Calculation of the branching ratio of $B^- \to h_c + K^-$ in perturbative QCD

Xue-Qian Li, Xiang Liu,* and Yu-Ming Wang

Department of physics, Nankai University, Tianjin 300071, China

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The branching ratio of $B^- \to h_c + K^-$ is re-evaluated in the PQCD approach. In this theoretical framework all the phenomenological parameters in the wave functions and Sudakov factor are priori fixed by fitting other experimental data, and in the whole numerical computations we do not introduce any new parameter. Our results are consistent with the upper bounds set by the *BABAR* and Belle measurements.

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I. INTRODUCTION

h_c of $^1P_1$ is the spin-partner of η_c in the charmonium family, however, it seems h_c behaves very differently from other family members. Searching for h_c, as well as h_b is a challenging task for both experimentalists and theorists of high energy physics [1]. h_c was first seen at the CERN Intersecting Storage Rings (ISR) [2], and later the E760 group at Fermilab reported observation of h_c by studying the $p\bar{p} \to h_c \to \pi^0 J/\psi$ [3]. However, the E835 group claimed that they did not see h_c in this channel. Instead, E835 reported observation of h_c in another channel $p\bar{p} \to h_c \to \gamma\eta_c \to \gamma\gamma\gamma$ [4]. Recently, the *BABAR* collaboration set an upper bound for the production rate of h_c as 3.4×10^{-6} for $BR(B^- \to h_c K^-) \times BR(h_c \to J/\psi \pi^+\pi^-)$ at 90% C.L. [5]. By contrast, the Belle collaboration reported null result on searching for h_c via $B^\pm \to h_c K^\pm$ and gave an upper bound on the branching ratio as $BR(B^+ \to h_c K^+) < 3.8 \times 10^{-5}$ [6]. The CLEO Collaboration has announced observation of h_c in the decay $\psi(2S) \to \pi^0 h_c \to \pi^0 \gamma\eta_c$ with a very small branching ratio [7,8]. As a comparison, so far, the BES II collaboration has not seen h_c yet, and searching for it may be an important issue for the upgraded stage BES III.

One needs to understand the experimental status of the production rate of h_c. Moreover, a relatively larger production rate may be associated with new physics [9–11], therefore study of h_c production may provide a chance to investigate effects of new physics, such as supersymmetry at lower energy scales. Of course, before invoking new physics, one needs to more accurately estimate the production rate of h_c in the standard model (SM).

Suzuki suggested to look for h_c at $B^+ \to h_c K^+ \to \gamma\eta_c K^+ \to \gamma(K\bar{K}\pi)K^+$ and if approximately $BR(B^+ \to h_c K^+) \approx BR(B^+ \to \chi_{c0} K^+)$, he estimated the cascade branching ratio as 2×10^{-5} [12]. Gu considered another decay chain and estimated the branching ratio $BR(B^+ \to h_c K^+ \to \gamma\eta_c K^+ \to \gamma(K_S^0 K^+ \pi^- + \text{c.c.})K^+ \to \gamma(\pi^+\pi^- K^+\pi^- + \text{c.c.})K^+)$ as 3.5×10^{-6} [13].

In the decay of B-meson into charmonium is the so-called internal emission process where due to the color matching the process is suppressed compared to the external emission. Moreover, the nonfactorizable effects would further change the contribution of the internal process besides a color factor of $1/3$ [14]. Therefore it seems that the smallness of $B \to h_c + K$ is natural. However, one could not reproduce experimental value of the $B^- \to \chi_{c0} K^-$ decay rate [15] in the QCD improved factorization, so that the authors of [15] suggested one consider rescattering effects in B meson decay. They claimed that [15] a larger branching ratio for $B^- \to \chi_{c0} K^-$ which is consistent with data, was obtained. Motivated by the same idea, they applied the same scenario to investigate the rescattering effects for $B^- \to h_c K^-$ case, i.e. supposing that the decay $B^- \to h_c K^-$ occurs via the subsequent rescattering effect of $D_s^{(*)} - D^{(*)0}$, the products of decay $B^- \to D_s^{(*)} D^{(*)0}$ [16]. They obtained a branching ratio ($BR(B^- \to h_c K^-) = (2 \sim 10) \times 10^{-4}$) which is much larger than the upper bound set by the Belle collaboration. It is very possible that the branching ratio of $B^- \to h_c K^-$ was overestimated in their work due to uncontrollable theoretical uncertainties, such as some input parameters and basic assumptions adopted in their calculation, which were comprehensively discussed in their paper.

Even though we may suppose that we have full knowledge on the weak and strong interactions at the quark level and can derive the quark-transition amplitude, the most difficult part is evaluation of the hadronic matrix elements of the exclusive processes. In fact, at present, a complete calculation of $B^\pm \to h_c K^\pm$ based on an underlying theoretical framework is absent. The perturbative QCD (PQCD) approach is believed to be successful for estimating transition rates of B and D into light mesons [17], even though there is still dispute about its applicability [18]. The authors of Ref. [19] applied the PQCD approach to study $B^- \to \chi_{c0} K^-$ and also obtained results which satisfactory comply with data. Therefore we have reason to believe that it is appropriate to analyze $B^\pm \to h_c K^\pm$ in this framework. In this work, we calculate decay rate of $B^- \to h_c K^-$ in the PQCD.

