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Solar system constraints on R^n gravity

A. F. Zakharov*
National Astronomical Observatories of Chinese Academy of Sciences, Beijing 100012, China

A. A. Nucita,† F. De Paolis,‡ and G. Ingrosso§
Department of Physics and INFN, University of Lecce, CP 193, I-73100 Lecce, Italy
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Recently, gravitational microlensing has been investigated in the framework of the weak field limit of fourth order gravity theory. However, solar system data (i.e. planetary periods and light bending) can be used to put strong constraints on the parameters of this class of gravity theories. We find that these parameters must be very close to those corresponding to the Newtonian limit of the theory.

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I. INTRODUCTION

Since many years different alternative approaches to gravity have been proposed in the literature such as MOND [1,2], scalar-tensor [3], conformal [4], Yukawa-like corrected gravity theories [5–7], and so on (see papers [8] for reviews). Very recently, it has been proposed [9–11], in the framework of higher order theories of gravity—also referred to as $f(R)$ theories—a modification of the gravity action with the form

$$\mathcal{A} = \int d^4x\sqrt{-g}[f(R) + \mathcal{L}_m], \tag{1}$$

where $f(R)$ is a generic function of the Ricci scalar curvature and $\mathcal{L}_m$ is the standard matter Lagrangian. For example, if $f(R) = R + 2\Lambda$ the theory coincides with general relativity (GR) with the Λ term. In particular, Capozziello *et al.* [10,11] considered power law function $f(R)$ theories of the form $f(R) = f_0R^n$. As a result, in the weak field limit [12], the gravitational potential is found to be [10,11]:

$$\Phi(r) = -\frac{Gm}{2r}\left[1 + \left(\frac{r}{r_c}\right)^\beta\right], \tag{2}$$

where

$$\beta = \frac{12n^2 - 7n - 1 - \sqrt{36n^4 + 12n^3 - 83n^2 + 50n + 1}}{6n^2 - 4n + 2}. \tag{3}$$

The dependence of the β parameter on the n power is shown in Fig. 1. Of course, for $n \to \infty$ it follows $\beta \to 1$, while for $n = 1$ the parameter β reduces to zero and the Newtonian gravitational field is recovered. On the other hand, while β is a universal parameter, r_c in principle is an arbitrary parameter, depending on the considered system and its typical scale. Consider, for example, the Sun as the source of the gravitational field and the Earth as the test particle. Since Earth velocity is $\simeq 30$ km s^{-1}, it has been found that the parameter r_c varies in the range $\simeq$ 1–10^4 AU. Once r_c and β has been fixed, Capozziello *et al.* [10] used them to study deviations from the standard Paczynski light curve for gravitational microlensing [14] and claimed that the implied deviation can be measured [15]. It is clear that for gravitational microlensing one could detect observational differences between GR and an alternative theory (the fourth order gravity, in particular), so that one should have different potentials at the scale R_E (the Einstein radius) of the gravitational microlensing. For the Galactic microlensing case R_E is about 1 AU. This is a reason why the authors [10] have selected r_c at a level of astronomical units to obtain observable signatures for nonvanishing β. The aim of the present paper is to show that solar system data (light bending and planetary periods)

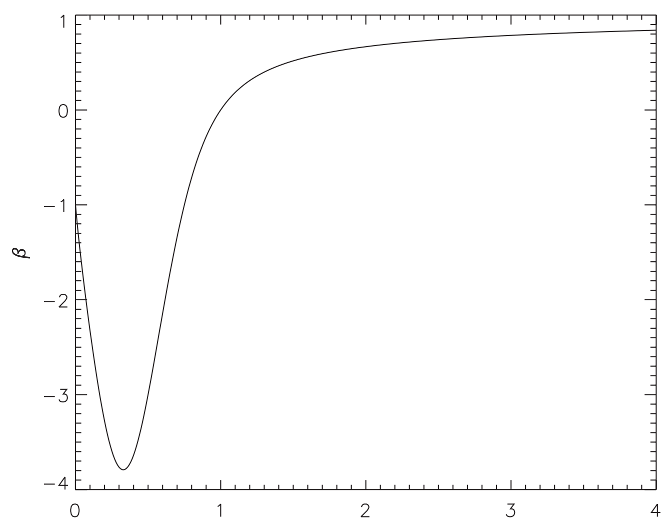

FIG. 1. The parameter β as a function of n for fourth order gravity.

