Annihilation type radiative decays of B meson in perturbative QCD approach

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With the perturbative QCD approach based on k_T factorization, we study the pure annihilation type radiative decays $B^0 \to \phi\gamma$ and $B^0 \to J/\psi\gamma$. We find that the branching ratio of $B^0 \to \phi\gamma$ is $(2.7^{+0.3+1.2}_{-0.6-0.6}) \times 10^{-11}$, which is too small to be measured in the current B factories of *BABAR* and Belle. The branching ratio of $B^0 \to J/\psi\gamma$ is $(4.5^{+0.6+0.7}_{-0.5-0.6}) \times 10^{-7}$, which is just at the corner of being observable in the B factories. A larger branching ratio $\mathrm{BR}(B_s^0 \to J/\psi\gamma) \simeq 5 \times 10^{-6}$ is also predicted. These decay modes will help us in testing the standard model and searching for new physics signals.

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B meson rare decays are interesting for testing the standard model and searching for new physics. However, due to our poor knowledge of nonperturbative QCD, predictions for many interesting exclusive decays are polluted by large hadronic uncertainties. The two body radiative B decays involve simpler hadronic dynamics with only one hadron in the final states, so they suffer much less pollution than nonleptonic decays. The radiative decays such as $B \to K^*\gamma$, $\rho(\omega)\gamma$ thus attract much attention [1]. The isospin breaking effects between the charged $B^\pm$ and neutral B^0 in these modes are mainly due to contributions from the annihilation type diagrams [1–5].

The importance of the annihilation type diagrams can also be shown from the pure annihilation radiative B decays. The color suppressed $B^0 \to J/\psi\gamma$ and $B^0 \to \phi\gamma$ modes are of this kind. The former is tree dominant while the latter is a pure penguin flavor changing neutral current decay. Despite the fact that they are annihilation type decays, these decay amplitudes can be factorized as the B meson to photon transition form factor $\langle\gamma|\bar{q}\gamma_\mu(1-\gamma_5)b|B\rangle$ times the decay constant $\langle\phi|\bar{s}\gamma_\mu s|0\rangle$ or $\langle J/\psi|\bar{c}\gamma_\mu c|0\rangle$ in the naive factorization approach.

Recently, the vertex corrections for the four-quark operators have been performed in the so-called QCD factorization approach [6], utilizing the light-cone wave functions. The branching ratios turn out to be 1 order of magnitude different from the naive factorization approach [7,8]. Such a large contribution from next-to-leading order corrections implies that the hadronic uncertainty in this kind of decay is as large as other hadronic annihilation type decays [9]. A recent study of soft collinear effective theory [10] also shows that the naive factorization contribution is not the only dominant contribution, which contradicts with QCD factorization claims. More theoretic study is needed before one can claim new physics effects in these decays.

In this paper, we will use an alternative approach—the perturbative QCD approach (PQCD) [11] to calculate the pure annihilation type decays $B^0 \to \phi\gamma$ and $B^0 \to J/\psi\gamma$. Based on k_T factorization, the PQCD approach has been proposed and applied to calculate two body nonleptonic B decays such as $B \to K\pi$ [12], $\pi\pi$ [13], $\rho\rho$ [14], the radiative decays $B \to K^*\gamma$ [4], $\rho(\omega)\gamma$ [5], etc. and the results are consistent with experimental data. In this approach, the quark transverse momentum k_T is kept in order to kill the end point singularity. Because of inclusion of transverse momenta, double logarithms from the overlap of two types of infrared divergences, soft and collinear, are generated in radiative corrections. The resummation of these double logarithms leads to a Sudakov form factor, which suppresses the long-distance contribution.

