D^0-$\bar{D}^0$ mixing in $\Upsilon(1S) \to D^0\bar{D}^0$ decay at a super-B factory

Hai-Bo Li* and Mao-Zhi Yang†

CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China
Institute of High Energy Physics, P.O. Box 918, Beijing 100049, China
(Received 3 October 2006; published 17 November 2006)

D^0-$\bar{D}^0$ mixing and significant CP violation in the charm system may indicate the signature of new physics. In this study, we suggest that the coherent $D^0\bar{D}^0$ events from the decay of $\Upsilon(1S) \to D^0\bar{D}^0$ can be used to measure both mixing parameters and CP violation in charm decays. The neutral D mesons from $\Upsilon(1S)$ decay are strongly boosted, so that it will offer the possibility to measure the proper-time interval Δt between the fully reconstructed D^0 and $\bar{D}^0$. Both coherent and time-dependent information can be used to extract D^0-$\bar{D}^0$ mixing parameters. The sensitivity of the measurement should be improved at B factories or at super-B.

DOI: 10.1103/PhysRevD.74.094016 PACS numbers: 13.25.Ft, 12.15.Ff, 14.40.Lb, 14.65.Dw

Because of the smallness of the standard model (SM) $\Delta C = 0$ amplitude, D^0-$\bar{D}^0$ mixing offers a unique opportunity to probe flavor-changing interactions which may be generated by new physics. The most promising place to produce $D^0\bar{D}^0$ pairs with low backgrounds is the $\psi(3770)$ resonance just above the $D^0\bar{D}^0$ threshold. The current experiments, such as CLEO-c and BESIII [1], are all symmetric D meson factories, on which the time information cannot be used. It is very hard to build an asymmetric τ-charm factory in order to separate the two D^0 decay vertices since we need a strong boost of the D meson to measure the mixing parameters. Although the time-dependent analyses have been done at B factories, the D mesons produced there are incoherent. In $\Upsilon(1S) \to D^0\bar{D}^0$ decay, both D mesons are strongly boosted in the rest frame of the $\Upsilon(1S)$ with the Lorentz boost factor of $((\beta\gamma)_D = 2.33)$, precise determination of the proper-time interval (Δt) between the two D meson decays is available. Both coherence and time information are essential to measure the D^0-$\bar{D}^0$ mixing and CP violation.

In this paper, we consider the possible observations of D^0-$\bar{D}^0$ mixing and CP violation in the $\Upsilon(1S) \to D^0\bar{D}^0$ decay, in which the coherent $D^0\bar{D}^0$ events are generated with a strong boost. Here we assume that possible strong multiquark effects that involve seaquarks play no role in $\Upsilon(1S) \to D^0\bar{D}^0$ decays [2]. The $\Upsilon(1S)$ decays will provide another opportunity to search for D^0-$\bar{D}^0$ mixing and understand the source of CP violation in the charm system. The amplitude for $\Upsilon(1S)$ decaying to $D^0\bar{D}^0$ is $\langle D^0\bar{D}^0|H|\Upsilon(1S)\rangle$, and the $D^0\bar{D}^0$ pair system is in a state with charge parity $C = -1$, which can be defined as [3]

$$|D^0\bar{D}^0\rangle^{C=-1} = \frac{1}{\sqrt{2}}[|D^0\rangle|\bar{D}^0\rangle - |\bar{D}^0\rangle|D^0\rangle]. \tag{1}$$

Although there is a weak current contribution in $\Upsilon(1S) \to D^0\bar{D}^0$ decay, which may not conserve charge parity, the $D^0\bar{D}^0$ pair cannot be in a state with $C = +1$. The reason is that the relative orbital angular momentum of the $D^0\bar{D}^0$ pair must be $l = 1$ because of angular momentum conservation. A boson pair with $l = 1$ must be in an antisymmetric state, the antisymmetric state of the particle-antiparticle pair must be in a state with $C = -1$.

We shall analyze the time evolution of the $D^0\bar{D}^0$ system produced in $\Upsilon(1S)$ decay.

In the assumption of CPT invariance, the weak eigenstates of the D^0-$\bar{D}^0$ system are $|D_L\rangle = p|D^0\rangle + q|\bar{D}^0\rangle$ and $|D_H\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$ with eigenvalues $\mu_L = m_L - \frac{i}{2}\Gamma_L$ and $\mu_H = m_H - \frac{i}{2}\Gamma_H$, respectively, where the m_L and Γ_L (m_H and Γ_H) are the mass and width of the "light (L)" D^0 ("heavy (H)" D^0) meson. Following the $\Upsilon(1S) \to D^0\bar{D}^0$ decay, the D^0 and $\bar{D}^0$ will go separately and the proper-time evolution of the particle states $|D^0_{\rm phys}(t)\rangle$ and $|\bar{D}^0_{\rm phys}(t)\rangle$ are given by

$$\begin{aligned} |D^0_{\rm phys}(t)\rangle &= g_+(t)|D^0\rangle - \frac{q}{p}g_-(t)|\bar{D}^0\rangle, \\ |\bar{D}^0_{\rm phys}(t)\rangle &= g_+(t)|\bar{D}^0\rangle - \frac{p}{q}g_-(t)|D^0\rangle, \end{aligned} \tag{2}$$

where

$$g_\pm = \frac{1}{2}(e^{-im_Ht-(1/2)\Gamma_Ht} \pm e^{-im_Lt-(1/2)\Gamma_Lt}), \tag{3}$$

with definitions

$$\begin{aligned} m &\equiv \frac{m_L + m_H}{2}, & \Delta m &\equiv m_H - m_L, \\ \Gamma &\equiv \frac{\Gamma_L + \Gamma_H}{2}, & \Delta\Gamma &\equiv \Gamma_H - \Gamma_L. \end{aligned} \tag{4}$$

Note that here Δm is positive by definition, while the sign of $\Delta\Gamma$ is to be determined by experiments.

