

New supersymmetric AdS₃ solutions

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We construct infinite new classes of supersymmetric solutions of $D = 11$ supergravity that are warped products of AdS₃ with an eight-dimensional manifold \mathcal{M}_8 and have nonvanishing four-form flux. In order to be compact, \mathcal{M}_8 is constructed as an S^2 bundle over a six-dimensional manifold B_6 which is either Kähler-Einstein or a product of Kähler-Einstein spaces. In the special cases that B_6 contains a two-torus, we also obtain new AdS₃ solutions of type IIB supergravity, with constant dilaton and only five-form flux. Via the anti-de Sitter (AdS)-conformal field theory (CFT) correspondence the solutions with compact \mathcal{M}_8 will be dual to two-dimensional conformal field theories with $N = (0, 2)$ supersymmetry. Our construction can also describe noncompact geometries and we discuss examples in type IIB which are dual to four-dimensional $N = 1$ superconformal theories coupled to stringlike defects.

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I. INTRODUCTION

In this paper we will construct infinite new classes of supersymmetric solutions of $D = 11$ supergravity and type IIB supergravity that contain AdS₃ factors and compact internal spaces. Via the anti-de Sitter (AdS)-conformal field theory (CFT) correspondence [1] these solutions are dual to two-dimensional conformal field theories with $N = (0, 2)$ supersymmetry. Note that a subclass of the type IIB solutions was already discussed in [2].

Our construction is directly inspired by the supersymmetric solutions found in [3]. Recall that these solutions are warped products of AdS₅ with a six-dimensional manifold M_6 and are all dual to $N = 1$ supersymmetric conformal field theories in four dimensions. These solutions were found in two steps. The first step consisted of classifying the most general supersymmetric solutions of this form using G -structure techniques. The second step involved imposing a suitable ansatz on M_6 , namely, that M_6 is a complex manifold, and then showing that all such compact M_6 could be constructed in explicit form. Specifically M_6 is an S^2 bundle over a four-dimensional base which is either (i) Kähler-Einstein with positive curvature i.e. $S^2 \times S^2$, CP^2 or a del-Pezzo dP_k with $k = 3, \dots, 8$, or (ii) a product: $S^2 \times S^2$, $S^2 \times H^2$, or $S^2 \times T^2$, each factor with its constant curvature metric. The last example with base $S^2 \times T^2$ is related, via dimensional reduction and T -duality, to a family of type IIB solutions AdS₅ \times $Y^{p,q}$ where $Y^{p,q}$ are new Sasaki-Einstein metrics on $S^2 \times S^3$ [4]. These five-dimensional Sasaki-Einstein metrics, and their generalizations [5], have been receiving much attention because the dual conformal field theories can be identified [6–10]. It is an important outstanding issue to elucidate the conformal field theories dual to the other M theory solutions found in [3].

It is natural to try and repeat the successful constructions of [3] in different contexts. In [11] a complete classification of the most general supersymmetric solutions of type IIB

supergravity consisting of warped products of AdS₅ with a five-dimensional manifold X_5 was successfully carried out.¹ The Pilch-Warner solution [13] was recovered using this formalism, but, as yet, it is unclear what additional ansatz one should impose upon X_5 in order to be able to construct new explicit solutions. Furthermore, a detailed classification of AdS₅, AdS₄, and AdS₃ solutions of $D = 11$ supergravity with various amounts of supersymmetry and vanishing electric four-form flux was given in [14]. While various known solutions were recovered, it again proved difficult to find new classes of solutions.

In this paper we will construct solutions of $D = 11$ supergravity that are the warped product of AdS₃ with an eight-dimensional manifold \mathcal{M}_8 which are dual to conformal field theories with $N = (0, 2)$ supersymmetry. The solutions, which have nonvanishing electric four-form flux, do not fall within the classes studied in [14], so require an appropriate generalization, which is discussed in detail in Appendix A. By considering a suitable ansatz, we will then show that we can indeed find infinite new classes of explicit AdS₃ solutions of $D = 11$ supergravity. We find solutions in which \mathcal{M}_8 is an S^2 bundle over a six-dimensional base space B_6 which is either a six-dimensional Kähler-Einstein space, KE_6 , a product of four- and two-dimensional Kähler-Einstein spaces, $KE_4 \times KE_2$, or a product of three two-dimensional Kähler-Einstein spaces, $KE_2 \times KE_2 \times KE_2$. There are various possibilities for the signs of the curvature of the Kähler-Einstein spaces, as we shall see. If these backgrounds are to be bona fide M theory solutions one must also ensure that the four-form flux is quantized. While we do not consider this in general, the expectation is that this will restrict most if not all of the parameters in the solutions to discrete values. We show how this is indeed the case in a particular class of solutions.

¹For some recent further developments see [12].

In the special case when \mathcal{M}_8 is an S^2 bundle over a $KE_4 \times T^2$ or a $KE_2 \times KE_2 \times T^2$ base space, we can dimensionally reduce and then T -dualise to give solutions of type IIB supergravity (this is analogous to how the $AdS_5 \times Y^{p,q}$ solutions were found in [3]). These type IIB solutions are warped products of AdS_3 with a seven manifold \mathcal{M}_7 , have constant dilaton and nonvanishing five-form flux. For those arising from $KE_4 \times T^2$, we will show that there are two families of regular solutions for any positively curved KE_4^+ . One family was first presented and analyzed in some detail in the type IIB context in [2]. Of the second family, taking the special case where $KE_4^+ = CP^2$, the IIB solution describes the near-horizon limit of D3-branes wrapping a holomorphic Riemann surface embedded in a Calabi-Yau four-fold that was first constructed in [15] (generalizing a construction of Maldacena and Núñez [16]). For this family we calculate the central charge of the dual CFT. Remarkably this is integral independent of the choice of KE_4^+ . The new $D = 11$ solutions arising from a $KE_2 \times KE_2 \times T^2$ base space give rise to new type IIB solutions generalizing these solutions and also those presented in [2].

Most of the paper will focus on solutions in which the internal space is compact. However, generically our ansatz also includes noncompact solutions. We will briefly discuss a class of these in the type IIB context, that is of the form of a warped product $AdS_3 \times \mathcal{M}_7$ with noncompact \mathcal{M}_7 . These solutions include backgrounds that can be interpreted as the backreacted geometry of probe D3-branes in $AdS_5 \times S^5$ with world volume $AdS_3 \times S^1$, and preserving $\frac{1}{16}$ th of the supersymmetry. The corresponding dual field theory is $N = 4$ super Yang-Mills coupled to stringlike defects preserving the superconformal group in two dimensions. More generally, the S^5 factor can be replaced by a Sasaki-Einstein manifold, and the backreacted geometry corresponds to some four-dimensional $N = 1$ SCFT coupled to stringlike defects.

The plan of the rest of the paper is as follows. We start by motivating the ansatz for the 11-dimensional supergravity fields in Sec. II. Section II B describes how the Killing spinor equations reduce to one single second-order differential equation, and uses it to comment on the conditions required to eliminate conical singularities in the metric. The following sections describe the explicit solutions that we have found. They are ordered in increasing complexity of the base B_6 : Sec. III deals with $B_6 = KE_6$, Sec. IV with $B_6 = KE_4 \times KE_2$, and Sec. V with $B_6 = KE_2 \times KE_2 \times KE_2$. The type IIB solutions that arise when one of the KE_2 factors is a T^2 are discussed in Sec. VI. A discussion of the new noncompact solutions of type IIB supergravity is presented in Sec. VII and we conclude in Sec. VIII. Finally, we have included three appendices. The technical study of the Killing spinor equations has been relegated from the text, and collected in Appendix A. In particular, this appendix contains the classification of the relevant

AdS_3 solutions using G -structures. Some properties of KE_4 spaces used in this paper are explained in Appendix B. Finally, Appendix C describes how to relate the orientations and parameters in M theory and IIB supergravity.

II. ANSATZ AND SOLUTIONS

To explain our ansatz in a bit more detail, we recall that the family of AdS_5 solutions of [3] with M_6 an S^2 bundle over an $H^2 \times S^2$ base space contain a limiting solution in which the geometry degenerates to $H^2 \times S^4$. This particular solution was constructed previously by Maldacena and Núñez and describes the near-horizon limit of M -fivebranes wrapping a holomorphic two-cycle (calibrated by the Kähler two-form) inside a Calabi-Yau threefold [16]. Here the two-cycle is the H^2 factor² while the S^4 factor corresponds to the four-sphere surrounding the wrapped fivebranes. The metric for this “2-in-6 Kähler” solution contains an S^4 factor twisted over the H^2 base in a specific manner, which can be deduced from probe fivebranes wrapping holomorphic cycles [16]. From this perspective, the solutions of [3] correspond to a generalization where the S^4 surrounding the brane is replaced by an S^2 bundle over S^2 .

For us, a key observation is that the structure of the AdS_5 2-in-6 Kähler solution [16] has many similarities with the AdS_3 solution of [17] corresponding to M -fivebranes wrapping a Kähler four-cycle in a Calabi-Yau four-fold, “4-in-8 Kähler.” In both cases, of the five directions transverse to the probe M -fivebranes wrapping such cycles, four of them are tangent to the Calabi-Yau. Combined with the fact that in both cases the fivebrane is wrapping a Kähler cycle, this implies that in the $D = 11$ solution the four-sphere is fibered over the base space in an almost identical way, the only difference being that the H^2 base is replaced by a four-dimensional Kähler-Einstein manifold KE_4 that is negatively curved. We have summarized the probe-brane configurations in Table I. Note that in the 4-in-8 case one could also consider a probe membrane wrapping directions orthogonal to the Calabi-Yau without further breaking the supersymmetry.

Thus, inspired by the fact that the 2-in-6 Kähler solution is a single member of a family of solutions where the S^4 is replaced by an S^2 bundle over S^2 , we are motivated to find new warped $AdS_3 \times \mathcal{M}_8$ solutions where \mathcal{M}_8 is a bundle over KE_4 , with fibres which are themselves S^2 bundles over S^2 . Or rather, in analogy with [3], the structure should actually simplify to an S^2 bundle over $S^2 \times KE_4$. In the specific 4-in-8 Kähler solution of [17] the KE_4 had negative curvature. However, following the success of [3] we can also seek solutions that are S^2 bundles over more

²It is possible to take a quotient of H^2 to obtain any compact Riemann surface with genus greater than one while still preserving supersymmetry [16].

TABLE I. Arrays describing the probe brane setups in M theory. In both situations, of the 5 transverse directions to the M5-branes, 4 are tangent to the CY and 1 lies in flat space.

“2-in-6”	0	1	2	3	4	5	6	7	8	9	10
CY ₃					-	-	-	-	-	-	
M5	-	-	-	-	-	-					
H_2											
“4-in-8”	0	1	2	3	4	5	6	7	8	9	10
CY ₄			-	-	-	-	-	-	-	-	
M5	-	-	-	-	-	-					
KE_4											

general bases B_6 . Specifically we will consider $B_6 = KE_6$, $B_6 = KE_2 \times KE_4$, or $B_6 = KE_2 \times KE_2 \times KE_2$, with the various factors having positive, negative, or zero curvature.

A. Ansatz

We start with the ansatz for the bosonic fields of $D = 11$ supergravity, discussing each choice for B_6 in turn. In all cases we assume the metric is a warped product:

$$ds^2 = \omega^2[ds^2(\text{AdS}_3) + ds^2(\mathcal{M}_8)], \quad (2.1)$$

where $ds^2(\text{AdS}_3)$ is the metric of constant curvature on AdS₃ with unit radius.

Case 1 $B_6 = KE_2 \times KE_2 \times KE_2$

In this case we assume that $ds^2(\mathcal{M}_8)$ has the form

$$ds^2(\mathcal{M}_8) = \sum_{i=1}^3 h_i ds^2(C_i) + f_3 dr^2 + f_4 (D\psi)^2, \quad (2.2)$$

and the four-form is given by

$$G_4 = g_3 J_1 \wedge J_2 + g_1 J_2 \wedge J_3 + g_2 J_3 \wedge J_1 + (g_4 J_1 + g_5 J_2 + g_6 J_3) \wedge dr \wedge D\psi + g_7 dr \wedge \text{vol}_{\text{AdS}_3}. \quad (2.3)$$

Here $ds^2(C_i)$ is locally a constant curvature metric on S^2 , H^2 , or T^2 for $k_i = 1, -1$, or 0 , respectively, and $J_i = \text{vol}_{C_i}$ is the corresponding Kähler-form on C_i . The ansatz depends on 13 functions $h_i, f_3, f_4, g_1, \dots, g_7, \omega$ which are functions of r only. Thus, in general, the isometry group will be, at least, $SO(2, 2) \times U(1)$, the first factor corresponding to the symmetries of AdS₃ and the latter to shifts of the fiber coordinate ψ (we will see later that, for compact \mathcal{M}_7 , ψ is a periodic coordinate). Note that using the freedom to change the r coordinate we can choose f_3 as we like. We also have

$$D\psi = d\psi + P \quad (2.4)$$

with

$$dP = \sum_{i=1}^3 \mathcal{R}_i = \sum_{i=1}^3 k_i J_i, \quad (2.5)$$

where \mathcal{R}_i is the Ricci-form for C_i . Thus, the twisting of the fiber with coordinate ψ is associated to the canonical $U(1)$ bundle over the six-dimensional base space B_6 given by $C_1 \times C_2 \times C_3$. Indeed in the compact complete solutions that we will construct r, ψ will parametrize a two-sphere and topologically we will have an S^2 bundle over $C_1 \times C_2 \times C_3$ that is obtained by adding a point to the fibers in the canonical line bundle. This is entirely analogous to the solutions in [3].

Case 2 $B = KE_4 \times KE_2$

In this case we assume

$$ds^2(\mathcal{M}_8) = h_1 ds^2(KE_4) + h_3 ds^2(C_3) + f_3 dr^2 + f_4 (D\psi)^2, \\ G_4 = \frac{g_3}{2} J \wedge J + g_1 J \wedge J_3 + (g_4 J + g_6 J_3) \wedge dr \wedge D\psi + g_7 dr \wedge \text{vol}_{\text{AdS}_3}, \quad (2.6)$$

where now J is the Kähler-form of KE_4 . Note that in terms of the functions h_i, g_i and parameters k_i , this can be viewed as a special case of the previous ansatz

$$k_1 = k_2, \quad h_1 = h_2, \\ g_1 = g_2, \quad g_4 = g_5. \quad (2.7)$$

The 4-in-8 Kähler solution of [17] is contained within this ansatz. More specifically, we shall show that it is recovered in the $B_6 = KE_4 \times KE_2$ class (2.7) when KE_4 has negative curvature ($k_1 = k_2 = -1$) and $C_3 = S^2$ ($k_3 = 1$). For this particular solution, the r, ψ coordinates parametrize a two-sphere but when combined with the two-sphere C_3 give rise to a four-sphere. The metric for this solution is then a warped product of the form $\text{AdS}_3 \times KE_4 \times S^4$. From a physical point of view the four-sphere surrounds the fivebrane that is wrapped on the negatively curved³ KE_4 .

