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Near-threshold enhancement in $J/\psi \to \gamma X$ with $X \to \omega\phi$

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We investigate the possibility of producing the enhancement observed in $J/\psi \to \gamma X$ with $X \to \omega\phi$ at BES by intermediate meson rescatterings through $f_0(1710) \to PP \to \omega\phi$, $f_0(1710) \to VV \to \omega\phi$, and $f_0(1710) \to SS \to \omega\phi$. We find that intermediate meson rescatterings can produce some enhancement near the $\omega\phi$ threshold. Implications about the property of this enhancement are discussed.

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I. INTRODUCTION

The observation of a near-threshold enhancement in $J/\psi \to \gamma X$; $X \to \omega\phi$ at BESII [1] immediately provokes discussions about its nature. This enhancement is reported to favor $J^P = 0^+$ in a partial wave analysis with a mass and width of $M = 1812^{+19}_{-26}(\text{stat}) \pm 18(\text{syst})$ MeV and $\Gamma = 105 \pm 20(\text{stat}) \pm 28(\text{syst})$ MeV, respectively, and a production branching ratio, $B(J/\psi \to \gamma X) \cdot B(X \to \omega\phi) = (2.61 \pm 0.27(\text{stat}) \pm 0.65(\text{syst})) \times 10^{-4}$.

The most interesting feature about $X(1810)$ is that it seems to have a mass different from the previously observed f_0 states, i.e. $f_0(1710)$ and/or $f_0(1790)$ [2], and "exclusively" couples to $\omega\phi$ channel, while those known scalars are generally below the $\omega\phi$ threshold. If one interprets the enhancement as a Breit-Wigner resonance, it may lead to implications of exotic meson productions [4–6] such as glueball [7,8], hybrid [9,10] and four-quark state [11].

However, the sparse experimental information makes it difficult to draw a decisive conclusion about its nature. A thorough investigation of all possible interpretations of the data is thus necessary, and mechanisms such as final state interactions should also be inspected. The established $f_0(1710)$ at the affinity makes it a candidate for such a consideration. PDG [12] quotes $M = 1714 \pm 5$ MeV and $\Gamma_{\text{tot}} = 140 \pm 10$ MeV as the average values of the mass and total width for the $f_0(1710)$. In contrast, the recent data from BES [13] with high statistics indicate a relatively larger mass pole (~ 1740 MeV) and total width (~ 166 MeV). In the $f_0(1710)$ rest frame, the production threshold for $\omega\phi$ will be at the upper edge of the $f_0(1710)$ mass tail. It is known that resonance off-shell effects can be significant due to the energy-dependence of its decay width in a relativistic Breit-Wigner form for a resonance propagating nonlocally. For the $f_0(1710)$ with the mass tail extended to about 1.8 GeV, the possibility that the $f_0(1710)$ contributes via $J/\psi \to \gamma f_0(1710) \to \gamma\omega\phi$ cannot be ruled out. Nonetheless, the $f_0(1790)$ [3], if indeed a resonance, may also be a candidate for producing the near-threshold enhancement in $\omega\phi$ invariant mass spectrum.

The tantalizing feature of this enhancement is its presence in $\omega\phi$ channel, which in general is OZI [14] suppressed compared with other VV decay channels such as $\omega\omega$, $\phi\phi$, $K^*\bar{K}^*$, and $\rho\rho$. Strong coupling to $\omega\phi$ simply implies the violation of the OZI rule or a completely different production mechanism for $X \to \omega\phi$. For the latter, proposals such as Refs. [7–11] are tackling some of those points, for which a coherent picture is still absent. For large OZI violations, the final state interactions due to intermediate meson rescatterings seem likely to explain why X only predominantly appears in $\omega\phi$, and is absent (at least not strongly show up) in other channels such as $K^*\bar{K}^*$, $\omega\omega$, $\rho\rho$ and $\phi\phi$. Certainly, this consideration should be tested by numerical studies, and our work in this paper is to provide some quantitative evaluation of such a possibility.

The importance of the intermediate meson rescatterings (also quoted as final state interactions) has been recognized in many cases. Early studies by Lipkin show that the OZI-rule violations can always proceed by a two-step process involving intermediate virtual mesons [15]. Detailed investigations by Geiger and Isgur in a quark model highlight such a correlation [16]. In particular, it is found that systematic cancellations among the hadronic loops occur for the $u\bar{u} \leftrightarrow s\bar{s}$ mixing in all nonets except 0^{++}, for which a general argument is given by Lipkin and Zou [17]. The heavy quarkonium decays into light hadrons provide a place for probing the intermediate meson rescattering mechanisms [18–20]. In the charmonium energy region, the decay of $J/\psi \to \omega f_0$ and ϕf_0 have significant contributions from $J/\psi \to K^*\bar{K} + \text{c.c.} \to \phi f_0$ and $J/\psi \to \rho\pi \to \omega f_0$ [21]. Similar phenomena are also observed in $\eta_c \to \omega\phi$ as a major mechanism for the OZI-rule violations [22].

For the $f_0(1710)$, its signal has been identified in different processes [3,23,24]. It has large branching ratios to $K\bar{K}$ from 0.38 [12] to 0.6 [3,23]. Taking into account that ω and ϕ both have sizeable couplings to $K\bar{K}$ and $K^*\bar{K} + \text{c.c.}$, it is natural to conjecture that significant contributions from the intermediate $K\bar{K}$ and/or $K^*\bar{K}^*$ could be possible. Nevertheless, the scalar meson exchange in VV rescattering can also contribute, such as $K^*\bar{K}^* \to \omega\phi$ via κ exchange. Since information about the VVS interactions still lacks, we try to gain some guidance about the couplings in the flavor SU(3) symmetry. Correlated to this, it is then possible that intermediate scalar meson rescatterings, such as $\kappa\kappa \to \omega\phi$ via K^* exchange, can contribute to the cross sections. Although we do not have any experimental information about the $f_0(1710) \to \kappa\kappa$ coupling, a test of this channel should be useful for gaining some insights into the $f_0(1710)$ properties. Similar to these considerations, the $f_0(1790)$ observed in $\pi\pi$ channel will allow contributions from $\pi\pi$ rescattering via ρ exchange to $\omega\phi$. Especially, note that $\phi \to \rho\pi + 3\pi$ has a sizeable branching ratio [12].

