Electroweak gauge boson production at hadron colliders through $\mathcal{O}(\alpha_s^2)$

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We describe a calculation of the $\mathcal{O}(\alpha_s^2)$ QCD corrections to the fully differential cross section for W and Z boson production in hadronic collisions. The result is fully realistic in that it includes spin correlations, finite width effects, $\gamma - Z$ interference and allows for the application of arbitrary cuts on the leptonic decay products of the W and Z. We have implemented this calculation into a numerical program. We demonstrate the use of this code by presenting phenomenological results for several future LHC analyses and recent Tevatron measurements, including the W cross section in the forward rapidity region and the central over forward cross section ratio.

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I. INTRODUCTION

The study of the electroweak gauge bosons W and Z is an important part of the physics programs at the Tevatron and the LHC. Their large production rates and clean experimental signatures facilitate several important measurements, such as the determination of the electroweak parameters M_W and $\sin^2\theta_W$, and the extraction of the parton distribution functions of the proton [1]. The idea to use W and Z production to monitor hadron collider luminosities has begun to be studied at the Tevatron and will continue to be investigated at the LHC [2]. During the initial stages of LHC running, W and Z bosons will be used to calibrate lepton energy scales, test the uniformity of the electromagnetic calorimeter and study the tracker alignment [3]. Searching for deviations from Standard Model predictions in di-lepton events with large invariant mass, missing energy, or transverse momentum probes extensions of the Standard Model which contain new gauge bosons or other exotic resonances.

The extensive experimental program described above requires accurate theoretical predictions, and many calculations describing electroweak gauge boson production are available. For example, the fully differential $\mathcal{O}(\alpha_s)$ next-to-leading order (NLO) QCD [4] and one-loop electroweak corrections [5] to W, Z production have been known for many years. The NLO QCD corrections are incorporated into the parton-shower event generator MC@NLO [6], which provides a consistent description of $\mathcal{O}(\alpha_s)$ hard emission effects together with a leading-logarithmic resummation of soft and collinear QCD radiation. The p_T-spectra of W and Z bosons are computed with next-to-leading-logarithmic accuracy and are incorporated into the program RESBOS [7]. The $\mathcal{O}(\alpha_s^2)$ QCD corrections in the large transverse momentum region are also known [8]. However, for many applications these theoretical results are insufficient. With an integrated luminosity per experiment at the Tevatron exceeding 1 fb^{-1} and an expected luminosity at the LHC of about 10 fb^{-1}/yr, the statistical error on W, Z production is becoming smaller than 1%. This statistical error sets the scale for the desired theoretical precision, so that the physics potential may be fully exploited. It is then easy to see that for percent level accuracy, next-to-next-to-leading order (NNLO) QCD computations for electroweak gauge boson production are required. The available results for $\mathcal{O}(\alpha_s^2)$ corrections to inclusive W, Z production [9] and to W, Z rapidity distributions [10] confirm that the NNLO QCD effects are at the level of a few percent, and that the remaining theoretical error after these corrections are included is at the percent level or lower.

The experimental identification of the W and Z production processes requires cuts on the pseudorapidities and transverse momenta of charged leptons and on the missing energy. The NNLO QCD results to the inclusive cross sections and to the rapidity distributions cannot be used to calculate the effects of these cuts, since they do not contain the spin correlations between the leptons and the initial-state partons arising from the spin-one nature of the electroweak gauge bosons [11]. The fully differential $\mathcal{O}(\alpha_s^2)$ corrections with spin correlations included are required to model these effects. To illustrate this point further, we discuss three examples.

(1) CDF and D0 have recently presented Run II measurements of the W/Z cross section ratio [12]. With only 72 pb^{-1} of integrated luminosity, CDF obtained

$$R = \frac{\sigma_W \times \mathrm{Br}(W \to l\nu)}{\sigma_{Z/\gamma^*} \times \mathrm{Br}(Z \to l^+l^-)} = 10.92 \pm 0.15_{\mathrm{stat}} \pm 0.14_{\mathrm{sys}}, \qquad l = e, \mu. \quad (1)$$

The systematic error was estimated to be only 1.5%. The actual measurement was performed with the cuts $p_T > 20$ GeV and $|\eta| < 1.8$ on the charged lepton, and with missing energy $\not{E}_T > 25$ GeV.