For convenience, we work in the light-cone coordinate, where the B meson momentum in its rest frame, is

$$P_B = (P_B^+, P_B^-, \vec{P}_{B\perp}) = \frac{M_B}{\sqrt{2}}(1, 1, \vec{0}_\perp). \tag{1}$$

By choosing the coordinate frame where the vector meson moves in the "−" direction and photon in the "+" direction, the momenta of final state particles are

$$\begin{aligned} P_V &= (P_V^+, P_V^-, \vec{P}_{V\perp}) = \frac{M_B}{\sqrt{2}}(r^2, 1, \vec{0}_\perp), \\ P_\gamma &= (P_\gamma^+, P_\gamma^-, \vec{P}_{\gamma\perp}) = \frac{M_B}{\sqrt{2}}(1-r^2, 0, \vec{0}_\perp), \end{aligned} \tag{2}$$

where $r = m_V/M_B$. The momentum of the light quark in the B meson is:

$$k_1 = (k_1^+, k_1^-, \vec{k}_{1T}) = \left(\frac{M_B}{\sqrt{2}}x_1, 0, \vec{k}_{1T}\right). \tag{3}$$

For the final state vector meson, we set the momentum of $q(q = s, c)$ as

$$k_2 = (k_2^+, k_2^-, \vec{k}_{2T}) = \left(\frac{M_B}{\sqrt{2}}x_2 r^2, \frac{M_B}{\sqrt{2}}x_2, \vec{k}_{2T}\right). \tag{4}$$

In the above functions, x_1 and x_2 are momentum fractions of the quarks.

In PQCD approach, the decay amplitude is factorized into the convolution of the mesons' wave functions, the

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hard scattering kernel, and the Wilson coefficients, which stand for the soft, hard, and harder dynamics, respectively. With transverse momentum and the Sudakov form factor, the formalism can be written as:

$$\int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \Phi_V(x_2, b_2)\Phi_B(x_1, b_1)C(t) \times H(x_1, x_2, b_1, b_2, t)\exp[-S(x_1, x_2, b_1, b_2, t)], \quad (5)$$

where b_i is the conjugate space coordinate of the transverse momentum k_{iT}, which represents the transverse interval of the meson. t is chosen as the largest energy scale in the hard scattering kernel H in order to suppress higher order corrections. The light-cone wave functions of mesons are not calculable in principle in PQCD, but they are universal for all the decay channels so that they can be constrained from the other measured decay channels, like decays $B \to K\pi$ [12], $B \to \pi\pi$ [13], etc.

Since the outgoing photon can be only transversely polarized, the decay amplitude can be decomposed into two parts as:

$$A = (\varepsilon_V^* \cdot \varepsilon_\gamma^*)M^S + \frac{i}{P_V \cdot P_\gamma}\epsilon_{\mu\nu\rho\sigma}\varepsilon_\gamma^{*\mu}\varepsilon_V^{*\nu}P_\gamma^\rho P_V^\sigma M^P, \quad (6)$$

where P_V and P_γ are the momenta of vector meson, and photon, respectively. ε_V^* and ε_γ^* are the relevant polarization vectors. The matrix element $M^{S(P)}$ can be calculated in the PQCD approach.

In principal, the leading order contributions for $B^0 \to V\gamma$ ($V = J/\psi, \phi$) decays involve only four-quark operators plus a photon emitted from any of the quark line. The effective weak Hamiltonian is formed by the 12 four-quark operators and the corresponding QCD corrected Wilson coefficients [15]. Very recently the electromagnetic penguin operator $O_{7\gamma}$ contribution through $B^0 \to \gamma\gamma$ with one photon connecting to the ϕ meson is studied in Ref. [16]. The branching ratio for $B \to \phi\gamma$ is found to be 1×10^{-11} which is larger than the four-quark operator contribution from the QCD factorization approach [7,8]. In PQCD language, the contribution from $O_{7\gamma}$ is next-to-leading order. Its contribution is still smaller than other contributions in the PQCD approach which will be shown later. The contribution of this kind of operator to the $B^0 \to J/\psi\gamma$ decay is negligibly small.