In practice, one defines the following mixing parameters

$$x \equiv \frac{\Delta m}{\Gamma}, \qquad y \equiv \frac{\Delta\Gamma}{2\Gamma}. \tag{5}$$

Then we consider a $D^0\bar{D}^0$ pair in $\Upsilon(1S)$ decay with definite charge-conjugation eigenvalue. The time-dependent wave function of the $D^0\bar{D}^0$ system with $C = -1$ can be written

*Electronic address: lihb@mail.ihep.ac.cn
†Electronic address: yangmz@mail.ihep.ac.cn

as

$$|D^0\bar{D}^0(t_1,t_2)\rangle = \frac{1}{\sqrt{2}}[|D^0_{\rm phys}(\mathbf{k_1},t_1)\rangle|\bar{D}^0_{\rm phys}(\mathbf{k_2},t_2)\rangle - |\bar{D}^0_{\rm phys}(\mathbf{k_1},t_1)\rangle|D^0_{\rm phys}(\mathbf{k_2},t_2)\rangle], \quad (6)$$

where $\mathbf{k_1}$ and $\mathbf{k_2}$ are the three-momentum vector of the two D mesons. We now consider decays of these correlated systems into various final states. An early study of correlated $D^0\bar{D}^0$ decays into specific flavor final states, at a τ-charm factory, was carried out by Bigi and Sanda [4]. Xing [5] had considered time-dependent decays into correlated pairs of states at $\psi(3770)$ and $\psi(4140)$ peaks. The amplitude of such joint decays, one D decaying to a final state f_1 at proper time t_1, and the other D to f_2 at proper time t_2, is given by [5]

$$A(\Upsilon(1S)\to D^0_{\rm phys}\overline{D}^0_{\rm phys}\to f_1f_2) \equiv \frac{1}{\sqrt{2}}\times\{a_-[g_-(t_1)g_+(t_2)-g_+(t_1)g_-(t_2)] + a_+[g_-(t_1)g_-(t_2)-g_+(t_1)g_+(t_2)]\}, \quad (7)$$

where

$$a_+ \equiv \bar{A}_{f_1}A_{f_2} - A_{f_1}\bar{A}_{f_2} = A_{f_1}A_{f_2}\frac{p}{q}(\lambda_{f_1}-\lambda_{f_2}),$$
$$a_- \equiv \frac{p}{q}A_{f_1}A_{f_2} - \frac{q}{p}\bar{A}_{f_1}\bar{A}_{f_2} = A_{f_1}A_{f_2}\frac{p}{q}(1-\lambda_{f_1}\lambda_{f_2}), \quad (8)$$

with $A_{f_i}\equiv\langle f_i|\mathcal{H}|D^0\rangle$, $\bar{A}_{f_i}\equiv\langle f_i|\mathcal{H}|\bar{D}^0\rangle$, and define

$$\lambda_{f_i} \equiv \frac{q}{p}\frac{\langle f_i|\mathcal{H}|\bar{D}^0\rangle}{\langle f_i|\mathcal{H}|D^0\rangle} = \frac{q}{p}\frac{\bar{A}_{f_i}}{A_{f_i}}, \quad (9)$$

$$\bar{\lambda}_{\bar{f}_i} \equiv \frac{p}{q}\frac{\langle \bar{f}_i|\mathcal{H}|D^0\rangle}{\langle \bar{f}_i|\mathcal{H}|\bar{D}^0\rangle} = \frac{p}{q}\frac{A_{\bar{f}_i}}{\bar{A}_{\bar{f}_i}}. \quad (10)$$

In the process $e^+e^-\to\Upsilon(1S)\to D^0\bar{D}^0$ the center-of-mass energy is far above the threshold of the $D^0\bar{D}^0$ pairs, so that the decay-time difference $(t=\Delta t_-=(t_2-t_1))$ between $D^0_{\rm phys}\to f_1$ and $\bar{D}^0_{\rm phys}\to f_2$ can be measured easily. From Eq. (7), one can derive the general expression for the time-dependent decay rate, in agreement with [6]:

$$\frac{d\Gamma(\Upsilon(1S)\to D^0_{\rm phys}\bar{D}^0_{\rm phys}\to f_1f_2)}{dt} = \mathcal{N}e^{-\Gamma|t|}\times[(|a_+|^2+|a_-|^2)\cosh(y\Gamma t) + (|a_+|^2-|a_-|^2)\cos(x\Gamma t) - 2\,\mathcal{R}e(a_+^*a_-) \times\sinh(y\Gamma t) + 2\,\mathcal{I}m(a_+^*a_-)\sin(x\Gamma t)], \quad (11)$$

where $\mathcal{N}$ is a common normalization factor, in Eq. (11), terms proportional to $|a_+|^2$ are associated with decays that occur without any net oscillation, while terms proportional to $|a_-|^2$ are associated with decays following a net oscillation. The other terms are associated with the interference between these two cases. In the following discussion, we define

$$R(f_1,f_2;t) \equiv \frac{d\Gamma(\Upsilon(1S)\to D^0_{\rm phys}\bar{D}^0_{\rm phys}\to f_1f_2)}{dt}. \quad (12)$$

