

Baryogenesis in $f(R)$ theories of gravity

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$f(R)$ -theories of gravity are reviewed in the context of the so called gravitational baryogenesis. The latter is a mechanism for generating the baryon asymmetry in the Universe, and relies on the coupling between the Ricci scalar curvature R and the baryon current. Gravity Lagrangians of the form $\mathcal{L}(R) \sim R^n$, where n differs from 1 (the case of the General Relativity) only for tiny deviations of a few percent, are consistent with the current bounds on the observed baryon asymmetry.

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The discovery of the accelerated expansion of the Universe [1] has motivated the developments of many models of gravity. These models are built up, typically, either in the framework of the conventional General Relativity or in the framework of its possible generalizations or modifications. In the last years, among the different approaches proposed to generalize Einstein's General Relativity, the so called $f(R)$ -theories of gravity received a growing attention. The reason relies on the fact that they allow to explain, via gravitational dynamics, the observed accelerating phase of the Universe, without invoking exotic matter as sources of dark matter. The gravity Lagrangian for these theories is a generic function of the Ricci scalar curvature R , and the corresponding action with the inclusion of matter reads

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m[g_{\mu\nu}, \psi_m] \quad (1)$$

where $\kappa^2 = 8\pi G$. It is a difficult task to deal with higher order terms in the scalar curvature, thus the forms of $f(R)$ that have been most widely studied in literature are $f(R) \sim R + \epsilon R^m$ and $f(R) \sim R^n$ [2,3].

The aim of this paper is to show that $f(R)$ -theories of gravity provide a framework in which the *gravitational* baryogenesis may occur, and yields the observed baryon asymmetry in the Universe. The origin of the baryon number asymmetry is still an open problem of the particle physics and cosmology. Big-Bang Nucleosynthesis (BBN) [4] and measurements of CMB combined with the large structure of the Universe [5] indicate that matter in the Universe is dominant over antimatter; the order of magnitude of such an asymmetry is

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{s} \lesssim 9 \cdot 10^{-11},$$

where n_B ($n_{\bar{B}}$) is the baryon (antibaryon) number density, and s the entropy of the Universe. As argued by Sakharov, the baryon asymmetry may be (dynamically) generated if the following conditions are satisfied [6]: (i) processes that violate baryon number; (ii) C and CP violation; (iii) out of the equilibrium. The last point is important because to have different numbers of baryons and antibaryons is different,

the reaction should freeze before particles and antiparticles achieve the thermodynamical equilibrium. However, as shown in [7], a dynamical violation of CPT may give rise to the baryon number asymmetry also in a regime of thermal equilibrium.

Recently, within Supergravity theories [8], Davoudiasl *et al.* [9] have proposed a mechanism for generating the baryon number asymmetry during the expansion of the Universe by means of a dynamical breaking of CPT (and CP). In this approach the thermal equilibrium is preserved. The interaction responsible for CPT violation is given by a coupling between the derivative of the Ricci scalar curvature R and the baryon current J^μ [10]

$$\frac{1}{M_*^2} \int d^4x \sqrt{-g} J^\mu \partial_\mu R, \quad (3)$$

where M_* is the cutoff scale characterizing the effective theory. For different approaches also based on gravitational baryogenesis, see [11].

If there exist interactions that violate the baryon number B in thermal equilibrium, then a net baryon asymmetry can be generated and gets frozen-in below the decoupling temperature T_D . From (3) it follows $M_*^{-2}(\partial_\mu R)J^\mu = M_*^{-2}\dot{R}(n_B - n_{\bar{B}})$. Therefore the effective chemical potential for baryons and antibaryons is $\mu_B = \dot{R}/M_*^2 = -\mu_{\bar{B}}$, and the net baryon number density at the equilibrium turns out to be (as $T \gg m_B$, where m_B is the baryon mass) $n_B = g_b \mu_B T^2/6$. $g_b \sim \mathcal{O}(1)$ is the number of intrinsic degrees of freedom of baryons. The baryon number to entropy ratio is [9]