The results were extrapolated to total cross sections using theoretically computed acceptances. Since no NNLO QCD calculation capable of modeling these cuts was available at the time of that analysis, the procedure for obtaining acceptances was to reweight the PYTHIA rapidity distributions for the electroweak gauge bosons with the NNLO QCD computation [10] to account for NNLO QCD effects. While it is unlikely that this procedure leads to drastically wrong results, a precision of a few percent *cannot be guaranteed* using this technique.

(2) The Tevatron and the LHC can potentially provide stringent constraints on parton distribution functions (PDFs). The current PDF extractions permit computations of LHC hard-scattering cross sections with $Q \approx 100$ GeV to an accuracy of 5% or better. Measurements such as the W, Z-charge asymmetries [13] can reduce these errors. These require precision predictions through NNLO in QCD for leptonic pseudorapidity distributions with cuts on transverse momenta and missing energy.

(3) With the high luminosity of the LHC, precise measurements of electroweak parameters are possible. An interesting example is the measurement of the effective electroweak mixing angle $\sin^2\theta_W$ through the forward-backward asymmetry in $Z \to l^+l^-$. A precision of 2×10^{-4}, competitive with the LEP analysis, can be achieved with 100 fb^{-1} of integrated luminosity, provided the following cuts on the leptons can be imposed: $|\eta_{e^+,e^-}| < 2.5$, $|Y(e^+, e^-)| > 1$, $p_T^l > 20$ GeV. Given the magnitude of the NLO QCD corrections for this set of cuts [14], the inclusion of NNLO QCD effects seems mandatory.

The above discussion illustrates the importance of having a fully differential description of electroweak gauge boson production through $\mathcal{O}(\alpha_s^2)$ in QCD. Unfortunately, such computations remain challenging. Their complexity lies in the intricate structure of soft and collinear singularities that plague individual contributions in QCD perturbation theory. While at NLO a number of approaches [15] can be used to isolate and subtract those singularities from complicated matrix elements in a process- and observable-independent way, an extension of this approach to NNLO is not complete [16]. We have formulated an alternative technique in a recent series of papers [17]. The central idea of this method is an automated extraction of infrared singularities from the real radiation matrix elements and a numerical cancellation of these divergences with the virtual corrections. We have previously applied this approach to the computation of the fully differential Higgs boson production cross section in gluon fusion and to $e^+e^- \to 2$ jets through NNLO [17]. We make extensive use of these references in this manuscript. Our goal is to describe a fully realistic calculation of single electroweak gauge boson production at the Tevatron and the LHC. The computation is valid through NNLO in perturbative QCD, includes spin correlations, finite widths effects, $\gamma - Z$ interference and is fully differential. A short version of this paper with initial results was presented in [18].

This manuscript is organized as follows. In the next Section we briefly recall the important features of the method and discuss some of the differences between the current calculation and the fully differential computation of $pp \to H \to \gamma\gamma$, reported in [17]. In Sec. III we demonstrate the possible uses of our numerical program by presenting phenomenological results for Tevatron and LHC measurements. We conclude in Sec. IV.

II. DETAILS OF THE COMPUTATION

We consider the production of a lepton pair in hadronic collisions,

$$h_1(P_1) + h_2(P_2) \to V + X \to l_1 + l_2 + X, \tag{2}$$

where $V = W$, Z and $l_{1,2}$ represent charged leptons or neutrinos, as appropriate. Within the framework of QCD factorization, the cross section for this process is

$$d\sigma^V = \sum_{ij} \int dx_1 dx_2 f_i^{h_1}(x_1) f_j^{h_2}(x_2) d\sigma_{ij\to V+X}(x_1, x_2), \tag{3}$$

where the f_i^h are parton distribution functions that describe the probability to find a parton i with momentum xP_h in the hadron h. The partonic cross sections $d\sigma_{ij}$ are computed perturbatively as an expansion in the strong coupling constant α_s:

$$d\sigma_{ij} = \sigma^{(0)}_{ij\to V} + \left(\frac{\alpha_s}{\pi}\right)\sigma^{(1)}_{ij\to V} + \left(\frac{\alpha_s}{\pi}\right)^2 \sigma^{(2)}_{ij\to V} + \mathcal{O}(\alpha_s^3). \tag{4}$$

The partonic processes $i + j \to l_1 + l_2 + X$ that contribute to electroweak gauge boson production differ at each order in the perturbative expansion. At leading order in α_s, only quark-antiquark annihilation channels contribute, while at NLO the (anti)quark-gluon channels also occur. At NNLO, the gluon-gluon fusion and (anti)quark-(anti)quark scattering channels also contribute. We note that all the relevant channels are catalogued in great detail in Ref. [9].