The time-dependent rate expression simplifies if one of the states (say, f_2) is a CP eigenstate S_η with eigenvalue $\eta=\pm$:

$$R(f_1,S_\eta;t) = \mathcal{N}|A_{S_\eta}|^2|A_{f_1}|^2e^{-\Gamma|t|} \times[2|\lambda_{f_1}+\eta|^2\cosh(y\Gamma t) - 2\eta(|\lambda_{f_1}+\eta|^2)\sinh(y\Gamma t)], \quad (13)$$

where $A_{S_\eta}=\langle S_\eta|\mathcal{H}|\bar{D}^0\rangle$, and we have used $CP|D^0\rangle = -|\bar{D}^0\rangle$ and $\lambda_{S_\eta}=-\eta=\mp$ by neglecting CP violation in decay, D^0-$\bar{D}^0$ mixing, and the interference of the decay with and without mixing.

Now we consider the following cases for the D meson decays to various final states, such as semileptonic, hadronic, and CP eigenstates.

(1) $(l^-X^+,K^+\pi^-;t)$:

$$R(l^-X^+,K^+\pi^-;t) = \mathcal{N}|A_l|^2|\bar{A}_{K^+\pi^-}|^2\left|\frac{q}{p}\right|^2e^{-\Gamma|t|}\times((1+|\bar{\lambda}_{K^+\pi^-}|^2)\cosh(y\Gamma t) - (1-|\bar{\lambda}_{K^+\pi^-}|^2)\cos(x\Gamma t) + 2\,\mathcal{R}e(\bar{\lambda}_{K^+\pi^-})\sinh(y\Gamma t) + 2\,\mathcal{I}m(\bar{\lambda}_{K^+\pi^-})\sin(x\Gamma t)). \quad (14)$$

(2) $(l^+X^-,K^-\pi^+;t)$:

$$R(l^+X^-,K^-\pi^+;t) = \mathcal{N}|A_l|^2|A_{K^-\pi^+}|^2\left|\frac{p}{q}\right|^2e^{-\Gamma|t|}\times((1+|\lambda_{K^-\pi^+}|^2)\cosh(y\Gamma t) - (1-|\lambda_{K^-\pi^+}|^2)\cos(x\Gamma t) + 2\,\mathcal{R}e(\lambda_{K^-\pi^+})\sinh(y\Gamma t) + 2\,\mathcal{I}m(\lambda_{K^-\pi^+})\sin(x\Gamma t)). \quad (15)$$

(3) $(l^+X^-,K^+\pi^-;t)$:

$$R(l^+X^-, K^+\pi^-; t) = \mathcal{N}|A_l|^2|\bar{A}_{K^+\pi^-}|^2 e^{-\Gamma|t|} \times ((1 + |\bar{\lambda}_{K^+\pi^-}|^2)\cosh(y\Gamma t) + (1 - |\bar{\lambda}_{K^+\pi^-}|^2)\cos(x\Gamma t) + 2\,\mathcal{R}e(\bar{\lambda}_{K^+\pi^-})\sinh(y\Gamma t) - 2\,\mathcal{I}m(\bar{\lambda}_{K^+\pi^-})\sin(x\Gamma t)). \tag{16}$$

(4) $(l^-X^+, K^-\pi^+; t)$:

$$R(l^-X^-, K^-\pi^+; t) = \mathcal{N}|A_l|^2|A_{K^-\pi^+}|^2 e^{-\Gamma|t|} \times ((1 + |\lambda_{K^-\pi^+}|^2)\cosh(y\Gamma t) + (1 - |\lambda_{K^-\pi^+}|^2)\cos(x\Gamma t) + 2\,\mathcal{R}e(\lambda_{K^-\pi^+})\sinh(y\Gamma t) - 2\,\mathcal{I}m(\lambda_{K^-\pi^+})\sin(x\Gamma t)). \tag{17}$$

(5) $(l_1^\pm X^\mp, l_2^\mp X^\pm; t)$:

$$R(l_1^\pm X^\mp, l_2^\mp X^\pm; t) = \mathcal{N}|A_{l_1}|^2|A_{l_2}|^2 e^{-\Gamma|t|} \times (\cosh(y\Gamma t) + \cos(x\Gamma t)), \tag{18}$$

where l_1 and l_2 could be electron or muon.

