Updated bounds on CP asymmetries in $B^0 \to \eta' K_S$ and $B^0 \to \pi^0 K_S$

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(Received 5 September 2006; published 8 November 2006)

New rate measurements of B^0 decays into $\pi^0\pi^0$, $\pi^0\eta$, $\pi^0\eta'$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$ and K^+K^- are used in conjunction with flavor $SU(3)$ to constrain the coefficients S and C of $\sin\Delta mt$ and $\cos\Delta mt$ in the time-dependent CP asymmetries of $B^0 \to \eta' K_S$ and $B^0 \to \pi^0 K_S$. Experimental values of $S_{\eta' K}$ are now seen to be closer to the Standard Model expectations, fully consistent with the new improved bounds.

DOI: 10.1103/PhysRevD.74.093003 PACS numbers: 12.15.Hh, 12.15.Ji, 13.25.Hw, 14.40.Nd

I. INTRODUCTION

Time-dependent CP asymmetries in B^0 decays to CP eigenstates dominated by the $b \to s$ penguin amplitude have for several years been fertile ground for exploring signatures of new physics [1]. The decay $B^0 \to \eta' K_S$, as one example, attracted attention because of the possible deviation of the coefficient $S_{\eta' K_S}$ of the $\sin\Delta mt$ term from its predicted value of $\sin 2\phi_1 = \sin 2\beta$, where $\beta \equiv \arg(-V_{tb}V_{td}^* V_{cd} V_{cb}^*)$ is one of the phases in the standard unitary triangle constructed from the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. A value $\sin 2\beta = 0.674 \pm 0.026$ is extracted from $B \to J/\psi K_{S,L}$ decays [2].

In Refs. [3,4] correlated bounds on S and C parameters in time-dependent decays $B^0 \to \eta' K_S$ and $B^0 \to \pi^0 K_S$ were obtained using branching ratio measurements of $SU(3)$-related B^0 decays. The *BABAR* Collaboration now has updated its measurements of a number of branching ratios which contributed to the bounds in Ref. [3], leading to a further strengthening of these bounds within the Standard Model. These new results include branching ratios for $B^0 \to \eta\eta'$, $\eta\pi^0$, $\eta'\pi^0$ based on 232×10^6 $\bar{B}B$ pairs [5], for $B^0 \to \eta\eta$, $\eta'\eta'$ based on 324×10^6 $\bar{B}B$ pairs [6], and $B^0 \to \pi^0\pi^0$ based on 347×10^6 $\bar{B}B$ pairs [7]. Belle has also updated its branching ratio for $B^0 \to \pi^0\pi^0$ based on 532×10^6 $\bar{B}B$ pairs [8]. At the same time *BABAR* has presented new values for $S_{\eta' K}$ and $C_{\eta' K}$ which are closer to the predictions of the standard model [9], while Belle has updated its values based on more data [2,10,11]. Finally, new measurements of $S_{\pi^0 K_S}$ and $C_{\pi^0 K_S}$ were presented by both Belle [10] and *BABAR* [12]. The purpose of this work is to compare the new predictions with the new measurements. The considerable improvements in bounds in comparison with our earlier treatments [3,4] deserve to be noted despite the fact that we break no new theoretical ground here. Where otherwise unspecified we use values of branching ratios quoted by the Heavy Flavor Averaging Group [13].

Alternative approaches for studying the asymmetries S and C in $b \to s$ penguin-dominated B^0 decays have been adopted in other works by calculating hadronic amplitudes for these processes within the frameworks of QCD Factorization [14], Soft Collinear Effective Theory (SCET) [15] and a model for final state interactions [16].

In Sec. II we briefly sketch the formalism for the case of time-dependent asymmetries in $B^0 \to \eta' K_S$, referring to [3] for details, and present the new bounds on $S_{\eta' K_S}$ and $C_{\eta' K_S}$. In Sec. III the updated bounds for $B^0 \to \pi^0 K_S$ will then be given. We summarize in Sec. IV, stressing the fact that our bounds may be approaching their optimum limits.