$$\frac{n_B}{s} \simeq -\frac{15g_b}{4\pi^2 g_*} \frac{\dot{R}}{M_*^2 T} \Big|_{T_D}, \quad (4)$$

where $s = 2\pi^2 g_{*s} T^3/45$, and g_{*s} counts the total degrees of freedom for particles that contribute to the entropy of the Universe. g_{*s} takes values very close to the total degrees of freedom of effective massless particles g_* , i.e. $g_{*s} \simeq g_* \sim 106$. n_B/s is different from zero provided that the time derivative of the Ricci scalar is nonvanishing. In the context of General Relativity, the Ricci scalar and the trace T_g of the energy-momentum tensor of matter ($T_g^{\mu\nu}$) are related

as follows

$$R = -8\pi GT_g = -8\pi G(1 - 3w)\rho,$$

where ρ is the matter density, p the pressure, $w = p/\rho$, and $T_g = T_{g\mu}^\mu$. \dot{R} is zero in the radiation dominated epoch of the standard Friedman-Robertson-Walker (FRW) cosmology, characterized (in the limit of exact conformal invariance) by $w = 1/3$. This way no net baryon number asymmetry can be generated. However, as we shall see, a net baryon asymmetry may be generated during the radiation dominated era in $f(R)$ -theories of gravity (see [9,12] for other scenarios)

The variation of the action (1) with respect to the metric yields the field equations

$$f'R_{\mu\nu} - \frac{f}{2}g_{\mu\nu} - \nabla_\mu \nabla_\nu f' + g_{\mu\nu} \square f' = \kappa^2 T_{g\mu\nu}, \quad (5)$$

where the prime stands for derivative with respect to R . The trace reads

$$3\square f' + f'R - 2f = \kappa^2 T_g. \quad (6)$$

In the spatially flat FRW's metric

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2], \quad (7)$$

Equations (5) and (6) become

$$-3\frac{\ddot{a}}{a}f' - \frac{f}{2} + 3\frac{\dot{a}}{a}f''\dot{R} = \kappa^2 \rho, \quad (8)$$

$$\left(\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2}\right)f' + \frac{f}{2} - 2\frac{\dot{a}}{a}f''\dot{R} - f''\dot{R}^2 - f''\ddot{R} = \kappa^2 p, \quad (9)$$

$$3f''\dot{R}^2 + 3f''\ddot{R} + 9\frac{\dot{a}}{a}f''\dot{R} + f'R - 2f = \kappa^2 T_g \quad (10)$$

where the dot denotes the derivative with respect to the cosmic time, $\rho = T_{g0}^0$, $p\delta_j^i = T_{gj}^i$, and T_g is the trace of the energy-momentum tensor, $T_g = \rho - 3p$. Moreover, the Bianchi identities give a further condition on the conservation of the energy

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0. \quad (11)$$

To derive a solution to Eqs. (8)–(11), an explicit form for $f(R)$ has to be specified. Following the models proposed in literature, we make the ansatz

$$f(R) = \left(\frac{R}{A}\right)^n, \quad (12)$$

where A is a constant with dimensions $m_p^{2-2/n}$ ($m_p \sim 1.2210^{19}$ GeV is the Planck mass). By using Eq. (12) and the ansatz $a(t) \sim t^\alpha$, Eqs. (8), (9), and (11) imply

$$n = 2\alpha. \quad (13)$$

One can easily check that the trace Eq. (10) is fulfilled.

The interaction (3) generates a net baryon asymmetry provided $\dot{R} \neq 0$. As already noted, in standard cosmology \dot{R} vanishes during the radiation era, deviations from the standard General Relativity prevent the Ricci curvature, as well as its first time derivative, to vanish. Directly from the definition of Ricci scalar curvature it follows $\dot{R} = -\frac{12\alpha(1-2\alpha)}{t^3}$. The net baryon asymmetry (4) turns out to be

$$\frac{n_B}{s} \simeq \frac{g_b}{g_*} \frac{45}{\pi^2} \frac{\alpha(1-2\alpha)}{t_D^3 T_D M_*}, \quad (15)$$

where t_D is the decoupling time. Equating the expression of ρ given by Eq. (8) to the usual expression of the energy density [13]

$$\rho = \frac{\pi^2}{30} g_* T^4, \quad (16)$$

one obtains

$$T = \left(\frac{15}{4\pi^3 g_*}\right)^{1/4} g_\alpha^{1/4} \frac{m_p^{1/2}}{t^\alpha A^{\alpha/2}}, \quad (17)$$

where

$$g_\alpha \equiv 6^{2\alpha} \alpha^{2\alpha} \frac{-10\alpha^2 + 8\alpha - 1}{2(1-2\alpha)^{1-2\alpha}}.$$

