PHYSICAL REVIEW D **74,** 094005 (2006)

Scalar glueball, scalar quarkonia, and their mixing

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(Received 21 July 2006; published 3 November 2006)

The isosinglet scalar mesons $f_0(1710)$, $f_0(1500)$, $f_0(1370)$ and their mixing are studied. We employ two recent lattice results as the starting point; one is the isovector scalar meson $a_0(1450)$ which displays an unusual property of being nearly independent of quark mass for quark masses smaller than that of the strange, and the other is the scalar glueball mass at 1710 MeV in the quenched approximation. In the SU(3) symmetry limit, $f_0(1500)$ turns out to be a pure SU(3) octet and is degenerate with $a_0(1450)$, while $f_0(1370)$ is mainly an SU(3) singlet with a slight mixing with the scalar glueball which is the primary component of $f_0(1710)$. These features remain essentially unchanged even when SU(3) breaking is taken into account. We discuss the sources of SU(3) breaking and their consequences on flavor-dependent decays of these mesons. The observed enhancement of $\omega f_0(1710)$ production over $\phi f_0(1710)$ in hadronic J/ψ decays and the copious $f_0(1710)$ production in radiative J/ψ decays lend further support to the prominent glueball nature of $f_0(1710)$.

DOI: 10.1103/PhysRevD.74.094005 PACS numbers: 14.40.Cs

I. INTRODUCTION

Despite of the fact that the $q\bar{q}$ and glueball contents of the isosinglet scalar mesons $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ have been studied extensively, it has been controversial as to which of these is the dominant glueball. Partly due to the fact that $f_0(1500)$, discovered in $p\bar{p}$ annihilation at LEAR, has decays to $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$ modes which are not compatible with a simple $q\bar{q}$ picture [1] and that the earlier quenched lattice calculations [2] predict the scalar glueball mass to be $\sim$1550 MeV, it has been suggested that $f_0(1500)$ is primarily a scalar glueball [3]. Furthermore, because of the small production of $\pi\pi$ in $f_0(1710)$ decay compared to that of $K\bar{K}$, it has been thought that $f_0(1710)$ is primarily $s\bar{s}$ dominated. On the other hand, the smaller production rate of $K\bar{K}$ relative to $\pi\pi$ in $f_0(1370)$ decay leads to the conjecture that $f_0(1370)$ is governed by the nonstrange light quark content.

Based on the above observations, a flavor-mixing scheme is proposed [3] to consider the glueball and $q\bar{q}$ mixing in the neutral scalar mesons $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$. χ^2 fits to the measured scalar meson masses and their branching ratios of strong decays have been performed in several references by Amsler, Close and Kirk [3], Close and Zhao [4], and He *et al.* [5]. A common feature of these analyses is that, before mixing, the $s\bar{s}$ mass M_S is larger than the pure glueball mass M_G which, in turn, is larger than the $N(\equiv (u\bar{u}+d\bar{d})/\sqrt{2})$ mass M_N, with M_G close to 1500 MeV and $M_S - M_N$ of the order of 200 $\sim$ 300 MeV. However, there are several serious problems with this scenario. First, the isovector scalar meson $a_0(1450)$ is confirmed to be a $q\bar{q}$ meson in lattice calculations [6–10] which will be discussed later. As such, the degeneracy of $a_0(1450)$ and $K_0^*(1430)$, which has a strange quark, cannot be explained if M_S is larger than M_N by $\sim$250 MeV. Second, the most recent quenched lattice calculation with improved action and lattice spacings extrapolated to the continuum favors a larger scalar glueball mass close to 1700 MeV [11,12] (see below for discussion). Third, if $f_0(1710)$ is dominated by the $s\bar{s}$ content, the decay $J/\psi \to \phi f_0(1710)$ is expected to have a rate larger than that of $J/\psi \to \omega f_0(1710)$. Experimentally, it is the other way around: the rate for $\omega f_0(1710)$ production is about 6 times that of $J/\psi \to \phi f_0(1710)$. Fourth, it is well known that the radiative decay $J/\psi \to \gamma f_0$ is an ideal place to test the glueball content of f_0. If $f_0(1500)$ has the largest scalar glueball component, one expects the $\Gamma(J/\psi \to \gamma f_0(1500))$ decay rate to be substantially larger than that of $\Gamma(J/\psi \to \gamma f_0(1710))$. Again, experimentally, the opposite is true.

Other scenarios have been proposed. Based on their lattice calculations of the quenched scalar glueball mass at 1625(94) MeV at the infinite volume and continuum limits [13] and the $s\bar{s}$ meson mass in the connected insertion (no annihilation) at $\sim$1500 MeV, Lee and Weingarten [14,15] considered a mixing scheme where $f_0(1500)$ is an almost pure $s\bar{s}$ meson and $f_0(1710)$ and $f_0(1370)$ are primarily the glueball and $u\bar{u}+d\bar{d}$ meson, respectively, but with substantial mixing between the two ($\sim$ 25% for the small component). With the effective chiral Lagrangian approach, Giacosa *et al.* [16] performed a fit to the experimental masses and decay widths of $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ and found four possible solutions, depending on whether the direct decay of the glueball component is considered. One of the solutions (see the appendix) gives $f_0(1710)$ as a pure glueball, while $f_0(1370)$ and $f_0(1500)$ are dominated by the quarkonia components, but with

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strong mixing between $(u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$. In this case, $M_S = 1452$ MeV, $M_N = 1392$ MeV and $M_G = 1712$ MeV.

In this work, we shall employ two recent lattice results as the input for the mass matrix which is essentially the starting point for the mixing model between scalar mesons and the glueball. First of all, an improved quenched lattice calculation of the glueball spectrum at the infinite volume and continuum limits based on much larger and finer lattices have been carried out [11]. The mass of the scalar glueball is calculated to be $m(0^{++}) = 1710 \pm 50 \pm 80$ MeV. The implicit assumption of the mixing models entails that the experimental glueball mass is reflected as a result of mixing of the glueball in the quenched approximation with the scalar $q\bar{q}$ mesons. This suggests that M_G should be close to 1700 MeV rather than 1500 MeV [2] from the earlier lattice calculations.[1] Second, the recent quenched lattice calculation of the isovector scalar meson a_0 mass has been carried out for a range of low quark masses [6]. With the lowest one corresponding to m_π as low as 180 MeV, it is found that, when the quark mass is smaller than that of the strange, a_0 mass levels off, in contrast to those of a_1 and other hadrons that have been calculated on the lattice. This confirms the trend that has been observed in earlier works at higher quark masses in both the quenched and unquenched calculations [7–10]. The chiral extrapolated mass $a_0 = 1.42 \pm 0.13$ GeV suggests that $a_0(1450)$ is a $q\bar{q}$ state. By virtue of the fact that lower state $a_0(980)$ is not seen, it is concluded that $a_0(980)$ is not a $q\bar{q}$ meson which is supposed to be readily accessible with the $\bar{\psi}\psi$ interpolation field. Furthermore, $K_0^*(1430)^+$, an $u\bar{s}$ meson, is calculated to be 1.41 ± 0.12 GeV and the corresponding scalar $\bar{s}s$ state from the connected insertion is 1.46 ± 0.05 GeV. This explains the fact that $K_0^*(1430)$ is basically degenerate with $a_0(1450)$ despite having one strange quark. This unusual behavior is not understood as far as we know and it serves as a challenge to the existing hadronic models.[2] We are aware that there is a recent $N_f = 2$ dynamical fermion calculation of a_0 with the $\bar{\psi}\psi$ interpolation field [22]. Extrapolating the mass difference between b_0 and a_0 to the chiral limit, it is claimed that $a_0(980)$ is a $q\bar{q}$ state. Even though there is no ghost state contamination in this calculation as there are in the above-mentioned quenched and partially quenched calculations, there is a physical $\pi\eta_2$ (η_2 is the η' in the $N_f = 2$ case) nearby which has not been taken into account. Also, since the quark masses in the present calculation are heavy, it would not be able to discern the possibility that b_0 and a_0 may cross each other toward the chiral limit. In addition, it is known that there are a host of problems assigning $a_0(980)$ to the $q\bar{q}$ state phenomenologically. Here are some of them:

(i) If the $q\bar{q}$ a_0 indeed goes down with the quark mass as is claimed in Ref. [22], the same calculation with a strange quark instead of an u quark would yield an $s\bar{d}$ meson around 1100 MeV which would place it far away from the two known mesons in this mass range. In other words, it is ~300 MeV below $K_0^*(1430)$ and ~300 MeV above $\kappa(800)$.

(ii) It cannot explain why the $K_0^*(1430)$ which, according to the review of scalar mesons in the particle data table, is a $q\bar{q}$ state in all the models, is higher than the axial-vector mesons $K_1(1270)$ and $K_1(1400)$. This is a situation which parallels to the case of nonstrange mesons where $a_0(1450)$ is higher than $a_1(1260)$. The authors admitted this is a problem in their paper [22], but did not offer any answer.

(iii) The widths of $a_0(980)$ and $f_0(980)$ are substantially smaller than those of $a_0(1450)$ and $f_0(1370)$. In particular, they are much smaller than that of $\kappa(800)$ which should be a nonet partner with $a_0(980)$ and $f_0(980)$.

(iv) The $\gamma\gamma$ widths of $a_0(980)$ and $f_0(980)$ are much smaller than expected of a $q\bar{q}$ state [23].

[1]We should note that the Monte Carlo results of these two set of calculations actually agree in lattice units within errors. The difference comes from the fact that Ref. [2] uses the string tension to set the scale, while Ref. [11] uses the Sommer scale [17] $r_0 = 0.5$ fm to set the scale. It is well known that the scale of the quenched approximation is uncertain from 10% to 20% depending on how it is set. The relation between the string tension scale and the r_0 scale in the quenched case has been examined for an extended range of couplings (Wilson $\beta = 6/g_0^2$ from 5.7 to 6.6) [18] and it is found that they consistently differ by ~10% which means that the string tension scale corresponds to $r_0 = 0.55$ fm in the Sommer scale and thus places the predicted glueball mass at 1550 MeV, 10% lower than the 1710 MeV set by the $r_0 = 0.5$ fm scale. The inconsistency of scales in the quenched case has prompted the work by the HPQCD-UKQCD-MILC-Fermilab collaboration to study the issue in both the quenched approximation and in full QCD [19]. When a set of quantities from both the light quark and heavy quark sectors are compared to experiments, they found that the few outliers in the quenched approximation (e.g. f_π is higher than the average by ~10% and the $1P - 1S$ splitting in the Upsilon is lower than the average by ~10%) would line up with the rest to give a common scale in full QCD. Since the string tension scale, like the f_π scale, in the quenched approximation is 10% higher than the average while the latter is closer to that in full QCD, we think it is essential to take $r_0 = 0.5$ fm and not the string tension to set the scale in the quenched approximation in order to fairly asses the quenched errors when compared with experiments.

[2]There are some attempts to understand the near degeneracy of $a_0(1450)$ and $K_0^*(1430)$. Since the 4-quark light scalar nonet is known to have a reversed ordering, namely, the scalar strange meson κ is lighter than the nonstrange one such as $f_0(980)$, it has been proposed in [20] to consider the mixing of the $q\bar{q}$ heavy scalar nonet with the light nonet to make $a_0(1450)$ and $K_0^*(1430)$ closer. It goes further in [21] to assume that the observed heavy scalar mesons form another 4-quark nonet. We note, however, the quenched lattice calculations in [6], which presumably gives the bare $q\bar{q}$ states before mixing with $q^2\bar{q}^2$ via sea quark loops in the dynamical fermion calculation, already seem to suggest the near degeneracy between $a_0(1450)$ and $K_0^*(1430)$.

(v) It is hard to understand why $a_0(980)$ and $f_0(980)$ are basically degenerate. The experimental data on $D_s^+ \to f_0(980)\pi^+$ [24] and $\phi \to f_0(980)\gamma$ [25] imply copious $f_0(980)$ production via its $s\bar{s}$ component. Yet, there cannot be an $s\bar{s}$ component in $a_0(980)$ since it is an $I = 1$ state.

(vi) The radiative decay $\phi \to a_0(980)\gamma$, which cannot proceed if $a_0(980)$ is a $q\bar{q}$ state, can be nicely described in the kaon loop mechanism [26]. This suggests a considerable admixture of the $K\bar{K}$ component.

For all these reasons, we do not take the claim in Ref. [22] seriously. We shall rely on the conclusion from the other calculations in both the quenched and unquenched calculations [6–10].

As we discussed above, the accumulated lattice results hint at an SU(3) symmetry in the scalar meson sector. Indeed, the near degeneracy of $K_0^*(1430)$, $a_0(1470)$, and $f_0(1500)$ implies that, to first order approximation, flavor SU(3) is a good symmetry for the scalar mesons above 1 GeV, much better than the pseudoscalar, vector, axial, and tensor sectors.

This work is organized as follows. In Sec. II, we discuss the mixing matrix of scalar mesons in the SU(3) limit and its implications on the strong decays of $f_0(1500)$. SU(3) breaking effects and chiral suppression in the scalar glueball decay into two pseudoscalar mesons are studied in Sec. III. Section IV is devoted to the numerical results for the mixing matrix and branching ratios. Conclusions are presented in Sec. V. The mixing matrices of the isosinglet scalar mesons $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ that have been proposed in the literature are summarized in the appendix.