(6) $(l^\pm X^\mp, S_\eta; t)$:

$$R(l^-X^+, S_\eta; t) = R(l^+X^-, S_\eta; t)_{q\Leftrightarrow p} = \mathcal{N}|A_l|^2|A_{S_\eta}|^2 e^{-\Gamma|t|} \times \left[2\cosh(y\Gamma t) - 2\eta\,\mathcal{R}e\left(\frac{q}{p}\right)\sinh(y\Gamma t) + 2\eta\,\mathcal{I}m\left(\frac{q}{p}\right)\sin(x\Gamma t)\right], \tag{19}$$

where $q \Leftrightarrow p$ indicates the exchange of q and p, and $|q/p| = 1$ is taken.

(7) $(K^-\pi^+, S_\eta; t)$: For this case, it is the same as the result in Eq. (13) for $f_1 = K^-\pi^+$ when CP violation is neglected.

(8) $(K^-\pi^+, K^+\pi^-; t)$: For the given final states $f_1 f_2 = (K^-\pi^+)(K^+\pi^-)$, the situation becomes more complicated; one can obtain the following expression after a lengthy calculation:

$$R(K^-\pi^+, K^+\pi^-; t) = \mathcal{N}|A_{K^-\pi^+}\bar{A}_{K^+\pi^-}|^2 e^{-\Gamma|t|} \times [(|1 - \lambda_{K^-\pi^+}\bar{\lambda}_{K^+\pi^-}|^2 + |\lambda_{K^-\pi^+} - \bar{\lambda}_{K^+\pi^-}|^2)\cosh(y\Gamma t) + (|1 - \lambda_{K^-\pi^+}\bar{\lambda}_{K^+\pi^-}|^2 - |\lambda_{K^-\pi^+} - \bar{\lambda}_{K^+\pi^-}|^2)\cos(x\Gamma t) + 2(\mathcal{R}e(\lambda_{K^-\pi^+}\bar{\lambda}_{K^+\pi^-} - 1)\mathcal{R}e(\lambda_{K^-\pi^+} - \bar{\lambda}_{K^+\pi^-}) + \mathcal{I}m(\lambda_{K^-\pi^+}\bar{\lambda}_{K^+\pi^-}) \times \mathcal{I}m(\lambda_{K^-\pi^+} - \bar{\lambda}_{K^+\pi^-}))\sinh(y\Gamma t) - 2(\mathcal{R}e(\lambda_{K^-\pi^+}\bar{\lambda}_{K^+\pi^-} - 1)\,\mathcal{I}m(\lambda_{K^-\pi^+} - \bar{\lambda}_{K^+\pi^-}) - \mathcal{I}m(\lambda_{K^-\pi^+}\bar{\lambda}_{K^+\pi^-})\,\mathcal{R}e(\lambda_{K^-\pi^+} - \bar{\lambda}_{K^+\pi^-}))\sin(x\Gamma t)]. \tag{20}$$

(9) $(K^-\pi^+, K^-\pi^+; t)$:

$$R(K^-\pi^+, K^-\pi^+; t) = \mathcal{N}|A_{K^-\pi^+}|^4\left|\frac{p}{q}\right|^2 e^{-\Gamma|t|} \times |\lambda^2_{K^-\pi^+} - 1|^2[\cosh(y\Gamma t) - \cos(x\Gamma t)]. \tag{21}$$

Mixing is the necessary condition for this process to occur.

(10) $(K^+\pi^-, K^+\pi^-; t)$:

$$R(K^+\pi^-, K^+\pi^-; t) = \mathcal{N}|\bar{A}_{K^+\pi^-}|^4\left|\frac{q}{p}\right|^2 e^{-\Gamma|t|} \times |\bar{\lambda}^2_{K^+\pi^-} - 1|^2[\cosh(y\Gamma t) - \cos(x\Gamma t)]. \tag{22}$$

(11) $(l_1^+X^-, l_2^+X^-, t)$:

$$R(l_1^+X^-, l_2^+X^-, t) = \mathcal{N}e^{-\Gamma|t|}\left|\frac{p}{q}\right|^2|A_{l_1^+X^-}|^2|A_{l_2^+X^-}|^2 \times (\cosh(y\Gamma t) - \cos(x\Gamma t)). \tag{23}$$

(12) $(l_1^-X^+, l_2^-X^+, t)$:

$$R(l_1^-X^+, l_2^-X^+, t) = \mathcal{N}e^{-\Gamma|t|}\left|\frac{q}{p}\right|^2|\bar{A}_{l_1^-X^+}|^2|\bar{A}_{l_2^-X^+}|^2 \times (\cosh(y\Gamma t) - \cos(x\Gamma t)). \tag{24}$$

In deriving the above formulas from Eqs. (14)–(24), we have assumed that: (1) $\Delta Q = \Delta C$ rule holds, $A_{l^-} = \langle l^- X^+ |\mathcal{H}| D^0\rangle = \bar{A}_{l^+} = \langle l^+ X^- |\mathcal{H}| \bar{D}^0\rangle = 0$; (2) CPT invariance holds. The results in Eqs. (14)–(17) are in agreement with those in Ref. [5].

