Renormalization of the baryon axial vector current in large-N_c chiral perturbation theory

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The baryon axial vector current is computed at one-loop order in heavy baryon chiral perturbation theory in the large-N_c limit, where N_c is the number of colors. Loop graphs with octet and decuplet intermediate states cancel to various orders in N_c as a consequence of the large-N_c spin-flavor symmetry of QCD baryons. These cancellations are explicitly shown for the general case of N_f flavors of light quarks. In particular, a new generic cancellation is identified in the renormalization of the baryon axial vector current at one-loop order. A comparison with conventional heavy baryon chiral perturbation theory is performed at the physical values $N_c = 3$, $N_f = 3$.

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I. INTRODUCTION

Despite the tremendous progress achieved in the understanding of the strong interactions with quantum chromodynamics (QCD), analytic calculations of the spectrum and properties of hadrons are not possible because the theory is strongly coupled at low energies, with no small expansion parameter. One thus has to resort to the implementation of alternative methods in order to extract low-energy consequences of QCD. Among these methods, chiral perturbation theory and the $1/N_c$ expansion (where N_c is the number of colors) have shed much light on the subject.

On the one hand, chiral perturbation theory exploits the symmetry of the QCD Lagrangian under $SU(3)_L \times SU(3)_R \times U(1)_V$ transformations of the three flavors of light quarks in the limit $m_q \to 0$. Chiral symmetry is spontaneously broken by the QCD vacuum to the vector subgroup $SU(3)_V \times U(1)_V$, giving rise to an octet of Goldstone bosons. Physical observables can be expanded order by order in powers of $p^2/\Lambda_\chi{}^2$ and $m_\Pi^2/\Lambda_\chi{}^2$, or equivalently, m_q/Λ_χ, where p is the meson momentum, m_Π is the mass of the Goldstone boson, and Λ_χ is the scale of chiral symmetry breaking. When chiral perturbation theory is extended to include baryons, it is convenient to introduce velocity-dependent baryon fields, so that the expansion of the baryon chiral Lagrangian in powers of m_q and $1/M_B$ (where M_B is the baryon mass) is manifest [1,2]. This so-called heavy baryon chiral perturbation theory was first applied to compute the chiral logarithmic corrections to the baryon axial vector current for baryon semileptonic decays due to meson loops [1,2]. While these corrections are large when only octet baryon intermediate states are kept [1], the inclusion of decuplet baryon intermediate states yields sizable cancellations between one-loop corrections [2]. This phenomenological observation can be rigorously explained in the context of the $1/N_c$ expansion [3–5] and will be illustrated in detail in the present paper for the case of the baryon axial vector current.

On the other hand, the generalization of QCD from $N_c = 3$ to $N_c \gg 3$ colors, known as the large-N_c limit, has also led to remarkable insights into the understanding of the nonperturbative QCD dynamics of hadrons. In the large-N_c limit the meson sector of QCD consists of a spectrum of narrow resonances and meson-meson scattering amplitudes are suppressed by powers of $1/\sqrt{N_c}$ [6]. The baryon sector of QCD, on the contrary, is more subtle to analyze [7] because in the large-N_c limit an exact contracted $SU(2N_f)$ spin-flavor symmetry (where N_f is the number of light quark flavors) emerges [3,8]. This symmetry can be used to classify large-N_c baryon states and matrix elements. It is then possible to consider physical quantities in the large-N_c limit, where corrections arise at relative orders $1/N_c$, $1/N_c^2$, and so on, which is precisely the origin of the $1/N_c$ expansion. Applications of this formalism to the computation of static properties of baryons range from masses [5,9,10], couplings [5,9,11,12] to magnetic moments [11,13], to name but a few.

In the present paper, we use a combined expansion in m_q and $1/N_c$. The $1/N_c$ chiral effective Lagrangian for the lowest-lying baryons was constructed in Refs. [14,15] and describes the interactions of the spin-$\frac{1}{2}$ baryon octet and the spin-$\frac{3}{2}$ baryon decuplet with the pion nonet. Within this framework we then compute the renormalization of the baryon axial vector current at the one-loop level. As already pointed out in Refs. [3–5,16], there are large-N_c cancellations between individual Feynman diagrams, provided one sums over all baryon states in a complete multiplet of the large-N_c $SU(6)$ spin-flavor symmetry, i.e., over both the octet and decuplet, and uses axial coupling ratios given by the large-N_c spin-flavor symmetry. In Ref. [16] the general structure of the various large-N_c cancellations was analyzed. In particular, a new large-N_c cancellation was identified. Our work goes beyond this global analysis as we explicitly evaluate the corresponding operator expressions that involve complicated structures of commutators and/or anticommutators of $SU(6)$ spin-flavor operators. Although straightforward in principle, the reduction of these operator products to a physical operator basis turns out to be quite tedious due to the considerable amount of group theory involved. Our final expressions explicitly demonstrate how these large-N_c cancellations occur. In particular, we show that the new large-N_c cancellation found in Ref. [16] is a generic feature of the corresponding commutator-anticommutator structure and does not just occur in the special case considered in this reference.

Our analysis also contains a comparison of the results obtained within the framework of large-N_c baryon chiral perturbation theory with conventional heavy baryon chiral perturbation theory (including both octet and decuplet baryons), where no $1/N_c$ expansion is involved. Both approaches agree—the large-N_c cancellations are guaranteed to occur as a consequence of the contracted $SU(6)$ spin-flavor symmetry present in the limit $N_c \to \infty$: No large numerical cancellations between loop diagrams with intermediate octet states and low-energy constants of the next-to-leading order effective Lagrangian, containing the effects of decuplet states, arise.

The present paper is organized as follows. In Sec. II we give a brief overview of the structure of the $1/N_c$ chiral effective Lagrangian for the lowest-lying baryons. In order to make the paper self-contained, Sec. II also contains the relevant large-N_c formalism. The renormalization of the baryon axial vector current is considered in Sec. III. Here, we present in detail our basic calculation, i.e., the reduction of complicated structures of commutators and/or anticommutators, and show how the various large-N_c cancellations occur. Formulas for the physically interesting case of three colors and three light quark flavors are given explicitly. In Sec. IV we discuss the renormalization of the baryon axial vector current within the framework of heavy baryon chiral perturbation theory in a form that allows us to then make the comparison with large-N_c baryon chiral perturbation theory in Sec. V; we close this latter section by performing a fit to the experimental data on baryon semileptonic decays. The inclusion of the η', which becomes a Goldstone boson in the limit $N_c \to \infty$, is performed in Sec. VI. Finally, we present our conclusions in Sec. VII. The paper contains three appendices. In Appendix A the most general expressions for the complicated reduced structures of commutators and/or anticommutators are given for an arbitrary number of colors and light quark flavors. Appendix B contains tables of matrix elements of spin-flavor operators relevant to discuss eight observed transitions between spin-$\frac{1}{2}$ baryons. In particular, we illustrate how one extracts the axial vector couplings for the semileptonic processes of physical interest. Finally, Appendix C lists the chiral coefficients occurring in the renormalization of the baryon axial vector current.

II. THE CHIRAL LAGRANGIAN FOR BARYONS IN THE $1/N_c$ EXPANSION

The formalism of heavy baryon chiral perturbation theory and the $1/N_c$ baryon chiral Lagrangian have been discussed in detail in Ref. [14]. In this section we restrict ourselves to presenting an overview and introducing our notation and conventions.

The $1/N_c$ baryon chiral Lagrangian which correctly implements nonet symmetry and contracted spin-flavor symmetry for baryons in the large-N_c limit can be written in the most general way as

$$\mathcal{L}_{\text{baryon}} = i\mathcal{D}^0 - \mathcal{M}_{\text{hyperfine}} + \text{Tr}(\mathcal{A}^k\lambda^c)A^{kc} + \frac{1}{N_c}\text{Tr}\left(\mathcal{A}^k\frac{2I}{\sqrt{6}}\right)A^k + \ldots, \qquad (1)$$

where

$$\mathcal{D}^0 = \partial^0 \mathbb{1} + \text{Tr}(\mathcal{V}^0\lambda^c)T^c. \qquad (2)$$

Each term in Eq. (1) involves a baryon operator which can be expressed as a polynomial in the $SU(6)$ spin-flavor generators [9]

$$J^k = q^\dagger \frac{\sigma^k}{2} q, \qquad T^c = q^\dagger \frac{\lambda^c}{2} q, \qquad G^{kc} = q^\dagger \frac{\sigma^k}{2}\frac{\lambda^c}{2} q, \qquad (3)$$

where $q^\dagger$ and q are $SU(6)$ operators that create and annihilate states in the fundamental representation of $SU(6)$, and σ^k and λ^c are the Pauli spin and Gell-Mann flavor matrices, respectively. In Eqs. (1)–(3) the flavor indices run from one to nine so the full meson nonet π, K, η, and η' is considered.

The baryon operator $\mathcal{M}_{\text{hyperfine}}$ denotes the spin splittings of the tower of baryon states with spins $1/2, \ldots, N_c/2$ in the flavor representations. Furthermore, the vector and axial vector combinations of the meson fields,

$$\mathcal{V}^0 = \frac{1}{2}(\xi\partial^0\xi^\dagger + \xi^\dagger\partial^0\xi), \qquad \mathcal{A}^k = \frac{i}{2}(\xi\nabla^k\xi^\dagger - \xi^\dagger\nabla^k\xi), \qquad (4)$$

couple to baryon vector and axial vector currents, respectively. Here $\xi = \exp[i\Pi(x)/f]$, where $\Pi(x)$ stands for the nonet of Goldstone boson fields (unless explicitly stated otherwise) and $f \approx 93$ MeV is the meson decay constant. In particular, the $\ell = 1$ flavor octet axial vector pion combination couples to the flavor octet baryon axial vector current, denoted by A^{kc} hereafter.

The QCD operators involved in $\mathcal{L}_{\text{baryon}}$ in Eq. (1) have well-defined $1/N_c$ expansions. Specifically, the baryon axial vector current A^{kc} is a spin-1 object, an octet under $SU(3)$, and odd under time reversal. Its $1/N_c$ expansion can be written as [9]

$$A^{kc} = a_1 G^{kc} + \sum_{n=2,3}^{N_c} b_n \frac{1}{N_c^{n-1}} \mathcal{D}_n^{kc} + \sum_{n=3,5}^{N_c} c_n \frac{1}{N_c^{n-1}} \mathcal{O}_n^{kc}, \tag{5}$$

where the $\mathcal{D}_n^{kc}$ are diagonal operators with nonzero matrix elements only between states with the same spin, and the $\mathcal{O}_n^{kc}$ are purely off-diagonal operators with nonzero matrix elements only between states with different spin. The first few terms in expansion (5) read

$$\mathcal{D}_2^{kc} = J^k T^c, \tag{6}$$

$$\mathcal{O}_2^{kc} = \epsilon^{ijk}\{J^i, G^{jc}\}, \tag{7}$$

$$\mathcal{D}_3^{kc} = \{J^k, \{J^r, G^{rc}\}\}, \tag{8}$$

$$\mathcal{O}_3^{kc} = \{J^2, G^{kc}\} - \tfrac{1}{2}\{J^k, \{J^r, G^{rc}\}\}. \tag{9}$$

Higher order terms can be obtained via $\mathcal{D}_n^{kc} = \{J^2, \mathcal{D}_{n-2}^{kc}\}$ and $\mathcal{O}_n^{kc} = \{J^2, \mathcal{O}_{n-2}^{kc}\}$ for $n \geq 4$. From the above definitions it is easy to verify that the operators $\mathcal{O}_{2m}^{kc}$ ($m = 1, 2, \ldots$) are forbidden in the expansion (5) because they are even under time reversal. Furthermore, the unknown coefficients a_1, b_n, and c_n in Eq. (5) have expansions in powers of $1/N_c$ and are order unity at leading order in the $1/N_c$ expansion. At the physical value $N_c = 3$ the series can be truncated as

$$A^{kc} = a_1 G^{kc} + b_2 \frac{1}{N_c} \mathcal{D}_2^{kc} + b_3 \frac{1}{N_c^2} \mathcal{D}_3^{kc} + c_3 \frac{1}{N_c^2} \mathcal{O}_3^{kc}. \tag{10}$$

The matrix elements of the space components of A^{kc} between $SU(6)$ symmetric states give the actual values of the axial vector couplings. For the octet baryons, the axial vector couplings are g_A, as conventionally defined in baryon β-decay experiments, with a normalization such that $g_A \approx 1.27$ and $g_V = 1$ for neutron decay.

Similarly, the baryon axial current A^k is a spin-1 object, a singlet under $SU(3)$ so its $1/N_c$ expansion can be written as [14]

$$A^k = \sum_{n=1,3}^{N_c} b_n^{1,1} \frac{1}{N_c^{n-1}} \mathcal{D}_n^k, \tag{11}$$

where $\mathcal{D}_1^k = J^k$ and $\mathcal{D}_{2m+1}^k = \{J^2, \mathcal{D}_{2m-1}^k\}$ for $m \geq 1$. The superscript on the operator coefficients of A^k denotes that they refer to the baryon singlet current. For $N_c = 3$, Eq. (11) reduces to

$$A^k = b_1^{1,1} J^k + b_3^{1,1} \frac{1}{N_c^2} \{J^2, J^k\}. \tag{12}$$

As for the baryon mass operator $\mathcal{M}$, its $1/N_c$ expansion can be written as [3,5,9,15]

$$\mathcal{M} = m_0 N_c \mathbb{1} + \sum_{n=2,4}^{N_c-1} m_n \frac{1}{N_c^{n-1}} J^n, \tag{13}$$

where m_n are unknown coefficients. The first term on the right-hand side of Eq. (13) is the overall spin-independent mass of the baryon multiplet and is removed from the chiral Lagrangian by the heavy baryon field redefinition [1]. The remaining terms are spin-dependent and define $\mathcal{M}_{\text{hyperfine}}$ introduced in Eq. (1). For $N_c = 3$ the hyperfine mass expansion reduces to a single operator

$$\mathcal{M}_{\text{hyperfine}} = \frac{m_2}{N_c} J^2. \tag{14}$$

III. RENORMALIZATION OF THE BARYON AXIAL VECTOR CURRENT

One of the earliest applications of Lagrangian (1) consisted in the calculation of nonanalytic meson-loop corrections in Ref. [14]. Specifically, the calculation of the flavor **27** contribution to the baryon masses was presented in this reference as an example.

The renormalization of the baryon axial vector current is another problem which can be analyzed within the formalism of Ref. [14]. Aspects of this problem have been discussed in the framework of heavy baryon chiral perturbation theory [1,2,17], the $1/N_c$ expansion [5,9,11], or in a simultaneous expansion in chiral symmetry breaking and $1/N_c$ [16,18]. This latter approach is implemented in the present work to the calculation of the renormalization of the baryon axial vector current at one-loop order, following the lines of Ref. [14]. There are, however, some aspects of this problem which have not been previously discussed and will be addressed here.

The baryon axial vector current A^{kc} is renormalized by the one-loop diagrams displayed in Fig. 1. These loop graphs have a calculable dependence on the ratio Δ/m_Π, where $\Delta \equiv M_\Delta - M_N$ is the decuplet-octet mass difference and m_Π is the meson mass. Let us discuss the dia-

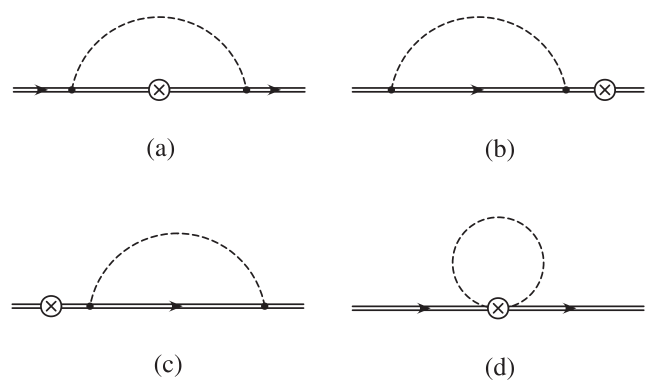

FIG. 1. One-loop corrections to the baryon axial vector current.

grams of Figs. 1(a)–1(d) separately, as they involve different commutator-anticommutator structures.

A. One-loop correction: Diagrams 1(a)–1(c)

We first consider the one-loop wave function renormalization graph Fig. 2, which is part of the diagrams 1(b) and 1(c). In this section we restrict ourselves to the computation of the octet meson corrections, such that Π denotes π, K, and η mesons. In Sec. VI we will then include the singlet η' correction into the analysis.

The Feynman diagram of Fig. 2 depends on the function $F(m_\Pi, \Delta, \mu)$ which is defined by the loop integral

$$\delta^{ij} F(m_\Pi, \Delta, \mu) = \frac{i}{f^2} \int \frac{d^4k}{(2\pi)^4} \times \frac{(\mathbf{k}^i)(-\mathbf{k}^j)}{(k^2 - m_\Pi^2)(k\cdot v - \Delta + i\epsilon)}. \quad (15)$$

This integral was solved using dimensional regularization in Ref. [19], so μ in Eq. (15) denotes the scale parameter. Therein, only the leading nonanalytic pieces were kept explicitly [20].