Generally, for two-body nonleptonic decays of B meson, both factorizable and nonfactorizable diagrams contribute to the transition amplitudes, however, for $B^- \to h_c K^-$, the contributions from factorizable diagrams disappear since

*Electronic address: liuxiang726@mail.nankai.edu.cn

the conservation of G parity leads to $\langle 0|\bar{c}\gamma_\mu\gamma_5 c|h_c\rangle = 0$ [20]. Therefore, the decay rate of $B^\pm \to h_c K^\pm$ is much different from that in $B^\pm \to J/\psi K^\pm$ [19,21] where both factorizable and nonfactorizable diagrams contribute.

To be more precise than the qualitative understanding, one needs to calculate the nonfactorizable contribution where the QCD effects are accounted. Moreover, since the process is not factorizable, the convolution integral would involve the initial B-meson and all the two produced mesons altogether.

Our numerical results indeed indicate that the order of magnitude of $B^- \to h_c K^-$ should be of order of 3.6×10^{-5}, which is smaller than the upper bound set by the *BABAR* collaboration [5] and slightly below the upper bound given by the Belle collaboration [6]. It is noticed that our result about $B(B^- \to h_c K^-)$ is smaller than that estimated by the authors of [12,13,15] and more consistent with data.

Indeed, a more decisive conclusion should be made as more accurate data are accumulated by BES III, CLEO, *BABAR*, Belle and even the LHCb.

The structure of this paper is organized as follows. After this introduction, we formulate the decay amplitude of $B^- \to h_c K^-$ in the PQCD approach. Then we present our numerical results along with all the input parameters in Sec. III. The last section is devoted to our conclusion and discussion. Some tedious expressions are collected in the Appendix.

II. FORMULATION

The quark-diagrams which contribute to the transition amplitude of $B^- \to ({}^1P_1)h_c + K^-$ are displayed in Fig. 1. As has been discussed in the introduction, the factorized diagrams Fig. 1(a) and 1(b) do not contribute to the amplitude because the G parities of axial meson h_c and axial current are mismatched and in the flavor SU(3) symmetry limit the corresponding hadronic matrix element is forbidden [15,20].

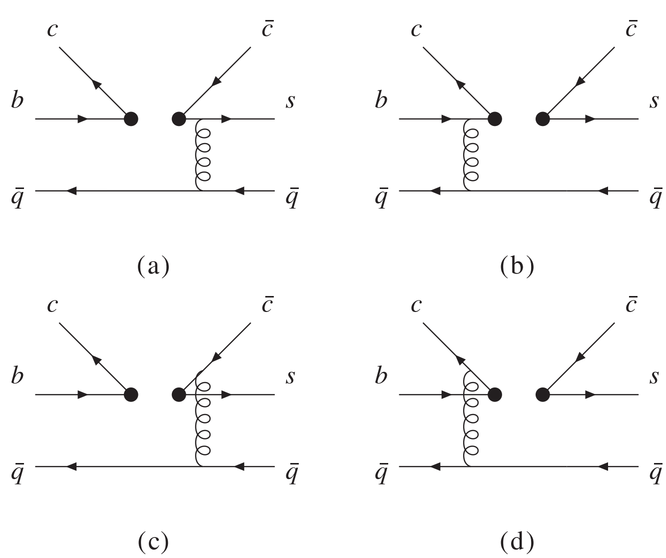

FIG. 1. Feynman diagrams correspond to the calculation of hard amplitudes in $B \to h_c K$.

The effective Hamiltonian relevant to $B^- \to h_c K^-$ decay in the SM is written as [22]

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[V_{cb}V_{cs}^*(\mathcal{C}_1(\mu)\mathcal{O}_1 + \mathcal{C}_2(\mu)\mathcal{O}_2) - V_{tb}V_{ts}^*\sum_{i=3}^{10}\mathcal{C}_i(\mu)\mathcal{O}_i\right], \tag{1}$$

where $C_i(\mu)$ are the Wilson coefficients and $\mathcal{O}_i$ are the relevant operators defined as

$$\begin{aligned}
\mathcal{O}_1 &= (\bar{s}_\alpha c_\beta)_{V-A}(\bar{c}_\beta b_\alpha)_{V-A},\\
\mathcal{O}_2 &= (\bar{s}_\alpha c_\alpha)_{V-A}(\bar{c}_\beta b_\beta)_{V-A},\\
\mathcal{O}_{3(5)} &= (\bar{s}_\alpha b_\alpha)_{V-A}\sum_q(\bar{q}_\beta q_\beta)_{V-A(V+A)},\\
\mathcal{O}_{4(6)} &= (\bar{s}_\alpha b_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V-A(V+A)},\\
\mathcal{O}_{7(9)} &= \frac{3}{2}(\bar{s}_\alpha b_\alpha)_{V-A}\sum_q e_q(\bar{q}_\beta q_\beta)_{V+A(V-A)},\\
\mathcal{O}_{8(10)} &= \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A}\sum_q e_q(\bar{q}_\beta q_\alpha)_{V+A(V-A)},
\end{aligned}$$

with α, β being the color indices. The explicit expressions of the Wilson coefficients appearing in the above equations can be found in Ref. [22].

