Nonlocality effects on color spin locking condensates

D. N. Aguilera,[1,*] D. Blaschke,[2,3,4,†] H. Grigorian,[3,5,6,‡] and N. N. Scoccola[7,8,9,§]

[1]*Department of Applied Physics, University of Alicante, Apartado Correus 99, 03080 Alicante, Spain*
[2]*Institute for Theoretical Physics, University of Wroclaw, Max Born place 3, 50-204 Wroclaw, Poland*
[3]*Institut für Physik, Universität Rostock, Universitätsplatz 3, 18051 Rostock, Germany*
[4]*Bogoliubov Laboratory of Theoretical Physics, JINR Dubna, Joliot-Curie Street 6, 141980 Dubna, Russia*
[5]*Laboratory for Information Technologies, JINR Dubna, Joliot-Burie Street 6, 141980 Dubna, Russia*
[6]*Department of Physics, Yerevan State University, Alex Manookian Street 1, 375047 Yerevan, Armenia*
[7]*Physics Department, Comisión Nacional de Energía Atómica, Avenida Libertador 8250, 1429 Buenos Aires, Argentina*
[8]*Universidad Favaloro, Solís 453, 1078 Buenos Aires, Argentina*
[9]*CONICET, Rivadavia 1917, 1033 Buenos Aires, Argentina*

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We consider the color spin locking (CSL) phase of two-flavor quark matter at zero temperature for nonlocal instantaneous separable interactions. We employ a Lorentzian-type form factor allowing a parametric interpolation between the sharp [Nambu-Jona-Lasinio (NJL) model] and very smooth (e.g. Gaussian) cutoff models for systematic studies of the influence on the CSL condensate the deviation from the NJL model entails. This smoothing of the NJL model form factor shows advantageous features for the phenomenology of compact stars: (i) a lowering of the critical chemical potential for the onset of the chiral phase transition as a prerequisite for stability of hybrid stars with extended quark matter cores and (ii) a reduction of the smallest pairing gap to the order of 100 keV, being in the range of values interesting for phenomenological studies of hybrid star cooling evolution.

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I. INTRODUCTION

Recently, the investigation of color superconducting phases in cold dense quark matter has received much attention [1–6], in particular, due to the possible consequences for the physics of compact stars [7,8]. From the point of view of observational constraints on quark matter and color superconductivity in compact stars, the cooling characteristics play a central role. It has been shown that the occurrence of a normal quark matter core would lead to a conflict with observations since the direct Urca (DU) process in normal quark matter would lead to enhanced cooling in disagreement with the data [9,10]. The DU conflict would be solved provided no ungapped quark modes occur in the quark core. This has been demonstrated on the example of a hypothetical pairing channel (X-gap) for the quark color which is ungapped in the 2SC phase (2 SC + X phase) [10,11]. However, the microscopic origin of the X-gap could not yet be specified. A microscopically well-defined pairing pattern which could solve the quark DU problem would be the color spin locking (CSL) phase [12] corresponding to a spin-one condensate [13–15]. A prerequisite for the realization of this pairing pattern in quark matter would be a sufficient flavor asymmetry to prevent the u-d pairing in the otherwise dominant scalar diquark channel of the 2SC phase. It has been demonstrated that under neutron star conditions the 2SC phase is indeed rather fragile and may not be realized for moderate coupling strengths [16]. Thus the CSL phase becomes particularly interesting for the solution of the quark DU cooling problem, and corresponding simulations will be performed as soon as the cooling regulators such as emissivities, specific heat and thermal conductivity are provided. The first steps in this direction have been made recently [17,18].

Most of the calculations of QCD superconducting phases have been done using the sharp cutoff NJL model (see Ref. [1] and references therein). However, lattice QCD calculations [19] indicate that quark interactions should act over a certain range in the momentum space, and various approaches to include nonlocality effects beyond the NJL model have been suggested [20]. We refer to nonlocal separable interaction models as introduced, e.g., in the works [21–26] and references therein, where it has been concluded that smoothing the cutoff leads to a reduction of the chiral condensate and a lowering of the critical temperature for the chiral phase transition. The question arises for the effects of nonlocality on the spin-one gaps, to be explored by varying the form factor of the quark interaction from a sharp cutoff in the NJL model to a smoothly decreasing form such as a Gaussian. The first exploratory calculations reported in [27] have shown that the nonlocality could lead to a sizable decrease of the energy gaps.