The lowest order Feynman diagrams of $B^0 \to \phi\gamma$ in PQCD are shown in Fig. 1. In principle, the photon can be emitted from any quark line of the four-quark operator. However, the contribution of Fig. 1(c) is canceled exactly by that of 1(d) because of topology symmetry. Thus, only the decay amplitudes from Fig. 1(a) and 1(b) are left as:

$$M^S = \frac{1}{\sqrt{3}}G_F e M_B^3 r f_V V_{\rm CKM}\int_0^1 dx_1 \int_0^\infty db_1 b_1 \phi_B(x_1, b_1) \times [(1 - r^2)C(t^a)K_0(b_1 A_a)e^{-S_B(t^a)} - (1 - r^2)C(t^b)K_0(b_1 B_a)e^{-S_B(t^b)}], \quad (7)$$

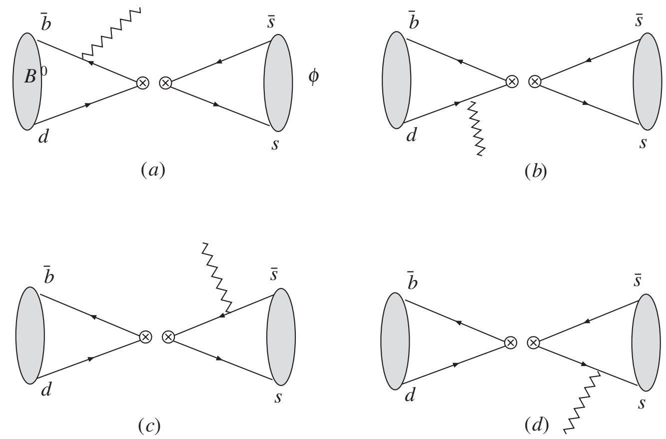

FIG. 1. Feynman diagrams for $B^0 \to \phi\gamma$ process in PQCD.

$$M^P = \frac{1}{\sqrt{3}}G_F e M_B^3 r f_V V_{\rm CKM}\int_0^1 dx_1 \int_0^\infty db_1 b_1 \phi_B(x_1, b_1) \times [C(t^a)K_0(b_1 A_a)e^{-S_B(t^a)} + C(t^b)K_0(b_1 B_a)e^{-S_B(t^b)}], \quad (8)$$

with

$$A_a^2 = (1 + x_1 - r^2)M_B^2, \qquad B_a^2 = x_1(1 - r^2)M_B^2, \qquad t^a = \max(A_a, 1/b_1), \qquad t^b = \max(B_a, 1/b_1). \quad (9)$$

$K_0(x)$ is the modified Bessel function which results from Fourier transformation of the quark propagator. $e^{-S_B(t)}$ is the Sudakov form factor, f_V is the vector meson decay constant, and $V_{\rm CKM}$ denotes Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. In the above calculations, the B meson is treated as a heavy-light system, whose wave function is defined as:

$$\Phi_B = \frac{i}{\sqrt{6}}(\not{P}_B + M_B)\gamma_5\phi_B(x_1, b_1), \quad (10)$$

and the expression of the distribution amplitude ϕ_B is shown in Refs. [12,13] with parameter $\omega_b = 0.4$ GeV, which is normalized as

$$\int_0^1 dx_1 \phi_B(x_1, b_1 = 0) = \frac{f_B}{2\sqrt{6}}, \quad (11)$$

where f_B is the decay constant of the B meson. Since we do not need the ϕ or J/ψ wave functions in the calculations of Feynman diagrams Fig. 1(a) and 1(b), we need not show them here.

