

Three-flavor Lorentz-violating solution to the LSND anomaly?André de Gouvêa¹ and Yuval Grossman²¹*Department of Physics & Astronomy, Northwestern University, 2145 Sheridan Road, Evanston, Illinois 60208, USA*²*Department of Physics, Technion-Israel Institute of Technology, Technion City, 32000 Haifa, Israel*

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We investigate whether postulating the existence of Lorentz-violating, *CPT*-conserving interactions allows three-neutrino solutions to the LSND anomaly that are also consistent with all other neutrino data. We show that Lorentz-violating interactions that couple only to one of the active neutrinos have the right properties to explain all the data. The details of the data make this solution unattractive. We find, for example, that a highly nontrivial energy dependence of the Lorentz-violating interactions is required.

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I. INTRODUCTION

In 1996, the LSND collaboration first presented statistically significant evidence for $\bar{\nu}_e$ appearance from antimuon decay at rest [1]. Further analysis of larger LSND data samples (which also included data from pion decay in flight) confirmed the original excess [2]. In spite of the fact that this so-called LSND anomaly is close to completing ten years of existence, there is no compelling physics explanation for it. It is very natural to try to understand the LSND anomaly by postulating that there is a probability for what was originally a muon antineutrino to be detected as an electron antineutrino. If this is the explanation for the LSND anomaly, the associated transition probability need not be very large: $P_{\mu e} \sim 0.2\%$ [2].

Neutrino oscillations are the leading candidate explanation for the LSND anomaly, for several reasons. First, neutrino oscillations are known to occur in nature—it is all but certain that mass-induced neutrino oscillations are the solution to the solar and atmospheric neutrino puzzles [3]. Second, the L dependence of the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation probability is crucial for reconciling the LSND anomaly with negative results from the KARMEN-2 experiment [4].

Mass-induced oscillations among the three known “active” neutrino species, however, are known not to provide a viable solution to the LSND anomaly. The reason is very easy to understand. In terms of two-flavor $\nu_\mu \rightarrow \nu_e$ oscillations, the LSND data (combined with other “short baseline” data) require $\Delta m^2 \sim 1 \text{ eV}^2$ and $\sin^2 2\theta_{e\mu} \sim 10^{-3}$. On the other hand, the well-established atmospheric and solar neutrino oscillations require $\Delta m^2 \sim 10^{-3} \text{ eV}^2$ and $\Delta m^2 \sim 10^{-4} \text{ eV}^2$, respectively. With three neutrino masses, it is impossible to obtain three mass-squared differences that differ by orders of magnitude. For this reason, the existence of so-called light sterile neutrinos (that couple to the active neutrinos via mixing) is often postulated when it comes to addressing the LSND anomaly. It is remarkable, however, that the addition of one sterile neutrino does not provide a good fit to all neutrino data [5,6], while the addition of two (or more) sterile neutrinos seems to provide a better fit [6]. The goodness-of-fit of $3 + 2$ models when one includes all

of the world’s neutrino data is, however, currently under detailed investigation [7], and, furthermore, concordance cosmology fits to all available “cosmological data” virtually rule out sterile neutrinos masses larger than $m_\nu \gtrsim 1 \text{ eV}$ if these mix significantly with the active neutrinos. This is precisely the case of sterile neutrinos that help resolve the LSND anomaly [8].

The fact that mass-induced oscillations do not provide a compelling solution to the LSND anomaly is best illustrated by the fact that there are several alternative solutions to the LSND anomaly [9–13]. While many of them [9] are now ruled out by more current data or more refined data analysis (see, for example, [5,14,15]), most of the ones that seem to work require new, very light degrees of freedom (say, sterile neutrinos or scalars) and extraordinary new physics (say, *CPT* violation) [10]. Potential exceptions include postulating that *CPT*-invariance is violated and that neutrino propagation is not described by the Schrödinger equation [11], or imposing Lorentz-violating interactions that include may direction-dependent effects [12,13,16]. It is yet to be properly demonstrated whether either of these two latter possibilities can indeed accommodate all the neutrino data.