II. MIXING MATRIX

We shall use $|U\rangle$, $|D\rangle$, $|S\rangle$ to denote the quarkonium states $|u\bar{u}\rangle$, $|d\bar{d}\rangle$ and $|s\bar{s}\rangle$, and $|G\rangle$ to denote the pure scalar glueball state. In this basis, the mass matrix reads

$$\mathrm{M} = \begin{pmatrix} M_U & 0 & 0 & 0 \\ 0 & M_D & 0 & 0 \\ 0 & 0 & M_S & 0 \\ 0 & 0 & 0 & M_G \end{pmatrix} + \begin{pmatrix} x & x & x_s & y \\ x & x & x_s & y \\ x_s & x_s & x_{ss} & y_s \\ y & y & y_s & 0 \end{pmatrix}, \tag{1}$$

where the mass parameters m_G is the mass of the scalar glueball in the pure gauge sector, and $m_{U,D,S}$ are the scalar quarkonia $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ before mixing which correspond to those from the connected insertion calculation. The parameter x denotes the mixing between different $q\bar{q}$ states through quark-antiquark annihilation and y stands for the glueball-quarkonia mixing strength. Possible SU(3) breaking effects are characterized by the subscripts "s" and "ss." As noticed in passing, lattice calculations [6] of the $a_0(1450)$ and $K_0^*(1430)$ masses indicate a good SU(3) symmetry for the scalar meson sector above 1 GeV. This means that M_S should be close to M_U or M_D. Also the glueball mass m_G should be close to the scalar glueball mass $1710 \pm 50 \pm 80$ MeV from the lattice QCD calculation in the pure gauge sector [11].

We shall begin by considering exact SU(3) symmetry as a first approximation, namely, $M_S = M_U = M_D = M$ and $x_s = x_{ss} = x$ and $y_s = y$. In this case, it is convenient to recast the mass matrix in Eq. (1) in terms of the basis $|a_0(1450)\rangle$, $|f_{\rm octet}\rangle$, $|f_{\rm singlet}\rangle$ and $|G\rangle$ defined by

$$\begin{aligned} |a_0(1450)\rangle &= \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle), \\ |f_{\rm octet}\rangle &= \frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle), \\ |f_{\rm singlet}\rangle &= \frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle). \end{aligned} \tag{2}$$

Then the mass matrix becomes

$$\mathrm{M} = \begin{pmatrix} M & 0 & 0 & 0 \\ 0 & M & 0 & 0 \\ 0 & 0 & M+3x & \sqrt{3}y \\ 0 & 0 & \sqrt{3}y & M_G \end{pmatrix}. \tag{3}$$

The first two eigenstates are identified with $a_0(1450)$ and $f_0(1500)$ which are degenerate with the mass M. Taking M to be the experimental mass of 1474 ± 19 MeV [27], we see it is a good approximation for the mass of $f_0(1500)$ at 1507 ± 5 MeV [27]. Thus, in the limit of exact SU(3) symmetry, $f_0(1500)$ is the SU(3) isosinglet octet state $|f_{\rm octet}\rangle$ and is degenerate with $a_0(1450)$. The diagonalization of the lower 2×2 matrix in (3) yields the eigenvalues

$$\begin{aligned} m_{f_0(1370)} &= \bar{M} - \sqrt{\Delta^2 + 3y^2}, \\ m_{f_0(1700)} &= \bar{M} + \sqrt{\Delta^2 + 3y^2}, \end{aligned} \tag{4}$$

where $\bar{M} = (M + 3x + M_G)/2$ and $\Delta = (M_G - M - 3x)/2$, and the corresponding eigenvectors are

$$\begin{aligned} |f_0(1370)\rangle &= N_{1370}\left(|f_{\rm singlet}\rangle - \frac{\sqrt{3}y}{\Delta + \sqrt{\Delta^2 + 3y^2}}|G\rangle\right), \\ |f_0(1710)\rangle &= N_{1710}\left(|G\rangle + \frac{\sqrt{3}y}{\Delta + \sqrt{\Delta^2 + 3y^2}}|f_{\rm singlet}\rangle\right), \end{aligned} \tag{5}$$

with N_{1370} and N_{1700} being the normalization constants.

Several remarks are in order. (i) In the absence of glueball-quarkonium mixing, i.e. $y = 0$, we see from Eq. (5) that $f_0(1370)$ becomes a pure SU(3) singlet $|f_{\rm singlet}\rangle$ and $f_0(1710)$ the pure glueball $|G\rangle$. The $f_0(1370)$ mass is given by $m_{f_0(1370)} = M + 3x$. Taking the experimental $f_0(1370)$ mass to be 1370 MeV, the quark-antiquark mixing matrix element x through annihilation is found to be -33 MeV. (ii) When the glueball-quarkonium mixing y is turned on, there will be some mixing between the glueball and the SU(3)-singlet $q\bar{q}$. If y has the same magnitude as x, i.e. 33 MeV, then $3y^2 \ll \Delta^2$ where Δ is half of the mass difference between M_G and $M + 3x$, which is $\sim$170 MeV. In this case, the mass shift of $f_0(1370)$ and $f_0(1710)$ due to mixing is only $\sim 3y^2/2\Delta = 9.6$ MeV. In the wave functions of the mixed states, the coefficient of the minor component is of order $\sqrt{3}y/(2\Delta) = 0.17$ which corresponds to $\sim$3% mixing.

We next proceed to consider the implications of the aforementioned mixing scheme to strong decays. We first discuss the $f_0(1500)$ meson, since its strong decays are better measured. If $f_0(1500)$ is the octet state $|f_{\rm octet}\rangle$, it will lead to the predictions

$$\frac{\Gamma(f_0(1500) \to K\bar{K})}{\Gamma(f_0(1500) \to \pi\pi)} \approx 0.21, \qquad \frac{\Gamma(f_0(1500) \to \eta\eta)}{\Gamma(f_0(1500) \to \pi\pi)} \approx 0.02, \tag{6}$$

where we have used the $\eta - \eta'$ mixing angle $\theta = -(15.4 \pm 1.0)°$ [28]. The corresponding experimental results are 0.246 ± 0.026 and 0.145 ± 0.027 [27]. We see that although the ratio of $K\bar{K}/\pi\pi$ is well accommodated, the predicted ratio for $\eta\eta/\pi\pi$ is too small. This can be understood as follows. Assuming no SU(3) breaking in the decay amplitude, we obtain[3]

$$\frac{\Gamma(f_0(1500) \to K\bar{K})}{\Gamma(f_0(1500) \to \pi\pi)} = \frac{1}{3}\left(1 + \frac{s_2}{u_2}\right)^2 \frac{p_K}{p_\pi}, \qquad \frac{\Gamma(f_0(1500) \to \eta\eta)}{\Gamma(f_0(1500) \to \pi\pi)} = \frac{1}{27}\left(2 + \frac{s_2}{u_2}\right)^2 \frac{p_\eta}{p_\pi}, \tag{7}$$

where p_h is the c.m. momentum of the hadron h, u_2 and s_2 are the $f_0(1500)$ wave function coefficients defined in the orthogonal transformation U

$$\begin{pmatrix} a_0(1450) \\ f_0(1500) \\ f_0(1370) \\ f_0(1710) \end{pmatrix} = U \begin{pmatrix} |U\rangle \\ |D\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix} = \begin{pmatrix} u_1 & d_1 & s_1 & g_1 \\ u_2 & d_2 & s_2 & g_2 \\ u_3 & d_3 & s_3 & g_3 \\ u_4 & d_4 & s_4 & g_4 \end{pmatrix} \begin{pmatrix} |U\rangle \\ |D\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}. \tag{8}$$

