$B_s - \bar{B}_s$ mixing, B decays and R-parity violating supersymmetry

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We discuss the implications of the recent measurement of the $B_s - \bar{B}_s$ oscillation frequency ΔM_s on the parameter space of R-parity violating supersymmetry. For completeness, we also discuss the bounds coming from leptonic, semileptonic, and nonleptonic B decay modes, and point out some possibly interesting channels at the Large Hadron Collider.

I. INTRODUCTION

The $B_s - \bar{B}_s$ mass difference, recently measured by the D0 [1] and the CDF [2] collaborations, is given by

$$17 \text{ ps}^{-1} < \Delta M_s < 21 \text{ ps}^{-1}(\text{D0}),$$
$$\Delta M_s = (17.31^{+0.33}_{-0.18} \pm 0.07) \text{ ps}^{-1}(\text{CDF}). \tag{1}$$

This result is consistent with the standard model (SM) prediction, which is estimated as 21.3 ± 2.6 ps^{-1} by the UTfit group [3] and as $20.9^{+4.5}_{-4.2}$ ps^{-1} by the CKMfitter group [4]. The implications of ΔM_s measurements on the parameter space of new physics (NP) have already been considered [5–8]. However, given the hadronic uncertainties in the SM prediction, along with additional uncertainties when NP is included, the present measurement of ΔM_s does not provide a really strong constraint on NP [5,6].

There have been some attempts to put bounds on the parameter space of R-parity conserving supersymmetry (RPC SUSY) from the ΔM_s data [9]. In this paper, we would like to put bounds on the R-parity violating (RPV) SUSY couplings. We will use not only the ΔM_s data but also the data on the leptonic, semileptonic, and nonleptonic branching ratios (BR) and CP asymmetries of B and B_s mesons that are affected by the particular RPV couplings. Such a work on B^0 mesons may be found in [10] and this is an extension of that work to the B_s sector. For the relevant formulas, we refer the reader to [10].

It has been shown in [11] that RPV couplings involving sleptons (λ and λ' type) generate nonzero neutrino mass and one can put stringent constraints on them from the Wilkinson microwave anisotropy probe data [12]. The exact bounds depend on the relation of the mass matrices with the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix. Anyway, such a study forces us to consider only those couplings which can still be relatively large, and in this paper we derive better bounds on some of these product couplings than those coming from [11].

A major motivation for this study is the B_s physics that is going to be probed at LHC-b, and even at CMS or ATLAS during the low-luminosity run of the Large Hadron Collider (LHC). The leptonic and semileptonic decays are clean and any enhancement over the SM expectations

will signify some NP. Also, the phase χ in $B_s - \bar{B}_s$ mixing comes in the subleading order of the CKM matrix and is expected to be very small (~ 0.03), so any CP-violating effect in B_s decays not involving an up quark will be interesting.

Effects of RPV SUSY on B physics have been discussed extensively in the literature [13–17]. Constraints coming from neutral meson mixing have been discussed in [10,18,19]. However, in these papers, the B_s sector could not be dealt with, since only the lower bound on ΔM_s existed then. This paper, in a sense, is the completion of the series. All the computational details that have been taken into account in [10,19] (e.g., the NLO QCD corrections for short-distance effects, inclusion of both SM and RPV) are incorporated in this paper.

The paper is arranged as follows. In Sec. II we outline the relevant formulas necessary for the analysis, and give the numerical inputs in Sec. III. The analysis on $B_s - \bar{B}_s$ box and the decay processes is in Sec. IV, and we conclude and summarize in Sec. V.

II. BASIC INPUTS

A. $B_s - \bar{B}_s$ mixing

The off-diagonal element M_{12} in the 2×2 effective Hamiltonian causes the $B_s - \bar{B}_s$ mixing. The mass difference between the two mass eigenstates ΔM_s is given by (following the convention of [20])

$$\Delta M_s = 2|M_{12}|, \tag{2}$$

with the approximation $|M_{12}| \gg |\Gamma_{12}|$ (this seems a good approximation even for the B_s system). If we have n number of NP amplitudes with weak phases θ_n, one can write the mass difference between mass eigenstates as

$$\Delta M_s = 2 \Bigg[|M_{12}^{\text{SM}}|^2 + \sum_i |M_{12}^i|^2$$
$$+ 2|M_{12}^{\text{SM}}| \sum_i |M_{12}^i| \cos 2(\theta_{\text{SM}} - \theta_i)$$
$$+ 2 \sum_i \sum_{j>i} |M_{12}^j||M_{12}^i| \cos 2(\theta_j - \theta_i) \Bigg]^{1/2}. \tag{3}$$

For $B_s - \bar{B}_s$ system, the short-distance SM amplitude is

$$M_{12}^{\text{SM}} \equiv \frac{\langle \bar{B}_s | H_{\text{eff}} | B_s \rangle}{2 m_B}$$

$$= \frac{G_F^2}{6\pi^2} (V_{ts} V_{tb}^*)^2 \eta_{B_s} m_{B_s} f_{B_s}^2 B_{B_s} m_W^2 S_0(x_t). \quad (4)$$

where generically $x_j = m_j^2/m_W^2$, f_{B_s} is the B_s meson decay constant, and η_{B_s} and B_{B_s} parametrize the short- and the long-distance QCD corrections, respectively. The function S_0 is given by

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3}. \quad (5)$$

If the NP amplitude has a nonzero phase, then there will be an effective phase in $B_s - \bar{B}_s$ mixing amplitude, whose presence may be tested in the hadronic B factories. In the presence of NP, the general $\Delta F = 2$ effective Hamiltonian can be written as

$$\mathcal{H}_{\text{eff}}^{\Delta F=2} = \sum_{i=1}^{5} c_i(\mu) O_i(\mu) + \sum_{i=1}^{3} \tilde{c}_i(\mu) \tilde{O}_i(\mu) + \text{H.c.} \quad (6)$$

where μ is the regularization scale. The effective operators O_i and $\tilde{O}_i$ are given in [10]. The Wilson coefficients c_i at $q^2 = m_W^2$ include NP effects, coming from couplings and internal propagators. However, for most of the NP models, and certainly for the case we are discussing here, all NP particles are heavier than m_W and hence the running of the coefficients between m_W and $\mu = \mathcal{O}(m_b)$ are controlled by the SM Hamiltonian alone. In other words, NP determines only the boundary conditions of the renormalization group (RG) equations. For the evolution of these coefficients down to the low-energy scale, we follow Ref. [21], which uses, for $B^0 - \bar{B}_0$ mixing, $\mu = m_b = 4.6$ GeV. The expectation values of these operators between $\bar{B}_s$ and B_s at the scale μ are analogous to those as given in [10]. The $B_{B_s}(\mu)$ parameters have been taken from [22].