It is interesting to explore whether one could resolve—at least in principle—the LSND anomaly without introducing any new, light degrees of freedom. Since new parameters need to be added to the ones present in the νSM ,¹ we will postulate that one of the neutrinos couples to a source of Lorentz invariance violation (LIV). By varying the flavor composition of this neutrino and the energy dependence of the LIV effect, we are able to probe for solutions that qualitatively accommodate all the neutrino data. The reason for this particular choice of new physics is straightforward, and will become clear in the next sections. It provides a minimal number of free parameters to fit the LSND data, and also contains a mechanism for allowing the rest of the neutrino data to be

¹The νSM refers to the standard model of particle physics, plus the addition of mass parameters for the light neutrinos [17].

properly interpreted in terms of “ordinary” mass-induced neutrino oscillations.

Our intentions are twofold. First, as already stated above, we wish to determine whether there is indeed such a solution, and how constrained it is. We found that when only one superposition of neutrinos is coupled to the Lorentz violating sector the general characteristic of the data can be reproduced. That is, the solar neutrino and atmospheric neutrino data are describe to a very good approximation by the ν SM. LSND data, however, is taken care of by the new LIV interaction. Second, we will use our LIV solution to estimate how much “fine-tuning” is required for a three-flavor solution to the LSND anomaly. Here, our main result is rather negative. We find that in order to accommodate all neutrino data, the LIV model has to be highly finely-tuned. Thus, while we do report here on our attempts to find a LIV solution to all neutrino data, our conclusion is that such a solution is unlikely to be realized in nature.

The paper is organized as follows. In Sec. II, we present the mechanism we wish to explore, and how it modifies neutrino oscillations. In Sec. III, we qualitatively fit all neutrino data with the model outlined in Sec. II. In Sec. IV, we speculate on possible origins for LIV and summarize our results.

Our study is qualitatively different compared to other studies of Lorentz invariance violation and neutrino oscillation experiments [12,13,16]. References [12,13] concentrate on observable consequences of LIV for “short” baseline neutrino experiments, including LSND. (Reference [13] very briefly comments on other neutrino oscillation experiments.) In Ref. [16], an attempt is made to fit all neutrino data. Various different scenarios of Lorentz invariance violation are discussed, but the fit is done for subsets of the neutrino data at a time. Moreover, only oscillation with L/E , L , and LE dependent were considered. Based on our results, we believe that none of the examples explored in [16] can fit all neutrino data once a global fit is performed. This conclusion is based on the energy dependency, regardless of whether neutrinos are assumed to be massive, whether the LIV effects are directionally dependent, or whether CPT is conserved. In contrast, our emphasis is on trying to qualitatively accommodate *all* neutrino data, including those from LSND. Unlike other studies, our main idea is to couple only one neutrino to the LIV source. As a specific model we concentrate on CPT -conserving, direction-independent effects. To fit all data we find ourselves forced to explore more exotic energy dependent LIV effects.

II. FORMALISM

We postulate that one of the neutrinos couples to a source of Lorentz-invariance violation (LIV). This assumption is motivated by minimality but turns out to be very important, as will be discussed later.

In the case of one neutrino with mass m , we postulate that LIV manifests itself via a modified dispersion relation of the form

$$E \sim |\vec{p}| + \frac{m^2}{2|\vec{p}|} + \frac{f(|\vec{p}|^2)}{2|\vec{p}|}. \quad (1)$$

Equation (1) is valid as long as $|\vec{p}| \gg m$, \sqrt{f} . f is some function of $|\vec{p}|^2$, so that CPT is conserved.² We assume that f can be expanded as

$$f = 2 \sum_{n=1}^{\infty} b_n \left(\frac{|\vec{p}|}{E_0} \right)^{2n}, \quad (2)$$

where E_0 is some convenient constant with dimensions of energy, while the coefficients b_n have dimensions of mass-squared.