Case 3 $B_6 = KE_6$

In this case the ansatz is even simpler

$$ds^2(\mathcal{M}_8) = h_1 ds^2(KE_6) + f_3 dr^2 + f_4 (D\psi)^2, \\ G_4 = \frac{g_1}{2} J \wedge J + g_4 J \wedge dr \wedge D\psi + g_7 dr \wedge \text{vol}_{\text{AdS}_3}, \quad (2.8)$$

where here J is the Kähler-form of KE_6 . Again in

³Note that solutions for fivebranes wrapping positively curved Kähler-Einstein spaces, i.e. $k_2 = 1$ were also found in [17] but they did not have an AdS₃ factor in the near-brane limit.

terms of the functions and parameters, this is a special case of the original ansatz with

$$\begin{aligned} k_1 = k_2 = k_3, \quad h_1 = h_2 = h_3, \\ g_1 = g_2 = g_3, \quad g_4 = g_5 = g_6. \end{aligned} \quad (2.9)$$

Finally, our ansatz leads to type IIB backgrounds when one of the cycles in B_6 is a torus. By reducing to IIA along one of the torus directions and T -dualizing along the other, we obtain type IIB solutions with constant dilaton and only five-form flux. To be explicit, let $C_3 = T^2$. The 11-dimensional four-form decomposes naturally as

$$\begin{aligned} G_4 &= F_4 + F_2 \wedge \text{vol}_{T^2}, \\ F_4 &= g_3 J_1 \wedge J_2 + (g_4 J_1 + g_5 J_2) \wedge dr \wedge D\psi \\ &\quad + g_7 dr \wedge \text{vol}_{\text{AdS}_3}, \\ F_2 &= g_2 J_1 + g_1 J_2 + g_6 dr \wedge D\psi. \end{aligned} \quad (2.10)$$

Note that both F_2 and F_4 are closed by virtue of the closure of G_4 . In particular, we can write locally $F_2 = dA_1$ for some one-form potential A_1 . Having made this decomposition, the IIB background reads (for a few more details see Appendix C)

$$\begin{aligned} ds^2 &= (\omega^6 h_3)^{1/2} [ds^2(\text{AdS}_3) + ds^2(\mathcal{M}_7)], \\ F_5 &= (1 + \star) F_4 \wedge Dz, \end{aligned} \quad (2.11)$$

where

$$\begin{aligned} ds^2(\mathcal{M}_7) &= h_1 ds^2(C_1) + h_2 ds^2(C_2) + f_3 dr^2 \\ &\quad + f_4 D\psi^2 + \frac{1}{\omega^6 h_3^2} Dz^2, \end{aligned} \quad (2.12)$$

and

$$Dz = dz + A_1. \quad (2.13)$$

For our ansatz, explicitly we have

$$\begin{aligned} \star [F_4 \wedge Dz] &= \frac{h_3 g_3 \omega^3 \sqrt{f_3 f_4}}{h_1 h_2} \text{vol}_{\text{AdS}_3} \wedge dr \wedge D\psi \\ &\quad + \frac{h_2 h_3 g_4 \omega^3}{h_1 \sqrt{f_3 f_4}} \text{vol}_{\text{AdS}_3} \wedge J_2 \\ &\quad + \frac{h_1 h_3 g_5 \omega^3}{h_2 \sqrt{f_3 f_4}} \text{vol}_{\text{AdS}_3} \wedge J_1 \\ &\quad + g_7 h_1 h_2 h_3 \omega^3 \sqrt{\frac{f_4}{f_3}} J_1 \wedge J_2 \wedge D\psi. \end{aligned} \quad (2.14)$$

Note that (as explained in Appendix A), taking $\omega^3 > 0$, we have $\sqrt{f_3 f_4} < 0$. Also note that these IIB backgrounds will have an isometry group at least as large as $SO(2, 2) \times U(1) \times U(1)$, the $U(1)$ factors acting as shifts of the coordinates ψ, z .

B. The BPS condition and global construction

Let us now turn to the conditions imposed by supersymmetry, the Bianchi identities, and the equations of motion. The derivation is technically involved and relies on a key gauge choice for the function f_3 . We have presented some details in Appendix A. Remarkably, we find that all conditions boil down to solving the second-order nonlinear equation for a function $H(r)$:

$$\begin{aligned} 0 &= -4(H')^2 + 4H(2H'' + 4k_i d_i + 4k_i c_i r - 3k_1 k_2 k_3 r^2) \\ &\quad + \prod_{i=1}^3 \left(\frac{k_1 k_2 k_3}{k_i} r^2 - 4rc_i - 4d_i \right). \end{aligned} \quad (2.15)$$

Here c_i and d_i are six integration constants that appear in the analysis. Given a solution of (2.15) one can construct the full 11-dimensional supergravity solution as follows

$$\begin{aligned} h_l &= \frac{2H'' + 4k_i d_i + 4k_i c_i r - 3k_1 k_2 k_3 r^2}{4(4d_l + 4c_l r - \frac{k_1 k_2 k_3}{k_l} r^2)}, \quad l = 1, 2, 3, \\ f_3 &= -\frac{2H'' + 4k_i d_i + 4k_i c_i r - 3k_1 k_2 k_3 r^2}{16H}, \\ f_4 &= -H \frac{2H'' + 4k_i d_i + 4k_i c_i r - 3k_1 k_2 k_3 r^2}{\prod_{i=1}^3 \left(\frac{k_1 k_2 k_3}{k_i} r^2 - 4rc_i - 4d_i \right)}, \\ \omega^6 &= \frac{4 \prod_{i=1}^3 \left(\frac{k_1 k_2 k_3}{k_i} r^2 - 4rc_i - 4d_i \right)}{(2H'' + 4k_i d_i + 4k_i c_i r - 3k_1 k_2 k_3 r^2)^2}. \end{aligned} \quad (2.16)$$

If we define the function f via

$$f = \frac{4H'}{2H'' + 4k_i d_i + 4k_i c_i r - 3k_1 k_2 k_3 r^2}, \quad (2.17)$$

then the first three components of the flux are given by

$$\begin{aligned} g_1 &= -\frac{1}{2} [f(k_2 h_3 + k_3 h_2) + k_2 k_3 r] + c_1, \\ g_2 &= -\frac{1}{2} [f(k_3 h_1 + k_1 h_3) + k_3 k_1 r] + c_2, \\ g_3 &= -\frac{1}{2} [f(k_1 h_2 + k_2 h_1) + k_1 k_2 r] + c_3. \end{aligned} \quad (2.18)$$

The next three components are given by

$$\begin{aligned} -4g_4 &= (2fh_1)' + k_1, \quad -4g_5 = (2fh_2)' + k_2, \\ -4g_6 &= (2fh_3)' + k_3, \end{aligned} \quad (2.19)$$

while the electric piece is

$$\begin{aligned} g_7 &= \frac{f_3}{2h_1 h_2 h_3} [(\omega^6 + f^2)(k_1 h_2 h_3 + \text{perm.}) \\ &\quad + f(k_1 k_2 h_3 + \text{perm.})r - 2fc_i h_i], \end{aligned}$$

where “+perm.” means adding the other two terms involving the obvious permutations of 1, 2, and 3. The explicit expression for the Killing spinors are given in the appendix. We find that the solution generically preserves 1/8 supersymmetry and is dual to a two-dimensional conformal field theory with $N = (0, 2)$ supersymmetry.

We have not managed to find the most general solution to the differential equation (2.16) for H . However, we have found a rich set of polynomial solutions that lead to regular and compact solutions, which we will discuss in detail in the next section.

Our principal interest is the construction of compact solutions. As mentioned earlier, our procedure will be to require r , ψ to parametrize a two-sphere fibered over a compact base B_6 :

$$\begin{array}{ccc} S^2_{r,\psi} & \rightarrow & \mathcal{M}_8 \\ & & \pi \downarrow \\ & & B_6 \end{array} \quad (2.20)$$

This is achieved if the range of r is restricted to lie in a suitable finite interval, $r_1 \leq r \leq r_2$, and ψ is a periodic coordinate. Clearly, the range of the coordinate r is restricted by the poles of f_3 which must be zeroes of the function H . Generically, the poles of f_3 are also the zeroes of f_4 and so the (r, ψ) part of the metric can indeed form an S^2 provided that one can remove potential singularities at each of the zeroes r_α of H . If such a generic H has a linear behavior at r_α we find that, after a change of coordinates $r = r(\rho)$, the (r, ψ) part of the metric takes the form

$$ds^2 = d\rho^2 + \gamma \rho^2 d\psi^2, \quad (2.21)$$

with

$$\gamma = \frac{4[H'(r_\alpha)]^2}{\prod_{i=1}^3 \left(\frac{k_1 k_2 k_3}{k_i} r_\alpha^2 - 4r_\alpha c_i - 4d_i \right)}, \quad (2.22)$$

where $H'(r_\alpha)$ is the value of H' at the corresponding zero. Now, using the differential equation (2.15), and evaluating it at a zero of H , one deduces the remarkable fact that $\gamma = 1$ at all poles. Thus, for such generic H , the condition for the absence of conical singularities at the poles of the S^2 formed by (r, ψ) is just $\Delta\psi = 2\pi$.

For such H , we have complete metrics on \mathcal{M}_8 , then, provided that the warp factor ω and the metric functions h_i remain finite and nonzero within the interval $r_1 \leq r \leq r_2$. Topologically, \mathcal{M}_8 is a two-sphere bundle over a six-dimensional base, B_6 , where $B_6 = KE_6, KE_4 \times KE_2$, or $KE_2 \times KE_2 \times KE_2$. This S^2 -bundle is obtained from the canonical line bundle over B_6 by simply adding a point ‘‘at infinity’’ to each of the fibers.

C. Flux quantization

Given a regular compact solution, the final condition that we have a bona fide solution of M theory is the quantization of the four-form flux G_4 . In our ansatz (2.1) we took AdS₃ to have unit radius, thus we should first reinstate dimensions by rescaling the metric and background by powers of L , the actual radius of the AdS₃ space,

$$ds_{11}^2 = L^2 ds_{11}^2, \quad \tilde{G}_4 = L^3 G_4. \quad (2.23)$$

In our conventions, the quantization condition [18] for the

four-form is that we have integer periods

$$N(\mathcal{D}) = \int_{\mathcal{D}} \left[\frac{1}{(2\pi l_p)^3} \tilde{G}_4 - \frac{1}{4} p_1(\mathcal{M}_{11}) \right] \in \mathbb{Z}, \quad (2.24)$$

where l_p is the 11-dimensional Planck length (see Appendix C), \mathcal{D} is any four-cycle of the 11-dimensional manifold M_{11} , and $p_1(\mathcal{M}_{11})$ is the first Pontryagin class of M_{11} . The subtlety here is the shift by $\frac{1}{4} p_1(\mathcal{M}_{11})$ which is not necessarily an integral class.

For our solutions $M_{11} = \text{AdS}_3 \times \mathcal{M}_8$, and the relevant four-cycles lie in \mathcal{M}_8 . Thus, written in terms of cohomology the condition becomes

$$\frac{1}{(2\pi l_p)^3} [\tilde{G}_4] - \frac{1}{4} p_1(\mathcal{M}_8) \in H^4(\mathcal{M}_8, \mathbb{Z}), \quad (2.25)$$

where $[\tilde{G}_4]$ denotes the cohomology class of \tilde{G}_4 on \mathcal{M}_8 . Given the fibration structure (2.20) it is relatively easy to find an expression for $p_1(\mathcal{M}_8)$ in terms of classes on B_6 . One way to view the fibration (2.20) is as a complex space formed by the anticanonical line bundle \mathcal{L} of B_6 together with a point $r = r_1$ added at infinity of each \mathbb{C} fiber to make them into spheres. One finds

$$\begin{aligned} p_1(\mathcal{M}_8) &= \pi^* p_1(B_6) + \pi^* p_1(\mathcal{L}) \\ &= \pi^* [c_1(B_6)^2 - 2c_2(B_6)] + \pi^* [c_1(B_6)^2], \end{aligned} \quad (2.26)$$

where the second term in the first line comes from the twisting of the fibration. In the second line we have used the complex structures on TB_6 and \mathcal{L} to rewrite the Pontryagin classes in terms of Chern classes.

As we will see in the next section all our new solutions will be of the form

$$B_6 = B_4 \times C, \quad (2.27)$$

where C is some Riemann surface. Given the projections $\pi_1: \mathcal{M}_8 \rightarrow B_4$ and $\pi_2: \mathcal{M}_8 \rightarrow C$ we then have $\pi^* c_1(B_6) = \pi_1^* c_1(B_4) + \pi_2^* c_1(C)$ and $\pi^* p_1(B_6) = \pi_1^* p_1(B_4) + \pi_2^* p_1(C) = \pi_1^* p_1(B_4)$. Hence

$$\begin{aligned} p_1(\mathcal{M}_8) &= \pi_1^* [2c_1(B_4)^2 - 2c_2(B_4)] + 2\pi_1^* c_1(B_4) \\ &\quad \wedge \pi_2^* c_1(C) \\ &= \pi_1^* [4c_1(B_4)^2 - 24\chi(\mathcal{O}_{B_4})] + 4\pi_1^* c_1(B_4) \\ &\quad \wedge \pi_2^* \chi(\mathcal{O}_C), \end{aligned} \quad (2.28)$$

where the alternating sum of Hodge numbers $\chi(\mathcal{O}_M) = \sum_p (-1)^p h^{0,p}(M)$ is the holomorphic Euler characteristic. (For B_4 [19,20] we have $\chi(\mathcal{O}_{B_4}) = \frac{1}{12} [c_1(B_4)^2 + c_2(B_4)]$, while for a Riemann surface it is half the Euler number $\chi(\mathcal{O}_C) = \frac{1}{2} \chi = 1 - g$, where g is the genus.)

From (2.28) we see that $\frac{1}{4} p_1(\mathcal{M}_8)$ is in fact integral for our new examples. Thus in the following we may neglect the $\frac{1}{4} p_1(\mathcal{M}_8)$ term in (2.25) and simply require that $[\tilde{G}_4]/(2\pi l_p)^3$ is integral.

III. THE CLASS $B_6 = KE_6$

Let us begin with the simplest case: $B_6 = KE_6$. As explained above, we need to impose the conditions (2.9), which imply $c_3 = c_2 = c_1$ and $d_3 = d_2 = d_1$. We are therefore left with only two integration constants, say c_1 , d_1 , and also the constant k_1 specifying the curvature of the KE_6 . We discuss the cases $k_1 \neq 0$ and $k_1 = 0$ separately.