The correction arising from the sum of the diagrams of Figs. 1(a)–1(c), containing the full dependence on the ratio Δ/m_Π, was derived in Ref. [16] and reads

$$\begin{aligned}\delta A^{kc} = {}& \tfrac{1}{2}[A^{ja},[A^{jb},A^{kc}]]\Pi^{ab}_{(1)} \\ &- \tfrac{1}{2}\{A^{ja},[A^{kc},[\mathcal{M},A^{jb}]]\}\Pi^{ab}_{(2)} \\ &+ \tfrac{1}{6}\Big([A^{ja},[[\mathcal{M},[\mathcal{M},A^{jb}]],A^{kc}]] \\ &- \tfrac{1}{2}[[\mathcal{M},A^{ja}],[[\mathcal{M},A^{jb}],A^{kc}]]\Big)\Pi^{ab}_{(3)} + \ldots \end{aligned} \quad (16)$$

Here $\Pi^{ab}_{(n)}$ is a symmetric tensor which contains meson-loop integrals with the exchange of a single meson: A

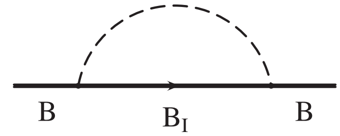

FIG. 2. One-loop wave function renormalization graph.

meson of flavor a is emitted and a meson of flavor b is reabsorbed. $\Pi^{ab}_{(n)}$ decomposes into flavor singlet, flavor **8**, and flavor **27** representations as [14]

$$\begin{aligned}\Pi^{ab}_{(n)} = {}& F^{(n)}_{\mathbf{1}}\delta^{ab} + F^{(n)}_{\mathbf{8}} d^{ab8} + F^{(n)}_{\mathbf{27}}[\delta^{a8}\delta^{b8} - \tfrac{1}{8}\delta^{ab} \\ &- \tfrac{3}{5}d^{ab8}d^{888}],\end{aligned} \quad (17)$$

where

$$\begin{aligned}F^{(n)}_{\mathbf{1}} = {}& \tfrac{1}{8}[3F^{(n)}(m_\pi,0,\mu) + 4F^{(n)}(m_K,0,\mu) \\ &+ F^{(n)}(m_\eta,0,\mu)],\end{aligned} \quad (18)$$

$$\begin{aligned}F^{(n)}_{\mathbf{8}} = {}& \tfrac{2\sqrt{3}}{5}[\tfrac{3}{2}F^{(n)}(m_\pi,0,\mu) - F^{(n)}(m_K,0,\mu) \\ &- \tfrac{1}{2}F^{(n)}(m_\eta,0,\mu)],\end{aligned} \quad (19)$$

$$\begin{aligned}F^{(n)}_{\mathbf{27}} = {}& \tfrac{1}{3}F^{(n)}(m_\pi,0,\mu) - \tfrac{4}{3}F^{(n)}(m_K,0,\mu) \\ &+ F^{(n)}(m_\eta,0,\mu).\end{aligned} \quad (20)$$

Note that Eqs. (18)–(20) are linear combinations of $F^{(n)}(m_\pi,0,\mu)$, $F^{(n)}(m_K,0,\mu)$, and $F^{(n)}(m_\eta,0,\mu)$, where $F^{(n)}(m_\Pi,0,\mu)$ represents the degeneracy limit $\Delta/m_\Pi = 0$ of the general function $F^{(n)}(m_\Pi,\Delta,\mu)$, defined as

$$F^{(n)}(m_\Pi,\Delta,\mu) \equiv \frac{\partial^n F(m_\Pi,\Delta,\mu)}{\partial\Delta^n}. \quad (21)$$

The first two derivatives of the function read

$$\begin{aligned}24\pi^2 f^2 F^{(1)}(m_\Pi,\Delta,\mu) = {}& 3\left[\Delta^2 - \frac{1}{2}m_\Pi^2\right]\ln\frac{m_\Pi^2}{\mu^2} - 6\Delta^2 - \frac{11}{2}m_\Pi^2 \\ &- \begin{cases} 3\Delta\sqrt{m_\Pi^2-\Delta^2}\left[\pi - 2\arctan\left(\frac{\Delta}{\sqrt{m_\Pi^2-\Delta^2}}\right)\right], & m_\Pi \geq |\Delta| \\ 3\Delta\sqrt{\Delta^2-m_\Pi^2}\ln\left[\frac{\Delta-\sqrt{\Delta^2-m_\Pi^2}}{\Delta+\sqrt{\Delta^2-m_\Pi^2}}\right], & m_\Pi \leq |\Delta| \end{cases}\end{aligned} \quad (22)$$

$$24\pi^2 f^2 F^{(2)}(m_\Pi,\Delta,\mu) = 6\Delta\left[\ln\frac{m_\Pi^2}{\mu^2} - 1\right] - \begin{cases} \frac{3(m_\Pi^2-2\Delta^2)}{\sqrt{m_\Pi^2-\Delta^2}}\left[\pi - 2\arctan\left(\frac{\Delta}{\sqrt{m_\Pi^2-\Delta^2}}\right)\right], & m_\Pi \geq |\Delta| \\ \frac{3(2\Delta^2-m_\Pi^2)}{\sqrt{\Delta^2-m_\Pi^2}}\ln\left[\frac{\Delta-\sqrt{\Delta^2-m_\Pi^2}}{\Delta+\sqrt{\Delta^2-m_\Pi^2}}\right]. & m_\Pi \leq |\Delta| \end{cases} \quad (23)$$

In the degeneracy limit $\Delta/m_\Pi = 0$ they thus reduce to

$$F^{(1)}(m_\Pi,0,\mu) = -\frac{m_\Pi^2}{16\pi^2 f^2}\left(\frac{11}{3} + \ln\frac{m_\Pi^2}{\mu^2}\right), \quad (24)$$

$$F^{(2)}(m_\Pi,0,\mu) = -\frac{m_\Pi}{8\pi f^2}. \quad (25)$$

In Eq. (24) the terms involving $11/3$ and $\ln(m_\Pi^2/\mu^2)$ are analytic and nonanalytic in the quark mass, respectively.

The former is scheme dependent and has the same form as higher dimension terms in the chiral Lagrangian whereas the latter is universal.

For $N_c = 3$, the baryon axial vector current A^{kc} has a $1/N_c$ expansion in terms of the four operators of Eq. (10). The correction δA^{kc}—Eq. (16)—contains n-body operators [21], with $n > N_c$, which are complicated commutators and/or anticommutators of the one-body operators J^k, T^c, and G^{kc}. All these higher order operators should be reduced and rewritten as linear combinations of the operator basis, with $n \leq N_c$. The fact that the operator basis is complete and independent facilitates this reduction [5,9]. In practice, however, dealing with these expressions becomes rather difficult. Before engaging ourselves in this task, it is convenient to have a useful $1/N_c$ power-counting scheme at hand to save a considerable effort.

There is a nontrivial N_c dependence of the matrix elements of the generators J^i, T^a, and G^{ia} in the weight diagrams for the $SU(3)$ flavor representations of the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons [9]. For instance, factors of T^a/N_c and G^{ia}/N_c are of order 1 somewhere in the weight diagram, whereas factors of J^i/N_c are of order $1/N_c$ everywhere. If we restrict ourselves to baryons with spins of order unity, the N_c counting rules can be summarized as [16]

$$T^a \sim N_c, \qquad G^{ia} \sim N_c, \qquad J^i \sim 1. \tag{26}$$

Note that factors of J^i/N_c are $1/N_c$ suppressed relative to factors of T^a/N_c and G^{ia}/N_c. Similarly, the meson decay constant $f \propto \sqrt{N_c}$, so the functions $F^{(n)}(m_\Pi, \Delta, \mu)$ introduce a $1/N_c$ suppression.

In order to evaluate the complicated expressions in Eq. (16), the mathematical groundwork developed in Ref. [9]—which involves a considerable amount of group theory—will be used here. First notice that the commutator of an m-body operator with an n-body operator is an $(m + n - 1)$-body operator, namely,

$$[\mathcal{O}^{(m)}, \mathcal{O}^{(n)}] = \mathcal{O}^{(m+n-1)}.$$

However, the anticommutator of an m-body operator and an n-body operator is in general an $(m + n)$-body operator. The $SU(2N_f)$ Lie algebra commutation relations between one-body operators are given in Table I. Along with these commutation relations, we will use the nontrivial two-body operator identities for $SU(2N_f)$ quark operators and their

TABLE I. $SU(2N_f)$ Commutation relations.

$[J^i, T^a] = 0$	
$[J^i, J^j] = i\epsilon^{ijk}J^k$	$[T^a, T^b] = if^{abc}T^c$
$[J^i, G^{ja}] = i\epsilon^{ijk}G^{ka}$	$[T^a, G^{ib}] = if^{abc}G^{ic}$
$[G^{ia}, G^{jb}] = \frac{i}{4}\delta^{ij}f^{abc}T^c + \frac{i}{2N_f}\delta^{ab}\epsilon^{ijk}J^k + \frac{i}{2}\epsilon^{ijk}d^{abc}G^{kc}$	

transformation properties under $SU(2) \times SU(N_f)$, which were derived in full in Ref. [9]. Let us now discuss the various terms occurring in the one-loop correction to the baryon axial vector current Eq. (16).

1. Diagrams 1(a)–1(c): Degeneracy limit $\Delta/m_\Pi = 0$

The first term in Eq. (16) is the double commutator

$$\frac{1}{2}[A^{ja}, [A^{jb}, A^{kc}]]\Pi^{ab}_{(1)}, \tag{27}$$

and corresponds to the degeneracy limit $\Delta/m_\Pi = 0$ for the correction to A^{kc}. Although this term has been already discussed in the literature [3,5,16,18], its explicit computation has not been presented in detail so far.

A crucial observation is the fact that the large-N_c consistency conditions derived in Ref. [3] set this double commutator to be $\mathcal{O}(N_c)$. Naively, one would expect the double commutator to be $\mathcal{O}(N_c^3)$: one factor of N_c from each A^{kc}. However, there are large-N_c cancellations between the Feynman diagrams of Figs. 1(a)–1(c), *provided all baryon states in a complete multiplet of the large-N_c $SU(6)$ spin-flavor symmetry are included in the sum over intermediate states and the axial coupling ratios predicted by this spin-flavor symmetry are used* [16]. We aim in this section to show explicitly how these cancellations occur.

For $N_c = 3$, it suffices taking the lowest-lying baryon states, which corresponds to the well-known **56**-dimensional representation of $SU(6)$, namely, octet and decuplet baryons. For larger N_c, there appear more complex representations containing unphysical states with spins greater than 3/2 and flavor representations bigger than the **8** and **10** [18]. It has already been shown in Ref. [16] that the terms GGG, $GG\mathcal{D}_2$, $G\mathcal{D}_2\mathcal{D}_2$, $GG\mathcal{D}_3$, and $GG\mathcal{O}_3$ in the product AAA contribute at the same order to the double commutator. In the present work we go one step further and also incorporate the terms $\mathcal{D}_2\mathcal{D}_2\mathcal{D}_2$, $G\mathcal{D}_2\mathcal{D}_3$, and $G\mathcal{D}_2\mathcal{O}_3$ into the analysis. This then means that in the correction to the baryon axial vector current (27) we will also include terms that represent $\mathcal{O}(1/N_c^2)$ corrections to the tree-level result $\mathcal{O}(N_c)$. Although our computation will be performed for an arbitrary number of light quark flavors N_f, without loss of generality, in this section we will present our results for the physically interesting case of three light flavors, $N_f = 3$. Results for arbitrary N_f are given in Appendix A for completeness.

In order to explicitly show the large-N_c cancellations in Eq. (27), it is useful to work out a few examples. At leading order in N_c, A^{kc} is given by a_1G^{kc} so that the double commutator $[a_1G^{ia}, [a_1G^{ib}, a_1G^{kc}]]$, for $N_f = 3$, yields

$$a_1^3[G^{ia}, [G^{ib}, G^{kc}]] = \frac{1}{12}a_1^3[3(-f^{bcd}f^{ade} + 2d^{bcd}d^{ade})G^{ke} + 4\delta^{bc}G^{ka} + 2d^{abc}J^k], \tag{28}$$

which is at most $\mathcal{O}(N_c)$ according to the counting rules (26), irrespective of the appropriate contractions of the flavor indices a and b: The contraction with either δ^{ab}, d^{ab8}, or $\delta^{a8}\delta^{b8}$ to construct an operator in the flavor singlet, octet, or **27** representations, respectively [see Eq. (17)], does not introduce any additional N_c-dependence.

At the next order in the $1/N_c$ expansion one has

$$
\begin{aligned}
&a_1^2 b_2 \frac{1}{N_c}([G^{ia},[G^{ib},\mathcal{D}_2^{kc}]] + [G^{ia},[\mathcal{D}_2^{ib},G^{kc}]] + [\mathcal{D}_2^{ia},[G^{ib},G^{kc}]]) \\
&\quad = a_1^2 b_2 \frac{1}{N_c}\Big(\frac{1}{6} i f^{abc} J^k - \frac{5}{4} f^{bcd} f^{ade} \mathcal{D}_2^{ke} + \frac{1}{3}\delta^{ab}\mathcal{D}_2^{kc} + \frac{1}{3}\delta^{ac}\mathcal{D}_2^{kb} + \frac{1}{3}\delta^{bc}\mathcal{D}_2^{ka} + d^{abe}G^{ke}T^c + d^{bce}G^{ke}T^a \\
&\qquad + d^{ace}G^{ke}T^b + \frac{1}{2}\epsilon^{kim}(f^{aed}d^{bce} + d^{aed}f^{bce})J^i G^{md} + \epsilon^{kim}(f^{bcd}G^{ma} + f^{acd}G^{mb} + f^{abd}G^{mc})G^{id}\Big), \qquad (29)
\end{aligned}
$$

which is also at most $\mathcal{O}(N_c)$. As for the additional subleading terms, the calculation is straightforward although tedious in practice in view of the considerable amount of group theory involved. The explicit expressions for arbitrary N_c and N_f may be found in Appendix A. To the order of approximation adopted here, the different flavor contributions originating from diagrams 1(a)–1(c), in the degeneracy limit can be organized as follows [22]:

(1) *Flavor singlet contribution*

$$
\begin{aligned}
[A^{ia},[A^{ia},A^{kc}]] &= \Big[\frac{23}{12}a_1^3 - \frac{2(N_c+3)}{3N_c}a_1^2 b_2 + \frac{N_c^2+6N_c-54}{6N_c^2}a_1 b_2^2 - \frac{N_c^2+6N_c+2}{N_c^2}a_1^2 b_3 \\
&\quad - \frac{N_c^2+6N_c-3}{N_c^2}a_1^2 c_3 - \frac{12(N_c+3)}{N_c^3}a_1 b_2 b_3\Big]G^{kc} + \frac{1}{N_c}\Big[\frac{101}{12}a_1^2 b_2 + \frac{4(N_c+3)}{3N_c}a_1 b_2^2 \\
&\quad - \frac{3(N_c+3)}{N_c}a_1^2 b_3 - \frac{N_c+3}{2N_c}a_1^2 c_3 + \frac{N_c^2+6N_c-18}{6N_c^2}b_2^3 + \frac{N_c^2+6N_c+2}{N_c^2}a_1 b_2 b_3 \\
&\quad - \frac{3(N_c^2+6N_c-24)}{2N_c^2}a_1 b_2 c_3\Big]\mathcal{D}_2^{kc} + \frac{1}{N_c^2}\Big[\frac{11}{4}a_1 b_2^2 + \frac{51}{4}a_1^2 b_3 + 2a_1^2 c_3 + \frac{17(N_c+3)}{3N_c}a_1 b_2 b_3 \\
&\quad - \frac{9(N_c+3)}{2N_c}a_1 b_2 c_3\Big]\mathcal{D}_3^{kc} + \frac{1}{N_c^3}\Big[\frac{5}{2}b_2^3 + \frac{11}{3}a_1 b_2 b_3 + 19 a_1 b_2 c_3\Big]\mathcal{D}_4^{kc} + \mathcal{O}(G\mathcal{D}_3\mathcal{D}_3). \qquad (30)
\end{aligned}
$$

The symbol $\mathcal{O}(G\mathcal{D}_3\mathcal{D}_3)$ means that, in the double commutator structure AAA, we have included all terms up to six-body operators, such as $G\mathcal{D}_2\mathcal{D}_3$, but have neglected contributions which are seven-body operators—like $G\mathcal{D}_3\mathcal{D}_3$—or higher.

(2) *Flavor octet contribution*

$$
\begin{aligned}
d^{ab8}[A^{ia},[A^{ib},A^{kc}]] &= \Big[\frac{11}{24}a_1^3 - \frac{2(N_c+3)}{3N_c}a_1^2 b_2 - \frac{9}{2N_c^2}a_1 b_2^2 - \frac{5}{N_c^2}a_1^2 b_3 + \frac{3}{2N_c^2}a_1^2 c_3 - \frac{6(N_c+3)}{N_c^3}a_1 b_2 b_3\Big]d^{c8e}G^{ke} \\
&\quad + \frac{1}{8N_c}\Big[23a_1^2 b_2 - \frac{2(N_c+3)}{N_c}(6a_1^2 b_3 + a_1^2 c_3) - \frac{12}{N_c^2}(b_2^3 + 2a_1 b_2 b_3 - 12 a_1 b_2 c_3)\Big]d^{ce8}\mathcal{D}_2^{ke} \\
&\quad - \frac{1}{6N_c}\Big[4a_1^2 b_2 + \frac{N_c+3}{N_c}(a_1 b_2^2 + 6a_1^2 b_3 + 6a_1^2 c_3) + \frac{36}{N_c^2}a_1 b_2 b_3\Big]\{G^{kc},T^8\} + \frac{1}{6N_c}\Big[11a_1^2 b_2 \\
&\quad + \frac{2(N_c+3)}{N_c}a_1 b_2^2 + \frac{48}{N_c^2}a_1 b_2 b_3\Big]\{G^{k8},T^c\} + \frac{1}{24N_c^2}\Big[27a_1 b_2^2 + 65a_1^2 b_3 + 8a_1^2 c_3 \\
&\quad + \frac{36(N_c+3)}{N_c}a_1 b_2 b_3 - \frac{46(N_c+3)}{N_c}a_1 b_2 c_3\Big]d^{c8e}\mathcal{D}_3^{ke} + \frac{1}{6N_c^2}\Big[3a_1 b_2^2 - 2a_1^2 b_3 + 30a_1^2 c_3
\end{aligned}
$$

$$+\frac{4(N_c+3)}{N_c}a_1b_2b_3\bigg]\{G^{kc},\{J^r,G^{r8}\}\}+\frac{1}{6N_c^2}\bigg[3a_1b_2^2+28a_1^2b_3-15a_1^2c_3+\frac{4(N_c+3)}{N_c}a_1b_2b_3\bigg]\{G^{k8},\{J^r,G^{rc}\}\}$$
$$+\frac{1}{3N_c^2}\bigg[12a_1^2b_3-2a_1^2c_3-\frac{2(N_c+3)}{N_c}a_1b_2c_3\bigg]\{J^k,\{G^{rc},G^{r8}\}\}+\frac{1}{12N_c^2}\bigg[2a_1b_2^2-9a_1^2b_3-\frac{3}{2}a_1^2c_3$$
$$-\frac{N_c+3}{N_c}(b_2^3-6a_1b_2b_3+9a_1b_2c_3)\bigg]\{J^k,\{T^c,T^8\}\}+\frac{1}{3N_c^3}(3b_2^3-21a_1b_2b_3+20a_1b_2c_3)\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}$$
$$+\frac{1}{6N_c^3}(24a_1b_2b_3-23a_1b_2c_3)\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}+\frac{1}{3N_c^3}(a_1b_2b_3-2a_1b_2c_3)\{J^2,\{G^{kc},T^8\}\}+\frac{1}{6N_c^3}(20a_1b_2b_3$$
$$+11a_1b_2c_3)\{J^2,\{G^{k8},T^c\}\}+\frac{1}{128N_c^3}(10a_1b_2b_3+71a_1b_2c_3)(\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}-\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}$$
$$+\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\})+\frac{1}{4N_c^3}(3b_2^3+6a_1b_2b_3+24a_1b_2c_3)d^{c8e}\mathcal{D}_4^{ke}+\mathcal{O}(G\mathcal{D}_3\mathcal{D}_3). \tag{31}$$