We define, in the rest frame of the B meson, p, p' and q to be the four-moneta of B, K and h_c, $k_{1(2)}$, $k'_{1(2)}$ and $q_{1(2)}$ to be the momenta of the valence quarks inside $B(b(\bar{q}))$, $K(s(\bar{q}))$, and h_c $(c(\bar{c}))$, respectively. Then we parametrize the light cone momenta with all the light quarks and mesons being treated as massless

$$\begin{aligned}
p &= \frac{m_B}{\sqrt{2}}(1, 1, \mathbf{0}_{\rm T}) = (p^+, p^-, \mathbf{0}_{\rm T}),\\
p' &= \frac{m_B}{\sqrt{2}}(0, 1 - r^2, \mathbf{0}_{\rm T}),\\
q &= \frac{m_B}{\sqrt{2}}(1, r^2, \mathbf{0}_{\rm T}) = (q^+, q^-, \mathbf{0}_{\rm T}),\\
k_1 &= (x_1 p^+, p^-, \mathbf{k}_{1\rm T}), \qquad k_2 = (\bar{x}_1 p^+, 0, -\mathbf{k}_{1\rm T}),\\
k'_1 &= (x'_1 p'^+, x'_1 p'^-, \mathbf{k}'_{1\rm T}), \qquad k'_2 = (\bar{x}'_1 p'^+, \bar{x}'_1 p'^-, -\mathbf{k}'_{1\rm T}),\\
q_1 &= (yq^+, yq^-, \mathbf{q}_T), \qquad q_2 = (\bar{y}q^+, \bar{y}q^-, -\mathbf{q}_{\rm T}),
\end{aligned}$$

where the mass ratio r is set as $r = m_{h_c}/m_B$. x_i and x'_i are the fractions of the longitudinal momenta of the valence quarks. The superscripts "+" (plus) and "−" (minus) mean that the three-momentum is parallel or antiparallel to the positive z direction which is defined as the direction of the three-momentum of the produced h_c. $\mathbf{k}_{1\rm T}$, $\mathbf{k}'_{1\rm T}$ and

$\mathbf{q}_T$ are the transverse momenta of the valence quarks inside B, K and h_c respectively.

Therefore, we concentrate on the calculations of the nonfactorizable diagrams Fig. 1(c) and 1(d). Then we divide the operators appearing in the effective Hamiltonian into two categories according to their chirality, i.e. $(V-A)\otimes(V-A)$ and $(V-A)\otimes(V+A)$.

For the type of $(V-A)\otimes(V\mp A)$, the hard kernels of Fig. 1(c) are, respectively, written as

$$H^{(c,1)}_{\alpha\beta\rho\alpha'\beta'\rho'}(p,p',q)=\left[ig_s\gamma_\nu\frac{i}{q_1-k_2+k_2'-m_c}\gamma_\mu(1-\gamma_5)\right]_{\rho\alpha}[\gamma^\mu(1-\gamma_5)]_{\alpha'\rho'}[ig_s\gamma^\nu]_{\beta\beta'}\frac{-i}{(k_2-k_2')^2},\tag{2}$$

$$H^{(c,2)}_{\alpha\beta\rho\alpha'\beta'\rho'}(p,p',q)=-2\left[ig_s\gamma_\nu\frac{i}{q_1-k_2+k_2'-m_c}(1-\gamma_5)\right]_{\rho\alpha}[(1+\gamma_5)]_{\alpha'\rho'}[ig_s\gamma^\nu]_{\beta\beta'}\frac{-i}{(k_2-k_2')^2},\tag{3}$$

where the factor "−2" in Eq. (3) comes from the Fierz transformation on the $(V-A)\otimes(V+A)$ operators.

Similarly, for Fig. 1(d), we have

$$H^{(d,1)}_{\alpha\beta\rho\alpha'\beta'\rho'}(p,p',q)=\left[\gamma_\mu(1-\gamma_5)]_{\rho\alpha}[ig_s\gamma^\mu(1-\gamma_5)\frac{i}{-(q_2-k_2+k_2')-m_c}\gamma_\nu\right]_{\alpha'\rho'}[ig_s\gamma^\nu]_{\beta\beta'}\frac{-i}{(k_2-k_2')^2},\tag{4}$$

$$H^{(d,2)}_{\alpha\beta\rho\alpha'\beta'\rho'}(p,p',q)=-2[\gamma_\mu(1-\gamma_5)]_{\rho\alpha}\left[ig_s\gamma^\mu(1+\gamma_5)\frac{i}{-(q_2-k_2+k_2')-m_c}\gamma_\nu\right]_{\alpha'\rho'}[ig_s\gamma^\nu]_{\beta\beta'}\frac{-i}{(k_2-k_2')^2}.\tag{5}$$

In the above expressions, indices 1, 2 denote the contributions from $(V-A)\otimes(V-A)$ and $(V-A)\otimes(V+A)$ operators, respectively.

Finally, for each type of operators we obtain the decay amplitude $M_i^{(j)}$ which is a sum of the two nonfactorizable diagrams. For saving the space, we collect them in Appendix A. The wave functions relevant to calculation is given in Appendix B.