A. $B_6 = KE_6^\pm$

From (2.18) we see that when $k_1 \neq 0$ we can shift the coordinate r to set $c_1 = 0$. Having done this, we are left with only one independent constant, say d_1 . Our polynomial solution for H is

$$H = k_1 \left(\frac{1}{4} r^4 - 2d_1 r^2 + 4d_1^2 \right), \quad (3.1)$$

which leads to the following metric

$$ds^2 = \omega^2 \left[ds^2(\text{AdS}_3) - \frac{4d_1 + 3r^2}{4(r^2 - 4d_1)} \left(k_1 ds^2(KE_6) + D\psi^2 + \frac{dr^2}{r^2 - 4d_1} \right) \right],$$

with

$$\omega^6 = \frac{4(r^2 - 4d_1)^3}{(3r^2 + 4d_1)^2}. \quad (3.2)$$

Demanding that the metric is positive definite implies that we must take $k_1 = 1$, choose $d_1 < 0$ and restrict the range of r so that $r^2 \leq 4|d_1|/3$. However, at $r^2 = 4|d_1|/3$, the warp factor ω diverges and hence there are no compact regular solutions in this class.

B. $B_6 = CY_3$

If $k_1 = 0$ we have $B_6 = CY_3$ (this includes $B_6 = T^6$ and $B_6 = CY_2 \times T^2$). We have only found one single regular solution, for which

$$H = -4r^2 + 1, \quad (3.3)$$

and $c_i = 0$, $d_i = -1$. This leads to constant metric functions:

$$h_1 = h_2 = h_3 = \omega = 1. \quad (3.4)$$

A redefinition $y = 2r$ then yields

$$ds^2 = ds^2(\text{AdS}_3) + ds^2(CY_3) + \frac{1}{4} \left[\frac{dy^2}{1-y^2} + (1-y^2)d\psi^2 \right],$$

$$G_4 = \frac{1}{2} J \wedge \text{vol}_{S^2}, \quad (3.5)$$

where $\text{vol}_{S^2} = dy \wedge d\psi$ is the volume form of the round S^2 parametrized by y, ψ . This solution is thus simply the well-known $\text{AdS}_3 \times S^2 \times CY_3$ solution that is dual to a (0, 4) superconformal field theory. In the special case that CY_3 is

$CY_2 \times T^2$ we can dimensionally reduce and T -dualise to obtain the type IIB solution of the form $\text{AdS}_3 \times CY_2 \times S^3$ with nonzero five-form flux, which is the near-horizon limit of two intersecting D3-branes and is dual to a (4, 4) superconformal field theory.

IV. THE CLASS $B_6 = KE_4 \times KE_2$

The next to simplest case is when the base is $B_6 = KE_4 \times KE_2$, which happens when we impose the conditions (2.7). These imply that we have to set $c_2 = c_1$ and $d_2 = d_1$, leaving a total of four independent constants, $\{c_1, c_3, d_1, d_3\}$ say, and the two curvature constants k_1 and k_3 of KE_4 and KE_2 , respectively. We proceed to consider the cases $k_1 = 0$ and $k_1 \neq 0$ separately.

The case $k_1 = 0$, which leads to $B_6 = K3 \times KE_2$ or $B_6 = T^4 \times KE_2$, is rather special, as we cannot set to zero any of the c_i by shifting r . Thus the general solution is specified by $\{c_1, c_3, d_1, d_3\}$ and the KE_2 curvature k_3 . We have found the following cubic polynomial solution for H :

$$H = -\frac{4c_3}{3k_3} r^3 - \frac{4(3c_1^2 + d_3)}{k_3} r^2 - \frac{4(6c_1 d_1 c_3^2 + 3d_3(4c_1^2 + d_3)c_3)}{3c_3^2 k_3} r - \frac{4(c_3^2 d_1^2 + 4c_1 c_3 d_3 d_1 + d_3^2(4c_1^2 + d_3))}{3c_3^2 k_3}, \quad (4.1)$$

which leads to the metric functions

$$h_1 = -\frac{3(4c_1^2 + d_3 + c_3 r)}{4k_3(d_1 + c_1 r)},$$

$$h_3 = -\frac{3(4c_1^2 + d_3 + c_3 r)}{4k_3(d_3 + c_3 r)},$$

$$\omega^6 = -\frac{16(d_1 + c_1 r)^2(d_3 + c_3 r)}{9(4c_1^2 + d_3 + c_3 r)^2}, \quad (4.2)$$

$$f_3 = \frac{3(4c_1^2 + d_3 + c_3 r)}{4Hk_3},$$

$$f_4 = -\frac{3H(4c_1^2 + d_3 + c_3 r)}{16k_3(d_1 + c_1 r)^2(d_3 + c_3 r)}.$$

Unfortunately, we have not been able to prove the existence of positive definite metrics leading to compact solutions for any choice of $\{c_1, c_3, d_1, d_3, k_3\}$.

However, if $k_1 \neq 0$, then we can set $c_3 = 0$ by shifting r . Thus the general solution is specified by $\{c_1, d_1, d_3\}$ and the KE_4 and KE_2 curvatures k_1 and k_3 . We have found the following quartic polynomial solution for H :

$$H = \sum_{n=0}^4 p_n r^n, \quad (4.3)$$

where

$$\begin{aligned}
p_4 &= \frac{1}{4}k_1^2k_3, \\
p_3 &= -\frac{2}{3}k_1c_1, \\
p_2 &= \frac{3d_1^2k_1^2}{2d_1k_1 + d_3k_3} - 2d_1k_1 - d_3k_3, \\
p_1 &= \frac{8c_1d_1d_3}{2d_1k_1 + d_3k_3}, \\
p_0 &= \frac{4d_3(4d_3c_1^2 + (2d_1k_1 + d_3k_3)^2)}{3k_1^2(2d_1k_1 + d_3k_3)}.
\end{aligned} \tag{4.4}$$

The corresponding expressions for the metric functions are:

$$\begin{aligned}
h_1 &= \frac{3k_1^2(4d_1^2 + k_3(2d_1k_1 + d_3k_3)r^2)}{4(2d_1k_1 + d_3k_3)(4d_1 + r(4c_1 - k_1k_3r))}, \\
h_3 &= -\frac{3k_1^2(4d_1^2 + k_3(2d_1k_1 + d_3k_3)r^2)}{4(2d_1k_1 + d_3k_3)(k_1^2r^2 - 4d_3)}, \\
\omega^6 &= \frac{4(2d_1k_1 + d_3k_3)^2(k_1^2r^2 - 4d_3)(4d_1 + r(4c_1 - k_1k_3r))^2}{9k_1^4(4d_1^2 + k_3(2d_1k_1 + d_3k_3)r^2)^2}, \\
f_3 &= -\frac{3k_1^2(4d_1^2 + k_3(2d_1k_1 + d_3k_3)r^2)}{16H(2d_1k_1 + d_3k_3)}, \\
f_4 &= \frac{3Hk_1^2(4d_1^2 + k_3(2d_1k_1 + d_3k_3)r^2)}{(2d_1k_1 + d_3k_3)(4d_3 - k_1^2r^2)(4d_1 + r(4c_1 - k_1k_3r))}.
\end{aligned} \tag{4.5}$$

We have found positive definite metrics that lead to compact solutions in the classes where $k_1k_3 = -1$, i.e. $KE_4^- \times S^2$ and $KE_4^+ \times H^2$, and $k_3 = 0$, i.e. $KE_4^+ \times T^2$. We now proceed to discuss them separately.

A. $B_6 = KE_4^- \times S^2$ and the solution of [17]

Let $(k_1, k_3) = (-1, 1)$. In this section we argue that there exists a range of the constants $\{c_1, d_1, d_3\}$ for which the expressions (4.5) yield positive definite metrics and compact \mathcal{M}_8 . To show this we first note that the special point in the $\{c_1, d_1, d_3\}$ space defined by

$$c_1 = 0, \quad d_1 = -9/16, \quad d_3 = -27/16, \tag{4.6}$$

is actually the 4-in-8 Kähler solution of [17] discussed in Sec. II. Indeed, defining $y = 2r/3$ we have

$$\begin{aligned}
\omega^6 &= 3 + y^2, & h_1 &= \frac{3}{4}, \\
h_3 = f_4 &= \frac{3(1 - y^2)}{4(3 + y^2)}, & f_3 &= \frac{3}{4(1 - y^2)}.
\end{aligned}$$

A further change $y = \cos\alpha$ leads to the 4-in-8 Kähler solution of [17]:

$$\begin{aligned}
ds^2(\mathcal{M}_8) &= \frac{3}{4} \left[ds^2(KE_4^-) + d\alpha^2 \right. \\
&\quad \left. + \frac{\sin^2\alpha}{3 + \cos^2\alpha} [ds^2(S^2) + D\psi^2] \right].
\end{aligned} \tag{4.7}$$

If the range of ψ is taken to be 4π then, at fixed α the S^2 and ψ parametrize a round three-sphere and together with α parametrize a four-sphere. Clearly, this solution is regular and compact (provided KE_4^- is). We can recover the solution in the coordinates used in [17] by defining constrained coordinates Y^A on the S^4 , satisfying $Y^AY^A = 1$, via

$$\begin{aligned}
Y^1 + iY^2 &= \sin\alpha \cos\frac{\theta}{2} e^{(i/2)(\nu - \psi)}, \\
Y^3 + iY^4 &= \sin\alpha \sin\frac{\theta}{2} e^{-(i/2)(\nu + \psi)}, & Y^5 &= \cos\alpha,
\end{aligned} \tag{4.8}$$

where θ, ν are polar coordinates for the S^2 appearing in (4.7).

It can be easily checked that for values of the constants sufficiently close to (4.6), the metric is still regular and positive definite for values of r between the two relevant roots of H . The topology of these nearby solutions is however not $KE_4^- \times S^4$. The function h_3 now does not go to zero. Instead, by virtue of the general properties discussed in Sec. II B, taking the period of ψ to be 2π , the ψ -circle fibers over r form a smooth S^2 . Globally the manifold is constructed by fibering this S^2 over $B_6 = KE_4^- \times S^2$. Noting that we can still use the scaling symmetry of the 11-dimensional equations of motion to set either d_1 or d_3 to a fixed value, we conclude that the expressions (4.5) (with $k_1 = -k_3 = -1$) lead to a two-parameter family of solutions that include the solution of [17] as a special case in which the topology changes. This family of solutions thus realizes expectations of Sec. II.

B. $B_6 = KE_4^+ \times H^2$

We repeat the analysis of the previous section in the $(k_1, k_3) = (1, -1)$ class. A very simple regular and compact solution occurs at the point

$$c_1 = 0, \quad d_1 = d_3 = -1/4, \tag{4.9}$$

where the metric is

$$\begin{aligned}
ds^2 &= \omega^2 \left[ds^2(\text{AdS}_3) + \frac{3(1 + r^2)}{4(1 - r^2)} ds^2(KE_4^+) + \frac{3}{4} ds^2(H^2) \right. \\
&\quad \left. + \frac{9(1 + r^2)}{4q(r)} dr^2 + \frac{q(r)}{4(1 - r^2)^2} D\psi^2 \right],
\end{aligned} \tag{4.10}$$

with

$$\omega^6 = \frac{4(1 - r^2)^2}{9(1 + r^2)}, \quad q(r) = 1 - 6r^2 - 3r^4. \tag{4.11}$$

It can be readily checked that the quartic polynomial $q(r)$ has only two real roots of opposite sign $r_2 = -r_1$, with absolute value less than one, and that $q(r)$ is positive

between these two. Although the analysis of Sec. II B does not apply because the expansion near the roots is not linear, one can still show that if the range of r is restricted to lie between them, the metric is positive definite, and r_1, r_2 are the north/south poles of the S^2 formed by (r, ψ) . Note that one can easily obtain compact solutions by the standard procedure of taking the quotient of H^2 by a discrete element of $SL(2, \mathbb{Z})$, to obtain a Riemann surface with genus greater than one. As shown in Appendix A 1, the Killing spinors are independent of the coordinates on H^2 , and are therefore preserved in the quotient procedure.

Having concluded that this solution is regular and compact, one can now check that for values of the constants sufficiently close to (4.9), the metric is still regular and positive definite between the two relevant roots of H . By virtue of the general properties discussed in Sec. II B, the ψ -circle always fibers over an interval parametrized by r to form a smooth S^2 . Thus, noting that we can still use the scaling symmetry of the 11-dimensional equations of motion to set either d_1 or d_3 to a fixed value, we conclude that the expressions (4.5) (with $k_1 = -k_3 = 1$) lead to a two-parameter family of deformations of the solution (4.10).

C. $B_6 = KE_4^+ \times T^2$

To study this class, we just need to set $k_3 = 0$ in the $KE_4 \times KE_2$ expressions (4.4) and (4.5). It can then be seen that we cannot have $d_1 = 0$, so it can be set to ± 1 by rescaling r . Further inspection reveals that we need to set $k_1 = 1, d_1 = -1$ in order to have a metric with the right signature and so we are led to consider only KE_4^+ spaces.

We will consider the cases when $c_1 = 0$ and when $c_1 \neq 0$ separately. The solutions in this section are of particular interest because one can use dimensional reduction and T -duality to relate them to new type IIB solutions. This will be discussed in Sec. VI below.

1. $c_1 = 0$ solutions

Setting $c_1 = 0$, the metric and the warp factor read

$$ds^2(\mathcal{M}_8) = \frac{3}{8} ds^2(KE_4^+) + \frac{3}{2(r^2 - 4d_3)} ds^2(T^2) + \frac{9dr^2}{4(3r^2 - 16d_3)} + \frac{(3r^2 - 16d_3)D\psi^2}{16(r^2 - 4d_3)}, \quad (4.12)$$

$$\omega^6 = \frac{16(r^2 - 4d_3)}{9}.$$

A further rescaling allows us to choose $d_3 = 0, \pm 1$. When $d_3 = 0$ the solution is singular. However, when $d_3 = \pm 1$ the solution is regular. For example, when $d_3 = 1$, the coordinate change $r = (4/\sqrt{3}) \cosh \alpha$ leads to the manifestly regular metric and warp factor

$$ds^2(\mathcal{M}_8) = \frac{3}{8} ds^2(KE_4^+) + \frac{9}{8(4\cosh^2 \alpha - 3)} ds^2(T^2) + \frac{3d\alpha^2}{4} + \frac{3\sinh^2 \alpha D\psi^2}{4(4\cosh^2 \alpha - 3)},$$

$$\omega^6 = \frac{64(4\cosh^2 \alpha - 3)}{27}.$$

2. $c_1 \neq 0$ solutions

When $c_1 \neq 0$ we can rescale it to 1. Defining

$$y = 1 - r, \quad a = 1 - 4d_3, \quad (4.13)$$

the 11-dimensional background reads

$$ds^2 = \omega^2 [ds^2(\text{AdS}_3) + ds^2(\mathcal{M}_8)], \quad (4.14)$$

$$G_4 = F_4 + F_2 \wedge \text{vol}_{T^2},$$

with

$$\omega^6 = \frac{16}{9} y^2 (y^2 - 2y + a),$$

$$\frac{8}{3} ds^2(\mathcal{M}_8) = \frac{1}{y} ds^2(KE_4^+) + \frac{4ds^2(T^2)}{(y^2 - 2y + a)} + \frac{6dy^2}{q(y)} + \frac{q(y)D\psi^2}{6y^2(y^2 - 2y + a)}, \quad (4.15)$$

$$F_4 = \frac{y-a}{8y} J \wedge J + \frac{a}{8y^2} J \wedge dy \wedge D\psi - \frac{8y}{3} dy \wedge \text{vol}_{\text{AdS}_3},$$

$$F_2 = \frac{a-y}{2(y^2 - 2y + a)} J + \frac{y^2 - 2ay + a}{2(y^2 - 2y + a)^2} dy \wedge D\psi,$$

and where $q(y)$ is the cubic polynomial

$$q(y) = 4y^3 - 9y^2 + 6ay - a^2. \quad (4.16)$$

Again one can show that this leads to a family of regular solutions parametrized by a . We will discuss this in more detail in the IIB dual formulation in Sec. VI.