In order to simplify the above formula, we make the following definitions:

$$\frac{q}{p} \equiv (1 + A_M)e^{-i\beta}, \tag{25}$$

where β is the weak phase in mixing and A_M is a real-valued parameter which indicates the magnitude of CP violation in the mixing, and for $f = K^-\pi^+$, we define

$$\frac{\bar{A}_{K^-\pi^+}}{A_{K^-\pi^+}} \equiv -\sqrt{r}e^{-i\alpha}; \qquad \frac{A_{K^+\pi^-}}{\bar{A}_{K^+\pi^-}} \equiv -\sqrt{r'}e^{-i\alpha'}, \tag{26}$$

where r and α (r' and α') are the ratio and relative phase of the doubly Cabibbo-suppressed (DCS) decay rate and the Cabibbo-favored (CF) decay rate. Then, $\lambda_{K^-\pi^+}$ and $\bar{\lambda}_{K^+\pi^-}$ can be parametrized as

$$\lambda_{K^-\pi^+} = -\sqrt{r}(1 + A_M)e^{-i(\alpha+\beta)}, \tag{27}$$

$$\bar{\lambda}_{K^+\pi^-} = -\sqrt{r'}\frac{1}{1 + A_M}e^{-i(\alpha'-\beta)}. \tag{28}$$

In order to demonstrate the CP violation in decay, we define $\sqrt{r} \equiv \sqrt{R_D}\frac{1}{1+A_D}$ and $\sqrt{r'} \equiv \sqrt{R_D}(1 + A_D)$. Thus, Eqs. (27) and (28) can be expressed as

$$\lambda_{K^-\pi^+} = -\sqrt{R_D}\frac{1 + A_M}{1 + A_D}e^{-i(\delta+\phi)}, \tag{29}$$

$$\bar{\lambda}_{K^+\pi^-} = -\sqrt{R_D}\frac{1 + A_D}{1 + A_M}e^{-i(\delta-\phi)}, \tag{30}$$

where $\delta = \frac{\alpha+\alpha'}{2}$ is the averaged phase difference between DCS and CF processes, and $\phi = \frac{\alpha-\alpha'}{2} + \beta$.

We can characterize the CP violation in the mixing amplitude, the decay amplitude, and the interference between mixing and decay, by real-valued parameters A_M, A_D, and ϕ as in Ref. [7]. In the limit of CP conservation, A_M, A_D, and ϕ are all zero. $A_M = 0$ means no CP violation in mixing, namely, $|q/p| = 1$; $A_D = 0$ means no CP violation in decay, for this case, $r = r' = R_D = |\bar{A}_{K^-\pi^+}/A_{K^-\pi^+}|^2 = |A_{K^+\pi^-}/\bar{A}_{K^+\pi^-}|^2$; $\phi = 0$ means no CP violation in the interference between decay and mixing.

Taking into account that $\lambda_{K^-\pi^+}$, $\bar{\lambda}_{K^+\pi^-} \ll 1$ and $x, y \ll 1$, keeping terms up to order x^2, y^2, and R_D in the expressions, neglecting CP violation in mixing, decay, and the interference between decay with and without mixing ($A_M = 0$, $A_D = 0$, and $\phi = 0$), expanding the time dependent for $xt, yt \lesssim \Gamma^{-1}$, we can write the results from Eqs. (14)–(24) as

(1) $(l^-X^+, K^+\pi^-; t)$:

$$R(l^-X^+, K^+\pi^-; t) = \mathcal{N}|A_l|^2|\bar{A}_{K^+\pi^-}|^2e^{-\Gamma|t|} \times (2R_D - 2\sqrt{R_D}y'\Gamma t + R_M\Gamma^2t^2), \tag{31}$$

where $R_M \equiv \frac{x^2+y^2}{2}$ is the mixing rate, and $y' \equiv y\cos\delta - x\sin\delta$.

(2) $(l^+X^-, K^-\pi^+; t)$:

$$R(l^+X^-, K^-\pi^+; t) = \mathcal{N}|A_l|^2|A_{K^-\pi^+}|^2e^{-\Gamma|t|} \times (2R_D - 2\sqrt{R_D}y'\Gamma t + R_M\Gamma^2t^2), \tag{32}$$

since the mixing in neutral D is tiny, it is much more likely that $x^2, y^2 \ll R_D$, cases (1) and (2) can be used for measuring R_D.

(3) $(l^+X^-, K^+\pi^-; t)$:

$$R(l^+X^-, K^+\pi^-; t) = \mathcal{N}|A_l|^2|\bar{A}_{K^+\pi^-}|^2e^{-\Gamma|t|}\left(2 - 2\sqrt{R_D}(y\cos(\delta) + x\sin(\delta))\Gamma t + \frac{y^2 - x^2}{2}\Gamma^2t^2\right). \tag{33}$$

(4) $(l^-X^+, K^-\pi^+; t)$:

$$R(l^-X^-, K^-\pi^+; t) = \mathcal{N}|A_l|^2|A_{K^-\pi^+}|^2e^{-\Gamma|t|} \times \left(2 - 2\sqrt{R_D}(y\cos(\delta) + x\sin(\delta))\Gamma t + \frac{y^2 - x^2}{2}\Gamma^2t^2\right) = R(l^+X^-, K^+\pi^-; t). \tag{34}$$

In the limit of no CP violation, case (3) is the same as (4).