Each partonic process contains several distinct contributions. We describe these components using Z-boson production in $q\bar{q}$ annihilation as an example. At NNLO, this process receives three distinct contributions: (i) the two-loop virtual corrections to $q\bar{q} \to Z$; (ii) the one-loop virtual corrections to $q\bar{q} \to Z + g$; (iii) the tree-level processes $q\bar{q} \to Z + gg$, $q\bar{q} \to Z + q\bar{q}$. We refer to these three mechanisms as the double-virtual, the real-virtual, and the double-real emission corrections. When computed separately, these three contributions exhibit numerous soft and collinear singularities. To produce a physically meaningful result, they should be combined in the presence of an infrared-safe measurement function. In addition, the collinear renormalization of the parton distribution functions f_i^h is required.

The double-virtual corrections are the simplest to calculate. They require the two-loop massless triangle diagrams that were obtained in [19]. If dimensional regularization is used to regularize ultraviolet, soft and collinear singularities, the result is given by a Laurant series in the regularization parameter $\epsilon = (d-4)/2$, where d is the dimensionality of space-time. The situation is more complex for the real-virtual and double-real corrections because they contain real emission matrix elements. These are finite for nonexceptional final-state momenta, but they diverge once an emitted parton becomes either soft or collinear to another parton. The challenge in performing NNLO computations is to extract the divergences from the real emission matrix elements *without* integrating over any kinematic parameter that describes the real emission process. The fully differential nature of the computation remains intact only if this can be achieved.

We have developed a technique to accomplish this in a previous series of papers [17]. We describe here the salient features of this method. Consider a double-real emission contribution to the production of a Z-boson in $q\bar{q}$ annihilation, $q(p_1) + \bar{q}(p_2) \to Z(p_Z) + g(p_3) + g(p_4)$. We choose a parameterization of the final-state momenta that maps the allowed phase-space onto the unit hypercube:

$$\int \mathrm{d}^d p_Z \mathrm{d}^d p_3 \mathrm{d}^d p_4 \delta^+(p_Z^2 - M_Z^2)\delta^+(p_3^2)\delta^+(p_4^2)\delta(p_1 + p_2 - p_3 - p_4 - p_Z) = \int_0^1 \prod_{i=1}^5 \mathrm{d}\lambda_i F(\{\lambda_i\}). \tag{5}$$

The function $F(\{\lambda_i\})$ depends on the details of the parameterization. The invariant masses of all particles that participate in the process, such as $(p_3 + p_4)^2, (p_3 + p_Z)^2$, etc., become functions of the parameters λ_i. Soft and collinear singularities in the matrix elements occur when some invariant masses reach zero or other exceptional values: $(p_3 + p_4)^2 \to 0$, $(p_3 + p_Z)^2 \to M_Z^2$, etc. Those limits correspond to the edges of phase-space and generally occur when a subset of the λ_i approaches zero or unity. Two things can happen in these limits. Preferably, the singular limits occur in a "factorized" form, and the singularities can be extracted using the simple prescription for plus-distributions:

$$\lambda^{-1+\epsilon} = \frac{\delta(\lambda)}{\epsilon} + \sum_{n=0}^{\infty} \frac{\epsilon^n}{n!}\left[\frac{\log(\lambda)}{\lambda}\right]_+. \tag{6}$$

If the singular limits do not factorize but instead appear in an "entangled" form, such as $1/(\lambda_1 + \lambda_2)$, the singularities are disentangled using iterated sector decomposition [20]. We find that at NNLO all singular limits can be reduced to one of these two forms. When applied to real emission diagrams, this procedure enables us to rewrite the real emission contribution as a Laurant series in the regularization parameter ϵ. The coefficients of this series can be integrated numerically over phase-space in the presence of arbitrary kinematic constraints. The double-virtual, real-virtual and the double-real emission contributions can then be combined and the cancellation of divergences for infrared-safe observables can be established numerically. A detailed discussion of the method with examples of parameterizations used in actual computations can be found in Ref. [17]. We describe here a few novel aspects of the current calculation.