In this paper, we investigate the robustness of CSL pairing against a modification of the sharp cutoff (NJL) in a systematic way by employing a separable, instantaneous interaction with a Lorentzian-type interaction which allows us to interpolate between the NJL case and very smooth interaction form factors of, e.g., the Gaussian type.

*Electronic address: deborah.aguilera@ua.es
†Electronic address: david.blaschke@uni-rostock.de
‡Electronic address: hovik.grigorian@uni-rostock.de
§Electronic address: scoccola@tandar.cnea.gov.ar

This investigation is performed on the basis of recently developed parametrizations for the instantaneous three flavor case [28].

II. NONLOCAL CHIRAL QUARK MODEL FOR THE CSL PHASE

We investigate a nonlocal chiral quark model in which the quark interaction is represented in a separable way by introducing form factor functions $g(p)$ in the bilinears of the current-current interaction terms in the Lagrangian [16,21,29]. It is assumed that this four-fermion interaction is instantaneous and therefore the form factors do not depend on the energy but only on the modulus of the three momentum $p = |\vec{p}|$. The ansatz for the s-wave, single-flavor diquark condensate characterizing the CSL phase as introduced in Ref. [12][1] is a scalar product (locking) of the three-vector of antisymmetric color matrices $(\lambda_2, \lambda_5, \lambda_7)$ with the three-vector of Dirac spin matrices $(\gamma_3, \gamma_2, \gamma_1)$. Thus, the corresponding gap matrix $\hat{\Delta}$ for the CSL phase reads

$$\hat{\Delta} = \Delta(\gamma_3\lambda_2 + \gamma_2\lambda_5 + \gamma_1\lambda_7). \tag{1}$$

Since the two flavor channels decouple, the quark thermodynamical potential can be decomposed into single-flavor components

$$\Omega_q(T, \{\mu_f\}) = \sum_{f=u,d} \Omega(T, \mu_f), \tag{2}$$

and it is sufficient to consider in the following the contribution of a single flavor only, which in the mean field approximation is given by

$$\Omega(T, \mu) = \frac{\phi^2}{8G} + 3\frac{\Delta^2}{8H_v} - T\sum_n \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \mathrm{Tr} \ln\left(\frac{1}{T} S^{-1}(i\omega_n, \vec{p})\right), \tag{3}$$

where μ stands for the chemical potential of that flavor. The first two terms are quadratic contributions of the mean field values ϕ and Δ of the order parameter fields that signal chiral symmetry breaking and CSL superconductivity, respectively. Their denominators contain the coupling constants G and H_v in the corresponding channel. In (3) the sum is over fermionic Matsubara frequencies $\omega_n = (2n + 1)\pi T$, and the trace is over Dirac, color and Nambu-Gorkov indices.

In our nonlocal extension, the inverse fermion propagator differs from the NJL model case by momentum-dependent form factors $g(p)$ modifying the mesonic and diquark mean fields

$$S^{-1}(p) = \begin{pmatrix} \not{p} + \mu\gamma^0 - M(p) & g(p)\hat{\Delta} \\ -g(p)\hat{\Delta}^\dagger & \not{p} - \mu\gamma^0 - M(p) \end{pmatrix}, \tag{4}$$

where $M(p)$ is the dynamical quark mass function

$$M(p) = m + g(p)\phi. \tag{5}$$

Note that, although the first two terms in Eq. (3) do not have any explicit dependence on the form factors, the quantities ϕ and Δ do depend implicitly on them through the gap equations; see Eq. (14) below.

After evaluation of the trace [12] and Matsubara summation the thermodynamical potential takes the form

$$\Omega(T, \mu) = \frac{\phi^2}{8G} + 3\frac{\Delta^2}{8H_v} - \sum_{k=1}^{6} \int \frac{d^3p}{(2\pi)^3} \times [E_k(p) + 2T \ln(1 + e^{-E_k(p)/T})], \tag{6}$$

where $E_k(p)$ denote the excitation energies for the modes $k = 1, \ldots, 6$. The odd (even) indices denote particle (antiparticle) excitations, each corresponding to a triplet of spin-one eigenstates. All modes have a gap in the excitation spectrum and can be brought into a standard form, which for $E_1(p)$ reads

$$E_1^2(p) = (\varepsilon_{\mathrm{eff}}(p) - \mu_{\mathrm{eff}}(p))^2 + \Delta_{\mathrm{eff}}^2(p), \tag{7}$$