B. *R*-parity violation

It is well known that in order to avoid rapid proton decay one cannot have both lepton number and baryon number violating RPV couplings, and we shall work with a lepton number violating model. This leads to both slepton (charged and neutral) and squark-mediated decays, and new amplitudes for $B_s - \bar{B}_s$ mixing, where particles flowing inside the box can be (i) charged sleptons and up-type quarks, (ii) sneutrino and down-type quarks, (iii) squarks and leptons. One or both of the scalar particles inside the box can be replaced by W bosons, charged Higgs bosons and Goldstone bosons (in a nonunitary gauge) (see Fig. 1). We follow the usual practice of avoiding the so-called ''pure SUSY'' contributions to the box amplitudes, i.e., those coming from charginos, neutralinos or gluinos inside the loop. Not only the strongly interacting superparticles
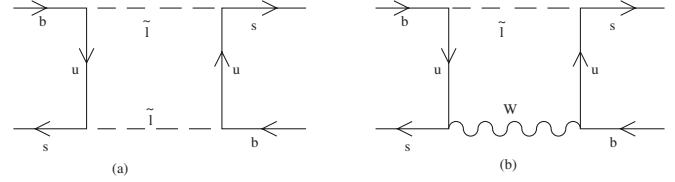

FIG. 1. *R*-parity violating contributions to $B_s - \bar{B}_s$ mixing. Figure (a) corresponds to L4, while figure (b) to L2 amplitudes (see text for their meanings). For L4, there are similar diagrams with squarks and leptons (both charged and neutral), as well as diagrams with left-chiral quarks as external legs and quarks and sneutrinos flowing in the box. For L2, there are diagrams where the W is replaced by the charged Higgs or the charged Goldstone. The internal slepton can be of any generation, and so can be the internal charge $+2/3$ quarks, generically depicted as u.

are expected to be heavier than the electroweak ones (and hence the contribution being suppressed), but also one can choose SUSY models where these contributions become negligible (e.g., alignment in the squark sector, or Higgsino-dominated lighter chargino, to kill off the respective boxes.) Since the current lower bound on the slepton mass is generally weaker than that on squark mass by a factor 2–3, the slepton-mediated boxes have greater chance to be numerically significant.

We start with the superpotential

$$\mathcal{W}_{\lambda'} = \lambda'_{ijk} L_i Q_j D_k^c, \quad (7)$$

where i, j, $k = 1$, 2, 3 are quark and lepton generation indices; L and Q are the $SU(2)$-doublet lepton and quark superfields and D^c is the $SU(2)$-singlet down-type quark superfield, respectively. Written in terms of component fields, this superpotential generates six terms, plus their Hermitian conjugates:

$$\mathcal{L}_{\text{LQD}} = \lambda'_{ijk} [\tilde{\nu}_L^i \bar{d}_R^k d_L^j + \tilde{d}_L^j \bar{d}_R^k \nu_L^i + (\tilde{d}_R^k)^* \bar{\nu}_L^i d_L^j - \tilde{e}_L^i \bar{d}_R^k u_L^j$$
$$- \tilde{u}_L^j \bar{d}_R^k e_L^i - (\tilde{d}_R^k)^* \bar{e}_L^i u_L^j] + \text{H.c.} \quad (8)$$

With such a term, one can have two different kind of boxes, shown in Fig. 1, that contribute to $B_s - \bar{B}_s$ mixing: first, the one where one has two sfermions flowing inside the loop, along with two SM fermions [23], and secondly, the one where one slepton, one W (or charged Higgs or Goldstone) and two up-type quarks complete the loop [18]. It is obvious that the first amplitude is proportional to the product of four λ' type couplings, and the second to the product of two λ' type couplings times G_F. We call them L4 and L2 boxes, respectively, for brevity, where L is a shorthand for λ'.

We will constrain only products of two λ'-type couplings at a time, and assume a hierarchical structure, i.e., only one product is, for all practical purpose, simultaneously nonzero (but can have a nontrivial phase). This may not be physically the most appealing scenario but

keeps the discussion free from unnecessary complications. This product can in general be complex. Any product is bounded by the product of the bounds on the individual terms, which we call the direct product bound (DPB). Interesting bounds are those which are numerically smaller, and hence stronger, than the corresponding DPBs. The DPBs are mostly taken from [11], and we highlight those products which are more tightly constrained than their respective DPBs. The detailed formulas of the box amplitudes may be found in [10].

C. Semileptonic and leptonic decay channels

The RPV couplings that may contribute to $B_s - \bar{B}_s$ mixing should also affect various B decay modes. Let us first consider the leptonic and semileptonic modes.

The expected BRs of leptonic flavor conserving $\Delta B = 1$ processes within SM are much below the experimental numbers (except $B \to K^{(*)}\ell^+\ell^-$), so one can safely ignore the SM effects as well as the R-conserving SUSY effects to put bounds on the RPV couplings. (The final state leptons must be the same if the product coupling contributes to $B_s - \bar{B}_s$ mixing.) The leptonic decay modes are theoretically clean and free from any hadronic uncertainties. The semileptonic modes have the usual form-factor uncertainties.

