Leptogenesis bound on neutrino masses in left-right symmetric models with spontaneous *D*-parity violation

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We study the baryogenesis via leptogenesis in a class of left-right symmetric models, in which D-parity is broken spontaneously. We first discuss the consequence of the spontaneous breaking of D-parity on the neutrino masses. Then we study the lepton asymmetry in various cases, from the decay of right-handed neutrinos as well as the triplet Higgs, depending on their relative masses they acquire from the symmetry breaking pattern. The leptogenesis bound on their masses is discussed by taking into account the low energy neutrino oscillation data. It is shown that a TeV scale leptogenesis is viable if there is an additional source of CP violation like CP-violating condensate in the left-right domain wall. This is demonstrated in a class of left-right symmetric models where D-parity breaks spontaneously at a high energy scale while allowing $SU(2)_R$ gauge symmetry to break at the TeV scale.

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I. INTRODUCTION

The matter antimatter asymmetry during the big bang nucleosynthesis era is required to be very tiny. Recent results from the Wilkinson Microwave Anisotropy Probe (WMAP) provides a fairly precise value for this asymmetry, given by [1]

$$\left(\frac{(n_B - n_{\bar{B}})}{n_\gamma}\right)_0 \equiv \left(\frac{n_B}{n_\gamma}\right)_0 = (6.1^{+0.3}_{-0.2}) \times 10^{-10}. \quad (1)$$

In recent years the most fascinating experimental result in particle physics came out in neutrino physics. The atmospheric neutrinos provided us the first evidence for a nonvanishing neutrino mass [2] and hence first indication for physics beyond the standard model. The mass-squared difference providing $\nu_\mu - \nu_\tau$ oscillations, as required by the atmospheric neutrinos, is given by

$$\Delta m_{\rm atm} \equiv \sqrt{|m_3^2 - m_2^2|} \simeq 0.05 \text{ eV}. \quad (2)$$

This result is further strengthened by the solar neutrino results [3] which require a mass-squared difference providing a $\nu_e - \nu_\mu$ oscillation. The mass splitting given by

$$\Delta m_\odot \equiv \sqrt{m_2^2 - m_1^2} \simeq 0.009 \text{ eV}, \quad (3)$$

where m_1, m_2, and m_3 are the masses of light physical neutrinos. Note that $\Delta m_\odot$ is positive as indicated by the SNO data while there is an ambiguity in the sign of $\Delta m_{\rm atm}$ to the date.

The above discoveries, the matter antimatter asymmetry of the present Universe (1) and the sub-eV neutrino masses (2) and (3), could be intricately related to each other. A most viable scenario to explain is the baryogenesis via leptogenesis [4,5]. The smallness of the neutrino masses compared to the charged fermions are best understood in terms of a seesaw mechanism [6]. Although the neutrinos are massless in the standard model, a minimal extension including right-handed neutrinos or triplet Higgs scalars or both can generate tiny Majorana masses for the neutrinos through the seesaw mechanism. The smallness of the neutrino masses depends on a large suppression by the lepton (L) number violating scales in the model, which is the scale of Majorana masses of the right-handed neutrinos or the masses and dimensional couplings of the triplet Higgs scalars. The L-number violating decays of the right-handed neutrinos or the triplet Higgs scalars at this large scale can then generate a L-asymmetry of the Universe, provided there is enough CP violation and the decays satisfy the out-of-equilibrium condition, the necessary criteria of Sakharov [7]. This L-asymmetry of the Universe is then converted to a baryon (B) asymmetry of the Universe through the sphaleron processes unsuppressed above the electroweak phase transition [8].

In the simplest type-I seesaw models the singlet right-handed neutrinos (N_R's) are added to the standard model (SM) gauge group, $SU(2)_L \times U(1)_Y$. The canonical seesaw then gives the light neutrino mass matrix:

$$m_\nu = m_\nu^{\rm I} = -m_D M_R^{-1} m_D^T, \quad (4)$$

where m_D is the Dirac mass matrix of the neutrinos connecting the left-handed neutrinos with the right-handed neutrinos and M_R is the Majorana mass matrix of the right-handed heavy neutrinos, which also sets the scale of L-number violation. Since the Majorana mass of the right-handed neutrinos violate L-number by two units, their out-of-thermal equilibrium decay to SM particles is a natural source of L-asymmetry [4]. The CP violation, which comes from the Yukawa couplings that give the Dirac mass matrix, resulted from the one loop radiative correction requiring at least two right-handed neutrinos.

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Assuming a strong hierarchy in the right-handed neutrino sector a successful L-asymmetry in these models requires the mass scale of the lightest right-handed neutrino to be $M_1 \geq O(10^9)$ GeV [9]. If the corresponding theory of matter is supersymmetric then this bound, being dangerously close to the reheat temperature, poses a problem. A modest solution was proposed in Ref.[10] by introducing an extra singlet. However, the success of the model is the reduction of the above bound [9] by an order of magnitude.

In the type-II seesaw models, on the other hand, triplet Higgs (Δ_L's) are added to the SM gauge group. The triplet seesaw [11] in this case gives the light neutrino mass matrix:

$$m_\nu = m_\nu^{\rm II} = f\mu \frac{v^2}{M_{\Delta_L}^2}, \tag{5}$$

where M_{Δ_L} is the mass of the triplet Higgs scalar Δ_L, f is the Yukawa coupling relating the triplet Higgs with the light leptons, μ is the coupling constant with mass dimension 1 for the trilinear term with the triplet Higgs and two standard model Higgs doublets, and v is the vacuum expectation value (*vev*) of the SM Higgs doublet. The L-asymmetry, in these models, is generated through the L-number violating decays of the Δ_L to SM lepton and Higgs. The CP-violation, originated from the one loop radiative correction, requires at least two triplets. Again the scale of L-number violation is determined by M_{Δ_L} and μ and required to be very high and larger than the type-I models [12].

An attractive scenario is the hybrid seesaw model (type-I + type-II), where both right-handed neutrinos as well as triplet Higgs scalars are present. So, there is no constraint on their number to have CP violation. The neutrino mass matrix in these models is given by

$$m_\nu = m_\nu^{\rm I} + m_\nu^{\rm II}, \tag{6}$$

where $m_\nu^{\rm I}$ and $m_\nu^{\rm II}$ are given by Eqs. (4) and (5) respectively. A natural extension of the SM to incorporate both type-I as well as type-II terms of the neutrino mass matrix is the left-right symmetric model [13] with the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. The advantages of considering this model are that 1) it has a natural explanation for the origin of parity violation, 2) it can be easily embedded in the $SO(10)$ Grand Unified Theory (GUT), and 3) $B-L$ is a gauge symmetry. Since $B-L$ is a gauge symmetry of the model, it is not possible to have any L-asymmetry before the left-right symmetry breaking. An L-asymmetry can be produced after the left-right symmetry breaking phase transition, either through the decay of right-handed neutrinos or through the decay of the triplet Higgs, or can be both depending on the relative magnitudes of their masses. Assuming a strong hierarchy in the right-handed neutrino sector and $M_1 < M_{\Delta_L}$, it is found that M_1 can be reduced to an order of magnitude in comparison to the type-I models [14–16]. Despite the success, this mechanism of producing L-asymmetry in these models can not bring down the scale of leptogenesis to the scale of the next generation accelerators.

The alternatives to these are provided by mechanisms which work at the TeV scale [17] either in supersymmetric extensions of the SM relying on the new particle content or finding the additional source of CP violation in the model [18]. It is worth investigating other possibilities, whether or not supersymmetry is essential to the mechanism. In the following we consider a class of left-right symmetric models in which the spontaneous breaking of D-parity occurs at a high energy scale ($\sim 10^{13}$ GeV) leaving the $SU(2)_R$ intact. In the left-right symmetric models, parity connects the left-handed gauge group with the right-handed gauge group. But the same need not be true for the scalar particles. In this class of left-right symmetric models, the spontaneous D-parity violation allows the scalars transforming under the group $SU(2)_L$ to decouple from the scalars transforming under the group $SU(2)_R$ and these scalars can have different masses and couplings. This allows the mass scale of the triplet Δ_L to be very high at the D-parity breaking scale [19] while leaving the mass of Δ_R to be as low as the $SU(2)_R$ symmetry breaking scale or vice versa. However, we will see that even in these models a successful leptogenesis does not allow either the mass of triplets or the mass of right-handed neutrinos less than 10^8 GeV if the L-asymmetry arises from their out-of-equilibrium decay. We then consider an alternative mechanism to bring down the mass scale of right-handed neutrinos to be in TeV scale. In the respective mechanism a net L-asymmetry arises through the preferential scattering of left-handed neutrino ν_L over its CP conjugate state ν_L^c from the left-right domain wall [20]. The survival of this asymmetry then requires the mass scale of lightest right-handed neutrino, assuming a normal mass hierarchy in the right-handed neutrino sector, to be in TeV scale [21,22]. In this class of models the TeV scale masses of the right-handed neutrinos result from the low scale (~ 10 TeV) breaking of $SU(2)_R$ gauge symmetry while D-parity breaks at a high energy scale ($\sim 10^{13}$ GeV). This is an important result pointed out in this paper.