*Electronic address: zakharov@itep.ru
Also at Institute of Theoretical and Experimental Physics, Moscow, 117259, Russia; Joint Institute for Nuclear Research, Dubna, Russia.
†Electronic address: achille.nucita@le.infn.it
‡Electronic address: Francesco.DePaolis@le.infn.it
§Electronic address: Gabriele.Ingrosso@le.infn.it

put extremely strong constraints on both r_c and β parameters making this alternative theory of gravity not so attracting.

II. SOLAR SYSTEM CONSTRAINTS

A. Light bending constraints

As stated above, we now discuss some observational consequences of the fourth order gravity, depending on the choice of the parameters β and r_c.

A constraint on the proposed theory can be derived by considering the light bending effect in the Sun limb. It is well known that in the parametrized post-Newtonian formalism the bending angle through which a electromagnetic light ray from a distant source is deflected by a body with mass m is [8,16]

$$\theta = \frac{(1+\gamma)Gm}{c^2 b}(1+\cos\phi), \tag{4}$$

where b is the impact parameter, ϕ is the solar elongation angle (between the Sun and the source as viewed from Earth) and γ is the post-Newtonian parameter. For GR, $\gamma = 1$ and for light rays at the Sun's limb, $\theta_{\rm GR} = 1.75''$. Recently, Shapiro *et al.* [17] measured the bending angles for distant compact sources and concluded that light bending angles follow GR with a very high precision ($\gamma = 0.9998 \pm 0.0004$). In other words, this means that the deflection of the light path is well described by the GR theory. In particular, as radio observations of distant sources have shown [18], the observed and expected bending angles are related by $\theta_{\rm obs} = (1.001 \pm 0.001)\theta_{\rm GR}$. In the framework of the fourth order gravity theory, the deflection angle of light rays at the Sun's limb depends on both the parameters β and r_c. We explore this dependence in Fig. 2, by requiring that the expected value for the bending angle is, at least, within 2σ (gray region) or within 5σ (light gray region) the observed value. Inspecting the same figure, it is clear that only β-values near zero (corresponding to a completely arbitrariness of r_c) are consistent with the observed deflection angles. We therefore emphasize that β-values considered in [10,11] (i.e. $\beta = 0.25$,0.43, 0.58, 0.75) are ruled out by light deflection data.

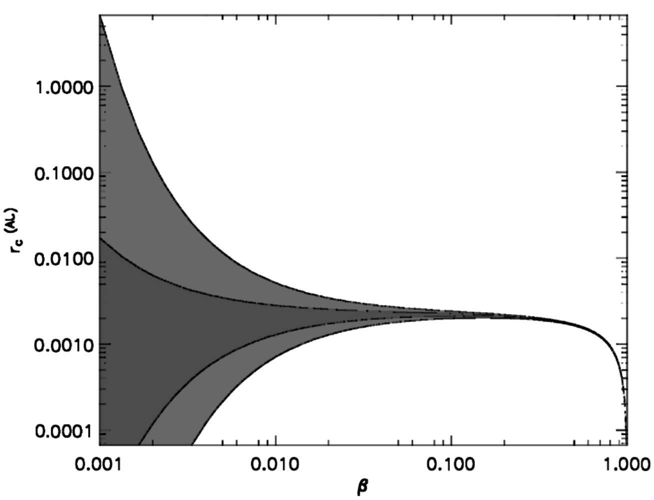

FIG. 2. Constraints on the fourth order theory parameters (β and r_c) arising from the deflection angle of light rays close to the solar limb. The gray and light gray regions embed the part of the parameter space allowed by solar system observations at the 2σ and 5σ confidence level, respectively. It is worth noticing that for scale reason we have not plotted values of r_c up to 10^4 AU. For these cases, the observations can be restored only for $\beta \rightarrow 0$.

