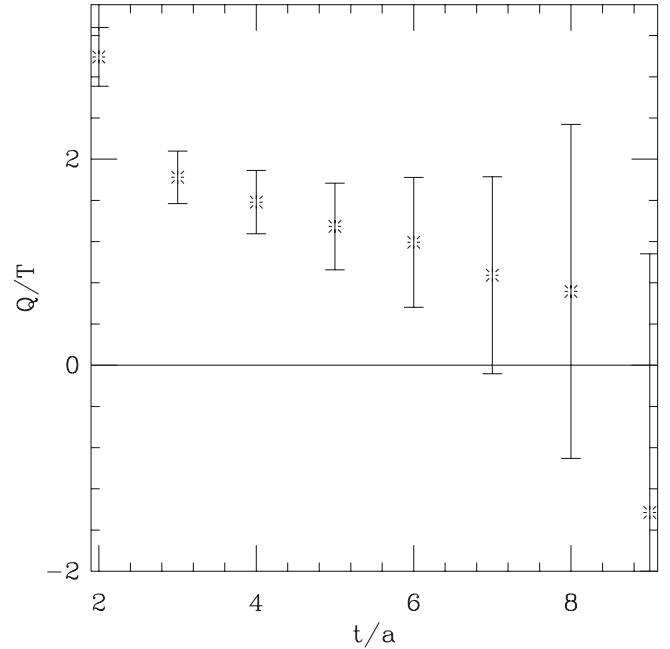


FIG. 10. The relative contribution of the disconnected contribution (Q) (vacuum subtracted) and the triangle contribution (T) to the transition $f_0 \rightarrow \pi\pi$ with all states at zero momentum for lattice U355.

Thus a thorough study of decays of FS scalar mesons must await a study with much larger statistics than that we have available.

VI. DISCUSSION AND CONCLUSIONS

The inclusion of the $\bar{\psi}\psi$ 0^{++} operators with the pure glue 0^{++} operators in the variational analysis has produced a suppression of the masses relative to the quenched results at an equivalent lattice spacing (see Fig. 5). There are two main possible interpretations of these results. It could be possible that the lattice errors ($O(a^2)$) in the FS 0^{++} masses are larger for unquenched QCD than for quenched QCD, because of the phase structure of the theory. Another possibility is that in the unquenched calculations the FS 0^{++} mesons couple to the $f_0(980)$ or even to the $f_0(400-1200)$, thus driving their mass way below the quenched 0^{++} glueball mass at 1600 MeV. We discuss these two possibilities in turn.

In quenched QCD the $O(a^2)$ lattice spacing errors on the mass of the 0^{++} glueball are large (see the quenched data in Fig. 5). There are lattice formalisms that add irrelevant operators to the lattice actions that in principle can remove the $O(a^2)$ errors (this is known as improvement). The first systematic use of the improvement formalism for the 0^{++} glueballs was by Morningstar and Peardon [8,9]. They also found large lattice spacing dependence in the mass of the 0^{++} glueball even with their improved gauge actions and this phenomena is colloquially known in lattice QCD circles as “the scalar dip”. Further studies of the 0^{++}

masses on the lattice with a variety of improved gauge actions by Necco [34] and Niedermayer *et al.* [40,49] still found a strong lattice dependence of the results. As shown by Morningstar and Peardon [8,9,50,51] the lattice spacing dependence of the 0^{++} glueball in quenched QCD is particularly large on coarse lattice spacings in the range 0.2–0.4 fm. The lattice spacing dependence of the 0^{++} mass is probably the best known example of the breakdown of the simplest application of the improvement and perfect action programs to reduce lattice spacing dependence.