(5) $(l_1^\pm X^\mp, l_2^\mp X^\pm; t)$:

$$R(l_1^\pm X^\mp, l_2^\mp X^\pm; t) = \mathcal{N}|A_{l_1}|^2|A_{l_2}|^2e^{-\Gamma|t|} \times \left(2 + \frac{y^2 - x^2}{2}\Gamma^2t^2\right). \tag{35}$$

(6) $(l^\pm, S_\eta; t)$:

$$R(l^{\pm}, S_{\eta}; t) = \mathcal{N}|A_l|^2|A_{S_\eta}|^2 e^{-\Gamma|t|} \times (2 - 2\eta(y\cos\beta \mp x\sin\beta)\Gamma t + y^2\Gamma^2 t^2), \tag{36}$$

where y may be determined because the phase $\beta = arg[(V_{us}V_{cs}^*/(V_{cs}V_{us}^*)] \sim 0$.

(7) $(K^-\pi^+, S_\eta; t)$:

$$R(K^-\pi^+, S_\eta; t) = \mathcal{N}|A_{K^-\pi^+}|^2|A_{S_\eta}|^2 e^{-\Gamma|t|} \times (\eta - \sqrt{R_D}\cos\delta)^2(1 - \eta y\Gamma t + \tfrac{1}{2}y^2(\Gamma t)^2), \tag{37}$$

where $\cos\delta$ can be measured in this case by combing $\eta = -1$ and $+1$ final states.

(8) $(K^-\pi^+, K^+\pi^-; t)$:

$$R(K^-\pi^+, K^+\pi^-; t) = \mathcal{N}|A_{K^-\pi^+}\bar{A}_{K^+\pi^-}|^2 e^{-\Gamma|t|} \times \left(2 + \frac{y^2 - x^2}{2}\Gamma^2 t^2 - 4R_D\cos(2\delta)\right). \tag{38}$$

(9) $(K^-\pi^+, K^-\pi^+; t)$:

$$R(K^-\pi^+, K^-\pi^+; t) = \mathcal{N}|A_{K^-\pi^+}|^4 e^{-\Gamma|t|} \times (1 - 2R_D\cos(2\delta))\frac{x^2 + y^2}{2}\Gamma^2 t^2. \tag{39}$$

This process is proportional to the mixing rate R_M, and can be used to measure the mixing parameter directly.

(10) $(K^+\pi^-, K^+\pi^-; t)$: The result is the same as $R(K^-\pi^+, K^-\pi^+; t)$ when CP violation is neglected.

(11) $(l_1^+X^-, l_2^+X^-, t)$:

$$R(l_1^+X^-, l_2^+X^-, t) = \mathcal{N}e^{-\Gamma|t|}|A_{l_1^+X^-}|^2|A_{l_2^+X^-}|^2 \times \frac{x^2 + y^2}{2}\Gamma^2 t^2. \tag{40}$$

One can also definitely measure the mixing rate in the like-sign processes as in case (9) and (10).

(12) $(l_1^-X^+, l_2^-X^+, t)$: The result is the same as $(l_1^+X^-, l_2^+X^-, t)$ when CP violation in mixing and decay is neglected.

Note that in all the above cases, when CP violation in decay is neglected, there is $|A_{K^-\pi^+}| = |\bar{A}_{K^+\pi^-}|$.

The experimental data from CLEO-c yield that the allowed values for the mixing parameters x and y are: $y = -0.058 \pm 0.066$, $x < 0.094$ [6]. The ratio of the DCS decay rate to the CF decay rate is $R_D \sim (V_{cs}V_{cd})^2 \approx 0.0026$. The illustrative plot of the decay rate $R(l^{\pm}X^{\mp}, K^{\mp}\pi^{\pm}; t)$ is shown in Fig. 1 by taking $\delta = 10°$. The decay rate $R(l^{\pm}X^{\mp}, K^{\mp}\pi^{\pm}; t)$ is very sensitive to the mixing parameters x, y and the ratio of R_D in the region $\Gamma t \sim 1 - 6$. The other decay rates in Eqs. (33)–(38) are not sensitive to the mixing parameters and R_D, because in these decay rates, the x, y and R_D are only a small correction to the dominant contributions.

In the Cabibbo-Kobayashi-Maskawa (CKM) framework CP violation in the neutral D system is very small and can be safely neglected. However extension of the SM could induce new physics of CP violation [4,8]. The most likely sizable effect is a possible new CP violation phase, $\phi = \arg(q\bar{A}/pA)$, occurring in the interference between mixing and decay amplitudes. Thus, in the presence of CP violation in the interference, we can construct the following CP observable based on the previous calculations. We can look at the difference between the cases (3) and (4), and define:

$$A_{CP}^{+-}(t) \equiv \frac{R(l^+X^-, K^+\pi^-) - R(l^-X^+, K^-\pi^+)}{R(l^+X^-, K^+\pi^-) + R(l^-X^+, K^-\pi^+)}. \tag{41}$$

As discussed in Ref. [5], the signal is due to the interplay of DCS decay and mixing. With the help of Eqs. (16) and (17), we obtain

$$A_{CP}^{+-}(t) = -\sqrt{R_D}(y\sin\delta - x\cos\delta)\sin\phi \times \Gamma t. \tag{42}$$