The rest of the article is arranged as follows. In Section II we briefly discuss the left-right symmetric models, elucidating the required Higgs structure for spontaneous breaking of D-parity. In Section III we discuss the parities in left-right symmetric models and their consequence on neutrino masses. Then we give a possible path for embedding the left-right symmetric models in the $SO(10)$ GUT. In Section IV we discuss the production of L-asymmetry through the decay of heavy Majorana neutrinos as well as the triplet Δ_L separately by taking into account the relative magnitudes of their masses. In Section V, by assuming a charge-neutral symmetry, we derive the neutrino mass matrices from the low energy neutrino data. Using this symmetry the L-asymmetry is estimated in Section VI by considering the relative masses of N_1 and the triplet Δ_L. In any case, it is found that the leptogenesis scale can not be lowered to a scale that can be accessible in the next generation accelerators. In Section VII, we therefore discuss an alternative mechanism which has the ability to explain the L-asymmetry at the TeV scale. In Section VIII we give a qualitative suggestion towards the density perturbations due to the presence of heavy singlet scalars. We summarize our results and conclude in Section IX.

II. LEFT-RIGHT SYMMETRIC MODELS

In the left-right symmetric model, the right-handed charged lepton of each family which was an isospin singlet under SM gauge group gets a new partner ν_R. These two form an isospin doublet under the $SU(2)_R$ of the left-right symmetric gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$, where P stands for the parity. Similarly, in the quark-sector, the right-handed up and down quarks of each family, which were isospin singlets under the SM gauge group, combine to form the isospin doublet under $SU(2)_R$. As a result before the left-right symmetry breaking both left- and right-handed leptons and quarks enjoy equal strength of interactions. This explains that the parity is a good quantum number in the left-right symmetric model in contrast to the SM where the left-handed particles are preferential under the electroweak interaction.

In the Higgs sector, the model consists of a $SU(2)$ singlet scalar field σ, two $SU(2)$ triplets Δ_L and Δ_R, and a bidoublet Φ which contains two copies of SM Higgs. Under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ the field contents and the quantum numbers of the Higgs fields are given as

$$\sigma \sim (1, 1, 0) \tag{7}$$

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \sim (2, 2, 0) \tag{8}$$

$$\Delta_L = \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix} \sim (3, 1, 2) \tag{9}$$

$$\Delta_R = \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix} \sim (1, 3, 2). \tag{10}$$

The most general renormalizable Higgs potential exhibiting left-right symmetry is given by [23]

$$\mathbf{V} = \mathbf{V}_\sigma + \mathbf{V}_\Phi + \mathbf{V}_\Delta + \mathbf{V}_{\sigma\Delta} + \mathbf{V}_{\sigma\Phi} + \mathbf{V}_{\Phi\Delta}, \tag{11}$$

where

$$\mathbf{V}_\sigma = -\mu_\sigma^2\sigma^2 + \lambda_\sigma\sigma^4,$$

$$\mathbf{V}_\Delta = -\mu_\Delta^2[\mathrm{Tr}(\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R^\dagger)] + \rho_1\{[\mathrm{Tr}(\Delta_L\Delta_L^\dagger)]^2 + [\mathrm{Tr}(\Delta_R\Delta_R^\dagger)]^2\} + \rho_2[\mathrm{Tr}(\Delta_L\Delta_L)\,\mathrm{Tr}(\Delta_L^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R)\,\mathrm{Tr}(\Delta_R^\dagger\Delta_R^\dagger)] + \rho_3[\mathrm{Tr}(\Delta_L\Delta_L^\dagger)\,\mathrm{Tr}(\Delta_R\Delta_R^\dagger)] + \rho_4[\mathrm{Tr}(\Delta_L\Delta_L)\,\mathrm{Tr}(\Delta_R^\dagger\Delta_R^\dagger) + \mathrm{Tr}(\Delta_L^\dagger\Delta_L^\dagger)\,\mathrm{Tr}(\Delta_R\Delta_R)],$$

$$\mathbf{V}_\Phi = -\mu_{\Phi 1}^2\,\mathrm{Tr}(\Phi^\dagger\Phi) - \mu_{\Phi 2}^2[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger\Phi)] + \lambda_1[\mathrm{Tr}(\Phi\Phi^\dagger)]^2 + \lambda_2\{[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)]^2 + [\mathrm{Tr}(\tilde{\Phi}^\dagger\Phi)]^2\} + \lambda_3[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)\,\mathrm{Tr}(\tilde{\Phi}^\dagger\Phi)] + \lambda_4\{\mathrm{Tr}(\Phi^\dagger\Phi)[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger\Phi)]\},$$

$$\mathbf{V}_{\sigma\Delta} = M\sigma[\mathrm{Tr}(\Delta_L\Delta_L^\dagger) - \mathrm{Tr}(\Delta_R\Delta_R^\dagger)] + \gamma\sigma^2(\mathrm{Tr}(\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R^\dagger)),$$

$$\mathbf{V}_{\sigma\Phi} = \delta_1\sigma^2\,\mathrm{Tr}(\Phi^\dagger\Phi) + M'\sigma[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger) - \mathrm{Tr}(\tilde{\Phi}^\dagger\Phi)] + \delta_2\sigma^2[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger\Phi)],$$

$$\mathbf{V}_{\Phi\Delta} = \alpha_1\{\mathrm{Tr}(\Phi^\dagger\Phi)[\mathrm{Tr}(\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R^\dagger)]\} + \alpha_2\{\mathrm{Tr}(\tilde{\Phi}^\dagger\Phi)\,\mathrm{Tr}(\Delta_R\Delta_R^\dagger) + \mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)\,\mathrm{Tr}(\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)\,\mathrm{Tr}(\Delta_R\Delta_R^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger\Phi)\,\mathrm{Tr}(\Delta_L\Delta_L^\dagger)\} + \alpha_3[\mathrm{Tr}(\Phi\Phi^\dagger\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\Phi^\dagger\Phi\Delta_R\Delta_R^\dagger)] + \beta_1[\mathrm{Tr}(\Phi\Delta_R\Phi^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\Phi^\dagger\Delta_L\Phi\Delta_R^\dagger)] + \beta_2[\mathrm{Tr}(\tilde{\Phi}\Delta_R\Phi^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger\Delta_L\Phi\Delta_R^\dagger)] + \beta_3[\mathrm{Tr}(\Phi\Delta_R\tilde{\Phi}^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\Phi^\dagger\Delta_L\tilde{\Phi}\Delta_R^\dagger)] + \beta_4[\mathrm{Tr}(\tilde{\Phi}\Delta_R\tilde{\Phi}^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger\Delta_L\tilde{\Phi}\Delta_R^\dagger)],$$

where $\tilde{\Phi} = \tau^2\Phi^*\tau^2$, τ^2 being the Pauli spin matrix and $\mu_a^2 > 0$, with $a = \sigma, \Delta, \Phi_1, \Phi_2$.

III. PARITIES IN LEFT-RIGHT SYMMETRIC MODELS AND CONSEQUENCES

Now we briefly discuss the parities, P and D, in left-right symmetric models. The main difference between a D parity and P parity is that the D parity acts on the groups $SU(2)_L \otimes SU(2)_R$, while the P parity acts on the Lorentz group. In the left-right symmetric models we identify both the parities with each other, so that when we break the $SU(2)_R$ group or the D parity, the Lorentz Pparity is also broken.

Under the operation of parity the fermions, scalars, and vector bosons transform as:

$$\begin{aligned} &\psi_{L,R} \to \psi_{R,L} \qquad \Phi \to \Phi^\dagger \qquad \Delta_{L,R} \to \Delta_{R,L} \\ &\qquad \sigma \to -\sigma \qquad W_{L,R} \to W_{R,L}. \end{aligned} \tag{12}$$

This implies that the combinations $W_L + W_R$ and $\Delta_L + \Delta_R$ are even under parity, while $W_L - W_R$ and $\Delta_L - \Delta_R$ are odd under parity. So, $W_L - W_R$ is axial vector and σ and $\Delta_L - \Delta_R$ are pseudoscalars. Thus the *vev* of the fields σ or Δ_R can break parity spontaneously.

It is possible to break the D parity spontaneously by breaking the group $SU(2)_R$ spontaneously by the *vev* of the field Δ_R, or by breaking it by the *vev* of σ. In general, σ could be a scalar or pseudoscalar. If we start with σ as a scalar, then it can break the D parity keeping the P parity invariant. However, if we consider σ to be a pseudoscalar, it can break both D and P parities spontaneously. Since it is conventional to identify P parity with the D parity, we consider σ to be a pseudoscalar. Then the *vev* of the field σ will break parity and the group $SU(2)_R$ at different scales. This will have some interesting phenomenology. This was proposed in Ref. [19]. Recently its phenomenological consequences using doublet and triplet Higgs were studied in Ref. [24].

We assume that $\mu_\sigma^2 > 0$ in Eq. (11). As a result, below the critical temperature $T_c \sim \langle\sigma\rangle$, the parity breaking scale, the singlet Higgs field acquires a *vev*

$$\eta_P \equiv \langle\sigma\rangle = \frac{\mu_\sigma}{\sqrt{2\lambda_\sigma}}. \tag{13}$$

Since σ does not possess any quantum number under $SU(2)_{L,R}$ and $U(1)_{B-L}$, these groups remain intact while P breaks. However it creates a mass splitting between the triplet fields Δ_L and Δ_R since it couples differently with them as given in Eq. (11). This leads to different effective masses for Δ_L and Δ_R

$$M_{\Delta_L}^2 = \mu_\Delta^2 - (M\eta_P + \gamma\eta_P^2), \tag{14}$$

$$M_{\Delta_R}^2 = \mu_\Delta^2 + (M\eta_P - \gamma\eta_P^2). \tag{15}$$

We now apply a fine tuning to set $M_{\Delta_R}^2 > 0$ so that Δ_R can acquire a *vev*

$$\langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}. \tag{16}$$

In order to restore the *SM* prediction, i.e., to restore the observed phenomenology at a low scale, Φ and $\tilde{\Phi}$ acquire *vevs*

$$\langle\Phi\rangle = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} \quad \text{and} \quad \langle\tilde{\Phi}\rangle = \begin{pmatrix} k_2 & 0 \\ 0 & k_1 \end{pmatrix}. \tag{17}$$

This breaks the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ down to $U(1)_{\rm em}$. However, this induces a nontrivial *vev* for the triplet Δ_L as

$$\langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}. \tag{18}$$

In the above v_L, v_R, k_1, and k_2 are real parameters. Further, the observed phenomenology requires that $v_L \ll k_1, k_2 \ll v_R$.