In quenched QCD the explanation of the strong lattice spacing dependence of the mass of the 0^{++} state is the influence of the bulk phase transition in the adjoint plane [50–53] close to the region where calculations are done. The way to reduce the lattice spacing dependence is to add an adjoint term to the gauge action with a negative coefficient to work in a region of the parameter space away from the phase transition [33,54]. This matters for unquenched calculations because it has been conjectured that the clover fermion action induces an adjoint gauge term that will increase the lattice spacing dependence of the FS 0^{++} mesons [31,32,35]. If this interpretation is correct, then the suppression of the FS 0^{++} masses in Fig. 5 is purely a lattice artifact. The effect of the quark action on the adjoint term in unquenched calculations has not been studied in the same detail as the effect of the adjoint term on the scaling of glueball masses in quenched QCD.

There is additional evidence that suggests that the suppression of the masses of FS 0^{++} mesons seen in this calculation is just not a lattice artifact. The induced adjoint term is also thought to be the cause of the phase transition seen by JLQCD [35] and Farchioni *et al.* [37]. The phase structure of the unquenched lattice calculations that use Wilson-like fermion actions have been reviewed by Shindler [55]. Both JLQCD [35] and Farchioni *et al.* [37] found that the phase transition was weakened or removed by the use of an improved gauge action, such as the Iwasaki action. In the unquenched case the conjecture is that the quark action induces an effective adjoint term, that is also thought to be the cause of the observed phase transitions, that increases the lattice spacing errors on the mass of the 0^{++} FS meson compared to the value from quenched QCD at the same lattice spacing. The observed lack of a phase transition for unquenched calculations that use the Iwasaki gauge action is suggestive that the lattice spacing errors between quenched and unquenched QCD should be similar in magnitude.

The comparison of the mass of the 0^{++} FS meson between quenched and unquenched QCD is complicated by the sea quark mass dependence of the 0^{++} FS meson mass, so Fig. 5 does not tell the full story. With only two quark masses for each type of action we cannot attempt a chiral extrapolation, but the data in Fig. 7 suggest the mass of the 0^{++} FS meson is between 1.75 and 2.25 in units of r_0

for both gauge actions in the chiral limit. The consistency of the suppression of the FS 0^{++} masses between calculations that use the Wilson and Iwasaki gauge actions, coupled with the lack of a phase transition for the Iwasaki gauge action, seems to us reasonable evidence that the suppression of the masses of the FS 0^{++} mesons is not just a lattice artifact.

There are other physical reasons why intuition based on the phase structure of quenched QCD may be not be a good guide for unquenched lattice calculations of the FS 0^{++} mesons. If the $f_0(980)$ or $f_0(400-1200)$ have any quark-antiquark components then they will couple to our interpolating operators and drive the ground state of the 0^{++} channel to the 1 GeV level. Although there have been recent claims by Mathur *et al.* [15] that the $a_0(980)$ is molecular, an unquenched calculation by UKQCD provided evidence for the $a_0(980)$ state to be a quark-antiquark state [29].

One of the advantages of lattice QCD calculations is that they can be used to understand the physical mechanisms behind the numbers. In the quenched theory the $\bar{q}q$ and glueball states couple to distinct 0^{++} states. Calculations that include very heavy dynamical quarks should be similar to the distinct quenched states. As the mass of the sea quarks is reduced the states will start to mix.

For example: taking a_0 masses from ref. [29], we get at U355 $r_0m(a_0) = 3.23(20)$. Then at this lattice spacing, the quenched glueball mass is around $r_0m(GB) = 3.6$. A mixing shift of $r_0\Delta E = 0.63$ would move these levels to 2.6 and 4.2, respectively, in excellent agreement with our two observed levels at 2.57(10) and 4.63(30) for U355. Furthermore, since the mixing shift is large compared to the initial splitting (0.37 in the above example), we would expect approximately maximal mixing in the observed spectrum—which is indeed what we find from the fit coefficients. Defining a mixing from

$$\tan^2 \alpha = -\frac{\langle 0|G|1\rangle\langle 0|\bar{q}q|2\rangle}{\langle 0|\bar{q}q|1\rangle\langle 0|G|2\rangle},$$

where G and $\bar{q}q$ refer to the operators used to create the states and 1 and 2 refer to the two lightest states observed on the lattice. The $\langle 0|G|1\rangle$ are one of the c_j^n coefficients in Eq. (3). For U355 with momentum $n.n = 1$, we obtain a mixing angle $\alpha = 57(10)^\circ$ where 45° would correspond to maximal mixing.