To construct four-fermion operators from λ' type couplings that mediate such semileptonic and leptonic B decays, one needs to integrate out the squark or slepton field. The product RPV coupling may in general be complex. However, since all the leptonic decays are one-amplitude processes (only RPV, for all practical purposes) there is no scope for CP-violation; one can only look at nonzero BRs. By the same argument, we can take all couplings to be real without any loss of generality.

The effective Hamiltonian is of the form [24]

$$
\begin{aligned}
\mathcal{H}_{\text{RPV}} &= \tfrac{1}{2} B_{jklm} [\bar{\ell}_j \gamma^\mu P_L \ell_l][\bar{u}_m \gamma_\mu P_R u_k] \\
&+ \tfrac{1}{2} B_{jklm} [\bar{\nu}_j \gamma^\mu P_L \nu_l][\bar{d}_m \gamma_\mu P_R d_k] \\
&- \tfrac{1}{2} C_{jklm} [\bar{\nu}_j \gamma^\mu P_L \nu_l][\bar{d}_k \gamma_\mu P_L d_m] + \text{H.c.}, \quad (9)
\end{aligned}
$$

where

$$
B_{jklm} = \sum_{i=1}^{3} \frac{\lambda'^*_{jik} \lambda'_{lim}}{m^2_{\tilde{L}_i}}, \qquad C_{jklm} = \sum_{i=1}^{3} \frac{\lambda'^*_{jki} \lambda'_{lmi}}{m^2_{\tilde{d}_{R_i}}}. \quad (10)
$$

We take any one to be nonzero at a time. $m_{\tilde{L}_i}$ is the left-chiral up/down squark mass, taken to be degenerate.

The entire leptonic B_s decay amplitude is solely due to new physics, as far as detectability is concerned. In RPV models, squark-mediated $\lambda'\lambda'$ type interactions are responsible for such purely leptonic decays. As already pointed out, the bounds are robust in a sense that they are free from any theoretical uncertainties (except for the decay constants of B_s), and do not depend on the phase of the RPV couplings. For the B mesons, no such leptonic mode has yet been observed. The corresponding upper limits on the BRs are of the order of 10^{-7} for $\ell = \mu$ and 10^{-5} for $\ell = e$ modes. With 300 GeV squarks, from the bounds that one obtains here, a BR at most of the order of 10^{-8} can be expected. Thus, we do not expect to see such leptonic channels before the next-generation hadronic machine or super e^+e^- B factories. However, a number of semileptonic modes $b \to s\ell^+\ell^-$ have been observed and the BRs are at the SM ballpark.

The decay width of $B_s \to \ell^-\ell^+$ is given in [24]. For the semileptonic decays, we use

$$
\langle K(p_2)|\bar{s}\gamma^\mu b|B(p_1)\rangle = P^\mu F_1(q^2) + q^\mu \frac{m_B^2 - m_K^2}{q^2}(F_0(q^2) - F_1(q^2)),
$$

$$
\langle \phi(p_2, \epsilon)|V_\mu \mp A_\mu|B_s(p_1)\rangle = \frac{1}{m_{B_s} + m_\phi}[-iV(q^2)\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}P^\alpha q^\beta \pm A_0(q^2)(P \cdot q)\epsilon^*_\mu \pm A_+(q^2)(\epsilon^* \cdot p_1)P_\mu
$$

$$
\pm A_-(q^2)(\epsilon^* \cdot p_1)q_\mu] \qquad (11)
$$

where m_{B_s} and m_ϕ are the meson masses, $p_1(p_2)$ is the momentum of the initial (final) meson, ϵ is the polarization vector of the vector meson ϕ, $P = p_1 + p_2$, $q = p_1 - p_2$, $V_\mu = \bar{q}_2 \gamma_\mu q_1$, $A_\mu = \bar{q}_2 \gamma_\mu \gamma_5 q_1$, $V, A_{0,\pm}$ and $F_{(1/0)}(q^2)$ are the form factors. The values of these form factors are taken from [25,26].

The RPV matrix elements for the decay mode $B_{(s/d)} \to (\phi/K)l^+l^-$ (where $l = e, \mu$):

$$
\begin{aligned}
&\mathcal{M}(B_{(s/d)} \to (\phi/K)\ell_i^+ \ell_i^-) \\
&= \tfrac{1}{2} B_{i3i2} (\bar{\ell}^+ \gamma^\mu P_L \ell^-)(\langle(\phi/K)|\overline{(s/d)}\gamma_\mu P_R b|B_{(s/d)}\rangle) \quad (12)
\end{aligned}
$$

D. Nonleptonic decay channels

There are four types of slepton-mediated nonleptonic decays that can proceed through the relevant RPV couplings. Among them, $b \to c\bar{c}s$ and $b \to s\bar{s}s$ rates are bound to be undetectably small if these couplings are to be compatible with the neutrino mass bounds [11]. We, therefore, focus on the $b \to u\bar{u}s$ and $b \to d\bar{d}s$ type transitions. They mediate the channels $B \to \pi K$, $B \to \rho K$. The corresponding B_s decay channels do not have data at comparable level. Note that both BRs and CP asymmetries for these channels have been measured [27,28]. While the data does not uniquely point to NP, the trend is encouraging.

We will use the conventional factorization (CF) model [26] to analyze the effect of RPV SUSY on the πK and ρK channels. While the validity of such a simple approach may be questioned, it is not wildly off the truth, at least for these channels. The effective Hamiltonian reads

$$H_{\text{RPV}} = d_{jkn}^R (\bar{d}_n \gamma^\mu P_L d_j)_8 (\bar{d}_k \gamma_\mu P_R b)_8 + d_{jkn}^L (\bar{d}_n \gamma^\mu P_L b)_8$$
$$\times (\bar{d}_k \gamma_\mu P_R d_j)_8 + u_{jnk}^R (\bar{u}_n \gamma^\mu P_L u_j)_8 (\bar{d}_k \gamma_\mu P_R b)_8 \tag{13}$$

where

$$d_{jkn}^R = \sum_i \frac{\lambda'_{ijk} \lambda'^*_{in3}}{2 m_{\tilde{\nu}_{Li}}^2}, \qquad d_{jkn}^L = \sum_i \frac{\lambda'_{i3k} \lambda'^*_{inj}}{2 m_{\tilde{\nu}_{Li}}^2}, \tag{14}$$
$$u_{jnk}^R = \sum_i \frac{\lambda'_{ijk} \lambda'^*_{in3}}{2 m_{\tilde{e}_{Li}}^2}.$$

Following the standard practice we shall assume that the RPV couplings are hierarchical i.e., only one combination of the couplings is numerically significant. Let us assume, to start with, only d_{112}^L and u_{112}^R to be nonzero. The QCD corrections are easy to implement: the short-distance QCD corrections enhance the $(S - P) \times (S + P)$ RPV operator by approximately a factor of 2 while running from the slepton mass scale (assumed to be at 100 GeV) to m_b [29].