Using Eqs. (16)–(18) in Eq. (11) we get the effective potential

$$\begin{aligned} \mathbf{V}_{\rm eff} = &-\mu_\sigma^2\eta_P^2 - [\mu_\Delta^2 - M\eta_P - \gamma\eta_P^2 - \alpha_1(k_1^2 + k_2^2) - \alpha_2(4k_1k_2) - \alpha_3k_2^2]v_L^2 \\ &- [\mu_\Delta^2 + M\eta_P - \gamma\eta_P^2 - \alpha_1(k_1^2 + k_2^2) - \alpha_2(4k_1k_2) - \alpha_3k_2^2]v_R^2 - \mu_{\Phi1}^2(k_1^2 + k_2^2) - \mu_{\Phi2}^2(4k_1k_2) + \lambda_\sigma\eta_P^4 \\ &+ \rho_1(v_L^4 + v_R^4) + \rho_3v_L^2v_R^2 + \lambda_1(k_1^2 + k_2^2) + (2\lambda_2 + \lambda_3)(4k_1^2k_2^2) + \lambda_4(k_1^2 + k_2^2)(4k_1k_2) + \delta_1\eta_P^2(k_1^2 + k_2^2) \\ &+ \delta_2\eta_P^2(4k_1k_2) + 2(\beta_1k_1k_2 + \beta_2k_1^2 + \beta_3k_2^2 + \beta_4k_1k_2)v_Lv_R. \end{aligned} \tag{19}$$

The electroweak phase transition occurs at a low energy scale and hence it is reasonable to assume that the parameters k_2^2, k_1k_2, $k_1^2 \ll \eta_P$. Using this approximation in Eq. (19) one can see that the effective masses of Δ_L and Δ_R coincide with Eqs. (14) and (15). Further assuming $M = \gamma\eta_P$ we get

$$M_{\Delta_R}^2 = \mu_\Delta^2 \quad \text{and} \quad M_{\Delta_L}^2 = M_{\Delta_R}^2 - 2\gamma\eta_P^2. \tag{20}$$

Thus a large cancellation between M_{Δ_R} and $\gamma\eta_P$ allows an effective mass scale of the triplet Δ_L to be very low and vice versa.

We now check the order of magnitude of the induced *vev* of the triplet Δ_L. This should be small (less than a GeV) in order for the theory to be consistent with Z-decay width. Further, the sub-eV masses of the light neutrinos require *vev* of Δ_L to be of the order of eV, because it gives masses through the type-II seesaw mechanism. From Eq. (19) we get

$$v_R \frac{\partial \mathbf{V}_{\text{eff}}}{\partial v_L} - v_L \frac{\partial \mathbf{V}_{\text{eff}}}{\partial v_R} = 0$$
$$= v_L v_R [4M\eta_P - 4\rho_1(v_R^2 - v_L^2) + 2\rho_3(v_R^2 - v_L^2)] + 2(\beta_1 k_1 k_2 + \beta_2 k_1^2 + \beta_3 k_2^2 + \beta_4 k_1 k_2)(v_R^2 - v_L^2). \tag{21}$$

In the quark sector the *vevs* k_1 and k_2 give masses to the up and down type quarks, respectively. Therefore, it is reasonable to assume

$$\frac{k_1}{k_2} = \frac{m_t}{m_b}. \tag{22}$$

With the approximation $v_L \ll k_1,\ k_2 \ll v_R \ll \eta_P$ and using the above assumption (22) in Eq. (21) we get

$$v_L \simeq \frac{-\beta_2 v^2 v_R}{2M\eta_P}, \tag{23}$$

where we have used $v = \sqrt{k_1^2 + k_2^2} \simeq k_1 = 174$ GeV. Notice that in the above equation the smallness of the *vev* of Δ_L is decided by the parity breaking scale, not the $SU(2)_R$ breaking scale. So there are no constraints on v_R from the seesaw point of view. After $SU(2)_R$ symmetry breaking the right-handed neutrinos acquire masses through the Majorana Yukawa coupling with the Δ_R. Depending on the strength of Majorana Yukawa coupling a possibility of TeV scale right-handed neutrino is unavoidable. We will discuss the consequences in context of L-asymmetry in Section IV.

Finally before going on to discuss the L-asymmetry in this model we give a most economic breaking scheme of $SO(10)$ GUT through the left-right symmetric path. Keeping in mind that the P and $SU(2)_R$ breaking scales are different, the breaking of $SO(10)$ down to $U(1)_{\text{em}}$ can be accomplished by using a set of Higgs: {210}, {126}, {10} of $SO(10)$. At the first stage $SO(10)$ breaks to $G_{224} \equiv SU(2)_L \otimes SU(2)_R \otimes SU(4)_C (\supset SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes SU(3)_C)$ through the *vev* of {210}. Under G_{224} its decomposition can be written as

$$\{210\} = (1,1,1) + (2,2,20) + (3,1,15) + (1,3,15) + (2,2,6) + (1,1,15), \tag{24}$$

where (1,1,1) is a singlet and it is odd under the D parity, which is a generator of the group $SO(10)$. Hence it can play the same role as σ discussed above. At a later epoch {126} of $SO(10)$ can get a *vev* and breaks $SU(2)_L \otimes SU(2)_R \otimes SU(4)_C$ to $G_{213} \equiv SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$. Under G_{224} the decomposition of {126} is given as

$$\{126\} = (3,1,10) + (1,3,10) + (2,2,15) + (1,1,6), \tag{25}$$

where (3,1,10) and (1,3,10) contain the fields Δ_L and Δ_R respectively as in the above discussion. Finally the *vev* of {10} breaks the gauge group $SU(2)_L \otimes U(1)_Y \times SU(3)_C$ down to $U(1)_{\text{em}} \otimes SU(3)_C$ which contains a (2,2,1) playing the role of Φ in our discussion.

IV. NEUTRINO MASSES AND LEPTOGENESIS IN LEFT-RIGHT SYMMETRIC MODELS

The relevant Yukawa couplings giving masses to the three generations of leptons are given by

$$\mathcal{L}_{yuk} = h_{ij}\overline{\psi_{Li}}\psi_{Rj}\Phi + \tilde{h}_{ij}\overline{\psi_{Li}}\psi_{Rj}\tilde{\Phi} + \text{H.c.} + f_{ij}[\overline{(\psi_{Li})^c}\psi_{Lj}\Delta_L + \overline{(\psi_{Ri})^c}\psi_{Rj}\Delta_R] + \text{H.c.}, \tag{26}$$

where $\psi_{L,R}^T = (\nu_{L,R}, e_{L,R})$. The discrete left-right symmetry ensures the Majorana Yukawa coupling f to be same for both left- and right-handed neutrinos. The breaking of left-right symmetry down to $U(1)_{\text{em}}$ results in the effective mass matrix of the light neutrinos being

$$m_\nu = f v_L - m_D \frac{f^{-1}}{v_R} m_D^T = m_\nu^{\text{II}} + m_\nu^{\text{I}}, \tag{27}$$

where $m_D = hk_1 + \tilde{h}k_2 \simeq hk_1$ and v_L is given by Eq. (23). In theories where both type-I and type-II mass terms originate at the same scale it is difficult to choose which of them contributes dominantly to the neutrino mass matrix. In contrast to it in the present case since the parity and the $SU(2)_R$ breaking scales are different and, in fact, $\eta_P \gg v_R$, it is reasonable to assume that the type-I neutrino mass dominantly contributes to the effective neutrino mass matrix. In what follows we assume

$$m_\nu = m_\nu^{\text{I}} = -m_D \frac{f^{-1}}{v_R} m_D^T. \tag{28}$$

In the previous section we showed that the $SU(2)_R$ breaking scale v_R can be much lower than the parity breaking scale η_P since the smallness of v_L does not depend on v_R. Conventionally this leads to the right-handed neutrino masses being smaller than those of the triplet Δ_L [19]. However, in the present case a large cancellation between $M_{\Delta_R}^2$ and $\gamma\eta_P^2$ allows an effective mass of the triplet Δ_L to be in low scale while leaving the mass of Δ_R at the D-parity breaking scale. Note that the source of smallness of the right-handed neutrinos and the triplet Δ_L are absolutely different. Unless the low energy observables constrain their masses one cannot predict which one is lighter. In the following we take leptogenesis as a tool to distinguish their mass scales.