V. THE CLASS $B_6 = KE_2 \times KE_2 \times KE_2$

In this section we consider the base space to be a product $B_6 = KE_2 \times KE_2 \times KE_2$ space. This case is the most general in the sense that it is associated with the most general polynomial solution of the differential equation (2.16). We have found that the integration constants $\{c_i, d_i\}$ have to be constrained in order for polynomial solutions to exist. This constraint reads

$$0 = d_1(k_1^2 d_1 + 4c_2 c_3)(k_2 c_2 - k_3 c_3) + 2k_1 d_1(k_2^2 c_2 d_2 - k_3^2 c_3 d_3) + \text{perm.} \quad (5.1)$$

The solution for H is then a quartic polynomial whose coefficients are rather long expressions involving the constants $\{c_i, d_i\}$ and the curvatures k_i . We know however that these expressions must be symmetric under permutations of the indices $\{1, 2, 3\}$. Indeed, the only symmetric combi-

nations appearing in H are

$$\begin{aligned} k &= k_1 k_2 k_3, & E_c &= k_i c_i, & E_d &= k_i d_i, & E_{cc} &= k_1 k_2 c_1 c_2 + \text{perm}, & E_{dd} &= k_1 k_2 d_1 d_2 + \text{perm}, \\ E_{cd} &= k_1 c_1 (k_2 d_2 + k_3 d_3) + \text{perm}, & E_{ccc} &= c_1 c_2 c_3, & E_{ccd} &= c_1 c_2 d_3 + \text{perm}, & E_{ddc} &= d_1 d_2 c_3 + \text{perm}. \end{aligned} \quad (5.2)$$

Using these definitions H reads

$$H = \sum_{n=0}^4 p_n r^n, \quad (5.3)$$

with

$$\begin{aligned} p_4 &= \frac{1}{4}k, \\ p_3 &= -\frac{2}{3}(E_c + c_4), \\ p_2 &= \{3E_{ddc}k^2 - 4[E_c(c_4 + 2E_c) - 6E_{cc}][4E_{ccc} - E_{cd}] + [3(4E_{ccc} - E_{cd})E_d + 2c_4(E_d^2 - 4E_{ccd} + E_{dd}) \\ &\quad + E_c(2E_d^2 - 4E_{ccd} + E_{dd})]k - 8[E_c^3 - 3E_{cc}E_c + c_4(E_c^2 - 2E_{cc})]E_d\}[4c_4(E_c^2 - 4E_{cc} - E_d k)]^{-1}, \\ p_1 &= \{(-2E_d^2 - 4E_{ccd} + E_{dd})E_c^2 + [E_d(-4E_{ccc} + E_{cd} - 2c_4E_d) + E_{ddc}k]E_c + 4E_{cc}(E_d^2 + 4E_{ccd} - E_{dd}) \\ &\quad + 2c_4(-4E_{ccc}E_d + E_{cd}E_d - E_{ddc}k)\}[c_4(E_c^2 - 4E_{cc} - E_d k)]^{-1}, \\ p_0 &= \{8E_{ddc}E_c^2 + (2E_d^3 - 4E_{ccd}E_d + E_{dd}E_d - 4c_4E_{ddc})E_c - 24E_{cc}E_{ddc} + E_d[12E_{ccc}E_d + 2c_4(E_d^2 + 4E_{ccd} - E_{dd}) \\ &\quad - 3(E_{cd}E_d + E_{ddc}k)]\}[c_4(-3E_c^2 + 12E_{cc} + 3E_d k)]^{-1}, \end{aligned} \quad (5.4)$$

where c_4 is either of the two roots of

$$\begin{aligned} c_4^2 &= k_1^2 c_1^2 + k_2^2 c_2^2 + k_3^2 c_3^2 - k_1 k_2 c_1 c_2 - k_2 k_3 c_2 c_3 \\ &\quad - k_3 k_1 c_3 c_1. \end{aligned} \quad (5.5)$$

One can readily check that all the solutions presented in the previous sections follow from this quartic polynomial by imposing the appropriate conditions discussed in Sec. II. In addition, (5.4) leads to interesting generalizations. Recall that in the class of solutions with $B_6 = KE_4 \times KE_2$, we found positive definite metrics in the cases $B_6 = KE_4^+ \times H^2$, $B_6 = KE_4^- \times S^2$, and $B_6 = KE_4^+ \times T^2$. For these, (5.4) leads to a generalization where the KE_4 splits into two KE_2 spaces with different radii (but with both still having the same sign of the curvature). We leave a more detailed analysis of these generalizations to future work.

On the other hand, the quartic polynomial (5.4) also provides a generalization of the KE_6^\pm solutions presented in Sec. III, where the KE_6 splits into three KE_2 spaces with different radii but still with the same sign of the curvature. Recall that we only found singular metrics in the KE_6^+ class. Unfortunately, the situation does not seem to improve by considering the more general solution (5.4) as we have not been able to find any regular metrics in the $KE_2^+ \times KE_2^+ \times KE_2^+$ class, nor any positive definite metrics in the $KE_2^- \times KE_2^- \times KE_2^-$ class. However, we have not carried out a systematic analysis of all possibilities.

VI. SOLUTIONS OF TYPE IIB STRING THEORY

In Sec. IV C above we presented two classes of solutions with $B_6 = KE_4^+ \times T^2$: the $c_1 = 0$ solutions with $d_3 = 0, \pm 1$ and the $c_1 = 1$ solutions parametrized by a . By dimensional reduction on one leg of T^2 , and T -duality on the other, these can be transformed into new solutions of type IIB supergravity with only the five-form flux excited. As such these should provide new examples of the AdS-CFT correspondence where the $N = (0, 2)$ two-dimensional CFT arises from a configuration of D3-branes.

The IIB duals of the second family with $c_1 = 1$ were discussed in some detail in [2]. So below we mostly focus on the $c_1 = 0$ solutions, analyzing the regularity of the solutions, the conditions for integral flux, and deriving an expression for the central charge of the dual CFT. We show that this family includes a solution first constructed in [15] which describes D3-branes wrapping a Kähler two-cycle in a Calabi-Yau four-fold. Turning to the $c_1 \neq 0$ solutions, we demonstrate how quantization of the G_4 flux in 11 dimensions is, as expected, related to the regularity and flux quantization in the dual type IIB configuration discussed in [2].

We also note here that the following analysis can also be applied to the analogous solutions in $B_6 = KE_2 \times KE_2 \times T^2$ class, but we shall leave the details to future work.

A. $c_1 = 0$ solutions

Dualizing the solution (4.12) to type IIB, using the formulae at the end of Sec. II, one finds the metric takes

the form

$$ds_{\text{IIB}}^2 = ds^2(\text{AdS}_3) + \frac{3}{4}ds^2(H_2) + \frac{9}{4}ds^2(SE_5), \quad (6.1)$$

where⁴

$$ds^2(H_2) = \left(r^2 - \frac{16d_3}{3}\right)^{-1} dr^2 + \left(r^2 - \frac{16d_3}{3}\right) dz^2, \quad (6.2)$$

is the constant curvature metric on H^2 (irrespective of whether $d_3 = 0, \pm 1$), and

$$ds^2(SE_5) = \frac{1}{6}[ds^2(KE_4^+) + \frac{2}{3}(D\psi + rdz)^2]. \quad (6.3)$$

Finally, the five-form flux reads, for the case $d_3 = 0$,

$$g_s F_5 = \frac{3}{32}(-J \wedge J + J \wedge dr \wedge dz) \wedge (D\psi + rdz) + \left(\frac{1}{4}J - 2dr \wedge dz\right) \wedge \text{vol}_{\text{AdS}_3}. \quad (6.4)$$

All other fluxes vanish.

We first observe that, at fixed z and for a given KE_4^+ manifold, $ds^2(SE_5)$ is a regular Sasaki-Einstein metric. We will see shortly that we can also consider quasiregular Sasaki-Einstein manifolds for which KE_4^+ is an orbifold. We also note that in order to obtain a compact solution, we need to take the quotient H^2/Γ by an element Γ of $SL(2, \mathbb{Z})$ (the Killing spinors are independent of the H^2 coordinates and are therefore preserved in the quotient procedure).

It is also interesting to point out that irrespective of whether $d_3 = 0, \pm 1$ we get the same type IIB solution. We noted earlier that in the $D = 11$ solutions $d_3 = 0$ was singular, but $d_3 = \pm 1$ were regular. This is not a contradiction, because in obtaining the $D = 11$ solutions from the type IIB solutions we are T -dualizing on different $U(1)$ directions of H^2 .

We now turn to examine the conditions for the general solutions to be globally well defined. Note that (6.1) is the metric on a compact seven-manifold \mathcal{M}_7 which is, locally, a $U(1)$ fibration over $\mathcal{B}_6 = (H^2/\Gamma) \times KE_4^+$. The $U(1)$ fibration is characterized by the first Chern class $c_1(\mathcal{M}_7)$. If we let ψ have period $\Delta\psi = 2\pi l$, then from (6.3) we find

$$c_1(\mathcal{M}_7) = l^{-1}\mathcal{R}_{KE_4^+} + l^{-1}\text{vol}_{H^2/\Gamma}. \quad (6.5)$$

We will have a proper $U(1)$ fibration if $c_1(\mathcal{M}_7) \in H^2(\mathcal{B}_6, \mathbb{Z})$. Since the cohomology of KE_4^+ and hence of \mathcal{B}_6 contains no torsion classes, this global condition is equivalent to the periods of $c_1(\mathcal{M}_7)$ being integral. Explicitly, suppose a convenient basis for $H_2(\mathcal{B}_6, \mathbb{Z})$ is provided by $\{H^2/\Gamma, \Sigma_a\}$, where Σ_a is a basis of $H_2(KE_4^+, \mathbb{Z})$, then we require

⁴We have rescaled the metric by a factor of $(3/8)^{1/2}$ and hence the five-form by $(3/8)^2$ with respect to our general IIB formula (2.11). We have also rescaled z by a factor of 2.

$$\begin{aligned} \frac{1}{2\pi} \int_{\Sigma_a} c_1(\mathcal{M}_7) &= \frac{mn_a}{l} \in \mathbb{Z}, \\ \frac{1}{2\pi} \int_{H^2/\Gamma} c_1(\mathcal{M}_7) &= \frac{\chi}{l} \in \mathbb{Z}, \end{aligned} \quad (6.6)$$

where χ is the Euler number of the Riemann surface H^2/Γ , m is the Fano index of the KE_4^+ space, and the integers n_a are coprime (see Appendix B for further details). Therefore, we deduce that the maximum value that l can take is

$$l = \text{hcf}\{m, |\chi|\}. \quad (6.7)$$

For example, if $KE_4^+ = \mathbb{C}\mathbb{P}^2$, then $m = 3$. If, furthermore, the Euler number of H^2/Γ is divisible by 3, then we can take $l = 3$. For fixed z , the S^1 fibration over the KE_4^+ is then, topologically, an S^5 . However, for a general choice of H^2/Γ , the largest possible l is $l = 1$, which leads to S^5/\mathbb{Z}_3 .

By considering, for example, the family of $Y_{p,q}$ Sasaki-Einstein metrics [4] on $S^3 \times S^2$, one can follow a similar argument to show that \mathcal{M}_7 is still regular when KE_4^+ is replaced by the orbifold base of a quasiregular $Y_{p,q}$. However, the construction does not appear to work for irregular $Y_{p,q}$ metrics. To check the regularity it is natural to change coordinates on $Y_{p,q}$ to the parameterization where $Y_{p,q}$ is manifestly a $U(1)$ bundle over a base which is itself an S^2 bundle over S^2 . Given the form of (6.1) and (6.3), in this coordinate system, \mathcal{M}_7 similarly becomes a $U(1)$ fibration over an S^2 fibration now over $S^2 \times H^2/\Gamma$. One can then check the regularity of the $U(1)$ and S^2 fibers over H^2/Γ . One finds that the whole space can be made regular, but generically one must make the size of the $U(1)$ fibration nonmaximal, that is consider $Y_{np,nq}$ for some positive integer n and (p, q) coprime (that is a \mathbb{Z}_n quotient of $Y_{p,q}$). One would expect that a similar construction would work for any quasiregular Sasaki-Einstein space.

Let us now compute the central charge for these solutions. We will follow the same steps as in [2]. We first reinstate dimensions by rescaling the metric and background

$$d\tilde{s}^2 = L^2 ds^2, \quad \tilde{F}_5 = L^4 F_5. \quad (6.8)$$

The quantization condition for the five-form in type IIB is

$$N(D) = \frac{1}{(2\pi l_s)^4} \int_D \tilde{F}_5 \in \mathbb{Z}, \quad (6.9)$$

where D is any five-cycle of \mathcal{M}_7 .

Now $H_5(\mathcal{M}_7, \mathbb{Z})$ is generated by two types of five-cycle: D_0 , the $U(1)$ fibration over the KE_4^+ space, and D_a , the five-cycles obtained from the $U(1)$ fibration over $H^2/\Gamma \times \Sigma_a$. From (6.9), we obtain

$$N(D_0) = -\frac{3L^4}{64\pi l_s^4 g_s} Ml, \quad N(D_a) = \frac{3L^4}{64\pi l_s^4 g_s} \chi l m n_a, \quad (6.10)$$

where

$$M = \frac{1}{(2\pi)^2} \int_{KE_4^+} \mathcal{R} \wedge \mathcal{R}. \quad (6.11)$$

Noting that M is always divisible by m^2 , and hence, in particular, by m , the condition that all the fluxes are the minimal possible integers becomes the following quantization condition on the AdS₃ radius:

$$\frac{3L^4}{64\pi g_s l_s^4} = \frac{n}{mlh}, \quad h = \text{hcf}\left\{\frac{M}{m}, |\chi|\right\}, \quad (6.12)$$

leaving $N(D_0) = -\frac{M}{mh}n$ and $N(D_a) = \frac{\chi}{h}n_a n$. For $n = 1$ we obtain the minimal D3-brane setup that creates the background, with higher values of n corresponding to n copies of this minimal system.