(i) Since electroweak gauge bosons couple to fermions chirally, we must specify our treatment of the axial current in d-dimensions. This issue arises from Dirac structures of the form $\mathrm{Tr_H}[\Gamma^{(1)}\gamma_5]\mathrm{Tr_L}[\Gamma^{(2)}\gamma_5]$, where $\Gamma^{(1,2)}$ denote generic products of Dirac matrices and $\mathrm{Tr_{H,L}}$ refer to traces over hadronic and leptonic degrees of freedom, respectively. Unlike in fully inclusive computations, these traces do not vanish when the final-state phase-space is sufficiently constrained. To deal with these terms we follow the prescription of Ref. [21]. We define the nonsinglet axial current by removing γ_5:

$$\bar{\psi}\gamma^\mu\gamma_5\psi \to \frac{-i\epsilon^{\mu\nu\alpha\beta}}{3!}\bar{\psi}\gamma_\nu\gamma_\alpha\gamma_\beta\psi. \tag{7}$$

These expressions are equivalent in the $d \to 4$ limit. Since the Levi-Civita tensor is a four-dimensional object, it must be combined with the matrix elements only after they are rendered finite. Although this seems to imply that computations should be performed with open Lorentz indices, which would be cumbersome in realistic calculations, this can be avoided. Since we are interested in the products of two traces which each contains a single γ_5, we only obtain products of two Levi-Civita tensors of the form $\epsilon_{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4}$. These can be simplified using the identity

$$\epsilon_{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4} = -\det|g_\mu^\nu|, \qquad \mu = \mu_1, \ldots \mu_4, \qquad \nu = \nu_1 \ldots \nu_4, \tag{8}$$

which can be easily continued to $d \neq 4$. We write the determinant in an explicit form and contract the free indices with the matrix elements. The resulting expressions become functions of scalar products of the particle momenta. The replacement in Eq. (7) violates the Ward identity that relates the renormalization of the axial and vector currents, since this definition of γ_5 does not anticommute with all of the γ_μ [21]. The following additional finite renormalization of the axial contribution must be performed:

$$Z_5^{ns} = 1 - \frac{\alpha_s}{\pi} C_F + \frac{\alpha_s^2}{16\pi^2}\left(22C_F^2 - \frac{107}{9} C_F C_A + \frac{2}{9} C_F N_f\right), \tag{9}$$

where N_f denotes the number of active fermion flavors and $C_{F,A}$ are the standard QCD Casimir invariants.

(ii) The differential cross section for the partonic process $q\bar{q} \rightarrow Z + X \rightarrow e^+e^- + X$ can be written as

$$|\mathcal{M}|^2 = \frac{H_{\mu\nu} L^{\mu\nu}}{(q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}, \tag{10}$$

where q^2 is the invariant mass of the di-lepton pair and $H_{\mu\nu}$ and $L_{\mu\nu}$ are the hadronic and leptonic tensors, respectively. Since we can define infrared-safe observables in QCD using leptonic momenta, the lepton tensor $L_{\mu\nu}$ is irrelevant for the cancellation of soft and collinear singularities. It is therefore natural to take the leptonic phase-space in four dimensions, rather than d dimensions, to simplify the calculation of matrix elements. However, we must take care when writing the scalar products of leptonic momenta with partonic momenta in the double-real emission corrections. We consider here the $q\bar{q} \rightarrow ggZ \rightarrow gge^+e^-$ process for illustration. The phase-space for the gge^+e^- final-state factorizes into the phase-space for $q\bar{q} \rightarrow ggZ$ and the phase-space for $Z \rightarrow e^+e^-$. We perform a Sudakov decomposition of the final-state momenta in terms of the incoming partonic momenta p_1, p_2. Denoting the momentum of one of the gluons as p_3, we have

$$p_3^\mu = a_3 p_1^\mu + b_3 p_2^\mu + p_{3T}^\mu. \tag{11}$$

Because of momentum conservation, we must define one relative angle between $\vec{p}_{3T}$ and $\vec{p}_{ZT}$ to parameterize the ggZ phase-space and one relative angle between $\vec{p}_{eT}$ and $\vec{p}_{ZT}$ to parameterize the $Z \rightarrow e^+e^-$ phase-space. In $d = 4$, each transverse phase-space is 2-dimensional, and these two angles determine the relative angle between $\vec{p}_{3T}$ and $\vec{p}_{eT}$. In $d \neq 4$, the ggZ transverse phase-space is $(d-2)$-dimensional; an additional angle ϕ is needed to define the relative orientation between the planes defined by $\vec{p}_{ZT}$, $\vec{p}_{3T}$ and by $\vec{p}_{ZT}$, $\vec{p}_{eT}$. The scalar product must therefore be written as

$$\vec{p}_{3T} \cdot \vec{p}_{eT} = p_{3T} p_{eT} (\cos\phi_{3Z} \cos\phi_{eZ} + \sin\phi_{eZ} \sin\phi_{3Z} \cos\phi). \tag{12}$$

In the limit $d \rightarrow 4$, we have $\cos\phi \rightarrow \pm 1$, and this expression reduces to $p_{3T} p_{eT} \cos(\phi_{3Z} \pm \phi_{eZ})$. This makes it explicit that given the two angles ϕ_{3Z} and ϕ_{eZ}, the orientation of $\vec{p}_{3T}$ and $\vec{p}_{eT}$ is completely determined in $d = 4$.