For $B^0 \to \phi\gamma$ decay, only penguin operators can contribute, and the CKM matrix elements are $V_{\rm CKM} = V_{\rm tb}^* V_{\rm td}$. The combination of the Wilson coefficients in Eqs. (7) and (8) are:

$$C_{\phi\gamma}^P = C_3 + \tfrac{1}{3}C_4 + C_5 + \tfrac{1}{3}C_6 - \tfrac{1}{2}C_7 - \tfrac{1}{6}C_8 - \tfrac{1}{2}C_9 - \tfrac{1}{6}C_{10}. \quad (12)$$

The Wilson coefficient of dominant QCD penguin operator C_3 cancels much with $C_4/3$, and C_5 cancels with $C_6/3$. This is a result of color suppression in this decay, since at least three gluons are needed for a vector ϕ meson produced from the penguin diagram [17]. For $B \to J/\psi\gamma$, both tree and penguin operators give contribution, and corresponding conclusions of the Wilson coefficients are:

$$C^T_{\psi\gamma} = C_1 + \tfrac{1}{3}C_2,$$
$$C^P_{\psi\gamma} = C_3 + \tfrac{1}{3}C_4 + C_5 + \tfrac{1}{3}C_6 + C_7 + \tfrac{1}{3}C_8 + C_9 + \tfrac{1}{3}C_{10}. \quad (13)$$

Again, C_1 cancels much with $C_2/3$ which is also a result of the color suppressed tree contribution. The $V_{\rm CKM}$ in tree (penguin) operators is $V^*_{\rm cb}V_{\rm cd}$ ($V^*_{\rm tb}V_{\rm td}$).

With the amplitudes M^S and M^P defined in Eq. (6), the decay width of $B^0 \to V\gamma$ is given by

$$\Gamma = \frac{|M^S|^2 + |M^P|^2}{8\pi M_B}(1 - r^2). \quad (14)$$

In our numerical calculations, the input parameters are summarized in Table I, where λ, A, ρ, and η are CKM parameters in Wolfenstein parametrization [19], and $\bar{\rho} = \rho(1 - \frac{1}{2}\lambda^2)$, $\bar{\eta} = \eta(1 - \frac{1}{2}\lambda^2)$. Their values can be found in Review of Particle Properties [18].

At the leading order, the main uncertainty for decay branching ratios comes from the B meson wave function. But it is constrained by the measured exclusive hadronic decays, like $B \to K\pi$ [12], $B \to \pi\pi$ [13] with parameter ω_B from 0.3 GeV to 0.5 GeV. On the other hand, there should be large uncertainty since we work only at leading order in α_s for the hard part and also for the Wilson coefficients. The missing next-to-leading order correction is a very important uncertainty for rare decays. To estimate it, we consider the hard scale t at a range of

$$\max\left(0.75A_a, \frac{1}{b_1}\right) < t^a < \max\left(1.25A_a, \frac{1}{b_1}\right), \quad (15)$$

$$\max\left(0.75B_b, \frac{1}{b_1}\right) < t^b < \max\left(1.25B_b, \frac{1}{b_1}\right). \quad (16)$$

With the above major uncertainties from the B meson wave function parameter ω_B and the different scale t, respectively, we give out the branching ratio of $B^0 \to \phi\gamma$:

$$\text{BR}\,(B^0 \to \phi\gamma) = (2.7^{+0.3+1.2}_{-0.6-0.6}) \times 10^{-11}. \quad (17)$$

There are many other uncertainties in our calculation such as decay constants and CKM matrix elements. However, the uncertainty induced by the above factors is not more than 10% [20]. With such a small branching ratio (17), this decay is too rare to be measured at the running B factories or even in a future LHC-b experiment. Our result is still much smaller than the recent upper limit 8.5×10^{-7} from experiments [21]. If some new physics particles enhance this ratio through tree or loop effects [7], it may be measurable in near future experiments.

After similar calculations, we also give the branching ratio of $B^0 \to J/\psi\gamma$ decay:

$$\text{BR}\,(B^0 \to J/\psi\gamma) = (4.5^{+0.6+0.7}_{-0.5-0.6}) \times 10^{-7}. \quad (18)$$

Because this process is tree diagram dominated, it is much larger than that of $B^0 \to \phi\gamma$. This result is still not big enough to be measured at the B factories but it is already around the corner of B factories capability. Currently, there is only an upper limit, which is 1.6×10^{-6} at 90% confidence level [22].