We can now determine the central charge c of the dual two-dimensional SCFT. It is well known [21] that c is fixed by the AdS₃ radius L and the Newton constant $G_{(3)}$ of the effective three-dimensional theory obtained by compactifying type IIB supergravity on \mathcal{M}_7 :

$$c = \frac{3L}{2G_{(3)}}. \quad (6.13)$$

Using the same conventions as in [2], we obtain the following expression:

$$c = \frac{36M|\chi|}{m^2 h^2 l} n^2, \quad (6.14)$$

which, remarkably, gives an integer number irrespective of the choice of KE_4^+ and H^2/Γ .

In the special case that we choose $KE_4^+ = \mathbb{C}\mathbb{P}^2$ we find that we have recovered the type IIB solution that corresponds to D3-branes wrapping a holomorphic H_2 -cycle (or H^2/Γ -cycle) inside a Calabi-Yau four-fold. This solution was first found in [15] and generalizes the solutions of [16]. From this perspective we have shown that the solutions of [15] (at least for noncompact H^2) can be dualized and uplifted to regular solutions in $D = 11$. Note that the S^5 in this solution is the S^5 that surrounds the D3-branes. We have thus also shown that we can replace⁵ this S^5 with any regular or quasiregular Sasaki-Einstein metric.

B. $c_1 \neq 0$ solutions

The reduction to type IIB of the $B_6 = KE_4^+ \times T^2$ solutions with $c_1 \neq 0$ gives precisely the solutions we presented in [2]. The metric (2.12) on the seven-dimensional

⁵We can analogously replace the S^7 in the $D = 11$ solutions describing membranes wrapping holomorphic curves in Calabi-Yau five-folds that were constructed in [22], by a seven-dimensional regular or quasiregular Sasaki-Einstein manifold.

IIB manifold \mathcal{M}_7 is given by

$$ds^2(\mathcal{M}_7) = \frac{3}{8y} ds_{KE_4}^2 + \frac{9dy^2}{4q(y)} + \frac{q(y)D\psi^2}{16y^2(y^2 - 2y + a)} + \frac{y^2 - 2y + a}{4y^2} Dz^2, \quad (6.15)$$

where, as above, $q(y) = 4y^3 - 9y^2 + 6ay - a^2$ and $Dz = dz + A_1$, with $F_2 = dA_1$ given in (4.15) or explicitly

$$Dz = dz - \frac{a - y}{2(y^2 - 2y + a)} D\psi. \quad (6.16)$$

The global analysis of [2] proceeded in two steps. First we showed that (y, ψ) form an S^2 fibration \mathcal{B}_6 over the base KE_4 . We then showed that z formed a $U(1)$ fibration over \mathcal{B}_6 .

In this section we will show how this analysis translates into conditions on the M theory solution. Note first that the S^2 of the M theory solution is only fibered over the KE_4^+ part of $B_6 = KE_4^+ \times T^2$. From this perspective we can write $\mathcal{M}_8 = \mathcal{B}_6 \times T^2$ where \mathcal{B}_6 is the same manifold that appears in the IIB solution: (y, ψ) form an S^2 fibration over KE_4 in 11 dimensions. Thus the global analysis of \mathcal{B}_6 is the same. Explicitly, it is regular and compact for values of $a \in (0, 1)$ if the range of y is restricted to lie between the first two roots of $q(y)$.

The second part of the IIB global analysis performed in [2] translates, however, not into geometry but quantization of the G_4 flux in 11 dimensions, as we will now show. To this aim, we first reinstate dimensions by rescaling the 11-dimensional metric and background,

$$d\tilde{s}_{11}^2 = L^2 ds_{11}^2, \quad \tilde{G}_4 = L^3 G_4. \quad (6.17)$$

Note that this implies an analogous rescaling of the IIB metric and five-form. As discussed in Sec. II C, for all our examples, the quantization condition for \tilde{G}_4 is that for any four-cycle \mathcal{D} we have

$$N(\mathcal{D}) = \frac{1}{(2\pi l_p)^3} \int_{\mathcal{D}} \tilde{G}_4 = \frac{L^3}{(2\pi l_p)^3} \int_{\mathcal{D}} G_4 \in \mathbb{Z}. \quad (6.18)$$

Recall that, upon reduction and T -duality along the T^2 , the surviving direction of the torus becomes the z -circle fibered over \mathcal{B}_6 . In [2] we denoted by l the resulting radius of this circle measured in units of L , and we studied the conditions for the fibration to be a proper $U(1)$ bundle; namely, that the periods of its first Chern class be integer numbers. Thus, in the IIB language, we require

$$P(C) = \frac{1}{2\pi l} \int_C F_2 \in \mathbb{Z}, \quad (6.19)$$

where C is a two-cycle in $H_2(\mathcal{B}_6, \mathbb{Z})$.

We can now use the standard relations between 11-dimensional and type IIB parameters (see Appendix C)

$$R_{\text{IIB}} = lL = l_p^3/R_1 R_2, \quad l_s^2 = l_p^3/R_1, \quad g_s = R_1/R_2. \quad (6.20)$$

From the first relation we have

$$\begin{aligned} P(C) &= \frac{LR_1 R_2}{2\pi l_p^3} \int_C F_2 = \frac{L^3 \text{Vol}(T^2)}{(2\pi l_p)^3} \int_C F_2 \\ &= \frac{L^3}{(2\pi l_p)^3} \int_{C \times T^2} G_4 = N(C \times T^2), \end{aligned} \quad (6.21)$$

where we have used $\text{Vol}(T^2) = (2\pi)^2 R_1 R_2 / L^2$ since the volume is measured in units of L . Thus the integrality of the IIB periods is equivalent to the quantization of the flux through the four-cycles $\mathcal{D} = C \times T^2$ in the M theory solution.

In the IIB solution we also needed to ensure that the flux of F_5 through any five-cycle D of \mathcal{M}_7 is appropriately quantized. The only nontrivial cycles arise as S^1 fibrations over a nontrivial four-cycle \mathcal{D} in $H_4(\mathcal{B}_6, \mathbb{Z})$. Now, recall that the IIB five-form is

$$g_s F_5 = L^4 (1 + \star)[F_4 \wedge Dz], \quad (6.22)$$

where z parametrizes the S^1 . The quantization condition reads

$$N^{\text{IIB}}(D) = \left(\frac{L}{2\pi l_s}\right)^4 \int_D g_s^{-1} F_4 \wedge dz = \frac{L^4 l}{(2\pi)^3 g_s l_s^4} \int_{\mathcal{D}} G_4, \quad (6.23)$$

for $\mathcal{D} \in H_4(\mathcal{B}_6, \mathbb{Z})$ and where in going to the final expression we have integrated over the S^1 fiber and used the expression (6.22). Using the relations (6.20) we directly see that

$$N^{\text{IIB}}(D) = N(\mathcal{D}), \quad (6.24)$$

so that the quantization of F_5 is equivalent to the quantization of G_4 through four-cycles \mathcal{D} in \mathcal{B}_6 . Notice that the ratio of the two radii of the torus is unfixed, corresponding to the fact that the IIB dilaton can take any constant value.

VII. NONCOMPACT SOLUTIONS

So far we have focused on solutions where the internal space is compact as this leads to new examples of the AdS-

CFT correspondence. However, our analysis can also be used to find new solutions where \mathcal{M}_7 is noncompact. In this section we will initiate a study of such solutions, restricting our attention to the class of type IIB solutions with $c_1 \neq 0$ that were first presented in [2] and briefly discussed above in Sec. VI B.

It is convenient to introduce the coordinates $\psi = \psi' - z'$, $z = 2z'$ so the local class of solutions (6.15) parametrized by a can be written

$$\begin{aligned} ds^2 &= \frac{9}{4} L^2 \left[\frac{4y}{9} ds^2(\text{AdS}_3) + \frac{y}{q(y)} dy^2 + \frac{q(y)}{9y^2} dz'^2 \right. \\ &\quad \left. + \frac{1}{6} ds_{KE_4}^2 + \frac{2}{3} (D\psi' - A)^2 \right], \\ A &= \frac{a}{y} dz', \end{aligned} \quad (7.1)$$

with

$$\begin{aligned} g_s L^{-4} F_5 &= J \wedge \left[\frac{3}{32} J \wedge (D\psi' - A) + \frac{3a}{34y^2} dy \wedge D\psi' \right. \\ &\quad \left. \wedge dz' \right] + \text{vol}_{\text{AdS}_3} \wedge \left[2y dy \wedge dz' - \frac{a}{4} J \right]. \end{aligned} \quad (7.2)$$

For the compact solutions, y ranged between y_1 and y_2 , the two smallest roots of the cubic $q(y)$. For the noncompact solutions, we instead take $y_3 \leq y < \infty$, where y_3 is the largest root of the cubic.

Let us first consider the case when $a = 0$ giving $y_1 = y_2 = 0$ and $y_3 = 9/4$. By implementing the coordinate change $y = (9/4)\cosh^2 \rho$, the metric becomes

$$\begin{aligned} ds^2 &= \frac{9}{4} L^2 [\cosh^2 \rho ds^2(\text{AdS}_3) + d\rho^2 + \sinh^2 \rho dz'^2 \\ &\quad + \frac{1}{6} (ds_{KE_4}^2 + \frac{2}{3} (D\psi')^2)]. \end{aligned} \quad (7.3)$$

Remarkably, we have just recovered the $\text{AdS}_5 \times SE_5$ solutions of type IIB supergravity, where SE_5 denotes a five-dimensional Sasaki-Einstein manifold. In particular, if $KE_4 = CP^2$ we obtain $\text{AdS}_5 \times S^5$.

We next observe that for general a , as $y \rightarrow \infty$ the solution behaves as if $a = 0$ and hence the solutions are all asymptotic to $\text{AdS}_5 \times SE_5$. Furthermore, for generic a (not equal to 0 or 1) when the three roots y_i are distinct, we see that as y approaches y_3 the potential conical singularity can be removed by taking the period of z' to be $2\pi[6y_3^{3/2}/q'(y_3)]$. With this period the noncompact solutions are regular: they are fibrations of SE_5 over a five-dimensional space which is a warped product of AdS_3 with a disc parametrized by y, z' .

To interpret these solutions we consider for simplicity the case when $SE_5 = S^5$. There are probe D3-branes in $\text{AdS}_5 \times S^5$ whose world volume is $\text{AdS}_3 \times S^1$. Following [23], in terms of intersecting branes, one such configuration [24] is just two flat D3-branes intersecting over a string, where the geometry $\text{AdS}_5 \times S^5$ corresponds to the near-horizon limit of one of the branes, while the second brane is treated as a probe. However, such a configuration preserves $\frac{1}{4}$ of the supersymmetry of Minkowski space, whereas our configurations preserve $\frac{1}{16}$ th. A configuration with the correct supersymmetry would be four D3-branes intersecting over a string, with the geometry corresponding again to the near-horizon limit of one of the branes. Specifically, if (z_1, z_2, z_3) are complex coordinates in the \mathbb{R}^6 space transverse to this background brane, the other three probe branes could lie in the orthogonal holomorphic two-planes $z_1 = z_2 = 0$, $z_2 = z_3 = 0$, and $z_3 = z_1 = 0$. These probe branes are a generalization of those studied in [23] which corresponded to defect CFTs. It is natural to interpret our new solutions as the backreacted geometry of such probe branes and dual to a four-dimensional $N = 4$ super Yang-Mills theory coupled to stringlike defects which preserve the $N = (0, 2)$ two-dimensional superconformal subgroup of $PSU(2, 2|4)$. One might expect the backreacted geometry of such branes to be localized in CP^2 corresponding to the positions of the three probe branes. However, in our solutions the CP^2 is still manifest. Hence our geometries seem to correspond to probe D3-branes that have been “smeared” over the CP^2 . In terms of intersecting branes, instead of three probe branes, one is considering a uniform superposition of flat probe branes spanning all holomorphic two-planes⁶ in \mathbb{C}^3 . More generally, the solutions where the S^5 factor is replaced by SE_5 can similarly be interpreted as the gravity duals of a general $N = 1$ SCFT coupled to stringlike defects.

We make a final observation about the $a = 1$ case, for which $q(y)$ has a double root at $y = 1$. By expanding the solution near $y = 1$ we find, again remarkably, that the solution is asymptotically approaching the solutions discussed in Sec. VI A. In particular, for the special case when $KE_4 = CP^2$, this is the solution found in [15] that describes the near-horizon limit of D3-branes wrapping a holomorphic H^2/Γ in a Calabi-Yau four-fold. Thus, in this special case, our full noncompact solution interpolates between $\text{AdS}_5 \times S^5$ and the solution [15], while preserving an AdS_3 factor.

VIII. SUMMARY AND CONCLUSIONS

In this work we have found new infinite classes of supersymmetric warped $\text{AdS}_3 \times \mathcal{M}_8$ solutions of 11-dimensional supergravity. The new compact solutions are

⁶A given plane is parameterized by $z_i = \lambda_i w$ for generic constants λ_i and parameter $w \in \mathbb{C}$.

all S^2 bundles over six-dimensional base spaces B_6 which are products of Kähler-Einstein spaces. The explicit solutions are obtained by solving the single second-order differential equation (2.15), for which we have found the most general *polynomial* solution. The most general polynomial solution arises for the case when $B_6 = KE_2^+ \times KE_2^+ \times KE_2^+$ and gives the quartic solution (5.4). The solutions for $B_6 = KE_4^+ \times KE_2^+$ and $B_6 = KE_6$ can then be obtained from this general solution as special cases. The new compact regular classes of solutions can be summarized as follows:

- (i) The first class is when $B_6 = KE_2^+ \times KE_2^+ \times H^2$. In the special limit where the two KE_2^+ radii coincide, the same class can also describe solutions with $B_6 = KE_4^+ \times H^2$, as discussed in Sec. IV B.
- (ii) The second class is when $B_6 = KE_2^- \times KE_2^- \times S^2$. In the special limit where the two KE_2^- radii coincide, this class can also describe solutions with $B_6 = KE_4^- \times S^2$, as discussed in Sec. IV A. The latter include the solution [17] originally found in gauged supergravity, describing M5 branes wrapping a Kähler four-cycle in a Calabi-Yau fourfold.
- (iii) The third class is when $B_6 = KE_2^+ \times KE_2^+ \times T^2$. In the special limit where the two KE_2^+ radii coincide, this class can also describe solutions with $B_6 = KE_4^+ \times T^2$, as discussed in Sec. IV C. This class is particularly interesting because it leads to type IIB backgrounds with constant dilaton and only five-form flux. Focusing on the IIB solutions arising from the $B_6 = KE_4^+ \times T^2$ case, in Sec. IV C 1, we showed how these lead to generalizations of the solutions corresponding to D3 branes wrapping an H^2/Γ in a CY_3 found in [15], whereas in Sec. IV C 2 we showed how to recover the infinite IIB families presented in [2]. The IIB solutions arising from the $B_6 = KE_2^+ \times KE_2^+ \times T^2$ case provide generalizations that would be interesting to explore further.