After all three components of the hard-scattering cross section are combined, additional counterterms are needed to remove initial-state collinear singularities. It is straightforward to extend the numerical approach described in [17] to obtain the desired results.

We have essentially two checks on our calculation. First, considering different cuts on the leptonic transverse momenta and rapidities and on the missing energy for W production, we verify cancellation of the divergences in the production cross sections. Because the divergences start at $1/\epsilon^4$ at NNLO, the cancellation of all divergences through $1/\epsilon$ provides a stringent check on the calculation. We also check that the vector and axial contributions are separately finite, as required. A second check is obtained by integrating fully over the final-state phase-space and comparing against known results for the inclusive cross section. We find excellent agreement with the results of [9] for all partonic channels.

III. PHENOMENOLOGICAL RESULTS

We have implemented our calculation into a numerical program, and we now discuss several phenomenological results obtained using this code. We first present the input parameters. We use the MRST parton distribution functions [22] at the appropriate order in α_s. We use $M_Z = 91.1875$ GeV, $\Gamma_Z = 2.4952$ GeV, $\mathrm{Br}(Z \rightarrow e^+e^-) = 0.0336$, $M_W = 80.451$ GeV, $\Gamma_W = 2.118$ GeV, $\mathrm{Br}(W \rightarrow e\nu) = 0.1068$. We set $|V_{\mathrm{ud}}| = 0.974$, $|V_{\mathrm{us}}| = |V_{\mathrm{cd}}| = 0.219$, and $|V_{\mathrm{cs}}| = 0.996$, and obtain $|V_{\mathrm{ub}}|$ and $|V_{\mathrm{cb}}|$ from unitarity of the CKM matrix. We neglect contributions from the top quark; these have been shown to be small in the inclusive cross section [9]. For electroweak input parameters, we use $\sin^2\theta_W = 0.2216$ and $\alpha_{\mathrm{QED}}(m_Z) = 1/128$. We set the factorization and renormalization scales to a common value, $\mu_r = \mu_f = \mu$, and employ various choices of μ in our numerical study. To perform the numerical integration we use the CUBA package [23].

The identification of electroweak gauge bosons at hadron colliders typically requires cuts on the transverse momenta and pseudorapidities of the charged leptons, as well as on the missing energy for W-boson production. For Z production, the invariant mass of the di-lepton pair is also restricted to suppress the importance of photon exchange. We first study the importance of the NNLO QCD effects for the cross sections and acceptances as a function of kinematic cuts for Z production at the LHC. In Fig. 1, the neutral current l^+l^- rate and acceptance at the LHC is studied as a function of a cut on the leptonic pseudorapidities. The NNLO results are absolutely stable with respect to scale variations, with residual uncertainties much less

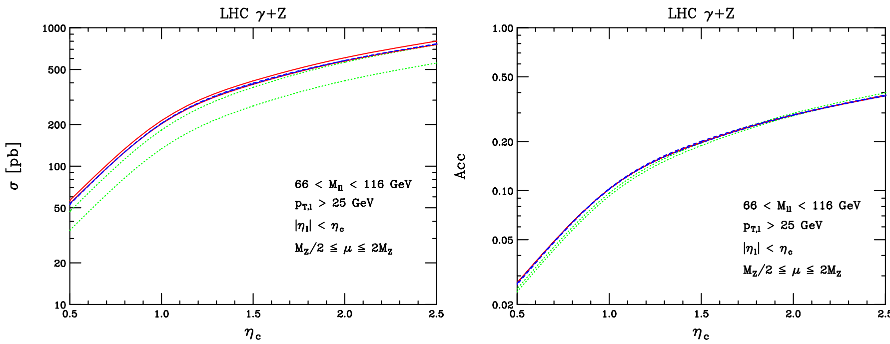

FIG. 1 (color online). The production cross section (left panel) and the acceptance (right panel) as functions of the lepton pseudorapidity cut for neutral current l^+l^- production at the LHC. The two charged leptons are required to have $p_T > 25$ GeV, and their invariant mass is constrained to $66 < M_{l^+l^-} < 116$ GeV. The dotted green lines refer to the LO result for $\mu = M_Z/2$ and $\mu = 2M_Z$, the solid red lines indicate the NLO result, and the dashed blue lines denote the NNLO result. We note that the $\mu = M_Z/2$ and $\mu = 2M_Z$ NNLO lines almost completely overlap and are nearly indistinguishable in both panels, and that both NNLO lines are completely contained within the NLO results.

than 1%, and are completely contained within the NLO uncertainty bands.