The RPV amplitude for $B \to \pi K$ is given by

$$M_{\pi^0 K^-} = \frac{1}{\sqrt{2}} \left[u_{112}^R \left(-R_1(A_{\pi K}^{(1)}) + A_{\pi K}^{(2)} \frac{1}{N_c} \right) \right.$$
$$\left. + d_{112}^L \frac{1}{N_c} (A_{\pi K}^{(2)}) \right] \tag{15}$$

$$M_{\pi^0 \bar{K}^0} = \frac{1}{\sqrt{2}} \left[d_{112}^L \left(-R_1(A_{\pi K}^{(1)}) + A_{\pi K}^{(2)} \frac{1}{N_c} \right) \right.$$
$$\left. + u_{112}^R \frac{1}{N_c} (A_{\pi K}^{(2)}) \right] \tag{16}$$

$$M_{\pi^- \bar{K}^0} = d_{112}^L (R_1 A_{\pi K}^{(1)}) \tag{17}$$

$$M_{\pi^+ K^-} = u_{112}^R (-R_1 A_{\pi K}^{(1)}) \tag{18}$$

The expressions for the $B \to \rho K$ amplitudes will be similar to those shown above, with R_1 replaced by R_2 and $A_{\pi K}^{(i)}$ replaced by $A_{\rho K}^{(i)}$. We use the shorthand

$$R_1 = 2 \frac{m_{K^0}^2}{(m_u + m_s)(m_b - m_u)},$$
$$R_2 = 2 \frac{m_\rho^2}{(m_u + m_s)(m_b - m_u)}, \tag{19}$$

and

$$A_{\pi K}^{(1)} = f_K F_0^{B \to \pi}(m_K^2)(m_B^2 - m_\pi^2),$$
$$A_{\pi K}^{(2)} = f_\pi F_0^{B \to K}(m_\pi^2)(m_B^2 - m_K^2),$$
$$A_{\rho K}^{(1)} = 2 f_K m_\rho A_0^{B \to \rho}(m_K^2)(\epsilon_\rho . p_K),$$
$$A_{\rho K}^{(2)} = 2 f_\rho m_\rho F_1^{B \to K}(m_\rho^2)(\epsilon_\rho . p_K). \tag{20}$$

III. NUMERICAL INPUTS

The major sources of the numerical inputs are: (i) the Heavy Flavor Averaging Group (HFAG) website [30] for the latest (summer 2006) updates on B physics; (ii) Particle Data Group 2006 edition [31]; and (iii) the inputs used in the CKMfitter package [4]. The quark masses and Wilson coefficients have been taken from [21,32]. We use the following numbers.

The masses for all the mesons B^0, B^-, π, ρ, and K are the corresponding central values as given in [31]. The meson decay constants (in GeV) are:

$$f_\pi = 0.133, \qquad f_K = 0.158, \qquad f_\rho = 0.210. \tag{21}$$

The transition form factors [33] at $q^2 = 0$ are given by

$$F_0(B \to K) = 0.38; \qquad F_0(B \to \pi) = 0.33;$$
$$A_0(B \to \rho) = 0.28, \tag{22}$$

and $F_0(0) = F_1(0)$.

The quark masses have been evaluated in the $\overline{MS}$ scheme. The pole mass for the top quark is about 5 GeV higher and the mass for the bottom quark is 4.6 GeV. The CKM elements are shown in Table I.

The leptonic and semileptonic BRs for the B meson, which are of interest to us, are as follows [1,30,31]:

TABLE I. Input parameters used for the numerical analysis, from [2,4,21,34].

Quantity	Value
ΔM_s	$(17.31^{+0.33}_{-0.18} \pm 0.07)$ ps^{-1}
γ	$50° - 72°$
η_{B_s}	0.55 ± 0.01
$m_t^{\overline{MS}}(m_t^{\overline{MS}})$	166 GeV
$m_b^{\overline{MS}}(m_b^{\overline{MS}})$	4.23 GeV
$m_b(m_b)$	4.6 GeV
$m_c(m_b)$	1.3 GeV
$m_d(m_b)$	5.4 MeV
$m_s(2 \text{ GeV})$	125 MeV
$f_B\sqrt{B_{B_s}}|_{\text{JLQCD}}$	$(0.245 \pm 0.021^{+0.003}_{-0.002})$ GeV
$|V_{us}| \times 10^1$	$2.272^{+0.01}_{-0.01}$
$|V_{cs}| \times 10^1$	9.73
$|V_{ts}| \times 10^3$	$41.61^{+0.12}_{-0.78}$
$|V_{ub}| \times 10^3$	4.4 ± 0.3
$|V_{cb}| \times 10^3$	42.0 ± 0.7
$|V_{tb}| \times 10^1$	9.99

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$\text{Br}(B \to K\ell^+\ell^-) < (0.57 \pm 0.07) \times 10^{-6}(\ell = e/\mu);$