For $a_0(1450)$, $s_1 = g_1 = 0$ and $u_1 = -d_1 = \frac{1}{\sqrt{2}}$. To derive Eq. (7) we have, for simplicity, applied the $\eta - \eta'$ mixing angle $\theta = -19.5°$ so that the wave functions of η and η' have the simple expressions:

$$|\eta\rangle = \frac{1}{\sqrt{3}}|u\bar{u} + d\bar{d} - s\bar{s}\rangle, \qquad |\eta'\rangle = \frac{1}{\sqrt{6}}|u\bar{u} + d\bar{d} + 2s\bar{s}\rangle. \tag{9}$$

Since $s_2 = -2u_2 = -2d_2$ for the octet $f_0(1500)$, it is evident from Eq. (7) that $f_0(1500)$ will not decay into $\eta\eta$ (or strongly suppressed) if SU(3) symmetry is exact. This implies that SU(3) symmetry must be broken in the mass matrix and/or in the decay amplitudes.

III. SU(3) BREAKING AND CHIRAL SUPPRESSION

As discussed before, SU(3) symmetry leads naturally to the near degeneracy of $a_0(1450)$, $K_0^*(1430)$ and $f_0(1500)$. However, in order to accommodate the observed branching ratios of strong decays, SU(3) symmetry must be broken to certain degree in the mass matrix and/or in the decay amplitudes. One also needs $M_S > M_U = M_D$ in order to lift the degeneracy of $a_0(1450)$ and $f_0(1500)$. Since the SU(3) breaking effect is expected to be weak, they will be treated perturbatively. In the mass matrix in Eq. (1), the glueball-quarkonia mixing has been computed in lattice QCD with the results [15]

$$y = 43 \pm 31 \text{ MeV}, \qquad y/y_s = 1.198 \pm 0.072, \tag{10}$$

which confirms that the magnitudes of y and x are about the same, as expected.

For strong decays, we consider a simple effective Hamiltonian of a scalar state decaying into two pseudoscalar mesons for the OZI allowed, OZI suppressed, and doubly OZI suppressed interactions[4]:

$$\mathcal{H}_{\rm SPP} = f_1 \,{\rm Tr}[X_F PP] + f_2 X_G \,{\rm Tr}[PP] + f_3 X_G \,{\rm Tr}[P]\,{\rm Tr}[P], \tag{11}$$

where

$$X_F = \begin{pmatrix} u\bar{u} & 0 & 0 \\ 0 & d\bar{d} & 0 \\ 0 & 0 & s\bar{s} \end{pmatrix} = \begin{pmatrix} \sum u_i F_i & 0 & 0 \\ 0 & \sum d_i F_i & 0 \\ 0 & 0 & \sum s_i F_i \end{pmatrix}, \qquad X_G = \sum g_i F_i, \tag{12}$$

with $F_i = (a_0(1450), f_0(1500), f_0(1370), f_0(1710))$. P is the pseudoscalar nonet

[3]Equation (7) also can be obtained from Eq. (16) by neglecting the glueball contribution from $f_0(1500)$.

[4]In principle, one can add more interaction terms such as ${\rm Tr}[X_F]{\rm Tr}[PP]$ terms (see [5] for detail). Because of the presumed narrowness of the glueball width, we assume its decay to hadrons is large N_c suppressed. We still call them OZI suppressed and doubly OZI suppressed in terms of the ways the mesons are formed from the quark lines.

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} \end{pmatrix} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{a_\eta \eta + a_{\eta'}\eta'}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{a_\eta \eta + a_{\eta'}\eta'}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & b_\eta \eta + b_{\eta'}\eta' \end{pmatrix}, \tag{13}$$

with

$$a_\eta = b_{\eta'} = \frac{\cos\theta - \sqrt{2}\sin\theta}{\sqrt{3}}, \qquad a_{\eta'} = -b_\eta = \frac{\sin\theta + \sqrt{2}\cos\theta}{\sqrt{3}}. \tag{14}$$

In the above equation, θ is the $\eta - \eta'$ mixing angle defined by

$$\eta = \eta_8 \cos\theta - \eta_0 \sin\theta, \qquad \eta' = \eta_8 \sin\theta + \eta_0 \cos\theta. \tag{15}$$

The invariant amplitudes squared for various strong decays are given by

$$\begin{aligned} |A(F_i \to K\bar{K})|^2 &= 2f_1^2(r_a u_i + s_i + 2\rho_s^{KK} g_i)^2, \\ |A(F_i \to \pi\pi)|^2 &= 6f_1^2(u_i + \rho_s^{\pi\pi} g_i)^2, \\ |A(F_i \to \eta\eta)|^2 &= 2f_1^2\Big(a_\eta^2 u_i + r_a b_\eta^2 s_i + \rho_s^{\eta\eta}(a_\eta^2 + b_\eta^2) g_i \\ &\quad + \rho_{ss}\Big(2a_\eta^2 + b_\eta^2 + \frac{4}{\sqrt{2}} a_\eta b_\eta\Big) g_i\Big)^2, \\ |A(a_0 \to \pi\eta)|^2 &= 4f_1^2 a_\eta^2 u_1^2, \end{aligned} \tag{16}$$

where $\rho_s = f_2/f_1$ and $\rho_{ss} = f_3/f_1$ are the ratios of the OZI suppressed and the doubly OZI suppressed couplings to that of the OZI allowed one. The invariant amplitudes squared for $f_2(1270) \to K\bar{K}, \pi\pi, \eta\eta$ are similar to that of F_i. (The quark content of $f_2(1270)$ is $(u\bar{u} + d\bar{d})/\sqrt{2}$.) Likewise, the invariant amplitudes squared for $a_2(1320) \to K\bar{K}$ and $\pi\eta$ are similar to that of $a_0(1450)$. The contribution characterized by the coupling f_3 or ρ_{ss} arises only from the SU(3)-singlet η_0. The parameter r_a denotes a possible SU(3) breaking effect in the OZI allowed decays when the $s\bar{s}$ pair is created relative to the $u\bar{u}$ and $d\bar{d}$ pairs. In principle, it can be determined from the ratios $\frac{\Gamma(a_0(1450)\to K\bar{K})}{\Gamma(a_0(1450)\to \pi\eta)}$, $\frac{\Gamma(a_2(1320)\to K\bar{K})}{\Gamma(a_2(1320)\to \pi\eta)}$, $\frac{\Gamma(f_2(1270)\to K\bar{K})}{\Gamma(f_2(1270)\to \pi\pi)}$ and $\frac{\Gamma(f_2(1270)\to \eta\eta)}{\Gamma(f_2(1270)\to \pi\pi)}$.