The decay width of $B^-\to h_cK^-$ is written as

$$\Gamma=\frac{G_F^2}{2}\frac{|\mathbf{p}_f|}{8\pi m_B^2}|\mathcal{M}|^2\tag{6}$$

with the total amplitude is the sum of $M_{1,2}(c,d)$ as

$$\mathcal{M}=M_1^{(c)}+M_2^{(c)}+M_1^{(d)}+M_2^{(d)}.$$

$\mathbf{p}_f$ denotes the three-momentum of the produced meson in the center-of-mass frame of B meson. All the explicit expressions of M_i are collected in Appendix A in order to shorten the text and focus on the physics contents.

III. NUMERICAL RESULTS

The input parameters used in the text are given below: $m_B=5.279$ GeV, $m_K=0.494$ GeV [23]. $m_{h_c}=3.524$ GeV [7,8]. For the CKM mixing parameters, we take $s_{12}=0.2243$, $s_{23}=0.0413$, $s_{13}=0.0037$ and $\delta_{13}=1.05$ [23]. The parameters appeared in the wave functions are put in the Appendix B following the corresponding wave functions.

Using the above parameters, one finally obtains the branching ratio of $B^-\to h_cK^-$ (in the numerical calculation, we take the central values listed in the data book for the input parameters)

$$BR(B^-\to h_cK^-)=3.6\times10^{-5}.$$

Since all the parameters adopted in our numerical computations are included in the wave functions of the concerned hadrons and the Sudakov factor which are obtained by fitting data of some well measured processes, thus the error of our final result is due to the uncertainties in such fitting. Because of the uncontrollable factors, one cannot expect to get very accurate value at the present stage. Therefore, we only keep two significant figures without explicitly marking out the error range. But in the last section, we will present a rough error estimate which may make sense.

There are some theoretical uncertainties in our calculations. One of them comes from the next to leading order corrections to the hard amplitudes [24]. In view of this point, we check the sensitivity of the decay rate with different choices of hard scales, i.e. we set the hard scales as

$$\max(0.75\sqrt{A_c},0.75\sqrt{B_c},1/b_1,1/b_1',1/b_q,1/|\mathbf{b_1}-\mathbf{b_q}|)\le t_c\le\max(1.25\sqrt{A_c},1.25\sqrt{B_c},1/b_1,1/b_1',1/b_q,1/|\mathbf{b_1}-\mathbf{b_q}|)$$
$$\max(0.75\sqrt{A_d},0.75\sqrt{B_d},1/b_1,1/b_1',1/b_q,1/|\mathbf{b_1}+\mathbf{b_q}|)\le t_d\le\max(1.25\sqrt{A_d},1.25\sqrt{B_d},1/b_1,1/b_1',1/b_q,1/|\mathbf{b_1}+\mathbf{b_q}|),$$

and other parameters are fixed. Then we can obtain the branching ratio of $B^- \to h_c K^-$ and the error may be a few percents and the whole result is not sensitive to the change of hard scales.

Another uncertainty comes from the nonperturbative parameters in meson wave functions, such as ω_b in B meson wave functions, although they are determined directly from previous experiments or some nonperturbative methods like QCD sum rules. Here we vary the value of ω_b at the range $0.32 \sim 0.48$ GeV, with other parameters fixed, as Ref. [24] did, then we find that the errors can be as large as 20%. It can be observed that the decay rate is relatively more dependent on the value of ω_b.

In Ref. [12], the author proposed that $h_c \to \eta_c \gamma$ is a promising mode to look for h_c and presented a theoretical estimation on the branching ratio as $BR(h_c \to \eta_c \gamma) = 0.50 \pm 0.11$. Combining this branching ratio with $BR(\eta_c \to K\bar{K}\pi) = (5.7 \pm 1.6)\%$, one could predict the branching ratio of the decay chain $B^- \to h_c K^- \to (\eta_c \gamma)K^- \to ((K\bar{K}\pi)\gamma)K^-$ as

$$BR(B^- \to h_c K^- \to (\eta_c \gamma)K^- \to ((K\bar{K}\pi)\gamma)K^-) = 1.0 \times 10^{-6}. \tag{7}$$

IV. DISCUSSION AND CONCLUSION

In this work, we employ the framework of PQCD approach to calculate the transition rate of $B^- \to h_c K^-$ and predict the branching ratio to be about 3.6×10^{-5}.

It is observed that only nonfactorizable diagrams contribute to the decay amplitude, because this transition is a G-parity violation process. So one can expect it to be relatively suppressed compared with $B^- \to J/\psi K^-$ which is about 1.00×10^{-3} [23]. In our work, we estimate the ratio and obtain results which are reasonably consistent with data. In this way, we not only naturally reproduce the production rate of h_c at *BABAR* and Belle, but we also have a chance to study the nonfactorizable diagrams. Usually, in most processes, both factorizable and nonfactorizable diagrams contribute and it is hard to separate their contributions. Thus when comparing with data, there is an obvious uncertainty. However, this case is an optimistic place where only nonfactorizable diagrams contribute. It enables us to uniquely investigate the nonfactorizable contributions to the weak transitions and testify applicability of the theory, PQCD.

Our prediction is consistent with the upper bound reported by the Belle and *BABAR* collaborations [6] at the level of order of magnitude. Thus we can conclude that the PQCD is applicable to the process, especially to deal with the nonfactorizable diagrams.