The general polynomial solution to the differential equation (2.15) thus gives rise to an extraordinarily rich five-parameter family of solutions. The most general solution involves eight integration constants and it would be interesting to know if this larger family includes any additional regular solutions.

Despite the long expressions involved in the most general polynomial solution, we gave a simple argument in Sec. II B (covering most cases) that the metric is regular. Therefore, the solutions presented here always lead to good 11-dimensional supergravity backgrounds. However, as M theory backgrounds, we still need to make sure that the fluxes of the four-form field strength are integral. We have only discussed the implications of this condition for the solutions where $B_6 = KE_4^+ \times T^2$. As we discussed in Sec. IV C 2 this requires appropriately discretizing both the volume of the torus and the parameter of the solution, leading to an infinite discrete series of $\text{AdS}_3/\text{CFT}_2$ examples. Though the most general case is more difficult to

analyze, we expect that most, if not all, of the parameters of the solution will also need to be discretized. It would be interesting to check this expectation since if it were not true, we would be led to predict the existence of exactly marginal deformations of the dual CFTs.

It would be very interesting if the dual conformal field theories to our new solutions could be identified. The solutions in the third class above that have a type IIB description seem the most promising, since the CFTs must arise from the gauge theories living on D3-branes. A key check of any proposal will be to recover the central charges that were calculated here and in [2] for the case when $B_6 = KE_4 \times T^2$.

We also found some intriguing noncompact solutions. In particular, there were type IIB solutions in the $B_6 = KE_4 \times T^2$ class that, in the simplest case, appear to be dual to the two-dimensional defect CFTs arising on probe D3-branes with world volume $\text{AdS}_3 \times S^1$ embedded in $\text{AdS}_5 \times S^5$. More generally, the S^5 factor could be replaced with SE_5 such that the geometries appear dual to defect CFTs in more general $N = 1$ field theories. This also seems to be a profitable avenue for further investigation.

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APPENDIX A: CONDITIONS FOR SUPERSYMMETRY

In this appendix we determine the conditions for the ansatz (2.1), (2.2), and (2.3) to be a supersymmetric solution to the equations of motion of $D = 11$ supergravity. For completeness and by way of comparison, we shall consider two equivalent approaches. First we substitute directly into the Killing spinor equations, given a particular ansatz for the Killing spinor. This is the most straightforward approach and gives the explicit dependence of the spinor on the coordinates. In the second approach we use G -structure techniques. In particular, we follow the general analysis of [14]. This has the advantage of giving generic conditions for a general class of AdS_3 compactifications. Substituting our particular ansatz then gives a comparatively easy way of obtaining the relevant differential equations. It also motivates a particular gauge choice for the function f_3 .

We shall use the conventions of [25]. In particular the Killing spinor equations are

$$(\nabla_b + \frac{1}{288}[\Gamma_b^{a_1 \dots a_4} - 8\delta_b^{a_1} \Gamma^{a_2 \dots a_4}]G_{a_1 \dots a_4})\epsilon = 0. \quad (\text{A1})$$

In addition we need to ensure that the Bianchi identity and the equations of motion for the four-form G_4 are satisfied

$$dG_4 = 0, \quad d * G_4 + \frac{1}{2}G_4 \wedge G_4 = 0. \quad (\text{A2})$$

As we will see below, the Killing spinors of the AdS_3 solutions we construct define a preferred local $SU(3)$ structure. Consequently, by the arguments of [26], it is sufficient to impose just the Bianchi identity, since all field equations are then identically implied by the supersymmetry conditions.

1. Killing spinor analysis

Let us start with the Bianchi identity. Given our ansatz, it implies the following three equations

$$\begin{aligned} g'_1 &= k_3 g_5 + k_2 g_6, & g'_2 &= k_3 g_4 + k_1 g_6, \\ g'_3 &= k_2 g_4 + k_1 g_5. \end{aligned} \quad (\text{A3})$$

The Killing spinor equation is more involved. We will use the following orthonormal frame

$$\begin{aligned} e^\alpha &= \omega \tilde{e}^\alpha, & e^a &= A_1 \tilde{e}^a, & e^i &= A_2 \tilde{e}^i, \\ e^m &= A_3 \tilde{e}^m, & e^r &= B dr, & e^\psi &= C(d\psi + \tilde{P}), \end{aligned}$$

with

$$A_i = \omega h_i^{1/2}, \quad B = \omega f_3^{1/2}, \quad C = \omega f_4^{1/2}, \quad (\text{A4})$$

where we have introduced the index notation

$$\begin{aligned} \text{AdS}_3: \alpha &= \{\tilde{0}, \tilde{1}, \tilde{2}\}, & C_1: a &= \{1, 2\}, \\ C_2: i &= \{3, 4\}, & C_3: m &= \{5, 6\}. \end{aligned}$$

The Kähler forms for the C_i are given by

$$J_1 = e^1 \wedge e^2, \quad J_2 = e^3 \wedge e^4, \quad J_3 = e^5 \wedge e^6. \quad (\text{A5})$$

Our orientation is fixed by $\epsilon_{\tilde{0}\tilde{1}\tilde{2}123456r\psi} = 1$, where the indices refer to the vielbein basis. It will be useful to define

$$\gamma_9 \equiv \Gamma_{123456r\psi} = \Gamma_{\tilde{0}\tilde{1}\tilde{2}}. \quad (\text{A6})$$

A straightforward calculation then shows that the Killing spinor equations take the following form, where all indices are tangent space indices:

$$\begin{aligned}
0 &= \left\{ \frac{1}{\omega} \hat{\nabla}_\alpha + \frac{\omega'}{2B\omega} \Gamma_{ar} + \frac{1}{12} \Gamma_\alpha \left[\frac{g_3}{4A_1^2 A_2^2} J_1 \Gamma J_2 \Gamma + \frac{g_1}{4A_2^2 A_3^2} J_2 \Gamma J_3 \Gamma + \frac{g_2}{4A_3^2 A_1^2} J_3 \Gamma J_1 \Gamma \right. \right. \\
&\quad \left. \left. + \frac{1}{BC} \left(\frac{g_4}{2A_1^2} J_1 \Gamma + \frac{g_5}{2A_2^2} J_2 \Gamma + \frac{g_6}{2A_3^2} J_3 \Gamma \right) \Gamma_{r\psi} \right] + \frac{g_7}{6B\omega^3} \Gamma_{ar} \gamma_9 \right\} \epsilon, \\
0 &= \left\{ \frac{1}{A_1} (\hat{\nabla}_a - P_a \partial_\psi) + \frac{A'_1}{2BA_1} \Gamma_{ar} - \frac{k_1 C}{4A_1^2} J_{ab} \Gamma_{\psi b} + \frac{1}{12} \Gamma_a \left[\frac{g_1}{4A_2^2 A_3^2} J_2 \Gamma J_3 \Gamma \right. \right. \\
&\quad \left. \left. + \frac{1}{BC} \left(\frac{g_5}{2A_2^2} J_2 \Gamma + \frac{g_6}{2A_3^2} J_3 \Gamma \right) \Gamma_{r\psi} - \frac{g_7}{B\omega^3} \Gamma_r \gamma_9 \right] - \frac{1}{6} J_{ab} \Gamma_b \left[\frac{g_3}{2A_1^2 A_2^2} J_2 \Gamma + \frac{g_2}{2A_3^2 A_1^2} J_3 \Gamma + \frac{g_4}{BCA_1^2} \Gamma_{r\psi} \right] \right\} \epsilon, \\
0 &= \left\{ \frac{1}{A_2} (\hat{\nabla}_i - P_i \partial_\psi) + \frac{A'_2}{2BA_2} \Gamma_{ir} - \frac{k_2 C}{4A_2^2} J_{ij} \Gamma_{\psi j} + \frac{1}{12} \Gamma_i \left[\frac{g_2}{4A_3^2 A_1^2} J_3 \Gamma J_1 \Gamma + \frac{1}{BC} \left(\frac{g_4}{2A_1^2} J_1 \Gamma + \frac{g_6}{2A_3^2} J_3 \Gamma \right) \Gamma_{r\psi} - \frac{g_7}{B\omega^3} \Gamma_r \gamma_9 \right] \right. \\
&\quad \left. - \frac{1}{6} J_{ij} \Gamma_j \left[\frac{g_3}{2A_1^2 A_2^2} J_1 \Gamma + \frac{g_1}{2A_2^2 A_3^2} J_3 \Gamma + \frac{g_5}{BCA_2^2} \Gamma_{r\psi} \right] \right\} \epsilon, \tag{A7} \\
0 &= \left\{ \frac{1}{A_3} (\hat{\nabla}_m - P_m \partial_\psi) + \frac{A'_3}{2BA_3} \Gamma_{mr} - \frac{k_3 C}{4A_3^2} J_{mn} \Gamma_{\psi n} + \frac{1}{12} \Gamma_m \left[\frac{g_3}{4A_1^2 A_2^2} J_1 \Gamma J_2 \Gamma + \frac{1}{BC} \left(\frac{g_4}{2A_1^2} J_1 \Gamma + \frac{g_5}{2A_2^2} J_2 \Gamma \right) \Gamma_{r\psi} \right. \right. \\
&\quad \left. \left. - \frac{g_7}{B\omega^3} \Gamma_r \gamma_9 \right] - \frac{1}{6} J_{mn} \Gamma_n \left[\frac{g_1}{2A_2^2 A_3^2} J_2 \Gamma + \frac{g_2}{2A_3^2 A_1^2} J_1 \Gamma + \frac{g_6}{BCA_3^2} \Gamma_{r\psi} \right] \right\} \epsilon, \\
0 &= \left\{ \frac{1}{B} \partial_r + \frac{1}{12} \Gamma_r \left[\frac{g_3}{4A_1^2 A_2^2} J_1 \Gamma J_2 \Gamma + \frac{g_1}{4A_2^2 A_3^2} J_2 \Gamma J_3 \Gamma + \frac{g_2}{4A_3^2 A_1^2} J_3 \Gamma J_1 \Gamma \right] \right. \\
&\quad \left. - \frac{1}{6BC} \left[\frac{g_4}{2A_1^2} J_1 \Gamma + \frac{g_5}{2A_2^2} J_2 \Gamma + \frac{g_6}{2A_3^2} J_3 \Gamma \right] \Gamma_\psi + \frac{g_7}{6B\omega^3} \gamma_9 \right\} \epsilon, \\
0 &= \left\{ \frac{1}{C} \partial_\psi + \frac{C'}{2BC} \Gamma_{\psi r} - \sum_i \frac{k_i C}{8A_i^2} J_i \Gamma + \frac{1}{12} \Gamma_\psi \left[\frac{g_3}{4A_1^2 A_2^2} J_1 \Gamma J_2 \Gamma + \frac{g_1}{4A_2^2 A_3^2} J_2 \Gamma J_3 \Gamma + \frac{g_2}{4A_3^2 A_1^2} J_3 \Gamma J_1 \Gamma - \frac{g_7}{B\omega^3} \Gamma_r \gamma_9 \right] \right. \\
&\quad \left. + \frac{1}{6BC} \left[\frac{g_4}{2A_1^2} J_1 \Gamma + \frac{g_5}{2A_2^2} J_2 \Gamma + \frac{g_6}{2A_3^2} J_3 \Gamma \right] \Gamma_r \right\} \epsilon.
\end{aligned}$$

The hats on the covariant derivatives indicate the covariant derivatives on the AdS₃ and KE₂ spaces.

To proceed, we assume that the Killing spinors are of the form

$$\epsilon = \alpha(r) e^{\beta(r) \Gamma_r \gamma_9} e^{(1/2) \psi \Gamma_{r\psi} \gamma_9} \epsilon_0, \tag{A8}$$

where ϵ_0 satisfies the projections

$$\Gamma_{12} \epsilon_0 = \Gamma_{34} \epsilon_0 = \Gamma_{56} \epsilon_0, \quad \gamma_9 \epsilon_0 = \epsilon_0. \tag{A9}$$

In addition ϵ_0 must satisfy

$$\begin{aligned}
\hat{\nabla}_\alpha \epsilon_0 &= \frac{1}{2} \Gamma_\alpha \gamma_9 \epsilon_0, & \hat{\nabla}_a \epsilon_0 &= \frac{1}{2} P_a \Gamma_{r\psi} \epsilon_0, \\
\hat{\nabla}_i \epsilon_0 &= \frac{1}{2} P_i \Gamma_{r\psi} \epsilon_0, & \hat{\nabla}_m \epsilon_0 &= \frac{1}{2} P_m \Gamma_{r\psi} \epsilon_0.
\end{aligned} \tag{A10}$$

The first of these equations can be solved using the Killing spinors on AdS₃, while the other three are solved using the Killing spinors on C_i . Note that the integrability conditions for the last three equations are consistent with the projections imposed in (A9). The projections imply that we preserve 1/8 of the supersymmetry. Two of the four supersymmetries are Poincaré supersymmetries and the other two are special conformal supersymmetries. Writing the metric on AdS₃ in horospherical coordinates, the former are eigenstates of Γ_ρ , the Γ -matrix along the AdS radial

direction [27]. But given that all the spinors preserved in our backgrounds are also eigenstates of γ_9 , we deduce that those that become Poincaré supersymmetries all have the same chirality with respect to the Minkowski conformal boundary, and hence the solutions are dual to conformal field theories with (0, 2) supersymmetry.

Another interesting property that follows from (A10), and that we have used repeatedly in the main text, is that *the Killing spinors are independent of the coordinates on the two-cycles C_i* . Essentially, the terms on the right-hand sides of (A10), which arise from the connection on the normal bundle to the cycles, cancel the spin connection terms inside the covariant derivatives $\hat{\nabla}$. Note that this statement ceases to be true, in general, when any two of the cycles are replaced with a KE₄ space.