A. Leptogenesis via heavy neutrino decay

Without loss of generality we work on a basis in which the mass matrix of the right-handed neutrinos is real and diagonal. In this basis the heavy Majorana neutrinos are defined as $N_i = (1/\sqrt{2})(\nu_{Ri} \pm \nu_{Ri}^c)$, where $i = 1, 2, 3$ representing the flavor indices. The corresponding masses of the heavy Majorana neutrinos are given by M_i. In this basis a net CP-asymmetry results from the decay of N_i to the SM fermions and the bidoublet Higgs and is given by the interference of tree level, one loop radiative correction and the self-energy correction diagrams as shown in Fig. 1. The resulting CP-asymmetry in this case is given by

$$\epsilon_i^I = \frac{1}{8\pi} \frac{\sum_l \mathrm{Im}[(h^{a\dagger}h^b)_{il}(h^{b\dagger}h^a)_{il}]}{(h^{a\dagger}h^a)_{ii}} \times \sqrt{x_l}[1-(1+x_l)\log(1+1/x_l)+1/(1-x_l)], \quad (29)$$

where $x_l = M_l^2/M_i^2$ and h^a, with $a = 1, 2$ stands for the Dirac Yukawa couplings of fermions with Φ and $\tilde{\Phi}$ respectively. That is $h^1 = h$ and $h^2 = \tilde{h}$ as given in Eq. (26). Now we assume a normal mass hierarchy, $M_1 \ll M_2 < M_3$, in the heavy Majorana neutrino sector. In this case while the heavier right-handed neutrinos N_2 and N_3 are decaying, the lightest one, N_1, is in thermal equilibrium. Any L-asymmetry thus produced by the decay N_2 and N_3 is erased by the L-number violating scatterings mediated by N_1. Therefore, it is reasonable to assume that the final L-asymmetry is given by the decay of N_1. Simplifying Eq. (29) we get a net CP-asymmetry coming from the decay of N_1 to be

$$\epsilon_1^{\rm I} = -\frac{3M_1}{16\pi} \frac{\sum_{i,j} \mathrm{Im}[(h^{a\dagger})_{1i}(h^b(M_{\rm dia})^{-1}(h^a)^T)_{ij}(h^{b*})_{j1}]}{(h^{a\dagger}h^a)_{11}}. \quad (30)$$

Expanding the above Eq. (30) and using the fact that $m_\nu \simeq -k_1^2(hM_{\rm dia}^{-1}h^T)$ we get

$$\epsilon_1^{\rm I} = \frac{3M_1}{16\pi v^2}\left\{\frac{\sum_{i,j}\mathrm{Im}[(h^\dagger)_{1i}(m_\nu^I)_{ij}(h^*)_{j1}]}{(h^{a\dagger}h^a)_{11}} + (h,\tilde{h})\,\text{terms}\right\}. \quad (31)$$

Unlike the type-I models [9] here we have additional terms contributing the CP-asymmetry in the decay of N_1. Note that if the strength of $\tilde{h}$ is comparable with h then the resulting CP-asymmetry enhances by a factor of 2 in comparison with the CP-asymmetry in the exclusive type-I models [9].

An additional contribution to CP-asymmetry also comes from the interference of tree level diagram in Fig. 1 and the one loop radiative correction diagram involving the virtual triplet Δ_L as shown in Fig. 2. The resulting CP-asymmetry in this case is given by [14,25]

$$\epsilon_i^{\rm II} = \frac{3}{8\pi}\frac{\sum_{k,j}\mathrm{Im}[(h^a)^*_{ji}(h^b)^*_{ki}f_{jk}(v_R\beta)_{ab}]}{(h^a h^{a\dagger})_{ii}M_i} \times\left(1-\frac{M_{\Delta_L}^2}{M_i^2}\log(1+M_i^2/M_{\Delta_L}^2)\right), \quad (32)$$

where

$$\beta = \begin{pmatrix} \beta_1 & \beta_3 \\ \beta_2 & \beta_4 \end{pmatrix}. \quad (33)$$

If we further assume that $M_1 \ll M_{\Delta_L}$ in addition to the normal mass hierarchy in the heavy Majorana neutrino sector, then the final L-asymmetry must be given by the CP-violating decay of N_1 to the SM lepton and the bidoublet Higgs. Now using (23) in Eq. (32) we get the CP-asymmetry parameter

$$\epsilon_1^{\rm II} = \frac{3M_1}{16\pi v^2}\left(\frac{2M\eta_P}{-\beta_2 M_{\Delta_L}^2}\right) \times \frac{\sum_{jk}\mathrm{Im}[(h^\dagger)_{1j}(m_\nu^{\rm II})_{jk}(h^b)^*_{k1}\beta_{ab}]}{(h^{a\dagger}h^a)_{11}}. \quad (34)$$

Note that this result differs from the usual type-II seesaw models [14,15] where only one triplet Δ_L is usually introduced into the SM in addition to the singlet heavy Majorana neutrinos.

The total CP-asymmetry coming from the decay of N_1 thus reads

$$\epsilon_1 = \epsilon_1^{\rm I} + \epsilon_1^{\rm II}, \quad (35)$$

where $\epsilon_1^{\rm I}$ and $\epsilon_1^{\rm II}$ are given by Eqs. (31) and (34) respectively. Unlike the existing literature [15,16] in the present

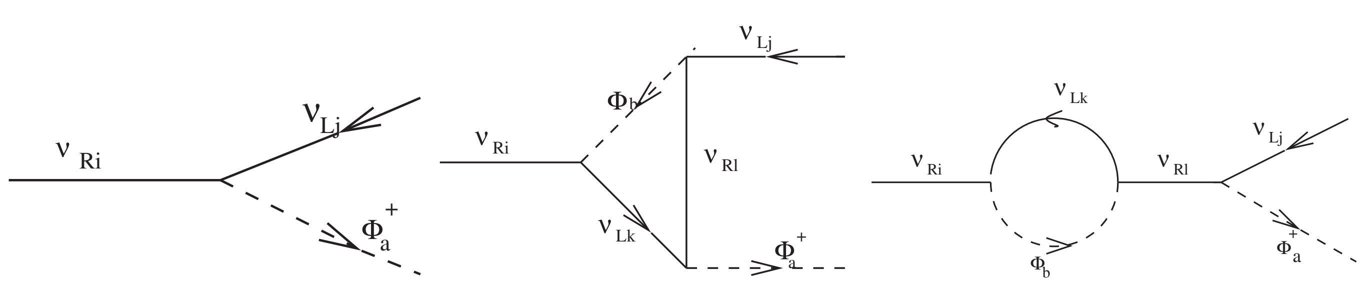

FIG. 1. The tree level, one loop radiative correction and the self-energy correction diagrams contributing to the CP-asymmetry in the decay of heavy Majorana neutrinos.

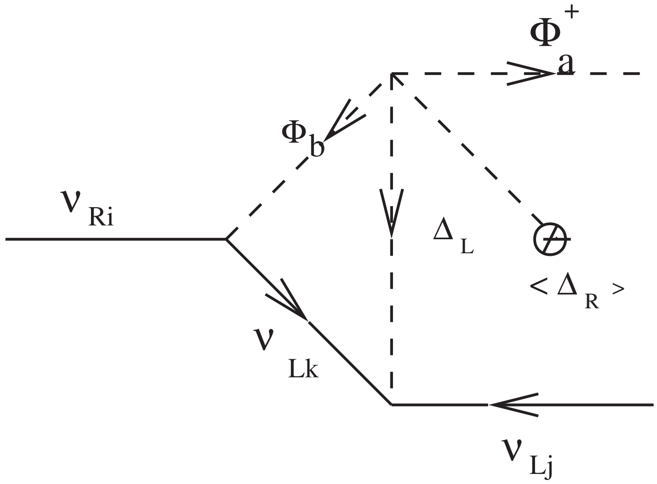

FIG. 2. The one loop radiative correction through the virtual triplet Δ_L in the decay of right-handed heavy Majorana neutrinos contributes to the CP-asymmetry.

case it is impossible to compare the magnitude of $\epsilon_1^{\rm I}$ and $\epsilon_1^{\rm II}$ through the type-I and type-II neutrino mass terms unless one takes the limiting cases.

1. Dominating type-I contribution

Let us first assume that $\epsilon_1^{\rm I}$ dominates in Eq. (35) and the neutrino Dirac Yukawa coupling $h \simeq \tilde{h}$. The resulting CP-asymmetry is then given by

$$\epsilon_1 = \epsilon_1^{\rm I} = 2\left\{\frac{3M_1}{16\pi v^2}\frac{\sum_{i,j}{\rm Im}[(h^\dagger)_{1i}(m_\nu^I)_{ij}(h^*)_{j1}]}{(h^\dagger h)_{11}}\right\}. \quad (36)$$

The maximum value of ϵ_1 then reads $\epsilon_1^{\rm max} = 2\epsilon_1^0$ [9], where

$$|\epsilon_1^0| = \frac{3M_1}{16\pi v^2}\sqrt{\Delta m^2_{\rm atm}}. \quad (37)$$

As a result we gain a factor of 2 in the lower bound on M_1 which is given as

$$M_1 \gtrsim 4.2\times 10^8\ {\rm GeV}\left(\frac{n_B/n_\gamma}{6.4\times 10^{-10}}\right)\left(\frac{10^{-3}}{\frac{n_{\nu_R}}{s}\delta}\right)\left(\frac{v}{174\ {\rm GeV}}\right)^2 \times\left(\frac{0.05\ {\rm eV}}{\sqrt{\Delta m^2_{\rm atm}}}\right), \quad (38)$$

where we have made use of the Eq. (1).

2. Dominating type-II contribution

Suppose $\epsilon_1^{\rm II}$ dominates in Eq. (35). In that case, assuming $\tilde{h} \simeq h$ and β_i's of order unity we get the maximum value of the CP-asymmetry parameter [16]

$$|\epsilon_1^{\rm max}| = \left(\frac{4M\eta_P}{M^2_{\Delta_L}}\right)\frac{3M_1}{16\pi v^2}m_3, \quad (39)$$

where $m_3 = \sqrt{\Delta m^2_{\rm atm}} \simeq 0.05$ eV. Following the same procedure in Section IVA 1 we gain a factor of $(M^2_{\Delta_L}/4M\eta_P)$ in the lower bound on M_1.