$\text{Br}(B \to Ke^+e^-) = (0.55 \pm 0.09) \times 10^{-7};$

$\text{Br}(B \to K\mu^+\mu^-) = (0.61 \pm 0.08) \times 10^{-7};$

$\text{Br}(B_s \to \phi\mu^+\mu^-) < 4.1 \times 10^{-6};$

$\text{Br}(B_s \to \mu^+\mu^-) < 1 \times 10^{-7};$

$\text{Br}(B_s \to e^+e^-) < 5.4 \times 10^{-5};$

For the πK and ρK modes, the data reads [30,31,35–37]:

$$\text{Br}(B^0 \to \pi^- K^+) = (18.9 \pm 0.7) \times 10^{-6}$$

$$\text{Br}(B^0 \to \pi^0 K^0) = (11.5 \pm 1.0) \times 10^{-6}$$

$$\text{Br}(B^+ \to \pi^+ K^0) = (24.1 \pm 1.3) \times 10^{-6}$$

$$\text{Br}(B^+ \to \pi^0 K^+) = (12.1 \pm 0.8) \times 10^{-6} \qquad (23)$$

$$\text{Br}(B^0 \to \rho^- K^+) = (9.9^{+1.6}_{-1.5}) \times 10^{-6}$$

$$\text{Br}(B^0 \to \rho^0 K^0) = (5.1 \pm 1.6) \times 10^{-6}$$

$$\text{Br}(B^+ \to \rho^0 K^+) = (4.23^{+0.56}_{-0.57}) \times 10^{-6}$$

$$A^{\text{dir}}_{CP}(B^0 \to \pi^\pm K^\mp) = 0.115 \pm 0.018.$$

To evaluate the QCD corrections, we take $\alpha_s(m_Z^2) = 0.1187$ [31], and take the SUSY scale $M_S = 500$ GeV. The precise value of this scale is not important, however, and we can take it to be at the squark mass scale (300 GeV) without affecting the final results. The exact evolution matrix can be found in [21,32]; for our purpose, it is sufficient to note that for the B_s system, the operator $\tilde{O}_1$ is multiplicatively renormalized by a factor 0.820 at the scale $\mu = 2$ GeV, and the operator O_4 at m_W changes to $(2.83 O_4 + 0.08 O_5)$. We again stress that theoretically the procedure is questionable for boxes with light quarks flowing in the loop. However, the numbers that we obtain are fairly robust and one can very well drop the NLO corrections altogether, if necessary, without compromising the results. Since the O_5 admixture is small, one can take the central values for these parameters without introducing too much error. The relevant B-parameters for B_s system are taken from [22]

We take all sleptons to be degenerate at 100 GeV, and all squarks at 300 GeV. We also take $\tan\beta(\equiv v_2/v_1) = 5$ (very low values are excluded by LEP, and the numbers are not sensitive to the precise choice of $\tan\beta$), and the charged Higgs boson mass as 200 GeV (lower values are disfavored from $b \to s\gamma$).

IV. ANALYSIS

A. $B_s - \bar{B}_s$ mixing

For the B_s system, the bounds are summarized in Table II. When the product coupling is complex, we show only the real part, since the bound on the imaginary part is almost equal to this. The reason is easy to under-

TABLE II. Bounds on $\lambda'\lambda'$ combinations from $B_s - \bar{B}_s$ mixing. The table displays the magnitudes only, and not the signs.

$\lambda'\lambda'$ combination	Only real	Complex, real part
$(i32)(i33)$	1.01×10^{-2}	1.0×10^{-2}
$(i22)(i23)$	8.2×10^{-3}	8.0×10^{-3}
$(i12)(i13)$	1.2×10^{-2}	3.7×10^{-2}
$(i22)(i33)$	1.8×10^{-1}	1.7×10^{-1}
$(i32)(i23)$	3.2×10^{-4}	3.0×10^{-4}
$(i22)(i13)$	3.47×10^{-2}	3.45×10^{-2}
$(i12)(i23)$	5.16×10^{-2}	7.56×10^{-2}
$(i32)(i13)$	1.4×10^{-3}	1.3×10^{-3}
$(i12)(i33)$	9.0×10^{-1}	9.0×10^{-1}
$(i23)(i33)$	1.66×10^{-2}	5.1×10^{-2}
$(i22)(i32)$	1.66×10^{-2}	5.1×10^{-2}
$(i21)(i31)$	1.66×10^{-2}	5.1×10^{-2}

stand: the bounds are obtained when the RPV coupling has a phase opposite to that of the SM coupling, so that the interference is destructive. At the limit where the RPV coupling determines the mixing amplitude, the phase is irrelevant. The effect of this destructive interference is clear in Fig. 2(a) and 2(b). For a more detailed explanation, we refer the reader to [10].

The relative magnitudes of the bounds are also easy to understand. For example let us consider the bounds on $\lambda'_{i32}\lambda'_{i23}$ vis-a-vis $\lambda'_{i22}\lambda'_{i33}$. The relevant box diagrams have the same particle content; but the first one is proportional to $V_{ts}V_{cb}$ [$\sim \mathcal{O}(\lambda^4)$], and the second one to $V_{tb}V_{cs}$ (~ 1). The relative suppression in λ enhances the limit on the RPV coupling.

Though most of the bounds are of the same order in magnitude, these are, theoretically, an improvement over those obtained earlier [18,19,23]. We have taken into account all possible amplitudes (and the interference patterns play a nontrivial role), including the SM one, but have systematically neglected the pure supersymmetric boxes coming from gaugino exchange. The reason is that those boxes decouple in the heavy squark limit, and one can always take an RPV model embedded in a minimal supersymmetric theory where such flavor-changing neutral-current (FCNC) processes are somehow forbidden. It was shown in [19] that the bounds are fairly robust even if one takes into account such SUSY contributions. Furthermore, the QCD corrections are implemented up to NLO.