In terms of a mass mixing matrix, this example would correspond to

$$\begin{pmatrix} r_0m(\bar{q}q) & r_0X \\ r_0X & r_0m(GB) \end{pmatrix} = \begin{pmatrix} 3.2 & 0.8 \\ 0.8 & 3.6 \end{pmatrix} \quad (5)$$

so that the mixing matrix element is $r_0X = 0.8$ and the resulting eigenvalues are those we observe (2.6 and 4.2) on the lattice. The mixing would be nearly maximal (actually 38°) in this example. This same mixing matrix element (X) and glueball mass with $r_0m(\bar{q}q) = 3.56(14)$ [29] for U350

would yield eigenstates at 2.8 and 4.4 where the former is in good agreement with our lattice result, 2.85(19), for U350. For the C410 data, at a coarser lattice spacing, a larger value of $r_0X \approx 1.1$ would give a splitting of the $r_0m(\bar{q}q) = 3.1(1)$ and $r_0m(GB) \approx 3.1$ values to agree with our lightest f_0 state at $r_0m_1 = 2.0(1)$. In an earlier study with $N_f = 2$ at a coarser lattice spacing, an estimate of $r_0X \approx 1.5$ ($3.65\sqrt{20.3}$) was quoted [26].

Thus we find that in our lattice spacing and quark mass range, the fermionic (with $N_f = 2$ degenerate sea-quarks) and gluonic operators create states which are maximally mixed, with a mixing matrix element of $r_0X \approx 0.8$ which corresponds to an energy of approximately 320 MeV. This value is smaller than that estimated previously using $N_f = 2$ sea-quarks with a coarser lattice spacing and than that we estimate from C410 with a similar coarse lattice spacing.

If the mixing matrix element X were to remain constant as the quark mass is reduced, then the estimate [29] of the a_0 mass (for $N_f = 2$) is 1 GeV while the glueball is around 1.6 GeV. These input masses would then be mixed to 0.86 and 1.74 GeV. This gives some indication of the magnitude of mixing effects which could be present in the experimental spectrum and require calculation with similar small lattice spacings.

In the experimental spectrum, however, there are expected to be significant effects arising from the $\bar{s}s$ scalar meson, which we have neglected here. This $\bar{s}s$ state should mix with gluonic operators and one will need a 3×3 model to incorporate this adequately. Lattice QCD is able to include $2 + 1$ flavours of sea-quark and this will be appropriate for a fuller confrontation with experiment.

In quenched QCD the first calculation [7] that computed the mass of the 0^{++} glueball in the continuum limit used a lattice spacings down to 0.05 fm to have a reliable continuum extrapolation. Unquenched calculations that use the Wilson gauge action may also have similar scaling violations.

As we have noted, there appears to be a significant lattice spacing dependence in the mixing. To quantify the amount of glueball to $\bar{q}q$ mixing in f_0 mesons will require a dynamical fermion calculation at even finer lattice spac-

ings such as 0.05 fm. For Wilson type fermions, recent algorithmic advances [56–58] means that this may now be just attainable on the current generation of machines.

The eventual definitive study of flavor-singlet scalar mesons will need fine lattice spacing, dynamical simulations with light quarks (plus a strange quark in the sea) and large statistics to enable the disconnected diagrams to be evaluated accurately. The methods we have used with $N_f = 2$ degenerate sea-quarks of mass down to about 50% of the strange quark mass show the way forward. We find substantial mixing of glueballs with $\bar{q}q$ states in the f_0 spectrum and we expect that feature to survive in a future study.