Assuming that all approximations above are valid, neutrino oscillations are described by the effective Schrödinger equation

$$i \frac{\partial}{\partial L} \nu_\alpha = H_{\alpha\beta} \nu_\beta, \quad (3)$$

where $\alpha, \beta = e, \mu, \tau$ and

$$H = U \begin{pmatrix} 0 & & \\ & \Delta_{12} & \\ & & \Delta_{13} \end{pmatrix} U^\dagger + \sum_{n=1}^{\infty} a_n \left(\frac{E}{E_0} \right)^{2n-1} \times \begin{pmatrix} \cos\zeta \cos\theta_L & & \\ \cos\zeta \sin\theta_L & & \\ \sin\zeta & & \end{pmatrix} \begin{pmatrix} \cos\zeta \cos\theta_L & \cos\zeta \sin\theta_L & \sin\zeta \end{pmatrix}. \quad (4)$$

Here, $\Delta_{ij} \equiv \Delta m_{ij}^2 / 2E$, $i, j = 1, 2, 3$ where Δm_{ij}^2 are the neutrino mass-squared differences and U is the leptonic mixing matrix, while $a_n \equiv b_n / E_0$ (the a_n coefficients have dimensions of mass). The mixing angles ζ and θ_L are defined by Eq. (4). They characterize the flavor content of ν_L , the neutrino that couples to the LIV sector. We have assumed, for simplicity, that there are no CP -odd parameters in the LIV sector.

All information we need to analyze the neutrino data is included in Eq. (4). Before proceeding, it is convenient to describe how we hope to accommodate all the neutrino data:

- (i) For large energies and short distances, $|\Delta_{ij}|L \ll 1$, all oscillation behavior is governed by the LIV terms. These will be fixed by the LSND anomaly plus other “short-baseline” constraints.

²More generally, we could have chosen f to also contain odd powers of $|\vec{p}|$. These, however, violate CPT , and lead to a different modified dispersion relation for neutrinos and antineutrinos. We find that this extra complication does not add significantly to our analysis, and we choose to work only with even powers of $|\vec{p}|$. This choice is natural in the sense that it is protected by a symmetry (CPT -invariance).

- (ii) For “solar” oscillations, we will take advantage of the energy dependence of the LIV term in order to make sure that these are negligible at typical solar neutrino energies. In particular, we will find that all a_n will vanish for sufficiently small values of n .
- (iii) For “atmospheric” oscillations, LIV effects will turn out to be very small. At large enough energies, the neutrino that couples to the LIV sector is a Hamiltonian eigenstate, while, because the LIV part of the Hamiltonian has rank one (two zero eigenvalues), the other two eigenvalues are still of order Δ_{12}, Δ_{13} . We will choose ζ and θ_L so that the “LIV neutrino” is mostly ν_e , and standard $\nu_\mu \leftrightarrow \nu_\tau$ oscillations governed by Δ_{13} remain virtually undisturbed for atmospheric-like neutrino energies.

III. CONSTRAINTS FROM THE NEUTRINO DATA

Here we explore choices for the LIV parameters that may allow one to accommodate all neutrino data. We do not aim at performing a global fit to all data, but concentrate on qualitatively understanding and satisfying the relevant constraints.

For LSND-like energies and baselines ($E \sim 50$ MeV and $L \sim 30$ m), the $\nu_\mu \rightarrow \nu_e$ transition probability is easy to compute. At this point, we further assume that the energy dependence is such that, at least for LSND-like energies,

$$\sum_{n=1}^{\infty} a_n \left(\frac{E}{E_0} \right)^{2n-1} \sim a_N \left(\frac{E}{E_0} \right)^{2N-1}, \quad (5)$$

where a_N is the smallest nonzero a_n term. The reason for this will become clear when we look at the solar neutrino data. We will comment on the behavior of the sum at large energies in due time. It is easy to compute all $P_{\mu\alpha}$ in the limit $|\Delta_{13}|L \ll 1$:

$$P_{\mu e} = \sin^2 2\theta_L \cos^2 \zeta \sin^2 \left(a_N \left(\frac{E}{E_0} \right)^{2N-1} \frac{L}{2} \right), \quad (6)$$

$$P_{\mu\tau} = \sin^2 2\zeta \sin^2 \theta_L \sin^2 \left(a_N \left(\frac{E}{E_0} \right)^{2N-1} \frac{L}{2} \right), \quad (7)$$

$$P_{\mu\mu} = 1 - 4\cos^2 \zeta \sin^2 \theta_L (1 - \cos^2 \zeta \sin^2 \theta_L) \times \sin^2 \left(a_N \left(\frac{E}{E_0} \right)^{2N-1} \frac{L}{2} \right). \quad (8)$$