B. Leptogenesis through triplet decay

In the left-right symmetric models the decay of the scalar triplets Δ_L and Δ_R violates L-number by two units and hence is potentially able to produce a net L-asymmetry. The efficient decay modes which violate L-number are

$$\Delta_{L,R} \to \nu_{L,R} + \nu_{L,R}, \qquad \Delta_{L,R} \to \Phi^{a\dagger} + \Phi^b. \quad (40)$$

However, the decay rate in the process $\Delta_R \to \Phi^{a\dagger} + \Phi^b$ is highly suppressed in comparison to $\Delta_L \to \Phi^{a\dagger} + \Phi^b$ because the proportionality constant is v_L in the former case, while it is of v_R in the latter case. Moreover, in the present case the effective mass scale of the triplet Δ_R is larger than the mass of Δ_L due to the large cancellation between $M^2_{\Delta_R}$ and $2\gamma\eta_P^2$. Therefore, in what follows we take only the decay modes of the triplet Δ_L. The decay rates are given as:

$$\Gamma_\nu(\Delta_L \to \nu_{Li}\nu_{Lj}) = \frac{|f_{ij}|^2}{8\pi}M_{\Delta_L}, \quad (41)$$

$$\Gamma_\Phi(\Delta_L \to \Phi^{a\dagger}\Phi^b) = \frac{|\beta_{ab}|^2}{8\pi}r^2 M_{\Delta_L}, \quad (42)$$

where β_{ab} is given in Eq. (33) and $r^2 = (v_R^2/M^2_{\Delta_L})$. A net asymmetry is produced when the decay rate of the triplet Δ_L fails to compete with the Hubble expansion rate of the Universe. This is given by the conditions:

$$\Gamma_\nu \lesssim H(T = M_{\Delta_L}), \quad (43)$$

$$\Gamma_\Phi \lesssim H(T = M_{\Delta_L}). \quad (44)$$

As shown in Eq. (20) a large cancellation can lead to a TeV scale of the triplet Δ_L. However, the standard model gauge interaction $W_L^\dagger + W_L \to \Delta_L^\dagger + \Delta_L$ keeps it in thermal equilibrium. The out-of-equilibrium of this process requires $\Gamma_W \lesssim H(T = M_{\Delta_L})$. Consequently we will get a lower bound on the mass of the triplet Δ_L to be $M_{\Delta_L} \gtrsim 4.8\times 10^{10}$ GeV.

The CP-asymmetry in this case arises from the interference of tree level diagrams in Fig. 3 with the one loop

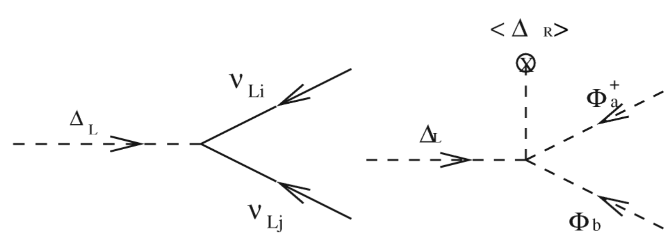

FIG. 3. The tree level diagrams of the decay of the triplet Δ_L contributing to the CP-asymmetry.

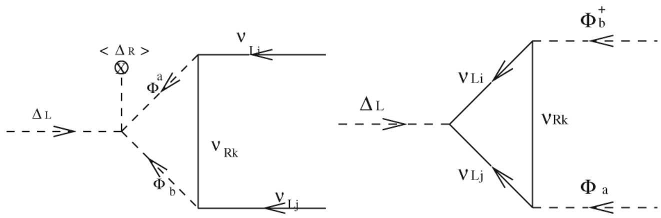

FIG. 4. The one loop radiative correction of the decay of the Δ_L through the exchange of virtual right-handed neutrinos contributing to the CP-asymmetry.

radiative correction diagrams involving the virtual right-handed neutrinos as shown in Fig. 4. The resulting CP-asymmetry in this case is given by [14,25]

$$\epsilon_\Delta = \frac{1}{8\pi}\sum_k M_k \frac{\sum_{ij}\mathrm{Im}[(h^a)^*_{ik}(h^b)^*_{jk}(\beta v_R)^*_{ab} f_{ij}]}{\sum_{ij}|f_{ij}|^2 M^2_{\Delta_L} + \sum_{cd}|\beta_{cd}|^2 v_R^2} \times \log\left(1+\frac{M^2_{\Delta_L}}{M_k^2}\right). \quad (45)$$

Assuming that $M_{\Delta_L} < M_1$ and $h = \tilde{h}$ the above equation can be simplified to

$$\epsilon_\Delta = \frac{1}{8\pi v^2}\frac{\sum_{ij}\mathrm{Im}[(m_\nu^{\mathrm{I}})^*_{ij}(M_R)_{ij}\sum\beta^*]}{\sum_{ij}|f_{ij}|^2 + \sum_{cd}|\beta_{cd}|^2 r^2}, \quad (46)$$

where m_ν^{I} is given by Eq. (28) which can be calculated from the low energy neutrino oscillation data.

V. CHARGE-NEUTRAL SYMMETRY AND NEUTRINO MASS MATRICES

The present neutrino oscillation data show that the neutrino mixing matrix up to a leading order in $\sin\theta_{13}$ is [26]

$$U_{\mathrm{PMNS}} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \epsilon e^{-i\delta} \\ \frac{-1}{\sqrt{6}} - \frac{1}{\sqrt{3}}\epsilon e^{i\delta} & \frac{1}{\sqrt{3}} - \frac{-1}{\sqrt{6}}\epsilon e^{i\delta} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}}\epsilon e^{i\delta} & \frac{-1}{\sqrt{3}} - \frac{-1}{\sqrt{6}}\epsilon e^{i\delta} & \frac{1}{\sqrt{2}} \end{pmatrix} \times \mathrm{dia}(1, e^{i\alpha}, e^{(\beta+\delta)}) \quad (47)$$

where we have used the best-fit parameters [27]; the atmospheric mixing angle $\theta_{23} = 45°$, the solar mixing angle $\theta_{12} \simeq 34°$, and the reactor angle $\sin\theta_{13} \equiv \epsilon$. Using (47) the neutrino mass matrix can be written as

$$m_\nu = U^*_{\mathrm{PMNS}} m_\nu^{\mathrm{dia}} U^\dagger_{\mathrm{PMNS}}, \quad (48)$$

where $m_\nu^{\mathrm{dia}} = \mathrm{dia}(m_1, m_2, m_3)$, with m_1, m_2, m_3 are the light neutrino masses. Using Eqs. (47) and (48) we get, up to an order of ϵ, the elements of the neutrino mass matrix:

$$\begin{aligned}
(m_\nu)_{11} &= \frac{m_2}{3} + \frac{2}{3}m_1 \\
(m_\nu)_{12} &= \epsilon e^{i\delta}\frac{m_3}{\sqrt{2}} + \frac{m_2}{\sqrt{3}}\left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}}\epsilon e^{-i\delta}\right) - \sqrt{\frac{2}{3}}m_1\left(\frac{1}{\sqrt{6}} + \frac{\epsilon e^{-i\delta}}{\sqrt{3}}\right) \\
(m_\nu)_{13} &= \epsilon e^{i\delta}\frac{m_3}{\sqrt{2}} - \frac{m_2}{\sqrt{3}}\left(\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{6}}\epsilon e^{-i\delta}\right) + \sqrt{\frac{2}{3}}m_1\left(\frac{1}{\sqrt{6}} - \frac{\epsilon e^{-i\delta}}{\sqrt{3}}\right) \\
(m_\nu)_{23} &= \frac{m_3}{2} - \frac{m_2}{3} - \frac{m_1}{6} \\
(m_\nu)_{22} &= \frac{m_3}{2} + \frac{m_1}{\sqrt{6}}\left(\frac{1}{\sqrt{6}} + \frac{2\epsilon e^{-i\delta}}{\sqrt{3}}\right) + \frac{m_2}{\sqrt{3}}\left(\frac{1}{\sqrt{3}} - \sqrt{\frac{2}{3}}\epsilon e^{-i\delta}\right) \\
(m_\nu)_{33} &= \frac{m_3}{2} + \frac{m_1}{\sqrt{6}}\left(\frac{1}{\sqrt{6}} - \frac{2\epsilon e^{-i\delta}}{\sqrt{3}}\right) + \frac{m_2}{\sqrt{3}}\left(\frac{1}{\sqrt{3}} + \frac{2\epsilon e^{-i\delta}}{\sqrt{6}}\right)
\end{aligned} \quad (49)$$

Inverting the seesaw relation (28) we get the right-handed neutrino mass matrix [28]

$$M_R = -m_D^T m_\nu^{-1} m_D, \quad (50)$$

where $M_R = f v_R$. The m_ν^{-1} in the above equation can be calculated from Eq. (48). Unless one assumes a texture of m_D it is difficult to link m_ν and M_R through Eq. (50). In general it is almost impossible to connect the low energy CP-phase and the CP-phase appearing in leptogenesis. So, by using some approximations for the neutrino Dirac mass matrix one can calculate the right-handed neutrino mass matrix M_R and hence the CP-asymmetry [29]. We assume a charge-neutral symmetry which is natural in the supersymmetric left-right symmetric models [30]. We take the neutrino Dirac mass

$$m_D = c m_l, \quad (51)$$

where m_l is the charged lepton mass matrix and c is a numerical factor. Further we assume the texture of the charged leptons mass matrix as [31]

$$m_l = \begin{pmatrix} 0 & \sqrt{m_e m_\mu} & 0 \\ \sqrt{m_e m_\mu} & m_\mu & \sqrt{m_e m_\tau} \\ 0 & \sqrt{m_e m_\tau} & m_\tau \end{pmatrix}. \quad (52)$$

We shall further assume that at a high energy scale, where the leptogenesis occurs, the PMNS matrix is given by [32]

$$U_{\mathrm{PMNS}} = U_l^\dagger U_0, \quad (53)$$

where U_l and U_0 are the diagonalizing matrix of m_l and m_ν respectively. At this scale we assume $U_l = I$ and a bimaximal structure for U_0 which is given by