(3) *Flavor* **27** *contribution*

$$[A^{i8},[A^{i8},A^{kc}]]=\bigg[\bigg(\frac{1}{4}a_1^3-\frac{1}{N_c^2}(2a_1b_2^2+2a_1^2b_3-a_1^2c_3)\bigg)f^{c8e}f^{8eg}+\frac{1}{2}\bigg(a_1^3+\frac{1}{N_c^2}(2a_1^2b_3-a_1^2c_3)\bigg)d^{c8e}d^{8eg}\bigg]G^{kg}$$
$$+\frac{1}{N_c}\bigg[\frac{1}{12}a_1^2b_2(4\delta^{cg}+21f^{c8e}f^{8eg})+\frac{1}{N_c^2}(-b_2^3+9a_1b_2b_3)f^{c8e}f^{8eg}\bigg]\mathcal{D}_2^{kg}$$
$$+\frac{1}{2N_c}a_1^2b_2(2d^{c8e}\{G^{ke},T^8\}+d^{88e}\{G^{ke},T^c\})+\frac{1}{N_c}\bigg(a_1^2b_2+\frac{4}{N_c^2}a_1b_2b_3\bigg)if^{c8e}[G^{k8},\{J^r,G^{re}\}]$$
$$-\frac{4}{N_c^3}a_1b_2b_3if^{c8e}[G^{ke},\{J^r,G^{r8}\}]+\frac{1}{12N_c^2}[9a_1b_2^2f^{c8e}f^{8eg}+a_1^2b_3(8\delta^{cg}+9f^{c8e}f^{8eg}+6d^{c8e}d^{8eg})$$
$$+6a_1^2c_3d^{c8e}d^{8eg}]\mathcal{D}_3^{kg}+\frac{1}{N_c^2}a_1^2c_3d^{88e}\{G^{kc},\{J^r,G^{re}\}\}+\frac{1}{2N_c^3}b_2^3\{\mathcal{D}_2^{kc},\{T^8,T^8\}\}+\frac{1}{2N_c^3}(-2a_1b_2b_3$$
$$+a_1b_2c_3)(2d^{c8e}\{\mathcal{D}_2^{k8},\{J^r,G^{re}\}\}+d^{88e}\{\mathcal{D}_2^{kc},\{J^r,G^{re}\}\})+\frac{1}{2N_c^2}a_1b_2^2(\{G^{kc},\{T^8,T^8\}\}$$
$$+2\{G^{k8},\{T^c,T^8\}\})+\frac{1}{N_c^2}(4a_1^2b_3-a_1^2c_3)d^{c8e}\{G^{ke},\{J^r,G^{r8}\}\}-\frac{1}{2N_c^2}(6a_1^2b_3+a_1^2c_3)d^{c8e}\{J^k,\{G^{re},G^{r8}\}\}$$
$$+\frac{1}{N_c^2}(2a_1^2b_3-a_1^2c_3)\{G^{rc},\{G^{r8},G^{k8}\}\}-\frac{1}{N_c^2}(2a_1^2b_3+a_1^2c_3)\{G^{kc},\{G^{r8},G^{r8}\}\}$$
$$+\frac{1}{2N_c^2}(-2a_1^2b_3+3a_1^2c_3)d^{c8e}\{G^{k8},\{J^r,G^{re}\}\}+\frac{1}{2N_c^2}(2a_1^2b_3-a_1^2c_3)(d^{88e}\{J^k,\{G^{rc},G^{re}\}\}$$
$$+d^{88e}\{G^{ke},\{J^r,G^{rc}\}\})+\frac{2}{N_c^3}a_1b_2b_3(\{\{J^r,G^{r8}\},\{G^{k8},T^c\}\}+\{\{J^r,G^{r8}\},\{G^{kc},T^8\}\}$$
$$+\{\{J^r,G^{rc}\},\{G^{k8},T^8\}\})-\frac{2}{N_c^3}a_1b_2c_3(2\{\mathcal{D}_2^{k8},\{G^{rc},G^{r8}\}\}+\{\mathcal{D}_2^{kc},\{G^{r8},G^{r8}\}\})+\frac{1}{2N_c^3}(2a_1b_2b_3$$
$$+a_1b_2c_3)(d^{88e}\{J^2,\{G^{ke},T^c\}\}+2d^{c8e}\{J^2,\{G^{ke},T^8\}\})+\frac{1}{N_c^3}\bigg[\frac{2}{3}a_1b_2c_3\delta^{cg}+\frac{1}{2}(b_2^3+9a_1b_2c_3)f^{c8e}f^{8eg}\bigg]$$
$$\times\mathcal{D}_4^{kg}+\frac{2}{N_c^3}a_1b_2c_3if^{c8e}\{J^2,[G^{k8},\{J^r,G^{re}\}]\}+\frac{1}{N_c^3}(2a_1b_2b_3-a_1b_2c_3)if^{c8e}\{J^2,[G^{ke},\{J^r,G^{r8}\}]\}$$
$$+\frac{2}{N_c^3}(a_1b_2b_3-a_1b_2c_3)if^{c8e}\{J^k,[\{J^i,G^{ie}\},\{J^r,G^{r8}\}]\}+\mathcal{O}(G\mathcal{D}_3\mathcal{D}_3). \tag{32}$$

In order for Eq. (32) to be a truly **27** contribution it is understood that flavor singlet and octet contributions should be subtracted off from this equation. For computational purposes the one-body operators T^8 and G^{i8} can be written in terms of the strange quark number operator N_s and the strange quark spin operator J_s^i as [9]

$$T^8=\frac{1}{2\sqrt{3}}(N_c-3N_s), \tag{33}$$

$$G^{i8}=\frac{1}{2\sqrt{3}}(J^i-3J_s^i). \tag{34}$$

These operators are order N_c and order 1, respectively.

Equations (30)–(32) have been rearranged to display leading and subleading terms in $1/N_c$ explicitly. Notice that only baryon operators with nonvanishing matrix elements between octet baryons have been kept in these equations (for the full expressions see Appendix A). Although the resulting expressions are rather lengthy, they are indeed enlightening. It is now evident that large-N_c cancellations occur in the evaluation of the double commutator so that it is at most $\mathcal{O}(N_c)$, according to the counting rules (26). The loop integrals are inversely proportional to f^2, which introduces a $1/N_c$ suppression. Therefore the net one-loop correction Eq. (27) is $\mathcal{O}(1)$, or $1/N_c$ times the tree-level value, which is $\mathcal{O}(N_c)$. The large-N_c cancellations in the renormalization of the baryon axial vector current to one-loop have thus been explicitly shown to occur in the degeneracy limit $\Delta/m_\Pi = 0$.

2. Diagrams 1(a)–1(c): Nondegeneracy case $\Delta/m_\Pi \neq 0$

Let us now discuss the additional terms that contribute to the renormalization of the baryon axial vector current for finite Δ/m_Π. The procedure for obtaining these terms is discussed in Ref. [16]. Specifically, let us consider the second term in Eq. (16),

$$\tfrac{1}{2}\{A^{ja},[A^{kc},[\mathcal{M},A^{jb}]]\}\Pi_{(2)}^{ab}. \tag{35}$$

This expression contains one insertion of the baryon mass matrix $\mathcal{M}$ introduced in Eq. (13) and thus represents the leading term in the nondegenerate case. The large-N_c counting rules imply that multiple insertions of the J^2 factor in $\mathcal{M}$ constitute the dominant $1/N_c$ corrections from the baryon mass splittings: In Ref. [16], it has been shown that one insertion of J^4 in the term linear in $\mathcal{M}$ is $1/N_c$ suppressed relative to two insertions of J^2 in the term quadratic in $\mathcal{M}$—the third term in Eq. (16). Moreover, in the same reference it was concluded that the quantities $GGGJ^2$ and $GG\mathcal{D}_2J^2$ in the product $AAA\mathcal{M}$ contribute at the same order in Eq. (35) and should be retained in the series Eq. (16).

Returning to Eq. (35), a new large-N_c cancellation for the specific commutator-anticommutator structure $GGGJ^2$ was found in Ref. [16]. Naively, one would expect this contribution to be of $\mathcal{O}(N_c^3)$: The two operators J may be eliminated with the two commutators, such that we are left with a product of three operators GGG, each one contributing a factor of N_c. However, the explicit calculation of the singlet contribution of the operator expression $GGGJ^2$ shows that it is of $\mathcal{O}(N_c^2)$, i.e., suppressed by one factor of N_c. We would like to see whether the same pattern repeats itself in the octet and the **27** piece of $GGGJ^2$, and whether new large-N_c cancellations also occur in the operator structure $GG\mathcal{D}_2J^2$. The expressions, when retaining both structures $GGGJ^2$ and $GG\mathcal{D}_2J^2$ in the product $AAA\mathcal{M}$, read:

(1) *Flavor singlet contribution*

$$\begin{aligned}\{A^{ja},[A^{kc},[\mathcal{M},A^{ja}]]\} = \frac{m_2}{2N_c}\Bigg\{&\left[-a_1^3+\frac{4(N_c+3)}{N_c}a_1^2b_2\right]G^{kc}+\left[(N_c+3)a_1^3+\frac{N_c^2+6N_c-29}{N_c}a_1^2b_2\right]\mathcal{D}_2^{kc}\\&+\left[-a_1^3+\frac{N_c+3}{N_c}a_1^2b_2\right]\mathcal{D}_3^{kc}-\frac{4}{N_c}a_1^2b_2\mathcal{D}_4^{kc}\Bigg\}+\ldots\end{aligned} \tag{36}$$

(2) *Flavor octet contribution*

$$\begin{aligned}d^{ab8}\{A^{ja},[A^{kc},[\mathcal{M},A^{jb}]]\} = \frac{m_2}{N_c}\Bigg\{&\left[\frac{1}{4}a_1^3+\frac{N_c+3}{N_c}a_1^2b_2\right]d^{c8e}G^{ke}+\frac{1}{4}\left[(N_c+3)a_1^3-\frac{25}{N_c}a_1^2b_2\right]d^{c8e}\mathcal{D}_2^{ke}\\&-\frac{1}{4}\left[a_1^3-\frac{N_c+3}{N_c}a_1^2b_2\right]d^{c8e}\mathcal{D}_3^{ke}-\frac{1}{2}a_1^3\{G^{kc},\{J^r,G^{r8}\}\}+\frac{1}{8}\left[a_1^3+\frac{2(N_c+3)}{N_c}a_1^2b_2\right]\\&\times\{J^k,\{T^c,T^8\}\}-\frac{1}{6}\left[2a_1^3-\frac{N_c+3}{N_c}a_1^2b_2\right](\{J^k,\{G^{rc},G^{r8}\}\}-\{G^{k8},\{J^r,G^{rc}\}\})\\&+\frac{1}{N_c}a_1^2b_2\bigg[-\frac{3}{2}\{T^c,G^{k8}\}+\{G^{kc},T^8\}-\frac{1}{2}d^{c8e}\mathcal{D}_4^{ke}+\frac{1}{2}\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}\\&-\frac{4}{3}\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}-\frac{1}{6}\{J^2,\{G^{k8},T^c\}\}\bigg]\Bigg\}+\ldots\end{aligned} \tag{37}$$

(3) *Flavor **27** contribution*

$$\{A^{j8},[A^{kc},[\mathcal{M},A^{j8}]]\} = \frac{m_2}{N_c}\left\{a_1^3\left[\frac{1}{3}G^{kc} - \frac{1}{2}(d^{c8e}d^{e8d} - d^{ced}d^{e88} - f^{c8e}f^{e8d})G^{kd} - \frac{1}{2}d^{c8e}\{G^{k8},\{J^r,G^{re}\}\} + \frac{1}{2}d^{c8e}\{J^k,\{G^{re},G^{r8}\}\}\right] + \frac{1}{N_c}a_1^2b_2\left[-\frac{15}{4}f^{c8e}f^{8eg}\mathcal{D}_2^{kg} + \frac{i}{2}f^{c8e}[G^{ke},\{J^r,G^{r8}\}] - if^{c8e}[G^{k8},\{J^r,G^{re}\}] - \frac{1}{2}f^{c8e}f^{8eg}\mathcal{D}_4^{kg} + \{\mathcal{D}_2^{kc},\{G^{r8},G^{r8}\}\} + \{\mathcal{D}_2^{k8},\{G^{rc},G^{r8}\}\} - \frac{1}{2}\{\{J^r,G^{rc}\},\{G^{k8},T^8\}\} - \frac{1}{2}\{\{J^r,G^{r8}\},\{G^{k8},T^c\}\} + \frac{i}{2}f^{c8e}\{J^k,[\{J^i,G^{ie}\},\{J^r,G^{r8}\}]\}\right]\right\} + \ldots \tag{38}$$

Again, in Eq. (38), the singlet and octet pieces need to be subtracted off in order to have a purely **27** contribution.

First of all, as can be seen in Appendix A, the new cancellation observed in the singlet piece of $GGGJ^2$ indeed repeats itself in the octet and the **27**: the three expressions (A25), (A27), and (A29) are indeed of $\mathcal{O}(N_c^2)$. Furthermore, it is evident from the expressions (A26), (A28), and (A30) in the same appendix, that the new large-N_c cancellation identified in $GGGJ^2$ does not occur in $GG\mathcal{D}_2J^2$. As one would expect, $GG\mathcal{D}_2J^2$ is of $\mathcal{O}(N_c^3)$: Eliminating two J's with the two commutators, one is left with the operator product $GGJT$, which is $\mathcal{O}(N_c^3)$, according to the counting rules (26).

This then means that the correction to δA^{kc} originating from Eq. (35) is $\mathcal{O}(1)$ and thus consistent with being a quantum correction: Naively, one would expect the operator expression $\{A^{ja},[A^{kc},[\mathcal{M},A^{jb}]]\}$ to be $\mathcal{O}(N_c^2)$ so that the correction Eq. (35) would be $\mathcal{O}(N_c)$, since $f \propto \sqrt{N_c}$. However, a close inspection of Eqs. (36)–(38) reveals that these equations exhibit at most a linear dependence in N_c, i.e., large-N_c cancellations occur in the structure of the operator factor in such a way that it is at most $\mathcal{O}(N_c)$. Therefore, the correction Eq. (35) is $\mathcal{O}(1)$, or $1/N_c$ times the tree-level value and contributes to the same order as Eq. (27). The general structure of these cancellations was analyzed in Ref. [16] and has been shown explicitly here.

Finally, there are the two remaining terms in Eq. (16) with two mass insertions,

$$\tfrac{1}{6}([A^{ja},[[J^2,[J^2,A^{jb}]],A^{kc}]] - \tfrac{1}{2}[[J^2,A^{ja}],[[J^2,A^{jb}],A^{kc}]])\Pi_{(3)}^{ab}, \tag{39}$$

which are both of $\mathcal{O}(N_c^3)$: eliminating the four J's with the four commutators, we are left with three G's, each one contributing a factor of N_c, according to the counting rules (26). Interestingly, as shown below for the singlet contribution, there is a new large-N_c cancellation in the first term of Eq. (39):

$$[G^{ia},[[J^2,[J^2,G^{ia}]],G^{kc}]] = -\tfrac{3}{2}(N_c+3)\mathcal{D}_2^{kc} + 2\mathcal{D}_3^{kc} + 3\mathcal{O}_3^{kc}. \tag{40}$$

The right-hand side is at most of $\mathcal{O}(N_c^2)$: The order N_c^3 part vanishes. We have checked that the same pattern repeats itself in the octet and the **27** piece—the explicit expressions will be given elsewhere.

As for the second term in Eq. (39), there is no new cancellation as can be seen in the singlet piece

$$[[J^2,G^{ia}],[[J^2,G^{ia}],G^{kc}]] = [-N_c(N_c+6)+3]G^{kc} + \tfrac{5}{2}(N_c+3)\mathcal{D}_2^{kc} - 2\mathcal{D}_3^{kc} - 2\mathcal{O}_3^{kc}, \tag{41}$$

where the right-hand side is of $\mathcal{O}(N_c^3)$, as one would naively expect. The octet and **27** pieces are of the same order N_c^3.

B. One-loop correction: Diagram 1(d)

The one-loop correction to the baryon axial vector current from the diagram of Fig. 1(d) is given by the expression

$$\delta A^{kc} = -\tfrac{1}{2}[T^a,[T^b,A^{kc}]]\Pi^{ab}, \tag{42}$$

where Π^{ab} is a symmetric tensor with a structure similar to the one introduced in Eq. (17), namely,

$$\Pi^{ab} = I_{\mathbf{1}}\delta^{ab} + I_{\mathbf{8}}d^{ab8} + I_{\mathbf{27}}[\delta^{a8}\delta^{b8} - \tfrac{1}{8}\delta^{ab} - \tfrac{3}{5}d^{ab8}d^{888}]. \tag{43}$$

Again, the flavor singlet, octet, and **27** tensors in Eq. (43) are proportional to flavor singlet $I_{\mathbf{1}}$, flavor octet $I_{\mathbf{8}}$, and flavor **27** $I_{\mathbf{27}}$ linear combinations of the loop integrals $I(m_\pi,\mu)$, $I(m_K,\mu)$, and $I(m_\eta,\mu)$, reading

$$I(m_\Pi,\mu) = \frac{i}{f^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_\Pi^2} = \frac{m_\Pi^2}{16\pi^2f^2}\left[\ln\frac{m_\Pi^2}{\mu^2} - 1\right]. \tag{44}$$

They enter the linear combinations as

$$I_{\mathbf{1}} = \tfrac{1}{8}[3I(m_\pi,\mu) + 4I(m_K,\mu) + I(m_\eta,\mu)], \tag{45}$$

$$I_{\mathbf{8}} = \frac{2\sqrt{3}}{5}\left[\frac{3}{2}I(m_\pi, \mu) - I(m_K, \mu) - \frac{1}{2}I(m_\eta, \mu)\right], \tag{46}$$

$$I_{\mathbf{27}} = \frac{1}{3}I(m_\pi, \mu) - \frac{4}{3}I(m_K, \mu) + I(m_\eta, \mu). \tag{47}$$

A straightforward computation yields the following flavor contributions for $N_f = 3$:

(1) *Flavor singlet contribution*

$$[T^a, [T^a, A^{kc}]] = 3A^{kc}, \tag{48}$$

(2) *Flavor octet contribution*

$$d^{ab8}[T^a, [T^b, A^{kc}]] = \frac{3}{2}d^{c8e}A^{ke}, \tag{49}$$

(3) *Flavor* **27** *contribution*

$$[T^8, [T^8, A^{kc}]] = f^{c8e}f^{8eg}A^{kg}. \tag{50}$$

The double commutators in Eqs. (48)–(50) are proportional to A^{kc} so they are $\mathcal{O}(N_c)$; thus the one-loop correction of Fig. 1(d) is at most $\mathcal{O}(1)$ since f^2 scales like N_c. Consequently, this correction is of the same order as the one arising from the sum of Figs. 1(a)–1(c), i.e., it is of order $1/N_c$ relative to the tree-level contribution and does not involve any cancellations between octet and decuplet states.

C. Total one-loop correction in the degeneracy limit $\Delta/m_\Pi = 0$

In the limit $\Delta/m_\Pi = 0$ the one-loop correction to A^{kc} becomes

$$\delta A^{kc} = \frac{1}{2}[A^{ja}, [A^{jb}, A^{kc}]]\Pi^{ab}_{(1)} - \frac{1}{2}[T^a, [T^b, A^{kc}]]\Pi^{ab}. \tag{51}$$

The matrix elements between spin-$\frac{1}{2}$ baryon states of the space components of the renormalized baryon axial vector current, $A^{kc} + \delta A^{kc}$, are discussed in detail in Appendix B. These matrix elements yield the coupling constants g_A. Our interest in computing these quantities relies on the fact that our calculations can be compared with results obtained within other approaches. Specifically, a direct comparison can be carried out with g_A obtained within the framework of heavy baryon chiral perturbation theory originally introduced in Refs. [1,2]. In these references the calculation was performed assuming $m_u = m_d = 0$ and vanishing decuplet-octet mass difference. In the next section we shall redo the calculation for arbitrary quark masses [23]. This will allow us to identify individual contributions of π, K, and η mesons in the loops.