In fact, all these phenomenological parameters in the wave functions of B, K, h_c are priori determined and the choice of the factorization scale follows the conventional way. In our numerical computations, we do not introduce any free parameter to adjust. Therefore the error of the final result obtained in this work is due to the uncertainties included in the wave functions of the concerned hadrons and the Sudakov factors which are expressed in terms of a few phenomenological parameters.

As the input parameters, which exist in the wave functions of the concerned hadrons, vary within a reasonable range, an error of about 20% is expected. On the other hand, it would be difficult to make a precise estimate on the uncertainty because it is determined by nonperturbative QCD effects, thus only the order of magnitude of the results can be trusted. Even then, our theoretical predictions are more consistent with the present data, i.e. the upper bounds set by the experiments. It seems that based on the PQCD approach, the obtained results are more consistent with data than the earlier analyses [12,13,16], thus one expects that the theoretical framework may reflect the real physics picture, at least works in this case.

If the future experiments further reduce the upper bound and the theoretical estimate on the branching ratio of $B^- \to h_c + K^-$ cannot tolerate the new data even considering the range for the input parameters, one would confront a serious challenge to our understanding of the physics mechanisms, therefore, we hope that by improvement to reduce both statistical and systematic errors, new measurements of the *BABAR* and Belle collaborations will provide important information towards the answer, otherwise we may need to wait for the data of the future LHC-b.

To be more accurate in both theory and experiment, we need to wait for more data and improvement in the method. Indeed, a favorable channel for observing h_c is decay of $\psi(2S)$. However, unfortunately, the BES collaboration has not observed this mode and one needs to wait for the upgraded stage, i.e BES III which would accumulate much larger database in future and then we can further investigate the consistency between theory and experiments.

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APPENDIX A: SOME RELEVANT FUNCTIONS APPEARED IN THE TEXT

The explicit expressions of decay amplitude $M_i^{(j)}$ appeared in the text can be written as

$$M_1^{(c)} = \frac{8}{(2N_c)^{3/2}} \int dx_1 \int dx_1' \int dy \int b_1 db_1 \int b_q db_q \int d\theta \Big[V_{cb}V_{cs}^* \mathcal{C}_2(\mu) - V_{tb}V_{ts}^* \Big(\mathcal{C}_4(\mu) + \frac{3}{2} e_c \mathcal{C}_{10}(\mu) \Big) \Big] \times \phi_B(x_1, b_1) \phi_{h_c}^{\parallel}(y) \times [\phi_K^A(x_1') \mathbb{KA}_1^{(c)} + \phi_K^P(x_1') \mathbb{KP}_1^{(c)} + \phi_K^T(x_1') \mathbb{KT}_1^{(c)}] \alpha_s(t_c) \exp[-S(t_c)] \Omega_c(x_1, x_1', y, b_1, b_q), \tag{A1}$$

$$M_2^{(c)} = \frac{-16}{(2N_c)^{3/2}} \int dx_1 \int dx_1' \int dy \int b_1 db_1 \int b_q db_q \int d\theta \Big[\mathcal{C}_6(\mu) + \frac{3}{2} e_c \mathcal{C}_8(\mu) \Big] (-V_{tb}V_{ts}^*) \phi_B(x_1, b_1) \phi_{h_c}^{\parallel}(y) \times [\phi_K^A(x_1') \mathbb{KA}_2^{(c)} + \phi_K^P(x_1') \mathbb{KP}_2^{(c)} + \phi_K^T(x_1') \mathbb{KT}_2^{(c)}] \alpha_s(t_c) \exp[-S(t_c)] \Omega_c(x_1, x_1', y, b_1, b_q), \tag{A2}$$

$$M_1^{(d)} = \frac{8}{(2N_c)^{3/2}} \int dx_1 \int dx_1' \int dy \int b_1 db_1 \int b_q db_q \int d\theta \Big[V_{cb}V_{cs}^* \mathcal{C}_2(\mu) - V_{tb}V_{ts}^* \Big(\mathcal{C}_4(\mu) + \frac{3}{2} e_c \mathcal{C}_{10}(\mu) \Big) \Big] \times \phi_B(x_1, b_1) \phi_{h_c}^{\parallel}(y) [\phi_K^A(x_1') \mathbb{KA}_1^{(d)} + \phi_K^P(x_1') \mathbb{KP}_1^{(d)} + \phi_K^T(x_1') \mathbb{KT}_1^{(d)}] \alpha_s(t_d) \exp[-S(t_d)] \Omega_d(x_1, x_1', y, b_1, b_q), \tag{A3}$$

$$M_2^{(d)} = \frac{-16}{(2N_c)^{3/2}} \int dx_1 \int dx_1' \int dy \int b_1 db_1 \int b_q db_q \int d\theta \Big[\mathcal{C}_6(\mu) + \frac{3}{2} e_c \mathcal{C}_8(\mu) \Big] (-V_{tb}V_{ts}^*) \phi_B(x_1, b_1) \phi_{h_c}^{\parallel}(y) \times [\phi_K^A(x_1') \mathbb{KA}_2^{(d)} + \phi_K^P(x_1') \mathbb{KP}_2^{(d)} + \phi_K^T(x_1') \mathbb{KT}_2^{(d)}] \alpha_s(t_d) \exp[-S(t_d)] \Omega_d(x_1, x_1', y, b_1, b_q), \tag{A4}$$

where the explicit expressions of $\mathbb{KA}_i^{(j)}$, $\mathbb{KP}_i^{(j)}$ and $\mathbb{KT}_i^{(j)}$ which come from the contraction of hard kernel and hadronic wave functions are given below. Here the meaning of indices i, j have been shown in the text.