with the effective quantities

$$\varepsilon_{\mathrm{eff}}(p) = \sqrt{p^2 + M_{\mathrm{eff}}^2(p)}, \tag{8}$$

$$M_{\mathrm{eff}}(p) = \frac{\mu}{\mu_{\mathrm{eff}}(p)} M(p), \tag{9}$$

$$\mu_{\mathrm{eff}}(p) = \mu\sqrt{1 + \Delta^2 g^2(p)/\mu^2}, \tag{10}$$

$$\Delta_{\mathrm{eff}}(p) = \frac{M(p)}{\mu_{\mathrm{eff}}(p)} \Delta g(p), \tag{11}$$

and for $E_{3,5}(p)$ is given by

$$E_{3,5}^2(p) = (\varepsilon(p) - \mu)^2 + a_{3,5}(p)\Delta^2 g^2(p), \tag{12}$$

with the momentum-dependent coefficients

$$a_{3,5}(p) = \frac{1}{2}\left[5 - \frac{p^2}{\varepsilon(p)\mu} \pm \sqrt{\left(1 - \frac{p^2}{\varepsilon(p)\mu}\right)^2 + 8\frac{M^2(p)}{\varepsilon^2(p)}}\right], \tag{13}$$

where $\varepsilon(p) = \sqrt{p^2 + M^2(p)}$. The remaining modes $E_{2,4,6}(p)$ are obtained from $E_{1,3,5}(p)$ by changing $\mu \rightarrow -\mu$ in Eqs. (7)–(13). Note that the modes $E_{1,2}(p)$ correspond to the vanishing z projection of the spin, $S_z = 0$, thus being inert against an external B field. The remaining modes corresponding to $S_z = \pm 1$ are expected to get shifted (Zeeman effect).

[1]Note that such ansatz differs from the one in [15], where the states are constructed with total angular momentum of 1.

For given values of T and μ, the global minimum of $\Omega(T, \mu)$ in the space of the order parameters ϕ and Δ corresponds to the thermodynamical equilibrium state. We obtain this state by comparing solutions of the gap equations

$$\frac{\delta\Omega(T,\mu)}{\delta\phi} = \frac{\delta\Omega(T,\mu)}{\delta\Delta} = 0. \tag{14}$$

We present results for the case of vanishing temperature and finite chemical potential in the next section.

III. MODEL CALCULATIONS

A. Form factors and their parameters

In (4) and (5) we have introduced the same form factors $g(p)$ to represent the nonlocality of the interaction in the meson ($q\bar{q}$) and diquark (qq) channels. In the calculations we use the sharp cutoff (NJL), Lorentzian with integer parameter α (Lα) and Gaussian (G), form factors defined as

$$g_{\rm NJL}(p) = \theta(1 - p/\Lambda), \tag{15}$$

$$g_{{\rm L}\alpha}(p) = [1 + (p/\Lambda)^{2\alpha}]^{-1}, \qquad \alpha \geq 2, \tag{16}$$

$$g_{\rm G}(p) = \exp(-p^2/\Lambda^2), \tag{17}$$

where Λ is a cutoff parameter. These form factors are plotted in Fig. 1. We achieve deviations from the NJL case (step function) by using the Lorentzian form factor with decreasing the α parameter. The Gaussian form factor appears on the other limit, having a very soft momentum dependence.

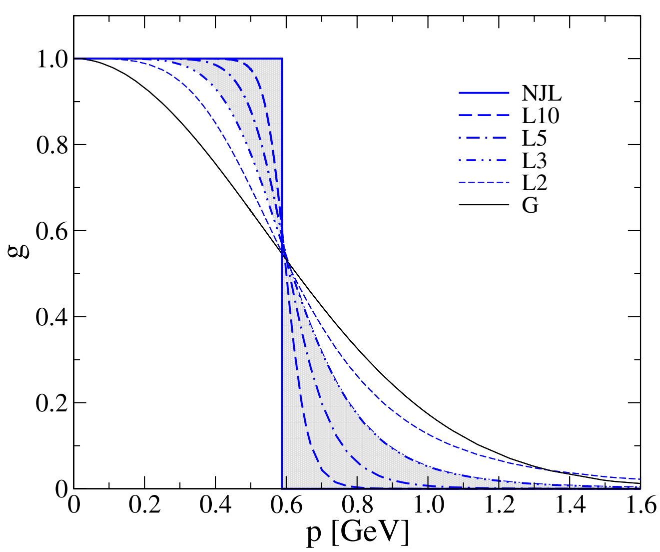

FIG. 1 (color online). Form factors used to represent the nonlocality in the momentum space. Decreasing α in the Lorentzian-type form factor (notation Lα) causes deviations from the NJL model in a systematic way.