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \epsilon e^{-i\delta} \\ \frac{-1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{-1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{54}$$

Now using (51) and (52) in Eq. (50) we get the elements in the right-handed neutrino mass matrix as:

$$\begin{aligned}
(M_R)_{11} &\simeq -c^2(m_e m_\mu)\Big(\frac{1}{4m_1}(1+2\epsilon e^{i\delta}) + \frac{1}{4m_2}(1-2\epsilon e^{i\delta}) + \frac{1}{2m_3}\Big)\\
(M_R)_{12} &\simeq -c^2(m_\mu\sqrt{m_e m_\mu})\Big(\frac{1}{4m_1}(1+2\epsilon e^{i\delta}) + \frac{1}{4m_2}(1-2\epsilon e^{i\delta}) + \frac{1}{2m_3}\Big)\\
(M_R)_{13} &\simeq -c^2(m_\tau\sqrt{m_e m_\mu})\Big(-\frac{1}{4m_1} - \frac{1}{4m_2} + \frac{1}{2m_3}\Big)\\
(M_R)_{22} &\simeq -c^2 m_\mu^2\Big(\frac{1}{4m_1}(1+2\epsilon e^{i\delta}) + \frac{1}{4m_2}(1-2\epsilon e^{i\delta}) + \frac{1}{2m_3}\Big)\\
(M_R)_{23} &\simeq -c^2(m_\mu m_\tau)\Big(-\frac{1}{4m_1} - \frac{1}{4m_2} + \frac{1}{2m_3}\Big)\\
(M_R)_{33} &\simeq -c^2 m_\tau^2\Big(\frac{1}{4m_1}(1+2\epsilon e^{i\delta}) + \frac{1}{4m_2}(1-2\epsilon e^{i\delta}) + \frac{1}{2m_3}\Big).
\end{aligned} \tag{55}$$

Below the electroweak phase transition the charged leptons are massive and the corresponding mass matrix is given by Eq. (52). So we can recover the PMNS matrix at low energy as given by Eq. (53) by attributing the small deviation from its bimaximal form to the diagonalizing matrix of the charged leptons U_l [32].

VI. LEPTON ASYMMETRY WITH CHARGE-NEUTRAL SYMMETRY

In this section we estimate the L-asymmetry from the decay of right-handed neutrinos as well as the triplet Δ_L, depending on the relative masses they acquire from the symmetry breaking pattern.

A. L-asymmetry with $M_1 < M_{\Delta_L}$ and dominating ϵ_1^I

Using (49) and (51) in Eq. (36) we get the resulting CP-asymmetry parameter from the decay of right-handed neutrino to be

$$\epsilon_1^{\rm I} \simeq -\frac{M_1}{16\pi v^2}[(2m_1+m_2)\epsilon^2\sin 2\delta + 2\sqrt{2}(m_1-m_2)\epsilon\sin\delta]. \tag{56}$$

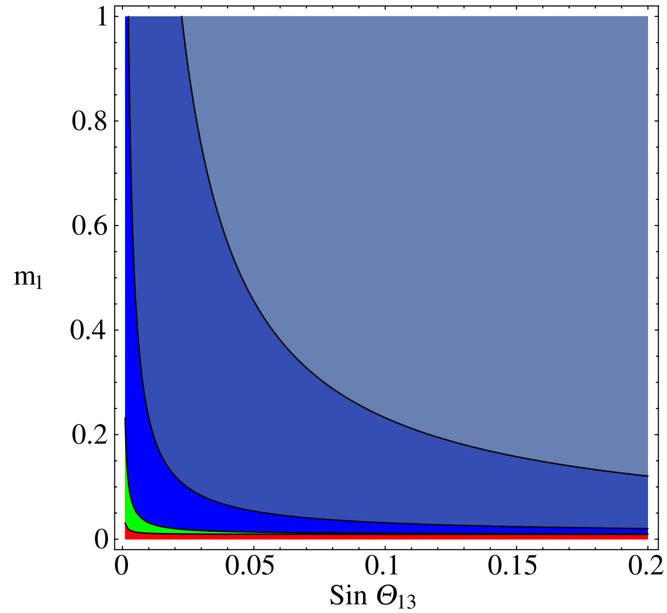

FIG. 5 (color online). Contours satisfying the required B-asymmetry are plotted in the $\sin\theta_{13}$ versus m_1 plane for $(4.2 \times 10^8\ \mathrm{GeV}/M_1) = 0.1, 0.01, 0.001, 0.0001$.

The L-asymmetry in a comoving volume is then given by

$$Y_L = \epsilon_1^{\rm I} Y_{N_1} d, \tag{57}$$

where $Y_{N_1} = (n_{N_1}/s)$, $s = (2\pi^2/45)g_* T^3$ is the entropy density, n_{N_1} is the number density of lightest right-handed neutrino in a physical volume, and d is the dilution factor which can be obtained by solving the required Boltzmann equations. A part of the L-asymmetry is then transferred to the B-asymmetry in a calculable way. As a result we get the net B-asymmetry

$$\frac{n_B}{n_\gamma} = 7Y_B = -3.5\epsilon_1^{\rm I} Y_{N_1} d. \tag{58}$$

With the maximal CP asymmetry, i.e., $\delta = \pi/2$, and using the best-fit parameter for $m_2 = 0.009$ eV we have shown the regions in the $\sin\theta_{13}$ versus m_1 plane for various values of M_1 as shown in Fig. 5. The upper most region represents 4.2×10^8 GeV $< M_1 < 4.2 \times 10^9$ GeV. As we go down the mass of N_1 increases by an order of magnitude per region. If we assume a normal mass hierarchy for the light physical neutrinos then only the bottom most region, i.e., $M_1 > 4.2 \times 10^{12}$ GeV, is allowed for all $m_1 < 0.001$ eV and $\sin\theta_{13} < 0.2$, the present experimentally allowed values.

B. L-asymmetry with $M_1 < M_{\Delta_L}$ and dominating $\epsilon_1^{\rm II}$

Assuming a normal mass hierarchy in the right-handed neutrino sector and the mass of lightest right-handed neutrino $M_1 < M_{\Delta_L}$, the CP-asymmetry parameter (32) can be rewritten as

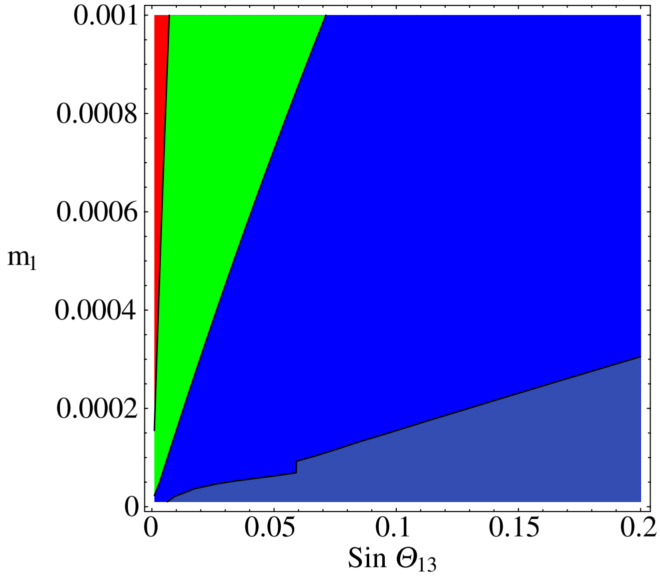

FIG. 6 (color online). Contours satisfying the required B-asymmetry in the $\sin\theta_{13}$ versus m_1 plane are plotted for $(4.2 \times 10^8 \text{ GeV}/M_1) = 0.01, 0.001, 0.0001$. We have used the parameters $\beta = 1$, $c = 1$, and $M_{\Delta_L} = 10^{13}$ GeV.

$$\epsilon_1^{\rm II} = \left(\frac{3M_1}{16\pi M_{\Delta_L}^2}\right)\frac{\mathrm{Im}[((m_D^a)^\dagger M_R (m_D^b)^*)_{11}\beta_{ab}]}{((m_D^a)^\dagger m_D^a)_{11}}. \qquad (59)$$

We further assume $m_D \simeq \tilde{m}_D$ and $\beta = O(1)$. Thus using the value of m_D and M_R from Eqs. (51) and (55) in the above equation we get

$$\epsilon_1^{\rm II} \simeq \left(-\frac{3M_1\beta c^2 m_\mu^2}{8\pi M_{\Delta_L}^2}\right)\frac{\epsilon \sin\delta}{2}\left(\frac{1}{m_1} - \frac{1}{m_2}\right). \qquad (60)$$

Following the same procedure in Section VI A we calculate the B-asymmetry by using $\epsilon_1^{\rm II}$. The corresponding regions in the $\sin\theta_{13}$ versus m_1 plane are shown in Fig. 6 for various values of M_1. In the bottommost region we have 4.2×10^9 GeV $< M_1 < 4.2 \times 10^9$ GeV. As we go up the mass of N_1 increases by an order of magnitude for each region. By taking the best-fit value for $m_2 = 0.009$ eV and using the maximal CP violation it is found that in a large allowed range of $\sin\theta_{13}$ the smaller values of M_1 are preferable for all $m_1 < 10^{-3}$ eV. That means a successful leptogenesis with $m_1 < 10^{-3}$ eV prefers only the values 4.2×10^8 GeV $\leq M_1 < 4.2 \times 10^{12}$ GeV. Note that these regions are exactly complementary to the dominant type-I case.