IV. THE BARYON AXIAL VECTOR CURRENT IN HEAVY BARYON CHIRAL PERTURBATION THEORY

The heavy baryon chiral Lagrangian was constructed [1,2] in terms of the octet meson field, the baryon octet B_v, and the baryon decuplet T^μ_{abc} fields. The lowest order Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{\text{baryon}} = {} & i\,\mathrm{Tr}\bar{B}_v(v\cdot\mathcal{D})B_v - i\bar{T}^\mu_v(v\cdot\mathcal{D})T_{v\mu} \\ & + \Delta\bar{T}^\mu_v T_{v\mu} + 2D\,\mathrm{Tr}\bar{B}_v S^\mu_v\{\mathcal{A}_\mu, B_v\} \\ & + 2F\,\mathrm{Tr}\bar{B}_v S^\mu_v[\mathcal{A}_\mu, B_v] + \mathcal{C}(\bar{T}^\mu_v\mathcal{A}_\mu B_v \\ & + \bar{B}_v\mathcal{A}_\mu T^\mu_v) + 2\mathcal{H}\bar{T}^\mu_v S^\nu_v\mathcal{A}_\nu T_{v\mu},\end{aligned} \tag{52}$$

where D, F, $\mathcal{C}$, and $\mathcal{H}$ are the baryon-pion couplings and Δ is the decuplet-octet mass difference as defined in the preceding sections.

Chiral corrections to the baryon axial vector current

The one-loop corrections to the axial vector current arise from the Feynman graphs displayed in Fig. 1. The renormalized current [24] can be written as

$$\begin{aligned}\langle B_j|J^A_\mu|B_i\rangle = \Big[& \alpha_{B_jB_i} - \sum_\Pi(\bar{\beta}^\Pi_{B_jB_i} \\ & - \bar{\lambda}^\Pi_{B_jB_i}\alpha_{B_jB_i})F^{(1)}(m_\Pi, 0, \mu) \\ & + \sum_\Pi \gamma^\Pi_{B_jB_i} I(m_\Pi, \mu)\Big]\bar{u}_{B_j}\gamma_\mu\gamma_5 u_{B_i},\end{aligned} \tag{53}$$

where $\alpha_{B_jB_i}$ is the tree-level result, $\bar{\beta}^\Pi_{B_jB_i} = \beta^\Pi_{B_jB_i} + \beta'^\Pi_{B_jB_i}$ is the contribution from the Feynman graph in Fig. 1(a), $\bar{\lambda}^\Pi_{B_jB_i} = \lambda^\Pi_{B_jB_i} + \lambda'^\Pi_{B_jB_i}$ is the one-loop correction due to wave function renormalization, Figs. 1(b) and 1(c),

$$\begin{aligned}\sqrt{Z_{B_j}Z_{B_i}} &= 1 - \sum_\Pi \bar{\lambda}^\Pi_{B_jB_i}F^{(1)}(m_\Pi, 0, \mu), \\ \bar{\lambda}^\Pi_{B_jB_i} &= \frac{1}{2}(\bar{\lambda}^\Pi_{B_i} + \bar{\lambda}^\Pi_{B_j}),\end{aligned} \tag{54}$$

and $\gamma^\Pi_{B_jB_i}$ is the correction arising from Fig. 1(d). Here Π stands for π, K, and η mesons and $F^{(1)}(m_\Pi, 0, \mu)$ and $I(m_\Pi, \mu)$ denote the loop functions defined in Eqs. (24) and (44). The unprimed and primed quantities are contributions with intermediate octet and decuplet baryons, respectively. Finally, u is a spinor referring to the initial and final baryon states B_i and B_j. The explicit formulas for the chiral coefficients $\alpha_{B_jB_i}$, $\bar{\beta}^\Pi_{B_jB_i}$, $\bar{\lambda}^\Pi_{B_jB_i}$, and $\gamma^\Pi_{B_jB_i}$ are listed in Appendix C for the sake of completeness. Observe that if we restrict ourselves to the case of nonanalytic corrections in the limit $m_u = m_d = 0$, and use the Gell-Mann-Okubo mass formula to rewrite m_η^2 as $(4/3)m_K^2$, Eq. (53) reduces to results already obtained [1,2].

In close analogy to Eq. (17), Eq. (53) can also be split into flavor singlet, flavor octet, and flavor **27** contributions in terms of flavor singlet, flavor octet, and flavor **27** linear combinations of $F^{(1)}(m_\Pi, 0, \mu)$ and $I(m_\Pi, \mu)$. Thus, in order to keep our formulas compact, the renormalized baryon axial vector current can be cast into the form

$$\langle B_j|J_\mu^A|B_i\rangle = [\alpha_{B_jB_i} + b_{\mathbf{1}}^{B_jB_i}F_{\mathbf{1}}^{(1)} + b_{\mathbf{8}}^{B_jB_i}F_{\mathbf{8}}^{(1)} + b_{\mathbf{27}}^{B_jB_i}F_{\mathbf{27}}^{(1)} + c_{\mathbf{1}}^{B_jB_i}I_{\mathbf{1}} + c_{\mathbf{8}}^{B_jB_i}I_{\mathbf{8}} + c_{\mathbf{27}}^{B_jB_i}I_{\mathbf{27}}]\bar{u}_{B_j}\gamma_\mu\gamma_5 u_{B_i}, \tag{55}$$

where the new coefficients are

$$b_{\mathbf{1}}^{B_jB_i} = -(a_{B_jB_i}^\pi + a_{B_jB_i}^K + a_{B_jB_i}^\eta), \tag{56}$$

$$b_{\mathbf{8}}^{B_jB_i} = -\frac{1}{\sqrt{3}}(a_{B_jB_i}^\pi - \frac{1}{2}a_{B_jB_i}^K - a_{B_jB_i}^\eta), \tag{57}$$

$$b_{\mathbf{27}}^{B_jB_i} = -\frac{3}{40}(a_{B_jB_i}^\pi - 3a_{B_jB_i}^K + 9a_{B_jB_i}^\eta), \tag{58}$$

$$c_{\mathbf{1}}^{B_jB_i} = \gamma_{B_jB_i}^\pi + \gamma_{B_jB_i}^K + \gamma_{B_jB_i}^\eta, \tag{59}$$

$$c_{\mathbf{8}}^{B_jB_i} = \frac{1}{\sqrt{3}}(\gamma_{B_jB_i}^\pi - \frac{1}{2}\gamma_{B_jB_i}^K - \gamma_{B_jB_i}^\eta), \tag{60}$$

$$c_{\mathbf{27}}^{B_jB_i} = \frac{3}{40}(\gamma_{B_jB_i}^\pi - 3\gamma_{B_jB_i}^K + 9\gamma_{B_jB_i}^\eta), \tag{61}$$

and the various $a_{B_jB_i}^\Pi$ are expressed in terms of the chiral coefficients as

$$a_{B_jB_i}^\Pi = \bar{\beta}_{B_jB_i}^\Pi - \bar{\lambda}_{B_jB_i}^\Pi \alpha_{B_jB_i}. \tag{62}$$

Equations (56)–(61) will be particularly useful in the comparison with the results obtained in the framework of large-N_c heavy baryon chiral perturbation theory. This will be done in the next section.

V. COMPARISON BETWEEN THE TWO APPROACHES IN THE LIMIT $\Delta/m_\Pi = 0$

The matrix elements of the space components of the renormalized baryon axial vector current between initial and final baryon states B_i and B_j can be denoted as

$$\langle B_j|\bar{\psi}\gamma^k\gamma_5 T^c\psi|B_i\rangle = [A_{\rm ren}^{kc}]_{B_jB_i}. \tag{63}$$

Here $A_{\rm ren}^{kc} = A^{kc} + \delta A^{kc}$, ψ are the QCD quark fields, and B_i and B_j are baryons in the lowest-lying irreducible representation of contracted $SU(6)$ spin-flavor symmetry, namely, the spin-$\frac{1}{2}$ octet and the spin-$\frac{3}{2}$ decuplet baryons. If the initial and final baryon states are restricted to the spin-$\frac{1}{2}$ octet baryons, the matrix elements $[A_{\rm ren}^{kc}]_{B_jB_i}$ yield the actual values of $g_A^{B_jB_i}$, the axial vector couplings of the baryons.

In the degeneracy limit the renormalization to the baryon axial vector current reads

$$\delta A_{\rm deg}^{kc} = \frac{1}{2}[A^{ja}, [A^{jb}, A^{kc}]]\Pi_{(1)}^{ab} - \frac{1}{2}[T^a, [T^b, A^{kc}]]\Pi^{ab}. \tag{64}$$

At the physical value $N_c = 3$, there is a one-to-one correspondence between the different flavor contributions of $[A_{\rm ren}^{kc}]_{B_jB_i}$ and those contained in Eq. (55). The comparison can be made through

$$[\frac{1}{2}[A^{ia}, [A^{ia}, A^{kc}]]]_{B_jB_i} = b_{\mathbf{1}}^{B_jB_i}, \tag{65}$$

$$[\frac{1}{2}d^{ab8}[A^{ia}, [A^{ib}, A^{kc}]]]_{B_jB_i} = b_{\mathbf{8}}^{B_jB_i}, \tag{66}$$

$$[\frac{1}{2}[A^{i8}, [A^{i8}, A^{kc}]]]_{B_jB_i} = b_{\mathbf{27}}^{B_jB_i}, \tag{67}$$

$$-[\frac{1}{2}[T^a, [T^a, A^{kc}]]]_{B_jB_i} = c_{\mathbf{1}}^{B_jB_i}, \tag{68}$$

$$-[\frac{1}{2}d^{ab8}[T^a, [T^b, A^{kc}]]]_{B_jB_i} = c_{\mathbf{8}}^{B_jB_i}, \tag{69}$$

$$-[\frac{1}{2}[T^8, [T^8, A^{kc}]]]_{B_jB_i} = c_{\mathbf{27}}^{B_jB_i}. \tag{70}$$

It is understood that flavor singlet and octet pieces must be subtracted off Eqs. (67) and (70) in order to have a truly **27** contribution.

For instance, for the process $n \to p + e + \bar{\nu}_e$, the singlet component of the renormalized axial vector coupling—diagrams 1(a)–1(c)—reads (see Appendix B),

$$[\frac{1}{2}[A^{ia}, [A^{ia}, A^{kc}]]]_{pn} = \frac{115}{144}a_1^3 + \frac{7}{48}a_1^2b_2 + \frac{19}{48}a_1b_2^2 - \frac{31}{432}a_1^2b_3 - \frac{11}{12}a_1^2c_3 + \frac{7}{144}b_2^2 + \frac{169}{216}a_1b_2b_3 - \frac{37}{36}a_1b_2c_3. \tag{71}$$

To the order of approximation implemented in this work, this corresponds exactly to $b_{\mathbf{1}}^{pn}$, Eq. (65), given in terms of α_{pn}, $\bar{\beta}_{pn}$, and $\bar{\lambda}_{pn}$, whose explicit expressions can be found in Appendix C. Note that, in order to make the comparison, the baryon-meson couplings have to be expressed in terms of the coefficients of the $1/N_c$ expansion at $N_c = 3$ as [14]

$$D = \frac{1}{2}a_1 + \frac{1}{6}b_3, \qquad F = \frac{1}{3}a_1 + \frac{1}{6}b_2 + \frac{1}{9}b_3,$$
$$\mathcal{C} = -a_1 - \frac{1}{2}c_3, \qquad \mathcal{H} = -\frac{3}{2}a_1 - \frac{3}{2}b_2 - \frac{5}{2}b_3. \tag{72}$$

The agreement between the two approaches can be seen term by term in all expressions given by Eqs. (65)–(70): Both approaches yield the same results. An analogous comparison for the baryon mass relations, using the above identifications, was performed in Ref. [14].

To close this section, a fit to baryon semileptonic decays by using the measured decay rates and g_A/g_V ratios [25] is performed. Our motivation here is not really to be definitive about the predictions of our expressions for g_A but rather to explore the quality of our working assumptions. To the order of approximation we implemented here, the fit

TABLE II. Values of g_A for various semileptonic processes.

Process	Total value	Tree level	Singlet piece	Octet piece	**27** piece
$n \to p e^- \bar{\nu}_e$	1.272	1.031	0.279	−0.040	0.002
$\Sigma^+ \to \Lambda e^+ \nu_e$	0.653	0.542	0.168	−0.057	0.000
$\Sigma^- \to \Lambda e^- \bar{\nu}_e$	0.624	0.542	0.113	−0.031	−0.000
$\Lambda \to p e^- \bar{\nu}_e$	−0.904	−0.720	−0.134	−0.055	0.005
$\Sigma^- \to n e^- \bar{\nu}_e$	0.375	0.298	0.080	−0.002	−0.001
$\Xi^- \to \Lambda e^- \bar{\nu}_e$	0.139	0.178	−0.034	−0.004	−0.001
$\Xi^- \to \Sigma^0 e^- \bar{\nu}_e$	0.869	0.729	0.128	0.014	−0.002
$\Xi^0 \to \Sigma^+ e^- \bar{\nu}_e$	1.312	1.031	0.246	0.041	−0.006

[26] gives $a_1 = 0.32 \pm 0.04$, $b_2 = -0.46 \pm 0.03$, $b_3 = 3.04 \pm 0.13$, and $c_3 = 2.49$, with $\chi^2 = 38.18$ for 11 degrees of freedom, or equivalently $F = 0.37 \pm 0.01$, $D = 0.66 \pm 0.01$, and $\mathcal{H} = -7.39 \pm 0.25$. The proton matrix element of the T^8 component of the axial vector current (which is equal to $3F - D$ in the $SU(3)$ symmetry limit) is found to be 0.45 ± 0.01, which is smaller than its $SU(6)$ symmetric value of 1. The coefficient c_3 was determined indirectly through the relation $|\mathcal{C}| \sim 1.6$, which was obtained by a fit to the $\Delta \to N\pi$ decay rate [2]. It should be pointed out that the coupling $\mathcal{H}$ obtained in the fit is not close to its $SU(6)$ value, which is $3D - 9F$; this is mainly due to the order of approximation used here.

The predicted values of g_A are listed in Table II, where the different flavor contributions are given separately. As one might have anticipated, the **27** contribution to g_A is suppressed relative to the octet contribution, which in turn is suppressed relative to the singlet one. It is also instructive to remark that the highest contributions to χ^2 come from the decay rate and g_A/g_V ratio of the process $\Xi^- \to \Lambda e^- \bar{\nu}_e$ (18.91 and 7.46, respectively), which might suggest some inconsistencies in these data.

Evidently, a more complete analysis which can yield a better fit should also incorporate seven-body operators—like $G\mathcal{D}_3\mathcal{D}_3$—or higher in the correction to the baryon axial vector current (27). These terms represent $\mathcal{O}(1/N_c^3)$ corrections or higher to the tree-level result $\mathcal{O}(N_c)$. Although a substantial improvement of the value of $\mathcal{H}$, for instance, is expected, the algebraic manipulations to reduce the double commutator $[A^{ia}, [A^{ib}, A^{kc}]]\Pi_{(1)}^{ab}$ to the operator basis require a formidable effort which goes beyond the scope of the present paper. One can also, of course, follow a more pragmatic approach and evaluate directly the matrix elements of the double commutator between octet baryon states and observe the agreement with heavy baryon chiral perturbation theory pointed out above. This procedure, however, does not allow to show the large-N_c cancellations explicitly.

VI. INCLUSION OF THE η'

So far, the renormalization of the baryon axial vector current has been performed by taking into account the contribution of the octet mesons in the loops, Eq. (16). In the large-N_c limit, however, the quark loop responsible for the axial $U(1)$ anomaly is suppressed and the chiral symmetry is extended from $SU(3)_R \times SU(3)_L \times U(1)_V$ to $U(3)_R \times U(3)_L$. As a consequence, the contribution from the η' should be included in the analysis.

Planar QCD flavor symmetry implies that the baryon $1/N_c$ chiral Lagrangian (1) possesses a $SU(2) \times U(3)$ spin-flavor symmetry at leading order in the $1/N_c$ expansion and constrains this Lagrangian by forming a nonet baryon axial vector current out of the singlet and octet baryon axial vector currents at leading order in the $1/N_c$ expansion [14], namely,

$$A^k = A^{k9} + \mathcal{O}(1/N_c), \tag{73}$$

where A^k is the flavor singlet baryon axial vector current given in Eq. (11). In Ref. [14] the constraint (73) was imposed through the relation

$$b_n^{1,1} \to \bar{b}_n^{1,1} + \frac{1}{N_c} b_n^{1,1}, \tag{74}$$

where the coefficients $\bar{b}_n^{1,1}$ are determined by exact nonet symmetry, whereas the others are not constrained and violate nonet symmetry at first subleading order $1/N_c$. Thus, for $N_c = 3$, nonet symmetry implies that

$$\bar{b}_1^{1,1} = \tfrac{1}{\sqrt{6}}(a_1 + b_2), \tag{75a}$$

$$\bar{b}_3^{1,1} = \tfrac{1}{\sqrt{6}}(2b_3), \tag{75b}$$

where a_1, b_2, and b_3 are the operator coefficients of the octet axial vector current expansion Eq. (10). The above relations can be easily obtained by using the ninth flavor components of G^{ia} and T^a given by [14]

$$G^{i9} = \tfrac{1}{\sqrt{6}} J^i, \qquad T^9 = \tfrac{1}{\sqrt{6}} N_c \mathbb{1}. \tag{76}$$

One should notice that the coefficients of the diagonal operators $\mathcal{D}_n^i$ in the singlet expansion do not depend on the coefficients c_n of the off-diagonal operators $\mathcal{O}_n^{ia}$ of the octet expansion.