II. BOUNDS FOR $S_{\eta' K_S}$ AND $C_{\eta' K_S}$

For $\eta' K_S$ the asymmetry has the form [17]:

$$A(t) \equiv \frac{\Gamma(\bar{B}^0(t) \to \eta' K_S) - \Gamma(B^0(t) \to \eta' K_S)}{\Gamma(\bar{B}^0(t) \to \eta' K_S) + \Gamma(B^0(t) \to \eta' K_S)} = -C_{\eta' K}\cos(\Delta mt) + S_{\eta' K}\sin(\Delta mt), \tag{1}$$

with

$$S_{\eta' K} \equiv \frac{2\,\mathrm{Im}(\lambda_{\eta' K})}{1 + |\lambda_{\eta' K}|^2}, \qquad C_{\eta' K} \equiv \frac{1 - |\lambda_{\eta' K}|^2}{1 + |\lambda_{\eta' K}|^2}, \qquad \lambda_{\eta' K} \equiv -e^{-2i\beta}\frac{A(\bar{B}^0 \to \eta' \bar{K}^0)}{A(B^0 \to \eta' K^0)}. \tag{2}$$

We decompose the $B^0 \to \eta' K^0$ amplitude into two terms A'_P and A'_C containing, respectively, the CKM factors $V_{cb}^* V_{cs}$ and $V_{ub}^* V_{us}$ [1]

$$A(B^0 \to \eta' K^0) = A'_P + A'_C = |A'_P|e^{i\delta} + |A'_C|e^{i\gamma}, \tag{3}$$

where δ and γ in the last equality are, respectively, the strong and the weak phase. In the diagrammatic language A'_P is the dominant $b \to s$ penguin amplitude and A'_C is a color-suppressed amplitude.

The asymmetries $S_{\eta' K}$ and $C_{\eta' K}$ are

[1]The normalization of $A'_{P,C}$ differs from the one in [3] by $\sqrt{6}$. This normalization cancels in the results for $S_{\eta' K}$, $C_{\eta' K}$.

$$S_{\eta' K} = \frac{\sin 2\beta + 2|A'_C/A'_P|\cos\delta\sin(2\beta+\gamma) - |A'_C/A'_P|^2\sin(2\alpha)}{R_{\eta' K}}, \tag{4}$$

$$C_{\eta' K} = \frac{2|A'_C/A'_P|\sin\delta\sin\gamma}{R_{\eta' K}}, \tag{5}$$

$$R_{\eta' K} \equiv 1 + 2|A'_C/A'_P|\cos\delta\cos\gamma + |A'_C/A'_P|^2. \tag{6}$$

The amplitudes A'_P and A'_C are expected to obey $|A'_C| \ll |A'_P|$ [18]. If A'_C were neglected one would have $S_{\pi K} = \sin 2\beta$, $C_{\pi K} = 0$. Keeping only linear terms in $|A'_C/A'_P|$ [17] one would have an allowed region in the $(S_{\eta' K}, C_{\eta' K})$ plane lying inside an ellipse centered at $(\sin 2\beta, 0)$. We use the exact expressions (4)–(6). Bounds on γ from global CKM analyses [19] lead to asymmetries in the approximately elliptical regions surrounding the Standard Model point.

Using the flavor-$SU(3)$ decomposition of Refs. [18,20–25] one can express the ratio A'_C/A'_P in terms of $SU(3)$-related amplitudes A_C/A_P for $\Delta S = 0$ B^0 decays as pointed out in [26]. The bounds on $\Delta S_{\eta' K} \equiv S_{\eta' K} - \sin 2\beta$ and $C_{\eta' K}$ then arise because $A'_C = \bar\lambda A_C$ is CKM-suppressed, while $A'_P = -\bar\lambda^{-1}A_P$ is CKM-enhanced compared to the $\Delta S = 0$ amplitudes (here $\bar\lambda = -V_{cd}/V_{cs} = 0.230$). Writing $A_{P,C}$ in terms of the $\Delta S = 0$ $B \to f$ amplitudes A_f

$$\Sigma_f a_f A(f) = A_P + A_C, \tag{7}$$

one then obtains the bounds (see [3] for details)

$$\frac{|\mathcal{R} - \bar\lambda^2|}{1+\mathcal{R}} \le |A'_C/A'_P| \le \frac{\mathcal{R}+\bar\lambda^2}{1-\mathcal{R}}. \tag{8}$$

The ratio $\mathcal{R}$ is

$$\mathcal{R}^2 \equiv \frac{\bar\lambda^2[|\Sigma_f a_f A(f)|^2 + |\Sigma_f a_f \bar A(f)|^2]}{|A(B^0 \to \eta' K^0)|^2 + |A(\bar B^0 \to \eta' \bar K^0)|^2}, \tag{9}$$

and is bounded by

$$\mathcal{R} \le \bar\lambda \Sigma_f |a_f| \sqrt{\frac{\bar{\mathcal{B}}_f}{\bar{\mathcal{B}}(\eta' K^0)}}. \tag{10}$$

For a given set of coefficients a_f, nonzero branching ratio measurements and upper limits on CP averaged branching ratios $\bar{\mathcal{B}}_f$ provide an upper bound on $\mathcal{R}$, for which the right-hand-side of (8) gives an upper bound on $|A'_C/A'_P|$.