If the size of the lattice spacing dependence of our unquenched results for the mass of the FS 0^{++} meson is similar to that in quenched QCD, then our results in Figs. 5 and 7 show that the effect of the sea quarks is to drive the mass of the FS state from the 1600 MeV of the quenched glueball towards 1 GeV. We have presented arguments to support our assumptions on the lattice spacing dependence, but as discussed above improved lattice calculations are required for definitive results.

This calculation does not favor the weak mixing of glue and $\bar{\psi}\psi$ operators proposed by Weingarten and Lee and other groups [59,60]. Looking at our data it is not clear that a mixing scheme based on only the states $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ is complete enough to determine the fate of the quenched glueball.

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- [1] E. Klempt, hep-ex/0101031.
 - [2] F.E. Close, Int. J. Mod. Phys. A **17**, 3239 (2002).
 - [3] T. Barnes, AIP Conf. Proc. 619 (AIP, New York, 2002), p. 447.
 - [4] F.E. Close and N.A. Tornqvist, J. Phys. G **28**, R249 (2002).
 - [5] F.E. Close, Int. J. Mod. Phys. A **20**, 5156 (2005).
 - [6] M.R. Pennington, J. Phys.: Conf. Ser. **18**, 1 (2005).
 - [7] G.S. Bali *et al.* (UKQCD), Phys. Lett. B **309**, 378 (1993).
 - [8] C.J. Morningstar and M.J. Peardon, Phys. Rev. D **56**, 4043 (1997).
 - [9] C.J. Morningstar and M.J. Peardon, Phys. Rev. D **60**, 034509 (1999).
 - [10] Y. Chen *et al.*, Phys. Rev. D **73**, 014516 (2006).
 - [11] M. Ablikim *et al.* (BES), Phys. Rev. Lett. **96**, 162002 (2006).
 - [12] I. Caprini, G. Colangelo, and H. Leutwyler, Phys. Rev. Lett. **96**, 132001 (2006).
 - [13] H. Leutwyler, hep-ph/0608218.
 - [14] T. Kunihiro *et al.* (SCALAR), Phys. Rev. D **70**, 034504 (2004).