TABLE I. Parameter sets for the nonlocal chiral quark model with Lorentzian and Gaussian form factors and for the NJL model. Sets for different fixed $M = 330$, 400 MeV are listed.

M (MeV)	Form factor	Λ (MeV)	$G\Lambda^2$	m (MeV)
330	NJL	629.5	2.17	5.28
	L10	649.2	2.36	4.71
	L5	666.5	2.49	4.09
	L3	685.8	2.59	3.25
	L2	703.4	2.58	2.37
	G	891.1	3.88	2.18
400	NJL	587.9	2.44	5.58
	L10	600.3	2.64	5.01
	L5	609.3	2.78	4.39
	L3	616.2	2.87	3.55
	L2	617.8	2.83	2.65
	G	756.1	4.22	2.60

To perform numerical calculations one has to specify, for each form factor, the following set of parameters: the light quark current mass (m), the coupling strength (G) and the range of the interaction (Λ). The diquark coupling constant H_v is fixed to the ratio $H_v/G = 8/3$ in accordance with the result of the Fierz transformation for a one-gluon exchange interaction. In this work we use the parametrizations recently given in Ref. [28] and listed in Table I. They have been obtained by fitting the vacuum properties of the pion ($f_\pi = 92.4$ MeV, $M_\pi = 135$ MeV) and the vacuum constituent quark mass at zero momentum, $M = m + \phi$. For the latter, the phenomenologically reasonable values $M = 330$ and 400 MeV have been used.

Note that the results to be presented below do not depend on the choice of the Lorentzian-type function as an interpolating form factor. In fact, similar results have been obtained using other interpolating functions such as, e.g., the Woods-Saxon form factors, the parametrization of which is given in [28].

B. Quark mass and CSL pairing gap

First, we analyze form factors which do not deviate strongly from the NJL case, i.e. Lα for $\alpha \geq 3$, shown as the gray area in Fig. 1. In Fig. 2 we compare the solutions obtained for the mass and the CSL gaps for two different sets of regularizations for fixed constituent mass: $M = 330$ MeV (left) and $M = 400$ MeV (right). For the parametrizations with a larger constituent mass in vacuum, one obtains a larger critical chemical potential μ_c for the phase transition from the chirally broken phase to the restored one, where the CSL pairing can occur. The gaps at the onset, $\Delta(\mu_c)$, are larger whereas the mass gaps after the chiral transition are smaller.

Like in the NJL case [12], the CSL gaps are strongly increasing functions of μ in the range that is relevant for compact stars, $\mu_c < \mu \lesssim 500$ MeV, where the upper limit is due to the threshold for the occurrence of strange quarks

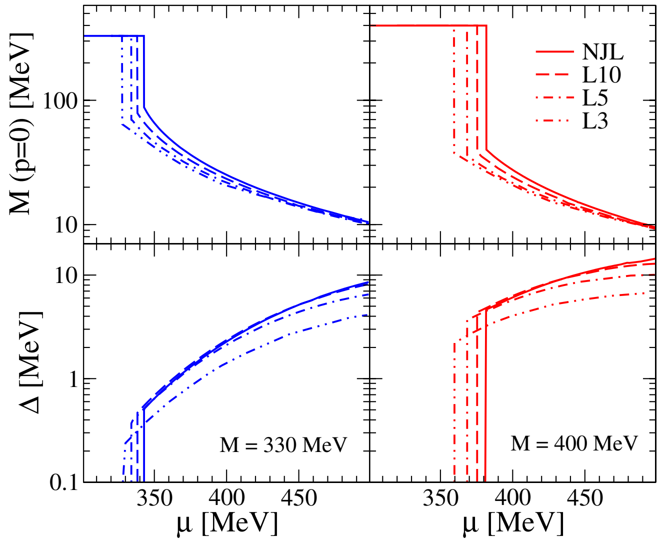

FIG. 2 (color online). The dependence of the dynamical mass $M(0)$ and the CSL pairing gap Δ on the chemical potential μ for different NJL-like form factor models (NJL and Lorentzian with $\alpha \geq 3$). The parametrizations correspond to a fixed constituent mass: $M = 330$ MeV on the left and $M = 400$ MeV on the right.

which allow pairing patterns like the CFL phase, energetically more favorable than CSL (for recent phase diagrams for neutral matter in the three flavor case, see [30–32]).