Figure 1 shows g_α vs α . The allowed range for the parameter α is $0.155 \leq \alpha \leq 1/2$ (the value $\alpha = 1/2$ corresponds to Einstein's theory of gravity). Nevertheless, as we shall discuss below, BBN imposes stringent restrictions on its possible values

Inserting (17) into (15), one has

$$\frac{n_B}{s} \leq 5.810^{-3} H_\alpha \left(\frac{A}{m_p^{2-(1/\alpha)}}\right)^{3/2} \left(\frac{T_D}{m_{\text{Pl}}}\right)^{(3/\alpha)-1} \left(\frac{m_{\text{Pl}}}{M_*}\right)^2 \quad (18)$$

where

$$H_\alpha \equiv \frac{1}{\sqrt{\alpha|1-2\alpha|}} \left[876, 5 \frac{2(1-2\alpha)}{-10\alpha^2 + 8\alpha - 1} \right]^{3/4\alpha}.$$

As pointed out in [9], a possible choice of the cutoff scale M_* is $M_* = m_{\text{Pl}}/\sqrt{8\pi}$ if $T_D = M_I$, where $M_I \sim 210^{16}$ GeV is the upper bound on the tensor mode fluctuation constraints in inflationary scale [14]. This choice is particularly interesting because implies that tensor mode fluctuations should be observed in the next generation of experiments. For $A \sim m_p^{2-1/\alpha}$, using the constraint on the observed baryon asymmetry $n_B/s \leq 9 \cdot 10^{-11}$, Eq. (18) can be recast in the form

$$\Lambda_\alpha \leq 1.6, \quad \text{where } \Lambda_\alpha \equiv 10^{12} H_\alpha (2.75 \cdot 10^{-3})^{3/\alpha}. \quad (19)$$

Equation (19) is the main result of our paper: It relates the estimation of the baryon number asymmetry inferred from observations to the parameter α , hence to the exponent of the Ricci scalar curvature entering in the generic

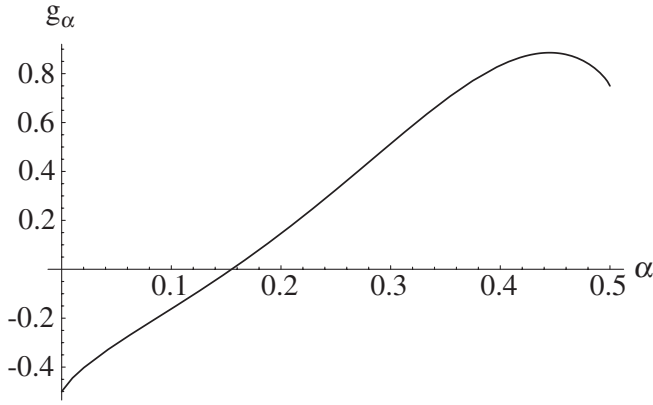
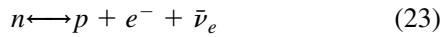
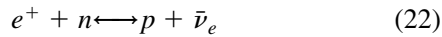


FIG. 1. Plot of the function g_α vs α . The range in which the function g_α is positive, hence T is defined, is $0.155 \leq \alpha \leq 1/2$.

function $f(R)$. This allows to fix the form of the $f(R)$ -Lagrangian.