Plugging our spinor ansatz into the Killing spinor equations (A7) leads to two differential equations for $\alpha(r)$, $\beta(r)$ plus a set of algebraic constraints on the spinor. The equation for $\alpha(r)$ can be solved exactly,

$$\alpha(r) = \omega^{1/2}(r), \tag{A11}$$

whereas the equation for β reads

$$\frac{1}{B}\beta' = \frac{1}{2\omega} - \frac{1}{4BC}\left(\frac{g_4}{A_1^2} + \frac{g_5}{A_2^2} + \frac{g_6}{A_3^2}\right). \quad (\text{A12})$$

AdS₃, C_i, and ψ directions) can all be written in the form

$$\mathcal{P}_{\text{AdS}_3}\epsilon = \mathcal{P}_{C_i}\epsilon = \mathcal{P}_\psi\epsilon = 0, \quad (\text{A13})$$

The remaining algebraic conditions (coming from the with

$$\begin{aligned} \mathcal{P}_{\text{AdS}_3} &= \frac{\omega'}{2B\omega} + \frac{g_7}{6B\omega^3}\gamma_9 - \frac{1}{12}\left[\frac{g_3}{A_1^2A_2^2} + \text{perm}\right]\Gamma_r + \left[\frac{1}{2\omega} - \frac{1}{12BC}\left(\frac{g_4}{A_1^2} + \text{perm}\right)\right]\Gamma_r\gamma_9, \\ \mathcal{P}_\psi &= \left(\frac{1}{2C} - \frac{Ck_i}{4A_i^2} + \frac{g_7}{12B\omega^3}\right) - \frac{C'}{2BC}\gamma_9 + \frac{1}{6BC}\left(\frac{g_4}{A_1^2} + \text{perm}\right)\Gamma_r - \frac{1}{12}\left(\frac{g_3}{A_1^2A_2^2} + \text{perm}\right)\Gamma_r\gamma_9, \\ \mathcal{P}_{C_1} &= \frac{1}{6}\left[\frac{-g_1}{A_2^2A_3^2} + \frac{2g_3}{A_1^2A_2^2} + \frac{2g_2}{A_3^2A_1^2}\right] - \frac{1}{6BC}\left[\frac{g_5}{A_2^2} + \frac{g_6}{A_3^2} - \frac{2g_4}{A_1^2}\right]\gamma_9 + \frac{A'_1}{BA_1}\Gamma_r - \left[\frac{g_7}{6B\omega^3} + \frac{k_1C}{2A_1^2}\right]\Gamma_r\gamma_9, \\ \mathcal{P}_{C_2} &= \frac{1}{6}\left[\frac{-g_2}{A_3^2A_1^2} + \frac{2g_3}{A_1^2A_2^2} + \frac{2g_1}{A_2^2A_3^2}\right] - \frac{1}{6BC}\left[\frac{g_4}{A_1^2} + \frac{g_6}{A_3^2} - \frac{2g_5}{A_2^2}\right]\gamma_9 + \frac{A'_2}{BA_2}\Gamma_r - \left[\frac{g_7}{6B\omega^3} + \frac{k_2C}{2A_2^2}\right]\Gamma_r\gamma_9, \\ \mathcal{P}_{C_3} &= \frac{1}{6}\left[\frac{-g_3}{A_1^2A_2^2} + \frac{2g_1}{A_2^2A_3^2} + \frac{2g_2}{A_3^2A_1^2}\right] - \frac{1}{6BC}\left[\frac{g_4}{A_1^2} + \frac{g_5}{A_2^2} - \frac{2g_6}{A_3^2}\right]\gamma_9 + \frac{A'_3}{BA_3}\Gamma_r - \left[\frac{g_7}{6B\omega^3} + \frac{k_3C}{2A_3^2}\right]\Gamma_r\gamma_9. \end{aligned} \quad (\text{A14})$$

Note that the algebraic equations (A14) have all the same structure

$$(a_0 + a_1\gamma_9 + a_2\Gamma_r + a_3\Gamma_r\gamma_9)e^{\beta\Gamma_r\gamma_9}e^{1/2\psi\Gamma_r\psi}\epsilon_0 = 0. \quad (\text{A15})$$

After multiplying from the left by $e^{-\beta\Gamma_r\gamma_9}$, we find that this is solved provided that

$$\begin{aligned} a_0 + a_1 \cos 2\beta + a_2 \sin 2\beta &= 0, \\ a_3 + a_2 \cos 2\beta - a_1 \sin 2\beta &= 0. \end{aligned} \quad (\text{A16})$$

In this way, we obtain ten equations from (A14). To solve the Killing spinor equation we also need to solve an 11th Eq. (A12). In addition to solving these 11 differential equations for the metric and four-form functions we also

have further three differential equations coming from the Bianchi identities (A3).

At this stage, the problem still seems formidable. However, we may progress as follows. To begin we use seven of the 11 equations to determine the flux functions g_1, \dots, g_7 in terms of the metric functions and their first derivatives. Specifically, we use the conditions arising from \mathcal{P}_{C_i} and \mathcal{P}_ψ . In making further progress we found it extremely useful to work in the gauge

$$f_3^{1/2} = \frac{1}{\omega^3 \sin 2\beta}. \quad (\text{A17})$$

In this gauge the expressions for the g_i are given by

$$\begin{aligned} g_1 &= \frac{\omega^3}{3f_3^{1/2}h_1}[\cot 2\beta((f_3f_4)^{1/2}(-2k_1h_2h_3 + k_2h_3h_1 + k_3h_1h_2) + \cos 2\beta[3h_1'h_2h_3 - (h_1h_2h_3)']) \\ &\quad + \sin 2\beta(3h_1'h_2h_3 - 2(h_1h_2h_3)' - 6h_1h_2h_3\omega'/\omega)], \\ &\vdots \\ g_{1+3} &= \frac{\omega^3 \csc 2\beta}{2f_4^{1/2}h_2h_3}[2(f_3f_4)^{1/2}h_1[-h_2h_3 + f_4(k_2h_3 + k_3h_2)] + \cos 2\beta[(f_4h_1)'h_2h_3 - f_4h_1(h_2h_3)']], \\ &\vdots \\ g_7 &= \frac{\omega^3}{h_1h_2h_3}[-(f_3f_4)^{1/2}[k_1h_2h_3 + \text{perm}] + \cos 2\beta[(h_1h_2h_3)' + 6h_1h_2h_3\omega'/\omega]], \end{aligned} \quad (\text{A18})$$

with the appropriate permutations to obtain g_2, g_3 and g_5, g_6 . We then used these to express the quantities

$$\left(\frac{g_4}{A_1^2} + \text{perm}\right), \quad \left(\frac{g_3}{A_1^2A_2^2} + \text{perm}\right), \quad (\text{A19})$$

in terms of the metric functions and their first derivatives. After inserting these expressions into the remaining four Killing spinor equations we obtain a system of four coupled first-order ordinary differential equations (ODEs) for the metric

functions:

$$\begin{aligned}
0 &= \frac{\omega^3}{12} \left[\sin^2 2\beta \log(\omega^{12} f_4^3 h_1 h_2 h_3)' + \cos^2 2\beta \log\left(\frac{f_4^3}{h_1 h_2 h_3}\right)' \right] + \cot 2\beta \left[\frac{f_4^{1/2}}{3} \sum_i \frac{k_i}{h_i} - \frac{1}{2f_4^{1/2}} \right], \\
1 &= \frac{\omega^3}{12} \left[\sin^2 2\beta \log(\omega^{12} f_4^3 h_1 h_2 h_3)' + \cos^2 2\beta \log\left(\frac{f_4^3}{h_1 h_2 h_3}\right)' \right] + \cot 2\beta \left[\frac{f_4^{1/2}}{3} \sum_i \frac{k_i}{h_i} - \frac{1}{2f_4^{1/2} \cos^2 2\beta} \right], \\
0 &= (\omega^3)' + \frac{\omega^3}{2} (1 + \cos^2 2\beta) \log(\omega^6 h_1 h_2 h_3)' - f_4^{1/2} \cot 2\beta \sum_i \frac{k_i}{h_i}, \\
0 &= \omega^3 \left[\sin 2\beta \beta' + \frac{\cos 2\beta}{8} \log\left(\frac{f_4^3}{h_1 h_2 h_3}\right)' \right] + \frac{1}{\sin 2\beta} \left[\frac{f_4^{1/2}}{2} \sum_i \frac{k_i}{h_i} - \frac{3}{4f_4^{1/2}} \right] - \frac{1}{2}.
\end{aligned}$$

These equations are not all independent, and in fact reduce to one algebraic condition⁷

$$f_4^{1/2} = -\frac{\sin 2\beta}{2}, \quad (\text{A20})$$

and two differential conditions:

$$\begin{aligned}
\log(\omega^{12} \sin^2 2\beta h_1 h_2 h_3)' &= -\frac{4 \cos 2\beta}{\omega^3 \sin^2 2\beta}, \\
(\omega^3 \cos 2\beta)' &= 2 - g_7.
\end{aligned} \quad (\text{A21})$$

It is convenient, therefore, to define

$$f = \omega^3 \cos 2\beta, \quad (\text{A22})$$

and trade β for f .

The next step is to integrate the Bianchi identities (2.18). From the expressions for the flux components one obtains

$$\begin{aligned}
k_3 g_5 + k_2 g_6 &= -\frac{1}{2} [f(k_2 h_3 + k_3 h_2) + k_2 k_3 r]', \\
g_1 &= -\frac{1}{2} f(k_2 h_3 + k_3 h_2) - (\omega^6 h_2 h_3)',
\end{aligned} \quad (\text{A23})$$

together with permutations of (1, 2, 3). We may therefore integrate the Bianchi identities twice to find

$$\begin{aligned}
g_1 &= -\frac{1}{2} [f(k_2 h_3 + k_3 h_2) + k_2 k_3 r] + c_1, \\
g_2 &= -\frac{1}{2} [f(k_3 h_1 + k_1 h_3) + k_3 k_1 r] + c_2, \\
g_3 &= -\frac{1}{2} [f(k_1 h_2 + k_2 h_1) + k_1 k_2 r] + c_3,
\end{aligned} \quad (\text{A24})$$

and

$$\begin{aligned}
0 &= \omega^6 h_1 h_2 - \frac{1}{4} k_1 k_2 r^2 + c_3 r + d_3, \\
0 &= \omega^6 h_2 h_3 - \frac{1}{4} k_2 k_3 r^2 + c_1 r + d_1, \\
0 &= \omega^6 h_3 h_1 - \frac{1}{4} k_3 k_1 r^2 + c_2 r + d_2,
\end{aligned} \quad (\text{A25})$$

for some constants c_i, d_i . The expression for g_7 now takes the form

⁷Note that (A20), together with (A17), imply that $\sqrt{f_3 f_4} = -1/2\omega^3$. This sign is important, for example, when checking the equations of motion for the four-form.

$$\begin{aligned}
g_7 &= \frac{f_3}{2h_1 h_2 h_3} [(\omega^6 + f^2)(k_1 h_2 h_3 + \text{perm.}) \\
&\quad + f(k_1 k_2 h_3 + \text{perm})r - 2f c_i h_i].
\end{aligned} \quad (\text{A26})$$

It thus remains to solve the two coupled first-order equations (A21). The function redefinitions

$$H = \omega^6 h_1 h_2 h_3 (\omega^6 - f^2), \quad I = 4\omega^6 h_1 h_2 h_3 f, \quad (\text{A27})$$

allows us to decouple them, at the expense of having to solve one single second-order differential equation

$$\begin{aligned}
I &= -H', \\
0 &= -4(H')^2 + 4H(2H'' + 4k_i d_i + 4k_i c_i r - 3k_1 k_2 k_3 r^2) \\
&\quad + \prod_{i=1}^3 \left(\frac{k_1 k_2 k_3}{k_i} r^2 - 4r c_i - 4d_i \right).
\end{aligned} \quad (\text{A28})$$

From any solution of this second-order ODE we can reconstruct the full solution. The explicit formulae are recorded in the main text.

2. G-structure analysis

In this appendix we use the G -structure techniques of [14] to derive, first, a set of general supersymmetry conditions for a class of AdS₃ backgrounds, and then consider the restriction to the specific ansatz (2.1), (2.2), and (2.3). The conditions are derived as a limit of class of warped $\mathbb{R}^{1,1}$ supersymmetric Minkowski backgrounds. The discussion follows exactly that in [14], except that here we include an electric flux component.

The class of Minkowski backgrounds is defined by the set of projections on the Killing spinors. The solutions of interest are related to M5-branes wrapping holomorphic four-cycles in a Calabi-Yau fourfold (4-in-8 Kähler solutions) with $N = (0, 2)$ supersymmetry. We start by considering the geometry with the M5-brane viewed as a probe in the special holonomy spacetime $\mathbb{R}^{1,2} \times \mathcal{M}_{SU(4)}$. We can choose a frame $\{e^+, e^-, e^1, \dots, e^9\}$, with $\mathbb{R}^{1,2}$ spanned by $\{e^+, e^-, e^9\}$, such that the four special holonomy Killing spinors satisfy

$$\Gamma^{1234} \epsilon = \Gamma^{3456} \epsilon = \Gamma^{5678} \epsilon = -\epsilon. \quad (\text{A29})$$

The $SU(4)$ structure can be written as

$$\begin{aligned}\tilde{J} &= e^{12} + e^{34} + e^{56} + e^{78}, \\ \tilde{\Omega} &= (e^1 + ie^2) \wedge (e^3 + ie^4) \wedge (e^5 + ie^6) \wedge (e^7 + ie^8),\end{aligned}\tag{A30}$$

where $e^{i_1 \dots i_n} = e^{i_1} \wedge \dots \wedge e^{i_n}$. The projection on the probe brane can be written $\Gamma^{+-1234} \epsilon = -\epsilon$ or equivalently

$$\Gamma^{+-} \epsilon = \epsilon.\tag{A31}$$

Together these projections define an $(SU(4) \times \mathbb{R}^8) \times \mathbb{R}$ structure [28], or equivalently, a pair of $(\text{Spin}(7) \times \mathbb{R}^8) \times \mathbb{R}$ structures $K = e^+$, $\Omega_M = K \wedge v$, $\Sigma = K \wedge \phi$ with $\phi = \phi_{\pm}$ and $v = e^9$ where

$$\phi_{\pm} = -\frac{1}{2} \tilde{J} \wedge \tilde{J} \mp \text{Re} \tilde{\Omega}.\tag{A32}$$

We define a class of ‘‘wrapped-brane’’ geometries as warped $\mathbb{R}^{1,1} \times \mathcal{M}_9$ backgrounds where the Killing spinors satisfy the same projections as the probe brane geometry. The metric is assumed to have the form

$$ds^2 = L^{-1} ds^2(\mathbb{R}^{1,1}) + ds^2(\mathcal{M}_9),\tag{A33}$$

so $e^+ = L^{-1} du$ and $e^- = dv$, while, preserving the Minkowski symmetries, we split the flux as

$$G_4 = e^{+-} \wedge E_2 + B_4.\tag{A34}$$

Note that unlike the discussion in [14] we keep some electric flux E_2 as well as magnetic flux B_4 .

A necessary condition for supersymmetry is that the $(\text{Spin}(7) \times \mathbb{R}^8) \times \mathbb{R}$ calibration conditions [25] are satisfied, namely,

$$\begin{aligned}dK &= \frac{2}{3} i_{\Omega_M} G_4 + \frac{1}{3} i_{\Sigma} \star G_4, & d\Omega_M &= i_K G_4, \\ d\Sigma &= i_K \star G_4 - \Omega_M \wedge G_4.\end{aligned}\tag{A35}$$

Substituting for the pair of structures ϕ_{\pm} one finds the conditions

$$\begin{aligned}L^{-1} dL &= \frac{2}{3} i_v E_2 - \frac{1}{6} i_{\tilde{J} \wedge \tilde{J}} \star_9 B_4, & \text{Re} \tilde{\Omega} \wedge B_4 &= 0, \\ Ld(L^{-1} v) &= E_2, & d(L^{-1} \text{Re} \tilde{\Omega}) &= 0, \\ \frac{1}{2} Ld(L^{-1} \tilde{J} \wedge \tilde{J}) &= \star_9 B_4 - v \wedge B_4,\end{aligned}\tag{A36}$$

where the orientation on \mathcal{M}_9 is defined by $\text{vol}_{\mathcal{M}_9} = e^{1 \dots 9}$. Generically, since the Killing spinors for supersymmetric spacetimes satisfying these conditions define a preferred local $(SU(4) \times \mathbb{R}^8) \times \mathbb{R}$ structure, given our metric ansatz

one must impose the Bianchi identity and the $+ - 9$ component of the four-form field equations to ensure that all field equations are satisfied [29].