We now explain why we put the superscripts $K\bar{K}$, $\pi\pi$ and $\eta\eta$ to the parameter ρ_s in Eq. (16). It is clear from Eq. (16) that, for a pure glueball decay into $\pi\pi$ and $K\bar{K}$, we have

$$\frac{\Gamma(G \to \pi\pi)}{\Gamma(G \to K\bar{K})} = \frac{3}{4}\left(\frac{\rho_s^{\pi\pi}}{\rho_s^{K\bar{K}}}\right)^2 \frac{p_\pi}{p_K}. \tag{17}$$

If the coupling f_2 (and hence ρ_s) between glueball and two pseudoscalar mesons is flavor independent, i.e. $\rho_s^{K\bar{K}} = \rho_s^{\pi\pi}$, then it is expected that $\Gamma(G \to \pi\pi)/\Gamma(G \to K\bar{K}) = 0.91$. Since $\Gamma(f_0(1710) \to \pi\pi)/\Gamma(f_0(1710) \to K\bar{K})$ is measured to be 0.20 ± 0.04 by WA102 [29], <0.11 by BES from $J/\psi \to \omega(K^+K^-, \pi^+\pi^-)$ decays [30] and $0.41^{+0.11}_{-0.17}$ from $J/\psi \to \gamma(K^+K^-, \pi^+\pi^-)$ decays [31],[5] this implies a relatively large suppression of $\pi\pi$ production relative to $K\bar{K}$ in scalar glueball decays if $f_0(1710)$ is a pure 0^{++} glueball. To explain the large disparity between $\pi\pi$ and $K\bar{K}$ production in scalar glueball decays, Chanowitz [32] advocated that a pure scalar glueball cannot decay into quark-antiquark in the chiral limit, i.e.

$$A(G \to q\bar{q}) \propto m_q. \tag{18}$$

Since the current strange quark mass is an order of magnitude larger than m_u and m_d, decay to $K\bar{K}$ is largely favored over $\pi\pi$. Furthermore, it has been pointed out that chiral suppression will manifest itself at the hadron level [33]. To this end, it is suggested in [33] that m_q in Eq. (18) should be interpreted as the scale of chiral symmetry breaking since chiral symmetry is broken not only by finite quark masses but is also broken spontaneously. Consequently, chiral suppression for the ratio $\Gamma(G \to \pi\pi)/\Gamma(G \to K\bar{K})$ is not so strong as the current quark mass ratio m_u^2/m_s^2. A pQCD calculation in [33] yields

$$\frac{A(G \to \pi^+\pi^-)}{A(G \to K^+K^-)} \approx \left(\frac{f_\pi}{f_K}\right)^2, \tag{19}$$

due mainly to the difference of the π and K light-cone distribution functions. Lattice calculations [34] seem to confirm the chiral suppression effect (see footnote 2 of [35]) with the results

$$\rho_s^{\pi\pi}:\rho_s^{K\bar{K}}:\rho_s^{\eta\eta} = 0.834^{+0.603}_{-0.579}:2.654^{+0.372}_{-0.402}:3.099^{+0.364}_{-0.423}, \tag{20}$$

which are in sharp contrast to the flavor-symmetry limit with $\rho_s^{\pi\pi}:\rho_s^{K\bar{K}}:\rho_s^{\eta\eta} = 1:1:1$. Although the errors are large, the lattice results show a sizable deviation from this limit.

From Eq. (16), the ratio of $\pi\pi$ and $K\bar{K}$ productions in $f_0(1710)$ decays is given by

$$\frac{\Gamma(f_0(1710) \to \pi\pi)}{\Gamma(f_0(1710) \to K\bar{K})} = 3\left(\frac{u_4 + \rho_s^{\pi\pi} g_4}{r_a u_4 + s_4 + 2\rho_s^{K\bar{K}} g_4}\right)^2 \frac{p_\pi}{p_K}. \tag{21}$$

At first sight, it appears that if ρ_s is negative, the destructive interference between the glueball and quark contents

[5] For the purpose of fitting, we will use $\Gamma(f_0(1710) \to \pi\pi)/\Gamma(f_0(1710) \to K\bar{K}) = 0.30 \pm 0.20$ as the experimental input.

of $f_0(1710)$ may lead to the desired suppression of $\pi\pi$ production even if the glueball decay is flavor blind, i.e. $\rho_s^{K\bar{K}} = \rho_s^{\pi\pi}$. That is, it seems possible that one does not need chiral suppression in order to explain the observed suppression of $\pi\pi$ relative to $K\bar{K}$. However, as we shall see below, chiral nonsuppression will lead to too small a width for $f_0(1710)$ which is several orders of magnitude smaller than the experimental width of 138 ± 9 MeV [27]. As stressed in [16], it is important to impose the condition that the total sum of the partial decay widths of $f_0(1710)$ into two pseudoscalar mesons to be comparable to but smaller than the total width. Without such a constraint, a local minimum for χ^2 can occur where $\Gamma[f_0(1710)]$ is either too large or too small compared to experiments.

IV. NUMERICAL RESULTS

To illustrate our mixing model with numerical results, we first fix the parameters $x = -44$ MeV, $x_{ss}/x_s = x_s/x = 0.82$ and $\theta = -14.4°$. For the chiral suppression in scalar glueball decay, we parametrize $\rho_s^{\pi\pi} = \rho_s$, $\rho_s^{K\bar{K}} = \rho_s r_s$ and $\rho_s^{\eta\eta}(a_\eta^2 + b_\eta^2) = \rho_s(a_\eta^2 + r_s^2 b_\eta^2)$ with r_s being an SU(3) breaking parameter in the OZI suppressed decays. We shall consider two cases of chiral suppression; (i) $r_s = 1.55$, so that $\rho_s^{\pi\pi}:\rho_s^{K\bar{K}}:\rho_s^{\eta\eta} = 1:1.55:1.59$ and (ii) $r_s = 3.15$ which corresponds to $\rho_s^{\pi\pi}:\rho_s^{K\bar{K}}:\rho_s^{\eta\eta} = 1:3.15:4.74$. They are in the region allowed by Eq. (20). Choosing $y = 64$ MeV and $y/y_s = 1.19$ in the range constrained by Eq. (10) and performing a best χ^2 fit to the experimental masses of $f_0(1710)$, $f_0(1500)$, $f_0(1370)$ and branching ratios of $f_0(1710)$, $f_0(1500)$, $a_0(1450)$, $a_2(1320)$ and $f_2(1270)$, we obtain the fitted parameters as shown in Table I. As noticed in passing, the quarkonium $n\bar{n}$ mass M_N is fixed by the $a_0(1450)$ state. The fitted masses and branching ratios are summarized in Table II, while the predicted decay properties of scalar mesons are exhibited in Table III.