In brief, the intermediate meson rescattering prescription has empirically accommodated as much as possible the available information, from which we expect to learn more about the largely unknown $X(1810)$. As follows, we will analyze three types of rescatterings: (i) intermediate pseudoscalar meson rescattering; (ii) intermediate vector meson rescattering; and (iii) intermediate scalar meson rescattering. We will study the lowest partial wave contributions as leading processes. This should be sufficient for gaining order-of-magnitude estimate at this stage. In Sec. II, the formalism will be provided. We will discuss the numerical results in Sec. III, and a brief summary is given in Sec. IV.

II. INTERMEDIATE PSEUDOSCALAR MESON RESCATTERING

The intermediate pseudoscalar meson rescattering is illustrated by Fig. 1, where (a) is for rescattering via vector meson exchange, and (b), via pseudoscalar meson exchange.

A. Rescattering via vector meson exchange

The amplitude for the transition of Fig. 1(a) can be expressed as

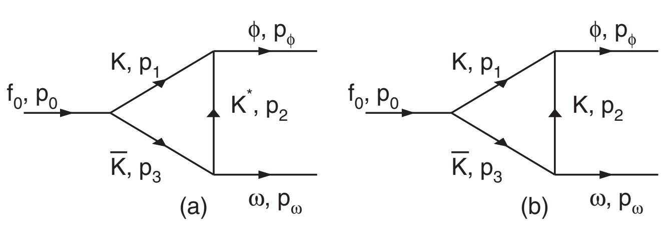

FIG. 1. Schematic diagrams for intermediate meson rescatterings via (a) $K\bar{K}$ with K^* exchange; and (b) $K\bar{K}$ with kaon exchange.

$$\mathcal{M} = \int \frac{d^4 p_2}{(2\pi)^4} \delta^4(P_0 - P_\phi - P_\omega) \sum_{K^* \text{pol.}} \frac{T_a T_b T_c}{a_1 a_2 a_3} \mathcal{F}(p_2^2), \tag{1}$$

with the vertex functions:

$$T_a \equiv i g_a M_0, \qquad T_b \equiv \frac{i g_b}{M_\phi} \epsilon_{\mu\nu\xi\tau} P_\phi^\mu \epsilon_\phi^\nu p_2^\xi \epsilon_2^\tau,$$
$$T_c \equiv \frac{i g_c}{M_\omega} \epsilon_{\lambda\iota\kappa\sigma} P_\omega^\lambda \epsilon_\omega^\iota p_2^\kappa \epsilon_2^\sigma, \tag{2}$$

where g_a, g_b and g_c are coupling constants at the meson interaction vertices; Note that the tensor part of the vector meson propagator will not contribute. The four-vectors, P_0, P_ϕ and P_ω, are momenta for the initial $f_0(1710)$ and final state ϕ and ω mesons, while p_1, p_2 and p_3 are four momenta for the intermediate mesons, respectively. Quantities, $a_1 = p_1^2 - m_1^2$, $a_2 = p_2^2 - m_2^2$ and $a_3 = p_3^2 - m_3^2$, are the denominators of the propagators of intermediate mesons.

By applying the Cutkosky rule, we have the decay amplitude

$$\mathcal{M} = \frac{i g_a g_b g_c |\mathbf{p}_3|}{64\pi^2 M_\phi M_\omega} (P_\phi \cdot P_\omega \epsilon_\phi \cdot \epsilon_\omega - P_\phi \cdot \epsilon_\omega P_\omega \cdot \epsilon_\phi) \mathcal{I}, \tag{3}$$

with

$$\mathcal{I} \equiv \int d\Omega \frac{p_2^2}{p_2^2 - m_2^2} \mathcal{F}(p_2^2), \tag{4}$$

where $\mathcal{F}(p_2^2)$ is the form factor introduced for the off-shell vector meson (K^* and ρ).

In Refs. [21,22], three cases have been studied for the integration: (i) with no form factor; (ii) with a monopole form factor, i.e. $\mathcal{F}(p_2^2) = (\Lambda^2 - m_2^2)/(\Lambda^2 - p_2^2)$, where Λ is the cut-off energy; and (iii) with a dipole form factor, i.e. $\mathcal{F}(p_2^2) = [(\Lambda^2 - m_2^2)/(\Lambda^2 - p_2^2)]^2$. Generally, the calculation results show large sensitivities to the value of Λ. Therefore, to determine Λ would require sufficient experimental constraints. Unfortunately, such constraints on $f_0 \to VV$ are unavailable. We hence apply the cut-off energy, $\Lambda = 1.2 \sim 1.8$ GeV, for the dipole form factor calculations.