C. L-asymmetry with $M_{\Delta_L} < M_1$

We now assume that $M_{\Delta_L} < M_1$. Hence the final L-asymmetry must be given by the decay of triplet Δ_L. The L-asymmetry from the decay of triplet Δ_L is defined as

$$Y_L = \epsilon_\Delta Y_\Delta d, \qquad (61)$$

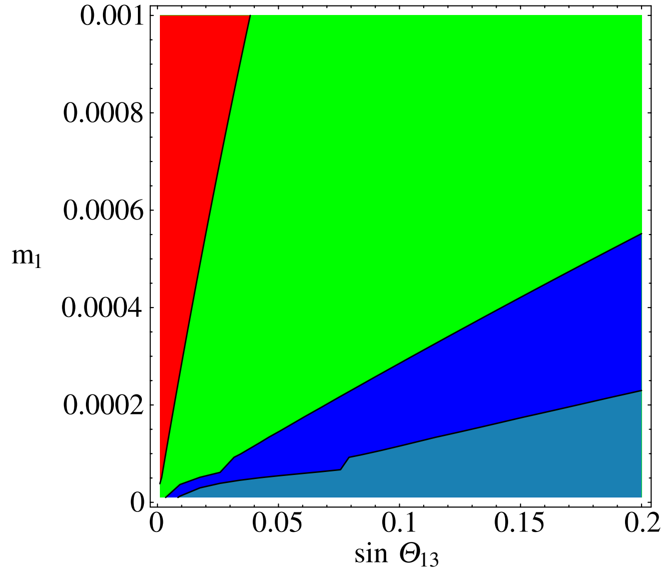

FIG. 7 (color online). Contours satisfying the required B-asymmetry in the $\sin\theta_{13}$ versus m_1 plane are shown for $r^2 = 1, 10, 25$. We have used the parameters $c = 0.1$, $\beta = 1$.

where $Y_\Delta = (n_{\Delta_L}/s)$, with $n_{\Delta_L} = n_{\Delta_L^{++}} + n_{\Delta_L^{+}} + n_{\Delta_L^{0}}$ the density of the triplets, s the entropy density, and d the dilution factor. Assuming β's of order unity and substituting ϵ_Δ from Eq. (46) we get the L-asymmetry

$$Y_L = \frac{1}{8\pi v^2}\frac{\mathrm{Im}(\mathrm{Tr}[(m_\nu^I)^* M_R]\sum\beta_i^*)}{\sum|\beta_i|^2 r^2} Y_\Delta d. \qquad (62)$$

Using the Eqs. (49) and (55) we evaluate Y_L. Again following the same procedure as given in Section VI A we calculate the B-asymmetry. With the maximal CP violation and using the best-fit parameters, $m_2 = 0.009$ eV and $m_3 = 0.05$ eV, the regions in the $\sin\theta_{13}$ versus m_1 plane are shown in Fig. 7 for various values of $r^2 = v_R^2/M_{\Delta_L}^2$. In the bottommost region we have $r^2 > 25$. r^2 values are decreased further towards the upper left (the red region which is not allowed because it represents $r^2 < 1$ which implies $M_{\Delta_L} > M_1$). Thus it is clear that for $\sin\theta_{13} < 0.2$ the only values of $m_1 < 10^{-4}$ eV are allowed for a successful leptogenesis.

D. Results and discussions

Assuming the neutrino Dirac mass matrix follows the same hierarchy of charged lepton mass matrix we studied the sensitivity of L-asymmetry on the mass scale of the lightest right-handed neutrino as well as the triplet Δ_L. In any case it is found that a successful L-asymmetry requires the mass of the lightest right-handed neutrino to satisfy $M_1 > O(10^8)$ GeV and that $M_{\Delta_L} > O(10^{10})$ GeV. Therefore, these mechanisms of producing L-asymmetry are far from hope of being verified in the next generation accelerators. On the other hand, the large masses of N_1 and

Δ_L satisfy a large range of parameters explored in the neutrino oscillations. In the following we study an alternative to explain the L-asymmetry at the TeV scale that is compatible with the low energy neutrino oscillation data.

VII. TRANSIENT LEFT-RIGHT DOMAIN WALLS, LEPTOGENESIS, AND TEV SCALE RIGHT-HANDED NEUTRINOS

A. Spontaneous breaking of *D*-parity and transient left-right domain walls

In the conventional low energy left-right symmetric model the discrete left-right symmetry as well as the gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ breaks at the same scale through the vev of Δ_R. As a result stable domain walls [33], interpolating between the L- and R-like regions, are formed. By L-like we mean regions favored by the observed phenomenology, while in the R-like regions the vacuum expectation value of Δ_R is zero. Unless some nontrivial mechanism prevents this domain structure, the existence of R-like domains would disagree with low energy phenomenology. Furthermore, the domain walls would quickly come to dominate the energy density of the Universe. Thus in this theory a departure from exact $L \leftrightarrow R$ symmetry is essential in such a way as to eliminate the phenomenologically disfavored R-like regions.

The domain walls formed can be *transient* if there exists a slight deviation from exact $L \leftrightarrow R$ symmetry. In other words we require $g_L \neq g_R$ before $SU(2)_L \times SU(2)_R$ breaking scale. In the present case this is achieved by breaking the D parity at a high scale, at around $\eta_P \sim 10^{13}$ GeV. This gives rise to $g_L \neq g_R$ before the breaking of gauge symmetry $SU(2)_L \times SU(2)_R$. As a result the spectrum of Higgs bosons exhibit the left-right *asymmetry* even though $SU(2)_R$ symmetry is unbroken. Therefore, the thermal perturbative corrections to the Higgs field free energy will not be symmetric and the domain walls will be unstable. The slight difference in the free energy between the two types of regions causes a pressure difference across the walls, converting all the R-like regions to L-like regions. Details of this dynamics can be found in Ref. [20].

B. Leptogenesis from transient domain walls

It was shown in [20] that within the thickness of the domain walls the net CP-violating phase becomes position dependent. Under these circumstances the preferred scattering of ν_L over its CP-conjugate state (ν_L^c) produces a net raw L-asymmetry [20]

$$\eta_L^{\rm raw} \cong 0.01 v_w \frac{1}{g_*} \frac{M_1^4}{T^5 \Delta_w} \tag{63}$$

where $\eta_L^{\rm raw}$ is the ratio of n_L to the entropy density s. In the right-hand side Δ_w is the wall width and g_* is the effective thermodynamic degrees of freedom at the epoch with temperature T. Using $M_1 = f_1 \Delta_T$, with Δ_T as the temperature dependent *vev* acquired by the Δ_R in the phase of interest, and $\Delta_w^{-1} = \sqrt{\lambda_{\rm eff}}\Delta_T$ in Eq. (63), we get

$$\eta_L^{\rm raw} \cong 10^{-4} v_w \left(\frac{\Delta_T}{T}\right)^5 f_1^4 \sqrt{\lambda_{\rm eff}}, \tag{64}$$

where we have used $g_* = 110$. Therefore, depending on the various dimensionless couplings, the raw asymmetry may lie in the range $O(10^{-4}\text{–}10^{-10})$. However, it may not be the final L-asymmetry, because the thermally equilibrated L-violating processes mediated by the right-handed neutrinos can erase the produced raw asymmetry. Therefore, a final L-asymmetry and hence the bound on right-handed neutrino masses can only be obtained by solving the Boltzmann equations [5]. We assume a normal mass hierarchy in the right-handed neutrino sector. In this scenario, as the temperature falls, first N_3 and N_2 go out of thermal equilibrium while N_1 is in thermal equilibrium. Therefore, it is the number density and mass of N_1 that are important in the present case and which enter into the Boltzmann equations. The relevant Boltzmann equations for the present purpose are [21,22]

$$\frac{dY_{N1}}{dZ} = -(D+S)(Y_{N1} - Y_{N1}^{\rm eq}) \tag{65}$$

$$\frac{dY_{B-L}}{dZ} = -WY_{B-L}, \tag{66}$$

where Y_{N_1} is the density of N_1 in a comoving volume, Y_{B-L} is the $B-L$ asymmetry, and the parameter $Z = M_1/T$. The various terms D, S, and W are representing the decay, scatterings, and the wash out processes involving the right-handed neutrinos. In particular, $D = \Gamma_D/ZH$, with

$$\Gamma_D = \frac{1}{16\pi v^2}\tilde{m}_1 M_1^2, \tag{67}$$

where $\tilde{m}_1 = (m_D^\dagger m_D)_{11}/M_1$ is called the effective neutrino mass parameter. Similarly $S = \Gamma_S/HZ$ and $W = \Gamma_W/HZ$. Here Γ_S and Γ_W receive the contribution from $\Delta_L = 1$ and $\Delta_L = 2$ L-violating scattering processes.

In an expanding universe these Γ's compete with the Hubble expansion parameter. In a comoving volume the dependence of $\Delta_{\rm L} = 1$ L-violating processes on the parameters $\tilde{m}_1$ and M_1 is given as

$$\left(\frac{\gamma_D}{sH(M_1)}\right), \left(\frac{\gamma_{\phi,s}^{N1}}{sH(M_1)}\right), \left(\frac{\gamma_{\phi,t}^{N1}}{sH(M_1)}\right) \propto k_1 \tilde{m}_1. \tag{68}$$

On the other hand, the dependence of the γ's in $\Delta_{\rm L} = 2$ L-number violating processes on $\tilde{m}_1$ and M_1 is given by

$$\left(\frac{\gamma_{N1}^l}{sH(M_1)}\right), \left(\frac{\gamma_{N1,t}^l}{sH(M_1)}\right) \propto k_2 \tilde{m}_1^2 M_1. \tag{69}$$

Finally there are also L-conserving processes whose dependence are given by

$$\left(\frac{\gamma_{Z'}}{sH(M_1)}\right) \propto k_3 M_1^{-1}. \tag{70}$$

In the above Eqs. (68)–(70), k_i, $i = 1, 2,3$ are dimensionful constants determined from other parameters. Since the L-conserving processes are inversely proportional to the mass scale of N_1, they rapidly bring the species N_1 into thermal equilibrium for all $T \gg M_1$. Furthermore, for smaller values of M_1, the washout effects (69) are negligible because of their linear dependence on M_1. We shall work in this regime while solving the Boltzmann equations.