From our calculation, we find that most of the contribution comes from Fig. 1(b), which indicates that photon emission from the light quark of the B meson is easier than that from the heavy quark. The reason is that the heavy quark is more difficult to become off shell than the light quark, while a nearly on-shell quark rarely emits photons.

Within QCD factorization approach, the branching ratios of these two decays have been calculated in Refs. [7,8]. The results are given as:

$$\text{BR}\,(B^0 \to J/\psi\gamma) = 7.65 \times 10^{-9}; \quad (19)$$

$$\text{BR}\,(B^0 \to \phi\gamma) = 3.6 \times 10^{-12}. \quad (20)$$

Compared with their results, we find that our results are 1 order of magnitude larger. In QCD factorization approach, some next-to-leading order contribution has been added in

TABLE I. Summary of input parameters [18].

CKM parameters and QCD constant					
λ	A	$\bar{\rho}$	$\bar{\eta}$	$\Lambda^{(f=4)}_{\overline{\rm MS}}$	τ_{B^0}
0.2196	0.819	0.20	0.33	250 MeV	1.54 ps
Meson decay constants					
f_B	f_{B_s}	f_ϕ	$f_{J/\psi}$		
216 MeV	236 MeV	254 MeV	405 ± 14 MeV		
Meson masses					
M_W	M_B	M_ϕ	$M_{J/\psi}$		
80.41 GeV	5.28 GeV	1.02 GeV	3.10 GeV		

addition to the naive factorization contribution. It shows that these presumably order α_s corrections change the branching ratios by 1 order of magnitude. In fact these two decays are a nonfactorizable contribution dominant in the naive factorization approach. Usually QCD factorization approach calculation for this kind of decay (such as $B^0 \to \bar{D}^0\pi^0$ [23]) is not stable. They receive dominant contribution from nonfactorizable diagrams, sometimes with an end point singularity in the collinear factorization. In fact, our numerical results are also much larger than the naive factorization approach, which is mainly due to a smaller energy scale ($\sqrt{\Lambda_{\rm QCD} m_b}$ other than m_b) used in the calculation of the Wilson coefficients. This uncertainty comes from part of the next-to-leading order effect in PQCD approach.

The $B_s \to J/\psi\gamma$ decay is the same as the $B^0 \to J/\psi\gamma$ decay within SU(3) symmetry. It is also a pure annihilation type decay. The decay formulas are exactly the same except replacing the corresponding CKM factor $V_{\rm cd}$, $V_{\rm td}$ by $V_{\rm cs}$, $V_{\rm ts}$, respectively. Taking into account the SU(3) breaking effect, we used $\omega_{B_s} = 0.5$ GeV [24], $f_{B_s} =$ 236 MeV, we can get

$$\mathrm{BR}\,(B_s \to J/\psi\gamma) \simeq 5 \times 10^{-6}, \tag{21}$$

which should be easy to be measured in the future LHC-b experiment.

In summary, we calculate the branching ratios of pure annihilation type radiative decays $B^0 \to \phi\gamma$ and $B^0 \to J/\psi\gamma$ within the standard model in PQCD approach. We find the branching ratio of $B^0 \to \phi\gamma$ is at the order of 10^{-11}. This small branching cannot be detected in the running B factories of *BABAR* and KEK, unless some new physics enhance these results sharply. In the future, this decay may be measured in a LHC-b experiment or other high luminosity experiments. For $B^0 \to J/\psi\gamma$ decay, the branching ratio is about 4×10^{-7}, which is just close to the B factory experiment capability of measurement. The experimental measurements of these decays would be very useful for understanding various QCD methods, like QCD factorization and PQCD approaches.

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