$$\mathbb{KA}_1^{(c)} = -16mb^4 r^2(-1 + r^2)(-x_1 + x_1' - y), \tag{A5}$$

$$\mathbb{KP}_1^{(c)} = -16mb^3 m_0^K r^2(-x_1 + x_1' - y), \tag{A6}$$

$$\mathbb{KT}_1^{(c)} = 16mb^3 m_0^K r^2(-x_1 + x_1' - y), \tag{A7}$$

$$\mathbb{KA}_2^{(c)} = \mathbb{KA}_1^{(d)} = 0, \tag{A8}$$

$$\mathbb{KP}_2^{(c)} = -8mb^3 m_0^K r^2(-x_1 + x_1' - y), \tag{A9}$$

$$\mathbb{KT}_2^{(c)} = -8mb^3 m_0^K r^2(-x_1 + x_1' - y), \tag{A10}$$

$$\mathbb{KP}_1^{(d)} = 16mb^3 m_0^K r^2(-1 - x_1 + x_1' + y), \tag{A11}$$

$$\mathbb{KT}_1^{(d)} = 16mb^3 m_0^K r^2(-1 - x_1 + x_1' + y), \tag{A12}$$

$$\mathbb{KA}_2^{(d)} = 8mb^4 r^2(-1 + r^2)(-1 - x_1 + x_1' + y), \tag{A13}$$

$$\mathbb{KP}_2^{(d)} = 8mb^3 m_0^K(-1 - x_1 + x_1' + y), \tag{A14}$$

$$\mathbb{KT}_2^{(d)} = -8mb^3 m_0^K(-1 - x_1 + x_1' + y). \tag{A15}$$

The explicit forms of $\Omega_{c,d}(x_1, x_1', y, b_1, b_q)$ which come from Fourier transformation to products of propagators corresponding to quark and gluon are listed as follow

$$\Omega_c(x_1, x_1', y, b_1, b_q) = \Big\{ K_0(\sqrt{\mathcal{A}_c}|\mathbf{b}_q|)\theta(\mathcal{A}_c) + \frac{\pi}{2}[-N_0(\sqrt{\mathcal{A}_c}|\mathbf{b}_q|) + iJ_0(\sqrt{\mathcal{A}_c}|\mathbf{b}_q|)]\theta(-\mathcal{A}_c) \Big\} K_0(\sqrt{\mathcal{B}_c}|\mathbf{b}_1 - \mathbf{b}_q|), \tag{A16}$$

$$\Omega_c(x_1, x_1', y, b_1, b_q) = \Big\{ K_0(\sqrt{\mathcal{A}_d}|\mathbf{b}_q|)\theta(\mathcal{A}_d) + \frac{\pi}{2}[-N_0(\sqrt{\mathcal{A}_d}|\mathbf{b}_q|) + iJ_0(\sqrt{\mathcal{A}_d}|\mathbf{b}_q|)]\theta(-\mathcal{A}_d) \Big\} K_0(\sqrt{\mathcal{B}_d}|\mathbf{b}_1 + \mathbf{b}_q|), \tag{A17}$$

with

$$A_c = m_c^2 - (x_1 + y - 1)((1 - x_1')(1 - r^2) + yr^2)m_b^2, \qquad B_c = (1 - x_1)(1 - x_1')(1 - r^2)m_b^2,$$
$$A_d = m_c^2 - (x_1 - y)((1 - x_1')(1 - r^2) + (1 - y)r^2)m_b^2, \qquad B_d = (1 - x_1)(1 - x_1')(1 - r^2)m_b^2.$$

Here J_i, N_i are 'ith' order Bessel functions of first and second kind, respectively, K_i denotes 'ith' order modified Bessel functions.

The explicit expressions of Sudakov factor coming from the resummation of double logarithm appeared in high order radiative corrections to the diagrams are also given below

$$S(t_c) = s((1-x_1)p^+, b_1) + s(x_1' p'^-, b_1') + s((1-x_1')p'^-, b_1') - \frac{1}{\beta_1}\left[\ln\frac{-\ln(t_c/\Lambda_{\rm QCD})}{-\ln(b_1\Lambda_{\rm QCD})} + \ln\frac{-\ln(t_c/\Lambda_{\rm QCD})}{\ln(b_1'\Lambda_{\rm QCD})} + \ln\frac{-\ln(t_c/\Lambda_{\rm QCD})}{\ln(b_q\Lambda_{\rm QCD})}\right], \quad (A18)$$

$$S(t_d) = s((1-x_1)p^+, b_1) + s(x_1' p'^-, b_1') + s((1-x_1')p'^-, b_1') - \frac{1}{\beta_1}\left[\ln\frac{-\ln(t_d/\Lambda_{\rm QCD})}{-\ln(b_1\Lambda_{\rm QCD})} + \ln\frac{-\ln(t_d/\Lambda_{\rm QCD})}{\ln(b_1'\Lambda_{\rm QCD})} + \ln\frac{-\ln(t_d/\Lambda_{\rm QCD})}{\ln(b_q\Lambda_{\rm QCD})}\right], \quad (A19)$$

where

$$t_c = \max(\sqrt{A_c}, \sqrt{B_c}, 1/b_1, 1/b_1', 1/b_q, 1/|\mathbf{b_1} - \mathbf{b_q}|), \qquad t_d = \max(\sqrt{A_d}, \sqrt{B_d}, 1/b_1, 1/b_1', 1/b_q, 1/|\mathbf{b_1} + \mathbf{b_q}|).$$