Since there are more physical amplitudes $A(f)$ than $SU(3)$ contributions, one may form a variety of combinations satisfying (7). We consider two of the cases noted in Ref. [3]:

(1) A combination involving pairs including π^0, η and η' in the final state was proposed in [26] by using a complete $SU(3)$ analysis, and in [27] by applying U-spin symmetry arguments:

$$\begin{aligned}\Sigma_f a_f A(f) = {}& \frac{1}{4\sqrt3}A(\pi^0\pi^0) - \frac13 A(\pi^0\eta) \\ &+ \frac{5}{6\sqrt2}A(\pi^0\eta') + \frac{2}{3\sqrt3}A(\eta\eta) \\ &- \frac{11}{12\sqrt3}A(\eta'\eta') - \frac{5}{3\sqrt3}A(\eta\eta').\end{aligned} \tag{11}$$

(2) Another superposition, satisfying (7) in the limit in which small amplitudes involving the spectator quark may be neglected, involves only three strangeness-conserving amplitudes:

$$\Sigma_f a_f A(f) = -\frac56 A(\pi^0\eta) + \frac{1}{3\sqrt2}A(\pi^0\eta') - \frac{\sqrt3}{2}A(\eta\eta'). \tag{12}$$

The coefficients a_f in these cases can be read off Eqs. (11) and (12).

As mentioned before, the upper bounds for a number of the relevant decays have been strengthened recently. In units of 10^{-6}, we use the value [13] $\mathcal{B}(\eta' K^0) = 64.9$ (we ignore the error ± 3.5 as in Ref. [3]) and the 90% c.l. upper limits $\mathcal{B}(\pi^0\pi^0) < 1.58$, $\mathcal{B}(\pi^0\eta) < 1.3$, $\mathcal{B}(\pi^0\eta') < 2.4$, $\mathcal{B}(\eta\eta) < 1.8$, $\mathcal{B}(\eta'\eta') < 2.4$, and $\mathcal{B}(\eta\eta') < 1.7$. These inputs are compared with those used in Ref. [3] in Table I. The bounds on $\mathcal{R}$ obtained in the above two cases are then as follows:

(1) Assuming exact $SU(3)$ and applying (11) we find, using the central value for $\bar{\mathcal{B}}(\eta' K^0)$,

$$\mathcal{R} < 0.116 \qquad (\text{formerly } 0.18). \tag{13}$$

(2) Using (12), which contains three processes, one finds

$$\mathcal{R} < 0.070 \qquad (\text{formerly } 0.10). \tag{14}$$

The approximation involved in deriving (14), where $SU(3)$ breaking and small amplitudes were neglected, is comparable to that associated with (13) which only neglects $SU(3)$ breaking effects.

TABLE I. Branching ratios in 10^{-6} and 90% C.L. upper limits on branching ratios.

Mode	$\eta' K^0$	$\pi^0\pi^0$	$\pi^0\eta$	$\pi^0\eta'$	$\eta\eta$	$\eta'\eta'$	$\eta\eta'$
This work	64.9 ± 3.5	1.31 ± 0.21	<1.3	$1.5^{+0.7}_{-0.6}$	<1.8	<2.4	<1.7
Ref. [3]	$65.2^{+6.0}_{-5.9}$	1.9 ± 0.5	<2.5	<3.7	<2.8	<10	<4.6

In order to study constraints in the $(S_{\eta' K}, C_{\eta' K})$ plane, we now apply the upper bounds (13) and (14). The exact expressions (4)–(6) imply correlated bounds on these two quantities associated with fixed values of $\mathcal{R}$. We scan over $-\pi \leq \delta \leq \pi$, taking a central value $\beta = 21.2°$, values of γ satisfying $52° \leq \gamma \leq 74°$ [19], and values of $|A'_C/A'_P|$ in the range (8), where $\mathcal{R}$ satisfies the bound (13) or (14). The bounds on $(S_{\eta' K}, C_{\eta' K})$ are shown in Fig. 1. The small plotted point corresponds to $(S_{\eta' K}, C_{\eta' K}) = (\sin 2\beta, 0)$ (see below). The large plotted points correspond to the most recent results reported by *BABAR* [9] and Belle [11]. These results are noted in Table II.