B. Planetary constraints

A stronger constraint on the fourth order gravity theory can be obtained from the motion of the solar system planets. Let us consider as a toy model a planet moving on circular orbit (of radius r) around the Sun. From Eq. (2), the planet acceleration $a = -\partial\Phi(r)/\partial r$ is given by

$$a = -\frac{Gm}{2r^2}\left[1 + \left(\frac{r}{r_c}\right)^\beta - \beta\left(\frac{r}{r_c}\right)^\beta\right]. \tag{5}$$

Accordingly, the planetary circular velocity v can be evaluated and, in turn, the orbital period P is given by

$$P = P_K\sqrt{2}\left[1 + \left(\frac{r}{r_c}\right)^\beta - \beta\left(\frac{r}{r_c}\right)^\beta\right]^{-1/2}, \tag{6}$$

where $P_K = [4\pi^2 r^3/(Gm)]^{1/2}$ is the usual Keplerian period. In order to compare the orbital period predicted by the fourth order theory with the Solar System observations, let us define the quantity

$$\frac{\Delta P}{P_K} = \frac{|P - P_K|}{P_K} = |f(\beta, r_c) - 1| \tag{7}$$

being $f(\beta, r_c)$ the factor appearing on the right-hand side of Eq. (6) and multiplying the usual Keplerian period. There is a question about a possibility to satisfy the planetary period condition—vanishing the Eq. (7)—with β parameter which is significantly different from zero. Vanishing the right-hand side of Eq. (7) we obtain the relation

$$\ln r = \ln r_c - \frac{\ln(1-\beta)}{\beta}, \tag{8}$$

so that Eq. (8) should be satisfied for all the planetary radii. This is obviously impossible since the fourth order theory defines β as a parameter, while the specific system under consideration (the Solar system in our case) allows us to specify the r_c parameter. Hence the right-hand side of Eq. (8) is fixed for the Solar system, implying that it is impossible to satisfy Eq. (8) even with two (or more) different planetary radii. Just for illustration we present the function $f(\beta, r_c)$ as a dependence on β parameter for fixed r_c and planetary radii r (see Fig. 3). As one can see from Eq. (8) (and Fig. 3 as well) for each planetary radius

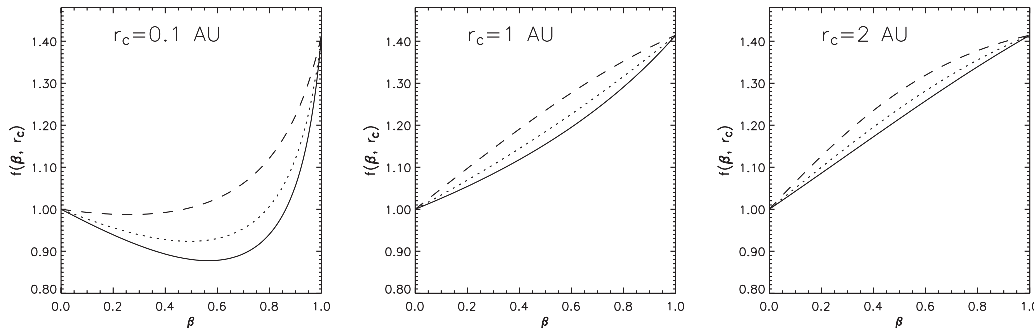

FIG. 3. The orbital period in units of the Keplerian one (the function $f(\beta, r_c)$) is given, as a function of β, in the case of Mercury (dashed line), Venus (dotted line), Earth (solid line) for different r_c (it is clear that $f(\beta, r_c) \to \sqrt{2}$ for $\beta \to 1$). If $r > r_c$ there is $\beta \in (0, 1)$ satisfying Eq. (8), but the β values depend on fixed r_c and r and they are different for a fixed r_c and different r, so that it is impossible to satisfy Eq. (8) if the number of planets is more than one. Moreover, if we have at least one radius $r \le r_c$, there is no solution of Eq. (8).

$r > r_c$ there is $\beta \in (0, 1)$ satisfying Eq. (8), but the β value depends on fixed r_c and r, so that they are different for a fixed r_c and different r. Moreover, if we have at least one radius $r \le r_c$, there is no solution of Eq. (8). Both cases imply that the β parameter should be around zero. In Fig. 4 (left panel), the factor $f(\beta, r_c)$ is given as a function of β for the two limiting values of r_c, 1 AU (dashed line) and $\simeq 10^4$ AU (solid line), considered in [10]. As one can note, only for β approaching zero it is expected to recover the value of the Keplerian period. In the above-mentioned figure, the calculation has been performed for the Earth orbit (i.e. $r = 1$ AU).