The inclusion of the η' meson into the renormalization of A^{kc} is now straightforwardly obtained in the degeneracy limit. Let us first discuss the contribution from diagrams 1(a)–1(c):

$$\delta A^{kc} = \tfrac{1}{2}[A^{i9}, [A^{i9}, A^{kc}]]F^{(1)}(m_{\eta'}, 0, \mu). \tag{77}$$

To the order of approximation implemented here, one has to evaluate the following commutator-anticommutator structures:

$$[J^i, [J^i, A^{kc}]] = 2A^{kc}, \tag{78}$$

$$[J^i, [\{J^2, J^i\}, G^{kc}]] + [\{J^2, J^i\}, [J^i, G^{kc}]] = 2\mathcal{D}_3^{kc} + 8\mathcal{O}_3^{kc}, \tag{79}$$

and

$$[J^i, [\{J^2, J^i\}, \mathcal{D}_2^{kc}]] + [\{J^2, J^i\}, [J^i, \mathcal{D}_2^{kc}]] = 4\mathcal{D}_4^{kc}. \tag{80}$$

The correction due to the inclusion of the η' thus amounts to

$$\begin{aligned}\delta A^{kc} = \frac{1}{6}\Big[& a_1(\bar{b}_1^{1,1})^2 G^{kc} + \frac{1}{N_c} b_2(\bar{b}_1^{1,1})^2\mathcal{D}_2^{kc} \\ & + \frac{1}{N_c^2} b_3(\bar{b}_1^{1,1})^2\mathcal{D}_3^{kc} + \frac{1}{N_c^2} c_3(\bar{b}_1^{1,1})^2\mathcal{O}_3^{kc} \\ & + \frac{1}{N_c^2} a_1(\bar{b}_1^{1,1})(\bar{b}_3^{1,1})(\mathcal{D}_3^{kc} + 4\mathcal{O}_3^{kc}) \\ & + \frac{2}{N_c^3} b_2(\bar{b}_1^{1,1})(\bar{b}_3^{1,1})\mathcal{D}_4^{kc}\Big]F^{(1)}(m_{\eta'}, 0, \mu). \end{aligned} \tag{81}$$

On the other hand, as far as conventional baryon chiral perturbation theory (i.e., without $1/N_c$-expansion) is concerned, the flavor singlet baryon-η' couplings can be incorporated into the chiral effective Lagrangian Eq. (52) by adding the two terms [14]

$$2S_B \,\mathrm{Tr}\mathcal{A}_\mu \,\mathrm{Tr}\bar{B}_v S_v^\mu B_v - 2S_T \,\mathrm{Tr}\mathcal{A}_\nu \bar{T}_v^\mu S_v^\nu T_{v\mu}, \tag{82}$$

where S_B and S_T are the singlet axial vector coupling constants of the octet and decuplet, respectively. The condition of nonet symmetry for the baryon axial vector couplings implies

$$S_B \to \tfrac{1}{3}(3F - D), \qquad S_T \to -\tfrac{1}{3}\mathcal{H}. \tag{83}$$

The contribution of the η' meson to the correction (53) can be written as

$$\delta\langle B_j|J_\mu^A|B_i\rangle = [\zeta_{B_jB_i}^{\eta'} F^{(1)}(m_{\eta'}, 0, \mu)]\bar{u}_{B_j}\gamma_\mu\gamma_5 u_{B_i}, \tag{84}$$

where $\zeta_{B_jB_i}^{\eta'}$ are the chiral coefficients which emerge from Fig. 1(a)–1(c) and can be found in Appendix C.

As in the previous section, a direct comparison between Eqs. (77) and (84) can be performed. In this case, the comparison can be made through

$$[\tfrac{1}{2}[A^{i9}, [A^{i9}, A^{kc}]]]_{B_jB_i} = \zeta_{B_jB_i}^{\eta'}, \tag{85}$$

by using the identifications (72) and (75). We have checked that, for the eight decays considered in the present study, the two approaches yield the same result.

Finally, we briefly discuss the remaining diagram 1(d). The corresponding one-loop correction to the baryon axial vector current in large-N_c chiral perturbation theory was derived in Sec. III, Eq. (42). Including the η' thus amounts to the extra term

$$\delta A^{kc} = -\tfrac{1}{2}[T^9, [T^9, A^{kc}]]I(m_{\eta'}, \mu). \tag{86}$$

However, the flavor operator T^9 is proportional to the unit matrix (76), such that the commutators are zero and there is thus no contribution from diagram 1(d). Likewise, in conventional baryon chiral perturbation theory, the additional piece in the axial vector current due to the term involving S_B in (82) does not contribute. Again, in the degeneracy limit, the two approaches agree.

VII. CONCLUSIONS

In this paper we have computed the renormalization of the baryon axial vector current in the framework of heavy baryon chiral perturbation theory in the large-N_c limit. The analysis was performed at one-loop order, where the correction to the baryon axial vector current is given by an infinite series, each term representing a complicated combination of commutators and/or anticommutators of the baryon axial vector current A^{kc} and mass insertions $\mathcal{M}$. We have explicitly evaluated the first four terms in this expansion: The contribution AAA in the degeneracy limit $\Delta/m_\Pi = 0$, the leading ($AAA\mathcal{M}$), and the two next-to-leading ($AAA\mathcal{M}\mathcal{M}$) order contributions for nonzero octet-decuplet mass difference, respectively. The general structure of these large-N_c cancellations was already discussed in Ref. [16], where also a new large-N_c cancellation in the singlet piece of the structure $AAA\mathcal{M}$ was identified.

Our motivation to go beyond this general analysis and to engage ourselves into the reduction of these rather involved operator products, including up to six $SU(6)$ spin-flavor operators J^k, T^c, and G^{kc}, was to explicitly demonstrate how these large-N_c cancellations occur. It has already been pointed out in Refs. [3–5,16], that there are large-N_c cancellations between individual Feynman diagrams in the degeneracy limit, provided one sums over all baryon states in a complete multiplet of the large-N_c $SU(6)$ spin-flavor symmetry, i.e., over both the octet and decuplet, and uses axial coupling ratios given by the large-N_c spin-flavor symmetry. Indeed, our final expressions referring to the degeneracy limit explicitly demonstrate that the double commutator AAA is of order N_c rather than of order N_c^3, as one would naively expect. As for the nondegenerate case we have shown that the new large-N_c cancellation found in Ref. [16] is a generic feature of the corresponding commutator-anticommutator structure $GGGJ^2$: The new cancellation observed in the singlet piece of $GGGJ^2$ indeed repeats itself in the octet and the **27**. On the other hand, in the structure $GG\mathcal{D}_2J^2$, no new large-N_c cancellations are detected: the expression is of order N_c^3, consistent with the global analysis of Ref. [16]. However, in one of the two commutator-anticommutator structures $GGGJ^2J^2$ with two mass insertions, a new large-N_c cancellation was identified: Although naively one would expect this structure to be of order N_c^3, our explicit calculation for the singlet, octet, and **27** piece shows that it is of order N_c^2.

In the degeneracy limit, we have also performed a comparison of the renormalized baryon axial vector current, obtained within two different schemes: Large-N_c baryon chiral perturbation theory on the one hand, and conventional heavy baryon chiral perturbation theory (including both octet and decuplet baryons), where no $1/N_c$ expansion is involved, on the other hand. Both approaches agree—the large-N_c cancellations are guaranteed to occur as a consequence of the contracted spin-flavor symmetry present in the limit $N_c \to \infty$. By keeping the large- N_c spin-flavor symmetry manifest, one thus avoids large numerical cancellations between loop diagrams with intermediate octet states and low-energy constants of the next-to-leading order effective Lagrangian, containing the effects of decuplet states [27].

In the present paper, we have taken into account the octet-decuplet mass difference, but neglected the $SU(3)$ splittings of the octet and decuplet baryons. Moreover, the comparison between large-N_c baryon chiral perturbation theory and conventional heavy baryon chiral perturbation theory, was performed for the degeneracy limit only. The extension to the nondegenerate case, as well as the incorporation of $SU(3)$ mass splittings is currently in progress.

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APPENDIX A: REDUCTION OF BARYON OPERATORS

Here we present the most general expressions, up to the order of approximation implemented in this work, for the two commutator-anticommutator structures involved in the analysis. The computation was performed by keeping N_f and N_c arbitrary, although the physical values $N_f = 3$ and $N_c = 3$ are used in the evaluation of g_A.

1. Degeneracy limit $\Delta/m_\pi = 0$

The flavor singlet, octet, and **27** contributions of the double commutator

$$[A^{ia}, [A^{ib}, A^{kc}]]$$

can be organized as follows:

(1) *Flavor singlet contribution*

$$[G^{ia}, [G^{ia}, G^{kc}]] = \frac{3N_f^2 - 4}{4N_f} G^{kc}, \tag{A1}$$

$$[G^{ia}, [G^{ia}, \mathcal{D}_2^{kc}]] + [G^{ia}, [\mathcal{D}_2^{ia}, G^{kc}]] + [\mathcal{D}_2^{ia}, [G^{ia}, G^{kc}]] = -\frac{2}{N_f}(N_c + N_f)G^{kc} + \frac{9N_f^2 + 8N_f - 4}{4N_f}\mathcal{D}_2^{kc}, \tag{A2}$$

$$[G^{ia}, [\mathcal{D}_2^{ia}, \mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{ia}, [G^{ia}, \mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{ia}, [\mathcal{D}_2^{ia}, G^{kc}]] = \frac{N_c(N_c + 2N_f)(N_f - 2) - 6N_f^2}{2N_f} G^{kc} + \frac{2}{N_f}(N_c + N_f)(N_f - 1)\mathcal{D}_2^{kc} + \frac{3N_f + 2}{4}\mathcal{D}_3^{kc} + \frac{N_f}{2}\mathcal{O}_3^{kc}, \tag{A3}$$

$$[G^{ia}, [G^{ia}, \mathcal{D}_3^{kc}]] + [G^{ia}, [\mathcal{D}_3^{ia}, G^{kc}]] + [\mathcal{D}_3^{ia}, [G^{ia}, G^{kc}]] = [-N_c(N_c + 2N_f) + 2N_f - 8]G^{kc} - 3(N_c + N_f)\mathcal{D}_2^{kc} + \frac{13N_f^2 + 16N_f - 12}{4N_f}\mathcal{D}_3^{kc} + \frac{N_f^2 + 2N_f - 8}{N_f}\mathcal{O}_3^{kc}, \tag{A4}$$

$$[G^{ia}, [G^{ia}, \mathcal{O}_3^{kc}]] + [G^{ia}, [\mathcal{O}_3^{ia}, G^{kc}]] + [\mathcal{O}_3^{ia}, [G^{ia}, G^{kc}]] = [-N_c(N_c + 2N_f) + N_f]G^{kc} - \frac{1}{2}(N_c + N_f)\mathcal{D}_2^{kc} + \frac{N_f + 1}{2}\mathcal{D}_3^{kc} + \frac{15N_f^2 + 12N_f - 4}{4N_f}\mathcal{O}_3^{kc}, \tag{A5}$$

$$[\mathcal{D}_2^{ia},[\mathcal{D}_2^{ia},\mathcal{D}_2^{kc}]] = \frac{N_c(N_c+2N_f)(N_f-2)-2N_f^2}{2N_f}\mathcal{D}_2^{kc} + \frac{N_f+2}{2}\mathcal{D}_4^{kc}, \tag{A6}$$

$$\begin{aligned}&[G^{ia},[\mathcal{D}_2^{ia},\mathcal{D}_3^{kc}]] + [G^{ia},[\mathcal{D}_3^{ia},\mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{ia},[G^{ia},\mathcal{D}_3^{kc}]] + [\mathcal{D}_2^{ia},[\mathcal{D}_3^{ia},G^{kc}]]\\ &\quad + [\mathcal{D}_3^{ia},[G^{ia},\mathcal{D}_2^{kc}]] + [\mathcal{D}_3^{ia},[\mathcal{D}_2^{ia},G^{kc}]]\\ &= -12(N_c+N_f)G^{kc} + [N_c(N_c+2N_f)-2N_f+8]\mathcal{D}_2^{kc} + \frac{7N_f-4}{N_f}(N_c+N_f)\mathcal{D}_3^{kc}\\ &\quad + \frac{2(3N_f-4)}{N_f}(N_c+N_f)\mathcal{O}_3^{kc} + \frac{3N_f^2-4N_f-4}{N_f}\mathcal{D}_4^{kc},\end{aligned} \tag{A7}$$

$$\begin{aligned}&[G^{ia},[\mathcal{D}_2^{ia},\mathcal{O}_3^{kc}]] + [G^{ia},[\mathcal{O}_3^{ia},\mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{ia},[G^{ia},\mathcal{O}_3^{kc}]] + [\mathcal{D}_2^{ia},[\mathcal{O}_3^{ia},G^{kc}]]\\ &\quad + [\mathcal{O}_3^{ia},[G^{ia},\mathcal{D}_2^{kc}]] + [\mathcal{O}_3^{ia},[\mathcal{D}_2^{ia},G^{kc}]]\\ &= -\frac{3}{2}[N_c(N_c+2N_f)-8N_f]\mathcal{D}_2^{kc} - \frac{9}{2}(N_c+N_f)\mathcal{D}_3^{kc} - \frac{2}{N_f}(N_c+N_f)\mathcal{O}_3^{kc} + (3N_f+10)\mathcal{D}_4^{kc}.\end{aligned} \tag{A8}$$

(2) *Flavor octet contribution*

$$d^{ab8}[G^{ia},[G^{ib},G^{kc}]] = \frac{3N_f^2-16}{8N_f}d^{c8e}G^{ke} + \frac{N_f^2-4}{2N_f^2}\delta^{c8}J^k, \tag{A9}$$

$$\begin{aligned}&d^{ab8}([G^{ia},[G^{ib},\mathcal{D}_2^{kc}]] + [G^{ia},[\mathcal{D}_2^{ib},G^{kc}]] + [\mathcal{D}_2^{ia},[G^{ib},G^{kc}]])\\ &= -\frac{2}{N_f}(N_c+N_f)d^{c8e}G^{ke} + \frac{5N_f+8}{8}d^{c8e}\mathcal{D}_2^{ke} - \frac{2}{N_f}\{G^{kc},T^8\} + \frac{N_f^2+2N_f-4}{2N_f}\{G^{k8},T^c\}\\ &\quad + \frac{N_f+2}{4}[J^2,[T^8,G^{kc}]] + \frac{(N_c+N_f)(N_f-2)}{N_f^2}\delta^{c8}J^k,\end{aligned} \tag{A10}$$

$$\begin{aligned}&d^{ab8}([G^{ia},[\mathcal{D}_2^{ib},\mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{ia},[G^{ib},\mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{ia},[\mathcal{D}_2^{ib},G^{kc}]])\\ &= -\frac{3}{2}N_f d^{c8e}G^{ke} + \frac{(N_c+N_f)(N_f-2)}{N_f}\{G^{k8},T^c\} + \frac{(N_c+N_f)(N_f-4)}{2N_f}\{G^{kc},T^8\} + \frac{3}{8}N_f d^{c8e}\mathcal{D}_3^{ke}\\ &\quad + \frac{N_f-2}{4}d^{c8e}\mathcal{O}_3^{ke} + \frac{1}{2}\{G^{kc},\{J^r,G^{r8}\}\} + \frac{1}{2}\{G^{k8},\{J^r,G^{rc}\}\} + \frac{N_f-2}{2N_f}\{J^k,\{T^c,T^8\}\}\\ &\quad + \frac{1}{4}(N_c+N_f)[J^2,[T^8,G^{kc}]],\end{aligned} \tag{A11}$$

$$\begin{aligned}&d^{ab8}([G^{ia},[G^{ib},\mathcal{D}_3^{kc}]] + [G^{ia},[\mathcal{D}_3^{ib},G^{kc}]] + [\mathcal{D}_3^{ia},[G^{ib},G^{kc}]])\\ &= (N_f-8)d^{c8e}G^{ke} - \frac{3}{2}(N_c+N_f)d^{c8e}\mathcal{D}_2^{ke} - (N_c+N_f)\{G^{kc},T^8\} + \frac{5N_f^2+12N_f-16}{8N_f}d^{c8e}\mathcal{D}_3^{ke}\\ &\quad + \frac{N_f^2+2N_f-24}{2N_f}d^{c8e}\mathcal{O}_3^{ke} - \frac{3}{4}\{J^k,\{T^c,T^8\}\} + (N_f+1)\{J^k,\{G^{rc},G^{r8}\}\} + \frac{N_f-4}{N_f}\{G^{kc},\{J^r,G^{r8}\}\}\\ &\quad + \frac{N_f^2+3N_f-4}{N_f}\{G^{k8},\{J^r,G^{rc}\}\} + \frac{3}{2}(N_c+N_f)[J^2,[T^8,G^{kc}]] - \frac{3N_c(N_c+2N_f)-8N_f+16}{2N_f}\delta^{c8}J^k\\ &\quad + \frac{N_f^2+3N_f-4}{N_f^2}\delta^{c8}\{J^2,J^k\},\end{aligned} \tag{A12}$$

$$d^{ab8}([G^{ia},[G^{ib},\mathcal{O}_3^{kc}]]+[G^{ia},[\mathcal{O}_3^{ib},G^{kc}]]+[\mathcal{O}_3^{ia},[G^{ib},G^{kc}]])$$
$$=\frac{N_f}{2}d^{c8e}G^{ke}-\frac{1}{4}(N_c+N_f)d^{c8e}\mathcal{D}_2^{ke}-(N_c+N_f)\{G^{kc},T^8\}+\frac{N_f^2+N_f-8}{4N_f}d^{c8e}\mathcal{D}_3^{ke}$$
$$+\frac{7N_f^2+8N_f-16}{8N_f}d^{c8e}\mathcal{O}_3^{ke}-\frac{1}{8}\{J^k,\{T^c,T^8\}\}-\frac{N_f^2+N_f-8}{2N_f}\{J^k,\{G^{rc},G^{r8}\}\}+(N_f+2)\{G^{kc},\{J^r,G^{r8}\}\}$$
$$-\frac{N_f+2}{2}\{G^{k8},\{J^r,G^{rc}\}\}-\frac{N_c(N_c+2N_f)}{4N_f}\delta^{c8}J^k+\frac{2N_f^2+N_f-8}{2N_f^2}\delta^{c8}\{J^2,J^k\}-(N_c+N_f)[J^2,[T^8,G^{kc}]], \tag{A13}$$

$$d^{ab8}[\mathcal{D}_2^{ia},[\mathcal{D}_2^{ib},\mathcal{D}_2^{kc}]]=-\frac{N_f}{2}d^{c8e}\mathcal{D}_2^{ke}+\frac{N_f-4}{4N_f}(N_c+N_f)\{J^k,\{T^c,T^8\}\}+\frac{N_f}{4}d^{c8e}\mathcal{D}_4^{ke}+\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}, \tag{A14}$$