For the dipole form factor, the integration gives

$$\mathcal{I} = 2\pi \frac{A_s(\Lambda^2 - m_2^2)^2}{B_s D_s^2} \left\{ \frac{2(A-D)}{(B-D)(1-D^2)} - \frac{A-B}{(B-D)^2} \ln\frac{(1+B)(1-D)}{(1-B)(1+D)} \right\}, \tag{5}$$

where

$$A_s \equiv M_\phi^2 + m_1^2 - 2E_\phi E_1, \qquad A \equiv 2|\mathbf{P}_\phi||\mathbf{p}_1|/A_s;$$
$$B_s \equiv A_s - m_2^2, \qquad B \equiv 2|\mathbf{P}_\phi||\mathbf{p}_1|/B_s; \qquad D_s \equiv A_s - \Lambda^2. \tag{6}$$

For $K\bar{K}$ rescattering, we adopt $B.R.(f_0(1710) \to K\bar{K}) = 0.6$ to determine $g_a = (8\pi\Gamma_{f_0(1710)\to K\bar{K}}/|\mathbf{p}_1|)^{1/2}$. Couplings $g_b = g_{\phi K^{*+}\bar{K}^-}$ and $g_c = g_{\omega K^{*+}\bar{K}^-}$ are determined in the SU(3) flavor symmetry limit: $g_b = \sqrt{2}g_c = 6.48$.

B. Rescattering via pseudoscalar meson exchange

In the pseudoscalar meson rescattering, it can also exchange pseudoscalar meson such as K and π and then couple to the final state ω and ϕ [see Fig. 1(b)]. In this process, considering that the couplings of $\phi K\bar{K}$ and $\omega K\bar{K}$ both are large, it is necessary to investigate the effects from this transition.

The VPP coupling has a form of

$$T_{VPP} = ig_{VPP}(p + p')\cdot\epsilon_v, \tag{7}$$

where p and p' are the four-vector momenta for the incoming and outgoing pseudoscalars, respectively. We determine the $g_{\phi K^+K^-}$ via $\phi \to K^+K^-$:

$$g_{\phi K^+K^-} = \frac{6\pi M_\phi^2}{|\mathbf{p}_K|^3}\Gamma^{\mathrm{exp}}_{\phi\to K^+K^-}, \tag{8}$$

where $\Gamma^{\mathrm{exp}}_{\phi\to K^+K^-} = 2.09$ MeV is given by the PDG [12]; $\mathbf{p}_K$ is the kaon momentum in the ϕ meson rest frame. We then deduce $g_{\omega K^+K^-} = g_{\phi K^+K^-}/\sqrt{2}$ in the SU(3) symmetry limit.

The exclusive transition amplitude is

$$\mathcal{M} = -\frac{ig_ag_bg_c|\mathbf{p}_3|}{32\pi^2}\epsilon_\phi\cdot\epsilon_\omega\mathcal{I}, \tag{9}$$

where the integral $\mathcal{I}$ has the same form as Eq. (4), but the mass m_2 is for the exchanged kaon. Also, in the above equation $g_b = g_{\phi K^+K^-}$ and $g_c = g_{\omega K^+K^-}$ are applied.

III. INTERMEDIATE VECTOR MESON RESCATTERING

In contrast with the pseudoscalar meson rescattering via vector and pseudoscalar meson exchanges [Fig. 1(a) and 1(b)], intermediate vector meson rescattering via pseudoscalar and scalar meson exchanges, i.e. Fig. 2(a) and 2(b), are also allowed.

A. Rescattering via pseudoscalar meson exchange

For transition Fig. 2(a), we consider the following couplings:

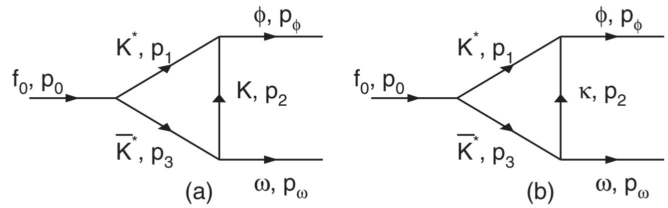

FIG. 2. Schematic diagrams for intermediate meson rescatterings via (a) $K^*\bar{K}^*$ with kaon exchanges; and (b) $K^*\bar{K}^*$ with κ exchange.

$$\begin{aligned} T_a &= -\frac{ig_a}{M_0}(p_1\cdot p_3\epsilon_1\cdot\epsilon_3 - p_1\cdot\epsilon_3 p_3\cdot\epsilon_1),\\ T_b &\equiv \frac{ig_b}{M_\phi}\epsilon_{\mu\nu\xi\tau}P_\phi^\mu\epsilon_\phi^\nu p_1^\xi\epsilon_1^\tau,\\ T_c &\equiv \frac{ig_c}{M_\omega}\epsilon_{\lambda\iota\kappa\sigma}P_\omega^\lambda\epsilon_\omega^\iota p_3^\kappa\epsilon_3^\sigma, \end{aligned} \tag{10}$$

where ϵ_1 and ϵ_3 are the polarization vectors for the rescattering vector mesons, and the coupling constants for the VVP vertices are the same as in Fig. 1(a). Similar to the treatment for the previous two transitions, we can derive the exclusive transition amplitude.

In the SU(3) symmetry, we can identify that the intermediate $K^*\bar{K}^* \to \omega\phi$ via kaon exchanges could be sizeable due to the large $\phi K^*\bar{K}$ and $\omega K^*\bar{K}$ couplings. The intermediate $\rho\rho$ scattering via pion exchange can also couple to $\omega\phi$. Because of SU(3) symmetry breaking, the $\phi\rho\pi$ coupling does not vanish. We adopt the branching ratios for $\phi \to \rho\pi + 3\pi$ as an upper limit to derive the $\phi\rho\pi$ coupling for the intermediate $\rho\rho$ scattering via pion exchanges.