In order to fit LSND and KARMEN-2 data, we must make sure that, at LSND, the oscillatory effects do not average out. Roughly speaking, we would like to choose a_N so that $P_{e\mu}^{\text{LSND}}/P_{e\mu}^{\text{KARMEN}} \sim (L^{\text{LSND}}/L^{\text{KARMEN}})^2$, as is

the case in successful mass-induced oscillation fits to LSND and KARMEN data [18]. Furthermore, we will assume that at experiments with larger energies and longer baselines, oscillation effects average out. Here, we will be particularly concerned with failed searches for $\nu_\mu \rightarrow \nu_e$ and $\nu_\mu \rightarrow \nu_\tau$ at NOMAD [19,20], CHORUS [21], and NuTeV [22]. These constrain, at the 90% confidence level,

$$\begin{aligned} \sin^2 2\theta_L \cos^2 \zeta &< 1.1 \times 10^{-3} \quad \text{and} \\ \sin^2 2\zeta \sin^2 \theta_L &< 3.3 \times 10^{-4}, \end{aligned} \quad (9)$$

while the LSND data require $\sin^2 2\theta_L \cos^2 \zeta \gtrsim 10^{-3}$. We therefore choose $\sin^2 2\zeta = 0$ to avoid the NOMAD constraint [20], and are forced to choose $\sin^2 2\theta_L = 1.1 \times 10^{-3}$ in order to avoid the NuTeV constraint [22] and to maintain hope that there is a passable fit to the LSND data.

Solar neutrino data, combined with those from KamLAND, are very well explained by solar matter-affected $\nu_e \rightarrow \nu_x$ oscillations (where ν_x is some linear combination of ν_μ and ν_τ). This is clearly visible in the fact that, for ^8B neutrinos, P_{ee} is significantly less than one half, which, given our current understanding of neutrino masses and mixing, can only be obtained if non-negligible solar matter effects are at work. In order to avoid spoiling this picture, we require that, for solarlike neutrino energies, LIV effects are negligible.

An order of magnitude estimate can be readily performed. The matter potential in the sun’s core is of order $A_{\text{sun}} \sim 5 \times 10^{-6} \text{ eV}^2/\text{MeV}$, so that significantly smaller a_N values guarantee that, for large N , P_{ee} is undisturbed for energies less than E_0 . This can also be deduce from Fig. 1 where the survival probability of solar electron neutrinos as a function of energy, for $E_0 = 15$ MeV and $N = 5$, is shown. The different curves correspond to $a_5 = 0$ (“normal” LMA behavior), $a_5 = 5 \times 10^{-5} \text{ eV}^2/\text{MeV}$, $a_5 = 5 \times 10^{-6} \text{ eV}^2/\text{MeV}$, and $a_5 = 5 \times 10^{-7} \text{ eV}^2/\text{MeV}$. The

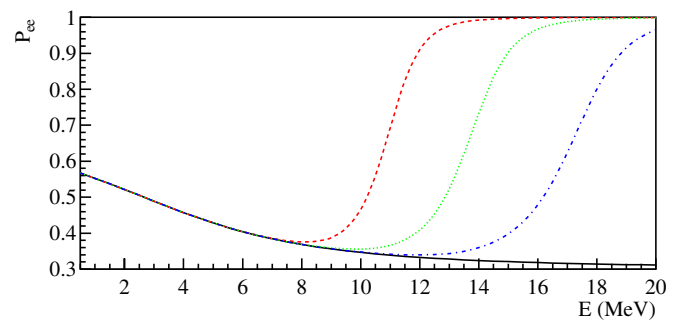


FIG. 1 (color online). Survival probability of solar neutrinos (P_{ee}) as a function of the solar neutrino energy (E), for $E_0 = 15$ MeV and different values of a_N . The different curves correspond to $N = 5$ and $a_5 = 0$ (solid, black line), $a_5 = 5 \times 10^{-5} \text{ eV}^2/\text{MeV}$ (dashed, red line), $a_5 = 5 \times 10^{-6} \text{ eV}^2/\text{MeV}$ (dotted, green line), and $5 \times 10^{-7} \text{ eV}^2/\text{MeV}$ (dashed-dotted, blue line). See text for details.

relevant standard neutrino parameters were set to $\Delta m_{12}^2 = 8 \times 10^{-5} \text{ eV}^2$, $\sin^2 \theta_{12} = 0.3$, and $\sin^2 \theta_{13} = 0$. In order to compute P_{ee} , we use the formalism outlined in [23], to which we refer for all the relevant details.