$$d^{ab8}([G^{ia},[\mathcal{D}_2^{ib},\mathcal{D}_3^{kc}]]+[G^{ia},[\mathcal{D}_3^{ib},\mathcal{D}_2^{kc}]]+[\mathcal{D}_2^{ia},[G^{ib},\mathcal{D}_3^{kc}]]+[\mathcal{D}_2^{ia},[\mathcal{D}_3^{ib},G^{kc}]]$$
$$+[\mathcal{D}_3^{ia},[G^{ib},\mathcal{D}_2^{kc}]]+[\mathcal{D}_3^{ia},[\mathcal{D}_2^{ib},G^{kc}]])$$
$$=-6(N_c+N_f)d^{c8e}G^{ke}-(3N_f-6)d^{c8e}\mathcal{D}_2^{ke}+2(N_f+1)\{G^{k8},T^c\}-6\{G^{kc},T^8\}+\frac{N_f^2+8}{N_f}[J^2,[T^8,G^{kc}]]$$
$$+\frac{3}{2}(N_c+N_f)d^{c8e}\mathcal{D}_3^{ke}+\frac{N_f-4}{N_f}(N_c+N_f)d^{c8e}\mathcal{O}_3^{ke}+\frac{3}{2}(N_f-2)d^{c8e}\mathcal{D}_4^{ke}+\frac{N_c+N_f}{2}\{J^k,\{T^c,T^8\}\}$$
$$+\frac{2}{N_f}(N_f-2)(N_c+N_f)\{G^{kc},\{J^r,G^{r8}\}\}+\frac{2}{N_f}(N_f-2)(N_c+N_f)\{G^{k8},\{J^r,G^{rc}\}\}$$
$$+\frac{N_f^2+3N_f-8}{N_f}\{J^2,\{G^{k8},T^c\}\}+\frac{3N_f-8}{N_f}\{J^2,\{G^{kc},T^8\}\}+4\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}-(N_f+4)\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}$$
$$+\frac{5}{64}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}-\frac{5}{64}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}-\frac{5}{64}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}+\frac{5}{64}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}$$
$$+\frac{5}{64}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}+\frac{N_f^2+4N_f-8}{2N_f}\{J^2,[J^2,[T^8,G^{kc}]]\}, \tag{A15}$$

$$d^{ab8}([G^{ia},[\mathcal{D}_2^{ib},\mathcal{O}_3^{kc}]]+[G^{ia},[\mathcal{O}_3^{ib},\mathcal{D}_2^{kc}]]+[\mathcal{D}_2^{ia},[G^{ib},\mathcal{O}_3^{kc}]]+[\mathcal{D}_2^{ia},[\mathcal{O}_3^{ib},G^{kc}]]$$
$$+[\mathcal{O}_3^{ia},[G^{ib},\mathcal{D}_2^{kc}]]+[\mathcal{O}_3^{ia},[\mathcal{D}_2^{ib},G^{kc}]])$$
$$=6N_fd^{c8e}\mathcal{D}_2^{ke}-\frac{5N_f+8}{4N_f}(N_c+N_f)d^{c8e}\mathcal{D}_3^{ke}-\frac{2}{N_f}(N_c+N_f)d^{c8e}\mathcal{O}_3^{ke}-\frac{2}{N_f}(N_f-2)(N_c+N_f)\{J^k,\{G^{rc},G^{r8}\}\}$$
$$-\frac{3}{4}(N_c+N_f)\{J^k,\{T^c,T^8\}\}+\frac{2}{N_f^2}(N_f-2)(N_c+N_f)\delta^{c8}\{J^2,J^k\}+\frac{N_f+9}{2}d^{c8e}\mathcal{D}_4^{ke}$$
$$+\frac{N_f^2+2N_f-4}{2N_f}\{J^2,\{G^{k8},T^c\}\}-\frac{2}{N_f}\{J^2,\{G^{kc},T^8\}\}-\frac{9N_f-4}{2N_f}\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}$$
$$+\frac{N_f^2+9N_f+4}{2N_f}\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}-\frac{71}{128}\{[J^2,G^{kc}],\{J^r,G^{r8}\}\}+\frac{71}{128}\{[J^2,G^{k8}],\{J^r,G^{rc}\}\}$$
$$+\frac{71}{128}\{J^k,[\{J^m,G^{mc}\},\{J^r,G^{r8}\}]\}+\frac{N_f+2}{4}\{J^2,[J^2,[T^8,G^{kc}]]\}-\frac{71}{128}\{J^2,[G^{kc},\{J^r,G^{r8}\}]\}$$
$$+\frac{71}{128}\{J^2,[G^{k8},\{J^r,G^{rc}\}]\}, \tag{A16}$$

(3) *Flavor* **27** *contribution*

$$[G^{i8},[G^{i8},G^{kc}]] = \frac{1}{4}(f^{c8e}f^{8eg} + 2d^{c8e}d^{8eg})G^{kg} + \frac{1}{N_f}\delta^{c8}G^{k8} + \frac{1}{2N_f}d^{c88}J^k, \tag{A17}$$

$$\begin{aligned}[G^{i8},[G^{i8},\mathcal{D}_2^{kc}]] + [G^{i8},[\mathcal{D}_2^{i8},G^{kc}]] + [\mathcal{D}_2^{i8},[G^{i8},G^{kc}]] = {} & \left[\frac{1}{N_f}\delta^{88}\delta^{cg} + \frac{7}{4}f^{c8e}f^{8eg}\right]\mathcal{D}_2^{kg} + \frac{2}{N_f}\delta^{c8}\mathcal{D}_2^{k8} \\ & + d^{c8e}\{G^{ke},T^8\} + \frac{1}{2}d^{88e}\{G^{ke},T^c\} \\ & + if^{c8e}[G^{k8},\{J^r,G^{re}\}],\end{aligned} \tag{A18}$$

$$\begin{aligned}[G^{i8},[\mathcal{D}_2^{i8},\mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{i8},[G^{i8},\mathcal{D}_2^{kc}]] + [\mathcal{D}_2^{i8},[\mathcal{D}_2^{i8},G^{kc}]] = {} & -2f^{c8e}f^{8eg}G^{kg} + \frac{3}{4}f^{c8e}f^{8eg}\mathcal{D}_3^{kg} + \frac{1}{2}f^{c8e}f^{8eg}\mathcal{O}_3^{kg} \\ & + \frac{1}{2}\{G^{kc},\{T^8,T^8\}\} + \{G^{k8},\{T^c,T^8\}\} \\ & - \frac{1}{2}f^{c8e}\epsilon^{kim}\{T^e,\{J^i,G^{m8}\}\},\end{aligned} \tag{A19}$$

$$\begin{aligned}&[G^{i8},[G^{i8},\mathcal{D}_3^{kc}]] + [G^{i8},[\mathcal{D}_3^{i8},G^{kc}]] + [\mathcal{D}_3^{i8},[G^{i8},G^{kc}]] \\ &\quad = (d^{c8e}d^{8eg} - 2f^{c8e}f^{8eg})G^{kg} + \frac{2}{N_f}\delta^{c8}G^{k8} + \frac{1}{N_f}d^{c88}J^k + \frac{2}{N_f}\delta^{88}\mathcal{D}_3^{kc} + \frac{1}{4}(3f^{c8e}f^{8eg} + 2d^{c8e}d^{8eg})\mathcal{D}_3^{kg} \\ &\qquad + \frac{1}{N_f}\delta^{c8}\mathcal{D}_3^{k8} + d^{c8e}d^{8eg}\mathcal{O}_3^{kg} - 2\{G^{kc},\{G^{r8},G^{r8}\}\} + 2\{G^{rc},\{G^{r8},G^{k8}\}\} + 4d^{c8e}\{G^{ke},\{J^r,G^{r8}\}\} \\ &\qquad - d^{c8e}\{G^{k8},\{J^r,G^{re}\}\} + d^{88e}\{G^{ke},\{J^r,G^{rc}\}\} - 3d^{c8e}\{J^k,\{G^{re},G^{r8}\}\} + d^{88e}\{J^k,\{G^{rc},G^{re}\}\} \\ &\qquad + \frac{1}{N_f}d^{c88}\{J^2,J^k\} - \frac{1}{2}f^{c8e}\epsilon^{kim}\{T^e,\{J^i,G^{m8}\}\},\end{aligned} \tag{A20}$$

$$\begin{aligned}&[G^{i8},[G^{i8},\mathcal{O}_3^{kc}]] + [G^{i8},[\mathcal{O}_3^{i8},G^{kc}]] + [\mathcal{O}_3^{i8},[G^{i8},G^{kc}]] \\ &\quad = -\frac{1}{2}(d^{c8e}d^{8eg} - 2f^{c8e}f^{8eg})G^{kg} - \frac{1}{N_f}\delta^{c8}G^{k8} - \frac{1}{2N_f}d^{c88}J^k + \frac{1}{2}d^{c8e}d^{8eg}\mathcal{D}_3^{kg} + \frac{1}{N_f}\delta^{c8}\mathcal{D}_3^{k8} + \frac{2}{N_f}\delta^{88}\mathcal{O}_3^{kc} \\ &\qquad + \frac{5}{N_f}\delta^{c8}\mathcal{O}_3^{k8} + \frac{1}{4}(3f^{c8e}f^{8eg} + 4d^{c8e}d^{8eg})\mathcal{O}_3^{kg} - \{G^{kc},\{G^{r8},G^{r8}\}\} - \{G^{rc},\{G^{r8},G^{k8}\}\} \\ &\qquad - d^{c8e}\{G^{ke},\{J^r,G^{r8}\}\} + \frac{3}{2}d^{c8e}\{G^{k8},\{J^r,G^{re}\}\} - \frac{1}{2}d^{88e}\{G^{ke},\{J^r,G^{rc}\}\} + d^{88e}\{G^{kc},\{J^r,G^{re}\}\} \\ &\qquad - \frac{1}{2}d^{c8e}\{J^k,\{G^{re},G^{r8}\}\} - \frac{1}{2}d^{88e}\{J^k,\{G^{rc},G^{re}\}\} + \frac{1}{N_f}d^{c88}\{J^2,J^k\} + \frac{3}{4}f^{c8e}\epsilon^{kim}\{T^e,\{J^i,G^{m8}\}\},\end{aligned} \tag{A21}$$

$$[\mathcal{D}_2^{i8},[\mathcal{D}_2^{i8},\mathcal{D}_2^{kc}]] = -f^{c8e}f^{8eg}\mathcal{D}_2^{kg} + \tfrac{1}{2}f^{c8e}f^{8eg}\mathcal{D}_4^{kg} + \tfrac{1}{2}\{\mathcal{D}_2^{kc},\{T^8,T^8\}\}, \tag{A22}$$

$$
\begin{aligned}
&[G^{i8},[\mathcal{D}_2^{i8},\mathcal{D}_3^{kc}]]+[G^{i8},[\mathcal{D}_3^{i8},\mathcal{D}_2^{kc}]]+[\mathcal{D}_2^{i8},[G^{i8},\mathcal{D}_3^{kc}]]+[\mathcal{D}_2^{i8},[\mathcal{D}_3^{i8},G^{kc}]]\\
&\quad+[\mathcal{D}_3^{i8},[G^{i8},\mathcal{D}_2^{kc}]]+[\mathcal{D}_3^{i8},[\mathcal{D}_2^{i8},G^{kc}]]\\
&=4if^{c8e}[G^{k8},\{J^r,G^{re}\}]-4if^{c8e}[G^{ke},\{J^r,G^{r8}\}]+2d^{c8e}\{J^2,\{G^{ke},T^8\}\}+d^{88e}\{J^2,\{G^{ke},T^c\}\}\\
&\quad-2d^{c8e}\{\mathcal{D}_2^{k8},\{J^r,G^{re}\}\}-d^{88e}\{\mathcal{D}_2^{kc},\{J^r,G^{re}\}\}+2\{\{J^r,G^{rc}\},\{G^{k8},T^8\}\}+2\{\{J^r,G^{r8}\},\{G^{kc},T^8\}\}\\
&\quad+2\{\{J^r,G^{r8}\},\{G^{k8},T^c\}\}+2if^{c8e}\{J^2,[G^{ke},\{J^r,G^{r8}\}]\}-2if^{c8e}\{\{J^r,G^{re}\},[J^2,G^{k8}]\}\\
&\quad+2if^{c8e}\{J^k,[\{J^i,G^{ie}\},\{J^r,G^{r8}\}]\},
\end{aligned}
\tag{A23}
$$

$$
\begin{aligned}
&[G^{i8},[\mathcal{D}_2^{i8},\mathcal{O}_3^{kc}]]+[G^{i8},[\mathcal{O}_3^{i8},\mathcal{D}_2^{kc}]]+[\mathcal{D}_2^{i8},[G^{i8},\mathcal{O}_3^{kc}]]+[\mathcal{D}_2^{i8},[\mathcal{O}_3^{i8},G^{kc}]]\\
&\quad+[\mathcal{O}_3^{i8},[G^{i8},\mathcal{D}_2^{kc}]]+[\mathcal{O}_3^{i8},[\mathcal{D}_2^{i8},G^{kc}]]\\
&=9f^{c8e}f^{8eg}\mathcal{D}_2^{kg}+\frac{2}{N_f}\delta^{88}\mathcal{D}_4^{kc}+\frac{9}{2}f^{c8e}f^{8eg}\mathcal{D}_4^{kg}+\frac{4}{N_f}\delta^{c8}\mathcal{D}_4^{k8}-2\{\mathcal{D}_2^{kc},\{G^{r8},G^{r8}\}\}-4\{\mathcal{D}_2^{k8},\{G^{rc},G^{r8}\}\}\\
&\quad+d^{c8e}\{\mathcal{D}_2^{k8},\{J^r,G^{re}\}\}+\frac{1}{2}d^{88e}\{\mathcal{D}_2^{kc},\{J^r,G^{re}\}\}+d^{c8e}\{J^2,\{G^{ke},T^8\}\}+\frac{1}{2}d^{88e}\{J^2,\{G^{ke},T^c\}\}\\
&\quad+2if^{c8e}\{J^2,[G^{k8},\{J^r,G^{re}\}]\}-if^{c8e}\{J^2,[G^{ke},\{J^r,G^{r8}\}]\}-if^{c8e}\{\{J^r,G^{r8}\},[J^2,G^{ke}]\}\\
&\quad+if^{c8e}\{\{J^r,G^{re}\},[J^2,G^{k8}]\}-2if^{c8e}\{J^k,[\{J^i,G^{ie}\},\{J^r,G^{r8}\}]\}.
\end{aligned}
\tag{A24}
$$

2. Nondegenerate case $\Delta/m_\pi \neq 0$

Similarly, the evaluation of the commutator-anticommutator structure

$$\{A^{ja},[A^{kc},[\mathcal{M},A^{jb}]]\},$$

which represents the leading contribution to the renormalized baryon axial vector current for finite octet-decuplet mass difference, yields the following terms:

(1) *Flavor singlet contribution*

$$\{G^{ia},[G^{kc},[J^2,G^{ia}]]\}=-\tfrac{1}{2}(N_f-2)G^{kc}+\tfrac{1}{2}(N_c+N_f)\mathcal{D}_2^{kc}-\tfrac{1}{2}\mathcal{D}_3^{kc}-\mathcal{O}_3^{kc},\tag{A25}$$

$$
\begin{aligned}
&\{G^{ia},[\mathcal{D}_2^{kc},[J^2,G^{ia}]]\}+\{G^{ia},[G^{kc},[J^2,\mathcal{D}_2^{ia}]]\}+\{\mathcal{D}_2^{ia},[G^{kc},[J^2,G^{ia}]]\}\\
&\quad=2(N_c+N_f)G^{kc}+\tfrac{1}{2}[N_c(N_c+2N_f)-9N_f-2]\mathcal{D}_2^{kc}+\tfrac{1}{2}(N_c+N_f)\mathcal{D}_3^{kc}-2\mathcal{D}_4^{kc},
\end{aligned}
\tag{A26}
$$

(2) *Flavor octet contribution*

$$
\begin{aligned}
d^{ab8}\{G^{ia},[G^{kc},[J^2,G^{ib}]]\}&=-\frac{1}{4}(N_f-4)d^{c8e}G^{ke}+\frac{1}{4}(N_c+N_f)d^{c8e}\mathcal{D}_2^{ke}-\frac{1}{4}d^{c8e}\mathcal{D}_3^{ke}-\frac{1}{2}d^{c8e}\mathcal{O}_3^{ke}\\
&\quad-\frac{1}{2}\{G^{kc},\{J^r,G^{r8}\}\}+\frac{1}{N_f}\{G^{k8},\{J^r,G^{rc}\}\}+\frac{1}{8}\{J^k,\{T^c,T^8\}\}-\frac{1}{N_f}\{J^k,\{G^{rc},G^{r8}\}\}\\
&\quad+\frac{1}{4}(N_c+N_f)[J^2,[T^8,G^{kc}]]+\frac{N_c(N_c+2N_f)-2N_f+4}{4N_f}\delta^{c8}J^k-\frac{1}{2N_f}\delta^{c8}\{J^2,J^k\},
\end{aligned}
\tag{A27}
$$

$$d^{ab8}(\{G^{ia},[\mathcal{D}_2^{kc},[J^2,G^{ib}]]\}+\{G^{ia},[G^{kc},[J^2,\mathcal{D}_2^{ib}]]\}+\{\mathcal{D}_2^{ia},[G^{kc},[J^2,G^{ib}]]\})$$
$$= (N_c+N_f)d^{c8e}G^{ke}-\frac{7N_f+4}{4}d^{c8e}\mathcal{D}_2^{ke}-\frac{N_f}{2}\{T^c,G^{k8}\}+\{G^{kc},T^8\}+\frac{1}{4}(N_c+N_f)d^{c8e}\mathcal{D}_3^{ke}$$
$$+\frac{N_f-2}{2N_f}(N_c+N_f)(\{J^k,\{G^{rc},G^{r8}\}\}-\{G^{k8},\{J^r,G^{rc}\}\})+\frac{1}{4}(N_c+N_f)\{J^k,\{T^c,T^8\}\}$$
$$-\frac{N_f^2+4}{4N_f}[J^2,[T^8,G^{kc}]]+\frac{1}{2}\{\mathcal{D}_2^{k8},\{J^r,G^{rc}\}\}-\frac{N_f-2}{2N_f}\{J^2,\{G^{k8},T^c\}\}-\frac{1}{2}d^{c8e}\mathcal{D}_4^{ke}$$
$$-\frac{N_f+1}{N_f}\{\mathcal{D}_2^{kc},\{J^r,G^{r8}\}\}, \tag{A28}$$

(3) *Flavor* **27** *contribution*

$$\{G^{i8},[G^{kc},[J^2,G^{i8}]]\}=\frac{1}{N_f}(\delta^{88}\delta^{bc}-\delta^{b8}\delta^{c8})G^{kb}-\frac{1}{2}(d^{c8e}d^{e8d}-d^{ced}d^{e88}-f^{c8e}f^{e8d})G^{kd}$$
$$-\frac{1}{2}d^{c8e}\{G^{k8},\{J^r,G^{re}\}\}+\frac{1}{2}d^{c8e}\{J^k,\{G^{re},G^{r8}\}\}-\frac{1}{4}\epsilon^{kij}f^{c8e}\{T^e,\{J^i,G^{j8}\}\}$$
$$+\frac{1}{2N_f}\delta^{c8}\{J^k,\{J^r,G^{r8}\}\}-\frac{1}{N_f}\delta^{c8}\{J^2,G^{k8}\}, \tag{A29}$$

$$\{G^{i8},[\mathcal{D}_2^{kc},[J^2,G^{i8}]]\}+\{G^{i8},[G^{kc},[J^2,\mathcal{D}_2^{i8}]]\}+\{\mathcal{D}_2^{i8},[G^{kc},[J^2,G^{i8}]]\}$$
$$=-\frac{15}{4}f^{c8e}f^{8eg}\mathcal{D}_2^{kg}+\frac{i}{2}f^{c8e}[G^{ke},\{J^r,G^{r8}\}]-if^{c8e}[G^{k8},\{J^r,G^{re}\}]-\frac{1}{2}f^{c8e}f^{8eg}\mathcal{D}_4^{kg}+\{\mathcal{D}_2^{kc},\{G^{r8},G^{r8}\}\}$$
$$+\{\mathcal{D}_2^{k8},\{G^{rc},G^{r8}\}\}-\frac{1}{2}\{\{J^r,G^{rc}\},\{G^{k8},T^8\}\}-\frac{1}{2}\{\{J^r,G^{r8}\},\{G^{k8},T^c\}\}+\frac{i}{2}f^{c8e}\{J^k,[\{J^i,G^{ie}\},\{J^r,G^{r8}\}]\}. \tag{A30}$$