The explicit expressions for the Sudakov factors are given in Ref. [25,26] as

$$s(\omega, Q) = \int_\omega^Q \frac{dp}{p}\left[\ln\left(\frac{Q}{p}\right)A[\alpha_s(p)] + B[\alpha_s(p)]\right],$$
$$A = C_F\frac{\alpha_s}{\pi} + \left[\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_0\ln\left(\frac{e^{\gamma_E}}{2}\right)\right] \times \left(\frac{\alpha_s}{\pi}\right)^2,$$
$$B = \frac{2}{3}\frac{\alpha_s}{\pi}\ln\left(\frac{e^{2\gamma_E-1}}{2}\right),$$
$$\gamma_q(\alpha_s(\mu)) = -\alpha_s(\mu)/\pi,$$
$$\beta_0 = \frac{33-2n_f}{12},$$

where γ_E is the Euler constant. n_f is the flavor number, and γ_q is the anomalous dimension. We will take n_f equal to 4 in our numerical calculations.

APPENDIX B: WAVE FUNCTIONS RELEVANT TO CALCULATION

The B meson light cone wave function is usually written as [27]

$$\int_0^1 \frac{d^4z}{(2\pi)^4} e^{ik\cdot z}\langle 0|\bar{q}_\beta(0)b_\alpha(z)|B(p)\rangle = -\frac{i}{\sqrt{2N_c}}\left\{(p + m_B)\gamma_5\left[\phi_B(\mathbf{k}) - \frac{\not{n} - \not{v}}{\sqrt{2}}\bar{\phi}_B(\mathbf{k})\right]\right\}_{\alpha\beta}, \quad (B1)$$

where $n \equiv (1, 0, \mathbf{0}_T)$ and $v \equiv (0, 1, \mathbf{0}_T)$ denote the unit vectors corresponding to the "plus" and "minus" directions, respectively. In Eq. (B1), two different Lorentz structures exist in the B meson wave functions. $\phi_B(\mathbf{k})$ and $\bar{\phi}_B(\mathbf{k})$ satisfy the following normalization conditions respectively

$$\int \frac{d^4k}{(2\pi)^4}\phi_B(\mathbf{k}) = \frac{f_B}{2\sqrt{2N_c}}, \qquad \int \frac{d^4k}{(2\pi)^4}\bar{\phi}_B(\mathbf{k}) = 0. \quad (B2)$$

In the numerical calculation, one usually ignores the contribution of $\bar{\phi}_B(\mathbf{k})$ [28,29] and only takes the contribution from

$$\Phi_B = \frac{1}{\sqrt{2N_c}}(\not{p} + m_B)\gamma_5\phi_B(\mathbf{k}). \quad (B3)$$

The wave function of B meson is given as [28,29]

$$\phi_B(x, b) = \frac{N_B}{2\sqrt{2N_c}}f_B x^2(1-x)^2\exp\left[-\frac{M_B^2x^2}{2\omega_b^2} - \frac{1}{2}(b\omega_b)^2\right], \quad (B4)$$

where $\omega_b = 0.4$ GeV and $N_B = 2.4\times 10^3$. The decay constant of B meson $f_B = 0.18$ GeV.

The twist-3 light cone distribution amplitude of K meson is expressed as

$$\langle \bar{K}^0(p')|\bar{s}_\alpha(z)q_\beta(0)|0\rangle = \frac{i}{\sqrt{2N_c}}\int_0^1 dx e^{ixp'\cdot z}\{\gamma_5 p'\phi_K^A(x) + m_0^K\gamma_5\phi_K^P(x) + m_0^K[\gamma_5(vn - 1)]\phi_K^T(x)\}_{\beta\alpha}, \quad (B5)$$

where $m_0^K = \frac{m_K^2}{m_s+m_d}$. In a recent work [30], the K meson wave function distribution amplitudes used in Eq. (B8) is given as

$$\phi_K^A(x) = \frac{f_K}{2\sqrt{2N_c}}\{6x(1-x)(1 + a_1^K C_1^{3/2}(2x-1) + a_2^K C_2^{3/2}(2x-1))\}, \quad (B6)$$