Since the ν SM successfully describes solar neutrinos up to the “end” of the ^8B neutrino spectrum (around 15 MeV), its predictions should not be dramatically modified for these neutrino energies. Then, from the figure, we see that, for $E_0 = 15 \text{ MeV}$, solar data require $a_5 \lesssim 10^{-6} \text{ eV}^2/\text{MeV}$. This estimate is qualitatively independent of N . For larger values of N , the same behavior is observed, except that the transition between LMA-like behavior and $P_{ee} = 1$ is more abrupt. We conclude that if we choose $E_0 \sim 15 \text{ MeV}$ and $a_N \sim 10^{-6} \text{ eV}^2/\text{MeV}$, observable effects in solar data should be safely absent. Once such parameter choices are made, KamLAND data is completely oblivious to LIV effects, given that relevant reactor neutrino energies are less than 10 MeV.

Next we consider atmospheric neutrinos. For the values of N in which we are interested, LIV effects for $E \geq 100 \text{ MeV}$ are very large and ν_L is basically decoupled from the other two neutrinos. Thus, we are left with a standard effective two-flavor oscillation scheme between these two states, governed by the oscillation frequency Δ_{13} . Since we choose ν_L to be mostly ν_e , the other two neutrinos are mostly linear combinations of ν_μ and ν_τ , such that $\nu_\mu \rightarrow \nu_\tau$ oscillations proceed as if there were no LIV effects. Effects due to the deviation of ν_L from a pure ν_e state can be readily computed, and turn out to be very small. We have checked that, for the parameter values considered here, such deviations are within the current experimental uncertainties.³

All considerations above constrain

$$P_{\mu e}^{\text{LSND}} \sim 10^{-3} \sin^2 \left[2.54 \times 10^{-6} \left(\frac{E}{15 \text{ MeV}} \right)^{2N-1} \left(\frac{L}{\text{m}} \right) \right], \quad (10)$$

so that N is the only leftover parameter. Since we wish to avoid averaged-out oscillations for typical LSND energies and baselines (if at all possible), the best one can do is to choose the oscillation phase to be close $\pi/2$ for typical LSND parameters, $L \sim 30 \text{ m}$ and $E \sim 50 \text{ MeV}$. This translates into $2N - 1 = 9$, or $N = 5$.⁴ This is far from providing a good fit to the LSND data, for a couple of reasons. $P_{\mu e}^{\text{LSND, max}} \lesssim 10^{-3}$, which, even in the case of constant $P_{\mu e}^{\text{LSND}}$, already provides a mediocre explanation to the LSND anomaly. To add insult to injury, because of

³These effects are indeed very small, and it is not clear whether one will be able to probe them in future atmospheric neutrino experiments.

⁴For smaller values of N , there is “no hope” of reconciling solar and LSND data. A detailed discussion was presented for $N = 1/2$ (energy independent LIV effect) in [23].

the acute energy dependence, $P_{\mu e}^{\text{LSND}}$ falls very rapidly with energy, so that, on average, $P_{\mu e} \ll 10^{-3}$.