On the other hand, Fig. 2 clearly shows that the pairing gaps in this nonlocal extension are reduced relative to the NJL ones: the smoother the form factor, the smaller the gap. The reduction could be up to a factor 3 in the case of the Lorentzian with $\alpha = 3$.

But perhaps one of the most important effects of nonlocality from the point of view of the phenomenology of compact stars is the shift of the chiral phase transition to lower values of μ. A lowering of the critical density for deconfinement makes stable hybrid star configurations with large quark matter cores possible [29,33]. On the other hand, results from lattice QCD simulations for the quark propagator [19] show a very smooth four-momentum dependence of the quark self-energies which is also a characteristic of confining quark models within the Dyson-Schwinger approach [34]. Smoother form factors could thus be more appropriate to model QCD interactions. Note, however, that the instantaneous nonlocal models with such smooth form factors lead to unrealistically large values of the chiral condensate (above 280 MeV) in the vacuum [28].

Therefore we have subdivided our discussion of different form factors into two groups: those which lead to deviations from the NJL results within 1 order of magnitude and those resulting in larger deviations from the NJL model case (Lorentzian with $\alpha = 2$ and Gaussian); see Fig. 3. For example, the shift in the critical chemical potential for the onset of the chiral phase transition relative to the NJL case is less than 20 MeV within the first group, but larger than 30 MeV for the second group; see Figs. 2 and 3. However, the qualitative behavior of the chiral and CSL gaps is not affected by the choice of the form factors.

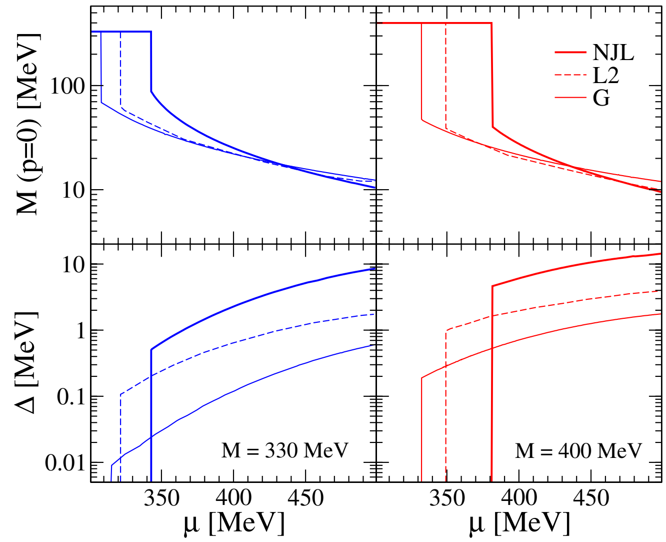

FIG. 3 (color online). Same as Fig. 2 for smooth form factor models (Lorentzian with $\alpha = 2$ and Gaussian).

To obtain the above results, we have kept fixed the ratio H_v/G at the standard value obtained from Fierz transforming the one-gluon exchange interaction. However, in order to estimate the effect of possible uncertainties in this value, we have also considered the situation in which this ratio is taken to be twice its Fierz value. The corresponding results for $2H_v$ and all Lorentzian-type form factors under consideration are shown in Fig. 4. It is worth noticing that,

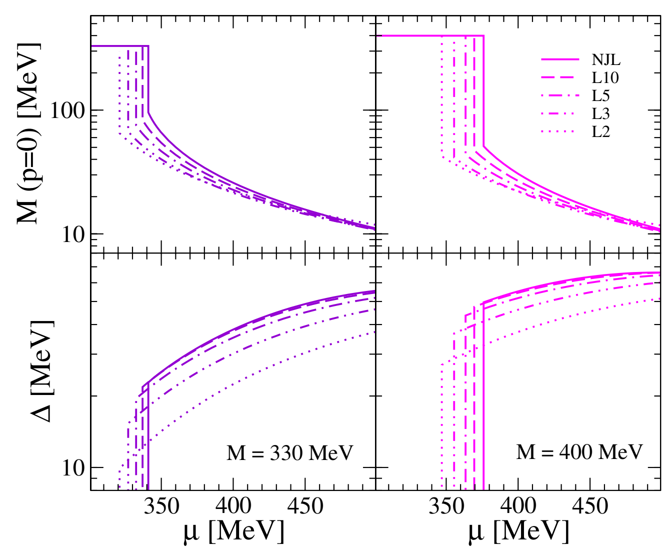

FIG. 4 (color online). Same as Figs. 2 and 3 but the coupling constant H_v in the diquark channel is doubled with respect to the usual value coming from Fierz transformed one-gluon exchange interaction.

while the increase of H_v by a factor 2 increases the CSL gaps by 1 ($M = 400$ MeV) or 2 ($M = 330$ MeV) orders of magnitude, the qualitative behavior of M and Δ as functions of μ remains practically unchanged. The lowering of μ_c and Δ as the form factor becomes smoother is also similar to the previous case.