$$\phi_K^p(x) = \frac{f_K}{2\sqrt{2N_c}}\Big\{1 + 3\rho_+^K(1 + 6a_2^K) - 9\rho_-^K a_1^K + C_1^{1/2}(2x-1)\Big[\frac{27}{2}\rho_+^K a_1^K - \rho_-^K\Big(\frac{3}{2} + 27a_2^K\Big)\Big] + C_2^{1/2}(2x-1)(30\eta_{3K} + 15\rho_+^K a_2^K - 3\rho_-^K a_1^K) + C_3^{1/2}(2x-1)\Big(10\eta_{3K}\lambda_{3K} - \frac{9}{2}\rho_-^K a_2^K\Big) - 3\eta_{3K}\omega_{3K}C_4^{1/2}(2x-1) + \frac{3}{2}(\rho_+^K + \rho_-^K)(1 - 3a_1^K + 6a_2^K)\ln x + \frac{3}{2}(\rho_+^K - \rho_-^K)(1 + 3a_1^K + 6a_2^K)\ln\bar{x}\Big\}, \tag{B7}$$

$$\phi_K^T = \frac{1}{6}\frac{d\phi_K^\sigma(x)}{dx}, \tag{B8}$$

$$\phi_K^\sigma(x) = \frac{f_K}{2\sqrt{2N_c}}\Big\{6x\bar{x}\Big[1 + \frac{3}{2}\rho_+^K + 15\rho_+^K a_2^K - \frac{15}{2}\rho_-^K a_1^K + \Big(3\rho_+^K a_1^K - \frac{15}{2}\rho_-^K a_2^K\Big)C_1^{3/2}(2x-1) + \Big(5\eta_{3K} - \frac{1}{2}\eta_{3K}\omega_{3K} + \frac{3}{2}\rho_+^K a_2^K\Big)C_2^{3/2}(2x-1) + \eta_{3K}\lambda_{3K}C_3^{3/2}(2x-1)\Big] + 9x\bar{x}(\rho_+^K + \rho_-^K)(1 - 3a_1^K + 6a_2^K)\ln x + 9x\bar{x}(\rho_+^K - \rho_-^K)(1 + 3a_1^K + 6a_2^K)\ln\bar{x}\Big\}, \tag{B9}$$

with

$$\rho_+^K = \frac{(m_s + m_q)^2}{m_K^2}, \qquad \rho_-^K = \frac{m_s^2 - m_q^2}{m_K^2}, \qquad \eta_{3K} = \frac{f_{3K}}{f_K}\frac{m_q + m_s}{m_K^2}, \qquad C_1^{1/2}(t) = t, \qquad C_2^{1/2}(t) = \frac{1}{2}(3t^2 - 1),$$

$$C_3^{1/2}(t) = \frac{1}{2}(5t^3 - t), \qquad C_4^{1/2}(t) = \frac{1}{8}(3 - 30t^2 + 35t^4), C_1^{3/2}(t) = 3t, \qquad C_2^{3/2}(t) = \frac{3}{2}(5t^2 - 1),$$

$$C_3^{3/2} = \frac{1}{2}(35t^3 + 3t),$$

where $a_1^K = 0.06 \pm 0.03$, $a_2^K = 0.25 \pm 0.15$, $f_{3K} = (0.45 \pm 0.15) \times 10^{-2}\ \mathrm{GeV}^2$, $\omega_{3K} = -1.2 \pm 0.7$, $\lambda_{3K} = 1.6 \pm 0.4$, $f_K = 0.16$ GeV, $m_s = 137 \pm 27$ MeV, $m_q = 5.6 \pm 1.6$ MeV.

The light cone distribution amplitude of h_c meson is proposed in [20]

$$\langle h_c(q, \epsilon)|\bar{c}_\alpha(z)c_\beta(0)|0\rangle = -\frac{i}{\sqrt{2N_c}}\int_0^1 du e^{iuq\cdot z}\Big\{f_{h_c}m_{h_c}\Big[\epsilon_\parallel^* + \frac{m_{h_c}^2 z\epsilon^*\cdot z}{2(q\cdot z)^2}\Big]\gamma_5\phi_{h_c}^\parallel(u) - f_{h_c}^\perp\epsilon_\perp^* q\gamma_5\phi_{h_c}^\perp(u)\Big\}_{\beta\alpha} \tag{B10}$$

with

$$\epsilon_{\parallel\mu}^* = \frac{\epsilon^*\cdot z}{q\cdot z}\Big[q_\mu - \frac{m_{h_c}^2 z_\mu}{q\cdot z}\Big], \qquad \epsilon_{\perp\mu}^* = \epsilon_\mu^* - \epsilon_{\parallel\mu}^*.$$

We have dropped out the twist-3 distribution amplitudes here, because h_c is heavy, the higher twist contributions are negligible. Because the produced h_c in the transition $B^- \to h_c K^-$ can only be longitudinally polarized, the wave function $\phi_{h_c}^\perp(u)$ does not contribute and we neglect its explicit form in the text. It is a good approximation to assume that $\phi_\parallel(u)$ is of the same form as χ_0^v which is the leading-twist distribution amplitude of χ_{c0} defined in Eq. (59) of Ref. [19]

$$\phi_{h_c}^\parallel(x) = 27.46\frac{f_{h_c}}{2\sqrt{2N_c}}(1 - 2x)\Big\{\frac{x(1-x)[1 - 4x(1-x)]}{[1 - 2.8x(1-x)]^2}\Big\}^{0.7}, \tag{B11}$$

where h_c decay constant f_{h_c} is set to be 0.335 GeV which is the same as $f_{\chi_{c1}}$ as has been done in Ref. [12].

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