The transition amplitude is

$$\mathcal{M}^{\lambda_\phi\lambda_\omega} = \frac{ig_ag_bg_c|\mathbf{p}_1|}{32\pi^2M_\phi M_\omega M_0^2}\int d\Omega\frac{f_{\lambda_\phi\lambda_\omega}}{p_2^2 - m_2^2}\mathcal{F}(p_2^2), \tag{11}$$

where λ_ϕ and λ_ω denote the helicity of the final state vector meson, and function $f_{\lambda_\phi\lambda_\omega}$ has the following expressions:

$$\begin{aligned} f_{00} &= -\frac{[(P_\phi\cdot P_\omega)^2 - M_0^2|\mathbf{P}_\phi|^2]}{2M_\phi M_\omega}p_2^2(p_2^2 - \Delta_0),\\ f_{11} &= f_{-1-1} = \frac{1}{2}P_\phi\cdot P_\omega p_2^2(p_2^2 - \Delta_1), \end{aligned} \tag{12}$$

with

$$\begin{aligned} \Delta_0 &\equiv \frac{1}{2}P_\phi\cdot P_\omega\left[3 - \frac{M_\phi^2M_\omega^2}{(P_\phi\cdot P_\omega)^2 - M_0^2|\mathbf{P}_\phi|^2}\right],\\ \Delta_1 &\equiv \frac{1}{2}P_\phi\cdot P_\omega\left[3 - \frac{M_\phi^2M_\omega^2}{(P_\phi\cdot P_\omega)^2}\right]. \end{aligned} \tag{13}$$

The integral in Eq. (11) has a typical form of

$$I = \int d\Omega \frac{p_2^2(p_2^2 - \Delta)}{p_2^2 - m_2^2} \mathcal{F}(p_2^2), \tag{14}$$

which has been given in Ref. [22].

B. Rescattering via scalar meson exchange

The intermediate vector meson rescattering via scalar meson exchange, Fig. 2(b), is described by the following couplings:

$$\begin{aligned}
T_a &= -\frac{ig_a}{M_0}(p_1 \cdot p_3 \epsilon_1 \cdot \epsilon_3 - p_1 \cdot \epsilon_3 p_3 \cdot \epsilon_1) \\
T_b &= \frac{ig_b}{M_\phi}(P_\phi \cdot p_1 \epsilon_\phi \cdot \epsilon_1 - P_\phi \cdot \epsilon_1 p_1 \cdot \epsilon_\phi) \\
T_c &= \frac{ig_c}{M_\omega}(P_\omega \cdot p_3 \epsilon_\omega \cdot \epsilon_3 - P_\omega \cdot \epsilon_3 p_3 \cdot \epsilon_\omega),
\end{aligned} \tag{15}$$

where g_a is the same SVV coupling adopted for the f_0VV interactions in the previous subsection. g_b and g_c are also SVV couplings for which we apply the SU(3) flavor relation as a constraint. For the $K^*\bar{K}^*$ rescattering, the exchanged scalar is κ meson. In the SU(3) symmetry, we have $g_{\phi K^{*+}\kappa^-} = \sqrt{2} g_{\omega K^{*+}\kappa^-} = -\sqrt{3/2} g_{\rho^0\rho^0\sigma}$ with $g_{\rho^0\rho^0\sigma} = 13.6$ broadly applied in the literature [25].

In the calculation, we assume that near the $\omega\phi$ threshold the decay of X is dominated by the S wave.

IV. INTERMEDIATE SCALAR MESON RESCATTERING

A. Rescattering via vector meson exchange

For the scalar meson rescattering, we consider the rescattering via vector meson exchanges (Fig. 3). This process has the same couplings for the ω and ϕ interaction vertices as in vector meson rescattering via scalar exchanges. The following couplings are hence applied:

$$\begin{aligned}
T_a &= ig_a M_0 \\
T_b &= \frac{ig_b}{M_\phi}(P_\phi \cdot p_2 \epsilon_\phi \cdot \epsilon_2 - P_\phi \cdot \epsilon_2 p_2 \cdot \epsilon_\phi) \\
T_c &= \frac{ig_c}{M_\omega}(P_\omega \cdot p_2 \epsilon_\omega \cdot \epsilon_2 - P_\omega \cdot \epsilon_2 p_2 \cdot \epsilon_\omega),
\end{aligned} \tag{16}$$

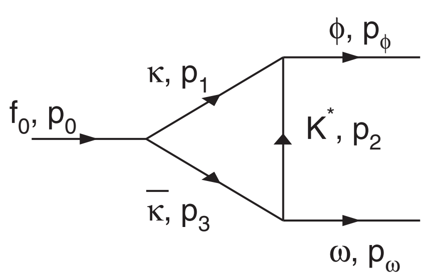

FIG. 3. Schematic diagrams for intermediate $\kappa\bar{\kappa}$ rescattering with K^* exchange.

where ϵ_2 is the polarization vectors of the exchanged vector meson.

Following the standard procedure, we derive the transition amplitude:

$$\mathcal{M}^{\lambda_\phi \lambda_\omega} = \frac{ig_a g_b g_c |\mathbf{p}_1|}{256\pi^2 M_\phi M_\omega M_2^2} h_{\lambda_\phi \lambda_\omega} \times \int d\Omega \frac{p_2^2(p_2^2 - 4m_2^2)}{p_2^2 - m_2^2} \mathcal{F}(p_2^2), \tag{17}$$

where the function $h_{\lambda_\phi \lambda_\omega}$ has the following form:

$$\begin{aligned}
h_{00} &= \frac{1}{M_\phi M_\omega}[(P_\phi \cdot P_\omega)^2 - M_0^2 |\mathbf{P}_\phi|^2], \\
h_{11} &= h_{-1-1} = -P_\phi \cdot P_\omega.
\end{aligned} \tag{18}$$