- (2004).
- [15] N. Mathur *et al.*, hep-ph/0607110.
- [16] M. J. Teper, hep-th/9812187.
- [17] S. Narison, Nucl. Phys. B, Proc. Suppl. **96**, 244 (2001).
- [18] M. R. Pennington, hep-ph/9811276.
- [19] J. Sexton, A. Vaccarino, and D. Weingarten, Phys. Rev. Lett. **75**, 4563 (1995).
- [20] C. Michael, Nucl. Phys. **B327**, 515 (1989).
- [21] T. A. DeGrand, Phys. Rev. D **43**, 2296 (1991).
- [22] C. McNeile, hep-lat/0307027.
- [23] C. W. Bernard *et al.*, Phys. Rev. D **64**, 054506 (2001).
- [24] C. Bernard *et al.*, Phys. Rev. D **68**, 074505 (2003).
- [25] C. Michael, Proc. Sci. LAT2005 (2005) 008 [hep-lat/0509023].
- [26] C. McNeile and C. Michael (UKQCD), Phys. Rev. D **63**, 114503 (2001).
- [27] C. R. Allton *et al.* (UKQCD), Phys. Rev. D **65**, 054502 (2002).
- [28] A. Hart and M. Teper (UKQCD), Phys. Rev. D **65**, 034502 (2002).
- [29] C. McNeile and C. Michael (UKQCD), Phys. Rev. D **74**, 014508 (2006).
- [30] A. Ali Khan *et al.* (CP-PACS), Phys. Rev. D **65**, 054505 (2002).
- [31] A. Hart (UKQCD), Nucl. Phys. B, Proc. Suppl. **106**, 575 (2002).
- [32] R. Sommer *et al.* (ALPHA), Nucl. Phys. B, Proc. Suppl. **129**, 405 (2004).
- [33] C. Morningstar and M. J. Peardon, Nucl. Phys. B, Proc. Suppl. **73**, 927 (1999).
- [34] S. Necco, Nucl. Phys. **B683**, 137 (2004).
- [35] S. Aoki *et al.* (JLQCD), Phys. Rev. D **72**, 054510 (2005).
- [36] K. Jansen, Nucl. Phys. B, Proc. Suppl. **129**, 3 (2004).
- [37] F. Farchioni *et al.*, Eur. Phys. J. C **42**, 73 (2005).
- [38] C. R. Allton *et al.* (UKQCD), Phys. Rev. D **70**, 014501 (2004).
- [39] A. Hart, C. McNeile, and C. Michael (UKQCD), Nucl. Phys. B, Proc. Suppl. **119**, 266 (2003).
- [40] F. Niedermayer, P. Rufenacht, and U. Wenger, Nucl. Phys. **B597**, 413 (2001).
- [41] M. Luscher and P. Weisz, Nucl. Phys. **B240**, 349 (1984).
- [42] C. Michael and A. McKerrell, Phys. Rev. D **51**, 3745 (1995).
- [43] E. B. Gregory, A. C. Irving, C. McNeile, S. Miller, and Z. Sroczynski, Proc. Sci. LAT2005 (2005) 027 [hep-lat/0510066].
- [44] C. McNeile and C. Michael (UKQCD), Phys. Lett. B **556**, 177 (2003).
- [45] C. McNeile, C. Michael, and P. Pennanen (UKQCD), Phys. Rev. D **65**, 094505 (2002).
- [46] M. Luscher, Nucl. Phys. **B364**, 237 (1991).
- [47] B. S. DeWitt, Phys. Rev. **103**, 1565 (1956).
- [48] U. J. Wiese, Nucl. Phys. B, Proc. Suppl. **9**, 609 (1989).
- [49] P. Rufenacht and U. Wenger, Nucl. Phys. **B616**, 163 (2001).
- [50] C. Morningstar and M. J. Peardon, Nucl. Phys. B, Proc. Suppl. **83**, 887 (2000).
- [51] C. Morningstar and M. J. Peardon, AIP Conf. Proc. 688 (AIP, New York, 2004), p. 220.
- [52] U. M. Heller, Phys. Lett. B **362**, 123 (1995).
- [53] T. Blum *et al.*, Nucl. Phys. **B442**, 301 (1995).
- [54] M. Hasenbusch and S. Necco, J. High Energy Phys. 08 (2004) 005.
- [55] A. Shindler, Proc. Sci. LAT2005 (2005) 014 [hep-lat/0511002].
- [56] M. Hasenbusch, Phys. Lett. B **519**, 177 (2001).
- [57] C. Urbach, K. Jansen, A. Shindler, and U. Wenger, Comput. Phys. Commun. **174**, 87 (2006).
- [58] M. Luscher, Proc. Sci. LAT2005 (2005) 002 [hep-lat/0509152].
- [59] W.-J. Lee and D. Weingarten, Phys. Rev. D **61**, 014015 (1999).
- [60] H.-Y. Cheng, C.-K. Chua, and K.-F. Liu, Phys. Rev. D **74**, 094005 (2006).
- [61] C. Michael and M. Teper, Nucl. Phys. **B314**, 347 (1989).
- [62] A. Vaccarino and D. Weingarten, Phys. Rev. D **60**, 114501 (1999).
- [63] H. Chen, J. Sexton, A. Vaccarino, and D. Weingarten, Nucl. Phys. B, Proc. Suppl. **34**, 357 (1994).
- [64] P. de Forcrand, G. Schierholz, H. Schneider, and M. Teper, Phys. Lett. **152B**, 107 (1985).