C. Quasiparticle excitation spectrum

In Fig. 5 we show the quasiparticle excitation spectrum at the critical chemical potential for both parametrizations, respectively. We observe that the quasiparticle mode with the lowest energy band corresponds to $E_1(p)$ with a minimum

$$E_{1,\min} = \min_p[E_1(p)], \tag{18}$$

being the most relevant quantity for possible applications of the CSL phase of quark matter to compact star cooling phenomenology. This minimum occurs at the Fermi momentum $p = p_F$. In a very good approximation, p_F can be represented by the lowest orders of a series expansion in the parameter $s = p_F g'_{L\alpha}(p_F)/g_{L\alpha}(p_F)$, which is a measure for the influence of the form factor

$$p_F^2 = \mu_{\rm eff}^2(p_F) - M_{\rm eff}^2(p_F) + 2\Delta_{\rm eff}^2(p_F) \times \frac{M(p_F)[M(p_F) - m] + M_{\rm eff}^2(p_F)}{M^2(p_F) - M_{\rm eff}^4(p_F)/\mu^2} s + O(s^2). \tag{19}$$

In the same order of the expansion in s, we obtain for the minimal excitation energy

$$E_{1,\min} = \Delta_{\rm eff}(p_F) + O(s^2). \tag{20}$$

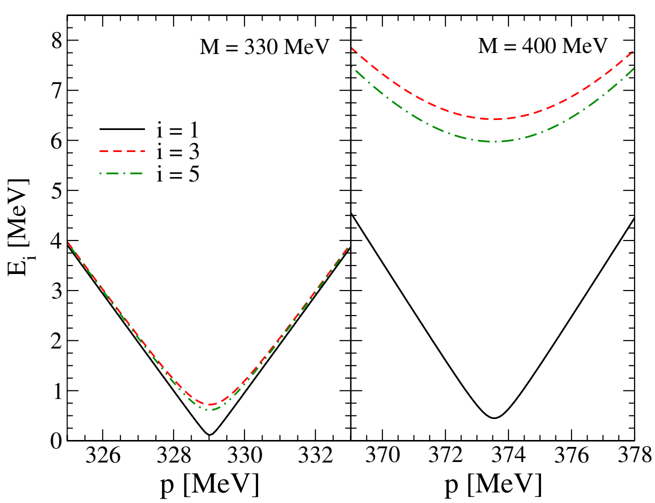

FIG. 5 (color online). Excitation energies in the CSL phase as a function of the momentum for both parametrizations of the L10 model from Table I at the corresponding critical values of the chemical potential, $\mu = \mu_c$. Left panel: $M = 330$ MeV, $\mu_c = 338$ MeV; right panel: $M = 400$ MeV, $\mu_c = 375$ MeV. Antiparticle modes E_2, E_4 and E_6 are not shown.

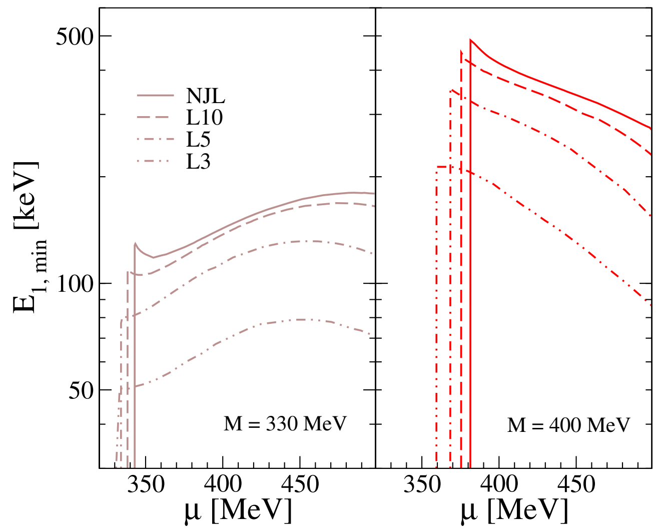

FIG. 6 (color online). Minimal excitation energies $E_{1,\min}$ in the CSL phase as functions of the chemical potential μ for different NJL-like form factors. Parametrizations correspond to fixed $M = 330$ MeV on the left and $M = 400$ MeV on the right.