One may wonder whether a qualitatively different set of constraints can be obtained if large LIV effects were present in solar neutrino oscillations. It turns out that such an assumption would not qualitatively modify the constraints obtained above. For ^8B neutrinos, the electron-type neutrino survival probability is significantly less than a half (at the 5σ level). This can be achieved in one of two ways. One way is that the effective oscillation length is of order the Earth-Sun distance. Assuming CPT invariance this possibility is ruled out by KamLAND data. Even if CPT -invariance were violated, solar data severely constrain “just-so” oscillations. Finally, a very long oscillation length would imply a very small value for the LIV effects and hence a much more constrained situation. The second option is that solar matter effects play a significant role in solar neutrino oscillations. In this case, for typical ^8B neutrino energies ($E \sim 5 - 10 \text{ MeV}$), the LIV are constrained to be, at most, of order A_{sun} . This must be true regardless of the value of Δm_{12}^2 and $\sin^2 \theta_{12}$, and regardless of the details of the LIV fit to the solar neutrino data, which may provide yet more constraints. Hence, assuming that solar neutrino oscillations are mostly dominated by LIV, we would obtain, for $E_0 = 15 \text{ GeV}$, $a_N \lesssim 10^{-5} \text{ eV}^2/\text{MeV}$. This, in turn, translates into $N \gtrsim 4$, which is qualitatively very similar to the case $N = 5$ discussed in detail above, and would not modify our conclusions in any way.

Even if one overlooks the fact that we failed to accommodate all neutrino data, there is another issue we need to deal with. For values of the LIV parameters discussed above, and assuming that all other a_n to vanish, the neutrino dispersion relation starts to differ significantly from that of a standard, ultrarelativistic particle for $|\vec{p}|$ values larger than a few GeV. One way to avoid as uncomfortable a situation is to postulate that, for large values of $|\vec{p}|^2$, f either stops increasing or decreases. We will now argue that both possibilities not only avoid a dramatic modification of the neutrino dispersion relation for GeV energies, but allow one to contemplate accommodating all neutrino data more comfortably.

We will briefly investigate two examples. One is to assume that

$$\sum_{n=1}^{\infty} a_n \left(\frac{E}{E_0} \right)^{2n-1} = \kappa \left[\tanh \left(\frac{E}{15 \text{ MeV}} \right) \right]^{2l+1}, \quad (11)$$

where l is a positive integer, κ is a constant coefficient and we chose to express the energy in “units” of 15 MeV, as before. In this case, the LIV potential is at most equal to κ (in the limit $E \gg 15 \text{ MeV}$), and we choose κ so that, for LSND-like neutrino energies and baselines, $\kappa L \sim 1$, which leads to $\kappa \sim 0.02 \text{ eV}^2/\text{MeV}$. In order to accommodate the solar data, we need to choose l so that $\kappa \tanh^{2l+1}(1) \lesssim 10^{-6} \text{ eV}^2/\text{MeV}$. This translates into $l \gtrsim 16$. It remains to

discuss the faith of NuTeV (plus CHORUS and NOMAD) neutrinos. If we fix $\zeta = 0$, $P_{\mu\tau}$ vanishes and, since here $E \gg 15$ MeV,

$$P_{\mu e} \approx \sin^2 2\theta_L \sin^2 \left[5.1 \left(\frac{\kappa}{0.02 \text{ eV}^2/\text{MeV}} \right) \left(\frac{L}{100 \text{ m}} \right) \right], \quad (12)$$

i.e., the oscillation length is energy independent. In reality, we anticipate that oscillatory effects average out because different neutrinos propagate different distances between production and detection. Given that decay tunnels and detectors are several tens of meters long, and the oscillation lengths we are interested in are of order 30 m, it sounds like a reasonable assumption. Under this assumption the constraints listed in Eq. (9) apply.⁵

Figure 2 depicts $P_{\mu e}$ as a function of energy for (a) $L = 18$ m (typical of KARMEN baselines) and (b) $L = 30$ m (typical of LSND baselines) for $\kappa = 0.02 \text{ eV}^2/\text{MeV}$, $l = 16$, and $\sin^2 2\theta_L = 1.1 \times 10^{-3}$. Also depicted [(c) + (d)] are equivalent two-flavor mass-induced oscillations for $\Delta m^2 = 1.5 \text{ eV}^2$ and $\sin^2 2\theta = 1.5 \times 10^{-3}$, parameter choices that should fit the combined LSND + KARMEN data at the 99% confidence level [18].⁶ This choice of parameters almost provides a good fit to all data, were it not for the fact that $\sin^2 2\theta_L$ is constrained to be a little too small.