V. NUMERICAL RESULTS AND DISCUSSIONS

To evaluate these rescatterings, we need information about the scalar couplings to pseudoscalar, vector and scalar meson pairs, which unfortunately are not available. If we treat the $X(1810)$ as the production of $f_0(1710)$ at higher mass tail, we then have some additional information about $B.R.(f_0(1710) \to PP)$ from experiment [3,23,24]. In particular, the branching ratio for $f_0(1710) \to K\bar{K}$ is found to be large. The PDG quote an average of 0.38 while the recent data from BES [3,23] show that $B.R.(f_0(1710) \to \pi\pi)/B.R.(f_0 \to K\bar{K})) < 15\%$, which lead to an estimate of $B.R.(f_0 \to K\bar{K})) \simeq 0.6$. As a result, a large coupling for $f_0(1710)K\bar{K}$ can be expected. So far, there is no information about the $f_0(1710)$ couplings to VV, $\kappa\bar{\kappa}$, and $\sigma\sigma$. Kinematic suppressions are also expected since the VV production thresholds are generally close to or above the $f_0(1710)$ mass. Because of these, in our numerical analysis, we will examine the ratio of the rescattering amplitude square over the corresponding tree processes.

As we know that an intermediate state can generally contribute to the transition amplitude via off-shell process. The differential partial decay width of $J/\psi \to \gamma X \to \gamma\omega\phi$ can thus be expressed as

$$\begin{aligned}
d\Gamma_{\omega\phi} = {} & \frac{1}{(2\pi)^5} \frac{1}{16 M_{J/\psi}^2} \frac{1}{2J+1} \sum_{\lambda_\gamma \lambda_\phi \lambda_\omega} |\mathbf{P}_\phi| \\
& \times |\mathcal{M}^{\lambda_\phi \lambda_\omega}(f_0 \to \omega\phi)|^2 |\mathbf{p}_\gamma| \\
& \times |\mathcal{M}^{\lambda_\gamma}(J/\psi \to \gamma f_0)|^2 \\
& \times [\mathrm{Re}^2(W) + \mathrm{Im}^2(W)] dW d\Omega_\gamma d\Omega_w,
\end{aligned} \tag{19}$$

where $\mathbf{P}_\phi$ is the momentum of ϕ meson in the rest frame of ω and ϕ; $\mathbf{p}_\gamma$ is the photon momentum in the J/ψ rest frame; W is the invariant mass of ω and ϕ system. The f_0 off-shell effects are taken care by the propagator of which the real and imaginary part are:

$$\mathrm{Re}\,(W) = \frac{W^2 - M_0^2}{(W^2 - M_0^2)^2 + M_0^2\Gamma_T^2}$$
$$\mathrm{Im}(W) = -\frac{M_0\Gamma_T}{(W^2 - M_0^2)^2 + M_0^2\Gamma_T^2} \tag{20}$$

where M_0 and Γ_T are the mass and total width of the f_0 state.

Similarly, we can express the differential partial decay width for $J/\psi \to \gamma f_0 \to \gamma K\bar{K}$ as

$$\begin{aligned} d\Gamma_{K\bar{K}} = {}& \frac{1}{(2\pi)^5}\frac{1}{16M_{J/\psi}^2}\frac{1}{2J+1}\sum_{\lambda_\gamma}|\mathbf{P}_k| \\ &\times |\mathcal{M}(f_0 \to K\bar{K})|^2|\mathbf{p}_\gamma||\mathcal{M}^{\lambda_\gamma}(J/\psi \to \gamma f_0)|^2 \\ &\times [\mathrm{Re}^2(W) + \mathrm{Im}^2(W)]dWd\Omega_\gamma d\Omega_w, \end{aligned} \tag{21}$$

Note that to produce the off-shell f_0 at mass W means that the invariant transition matrix element $\mathcal{M}^{\lambda_\gamma}(J/\psi \to \gamma f_0)$ is W-dependent. To obtain the partial decay width we also need to know the information about the $J/\psi \to \gamma f_0$ transition. There are phenomenological studies available in the literature for $J/\psi \to \gamma f_0(1710)$ [26]. Here, we are interested in the decays of $f_0 \to \omega\phi$ via different rescattering processes. To eliminate the ambiguities from the first vertex, we take the ratio between these two differential partial decay widths for those above-listed meson rescatterings, and define, for example, for the pseudoscalar meson rescattering via vector meson exchange:

$$R_V^{PP} \equiv \frac{|\mathbf{P}_\phi|\sum_{\lambda_\phi\lambda_\omega}|\mathcal{M}_V^{\lambda_\phi\lambda_\omega}(f_0 \to PP \to \omega\phi)|^2}{|\mathbf{P}_k||\mathcal{M}(f_0 \to PP)|^2}, \tag{22}$$

where the superscript PP denotes the rescattered pseudoscalar meson pair and the subscript V denotes the exchanged vector meson; $\mathbf{P}_k$ is the three momentum carried by the pseudoscalar meson in the c.m. frame of f_0 with mass W. By scanning over a range of W, the energy dependence of the ratio will reveal the evolution of the intermediate meson loop.