Although $E_{1,\min}$ might be quite small (see below) it never vanishes. In fact, as in the NJL case [12], in the present class of models none of the dispersion relations lead to gapless modes. It is interesting to note that this lowest energy mode $E_1(p)$, relevant for compact star cooling phenomenology, is the one which is inert against the influence of a strong external magnetic field typical for neutron stars since it belongs to vanishing spin projection, $S_z = 0$.

In Figs. 6 and 7 we plot $E_{1,\min}$ as a function of μ for different models from the NJL-like and the smooth form factor groups, respectively. The calculations are made for

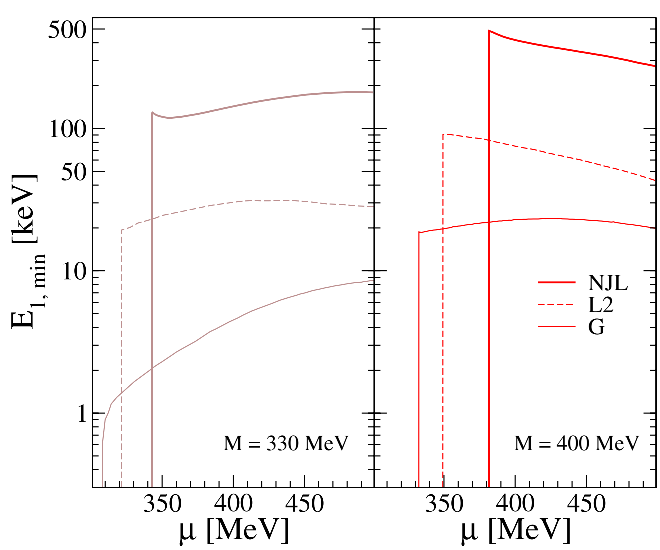

FIG. 7 (color online). Same as Fig. 6 for models that present a larger deviation from NJL models (Lorentzian with $\alpha = 2$ and Gaussian).

both sets of parametrizations of Table I, corresponding to constituent masses of $M = 330$ MeV and $M = 400$ MeV. According to the analytical approximative result of Eq. (20), the behavior of the minimal excitation energies can be understood as a product of the increasing μ dependence of the CSL pairing gaps and the decreasing one of the other factors in Eq. (11). For the parametrizations with $M = 400$ MeV in the right panel, we obtain that $E_{1,\min}$ is a decreasing function of μ since the increase in $\Delta(\mu)$ cannot overcompensate the decrease in $M(p_F)g(p_F)/\mu_{\rm eff}(p_F)$. A similar effect has been reported for NJL models [12]. On the other hand, for parametrizations with $M = 330$ MeV the interplay between $\Delta(\mu)$ and the other factors in Eq. (11) is density dependent: a slightly increasing behavior of $E_{1,\min}$ at low densities is followed by a tendency to a saturation or even decreasing behavior at high densities. For the group of NJL-like form factors, our results for $E_{1,\min}$ lie in the range of 50–500 keV, and for the case of smooth form factors they are between 1 and 100 keV.

Our main results are summarized in Fig. 8. In the upper panel we show the scaling of the critical chemical potential $\mu_c^{\rm L\alpha}$ for the onset of the CSL phase for the Lorentztian-type model Lα with the smoothness parameter $1/\alpha$ normalized to the NJL limit case, $\mu_c^{\rm NJL}$. In the lower panel we show a comparative plot of the minimal excitation energies $E_{1,\min}^{\rm L\alpha}$ in units of the corresponding NJL counterpart $E_{1,\min}^{\rm NJL}$, evaluated at $\mu_c^{\rm L\alpha}$ and $\mu_c^{\rm NJL}$, respectively. The corresponding results for $2H_v$ are also shown in Fig. 8 as open