A much more satisfactory fit could be obtained if the constraints listed in Eq. (9) did not apply. In that case a good fit to the LSND data can already be obtained for $\sin^2 2\theta_L \approx 2 \times 10^{-3}$. It is plausible that a more detailed analysis of NuTeV, NOMAD, and CHORUS data, beyond the intentions of this short paper, would reveal that it is possible to loosen Eq. (9) by some factor of order one, but understanding whether this is indeed the case is far from trivial. Among the issues that need to be considered carefully is the fact that detectors and decay pipes are not negligibly small compared with the typical baselines, and one needs to average over their dimensions taking into account the exponential decay profile of the parent pions and kaons that originate the different neutrino beams.

Finally, we mention the most radical change. Instead of considering LIV effects that behave like Eq. (11), we postulate that these are best represented by a function that is negligible at $E \lesssim 15$ MeV and $E \gtrsim 1$ GeV, while relevant for LSND energies “in between.” In this case, almost by construction, all neutrino data can be comfortably accommodated. There is one concern that arises from atmospheric data. If $|f|$ falls for energy values above 100 MeV, at some energy one would encounter resonant

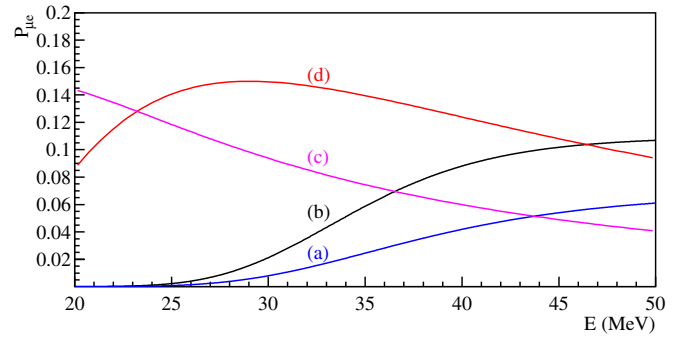


FIG. 2 (color online). $\nu_\mu \rightarrow \nu_e$ transition probability ($P_{\mu e}$) as a function of energy (E) at (a) KARMEN ($L = 18$ m) and (b) LSND ($L = 30$ m), using the tanh ansatz [Eq. (11)], compared with those obtained with marginal mass-induced oscillation solutions [(c) at KARMEN and (d) at LSND]. See text for details.

ν_μ to ν_e transitions. While the resonance is expected to be very narrow and hence “harmless,” it can be avoided altogether if one chooses ζ and θ_L so that ν_L is orthogonal to ν_3 .

Clearly, this possibility is even more tailor-made toward resolving the LSND anomaly. Moreover, as we will briefly comment later, we have no simple argument that justifies suppressing LIV effects at large energies other than very nontrivial fine-tuning.

IV. SUMMARY AND REMARKS

We have explored a new three-flavor fit to all neutrino data, including those from the LSND experiment. We postulate that one of the neutrinos couples to a source of Lorentz-invariance violation (LIV), so that its dispersion relation is modified in such a way that oscillation effects proportional to $\sin^2(E^n L)$ are present for LSND neutrinos [see Eq. (4)]. For positive n values, we see that it is possible to accommodate solar and LSND data, while atmospheric data is easily accommodated as long as the LSND mixing angle and “oscillation length” are small. Curiously enough, the strongest constraints to this approach are provided by searches for $\nu_\mu \rightarrow \nu_e$ oscillations at “small” L/E . Constraints obtained by NOMAD, CHORUS and NuTeV [19,21,22] seem to allow only a mediocre fit to all neutrino data, unless the LIV parameters are chosen so that LIV oscillations are only relevant at LSND-like energies ($30 \text{ MeV} \lesssim E \lesssim 100 \text{ MeV}$).

The model explored here is, by construction, very finely-tuned. Indeed, all manifestations of it are. The point where the LIV term becomes important is immediately below LSND energies and above typical solar neutrino energies. We were forced to choose very steep functions of the neutrino energy for energies between few MeV to tens of MeV. Clearly, this class of models is not motivated, but just serves as an example of how nontrivial it is to accommodate the LSND anomaly when the rest of the world neu-

⁵If oscillation effects did not average out, we would be allowed to choose κ so that ν_μ to ν_e oscillation effects at these experiments are minimized, hence loosening the constraints.