In Fig. 2 we present the neutral current l^+l^- rate and acceptance at the LHC as a function of a minimum lepton p_T cut which we refer to as $p_{T,c}$. Several comments regarding these results are required.

(i) There is a kinematic boundary at $p_T^b = M_Z/2$ above which the pure Z contribution to the LO cross section vanishes in the limit $\Gamma_Z \longrightarrow 0$. At higher orders, soft gluon effects are important near this boundary. We expect the fixed-order result to be very accurate for values of $p_{T,c}$ away from this boundary. Evidence for this is provided by the close agreement between NLO and MC@NLO for a similar boundary at $p_T^b = M_W/2$ in W production [11].

(ii) Below $p_{T,c} = 40$ GeV, the NNLO results are absolutely stable with respect to scale variations, with residual uncertainties less than 1%, and are almost completely contained within the NLO uncertainty bands.

(iii) For higher values of $p_{T,c}$, there are large shifts when going from NLO to NNLO, and the scale uncertainties underestimate the corrections. This is not too surprising; in the limit $\Gamma_Z \longrightarrow 0$ the LO result vanishes in this region since an additional radiated gluon is needed to have $p_T > M_Z/2$, and what we call NLO is the first term in the perturbative expansion. The absolute magnitude of the shift is also consistent with an $\mathcal{O}(\alpha_s^2)$ effect.

We now discuss a Tevatron analysis of the W-boson cross section. CDF recently presented a measurement of the $W \longrightarrow e\nu$ cross section in the forward rapidity region, $1.2 < |\eta| < 2.8$, and compared this result to the central cross section [24]. Different values of Bjorken-x contribute to each rapidity region, and measuring the central/forward cross section ratio may provide a useful constraint on parton distribution functions. The geometric and kinematic cuts in each region on the charged lepton pseudorapidity and transverse momentum, and on the missing energy, are listed below.

(i) Forward: $1.2 < |\eta| < 2.8$, $E_T > 20$ GeV, $\not{E}_T > 25$ GeV

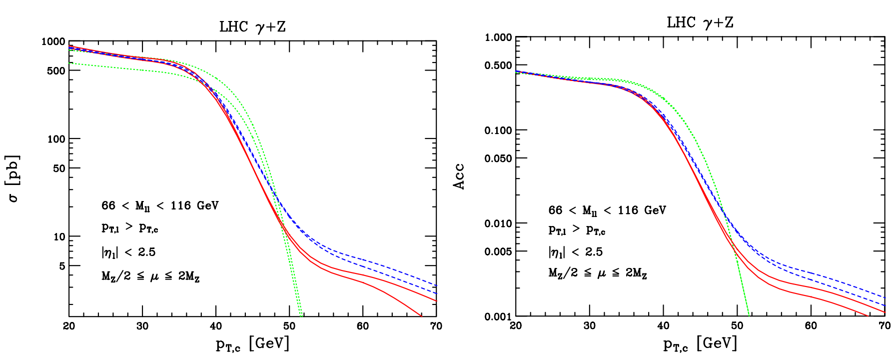

FIG. 2 (color online). The production cross section (left panel) and the acceptance (right panel) as functions of the lepton transverse momentum cut for neutral current l^+l^- production at the LHC. The two charged leptons are required to have $|\eta| < 2.5$, and their invariant mass is constrained to $66 < M_{l^+l^-} < 116$ GeV. The dotted green lines refer to the LO result for $\mu = M_Z/2$ and $\mu = 2M_Z$, the solid red lines indicate the NLO result, and the dashed blue lines denote the NNLO result.

TABLE I. The theoretical predictions for the forward region acceptance $A_{\rm for}$, the central region acceptance $A_{\rm cen}$, and the ratio of central/forward acceptances $R_{c/f}$, together with their associated uncertainties at NLO and NNLO. We note that both the geometric acceptance and the factor $A_{\rm cor}$ have been included in the central region result.