B. Nonleptonic decay channels

As we have mentioned, the $B \to \pi K$ and $B \to \rho K$ numbers are encouraging for NP enthusiasts. However, the hadronic uncertainties are significant. Also, one must have a nonzero strong phase between the SM tree and the SM penguin amplitudes to explain the direct CP asymmetry data on $B \to \pi^+ K^-$. Thus, it is of importance to explore the data in conjunction with $B_s - \bar{B}_s$ mixing. In our analysis, we use the CF model, as mentioned, and vary
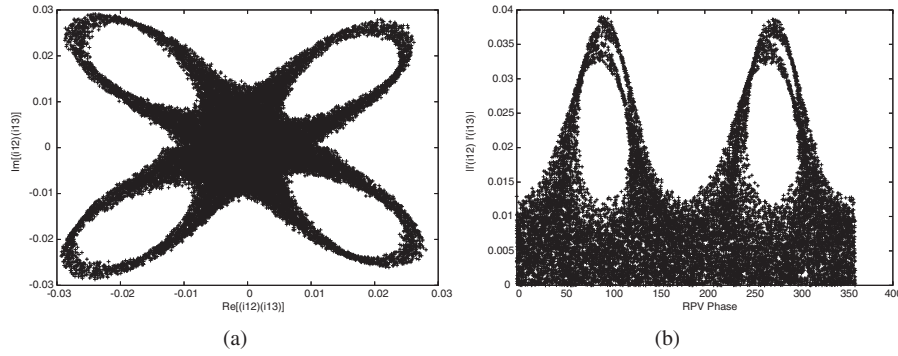
FIG. 2.   (a) Allowed parameter space for $\lambda'_{i12}\lambda'_{i13}$ (b) The allowed parameter space of RPV phase for $|\lambda'_{i12}\lambda'_{i13}|$, which can gives *CP* asymmetries in the $B_s \rightarrow K\pi$ decay.

the strong phase difference between the SM tree and the RPV amplitudes from 0 to 2π. To take into account the hadronic uncertainties, we (i) vary the SM amplitude from its CF value by 20%, and (ii) vary the RPV weak phase in the range $[0:2\pi]$. The allowed parameter space of RPV couplings, in the magnitude-phase plane, comes out with separate island-like structures. For example, $\lambda'_{i31}\lambda'_{i21}$ lies between 2.05×10^{-3} to 2.45×10^{-3} and the corresponding phase lies between 104° to 120°, whereas $\lambda'_{i13}\lambda'_{i12}$ lies between 2.31×10^{-3} to 2.82×10^{-3} and the corresponding phase lies between 85° to 105°. We quoted the highest value in Table III. These bounds are much stronger then that coming from $B_s - \bar{B}_s$ mixing.

We have not considered the $B \rightarrow (\eta, \eta')K$ modes. Though they are mediated by $b \rightarrow s\bar{q}q$ $(q = u, d)$ transitions, the BRs for those modes cannot be explained simultaneously by such a simple new physics structure [38].

Similarly, the mode $B \rightarrow \phi K^*$ has not been considered, since the longitudinal polarization anomaly for this mode cannot be explained without a contribution from tensor current but RPV SUSY do not provide for such tensor current structures, at least at the tree-level [39]. However, we note that unless a product coupling is at least of the order of 10^{-3}, the RPV contribution is unlikely to affect the SM amplitude.

C. Leptonic and semileptonic decay channels

The detail procedure of the leptonic and semileptonic decay channels are given in [24]. While their bounds were

TABLE III.   Some of the possible nonleptonic transitions mediated by the RPV couplings discussed in the paper.

$\lambda'\lambda'$ combination	Quark level	Meson level	Bound
$(i21)(i31)$	$b \rightarrow d\bar{d}s$	$B_d \rightarrow \pi^0\bar{K}^0$ $B^- \rightarrow \pi^0 K^-$	2.45×10^{-3}
$(i12)(i13)$	$b \rightarrow d\bar{d}s$ $b \rightarrow u\bar{u}s$	$B^- \rightarrow \pi^0 K^-$ $B_d \rightarrow \pi^+ K^-$	2.82×10^{-3}
$(i22)(i32)$	$b \rightarrow s\bar{s}s$	$B_s \rightarrow \phi K_s$	2.33×10^{-3}

TABLE IV.   Some of the possible leptonic and semileptonic transitions mediated by the RPV coupling relevant with $B_s - \bar{B}_s$ mixing are given here. It has shown that in many cases the bounds coming from this decays are much better then coming from mixing.

$\lambda'\lambda'$ combination	Quark level	Meson level	Bound
$(2i2)(2i3)$	$b \rightarrow s\mu^+\mu^-$	$B_s \rightarrow \mu^+\mu^-$	7.7×10^{-3}
$(2i2)(2i3)$	$b \rightarrow s\mu^+\mu^-$	$B_s \rightarrow \phi\mu^+\mu^-$	4.9×10^{-3}
$(2i2)(2i3)$	$b \rightarrow s\mu^+\mu^-$	$B \rightarrow K\mu^+\mu^-$	6.6×10^{-4}
$(1i2)(1i3)$	$b \rightarrow se^+e^-$	$B \rightarrow Ke^+e^-$	7.7×10^{-4}

TABLE V.   Bounds on real $\lambda'\lambda'$ combinations from $B_s - \bar{B}_s$ mixing and correlated decay channels. The DPBs, displayed in the last column, occur from neutrino constraints with no mixing scenario [11]. The product marked with a dagger is bounded from tree-level $B_s - \bar{B}_s$ mixing ($\sim \mathcal{O}(10^{-6})$).