Similarly, we define the ratios for other rescatterings relative to the corresponding tree processes:

$$R_P^{PP} \equiv \frac{|\mathbf{P}_\phi|\sum_{\lambda_\phi\lambda_\omega}|\mathcal{M}_P^{\lambda_\phi\lambda_\omega}(f_0 \to PP \to \omega\phi)|^2}{|\mathbf{P}_k||\mathcal{M}(f_0 \to PP)|^2} \tag{23}$$

for the pseudoscalar meson rescattering via pseudoscalar meson exchange;

$$R_P^{VV} \equiv \frac{|\mathbf{P}_\phi|\sum_{\lambda_\phi\lambda_\omega}|\mathcal{M}_P^{\lambda_\phi\lambda_\omega}(f_0 \to VV \to \omega\phi)|^2}{|\mathbf{P}_v||\mathcal{M}(f_0 \to VV)|^2} \tag{24}$$

for vector meson rescattering via pseudoscalar meson exchange;

$$R_S^{VV} \equiv \frac{|\mathbf{P}_\phi|\sum_{\lambda_\phi\lambda_\omega}|\mathcal{M}_S^{\lambda_\phi\lambda_\omega}(f_0 \to VV \to \omega\phi)|^2}{|\mathbf{P}_v||\mathcal{M}(f_0 \to VV)|^2} \tag{25}$$

for vector meson rescattering via scalar meson exchange; and

$$R_S^{SS} \equiv \frac{|\mathbf{P}_\phi|\sum_{\lambda_\phi\lambda_\omega}|\mathcal{M}_S^{\lambda_\phi\lambda_\omega}(f_0 \to SS \to \omega\phi)|^2}{|\mathbf{P}_s||\mathcal{M}(f_0 \to SS)|^2} \tag{26}$$

for scalar meson rescattering via scalar meson exchange.

It is also interesting to examine the evolution of the meson loop in terms of W compared with the tree processes for $f_0 \to K\bar{K}$. Therefore, we define ratio Q as follows:

$$Q_P^{VV} \equiv \frac{|\mathbf{P}_\phi|\sum_{\lambda_\phi\lambda_\omega}|\mathcal{M}_P^{\lambda_\phi\lambda_\omega}(f_0 \to VV \to \omega\phi)|^2}{|\mathbf{P}_k||\mathcal{M}(f_0 \to PP)|^2}, \tag{27}$$

$$Q_S^{VV} \equiv \frac{|\mathbf{P}_\phi|\sum_{\lambda_\phi\lambda_\omega}|\mathcal{M}_S^{\lambda_\phi\lambda_\omega}(f_0 \to VV \to \omega\phi)|^2}{|\mathbf{P}_k||\mathcal{M}(f_0 \to PP)|^2}, \tag{28}$$

$$Q_S^{SS} \equiv \frac{|\mathbf{P}_\phi|\sum_{\lambda_\phi\lambda_\omega}|\mathcal{M}_S^{\lambda_\phi\lambda_\omega}(f_0 \to SS \to \omega\phi)|^2}{|\mathbf{P}_k||\mathcal{M}(f_0 \to PP)|^2}, \tag{29}$$

where $\mathbf{P}_k$ is the three momenta for $f_0 \to PP$ in the f_0 rest frame with mass W. Because of lack of information about the f_0VV and f_0SS couplings, the ratio Q will possess large uncertainties.

The numerical results for ratio R are presented in Fig. 4. The $K^*\bar{K}^*$ rescattering via scalar meson exchange turns out

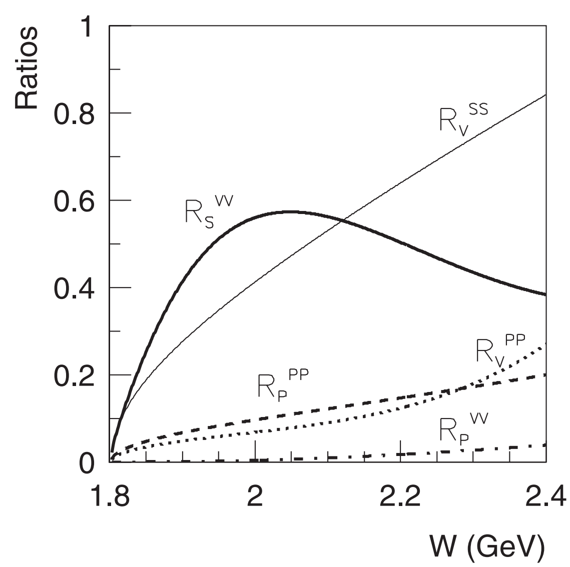

FIG. 4. The evolution of the ratios of the invariant amplitude square of the meson loops to the corresponding tree processes in terms of the initial scalar meson masses. Definition of the ratios is given in the text.

to be the largest contribution as denoted by the thick solid line. A strong enhancement is produced with an increasing invariant mass for the $\omega\phi$ system. Since this is the ratio between the $K^*\bar{K}^*\kappa$ loop and the tree process for $f_0 \to K^*\bar{K}^*$, it is independent of the $f_0 K^*\bar{K}^*$ coupling. As shown by the solid curve, the $K^*\bar{K}^*$ rescattering via κ exchange has a sizeable fraction compared with the $f_0 \to K^*\bar{K}^*$ transition.

To estimate this channel's contributions to the $\omega\phi$ partial width, we assume that $\mathcal{M}^{\lambda_\gamma}(J/\psi \to \gamma f_0)$ is insensitive to W. Equation (19) can then be expressed as

$$\Gamma_{\omega\phi} = \Gamma_{J/\psi\to\gamma f_0} \times B.R.(f_0 \to \omega\phi), \tag{30}$$

where

$$B.R.(f_0 \to \omega\phi) \equiv \iint \frac{|\mathbf{P}_\phi|}{16\pi^3} \sum_{\lambda_\phi\lambda_\omega} |\mathcal{M}^{\lambda_\phi\lambda_\omega}(f_0 \to \omega\phi)|^2 \times [\mathrm{Re}^2(W) + \mathrm{Im}^2(W)] dW d\Omega_w. \tag{31}$$

For $K^*\bar{K}^*$ rescattering via scalar meson exchange, we examine the following conditions:

(i) With $M_\kappa = 0.7$ GeV and $B.R.(f_0 \to K^*\bar{K}^*) = 0.1$, we find $\Gamma_{\omega\phi} = 1.36 \sim 3.01$ MeV for a mass range of f_0 from $1.74 \sim 1.81$ GeV.