	NLO	NNLO
$A_{\rm for}$	0.2616(2)	0.2614(2)
$A_{\rm cen}$	0.2458(28)	0.2422(5)
$R_{c/f}$	0.940(12)	0.9266(19)

(ii) Central: $|\eta| < 1.1$, $E_T > 25$ GeV, $\not{E}_T > 25$ GeV.

In the central cross section analysis there are additional selection cuts requiring the electron to be in the fiducial region of the calorimeter, and for the tracker to find an electron with $p_T > 10$ GeV consistent with the energy deposition in the calorimeter [12]. These cuts give an additional factor $A_{\rm cor} = 0.6985$, so that the acceptance in the central region is $A_{\rm cen} = A_{\rm geom} \times A_{\rm cor}$. We compute $A_{\rm geom}$ through NNLO in perturbative QCD and use the given $A_{\rm cor}$ to determine the acceptance in the central region.

We present in Table I the predictions for the acceptances in the central and forward regions, and for the central/forward ratio $R_{c/f}$. The uncertainties in this table have been obtained by computing the results for the scale choices $M_W/2 \le \mu \le 2M_W$ and equating the spread with the residual uncertainty. This procedure is supported by the NNLO results lying within the ranges indicated by the NLO scale variation. The NNLO theoretical uncertainties are at the 0.25% level or less, and are completely negligible. The forward region acceptance, in particular, is absolutely stable against radiative corrections. In the central region we observe an error reduction of a factor of 5 when the NNLO QCD effects are included. Only experimental errors and parton distribution function uncertainties remain, indicating that this measurement can potentially provide useful constraints on parton distribution functions. Our result $R^{\rm th}_{c/f} = 0.9266(19)$ is in good agreement with the preliminary value obtained by CDF, $R^{\rm exp}_{c/f} = 0.925(33)$ [24].

IV. CONCLUSIONS

In this paper we described a computation of the fully differential cross section through NNLO in QCD for W and Z boson production in hadronic collisions. Our result includes spin correlations, finite width effects, $\gamma - Z$ interference and allows for the application of arbitrary cuts on the final-state decay products. We have incorporated our result into a numerical code FEWZ available at the web site http://www.phys.hawaii.edu/~kirill/FEHiP.htm. We believe this program will be invaluable for precision electroweak studies at both the Tevatron and the LHC.

We studied several LHC and Tevatron examples where precise predictions for gauge boson acceptances are required. The theoretical prediction for neutral current l^+l^- production at the LHC is absolutely stable with respect to residual scale dependence, with a remaining theoretical uncertainty much less than 1%, as long as the minimum p_T cut on the leptons is less than the kinematic boundary value $p_T^b = M_Z/2$. For momenta above p_T^b the LO result vanishes in the limit $\Gamma_Z \to 0$. The NNLO calculation provides the first radiative correction in this region, and a significant scale variation remains.

We also studied the W cross section in the central and forward regions, recently analyzed by the CDF collaboration. The theoretical predictions for the acceptances in each region have residual uncertainties less than 0.25% Our calculation of the ratio of central and forward cross sections, $R^{\rm th}_{c/f} = 0.9266(19)$, is in good agreement with the preliminary CDF result $R^{\rm exp}_{c/f} = 0.925(33)$.

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[1] For reviews, see P. M. Nadolsky, AIP Conf. Proc. **753**, 158 (2005); U. Baur, hep-ph/0511064.
[2] M. Dittmar, F. Pauss, and D. Zurcher, Phys. Rev. D **56**, 7284 (1997); V. A. Khoze, A. D. Martin, R. Orava, and M. G. Ryskin, Eur. Phys. J. C **19**, 313 (2001); W. T. Giele and S. A. Keller, hep-ph/0104053.
[3] For a summary of these issues, see F. Gianotti and M. L. Mangano, hep-ph/0504221.
[4] G. Altarelli, R. K. Ellis, and G. Martinelli, Nucl. Phys. **B157**, 461 (1979); J. Kubar-Andre and F. E. Paige, Phys. Rev. D **19**, 221 (1979); K. Harada, T. Kaneko, and N. Sakai, Nucl. Phys. **B155**, 169 (1979); **165**, 545(E) (1980); P. Aurenche and J. Lindfors, Nucl. Phys. **185**, 274 (1981).
[5] S. Dittmaier and M. Kramer, Phys. Rev. D **65**, 073007 (2002); U. Baur, S. Keller, and D. Wackeroth, Phys. Rev. D **59**, 013002 (1998).