APPENDIX B: MATRIX ELEMENTS OF BARYON OPERATORS

In order to produce results of straightforward applicability, here we present the evaluation of the matrix elements of the baryon operators that constitute A^{kc}. A glance at Eqs. (30)–(32) reveals that one can identify the basic operators

$$X_0^c=\{J^r,G^{rc}\},\quad X_1^{kc}=G^{kc},\quad X_2^{kc}=\mathcal{D}_2^{kc},\quad X_3^{kc}=\mathcal{D}_3^{kc},\quad X_4^{kc}=\mathcal{O}_3^{kc},\quad X_5^{kc}=\{G^{kc},T^8\},$$
$$X_6^{kc}=\{G^{k8},T^c\},\quad X_7^{kc}=\{G^{kc},\{J^r,G^{r8}\}\},\quad X_8^{kc}=\{G^{k8},\{J^r,G^{rc}\}\},\quad X_9^{kc}=\{J^k,\{G^{rc},G^{r8}\}\},$$
$$X_{10}^{kc}=\{J^k,\{T^c,T^8\}\},\quad X_{11}^{kc}=[G^{k8},\{J^r,G^{rc}\}],\quad X_{12}^{kc}=[G^{kc},\{J^r,G^{r8}\}],\quad X_{13}^{kc}=\{G^{kc},\{T^8,T^8\}\},$$
$$X_{14}^{kc}=\{G^{k8},\{T^c,T^8\}\},\quad X_{15}^{kc}=\{G^{rc},\{G^{r8},G^{k8}\}\},\quad X_{16}^{kc}=\{G^{kc},\{G^{r8},G^{r8}\}\},$$
$$X_{17}^{kc}=\{\mathcal{D}_2^{kc},\{G^{r8},G^{r8}\}\},\quad X_{18}^{kc}=\{\mathcal{D}_2^{k8},\{G^{rc},G^{r8}\}\}.$$

Among all the allowed operators, $\mathcal{O}_3^{kc}$ and $[J^2,G^{kc}]$ connect states of different spin only, whereas $[J^2,[T^8,G^{kc}]]$ connects states which change both spin and strangeness and along with $f^{c8e}\epsilon^{kim}\{T^e,\{J^i,G^{m8}\}\}$, they do not contribute to any observed decay. Thus, the nonvanishing matrix elements of the operators X_m^{kc} for initial and final spin-$\frac{1}{2}$ baryon states for eight physically relevant processes are listed in Table III. Notice that operators of the form $f^{c8e}X_m^{ke}$, $d^{c8e}X_m^{ke}$, $f^{c8d}d^{d8e}X_m^{ke},\ldots$, can be trivially obtained from X_m^{kc} and are not listed in Table III.

We now proceed further to obtain theoretical expressions for the axial vector couplings $g_A^{B_jB_i}$. For any given process, $g_A^{B_jB_i}$ is composed of three terms. The first one is the tree-level value $\alpha_{B_jB_i}$; the next one is the contribution of Figs. 1(a)–1(c); and the last one is the contribution of Fig. 1(d). The tree-level value can be written as a sum of the three parameters a_1, b_2, and b_3 times coefficients obtained from the appropriate matrix elements of the baryon operators that accompany them; these coefficients are listed in Table IV for the processes of interest here. The

TABLE III. Matrix elements of the operators X_m^{kc} for some observed transitions between spin-$\frac{1}{2}$ baryons.

	pn	$\Lambda\Sigma^\pm$	$\Xi^0\Xi^-$	$p\Lambda$	$n\Sigma^-$	$\Lambda\Xi^-$	$\Sigma^0\Xi^-$	$\Sigma^+\Xi^0$
$[X_0^c]_{B_jB_i}$	$5/2$	$\sqrt{3/2}$	$1/2$	$-\sqrt{27/8}$	$1/2$	$\sqrt{3/8}$	$5\sqrt{2}/4$	$5/2$
$[X_1^{kc}]_{B_jB_i}$	$5/6$	$1/\sqrt{6}$	$1/6$	$-\sqrt{3/8}$	$1/6$	$1/\sqrt{24}$	$5/\sqrt{72}$	$5/6$
$[X_2^{kc}]_{B_jB_i}$	$1/2$	0	$-1/2$	$-\sqrt{3/8}$	$-1/2$	$\sqrt{3/8}$	$1/\sqrt{8}$	$1/2$
$[X_3^{kc}]_{B_jB_i}$	$5/2$	$\sqrt{3/2}$	$1/2$	$-\sqrt{27/8}$	$1/2$	$\sqrt{3/8}$	$5/\sqrt{8}$	$5/2$
$[X_5^{kc}]_{B_jB_i}$	$5/\sqrt{12}$	0	$-1/\sqrt{12}$	$-3/\sqrt{32}$	$1/\sqrt{48}$	$-1/\sqrt{32}$	$-5/\sqrt{96}$	$-5/\sqrt{48}$
$[X_6^{kc}]_{B_jB_i}$	$1/\sqrt{12}$	0	$\sqrt{3}/2$	$1/\sqrt{32}$	$-\sqrt{3}/4$	$-5/\sqrt{32}$	$-1/\sqrt{96}$	$-1/\sqrt{48}$
$[X_7^{kc}]_{B_jB_i}$	$5/\sqrt{48}$	0	$-\sqrt{3}/4$	$3\sqrt{2}/16$	$\sqrt{3}/8$	$-5\sqrt{2}/16$	$-5\sqrt{6}/48$	$-5\sqrt{3}/24$
$[X_8^{kc}]_{B_jB_i}$	$5/\sqrt{48}$	0	$-\sqrt{3}/4$	$3\sqrt{2}/16$	$\sqrt{3}/8$	$-5\sqrt{2}/16$	$-5\sqrt{6}/48$	$-5\sqrt{3}/24$
$[X_9^{kc}]_{B_jB_i}$	$5/\sqrt{48}$	$-1/\sqrt{2}$	$-11/\sqrt{48}$	$3\sqrt{2}/16$	$11\sqrt{3}/24$	$-13\sqrt{2}/16$	$-5\sqrt{6}/48$	$-5\sqrt{3}/24$
$[X_{10}^{kc}]_{B_jB_i}$	$\sqrt{3}$	0	$\sqrt{3}$	$-3/\sqrt{8}$	$-\sqrt{3}/2$	$-3/\sqrt{8}$	$-\sqrt{3/8}$	$-\sqrt{3}/2$
$[X_{11}^{kc}]_{B_jB_i}$	0	$1/\sqrt{2}$	0	$-9\sqrt{2}/16$	$-\sqrt{3}/24$	$\sqrt{2}/16$	$25\sqrt{6}/48$	$25\sqrt{3}/24$
$[X_{12}^{kc}]_{B_jB_i}$	0	$-1/\sqrt{2}$	0	$9\sqrt{2}/16$	$\sqrt{3}/24$	$-\sqrt{2}/16$	$-25\sqrt{6}/48$	$-25\sqrt{3}/24$
$[X_{13}^{kc}]_{B_jB_i}$	$5/2$	0	$1/2$	$-\sqrt{27/32}$	$1/4$	$\sqrt{3/32}$	$5/\sqrt{32}$	$5/4$
$[X_{14}^{kc}]_{B_jB_i}$	$1/2$	0	$-3/2$	$\sqrt{6}/16$	$-3/8$	$5\sqrt{6}/16$	$\sqrt{2}/16$	$1/8$
$[X_{15}^{kc}]_{B_jB_i}$	$5/72$	$-\sqrt{6}/36$	$97/72$	$-5\sqrt{6}/96$	$53/144$	$101\sqrt{6}/288$	$25\sqrt{2}/288$	$25/144$
$[X_{16}^{kc}]_{B_jB_i}$	$5/24$	$\sqrt{2/3}$	$17/24$	$-5\sqrt{6}/32$	$13/48$	$7\sqrt{6}/32$	$145\sqrt{2}/96$	$145/48$
$[X_{17}^{kc}]_{B_jB_i}$	$1/8$	0	$-17/8$	$-5\sqrt{6}/32$	$-13/16$	$21\sqrt{6}/32$	$29\sqrt{2}/32$	$29/16$
$[X_{18}^{kc}]_{B_jB_i}$	$5/8$	0	$11/8$	$3\sqrt{6}/64$	$11/32$	$13\sqrt{6}/64$	$5\sqrt{2}/64$	$5/32$

contribution of Fig. 1(a)–1(c) contains cubic products of a_1, b_j, and c_k, but to the order of approximation implemented here this contribution can be expressed as a sum of the eight quantities a_1^3, $a_1^2b_2$, $a_1b_2^2$, $a_1^2b_3$, $a_1^2c_3$, b_2^3, $a_1b_2b_3$, and $a_1b_2c_3$ times coefficients arising from the matrix elements of their respective operators, multiplied by a global factor containing the integrals over the loops; in Table V we have listed these coefficients. Finally, the contribution of Fig. 1(d) can be expressed as a sum of the three parameters a_1, b_2, and b_3 times coefficients from the matrix elements of the corresponding operators, also multiplied by a global factor containing the integrals over the loops. For completeness these coefficients can be found in Table VI. However, a few clarifying notes are instructive here. In Tables V and VI, the singlet, octet, and **27** contributions are explicitly separated so that the interested reader can reproduce our results. Besides, the singlet and octet pieces have been subtracted from the entries corresponding to the **27** piece so that it is a purely **27** contribution. In order to simplify our notation, a coefficient that multiplies the entries of each flavor representation has been factored out.

Accordingly, for the process $n \to pe^-\bar{\nu}_e$ for instance, g_A^{pn} can be constructed by reading off the appropriate coefficients from Tables IV, V, and VI, namely,

$$\begin{aligned} g_A^{pn} = {}& \alpha_{pn} + C_{\mathbf{1}}^{pn}(345a_1^3 + 63a_1^2b_2 + 171a_1b_2^2 - 31a_1^2b_3 \\ & - 396a_1^2c_3 + 21b_2^3 + 338a_1b_2b_3 - 444a_1b_2c_3)F_{\mathbf{1}}^{(1)} \\ & + C_{\mathbf{8}}^{pn}(165a_1^3 - 381a_1^2b_2 - 33a_1b_2^2 - 419a_1^2b_3 \\ & - 564a_1^2c_3 - 15b_2^3 + 218a_1b_2b_3 - 708a_1b_2c_3)F_{\mathbf{8}}^{(1)} \\ & + C_{\mathbf{27}}^{pn}(45a_1^3 + 267a_1^2b_2 + 231a_1b_2^2 - 107a_1^2b_3 \\ & - 92a_1^2c_3 + 25b_2^3 + 314a_1b_2b_3 - 204a_1b_2c_3)F_{\mathbf{27}}^{(1)} \\ & + D_{\mathbf{1}}^{pn}(15a_1 + 3b_2 + 5b_3)I_{\mathbf{1}} + D_{\mathbf{8}}^{pn}(15a_1 + 3b_2 \\ & + 5b_3)I_{\mathbf{8}} + D_{\mathbf{27}}^{pn}(15a_1 + 3b_2 + 5b_3)I_{\mathbf{27}}, \end{aligned} \tag{B1}$$

where the tree-level value reads

$$\alpha_{pn} = \tfrac{5}{6}a_1 + \tfrac{1}{6}b_2 + \tfrac{5}{18}b_3. \tag{B2}$$

Note that the coefficients $C_{\mathbf{1}}^{pn} = 1/432$, $C_{\mathbf{8}}^{pn} = \sqrt{3}/2592$, $C_{\mathbf{27}}^{pn} = 1/5760$, $D_{\mathbf{1}}^{pn} = -1/12$, $D_{\mathbf{8}}^{pn} = -\sqrt{3}/72$, and $D_{\mathbf{27}}^{pn} = 1/480$ are the common factors that multiply each entry referred to above. Analogous expressions can be obtained for the axial vector couplings of the remaining processes.

TABLE IV. Coefficients for the axial vector couplings of the baryons: tree-level values.

B_jB_i	a_1	b_2	b_3
pn	$5/6$	$1/6$	$5/18$
$\Lambda\Sigma^\pm$	$1/\sqrt{6}$	0	$\sqrt{6}/18$
$\Xi^0\Xi^-$	$1/6$	$-1/6$	$1/18$
$p\Lambda$	$-\sqrt{3/8}$	$-\sqrt{6}/12$	$-\sqrt{6}/12$
$n\Sigma^-$	$1/6$	$-1/6$	$1/18$
$\Lambda\Xi^-$	$\sqrt{6}/12$	$\sqrt{6}/12$	$\sqrt{6}/36$
$\Sigma^0\Xi^-$	$5/\sqrt{72}$	$\sqrt{2}/12$	$5\sqrt{2}/36$
$\Sigma^+\Xi^0$	$5/6$	$1/6$	$5/18$

TABLE V. Coefficients for the axial vector couplings of the baryons, Figs. 1(a)–1(c).

					Singlet				
B_jB_i	$C_{\mathbf{1}}^{B_jB_i}$	a_1^3	$a_1^2b_2$	$a_1b_2^2$	$a_1^2b_3$	$a_1^2c_3$	b_2^3	$a_1b_2b_3$	$a_1b_2c_3$
pn	$1/432$	345	63	171	−31	−396	21	338	−444
$\Lambda\Sigma^\pm$	$\sqrt{6}/432$	69	−48	15	37	−72	0	40	−108
$\Xi^0\Xi^-$	$1/432$	69	−351	−81	253	−36	−21	−98	−204
$p\Lambda$	$\sqrt{6}/288$	−69	−53	−47	35	84	−7	−86	76
$n\Sigma^-$	$1/432$	69	−351	−81	253	−36	−21	−98	−204
$\Lambda\Xi^-$	$\sqrt{6}/864$	69	255	111	−179	−108	21	178	−12
$\Sigma^0\Xi^-$	$\sqrt{2}/864$	345	63	171	−31	−396	21	338	−444
$\Sigma^+\Xi^0$	$1/432$	345	63	171	−31	−396	21	338	−444
					Octet				
B_jB_i	$C_{\mathbf{8}}^{B_jB_i}$	a_1^3	$a_1^2b_2$	$a_1b_2^2$	$a_1^2b_3$	$a_1^2c_3$	b_2^3	$a_1b_2b_3$	$a_1b_2c_3$
pn	$\sqrt{3}/2592$	165	−381	−33	−419	−564	−15	218	−708
$\Lambda\Sigma^\pm$	$\sqrt{2}/864$	33	−96	−9	−71	36	0	−24	−60
$\Xi^0\Xi^-$	$\sqrt{3}/2592$	33	141	147	−407	180	15	86	12
$p\Lambda$	$\sqrt{2}/3456$	99	195	105	699	540	33	−118	372
$n\Sigma^-$	$\sqrt{3}/5184$	−33	−141	−147	407	−180	−15	−86	−12
$\Lambda\Xi^-$	$\sqrt{2}/1152$	−11	−241	−97	−147	44	−11	−126	−52
$\Sigma^0\Xi^-$	$\sqrt{6}/10368$	−165	381	33	419	564	15	−218	708
$\Sigma^+\Xi^0$	$\sqrt{3}/5184$	−165	381	33	419	564	15	−218	708
					27				
B_jB_i	$C_{\mathbf{27}}^{B_jB_i}$	a_1^3	$a_1^2b_2$	$a_1b_2^2$	$a_1^2b_3$	$a_1^2c_3$	b_2^3	$a_1b_2b_3$	$a_1b_2c_3$
pn	$1/5760$	45	267	231	−107	−92	25	314	−204
$\Lambda\Sigma^\pm$	$\sqrt{6}/17280$	27	−144	−111	−69	264	0	−296	300
$\Xi^0\Xi^-$	$1/5760$	9	213	−69	609	−340	−25	118	36
$p\Lambda$	$\sqrt{6}/11520$	81	225	195	−39	−60	27	238	−132
$n\Sigma^-$	$1/5760$	−27	−159	−33	13	−140	−5	46	−228
$\Lambda\Xi^-$	$\sqrt{6}/11520$	−27	63	−129	381	−132	−27	−62	156
$\Sigma^0\Xi^-$	$\sqrt{2}/11520$	−135	−321	27	−719	236	5	−62	−228
$\Sigma^+\Xi^0$	$1/5760$	−135	−321	27	−719	236	5	−62	−228

TABLE VI. Coefficients for the axial vector couplings of the baryons. Figure 1(d).