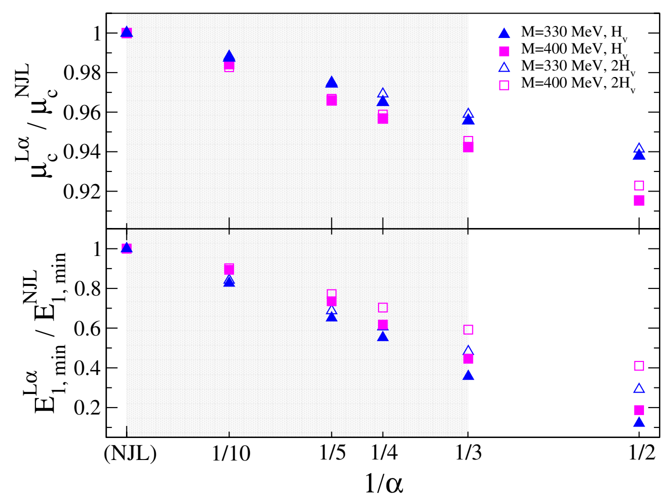

FIG. 8 (color online). Two nonlocality effects on CSL condensates as a function of the smoothness parameter $1/\alpha$ of Lorentzian-type models, for $\alpha = 2, 3, 4, 5, 10$, normalized to the NJL limit ($\alpha \to \infty$). First: the lowering of the critical chemical potentials $\mu_c^{\rm L\alpha}$ with respect to $\mu_c^{\rm NJL}$ (upper panel). Second: the reduction of the minimal excitation energies $E_{1,\min}^{\rm L\alpha}$ with respect to $E_{1,\min}^{\rm NJL}$ (upper panel; values calculated at $\mu_c^{\rm L\alpha}$ and $\mu_c^{\rm NJL}$, respectively). Results are shown as a function of $1/\alpha$ for two different quark mass parametrizations in the vacuum: $M = 330$ MeV (triangles) and $M = 400$ MeV (squares). The open symbols show results for doubled CSL coupling strength ($2H_v$).

symbols. As we see, the qualitative behavior of both $\mu_c^{\rm L\alpha}/\mu_c^{\rm NJL}$ and $E_{1,\min}^{\rm L\alpha}/E_{1,\min}^{\rm NJL}$ as a function of $1/\alpha$ remains unchanged.

It is remarkable that, as it would be expected from an expansion to lowest order in s, both quantities scale almost linearly with $1/\alpha$. In fact, a very good approximation to our numerical results is obtained with

$$\mu_c^{\rm L\alpha} \approx \left(1 - \frac{\xi'}{\alpha}\right)\mu_c^{\rm NJL}, \tag{21}$$

$$E_{1,\min}^{\rm L\alpha} \approx \left(1 - \frac{\xi}{\alpha}\right)E_{1,\min}^{\rm NJL}, \tag{22}$$

for α down to 2, where the slope parameters ξ and ξ' only moderately depend on the model parametrization (M) and the CSL coupling strength (H_v). For $M = 330$ MeV we get $\xi = 1.8\,(1.5)$ and $\xi' = 0.13\,(0.12)$, while for $M = 400$ MeV the corresponding values are $\xi = 1.6\,(1.2)$, $\xi' = 0.17\,(0.16)$. The numbers in parentheses are obtained by doubling H_v.

IV. CONCLUSION

We have studied the effect of instantaneous nonlocal interactions in the CSL phase of quark matter. We have introduced momentum-dependent form factors to model the nonlocality and compared systematically with the local NJL counterpart.

We have shown that there is a systematic lowering of the critical chemical potential for the onset of the CSL phase as well as for the minimal excitation energy (effective CSL gap) as a function of the nonlocality which can be represented as a linear dependence on the smoothness parameter $1/\alpha$ of the Lorentz-type form factor. These qualitative effects are shown to be robust under changes in the coupling constant used to represent the CSL interaction.

It has been found that hybrid star cooling requires all quark modes to be paired with a minimal pairing of the order of 10–100 keV to suppress the direct Urca process in quark matter. The present model for the CSL phase meets this requirement and calls for a more detailed analysis of the cooling phenomenology based on this microscopically justified pairing pattern.

The smallest gap which governs the cooling phenomenology corresponds to the zero z projection of the spin and thus remains unaffected by the external magnetic field of a compact star. Moreover, the CSL pairing pattern is a flavor singlet and insensitive to the flavor asymmetry in a compact star under β equilibrium.

Therefore, the CSL phase with nonlocal instantaneous interactions is particularly interesting for applications in compact stars and allows us to achieve a suitable description of quark matter properties by choosing the appropriate form factor models. Although it remains to be shown that hybrid star configurations with the CSL quark matter phase could be stable, due to the small gaps, we expect to have results reproducing those of the normal quark matter case, where stable quark matter cores in nonlocal models have been found.

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