The greatest range of $\Delta S_{\eta' K_S}$ is obtained for $C_{\eta' K_S} = 0$. For the inner ellipse in Fig. 1, based on Eq. (14), one finds

$$-0.046 < \Delta S_{\eta' K_S} < 0.094, \tag{15}$$

while for the outer ellipse based on Eq. (13), the limits are

$$-0.133 < \Delta S_{\eta' K_S} < 0.152, \tag{16}$$

Note that the conservative bound (16) uses only $SU(3)$ symmetry. In obtaining the more restrictive bound (15) further dynamical assumptions were made: that the annihilationlike amplitudes pa and e [18] can be neglected (this can be justified by taking the $m_b \to \infty$ limit [15,28]) and furthermore that the singlet annihilationlike amplitudes c_s and s_0 [15] that depend on the gluonic content of η' can be neglected (the latter do not vanish in the $m_b \to \infty$ limit, while it is not clear whether or not they are small numerically [15]). The explicit calculations in QCD Factorization [14], SCET [15], and a model for final state interactions [16] give results that lie well within both of the above ranges.

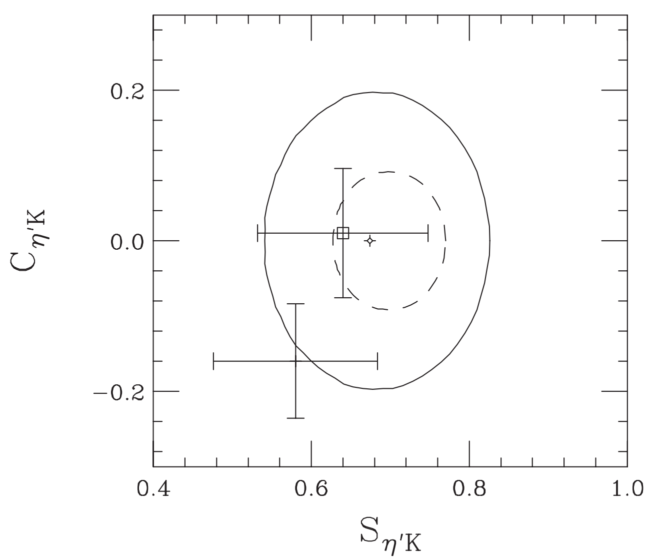

FIG. 1. Regions in the $(S_{\eta' K}, C_{\eta' K})$ plane satisfying limits on the ratio $|A'_C/A'_P|$ and bounds (13) (region enclosed by the solid curve) or (14) (region enclosed by the dashed curve). The small plotted point denotes $(S_{\eta' K}, C_{\eta' K}) = (\sin 2\beta, 0)$. The points with experimental errors denote values from *BABAR* [9] (plain point) and Belle [11] (small square).

TABLE II. Time-dependent asymmetries in $B^0 \to \eta' K_S$.

Parameter	*BABAR* [9]	Belle [11]
$S_{\eta' K_S}$	$0.58 \pm 0.10 \pm 0.03$	$0.64 \pm 0.10 \pm 0.04$
$C_{\eta' K_S}$	$-0.16 \pm 0.07 \pm 0.03$	$0.01 \pm 0.07 \pm 0.05$

III. BOUNDS FOR $S_{\pi^0 K_S}$ AND $C_{\pi^0 K_S}$

We next turn to $B \to \pi^0 K_S$ decay. Measured asymmetries are summarized in Table III. The analysis is similar to the one presented above, with the details given in [4]. Using the same notation as for $B \to \eta' K_S$ we have

$$\sum_f a_f A_f = A(B^0 \to \pi^0 \pi^0) + A(B^0 \to K^+ K^-)/\sqrt{2}, \tag{17}$$

so that (9) gives now

$$\mathcal{R}^2_{\pi/K} \equiv \frac{\bar{\lambda}^2[|A_{\pi\pi} + A_{KK}/\sqrt{2}|^2 + |\bar{A}_{\pi\pi} + \bar{A}_{KK}/\sqrt{2}|^2]}{|A_{\pi K}|^2 + |\bar{A}_{\pi K}|^2}. \tag{18}$$