To obtain conditions for supersymmetric $\text{AdS}_3 \times \mathcal{M}_8$ spacetimes, one specializes the conditions (A36) by assuming the warping and metric have the form

$$L = e^{2R} \omega^{-2}, \quad ds^2(\mathcal{M}_9) = \omega^2 [dR^2 + ds^2(\mathcal{M}_8)].\tag{A37}$$

This matches the ansatz (2.1) with the radial coordinate R combining with the $\mathbb{R}^{1,1}$ factor to give a unit radius AdS_3 in Poincaré coordinates. To preserve the AdS_3 symmetries one assumes in addition that

$$E_2 = \omega dR \wedge E_1\tag{A38}$$

and that B_4 has no component along dR .

In general the radial direction will only lie partly along v . If the orthogonal component lies along a unit one-form \hat{u} then writing $v = \omega \hat{v}$ we have

$$dR = \cos 2\beta \hat{v} + \sin 2\beta \hat{u}, \quad \hat{\rho} = -\sin 2\beta \hat{v} + \cos 2\beta \hat{u},\tag{A39}$$

where we have also introduced $\hat{\rho}$, the unit one-form orthogonal to dR . Note that the angle β is the same angle defined in the Killing spinor analysis above. This decomposition defines a (local) $SU(3)$ structure on \mathcal{M}_8 , defined by $\hat{\rho}$, together with a second unit one-form $\hat{w} = J\hat{u}$, a two-form J , and three-form Ω . The relation to the original $SU(4)$ structure is

$$\omega^{-2} \tilde{J} = J + \hat{w} \wedge \hat{u}, \quad \omega^{-4} \tilde{\Omega} = \Omega \wedge (\hat{w} + i\hat{u}).\tag{A40}$$

The metric on \mathcal{M}_8 can be written as

$$ds^2(\mathcal{M}_8) = ds^2(\mathcal{M}_{SU(3)}) + \hat{\rho}^2 + \hat{w}^2,\tag{A41}$$

where $ds^2(\mathcal{M}_{SU(3)})$ is the metric defined by the $SU(3)$ structure (J, Ω) .

Given the relations (A40), reducing the conditions (A36) one finds

$$d(\omega^3 \sin 2\beta \hat{\rho}) = 0,\tag{A42}$$

$$d(\omega^6 \sin 2\beta \text{Im} \Omega) = -2\omega^6 (\text{Re} \Omega \wedge \hat{w} - \cos 2\beta \text{Im} \Omega \wedge \hat{\rho})\tag{A43}$$

together with

$$\begin{aligned}d(\omega^3 \cos 2\beta) - 2\omega^3 \sin 2\beta \hat{\rho} &= -\omega^3 E_1, & 6\omega^{-1} d\omega &= -2 \cos 2\beta E_1 + \omega^{-3} \sin 2\beta i_{J \wedge \hat{w}} \star_8 B_4, \\ d\left[\omega^6 \left(\frac{1}{2} J \wedge J + \cos 2\beta J \wedge \hat{w} \wedge \hat{\rho}\right)\right] &= \omega^3 \sin 2\beta \hat{\rho} \wedge B_4, \\ -d(\omega^6 \sin 2\beta J \wedge \hat{w}) + 2\omega^6 \left(\frac{1}{2} J \wedge J + \cos 2\beta J \wedge \hat{w} \wedge \hat{\rho}\right) &= \omega^3 \star_8 B_4 + \omega^3 \cos 2\beta B_4,\end{aligned}\tag{A44}$$

and the algebraic constraints

$$\begin{aligned} \text{Im}\Omega \wedge B_4 &= 0, & \hat{w} \wedge \text{Re}\Omega \wedge B_4 &= 0, \\ 4\omega^3 \sin 2\beta i_{\hat{\rho}} E_1 + (i_{J\wedge J} + 2\cos 2\beta i_{J\wedge \hat{w}\wedge \hat{\rho}}) \star_8 B_4 &= 12\omega^3, \end{aligned} \quad (\text{A45})$$

where the orientation on \mathcal{M}_8 is given by $\text{vol}_{\mathcal{M}_8} = \frac{1}{6} J \wedge J \wedge \hat{\rho} \wedge \hat{w}$. Note that this is not necessarily a minimal set of conditions: one expects that there is redundancy among these relations.

In addition, one must impose the Bianchi identity for G_4 , which, given (A44) requires that we impose

$$dB_4 = 0. \quad (\text{A46})$$

For the specialization to AdS₃, the preferred local structure group defined by the Killing spinors of the wrapped-brane spacetime is reduced from $(SU(4) \ltimes \mathbb{R}^8) \times \mathbb{R}$ to $SU(3)$. We may read off from the results of [26] that for the AdS₃ spacetimes it is sufficient to impose just the Bianchi identity in addition to the supersymmetry conditions, and all field equations are then identically satisfied.

Let us now compare our general conditions with the specific ansatz (2.1), (2.2), and (2.3) relevant to our solutions. The identification is

$$\begin{aligned} J &= h_1 J_1 + h_2 J_2 + h_3 J_3, \\ \Omega &= (h_1 h_2 h_3)^{1/2} \Omega_1 \wedge \Omega_2 \wedge \Omega_3, \\ \hat{\rho} &= f_3^{1/2} dy, & \hat{w} &= f_4^{1/2} D\psi, \end{aligned} \quad (\text{A47})$$

where Ω_i are the holomorphic one-forms on Kähler-Einstein spaces C_i . Substituting into the conditions (A42)–(A46) one can relatively quickly derive the supersymmetry conditions given in the previous section. Rather than repeat that calculation in detail, let us make a couple of observations. From (A42), one notes that, quite generically for any AdS₃ solution, one can introduce a coordinate y such that

$$\hat{\rho} = \frac{dy}{\omega^3 \sin 2\beta}. \quad (\text{A48})$$

This is precisely the gauge condition (A17) we chose in analyzing the Killing spinor equation and was the motivation for this choice. Using this gauge, it is easy to see, for instance, that the $\hat{\rho}$ component of (A43) gives the first differential equation in (A28).

APPENDIX B: SOME PROPERTIES OF KE_4 SPACES

In the main text we make use of a few properties of four-dimensional Kähler-Einstein spaces. Let us summarize and explain them here. For some more details see Refs. [19,20,30]

Given a Kähler metric on a complex manifold \mathcal{M} one can always construct the Ricci form \mathcal{R} by contracting the Riemann tensor with J . This two-form gives the curvature of a holomorphic line bundle \mathcal{L} known as the anticanonical

bundle. The first Chern class $c_1(\mathcal{M})$ of \mathcal{M} is just the first Chern class of \mathcal{L} . It is given by $\frac{1}{2\pi} \mathcal{R}$, is necessarily integral, and depends only on the choice of complex structure on \mathcal{M} . In the special case where the metric is also Einstein it is easy to show that the Ricci form is proportional to the Kähler-form J ,

$$\mathcal{R} = kJ. \quad (\text{B1})$$

In this paper we normalize the metric such that $k = 0, \pm 1$. If $k = 0$ the metric is Calabi-Yau.

Suppose we have a minimal set of two-cycles $\{\Sigma_a\}$ which generate $H_2(\mathcal{M}, \mathbb{Z})$. This means that any element of the homology can be written as $m^a \Sigma_a$ for some set of integers m^a . It is possible that $H_2(\mathcal{M}, \mathbb{Z})$ includes torsion elements. These are elements Σ such that Σ itself is non-trivial but $p\Sigma = 0$ in cohomology for some $p \in \mathbb{Z}$. If we ignore this subtlety (in fact none of the manifolds we are interested in will have torsion) then the integrality of $c_1(\mathcal{M})$ is equivalent to the integrality of the periods

$$n(\Sigma_a) = \int_{\Sigma_a} c_1(\mathcal{M}) = \frac{1}{2\pi} \int_{\Sigma_a} \mathcal{R} \in \mathbb{Z}. \quad (\text{B2})$$

In some cases, the anticanonical bundle can be written as a (positive) multiple tensor product of another line bundle, that is $\mathcal{L} = \mathcal{N}^m$ for $m \in \mathbb{Z}^+$. In general the maximal possible m is known as the Fano index. It implies that $c_1(\mathcal{M}) = mc_1(\mathcal{N})$ and hence $n(\Sigma_a) = mn_a$ where $n_a = \int_{\Sigma_a} c_1(\mathcal{N})$. Since m is maximal, the n_a must be coprime.

Recall that, in d dimensions, $c_1(\mathcal{M}) \in H^2(\mathcal{M}, \mathbb{Z})$ is Poincaré dual to a $(d-2)$ -cycle $\Sigma_{\mathcal{L}}$ in $H_{d-2}(\mathcal{M}, \mathbb{Z})$. This means by definition that the integers $n(\Sigma_a)$ are given by the intersection number of $\Sigma_{\mathcal{L}}$ with Σ_a

$$n(\Sigma_a) = \Sigma_{\mathcal{L}} \cdot \Sigma_a. \quad (\text{B3})$$

Note that in $d = 4$, $\Sigma_{\mathcal{L}} \in H_2(\mathcal{M}, \mathbb{Z})$ and so can be written as $\Sigma_{\mathcal{L}} = m s^a \Sigma_a$ for some set of integers $s^a \in \mathbb{Z}$.

Of specific interest here are those four-dimensional complex manifolds KE_4^+ which admit a positive-curvature Kähler-Einstein metric. The list is in fact finite: only CP^2 , $S^2 \times S^2$ and the del-Pezzo surfaces dP_k for $k = 3, \dots, 8$ are allowed. These latter spaces are CP^2 blown-up at k distinct points. None of these spaces have torsion classes. In four dimensions, in addition to the Fano index m and the periods n_a , one can also consider the integer

$$\begin{aligned} M &= \int_{KE_4^+} c_1(KE_4^+) \wedge c_1(KE_4^+) \\ &= m^2 \int_{KE_4^+} c_1(\mathcal{N}) \wedge c_1(\mathcal{N}) = \Sigma_{\mathcal{L}} \cdot \Sigma_{\mathcal{L}} \in \mathbb{Z}. \end{aligned} \quad (\text{B4})$$

Note that by construction M is always divisible by m^2 . These are all the quantities which will enter our discussion.

For completeness, it is straightforward to list $m, \{n_a\}$ and M for each of the allowed KE_4^+ spaces. For CP^2 the situation is very simple: $H_2(CP^2, \mathbb{Z}) = \mathbb{Z}$ and is generated

by a single class, known as the hyperplane class H . A representative curve in this class is just the S^2 given by say $z_1 = 0$ in homogeneous coordinates (z_1, z_2, z_3) . It is easy to see that $H \cdot H = 1$. One finds that $\Sigma_{\mathcal{L}} = 3H$ and so $m = 3$, $n_1 = 1$, and $M = 9$.

If $KE_4^+ = S^2 \times S^2$ things are equally straightforward. Now $H_2(S^2 \times S^2, \mathbb{Z}) = \mathbb{Z}^2$, and is simply generated by the classes H_1 and H_2 corresponding to each of the two-spheres. The only nonzero intersection is $H_1 \cdot H_2 = 1$. One finds $\Sigma_{\mathcal{L}} = 2(H_1 + H_2)$ and so $m = 2$, $n_1 = n_2 = 1$, and $M = 8$.

For dP_k recall that in four dimensions blowing up replaces a point on the manifold with a two-sphere. Thus $H_2(dP_k, \mathbb{Z}) = \mathbb{Z}^{k+1}$ and is generated by H , the image of the hyperplane class after blowing up CP^2 , together with the exceptional two-spheres E_i , for $i = 1, \dots, k$, at the k blown-up points. The only nonzero intersections are $H \cdot H = 1$ and $E_i \cdot E_j = -\delta_{ij}$. One finds $\Sigma_{\mathcal{L}} = 3H - E_1 - \dots - E_k$ and so $m = 1$ while $n_i = 1$ and $n_{k+1} = 3$ (if we label $\Sigma_i = E_i$ and $\Sigma_{k+1} = H$). In addition $M = 9 - k$.

APPENDIX C: REDUCTION TO TYPE IIB

Consider a solution of 11-dimensional supergravity invariant under a T^2 action of the form

$$ds_{11}^2 = ds_9^2 + f^2 ds^2(T^2), \quad G_4 = F_4 + F_2 \wedge \text{vol}_{T^2}, \quad (\text{C1})$$

with orientation

$$\epsilon_{11} = \epsilon_9 \wedge \text{vol}_{T^2}, \quad (\text{C2})$$

where ϵ_9 defines an orientation on ds_9^2 . The Bianchi identity, $dG_4 = 0$, and the 11-dimensional equation of motion for the four-form, $d \star G_4 + \frac{1}{2} G_4 \wedge G_4 = 0$, decompose as

$$\begin{aligned} dF_2 &= 0, & dF_4 &= 0, \\ d(f^2 \star_9 F_4) + F_2 \wedge F_4 &= 0, & (\text{C3}) \\ d(f^{-2} \star_9 F_2) + \frac{1}{2} F_4 \wedge F_4 &= 0, \end{aligned}$$

where \star_9 is the Hodge dual with respect to ds_9^2 .

After dimensional reduction and T -duality, we find the following metric and five-form for type IIB supergravity

$$\begin{aligned} ds_{10}^2 &= f ds_9^2 + \frac{1}{f^3} (dz + A_1)^2, \\ F_5 &= (1 + \star) F_4 \wedge (dz + A_1) \\ &= F_4 \wedge (dz + A_1) + f^2 \star_9 F_4 \end{aligned} \quad (\text{C4})$$

with $dA_1 = F_2$. Observe that the orientation in ten dimensions is given by

$$\epsilon_{10} = \epsilon_9 \wedge dz, \quad (\text{C5})$$

so that closure of F_5 indeed follows from (C3).

It is also useful to note here the standard relations between 11-dimensional M theory parameters and the ten-dimensional type IIB parameters [31]. Following Polchinski [32], we define the 11-dimensional Planck length l_P by $2\kappa_{11}^2 = (2\pi)^8 l_P^9$. For a square torus, with sides length $2\pi R_1$ and $2\pi R_2$ the string length l_s , and type IIB coupling are given by

$$l_s^2 = l_P^3 / R_1, \quad g_s = R_1 / R_2. \quad (\text{C6})$$

The radius R_{IIB} of the IIB circle is given by

$$R_{\text{IIB}} = l_P^3 / R_1 R_2. \quad (\text{C7})$$

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