$\lambda'\lambda'$ combination	Related process	Current bound	Previous bound
$(112)(113)$	$B_s \rightarrow Ke^+e^-$	7.74×10^{-4}	$1.52 \times 10^{-1}{}_\nu$
$(122)(123)$	$B_s \rightarrow Ke^+e^-$	7.74×10^{-4}	$9.7 \times 10^{-6}{}_\nu$
$(132)(133)$	$B_s \rightarrow Ke^+e^-$	7.74×10^{-4}	$6.0 \times 10^{-5}{}_\nu$
$(212)(213)$	$B_s \rightarrow K\mu^+\mu^-$	6.57×10^{-4}	$1.52 \times 10^{-1}{}_\nu$
$(222)(223)$	$B_s \rightarrow K\mu^+\mu^-$	7.74×10^{-4}	$9.7 \times 10^{-6}{}_\nu$
$(232)(233)$	$B_s \rightarrow K\mu^+\mu^-$	7.74×10^{-4}	$6.0 \times 10^{-5}{}_\nu$
$(312)(313)$	$B^- \rightarrow \pi^0 K^-$ $B_d \rightarrow \pi^+ K^-$	2.8×10^{-3}	$1.52 \times 10^{-1}{}_\nu$
$(322)(323)$	$B_s - \bar{B}_s$	8.2×10^{-3}	$9.7 \times 10^{-6}{}_\nu$
$(332)(333)$	$B_s - \bar{B}_s$	1.01×10^{-2}	$6.0 \times 10^{-5}{}_\nu$
$(i22)(i33)$	$B_s - \bar{B}_s$	1.8×10^{-1}	$3.75 \times 10^{-9}{}_\nu$
$(i32)(i23)$†	$B_s - \bar{B}_s$	3.2×10^{-4}	$1.56 \times 10^{-1}{}_\nu$
$(i22)(i13)$	$B_s - \bar{B}_s$	3.47×10^{-2}	$9.75 \times 10^{-6}{}_\nu$
$(i12)(i23)$	$B_s - \bar{B}_s$	5.16×10^{-2}	$1.52 \times 10^{-1}{}_\nu$
$(i32)(i13)$	$B_s - \bar{B}_s$	1.4×10^{-3}	$1.56 \times 10^{-1}{}_\nu$
$(i12)(i33)$	$B_s - \bar{B}_s$	9.0×10^{-1}	$5.8 \times 10^{-5}{}_\nu$
$(i23)(i33)$	$B_s - \bar{B}_s$	1.66×10^{-2}	$5.8 \times 10^{-5}{}_\nu$
$(i22)(i32)$	$B_s \rightarrow \phi K_s$	2.3×10^{-3}	$1.0 \times 10^{-5}{}_\nu$
$(i21)(i31)$	$B_d \rightarrow \pi^0\bar{K}^0$ $B^- \rightarrow \pi^0\bar{K}^-$	2.45×10^{-3}	$1.56 \times 10^{-1}{}_\nu$

for 100 GeV squarks, we scale the numbers for 300 GeV squarks. The bounds are shown in Table IV. Note that the channel $B \rightarrow K\mu^+\mu^-$ gives the best bound. Also, there is no bound involving τs in the final state, but we can estimate the number of $B_s \rightarrow \tau^+\tau^-$ decays. The relevant RPV coupling is $\lambda'_{3i2}\lambda'_{3i3}$. It can easily be checked that unless $i = 1$, the product is so constrained from neutrino mass [11] that even at LHC-b, there is no hope to detect a RPV signal in this channel. For $i = 1$, the coupling $\lambda'_{312}\lambda'_{313}$ should be less than 2.8×10^{-3}, which in turn translates into a bound on the BR to be less than 2.7×10^{-6}. Note that the SM expectation is about 7×10^{-7}.

D. Comparison between bounds coming from mixing and decay

In Table V we summarize our results, displaying the bounds on all $\lambda'\lambda'$ type products that may be responsible for $B_s - \bar{B}_s$ mixing. We find that a number of them may have better bounds from semileptonic or nonleptonic decay modes. While the neutrino constraints are indeed tight, we obtain a tighter constraint for most of the products.

V. SUMMARY AND CONCLUSIONS

In this paper we have computed the bounds on the product couplings of the type $\lambda'\lambda'$ coming from $B_s - \bar{B}_s$ mixing. Though such an analysis is not new, we have implemented several features in the analysis which have not been taken into account in earlier studies. Previously there was a lower limit on ΔM_s, here we have used the current bound on it and considered the exact expression for the box amplitude taking all possible processes, including that from SM. The QCD corrections to the amplitudes have been taken up to the NLO level. We have considered the possibility that the RPV product couplings may be complex. The analysis is done in the benchmark point $m_{H^+} = 200$ GeV, $\tan\beta = 5$, all sleptons degenerate at 100 GeV and all squarks degenerate at 300 GeV, and neglecting the pure MSSM contribution to the box amplitudes (by possibly applying some underlying FCNC suppression principle, like alignment of the squark mass matrices).

It is to be observed that in some cases, our bounds are actually weaker than those obtained earlier by saturating the mass difference with RPV alone. The reason is that destructive interference with the SM amplitude plays a very crucial role in determining the bounds, particularly when the phase of the RPV coupling is arbitrary. There is an intricate interplay among different amplitudes as can be seen in Fig. 2.

In some cases the bounds obtained from semileptonic B decays are better. Of course, one can enhance the squark mass to a limit where these bounds become weaker then those obtained from the box (the latter is not much affected by decoupling the squarks), but such extremely massive squarks are not interesting, even for the LHC.

However, some of these couplings may affect the nonleptonic decay modes (which, being slepton mediated, cannot be suppressed by decoupling the squarks). For some cases, the bounds coming from such decays are tighter than those coming from mixing.

[1] V. Abazov (D0 Collaboration), Phys. Rev. Lett. **97**, 021802 (2006).

[2] CDF Collaboration, Phys. Rev. Lett. **97**, 062003 (2006).

[3] M. Bona *et al.* (UTfit Collaboration), J. High Energy Phys. 07 (2005) 028; updated results available at http://utfit.roma1.infn.it/; M. Bona *et al.* (UTfit Collaboration), hep-ph/0605213.

[4] J. Charles *et al.* (CKMfitter Group), Eur. Phys. J. C **41**, 1 (2005); updated results available at http://ckmfitter.in2p3.fr/.

[5] P. Ball and R. Fleischer, hep-ph/0604249; A. Datta, Phys. Rev. D **74**, 014022 (2006).

[6] Z. Ligeti, M. Papucci, and G. Perez, Phys. Rev. Lett. **97**, 101801 (2006); Y. Grossman, Y. Nir, and G. Raz, Phys. Rev. Lett. **97**, 151801 (2006).