Current observations allow also to evaluate the distances between the Sun and the planets of the Solar System with a great accuracy. In particular, differences in the heliocentric distances do not exceed 10 km for Jupiter and amount to 180, 410, 1200 and 14000 km for Saturn, Uranus, Neptune and Pluto, respectively [19]. Errors in the semimajor axes of the inner planets are even smaller (see e.g. Table 2 in [20]) so that the relative error in the orbital period determination is extremely low. As an example, the orbital period of Earth is $T = 365.256\,363\,051$ days with an error of $\Delta T = 5.0 \times 10^{-10}$ d, corresponding to a relative error of $\Delta T/T$ less than 10^{-12}. These values can be used in order to constrain the possible values of both the parameters β and r_c introduced by the fourth order gravity theory. This can be done by requiring that $\Delta P/P_K \lesssim \Delta T/T$ so that, in the case of Earth, $|f(\beta, r_c) - 1| \lesssim 10^{-12}$ which can be solved with respect to β once the r_c parameter has been fixed to some value. For $r_c = 1$ AU and $r_c = 10^4$ AU (i.e. the two limiting cases considered by Capozziello *et al.* [10]) we find the allowed upper limits on the β parameter to be 4.0×10^{-12} and 3.9×10^{-13}, respectively, (since $\Delta P/P_K = \Delta\beta[-1 + \ln(r/r_c)]/4$). These results can also be inferred from the middle and right panels of Fig. 4.

A more precise analysis which takes into account the planetary semimajor axes and eccentricities leads to variations of at most a few percent with respect to the results in Fig. 4, since the planetary orbits are nearly circular. Therefore, in spite of the fact that orbital periods of planets are not generally used to test alternative theories of gravity (since it is taken for granted that the weak field approximation of these theories gives the Newtonian limit), we found that these data are important to constrain parameters of the fourth order gravity theory.

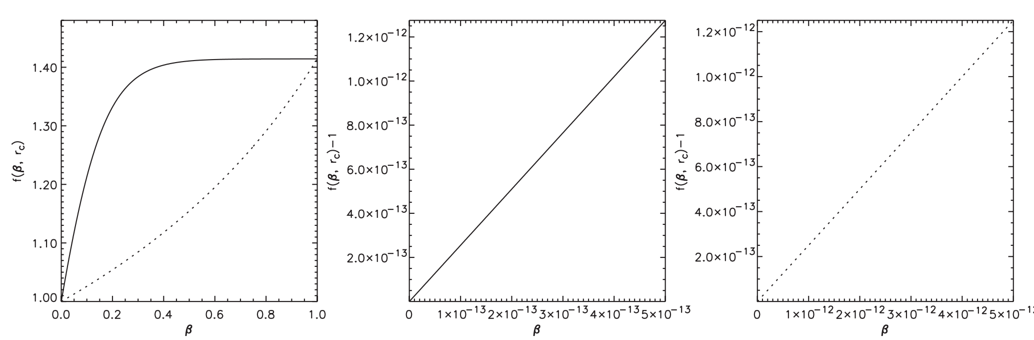

FIG. 4. The factor $f(\beta, r_c)$ is given as a function of β (left panel) for the two limiting values of r_c, 1 AU (dashed line) and $\simeq 10^4$ AU (solid line), respectively. As one can note, only for β approaching 0 it is expected to recover the value of the Keplerian period. Here, the calculation have been performed at Earth (i.e. $r = 1$ AU). In the middle and right panel, the quantity $f(\beta, r_c) - 1$ is given as a function of β for $r_c = 10^4$ AU and $r_c = 1$ AU. Note that only for values of β close to 0 the Solar System observation can be restored (see text for more details).

III. DISCUSSION

GR and Newtonian theory (as its weak field limit) were verified by a very precise way at different scales. There are observational data which constrain parameters of alternative theories as well. As a result, the parameter β of fourth order gravity should be very close to zero (it means that the gravitational theory should be very close to GR). In particular, the β parameter values considered for microlensing [10], for rotation curves [11] and cosmological SN type Ia [21] are ruled out by solar system data.

No doubt that one could also derive further constraints on the fourth order gravity theory by analyzing other physical phenomena such as Shapiro time delay, frequency shift of radio photons [22], laser ranging for distant objects in the solar system, deviations of trajectories of celestial bodies from ellipses, parabolas and hyperbolas and so on. But our aim was only to show that only $\beta \simeq 0$ values are not in contradiction with solar system data in spite of the fact that there are a lot of speculations to fit observational data with β values significantly different from zero.

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