Here we keep both x and y terms since the current experiments indicate that they may be at the same order, this is different from Ref. [5]. The above asymmetry depends on the nonvanishing phase ϕ, and also the mixing parameters. Within the SM, it is of order $\mathcal{O}(10^{-3})$ [9], which makes such an asymmetry unmeasurably tiny unless there is new

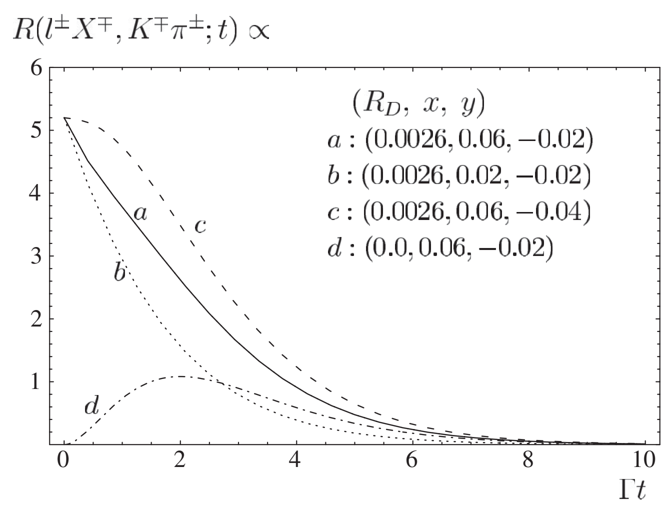

FIG. 1. Illustrative plot of the sensitivity of the decay rate $R(l^{\pm}X^{\mp}, K^{\mp}\pi^{\pm}; t)$ to the mixing parameters of x, y and the ratio R_D, where $\delta = 10°$ is taken.

physics [10]. By looking at the difference between $R(l^+, S_\eta; t)$ and $R(l^+, S_\eta; t)$ in case (6), we get the following asymmetry

$$A_{CP}^{S_\eta}(t) \equiv \frac{R(l^+, S_\eta) - R(l^-, S_\eta)}{R(l^+, S_\eta) + R(l^-, S_\eta)}. \quad (43)$$

With the help of Eq. (19), we obtain:

$$A_{CP}^{S_\eta}(t) = \eta x \sin\beta \times \Gamma t, \quad (44)$$

where β is defined in Eq. (25). This CP term depends on the mixing parameter x and the phase in the mixing amplitude.

In experiments, the KEK-B can move to the $\Upsilon(1S)$ peak without losing luminosity, more than 330 fb^{-1} data per year could be taken [11]. The measured cross section at the $\Upsilon(1S)$ peak is $\sigma_R = 21.5 \pm 0.2 \pm 0.8$ nb by CLEO [12]. One can estimate the total $\Upsilon(1S)$ events with 1 yr running of KEK-B are about 7.1×10^9. While, at super-KEK-B, about 10^{11} $\Upsilon(1S)$ events can be obtained with 1 yr data taking if the design luminosity is 8×10^{35} [13]. More recently, a super-B factory based on the concepts of the linear collider (LC) is proposed [14], the low energy beam and high energy beam are 4.0 and 7.0 GeV, respectively. The machine can run at both $\Upsilon(4S)$ and $\Upsilon(1S)$ peaks with luminosity about 1.0×10^{36}. Then about 10^{12} $\Upsilon(1S)$ events can be collected with 1 yr's run. In Ref. [15], one had estimated the ratio of $\mathcal{BR}(\Upsilon(1S) \to D^+D^-)$ and $\mathcal{BR}(\Upsilon(1S) \to K^+K^-)$ as

$$\frac{\mathcal{BR}(\Upsilon(1S) \to D^+D^-)}{\mathcal{BR}(\Upsilon(1S) \to K^+K^-)} \cong \left(\frac{1}{0.2}\right)^2 \gg 1.0, \quad (45)$$

where the current upper limit of $\mathcal{BR}(\Upsilon(1S) \to K^+K^-)$ is 5.0×10^{-4} at 90% confidence level. One can expect that the decay rate of $\Upsilon(1S) \to K^+K^-)$ is at the order of 10^{-6} [15], so that we can estimate $\mathcal{BR}(\Upsilon(1S) \to D^+D^-) \sim 10^{-4} - 10^{-5}$. At the super-$B$ factory, around 10^7–10^8 $D^0\bar{D}^0$ pairs can be collected in 1 yr's data taking, which is comparable with that at the BESIII with four years integrated luminosity [16].