[7] M. Blanke, A. J. Buras, D. Guadagnoli, and C. Tarantino, hep-ph/0604057; B. Dutta and Y. Mimura, hep-ph/0607147; S. Chang, C. S. Kim, and J. Song, hep-ph/

0607313; S. Baek, J. Jeon, and C. S. Kim, Phys. Lett. B **641**, 183 (2006).

[8] K. Cheung, C. W. Chiang, N. G. Deshpande, and J. Jiang, hep-ph/0604223; X.-G. He and G. Valencia, Phys. Rev. D **74**, 013011 (2006); J. K. Parry, hep-ph/0608192.

[9] M. Ciuchini and L. Silvestrini, Phys. Rev. Lett. **97**, 021803 (2006); J. Foster, K. i. Okumura, and L. Roszkowski, hep-ph/0604121; G. Isidori and P. Paradisi, Phys. Lett. B **639**, 499 (2006); S. Khalil, Phys. Rev. D **74**, 035005 (2006); M. Endo and S. Mishima, Phys. Lett. B **640**, 205 (2006); S. Baek, J. High Energy Phys. 09 (2006) 077.

[10] J. P. Saha and A. Kundu, Phys. Rev. D **70**, 096002 (2004).

[11] B. C. Allanach, A. Dedes, and H. K. Dreiner, Phys. Rev. D **60**, 075014 (1999).

[12] M. Tegmark *et al.* (SDSS Collaboration), Phys. Rev. D **69**, 103501 (2004).

[13] R. Barbieri and A. Masiero, Nucl. Phys. **B267**, 679 (1986).

[14] R. Barbieri, A. Strumia, and Z. Berezhiani, Phys. Lett. B **407**, 250 (1997).

[15] K. Agashe and M. Graesser, Phys. Rev. D **54**, 4445 (1996).

[16] S. A. Abel, Phys. Lett. B **410**, 173 (1997).

[17] J.-H. Jang, Y. G. Kim, and J. S. Lee, Phys. Lett. B **408**, 367 (1997); Phys. Rev. D **58**, 035006 (1998); J.-H. Jang, J. K. Kim, and J. S. Lee, Phys. Rev. D **55**, 7296 (1997); D. Guetta, Phys. Rev. D **58**, 116008 (1998); K. Huitu *et al.*, Phys. Rev. Lett. **81**, 4313 (1998); D. Choudhury, B. Dutta, and A. Kundu, Phys. Lett. B **456**, 185 (1999); G. Bhattacharyya and A. Datta, Phys. Rev. Lett. **83**, 2300 (1999); G. Bhattacharyya, A. Datta, and A. Kundu, Phys. Lett. B **514**, 47 (2001); D. Chakraverty and D. Choudhury, Phys. Rev. D **63**, 075009 (2001); **63**, 112002 (2001); H. Dreiner, G. Polesello, and M. Thormeier, Phys. Rev. D **65**, 115006 (2002); J. P. Saha and A. Kundu, Phys. Rev. D **66**, 054021 (2002); A. Datta, Phys. Rev. D **66**, 071702 (2002); A. Akeroyd and S. Recksiegel, Phys. Lett. B **541**, 121 (2002); B. Dutta, C. S. Kim, and S. Oh, Phys. Rev. Lett. **90**, 011801 (2003); A. Kundu and T. Mitra, Phys. Rev. D **67**, 116005 (2003); B. Dutta *et al.*, Eur. Phys. J. C **37**, 273 (2004); Phys. Lett. B **601**, 144 (2004).

[18] G. Bhattacharyya and A. Raychaudhuri, Phys. Rev. D **57**, R3837 (1998).

[19] J. P. Saha and A. Kundu, Phys. Rev. D **69**, 016004 (2004).

[20] A. J. Buras and R. Fleischer, Adv. Ser. Dir. High Energy Phys. **15**, 65 (1998); *Heavy Flavours II*, edited by A. J. Buras and M. Lindner (World Scientific, Singapore, 1997).

[21] D. Bećirević *et al.*, Nucl. Phys. **B634**, 105 (2002).

[22] D. Bećirević *et al.*, J. High Energy Phys. 04 (2002) 025.

[23] B. de Carlos and P. White, Phys. Rev. D **55**, 4222 (1997).

[24] J. P. Saha and A. Kundu, in [17].

[25] C. Q. Geng and C. C. Liu, J. Phys. G **29**, 1103 (2003).

[26] A. Ali, G. Kramer, and C.-D. Lü, Phys. Rev. D **58**, 094009 (1998).

[27] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **89**, 281802 (2002).

[28] K. Abe *et al.* (Belle Collaboration), Phys. Rev. D **68**, 012001 (2003).

[29] J. L. Bagger, K. T. Matchev, and R. J. Zhang, Phys. Lett. B **412**, 77 (1997); M. Ciuchini *et al.*, Nucl. Phys. **B523**, 501 (1998).

[30] See http://www.slac.stanford.edu/xorg/hfag/, the website of the Heavy Flavor Averaging Group, for the Particle Data Group 2003 update of the rare decay (hadronic, charmless) data, averaged over *BABAR*, Belle, and CLEO collaborations.

[31] Yao *et al.* (Particle Data Group Collaboration), J. Phys. G **33**, 1 (2006).

[32] M. Ciuchini *et al.*, J. High Energy Phys. 10 (1998) 008.

[33] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985); Phys. Rev. D **73**, 073008 (2006).

[34] S. Aoki *et al.* (JLQCD Collaboration), Phys. Rev. Lett. **91**, 212001 (2003).

[35] R. Aleksan *et al.*, Phys. Rev. D **67**, 094019 (2003).

[36] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **65**, 051101 (2002).

[37] K. F. Chen *et al.* (Belle Collaboration), Phys. Lett. B **546**, 196 (2002).

[38] A. Kundu, S. Nandi, and J. P. Saha, Phys. Lett. B **622**, 102 (2005).

[39] P. K. Das and K. C. Yang, Phys. Rev. D **71**, 094002 (2005); S. Nandi and A. Kundu, J. Phys. G **32**, 835 (2006).