Some of the strong decay modes are not used for the fit. The experimental measurements of $\Gamma(f_0(1370) \to K\bar{K})/\Gamma(f_0(1370) \to \pi\pi)$ range from 1.33 ± 0.67 [39], 0.91 ± 0.20 [40], $0.46 \pm 0.15 \pm 0.11$ [36] to 0.12 ± 0.06 [41] and 0.08 ± 0.08 [37]. Likewise, the result for $f_0(1370) \to \eta\eta$ spans a large range. Consequently, the decays of $f_0(1370)$ are not employed as the fitting input. The decay $f_0(1500) \to \eta\eta'$ is also not used for the fit since $\eta\eta'$ is produced at threshold and hence its measurement could be subject to large uncertainties.

TABLE I. Fitted parameters for two cases of chiral suppression in $G \to PP$ decay: (i) $r_s = 1.55$ and (ii) $r_s = 3.15$. Parameters denoted by "*" are input ones.

	M_N (MeV)*	M_S (MeV)	M_G (MeV)	x_s/x*	y_s/y*	r_a	ρ_s	ρ_{ss}
(i)	1474	1507	1665	0.82	0.82	1.21	−0.48	0
(ii)	1474	1498	1666	0.82	0.82	1.22	0.10	0.12

TABLE II. Fitted masses and branching ratios for two cases of chiral suppression in $G \to PP$ decay: (i) $r_s = 1.55$ and (ii) $r_s = 3.25$.

	Experiment	fit (i)	fit (ii)
$M_{f_0(1710)}$ (MeV)	1714 ± 5 [27]	1715	1715
$M_{f_0(1500)}$ (MeV)	1507 ± 5 [27]	1510	1504
$M_{f_0(1370)}$ (MeV)	1350 ± 150 [27]	1348	1346
$\frac{\Gamma(f_0(1500)\to\eta\eta)}{\Gamma(f_0(1500)\to\pi\pi)}$	0.145 ± 0.027 [27]	0.068	0.081
$\frac{\Gamma(f_0(1500)\to K\bar{K})}{\Gamma(f_0(1500)\to\pi\pi)}$	0.246 ± 0.026 [27]	0.26	0.27
$\frac{\Gamma(f_0(1710)\to\pi\pi)}{\Gamma(f_0(1710)\to K\bar{K})}$	0.30 ± 0.20 (see text)	0.21	0.34
$\frac{\Gamma(f_0(1710)\to\eta\eta)}{\Gamma(f_0(1710)\to K\bar{K})}$	0.48 ± 0.15 [29]	0.26	0.51
$\frac{\Gamma(a_0(1450)\to K\bar{K})}{\Gamma(a_0(1450)\to\pi\eta)}$	0.88 ± 0.23 [27]	1.10	1.12
$\frac{\Gamma(a_2(1320)\to K\bar{K})}{\Gamma(a_2(1320)\to\pi\eta)}$	0.34 ± 0.06 [27]	0.45	0.46
$\frac{\Gamma(f_2(1270)\to K\bar{K})}{\Gamma(f_2(1270)\to\pi\pi)}$	$0.054^{+0.005}_{-0.006}$ [27]	0.056	0.057
$\frac{\Gamma(f_2(1270)\to\eta\eta)}{\Gamma(f_2(1270)\to\pi\pi)}$	0.003 ± 0.001 [36]	0.005	0.005
χ^2/d.o.f.		2.6	2.5

The mixing matrices obtained in both cases have the similar results:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.78 & 0.51 & -0.36 \\ -0.54 & 0.84 & 0.03 \\ 0.32 & 0.18 & 0.93 \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}. \tag{22}$$

It is evident that $f_0(1710)$ is composed primarily of the scalar glueball, $f_0(1500)$ is close to an SU(3) octet, and $f_0(1370)$ consists of an approximated SU(3) singlet with some glueball component ($\sim 10\%$). Unlike $f_0(1370)$, the glueball content of $f_0(1500)$ is very tiny because an SU(3) octet does not mix with the scalar glueball.

To compute the partial decay widths of $f_0(1710)$ and $f_0(1370)$ we have used the measured $\Gamma(f_0(1500) \to \pi\pi) = 34 \pm 4$ MeV [27] to fix the strong coupling f_1.

TABLE III. Predicted decay properties of scalar mesons for (i) $r_s = 1.55$ and (ii) $r_s = 3.25$. For partial widths of $f_0(1710)$ and $f_0(1500)$, we have summed over $PP = K\bar{K}, \pi\pi, \eta\eta$ states.

	Experiment	fit (i)	fit (ii)
$\frac{\Gamma(f_0(1370)\to K\bar{K})}{\Gamma(f_0(1370)\to\pi\pi)}$	see text	1.27	0.79
$\frac{\Gamma(f_0(1370)\to\eta\eta)}{\Gamma(f_0(1370)\to K\bar{K})}$	0.35 ± 0.30 [27]	0.21	0.12
$\frac{\Gamma(J/\psi\to\omega f_0(1710))}{\Gamma(J/\psi\to\phi f_0(1710))}$	6.6 ± 2.7 [30,37,38]	3.8	4.1
$\frac{\Gamma(J/\psi\to\omega f_0(1500))}{\Gamma(J/\psi\to\phi f_0(1500))}$		0.44	0.47
$\frac{\Gamma(J/\psi\to\omega f_0(1370))}{\Gamma(J/\psi\to\phi f_0(1370))}$		2.85	2.56
$\Gamma_{f_0(1710)\to PP}$ (MeV)	$<138 \pm 9$ [27]	84	133
$\Gamma_{f_0(1370)\to PP}$ (MeV)		406	146

We see from Table III that the predicted 2-body PP decay width of $f_0(1710)$ in case (i) is smaller than that in case (ii). This is because, given a smaller $r_s(= 1.55)$, the parameter ρ_s has to be negative in order to fit the ratio of $\Gamma(\pi\pi)/\Gamma(K\bar{K})$ in $f_0(1710)$ decay. This in turn leads to a suppressed $f_0(1710)$ width due to the destructive interference between the glueball and quarkonia contributions. Note that, apart from the two-body decay modes $K\bar{K}$, $\pi\pi$ and $\eta\eta$, none of the multihadron modes in $f_0(1710)$ decay has been seen, though theoretically $G \to q\bar{q}g$ and $G \to qq\bar{q}\,\bar{q}$ are not chirally suppressed [32]. In our favored scenario (ii), the calculated partial width of 133 MeV is in agreement with the scenario that the decay of $f_0(1710)$ is saturated by the PP pairs.