	Singlet				Octet				27			
B_jB_i	$D_{\mathbf{1}}^{B_jB_i}$	a_1	b_2	b_3	$D_{\mathbf{8}}^{B_jB_i}$	a_1	b_2	b_3	$D_{\mathbf{27}}^{B_jB_i}$	a_1	b_2	b_3
pn	$-1/12$	15	3	5	$-\sqrt{3}/72$	15	3	5	$1/480$	15	3	5
$\Lambda\Sigma^\pm$	$-\sqrt{6}/12$	3	0	1	$-\sqrt{2}/24$	3	0	1	$\sqrt{6}/480$	3	0	1
$\Xi^0\Xi^-$	$-1/12$	3	−3	1	$-\sqrt{3}/72$	3	−3	1	$1/480$	3	−3	1
$p\Lambda$	$\sqrt{6}/8$	3	1	1	$-\sqrt{2}/32$	3	1	1	$3\sqrt{6}/320$	3	1	1
$n\Sigma^-$	$-1/12$	3	−3	1	$\sqrt{3}/144$	3	−3	1	$-1/160$	3	−3	1
$\Lambda\Xi^-$	$-\sqrt{6}/24$	3	3	1	$\sqrt{2}/96$	3	3	1	$-\sqrt{6}/320$	3	3	1
$\Sigma^0\Xi^-$	$-\sqrt{2}/24$	15	3	5	$\sqrt{6}/288$	15	3	5	$-\sqrt{2}/320$	15	3	5
$\Sigma^+\Xi^0$	$-1/12$	15	3	5	$\sqrt{3}/144$	15	3	5	$-1/160$	15	3	5

APPENDIX C: CHIRAL COEFFICIENTS

In this appendix, for completeness, the explicit formulas for the chiral coefficients introduced in Eq. (53) are given. The lowest order coefficients $\alpha_{B_jB_i}$ are

$$\alpha_{pn} = D + F, \qquad \alpha_{\Lambda\Sigma^\pm} = \frac{2}{\sqrt{6}}D, \qquad \alpha_{p\Lambda} = -\frac{1}{\sqrt{6}}(D + 3F), \qquad \alpha_{n\Sigma^-} = D - F,$$
$$\alpha_{\Lambda\Xi^-} = -\frac{1}{\sqrt{6}}(D - 3F), \qquad \alpha_{\Xi^0\Xi^-} = D - F, \qquad \alpha_{\Sigma^0\Xi^-} = \frac{1}{\sqrt{2}}(D + F) = \frac{1}{\sqrt{2}}\alpha_{\Sigma^+\Xi^0}.$$

The coefficients $\bar{\lambda}_{B_i}^{\Pi}$ arising from the one-loop correction due to wave function renormalization, Figs. 1(b) and 1(c), are for the octet baryons

$$\bar{\lambda}_N^\pi = \frac{9}{4}(F+D)^2 + 2\mathcal{C}^2, \quad \bar{\lambda}_\Sigma^\pi = 6F^2 + D^2 + \frac{1}{3}\mathcal{C}^2, \quad \bar{\lambda}_N^K = \frac{1}{2}(9F^2 - 6FD + 5D^2 + \mathcal{C}^2), \quad \bar{\lambda}_\Sigma^K = 3(F^2+D^2) + \frac{5}{3}\mathcal{C}^2,$$
$$\bar{\lambda}_N^\eta = \frac{1}{4}(3F-D)^2, \quad \bar{\lambda}_\Sigma^\eta = D^2 + \frac{1}{2}\mathcal{C}^2, \quad \bar{\lambda}_\Xi^\pi = \frac{9}{4}(F-D)^2 + \frac{1}{2}\mathcal{C}^2, \quad \bar{\lambda}_\Lambda^\pi = 3D^2 + \frac{3}{2}\mathcal{C}^2, \quad \bar{\lambda}_\Xi^K = \frac{1}{2}(9F^2 + 6FD + 5D^2 + 3\mathcal{C}^2),$$
$$\bar{\lambda}_\Lambda^K = 9F^2 + D^2 + \mathcal{C}^2, \quad \bar{\lambda}_\Xi^\eta = \frac{1}{4}(3F+D)^2 + \frac{1}{2}\mathcal{C}^2, \quad \bar{\lambda}_\Lambda^\eta = D^2,$$

and for the decuplet baryons

$$\bar{\lambda}_\Delta^\pi = \frac{25}{36}\mathcal{H}^2 + \frac{1}{2}\mathcal{C}^2, \quad \bar{\lambda}_{\Xi^*}^\pi = \frac{5}{36}\mathcal{H}^2 + \frac{1}{4}\mathcal{C}^2, \quad \bar{\lambda}_\Delta^K = \frac{5}{18}\mathcal{H}^2 + \frac{1}{2}\mathcal{C}^2, \quad \bar{\lambda}_{\Xi^*}^K = \frac{5}{6}\mathcal{H}^2 + \frac{1}{2}\mathcal{C}^2,$$
$$\bar{\lambda}_\Delta^\eta = \frac{5}{36}\mathcal{H}^2, \quad \bar{\lambda}_{\Xi^*}^\eta = \frac{5}{36}\mathcal{H}^2 + \frac{1}{4}\mathcal{C}^2, \quad \bar{\lambda}_{\Sigma^*}^\pi = \frac{10}{27}\mathcal{H}^2 + \frac{5}{12}\mathcal{C}^2, \quad \bar{\lambda}_{\Omega^-}^\pi = \frac{10}{27}\mathcal{H}^2,$$
$$\bar{\lambda}_{\Sigma^*}^K = \frac{20}{27}\mathcal{H}^2 + \frac{1}{3}\mathcal{C}^2, \quad \bar{\lambda}_{\Omega^-}^K = \frac{5}{9}\mathcal{H}^2 + \mathcal{C}^2, \quad \bar{\lambda}_{\Sigma^*}^\eta = \frac{1}{4}\mathcal{C}^2, \quad \bar{\lambda}_{\Omega^-}^\eta = \frac{5}{9}\mathcal{H}^2 + \mathcal{C}^2.$$

The coefficients $\bar{\lambda}_{B_jB_i}^{\Pi}$ are thus written as

$$\bar{\lambda}_{pn}^\pi = \frac{9}{4}(F+D)^2 + 2\mathcal{C}^2, \quad \bar{\lambda}_{\Lambda\Sigma^\pm}^\pi = 3F^2 + 2D^2 + \frac{11}{12}\mathcal{C}^2, \quad \bar{\lambda}_{pn}^K = \frac{1}{2}(9F^2 - 6FD + 5D^2 + \mathcal{C}^2),$$
$$\bar{\lambda}_{\Lambda\Sigma^\pm}^K = 6F^2 + 2D^2 + \frac{4}{3}\mathcal{C}^2, \quad \bar{\lambda}_{pn}^\eta = \frac{1}{4}(3F-D)^2, \quad \bar{\lambda}_{\Lambda\Sigma^\pm}^\eta = D^2 + \frac{1}{4}\mathcal{C}^2, \quad \bar{\lambda}_{p\Lambda}^\pi = \frac{3}{8}(3F^2 + 6FD + 7D^2) + \frac{7}{4}\mathcal{C}^2,$$
$$\bar{\lambda}_{n\Sigma^-}^\pi = \frac{1}{8}(33F^2 + 18FD + 13D^2) + \frac{7}{6}\mathcal{C}^2, \quad \bar{\lambda}_{p\Lambda}^K = \frac{1}{4}(27F^2 - 6FD + 7D^2) + \frac{3}{4}\mathcal{C}^2,$$
$$\bar{\lambda}_{n\Sigma^-}^K = \frac{1}{4}(15F^2 - 6FD + 11D^2) + \frac{13}{12}\mathcal{C}^2, \quad \bar{\lambda}_{p\Lambda}^\eta = \frac{1}{8}(9F^2 - 6FD + 5D^2), \quad \bar{\lambda}_{n\Sigma^-}^\eta = \frac{1}{8}(9F^2 - 6FD + 5D^2) + \frac{1}{4}\mathcal{C}^2,$$
$$\bar{\lambda}_{\Lambda\Xi^-}^\pi = \frac{3}{8}(3F^2 - 6FD + 7D^2) + \mathcal{C}^2, \quad \bar{\lambda}_{\Sigma^0\Xi^-}^\pi = \frac{1}{8}(33F^2 - 18FD + 13D^2) + \frac{5}{12}\mathcal{C}^2,$$
$$\bar{\lambda}_{\Lambda\Xi^-}^K = \frac{1}{4}(27F^2 + 6FD + 7D^2) + \frac{5}{4}\mathcal{C}^2, \quad \bar{\lambda}_{\Sigma^0\Xi^-}^K = \frac{1}{4}(15F^2 + 6FD + 11D^2) + \frac{19}{12}\mathcal{C}^2,$$
$$\bar{\lambda}_{\Lambda\Xi^-}^\eta = \frac{1}{8}(9F^2 + 6FD + 5D^2) + \frac{1}{4}\mathcal{C}^2, \quad \bar{\lambda}_{\Sigma^0\Xi^-}^\eta = \frac{1}{8}(9F^2 + 6FD + 5D^2) + \frac{1}{2}\mathcal{C}^2,$$
$$\bar{\lambda}_{\Xi^0\Xi^-}^\pi = \frac{9}{4}(F-D)^2 + \frac{1}{2}\mathcal{C}^2, \quad \bar{\lambda}_{\Sigma^+\Xi^0}^\pi = \bar{\lambda}_{\Lambda\Xi^-}^\pi, \quad \bar{\lambda}_{\Xi^0\Xi^-}^K = \frac{1}{2}(9F^2 + 6FD + 5D^2) + \frac{3}{2}\mathcal{C}^2,$$
$$\bar{\lambda}_{\Sigma^+\Xi^0}^K = \bar{\lambda}_{\Lambda\Xi^-}^K, \quad \bar{\lambda}_{\Xi^0\Xi^-}^\eta = \frac{1}{4}(3F+D)^2 + \frac{1}{2}\mathcal{C}^2, \quad \bar{\lambda}_{\Sigma^+\Xi^0}^\eta = \bar{\lambda}_{\Lambda\Xi^-}^\eta.$$

The coefficients $\bar{\beta}_{B_jB_i}^{\Pi}$ evaluated from the graph in Fig. 1(a) are

$$\bar{\beta}_{pn}^\pi = \frac{1}{4}(F+D)^3 + \frac{16}{9}(F+D)\mathcal{C}^2 - \frac{50}{81}\mathcal{H}\mathcal{C}^2, \quad \bar{\beta}_{pn}^K = \frac{1}{3}(-3F^3 + 3F^2D - FD^2 + D^3) + \frac{2}{9}(F+3D)\mathcal{C}^2 - \frac{10}{81}\mathcal{H}\mathcal{C}^2,$$
$$\bar{\beta}_{pn}^\eta = -\frac{1}{12}(F+D)(3F-D)^2, \quad \bar{\beta}_{\Lambda\Sigma^\pm}^\pi = \frac{2}{3\sqrt{6}}D(6F^2 - D^2) + \frac{2}{3\sqrt{6}}\left(2F + \frac{1}{3}D\right)\mathcal{C}^2 - \frac{10}{27\sqrt{6}}\mathcal{H}\mathcal{C}^2,$$
$$\bar{\beta}_{\Lambda\Sigma^\pm}^K = -\frac{1}{\sqrt{6}}D(F^2 - D^2) + \frac{8}{3\sqrt{6}}\left(F + \frac{2}{3}D\right)\mathcal{C}^2 - \frac{5}{27\sqrt{6}}\mathcal{H}\mathcal{C}^2, \quad \bar{\beta}_{\Lambda\Sigma^\pm}^\eta = \frac{2}{3\sqrt{6}}D(D^2 + \mathcal{C}^2),$$
$$\bar{\beta}_{p\Lambda}^\pi = \frac{3}{2\sqrt{6}}D(F^2 - D^2) - \frac{1}{3\sqrt{6}}(11D + 3F)\mathcal{C}^2 + \frac{10}{9\sqrt{6}}\mathcal{H}\mathcal{C}^2,$$
$$\bar{\beta}_{p\Lambda}^K = \frac{1}{6\sqrt{6}}(27F^3 - 9F^2D - 15FD^2 + 5D^3) - \frac{1}{\sqrt{6}}(F+D)\mathcal{C}^2 + \frac{5}{9\sqrt{6}}\mathcal{H}\mathcal{C}^2, \quad \bar{\beta}_{p\Lambda}^\eta = -\frac{1}{6\sqrt{6}}D(9F^2 - D^2),$$
$$\bar{\beta}_{n\Sigma^-}^\pi = \frac{1}{6}(6F^3 + 3F^2D - 2FD^2 + D^3) + \frac{2}{9}(5F + D)\mathcal{C}^2 + \frac{10}{81}\mathcal{H}\mathcal{C}^2,$$
$$\bar{\beta}_{n\Sigma^-}^K = \frac{1}{6}(3F^3 + 3F^2D + FD^2 + D^3) + \frac{1}{9}(5F + D)\mathcal{C}^2 + \frac{5}{81}\mathcal{H}\mathcal{C}^2, \quad \bar{\beta}_{n\Sigma^-}^\eta = \frac{1}{6}D(3F^2 - 4FD + D^2) + \frac{1}{9}(3F - D)\mathcal{C}^2,$$
$$\bar{\beta}_{\Lambda\Xi^-}^\pi = \frac{3}{2\sqrt{6}}D(F^2 - D^2) - \frac{1}{3\sqrt{6}}(3F - D)\mathcal{C}^2 - \frac{5}{9\sqrt{6}}\mathcal{H}\mathcal{C}^2,$$
$$\bar{\beta}_{\Lambda\Xi^-}^K = \frac{1}{6\sqrt{6}}(-27F^3 - 9F^2D + 15FD^2 + 5D^3) - \frac{1}{\sqrt{6}}(F - D)\mathcal{C}^2 - \frac{5}{9\sqrt{6}}\mathcal{H}\mathcal{C}^2, \quad \bar{\beta}_{\Lambda\Xi^-}^\eta = -\frac{1}{6\sqrt{6}}D(9F^2 - D^2) + \frac{2}{3\sqrt{6}}D\mathcal{C}^2,$$
$$\bar{\beta}_{\Sigma^0\Xi^-}^\pi = \frac{1}{6\sqrt{2}}(-6F^3 + 3F^2D + 2FD^2 + D^3) + \frac{2}{9\sqrt{2}}(F + 2D)\mathcal{C}^2 - \frac{10}{81\sqrt{2}}\mathcal{H}\mathcal{C}^2$$
$$\bar{\beta}_{\Sigma^0\Xi^-}^K = \frac{1}{6\sqrt{2}}(-3F^3 + 3F^2D - FD^2 + D^3) + \frac{1}{9\sqrt{2}}(13F + 15D)\mathcal{C}^2 - \frac{35}{81\sqrt{2}}\mathcal{H}\mathcal{C}^2$$
$$\bar{\beta}_{\Sigma^0\Xi^-}^\eta = \frac{1}{6\sqrt{2}}D(3F^2 + 4FD + D^2) + \frac{1}{3\sqrt{2}}(F + D)\mathcal{C}^2 - \frac{5}{27\sqrt{2}}\mathcal{H}\mathcal{C}^2 \quad \bar{\beta}_{\Xi^0\Xi^-}^\pi = -\frac{1}{4}(F - D)^3 + \frac{2}{9}(F - D)\mathcal{C}^2 - \frac{5}{162}\mathcal{H}\mathcal{C}^2,$$
$$\bar{\beta}_{\Xi^0\Xi^-}^K = \frac{1}{3}(3F^3 + 3F^2D + FD^2 + D^3) + \frac{2}{9}(5F + D)\mathcal{C}^2 + \frac{10}{81}\mathcal{H}\mathcal{C}^2,$$
$$\bar{\beta}_{\Xi^0\Xi^-}^\eta = \frac{1}{12}(F - D)(3F + D)^2 + \frac{2}{9}(3F + D)\mathcal{C}^2 + \frac{5}{54}\mathcal{H}\mathcal{C}^2,$$

and, due to isospin symmetry, one also has

$$\bar{\beta}^{\Pi}_{\Sigma^+\Xi^0} = \sqrt{2}\bar{\beta}^{\Pi}_{\Sigma^0\Xi^-}. \qquad (\Pi = \pi, K, \eta)$$

Now, the coefficients $\gamma^{\Pi}_{B_jB_i}$ from Fig. 1(d) are

$$\gamma^{\pi}_{pn} = -F - D, \quad \gamma^{\pi}_{\Lambda\Sigma^\pm} = -\frac{2}{\sqrt{6}}D, \quad \gamma^{K}_{pn} = -\frac{1}{2}(F+D), \quad \gamma^{K}_{\Lambda\Sigma^\pm} = -\frac{1}{\sqrt{6}}D, \quad \gamma^{\eta}_{pn} = 0, \quad \gamma^{\eta}_{\Lambda\Sigma^\pm} = 0,$$

$$\gamma^{\pi}_{p\Lambda} = \frac{3}{8\sqrt{6}}(3F+D), \quad \gamma^{\pi}_{n\Sigma^-} = \frac{3}{8}(F-D), \quad \gamma^{K}_{p\Lambda} = \frac{3}{4\sqrt{6}}(3F+D), \quad \gamma^{K}_{n\Sigma^-} = \frac{3}{4}(F-D), \quad \gamma^{\eta}_{p\Lambda} = \frac{3}{8\sqrt{6}}(3F+D),$$

$$\gamma^{\eta}_{n\Sigma^-} = \frac{3}{8}(F-D), \quad \gamma^{\pi}_{\Lambda\Xi^-} = -\frac{3}{8\sqrt{6}}(3F-D), \quad \gamma^{\pi}_{\Sigma^0\Xi^-} = -\frac{3}{8\sqrt{2}}(F+D), \quad \gamma^{K}_{\Lambda\Xi^-} = -\frac{3}{4\sqrt{6}}(3F-D),$$

$$\gamma^{K}_{\Sigma^0\Xi^-} = -\frac{3}{4\sqrt{2}}(F+D), \quad \gamma^{\eta}_{\Lambda\Xi^-} = -\frac{3}{8\sqrt{6}}(3F-D), \quad \gamma^{\eta}_{\Sigma^0\Xi^-} = -\frac{3}{8\sqrt{2}}(F+D), \quad \gamma^{\pi}_{\Xi^0\Xi^-} = F - D,$$

$$\gamma^{\pi}_{\Sigma^+\Xi^0} = -\frac{3}{8}(F+D), \quad \gamma^{K}_{\Xi^0\Xi^-} = \frac{1}{2}(F-D), \quad \gamma^{K}_{\Sigma^+\Xi^0} = -\frac{3}{4}(F+D), \quad \gamma^{\eta}_{\Xi^0\Xi^-} = 0, \quad \gamma^{\eta}_{\Sigma^+\Xi^0} = -\frac{3}{8}(F+D).$$

The chiral coefficients listed above include contributions from intermediate octet and decuplet baryons. The corresponding distinction between primed and unprimed coefficients as defined in Eq. (53) is straightforward.

Finally, the coefficients $\zeta^{\eta'}_{B_jB_i}$ from Fig. 1(a)–1(c) read

$$\zeta^{\eta'}_{pn} = \frac{1}{9}(F+D)(3F-D)^2, \quad \zeta^{\eta'}_{\Lambda\Sigma^\pm} = \frac{1}{9}\sqrt{\frac{2}{3}}D(3F-D)^2, \quad \zeta^{\eta'}_{\Xi^0\Xi^-} = \frac{1}{9}(D-F)(3F-D)^2,$$

$$\zeta^{\eta'}_{p\Lambda} = -\frac{1}{9\sqrt{6}}(3F+D)(3F-D)^2, \quad \zeta^{\eta'}_{n\Sigma^-} = \frac{1}{9}(D-F)(3F-D)^2, \quad \zeta^{\eta'}_{\Lambda\Xi^-} = \frac{1}{9\sqrt{6}}(3F-D)^3,$$

$$\zeta^{\eta'}_{\Sigma^0\Xi^-} = \frac{1}{9\sqrt{2}}(F+D)(3F-D)^2, \quad \zeta^{\eta'}_{\Sigma^+\Xi^0} = \frac{1}{9}(F+D)(3F-D)^2.$$

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[20] Note that in Refs. [17,28], in dealing with loop integrals, other regularization schemes were proposed. The central idea in these works was to emphasize the long distance effects of the integrals and reduce the short distance contributions, which arise from the propagation of Goldstone bosons over distances smaller than a typical hadronic size. In the present work, however, we will concentrate on the structure of the diagrams themselves and use the former results obtained in dimensional regularization.
[21] An n-body operator is one with n q's and n $q^\dagger$'s, namely, it can be written as a polynomial of order n in J^i, T^a, and G^{ia} [9].
[22] Note that, in this section, we restrict ourselves to operators that have nonzero matrix elements between *octet* baryons—the full expressions can be found in Appendix A.
[23] Of course, the quark masses have still to be small with respect to the scale of chiral symmetry breaking.
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[26] The quoted errors of the best fit parameters will be from the χ^2 fit only and will not include any theoretical uncertainties.
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