(ii) With $M_\kappa = 0.7$ GeV and fixing the mass of the f_0 at 1.74 GeV, for a range of $B.R.(f_0 \to K^*\bar{K}^*) = 0.1 \sim 0.3$, we obtain $\Gamma_{\omega\phi} = 1.36 \sim 4.09$ MeV, which correspond to $B.R.(f_0 \to \omega\phi) = (0.97 \sim 2.92)\%$. For the PDG value $B.R.(J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K}) = 8.5 \times 10^{-4}$ and BES estimate of $B.R.(f_0(1710) \to K\bar{K}) = 0.6$, we derive $B.R.(J/\psi \to \gamma f_0(1710)) = 1.4 \times 10^{-3}$. Thus, we estimate $B.R.(J/\psi \to \gamma f_0 \to \gamma\omega\phi) \simeq (1.36 \sim 4.09) \times 10^{-5}$.

(iii) By fixing the mass of the f_0 at 1.74 GeV, and $B.R.(f_0 \to K^*\bar{K}^*) = 0.1$, and then varying the mass of the κ from 0.8–0.6 GeV, we obtain $\Gamma_{\omega\phi} = 0.45 \sim 3.61$ MeV corresponding to $B.R.(J/\psi \to \gamma f_0 \to \gamma\omega\phi) \simeq (0.45 \sim 3.61) \times 10^{-5}$. The calculation results turn to be sensitive to the mass of the exchanged κ meson, which still has large uncertainties [12]. We also find that $\Gamma_{\omega\phi}$ drops fast with an increasing κ mass. With $M_\kappa > 0.9$ GeV, the branching ratio for $f_0 \to \omega\phi$ will be just about 0.01%. On the other hand, although a smaller mass for κ can produce relatively large branching ratios for $\omega\phi$ channel, $M_\kappa = 0.6$ GeV can be regarded as the lower bound for the κ mass, which gives $B.R.(f_0 \to \omega\phi) \simeq 2.6\%$.

In Fig. 4 the dashed line is the ratio R_P^{PP} for pseudoscalar meson rescattering via pseudoscalar meson exchange, and the dotted and dot-dashed line are for R_V^{PP} and R_P^{VV}, respectively. These ratios are found flat in terms of the increasing invariant mass. In particular, R_P^{VV} is found negligible at low W. After considering the Breit-Wigner factor, their contributions to the $\omega\phi$ width are negligible.

The thin solid line denotes R_V^{SS} for the scalar meson rescattering via vector meson exchange. It shows a rapid increase along the invariant mass W. We also derive its contributions to $f_0 \to \omega\phi$ partial width, and find $B.R.(f_0 \to \omega\phi) < 1\%$. This suggests that the Breit-Wigner factor will kill the invariant amplitudes quickly when the scalar goes off-shell. It should be noted that so far there are no data available for $f_0(1710) \to \kappa\bar{\kappa}$ in experiment. We have assumed $B.R.(f_0 \to \kappa\bar{\kappa}) = 0.1 \sim 0.2$ in the calculation. However, this does not bring significant contributions to the $\omega\phi$ final state. Although the $\kappa\bar{\kappa}$ rescattering is unlikely to have large contributions to $\omega\phi$, it may becomes important in heavier scalar decays. The recent BES analysis shows κ signals in $J/\psi \to \bar{K}^*\kappa$ [27,28], which suggests strong κ couplings in some channels. As shown in Fig. 4, its increasing contributions in the rescatterings imply that it may play a role in OZI violation processes at higher energies.

The ratio Q for the intermediate pseudoscalar meson rescatterings are the same as R. So we only present the ratios Q_S^{VV}, Q_P^{VV} and Q_V^{SS} in Fig. 5. It shows that the $K^*\bar{K}^*$ rescattering via κ exchange is the major contribution to the $\omega\phi$ partial width since it has a steep increase at low W where the off-shell effects are relatively small. The $\kappa\kappa$ rescattering increases with the increasing W as shown by the dotted curve. However, due to the suppression from the off-shell factors, it does not significantly contribute to the $f_0 \to \omega\phi$ partial width. Ratio Q_P^{VV} illustrated by the dashed curve also turns out to be negligible.

We also study the $\omega\phi$ invariant mass spectrum to examine the behavior of the intermediate meson rescatterings and compare it with the BES data [1]. In Fig. 6, the solid curve denotes the $\omega\phi$ invariant mass distribution with

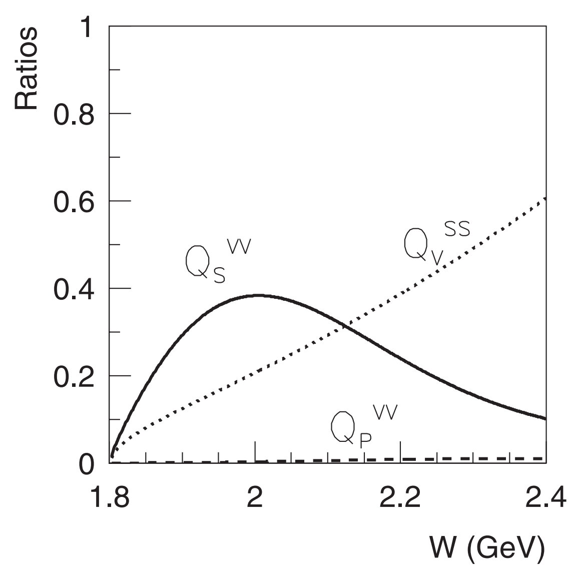

FIG. 5. The evolution of the ratios of the invariant amplitude square of the meson loops to $f_0 \to K\bar{K}$ in terms of the initial scalar meson masses. Definition of the ratios is given in the text.