[6] S. Frixione and B. R. Webber, J. High Energy Phys. 06 (2002) 029; S. Frixione, P. Nason, and B. R. Webber, J. High Energy Phys. 08 (2003) 007.
[7] C. Balazs and C. P. Yuan, Phys. Rev. D **56**, 5558 (1997); F. Landry, R. Brock, P. M. Nadolsky, and C. P. Yuan, Phys. Rev. D **67**, 073016 (2003).
[8] R. K. Ellis, G. Martinelli, and R. Petronzio, Nucl. Phys. **B211**, 106 (1983); R. J. Gonsalves, J. Pawlowski, and C. F. Wai, Phys. Rev. D **40**, 2245 (1989); P. B. Arnold and M. H. Reno, Nucl. Phys. **B319**, 37 (1989); **B330**, 284(E) (1990).
[9] R. Hamberg, W. L. van Neerven, and T. Matsuura, Nucl. Phys. **B359**, 343 (1991); **B644**, 403(E) (2002); R. V. Harlander and W. B. Kilgore, Phys. Rev. Lett. **88**, 201801 (2002).
[10] C. Anastasiou, L. J. Dixon, K. Melnikov, and F. Petriello, Phys. Rev. Lett. **91**, 182002 (2003); C. Anastasiou, L. J. Dixon, K. Melnikov, and F. Petriello, Phys. Rev. D **69**, 094008 (2004).
[11] S. Frixione and M. L. Mangano, J. High Energy Phys. 05 (2004) 056.
[12] D. Acosta *et al.* (CDF II Collaboration), Phys. Rev. Lett. **94**, 091803 (2005); A. Abulencia *et al.* (CDF Collaboration), hep-ex/0508029; P. Petrov (CDF and D0 Collaborations), AIP Conf. Proc. **815**, 49 (2006).
[13] D. Acosta *et al.* (CDF Collaboration), Phys. Rev. D **71**, 052002 (2005); **71**, 051104 (2005).
[14] See, for example, S. Haywood *et al.*, hep-ph/0003275.
[15] R. K. Ellis, D. A. Ross, and A. E. Terrano, Nucl. Phys. **B178**, 421 (1981); W. T. Giele and E. W. N. Glover, Phys. Rev. D **46**, 1980 (1992); Z. Kunszt and D. E. Soper, Phys. Rev. D **46**, 192 (1992); S. Frixione, Z. Kunszt, and A. Signer, Nucl. Phys. **B467**, 399 (1996); S. Catani and M. H. Seymour, Nucl. Phys. **B485**, 291 (1997); **B510**, 503(E) (1997).
[16] S. Weinzierl, J. High Energy Phys. 03 (2003) 062; W. B. Kilgore, Phys. Rev. D **70**, 031501 (2004); S. Frixione and M. Grazzini, J. High Energy Phys. 06 (2005) 010; G. Somogyi, Z. Trocsanyi, and V. Del Duca, J. High Energy Phys. 06 (2005) 024; A. Gehrmann-De Ridder, T. Gehrmann, and E. W. N. Glover, J. High Energy Phys. 09 (2005) 056.
[17] C. Anastasiou, K. Melnikov, and F. Petriello, Phys. Rev. D **69**, 076010 (2004); C. Anastasiou, K. Melnikov, and F. Petriello, Phys. Rev. Lett. **93**, 032002 (2004); **93**, 262002 (2004); Nucl. Phys. **B724**, 197 (2005).
[18] K. Melnikov and F. Petriello, Phys. Rev. Lett. **96**, 231803 (2006).
[19] R. J. Gonsalves, Phys. Rev. D **28**, 1542 (1983); G. Kramer and B. Lampe, Z. Phys. C **34**, 497 (1987); **42**, 504(E) (1989).
[20] T. Binoth and G. Heinrich, Nucl. Phys. **B585**, 741 (2000); see also K. Hepp, Commun. Math. Phys. **2**, 301 (1966); M. Roth and A. Denner, Nucl. Phys. **B479**, 495 (1996).
[21] S. A. Larin, Phys. Lett. B **303**, 113 (1993).
[22] A. D. Martin, R. G. Roberts, W. J. Stirling, and R. S. Thorne, Phys. Lett. B **531**, 216 (2002).
[23] T. Hahn, Comput. Phys. Commun. **168**, 78 (2005).
[24] CDF Collaboration, Report No. CDF-note 8119 (unpublished); M. Lancaster, Proceedings of the 33rd International Conference On High Energy Physics, 2006, Moscow.