⁶Recall, however, that the smallest allowed value of $\sin^2 2\theta$ at a given confidence level depends on the statistical analysis method employed [18].

trino data is included. In order to obtain a “natural” fit, we are all but forced into postulating that LIV effects are only present at LSND.

The character of the LSND anomaly will, hopefully, be clarified with new experimental data. The MiniBooNE experiment [24], currently taking data at Fermilab, was constructed in order to test whether the LSND anomaly is a manifestation of neutrino flavor transitions. As far as the different scenarios discussed here are concerned, MiniBooNE, which studies muon-type neutrinos with energies of several hundreds of MeV, is expected see either no “new physics” effect—if the LIV are indeed concentrated at LSND energies only—or a rather small signal consistent with “averaged-out” $\nu_\mu \rightarrow \nu_e$ oscillations. On the other hand, if MiniBooNE were to see a significant, energy-dependent signal for ν_e appearance, LIV models of the type discussed here would be significantly disfavored. Hence, in the case of no (significant) sign of new physics at MiniBooNE, we would be unable to rule out the kind of LIV effects discussed here. Conclusive information capable of resolving the LSND anomaly and addressing the models discussed here could be obtained, on the other hand, in new experiments with lower energy neutrinos, such as “long-baseline” studies of muon or pion decay at rest (see, for example, [25]).

We find it remarkable that so many distinct attempts to address the LSND anomaly by augmenting the ν SM fail because of distinct aspects of the world neutrino data (see, for example, [14], for a nice overview). Some L/E effects, for example, are disfavored by atmospheric and solar data (as in the case of “2 + 2” spectra) or by combined failed searches for ν_μ and ν_e disappearance at short baselines (as in the case of “3 + 1” spectra). L -independent solutions are disfavored by the KARMEN experiment (see, for example, [26] for a specific example), and we find that $L \times E^n$ effects are ultimately disfavored by failed $\nu_\mu \rightarrow \nu_e$ searches at NOMAD, CHORUS, and NuTeV (this was also mentioned in [14]).

A few model-building comments are in order. We need to introduce a term that violates Lorentz invariance and couples to neutrinos. Instead of providing a full model, we simply make the following general remarks. Consider a scalar field ϕ whose time-derivative (in the preferred reference frame) acquires a vacuum expectation value:

$$\langle \partial_\mu \phi \rangle = \delta_{0\mu} M, \quad (13)$$

and we further impose a Z_{2n} symmetry on ϕ . This symmetry may be softly broken in the ϕ potential, but is assumed to be (almost) exact when considering the couplings to neutrinos, so that only terms that scale like E^{2n} (and its powers) are allowed. Hence, symmetry arguments can be used to explain why only large N values contribute. A similar argument applies for LIV effects proportional to $[\tanh(E)]^m$ as far as the exponent m is concerned, but we cannot use symmetry arguments to “explain” why infinite series of higher dimensional LIV operators would combine into either a $\tanh(E)$ form, or why effects should be enhanced at neutrino energies close to 50 MeV (but severely suppressed at energies above or below 50 MeV).

In summary, we succeed in identifying a three-neutrino, LIV solution capable of accommodating all neutrino data. The price we had to pay in order to do that was quite high: not only were we forced to impose that the LIV effects are very strongly energy-dependent for energies around 20 MeV, we were also forced to postulate that LIV effects either “flatten out” for energies larger than 100 MeV (in which case the fit to LSND data is mediocre) or peak at LSND energies and quickly “disappear” for $E \gtrsim 1$ GeV.

We do not advocate our model as a solution to the LSND anomaly, but rather as another evidence for the “real” LSND puzzle: how can the effect observed at LSND be due to physics beyond the ν SM if all other experiments (“neutrino” or otherwise) have failed to observe related “new physics” effects? If the integrated literature on the subject is to be taken as a good indicator, one would, naively, conclude that the answer to the question above is “it cannot.” It seems unfair, however, to conclude that a new physics interpretation of the LSND anomaly does not exist simply because there seems to be no elegant solution to the LSND anomaly. It may well be we just have not found one yet.

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