As in [4] we now distinguish two cases:

(1) Neglect the $1/m_b$ suppressed $B^0 \to K^+K^-$ amplitude for which the experimental value is $\bar{\mathcal{B}}(B^0 \to K^+K^-) = (0.07^{+0.12}_{-0.11}) \cdot 10^{-6}$ [13]. Then with $\bar{\mathcal{B}}(B^0 \to \pi^0\pi^0) = (1.31 \pm 0.21) \times 10^{-6}$ and $\bar{\mathcal{B}}(B^0 \to \pi^0 K^0) = (10.0 \pm 0.6) \times 10^{-6}$, we find

$$\mathcal{R}_{\pi/K} = \bar{\lambda}\sqrt{\frac{\bar{\mathcal{B}}(B^0 \to \pi^0\pi^0)}{\bar{\mathcal{B}}(B^0 \to \pi^0 K^0)}} = (8.3 \pm 0.7) \cdot 10^{-2}, \tag{19}$$

to be compared with $\mathcal{R}_{\pi/K} = (9.1 \pm 1.2) \cdot 10^{-2}$ in [4].

(2) Keeping $A(B^0 \to K^+K^-)$ increases the error on $\mathcal{R}_{\pi/K}$ which now lies in the range

$$\mathcal{R}_- \leq \mathcal{R}_{\pi/K} \leq \mathcal{R}_+, \tag{20}$$

TABLE III. Time-dependent asymmetries in $B^0 \to \pi^0 K_S$.

Parameter	*BABAR* [12]	Belle [2]
$S_{\pi^0 K_S}$	$0.33 \pm 0.26 \pm 0.04$	$0.33 \pm 0.35 \pm 0.08$
$C_{\pi^0 K_S}$	$0.20 \pm 0.16 \pm 0.03$	$0.05 \pm 0.14 \pm 0.05$

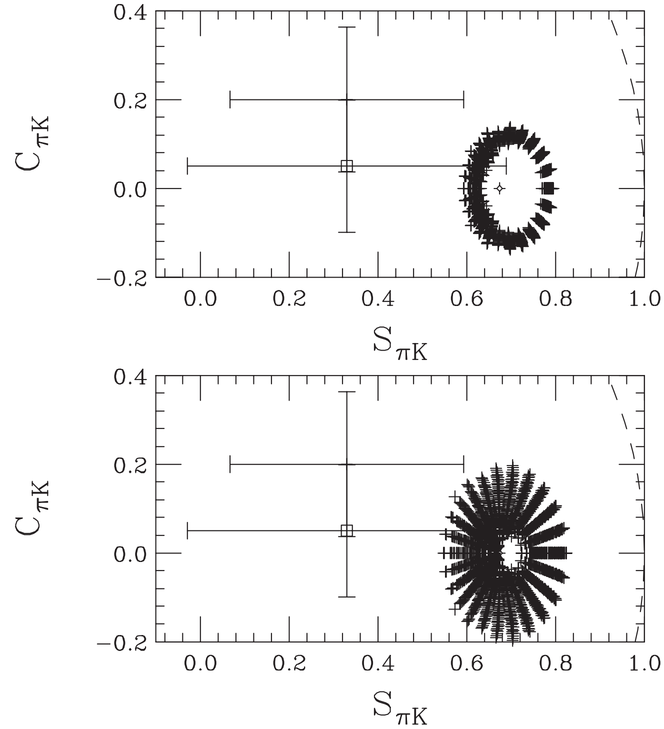

FIG. 2. Top: points in the $S_{\pi K}$-$C_{\pi K}$ plane satisfying $\pm 1\sigma$ limits (19) on the ratio $R_{\pi/K}$, with the small $B^0 \to K^+K^-$ contribution ignored. The small plotted point denotes the pure-penguin value $S_{\pi K} = \sin 2\beta$, $C_{\pi K} = 0$. Points with experimental errors denote values from *BABAR* [12] (plain point) and Belle [10] (small square). The dashed arc denotes the boundary of allowed values: $S^2_{\pi K} + C^2_{\pi K} \leq 1$. Bottom: small $B^0 \to K^+K^-$ contribution included.