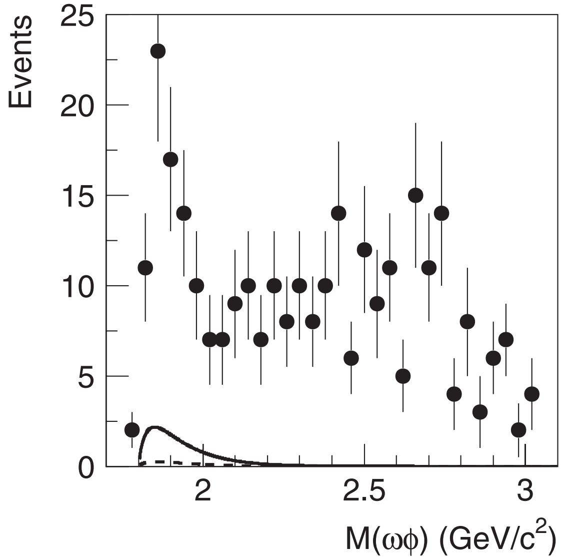

FIG. 6. Invariant mass spectrum for $\omega\phi$ in comparison with the BES experimental data [1]. The solid curve denotes the $\omega\phi$ invariant mass distribution with $B.R.(f_0 \to K^*\bar{K}^*) = 0.3$ and $M_\kappa = 0.6$ GeV, while the dashed curve with $B.R.(f_0 \to K^*\bar{K}^*) = 0.1$ and $M_\kappa = 0.7$ GeV.

$B.R.(f_0 \to K^*\bar{K}^*) = 0.3$ and $M_\kappa = 0.6$ GeV, which is the higher bound for $K^*\bar{K}^*$ rescattering via κ exchange. The dashed curve denotes the results with $B.R.(f_0 \to K^*\bar{K}^*) = 0.1$ and $M_\kappa = 0.7$ GeV. The peak of the enhancement sits around 1.85 GeV which seems to be consistent with the data. Note that though the partial width, $\Gamma(f_0 \to \omega\phi) = 10.8$ MeV (derived with $B.R.(f_0 \to K^*\bar{K}^*) = 0.3$ and $M_\kappa = 0.6$ GeV) is still significantly smaller than the Breit-Wigner fit $\Gamma(X(1810) \to \omega\phi) = 105 \pm 20$ MeV, it gives $B.R.(J/\psi \to \gamma f_0 \to \gamma\omega\phi) \simeq 1.08 \times 10^{-4}$, which is comparable with the experimental data [1]. This shows that if the enhancement does originate from the $f_0(1710)$ due to intermediate meson rescatterings, it will exhibit a rather narrow width. In this sense, a broad width at order of 100 MeV may favor its being a real Breit-Wigner resonance.

VI. SUMMARY

In summary, based on the present experimental information we examine the intermediate meson rescattering contributions to $J/\psi \to \gamma X \to \gamma\omega\phi$ by assuming that $X = f_0(1710)$ with a mass at $1.74 \sim 1.81$ GeV. We find that the contributions from the vector meson $K^*\bar{K}^*$ rescattering via scalar meson exchange can produce some enhancement near the $\omega\phi$ threshold. The other intermediate meson rescatterings, such as pseudoscalar and scalar meson rescatterings, are all found relatively small.

The calculation results for $B.R.(J/\psi \to \gamma f_0 \to \gamma\omega\phi)$ range from $(1.36 \sim 10.8) \times 10^{-5}$ for different values of $B.R.(f_0 \to K^*\bar{K}^*)$ and the κ mass. The derived partial decay width for $f_0 \to \omega\phi$ turns out to be at least one order-of-magnitude smaller than the observed partial width for $X(1810) \to \omega\phi$ [1]. This seems to make it unlikely that the observed enhancement in $\omega\phi$ invariant mass spectrum is from intermediate meson rescatterings in $f_0(1710)$ decays. However, due to lack of information about the $f_0(1710) \to SS$ and VV, we find it does not suffice to conclude on the nature of the enhancement. There still exist uncertainties in our calculations which should be cautioned: (i) the value of $\Gamma_{\omega\phi}$ turns to be sensitive to the choice of cut-off energies for the dipole form factor, and it can lead to a change about a factor of 2; (ii) the contributions from the intermediate meson rescatterings may be underestimated due to the application of the on-shell approximation. Under such an approximation, only the imaginary part of the transition amplitudes is picked up. For the VV rescattering, since the vector meson pair is close to the f_0 threshold, the on-shell approximation will introduce double kinematic suppressions to the imaginary part, while the real part will not suffer such a suppression. Because of this, the inclusion of the real part could enhance the partial width to $\omega\phi$ via $K^*\bar{K}^*$ rescatterings. This issue should be studies in the future with more knowledge about the VVS couplings available.

It is necessary to have more experimental information about the signals of $X(1810)$ in PP and VV (i.e. $K^*\bar{K}^*$, $\omega\omega$ and $\rho\rho$). In particular, a direct analysis of $J/\psi \to \phi X \to \phi\omega\phi$ and $J/\psi \to \omega X \to \omega\omega\phi$ could be useful for establishing the $X(1810)$ as a real resonance. In case that $X(1810)$ is a new scalar, it will be a challenge for theory in the understanding of the scalar meson spectrum, and may also be a chance for us to gain more insights into the underlying dynamics.

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about 270 MeV. Further experimental confirmation is needed.
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