It should be stressed that, in the absence of chiral suppression in $G \to PP$ decay, namely, $\rho_s^{\pi\pi} = \rho_s^{K\bar{K}}$, the $f_0(1710)$ width is predicted to be less than 1 MeV and hence is ruled out by experiment. This is a strong indication in favor of chiral suppression of $G \to \pi\pi$ relative to $G \to K\bar{K}$. We note that fitted $|\rho_s|$ and $|\rho_{ss}|$ are less than unity in both (i) and (ii). This supports the supposition that the OZI suppressed decays via the f_2 and f_3 terms in Eq. (24) are smaller than the OZI allowed decay via the f_1 term.

Apart from the partial widths of $f_0(1710)$ and $f_0(1370)$, scenarios (i) and (ii) also differ in the predictions of $f_0(1370) \to K\bar{K}/\pi\pi$ (see Table III) and $f_0(1710) \to \eta\eta/K\bar{K}$ (Table II). Because the glueball and quark contents in the wave function of $f_0(1370)$ have an opposite sign and the parameter ρ_s is negative in the case of (i) as noted in passing, the interference between $q\bar{q}$ and glueball amplitudes turns out to be constructive in $f_0(1370) \to (\pi\pi, K\bar{K})$ decays [see Eq. (21) with $f_0(1700)$ replaced by $f_0(1370)$]. Consequently, the ratio $R \equiv \Gamma(f_0(1370) \to K\bar{K})/\Gamma(f_0(1370) \to \pi\pi)$ is larger than unity in case (i). In our favored case (ii), R is predicted to be 0.79. The χ^2 value in both cases is almost entirely governed by the ratio $\frac{\Gamma(f_0(1500)\to\eta\eta)}{\Gamma(f_0(1500)\to\pi\pi)}$ whose measurement ranges from 0.230 ± 0.097 [42] to 0.18 ± 0.03 [36] and 0.080 ± 0.033 [43]. If the measured ratio is close to our prediction of 0.08, the χ^2 value will be greatly reduced.

In our scheme, it is easy to understand why $J/\psi \to \omega f_0(1710)$ has a rate larger than $J/\psi \to \phi f_0(1710)$ This is because the $n\bar{n}$ content is more copious than $s\bar{s}$ in $f_0(1710)$. Just as the scalar meson decay into two pseudoscalar mesons, we can use the similar Hamiltonian for the vector-vector-scalar interaction as in Eq. (23)

$$\mathcal{H}_{S(J/\psi)V} = h_1 \mathrm{Tr}[X_F V] + h_2 X_G \mathrm{Tr}[V] + h_3 \mathrm{Tr}[X_F]\mathrm{Tr}[V] \qquad (23)$$

to write

$$|A(J/\psi \to \phi F_i)|^2 = h_1^2 s_i^2, \quad |A(J/\psi \to \omega F_i)|^2 = 2h_1^2 u_i^2, \qquad (24)$$

where we have neglected the h_2 and h_3 terms which are presumably OZI suppressed. Our prediction of $\Gamma(J/\psi \to \omega f_0(1710))/\Gamma(J/\psi \to \phi f_0(1710)) = 4.1$ is consistent with the observed value of 6.6 ± 2.7.[6] If $f_0(1710)$ is dominated by $s\bar{s}$ as advocated before [3,4], one will naively expect a suppression of the $\omega f_0(1710)$ production relative to $\phi f_0(1701)$. One way to circumvent this apparent contradiction with experiment is to assume a large OZI violating effects in the scalar meson production [4]. That is, the doubly OZI suppressed process (i.e. doubly disconnected diagram) is assumed to dominate over the singly OZI suppressed (singly disconnected) process [4]. In contrast, a larger $\Gamma(J/\psi \to \omega f_0(1710))$ rate over that of $\Gamma(J/\psi \to \phi f_0(1710))$ is naturally accommodated in our scheme without resorting to large OZI violating effects.

The radiative decay $J/\psi \to \gamma f_0$ is an ideal place to test the scalar glueball content of f_0 since the leading short-distance mechanism for inclusive $J/\psi \to \gamma + X$ is $J/\psi \to \gamma + gg$. Its flavor-independence in J/ψ decays as well as in hadronic and $\gamma\gamma$ productions has been explored [44]. If $f_0(1710)$ is composed mainly of the scalar glueball, it should be the most prominent scalar produced in radiative J/ψ decay. Hence, it is expected that

$$\Gamma(J/\psi \to \gamma f_0(1710)) \gg \Gamma(J/\psi \to \gamma f_0(1500)). \qquad (25)$$

As for $J/\psi \to \gamma f_0(1370)$, it has a destructive interference between the glueball and $q\bar{q}$ components. From the Particle Data Group [27], $\mathcal{B}(J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K}) = (8.5^{+1.2}_{-0.9}) \times 10^{-4}$. Combining with the WA102 measurements [29]: $\mathcal{B}(f_0 \to \pi\pi)/\Gamma(f_0 \to K\bar{K}) = 0.20 \pm 0.04$ and $\Gamma(f_0 \to \eta\eta)/\Gamma(f_0 \to K\bar{K}) = 0.48 \pm 0.15$ yields $\mathcal{B}(J/\psi \to \gamma f_0(1710)) \sim 1.4 \times 10^{-3}$. For $f_0(1500)$, the BES result $\mathcal{B}(J/\psi \to \gamma f_0(1500) \to \gamma\pi\pi) = (6.7 \pm 2.8) \times 10^{-5}$ [31] together with $\mathcal{B}(f_0(1500) \to \pi\pi) = 0.349 \pm 0.023$ [27] gives $\mathcal{B}(J/\psi \to \gamma f_0(1500)) = (2.9 \pm 1.2) \times 10^{-4}$. Therefore, $\Gamma(J/\psi \to \gamma f_0(1710)) \sim 5\Gamma(J/\psi \to \gamma f_0(1500))$. This is consistent with the expectation from Eq. (25).

Finally we comment on the strange quark content of $f_0(1370)$. Although $\rho\rho$ and 4π are the dominant decay modes of $f_0(1370)$ [27], it does not necessarily imply that $f_0(1370)$ is mostly $n\bar{n}$. In principle, the $s\bar{s}$ content relative to $n\bar{n}$ can be determined from the ratio $R = \Gamma(f_0(1370) \to K\bar{K})/\Gamma(f_0(1370) \to \pi\pi)$. If $f_0(1370)$ is a pure $n\bar{n}$, R turns out to be 0.23. As noticed in passing, the measured ratio ranges from 1.33 ± 0.67 down to 0.08 ± 0.08. In our scheme, the $s\bar{s}$ and $u\bar{u}$ or $d\bar{d}$ components are similar [see Eq. (22)]. Because of the opposite sign between the glueball and quark contents in the wavefunction of $f_0(1370)$, R is predicted to be around 0.79 in our scheme. Another ideal