The Eqs. (65) and (66) are solved numerically. The initial $B - L$ asymmetry is the net raw asymmetry produced through the domain wall mechanism as discussed above. We impose the following initial conditions:

$$Y_{N1}^{\rm in} = Y_{N1}^{\rm eq} \quad \text{and} \quad Y_{B-L}^{\rm in} = \eta_{B-L}^{\rm raw}, \tag{71}$$

assuming that there are no other processes creating L-asymmetry below the $B - L$ symmetry breaking scale. This requires $\Gamma_D \leq H$ at an epoch $T \geq M_1$ and hence leads to a bound [34]

$$m_\nu < m_* \equiv 4\pi g_*^{1/2} \frac{G_N^{1/2}}{\sqrt{2}G_F} = 6.5 \times 10^{-4} \text{ eV}. \tag{72}$$

Alternatively in terms of Yukawa couplings this bound reads

$$h_\nu \leq 10x, \quad \text{with } x = (M_1/M_{\rm pl})^{1/2}. \tag{73}$$

At any temperature $T \geq M_1$, wash out processes involving N_1 are kept under check due to the $\tilde{m}_1^2$ dependence in (69) for small values of $\tilde{m}_1$. As a result a given raw asymmetry suffers limited erasure. As the temperature falls below the mass scale of N_1 the wash out processes become negligible leaving behind a final L-asymmetry. Figure 8 shows the result of solving the Boltzmann equations for different values of M_1. An important conclusion from this figure is that for smaller values of M_1 the wash out effects are tiny. Hence by demanding that the initial raw asymmetry is the required asymmetry of the present Universe we can conspire the mass scale of N_1 to be as low as 1 TeV. For this value of M_1, using Eq. (73), we get the constraint on the neutrino Dirac Yukawa coupling to be $h_\nu \leq 10^{-7}$. It is shown in Ref. [22] that $h_\nu = 10^{-7}$ is reasonable to suppress the flavor changing neutral current in the conventional left-right symmetric model.

We assume that in Eq. (66) there are no other sources that produce L-asymmetry below the $SU(2)_R \times U(1)_{B-L}$ symmetry breaking phase transition. This can be justified by considering small values of h_ν, since the CP asymmetry parameter ϵ_1 depends quadratically on h_ν. For $h_\nu \leq 10^{-7}$ the L-asymmetry $Y_L \leq O(10^{-14})$, which is far less than the raw asymmetry produced by the scatterings of neutrinos on the domain walls. This explains the absence of any L-asymmetry generating terms in Eq. (66).

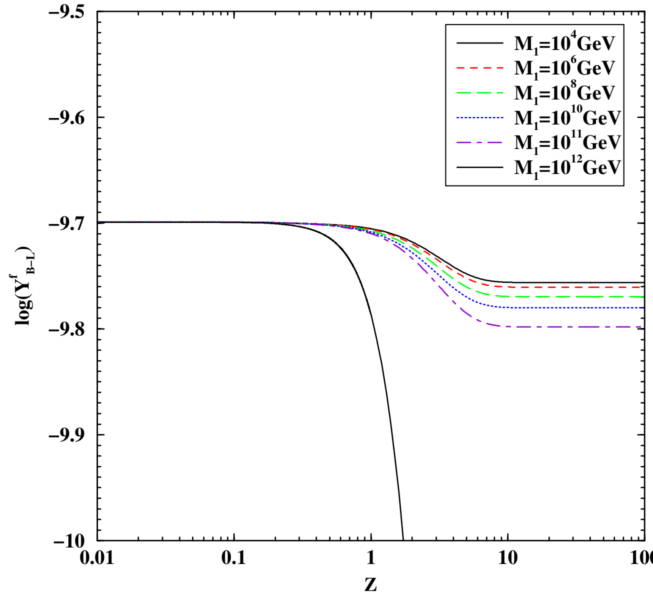

FIG. 8 (color online). The evolution of $B - L$ asymmetry for different values of M_1 shown against $Z(= M_1/T)$ for $\tilde{m}_1 = 10^{-4}$ eV and $\eta_{B-L}^{\rm raw} = 2.0 \times 10^{-10}$.

VIII. SPONTANEOUS BREAKING OF *D*-PARITY AND IMPLICATIONS FOR COSMOLOGY

An important aspect of the particle physics models is that the out-of-equilibrium decay of heavy scalar condensations gives rise to density perturbations in the early Universe [35]. In such a scenario, the cosmic microwave background radiation originating from the decay products of the scalar condensation and hence its anisotropy can be affected by the fluctuation of the scalar condensates. The observed anisotropy then constrains the mass scale of the heavy Higgs which induces the density perturbations. In the present model the fluctuation of the amplitude of late decaying condensation σ (the so called curvaton scenario) can give rise to density perturbations if the energy density of σ dominates the Universe for some time before its decay. Thus the models where inflaton does not generate sufficient perturbations can be rescued.

One possibility is that the σ can be abundantly produced from the decay of the inflaton field and dominates before its decay. Note that σ is a singlet field under the gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Therefore, the domination of σ before its decay is natural in this model more than any other scalar fields which have the gauge interactions. This is possible if $\Gamma_{\rm inf} \gg \Gamma_\sigma$, where $\Gamma_{\rm inf}$ and Γ_σ are, respectively, the decay rates of inflaton and σ fields. The Universe will then go through a radiation dominated era with a reheating temperature $T_I \simeq g_*^{-1/4}(M_{\rm pl}\Gamma_{\rm inf})^{1/2}$ when the inflaton field decays completely, i.e., $\Gamma_{\rm inf} \sim H$. If the initial amplitude of σ is substantial then it will reheat the Universe at a latter epoch $H \sim \Gamma_\sigma$ characterized by the reheat temperature

$T_{\rm II} \simeq g_*^{-1/4}(M_{\rm pl}\Gamma_\sigma)^{1/2}$ when σ decays completely. Therefore, the final density perturbation is mostly given by the σ field [35].

Obtaining an acceptable perturbation of the correct size (about 1 in 10^5) requires that the *vev* of σ field $\eta_P \sim 10^5 H_{\rm I}$ [35], where H_I is the Hubble expansion rate during inflation. For $\eta_P \sim 10^{13}$ GeV (which is required to suppress the type-II contribution of the neutrino mass matrix) one can have $H_{\rm I} \sim 10^8$ GeV.

IX. CONCLUSIONS

We studied baryogenesis via leptogenesis from the decay of right-handed heavy Majorana neutrinos as well as the triplet Δ_L in a class of left-right symmetric models with spontaneous D-parity violation. While in a generic type-I seesaw model, assuming normal mass hierarchy in the right-handed neutrino sector, one requires $M_1 > 4.2 \times 10^8$ GeV for successful thermal leptogenesis, with D parity this bound can be lowered up to a factor of $(M_{\Delta_L}^2/4M\eta_P)$. Thus the lowering factor depends on the model parameters in the present case. On the other hand, in the case $M_{\Delta_L} < M_1$ the lower bound on the mass scale of Δ_L is of the order 10^{10} GeV to produce the required lepton asymmetry. In any case the thermal leptogenesis scale can not be lowered up to a TeV scale if the lepton asymmetry is produced through the out-of-equilibrium decay of these heavy particles (either right-handed neutrinos or triplet Higgs). However, this is not true if the production and decay channels of these heavy particles in a thermal bath are different.

The large masses of these heavy particles satisfy a large range of low energy neutrino oscillation data as we saw in Figs. 5–7. In particular, we found that in case $M_1 < M_{\Delta_L}$ (1) the dominating ϵ_1 favors $M_1 > 4.2 \times 10^{12}$ GeV for all $m_1 < 10^{-3}$ eV, (2) the dominating $\epsilon_1^{\rm II}$, on the other hand, favors 4.2×10^8 GeV $\leq M_1 < 4.2 \times 10^{12}$ GeV for all $m_1 < 10^{-3}$ eV. In the case $M_{\Delta_L} < M_1$ we found that $m_1 < 10^{-4}$ eV are the only allowed values to give rise to a successful leptogenesis.

Despite the success, the out-of-equilibrium decay production of L-asymmetry suffers a serious problem as far as the collider energy concern. Therefore, we considered an alternative mechanism of producing L-asymmetry by considering the extra source of CP violation in the model. In particular, the complex condensate inside left-right domain wall gives rise to CP violation. Under these circumstance the preferred scattering of ν_L over its CP-conjugate state ν_L^c produce a net L-asymmetry. The survival of this asymmetry then requires the mass scale of N_1 to be very small, say 10 TeV. This is compatible with the low energy neutrino oscillation data if the Dirac mass matrix of the neutrinos follows 2 orders of magnitude less than the charged lepton mass matrix. Moreover, the TeV scale masses of the right-handed neutrinos are explained through the breaking of $SU(2)_R$ gauge symmetry at a few TeV scale while leaving the D-parity breaking scale as high as 10^{13} GeV.

Since σ is a singlet scalar field under the gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, we conjecture that its late decay can produce a density perturbation in the early Universe. However, in this work, we have not explored the details of density perturbations due to its out-of-equilibrium decay. This is under investigation and will be reported elsewhere.

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