It is known that one has to fit the proper-time distribution in experiments to extract both the mixing and the CP parameters; we discuss the following two cases: (1) Case-I: at a symmetric $\Upsilon(1S)$ factory, namely, the $\Upsilon(1S)$ is at rest in the central-mass (CM) frame. Then, the proper-time interval between the two D mesons is calculated as

$$\Delta t = (r_{D^0} - r_{\bar{D}^0}) \frac{m_D}{c|\mathbf{P}|}, \quad (46)$$

where r_{D^0} and $r_{\bar{D}^0}$ are the D^0 and $\bar{D}^0$ decay length, respectively, and $\mathbf{P}$ is the three-momentum vector of D^0. Since the momentum can be calculated with $\Upsilon(1S)$ decay in the CM frame, all the joint $D^0\bar{D}^0$ decays in this paper can be used to study D^0-$\bar{D}^0$ mixing and the CP violation in the symmetric $\Upsilon(1S)$ factory. (2) Case-II: while, at an asymmetric $\Upsilon(1S)$ factory, the $\Upsilon(1S)$ will be produced with a boost. In this case, the momentum of D^0 and $\bar{D}^0$ will be different from each other, and one has to fully reconstruct at least one of the two D mesons, since the proper-time interval between the two D mesons is calculated as

$$\Delta t = r_{D^0} \frac{m_D}{c|\mathbf{P_{D^0}}|} - r_{\bar{D}^0} \frac{m_D}{c|\mathbf{P_{\bar{D}^0}}|}, \quad (47)$$

where $|\mathbf{P_{D^0}}|$ and $|\mathbf{P_{\bar{D}^0}}|$ are the momentum of D^0 and $\bar{D}^0$, respectively. In this case, the joint decays to dilepton in Eq. (18) will not work, since both the D mesons cannot be fully reconstructed. One cannot obtain the proper time from such a kind of experiment.

The average decay length of the D^0 meson in the rest frame of $\Upsilon(1S)$ is $c\tau_{D^0} \times (\beta\gamma)_{D^0} \approx 290$ μm. At B factories, such as the Belle detector, the impact parameter resolution of the vertex detector, which is directly related to the decay vertex resolution of D^0, is described in Ref. [17], from which we can get that the resolution for the reconstructed D^0 decay length should be less than 100 μm within the coverage of the detector. This means the Belle detector is good enough to separate the two D^0 decay vertices, so that the mixing parameters can be measured by using time information.

All the results in this paper can also be applied to the following processes,

$$e^+e^- \to \Upsilon(1S) \to \pi^0(D^0\bar{D}^0)_{C=-1}, \quad (48)$$

where the $D^0\bar{D}^0$ pair are in a $C = -1$ state, for example, $\Upsilon(1S) \to D^{*0}\bar{D}^0 \to \pi^0 D^0\bar{D}^0$.

In conclusion, we suggest that the coherent $D^0\bar{D}^0$ events from the decay of $\Upsilon(1S) \to D^0\bar{D}^0$ can be used to measure both the mixing parameters and the CP violation in charm decays. The neutral D mesons from the $\Upsilon(1S)$ decay are strongly boosted, so that it will offer the possibility to measure the proper-time interval Δt between the fully reconstructed D^0 and $\bar{D}^0$. Both coherent and time-dependent information can be used to extract D^0-$\bar{D}^0$ mixing parameters, in which the sensitivity of the measurements could be improved by comparing to the future measurement at the BESIII with the same amount of $D^0\bar{D}^0$ pairs.

One of the authors (H. B. Li) would like to thank Dr. Rong-Gang Ping for helpful discussion. This work is supported in part by the National Natural Science Foundation of China under Contract No. 10205017, No. 10575108, and the Knowledge Innovation Project of CAS under Contract No. U-612 and No. U-530 (IHEP).

[1] BESIII Collaboration, "The Preliminary Design Report of the BESIII Detector," Report No. IHEP-BEPCII-SB-13.
[2] M. B. Voloshin, Phys. Rev. D **71**, 114003 (2005).
[3] M. Goldhaber and J. L. Rosner, Phys. Rev. D **15**, 1254 (1977); D. Atwood and A. A. Petrov, Phys. Rev. D **71**, 054032 (2005); D. M. Asner and W. M. Sun, Phys. Rev. D **73**, 034024 (2006).
[4] I. Bigi and A. Sanda, Phys. Lett. B **171**, 320 (1986).
[5] Z. Z. Xing, Phys. Rev. D **55**, 196 (1997); Phys. Lett. B **372**, 317 (1996).
[6] W. M. Yao *et al.* (Partcle Data Group), J. Phys. G **33**, 1 (2006).
[7] A. F. Falk, Y. Nir, and A. Petrov, J. High Energy Phys. 12, 019 (1999).
[8] Y. Grossman, A. L. Kagan, and Y. Nir, hep-ph/0609178.
[9] I. I. Bigi, in Proceedings of the First International Workshop on τ-Charm Physics (Charm 2006), Beijing, 2006 (unpublished).
[10] K. Agashe, G. Perez, and A. Soni, Phys. Rev. D **71**, 016002 (2005).
[11] Y. Funakoshi, in Proceeding of the 8th ICFA Seminar on Future Perspectives in High Energy Physics, Kyungpook National University, Daegu, Korea, 2005 (unpublished).
[12] D. Andrews *et al.*, Phys. Rev. Lett. **50**, 807 (1983).
[13] Masashi Hazumi (private communications).
[14] J. Albert *et al.*, Report No. SLAC-PUB-11676 and INFN-AE-05-08.
[15] V. L. Chernyark and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).
[16] Haibo Li, hep-ex/0605004 [Nucl. Phys. B (to be published)].
[17] M. Hazumi, Nucl. Instrum. Methods Phys. Res., Sect. A **473**, 1 (2001); J. Li *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **553**, 483 (2005).