[6]The published BES measurements are $\mathcal{B}(J/\psi \to \phi f_0(1710) \to \phi K\bar{K}) = (2.0 \pm 0.7) \times 10^{-4}$ [37] and $\mathcal{B}(J/\psi \to \omega f_0(1710) \to \omega K\bar{K}) = (6.6 \pm 1.3) \times 10^{-4}$ [30]. For the latter, we shall use the updated value of $(13.2 \pm 2.6) \times 10^{-4}$ [38].

place for determining the strange quark component in $f_0(1370)$ is the decay $D_s^+ \to f_0(1370)\pi^+$ [45]. If $f_0(1370)$ is purely a $n\bar{n}$ state, it can proceed only via the W-annihilation diagram. In contrast, if $f_0(1370)$ has an $s\bar{s}$ content, the decay $D_s^+ \to f_0(1370)\pi^+$ will receive an external W-emission contribution. In practice, one can compare $\Gamma(D_s^+ \to f_0(1370)\pi^+ \to \pi^+\pi^+\pi^-)$ with $\Gamma(D^+ \to f_0(1370)\pi^+ \to \pi^+\pi^+\pi^-)$ without the information of $\mathcal{B}(f_0(1370) \to \pi\pi)$. Unfortunately, the experimental measurement of $D^+ \to f_0(1370)\pi^+ \to \pi^+\pi^+\pi^-$ is not yet available.

V. CONCLUSIONS

We have studied the isosinglet scalar mesons $f_0(1710)$, $f_0(1500)$, $f_0(1370)$ and their mixing. We employ two recent lattice results as the input for the mass matrix which is essentially the starting point for the mixing model between scalar mesons and the glueball; one is the isovector scalar meson $a_0(1450)$ which displays an unusual property of being nearly independent of quark mass for quark masses smaller than that of the strange, and the other is the scalar glueball mass at 1710 MeV in the quenched approximation. The former implies that, to first order approximation, flavor SU(3) is a good symmetry for the scalar mesons above 1 GeV. The latter indicates that the scalar glueball mass before mixing should be close to 1700 MeV rather than 1500 MeV.

Our main results are the following: (i) In the SU(3) symmetry limit, $f_0(1500)$ turns out to be a pure SU(3) octet and is degenerate with $a_0(1450)$, while $f_0(1370)$ is mainly an SU(3) singlet with a small mixing with $f_0(1710)$ which is composed primarily of a scalar glueball. These features remain essentially unchanged even when SU(3) breaking is taken into account when the glueball-$q\bar{q}$ mixing is about the same as that between $q\bar{q}$, i.e. $|y| \sim |x|$. (ii) Sources of SU(3) breaking in the mass matrix and in the decay amplitudes are discussed. Their effects are weak and can be treated perturbatively. (iii) Chiral suppression in the scalar glueball decay into two pseudoscalar mesons is essential for explaining the width and strong decays of $f_0(1710)$. (iv) The observed enhancement of $J/\psi \to \omega f_0(1710)$ production relative to $\phi f_0(1710)$ in hadronic J/ψ decays and the copious $f_0(1710)$ production in radiative J/ψ decays lend further support to the prominent glueball nature of $f_0(1710)$.

ACKNOWLEDGMENTS

We wish to thank M. Chanowitz, Xiao-Gang He, Shan Jin, J. Schechter and Q. Zhao for useful discussions. This research was supported in part by the National Science Council of R.O.C. under Grant Nos. NSC94-2112-M-001-016, NSC94-2112-M-001-023, NSC94-2811-M-001-059, and the USDOE grant No. DE-FG05-84ER40154.

APPENDIX: MIXING MATRIX OF NEUTRAL SCALAR MESONS

In this appendix we collect the mixing matrices of the isosinglet scalar mesons $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ that have been proposed in the literature. A typical result of the mixing matrices obtained by Amsler, Close and Kirk [3], Close and Zhao [4], and He *et al.* [5] is the following

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} -0.91 & -0.07 & 0.40 \\ -0.41 & 0.35 & -0.84 \\ 0.09 & 0.93 & 0.36 \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}, \tag{A1}$$

taken from [4].

A common feature of these analyses is that $M_S > M_G > M_N$ with M_G close to 1500 MeV and $M_S - M_N$ of the order $200 \sim 300$ MeV. Furthermore, $f_0(1710)$ is considered mainly as a $s\bar{s}$ state, while $f_0(1370)$ is dominated by the $n\bar{n}$ content and $f_0(1500)$ is composed primarily of a glueball with possible large mixing with $q\bar{q}$ states.

Based on the lattice calculations, Lee and Weingarten [15] found that $f_0(1710)$ is composed mainly of the scalar glueball, $f_0(1500)$ is dominated by the $s\bar{s}$ quark content, and $f_0(1370)$ is mainly governed by the $n\bar{n}$ component, but it also has a glueball content of 25%. Their result is

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.819(89) & 0.290(91) & -0.495(118) \\ -0.399(113) & 0.908(37) & -0.128(52) \\ 0.413(87) & 0.302(52) & 0.859(54) \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}. \tag{A2}$$

In this scheme, $M_S = 1514 \pm 11$ MeV, $M_N = 1470 \pm 25$ MeV and $M_G = 1622 \pm 29$ MeV.

With the chiral Lagrangian approach, Giacosa *et al.* [16] performed a fit to the experimental masses and decay widths of $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ and found four possible solutions, depending on whether the direct decay of the glueball component is considered. The first two solutions are obtained without direct glueball decay:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.86 & 0.24 & 0.45 \\ -0.45 & -0.06 & 0.89 \\ -0.24 & 0.97 & -0.06 \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}, \tag{A3}$$

and

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.81 & 0.19 & 0.54 \\ -0.49 & 0.72 & 0.49 \\ -0.30 & 0.67 & -0.68 \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}, \tag{A4}$$

while the last two solutions are phenomenological fits with direct glueball decay:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.79 & 0.26 & 0.56 \\ -0.58 & 0.02 & 0.81 \\ -0.20 & 0.97 & -0.16 \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}, \tag{A5}$$

and

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.82 & 0.57 & -0.07 \\ -0.57 & 0.82 & \sim 0 \\ -0.06 & 0.04 & 0.99 \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}. \tag{A6}$$

Among those four solutions, (A3) and (A5) are similar to the mixing matrix (A1).

A solution in which $f_0(1710)$ is dominated by a glueball state is also found by Burakovsky and Page [35]

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.908(50) & 0.133(50) & -0.397(80) \\ -0.305(80) & 0.860(20) & -0.410(40) \\ 0.287(50) & 0.493(20) & 0.821(20) \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}. \tag{A7}$$

Although this solution is similar to (A2) and (A6) and ours in (22), the mass difference of their M_S and M_N is of order 250 MeV. Consequently, the mass of the $f_0(1370)$ state is predicted to be 1218 MeV in [35]. In our case, M_S is larger than M_N by only ~25 MeV which reflects the result from the recent lattice calculation [6].

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