where

$$\mathcal{R}_{\pm} \equiv \bar{\lambda}\left(\sqrt{\frac{\bar{\mathcal{B}}(B^0 \to \pi^0\pi^0)}{\bar{\mathcal{B}}(B^0 \to \pi^0 K^0)}} \pm \sqrt{\frac{\bar{\mathcal{B}}(B^0 \to K^+K^-)}{2\bar{\mathcal{B}}(B^0 \to \pi^0 K^0)}}\right) \equiv \mathcal{R}_{\pi/K}(1 \pm r). \quad (21)$$

With $\bar{\mathcal{B}}(B^0 \to K^+K^-) < (0.224 \times 10^{-6}$ (90% c.l.) and the central value of $\bar{\mathcal{B}}(B^0 \to \pi^0\pi^0)$ we find $r < r_{\max} = 0.292$. Then the lower limit on $\mathcal{R}_{\pi/K}$ becomes $\mathcal{R}_- = (0.076)(1 - r_{\max}) = 0.054$, while the upper limit becomes $\mathcal{R}_+ = (0.090)(1 + r_{\max}) = 0.117$. These are to be compared with $\mathcal{R}_- = 0.055$, $\mathcal{R}_+ = 0.126$ obtained in [4] using central values for $\bar{\mathcal{B}}(B^0 \to \pi^0 K^0)$, $\bar{\mathcal{B}}(B^0 \to \pi^0\pi^0)$ and the upper limit on $\bar{\mathcal{B}}(B^0 \to K^+K^-)$.

The results of these two cases are shown in Figs. 2. A small region of parameter space near the value $(S, C) = (\sin 2\beta, 0)$ is actually *excluded*, as in the case considered in Ref. [4] when the small $B^0 \to K^+K^-$ decay amplitude was ignored. Here, a small region is excluded even when $B^0 \to K^+K^-$ is taken into account. This is due in part to the improved upper bounds on this process but also to the more restricted range assumed for γ: $52° \leq \gamma \leq 74°$ [19] compared with $38° \leq \gamma \leq 80°$ taken in Ref. [4].

IV. SUMMARY

$SU(3)$ bounds on the time-dependent CP asymmetries in $B^0 \to \eta' K_S$ and $B^0 \to \pi^0 K_S$ continue to improve as one incorporates improved bounds on rare $B^0 \to \pi^0\eta^{(\prime)}$, $\eta^{(\prime)}\eta^{(\prime)}$, and K^+K^- decay branching ratios. The bounds presented in this work will thus reach their minimal values once all the above decay branching ratios are measured. These minimal bounds can be estimated using theoretical predictions for $B^0 \to \pi^0\eta^{(\prime)}$, $\eta^{(\prime)}\eta^{(\prime)}$, and K^+K^- within QCD Factorization [14], SCET [15] and perturbative QCD (PQCD) [29]. While the central values in these calculations are typically smaller than the current experimental upper bounds, their theoretical uncertainties are large, permitting values close to these bounds. For example, gluonic contributions to $B \to \eta^{(\prime)}$ form factors may enhance the relevant branching ratios. Global $SU(3)$ fits for B decays into two pseudoscalars obtain values which are within errors near the upper bounds [27,30,31]. A first indication that the actual branching ratios are not far below current bounds is the measurement $\mathcal{B}(B^0 \to \pi^0\eta') = (1.5^{+0.7}_{-0.3}) \times 10^{-6}$, lying significantly higher than central values calculated in QCD Factorization and PQCD. This may indicate that the bounds (13) and (14) will not improve significantly in the future.

The present constraint on the region around $(S, C) = (\sin 2\beta, 0)$ consistent with the Standard Model is shown in Figs. 1 and 2. With the new measurements the experimental deviations from the Standard Model for $B^0 \to \eta' K_S$ have decreased, while those for $B^0 \to \pi^0 K_S$ are not yet statistically compelling.

ACKNOWLEDGMENTS

We thank Bill Ford, Fernando Palombo, Jim Smith, and Denis Suprun for helpful comments. M. G. is grateful to the SLAC Theory Group for its kind hospitality. J. L. R. thanks the Newman Laboratory of Elementary-Particle Physics, Cornell University, for its kind hospitality during part of this investigation. This work was supported in part by the United States Department of Energy, High Energy Physics Division, through Grants No. DE-FG02-90ER40560, DOE-ER-40682-143, and DEAC02-6CH03000, and contract No. DE-AC02-76SF00515; by the Israel Science Foundation under Grant No. 1052/04; and by the German-Israeli Foundation under Grant No. I-781-55.14/2003.

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