Let us now discuss $f(R)$ -theories of gravity in the context of BBN. According to BBN, the formation of light elements in the early time occurred when the temperature of the Universe was $T \leq 100$ MeV and the energy and the number density were dominated by relativistic particles. At this stage of the evolution of the Universe, the smattering of neutrons and protons does not contribute in a relevant way to the total energy density. All these particles are in thermal equilibrium owing to their rapid collisions. Besides, protons and neutrons are kept in thermal equilibrium by their interactions with leptons



The weak interaction rate $\Lambda(T)$ is determined by means of the conversion rates of protons into neutrons, $\lambda_{pn}(T)$, and the inverse ones $\lambda_{np}(T)$. The rate $\lambda_{np}(T)$ is expressed in terms of the sum of rates associated to the individual processes (21)–(23) [13,15]

$$\lambda_{np} = \lambda_{n+\nu_e \rightarrow p+e^-} + \lambda_{n+e^+ \rightarrow p+\bar{\nu}_e} + \lambda_{n \rightarrow p+e^-+\bar{\nu}_e}. \quad (24)$$

Similarly for $\lambda_{pn}(T)$. At enough high temperatures ($T \gg Q$ and $T \gg m_e$, where $Q = m_n - m_p \simeq 1.293$ MeV, and $m_{n,p,e}$ stands for the mass of the neutron, proton and electron, respectively) the weak interaction rate is given by [13,15]

$$\Lambda(T) = \lambda_{np}(T) + \lambda_{pn}(T) \simeq \frac{7\pi(1+3g_A^2)}{60} G_F^2 T^5, \quad (25)$$

where $G_F \simeq 1.166 \cdot 10^{-5} \text{ GeV}^{-2}$ is the Fermi coupling constant and $g_A \simeq 1.26$ is the axial-vector coupling constant of the nucleon. To compute the freeze-out temperature T_f (the temperature at which baryons decouple from leptons), one

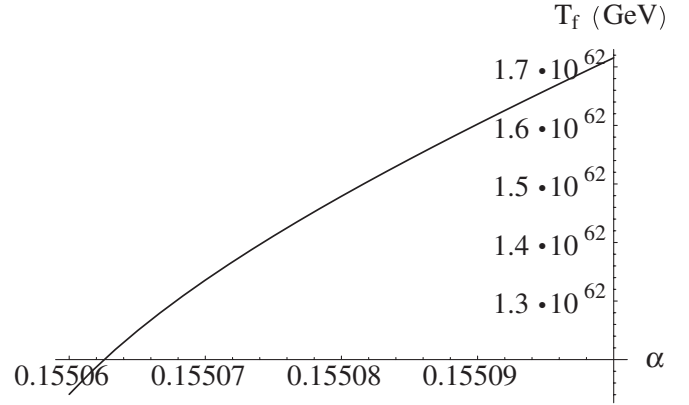


FIG. 2. T_f vs α . In this plot the parameter α assumes values ≥ 0.15506 .

has to equate $\Lambda(T)$ given by (25) to the expansion rate of the Universe $H = \dot{a}/a$: $\Lambda(T) \simeq H$. Using (17) with g_* replaced by $g_*^{(\text{BBN})} = 10.75$, it follows

$$T_f = \left[6.410^{28-(19/\alpha)} \frac{\alpha}{g_\alpha^{1/4}} \left(\frac{4\pi^3 g_*^{(\text{BBN})}}{15} \right)^{1/4\alpha} \right]^{\alpha/(5\alpha-1)} \text{ GeV}. \quad (26)$$

Coming back to Eqs. (19), the analysis of the gravitational baryogenesis indicates that the value $\alpha \simeq 0.15506$ is compatible with the estimation (19). Nevertheless, such a result is inconsistent with BBN. As matter of fact, the plot of T_f vs α in Fig. 2 shows that values of α close to 0.155 have to be discarded because they lead to temperatures incompatible with the typical temperatures of BBN ((0.1–100) MeV). This is not the case for those values of α close to 0.5, as arises from Fig. 3. Thus we shall be interested only in the gravitational baryogenesis for $\alpha \leq 0.5$. The results are reported in Fig. 4. As we can see, a narrow region for the parameter α , $0.46 \leq \alpha \leq 0.49$, is consistent with the order of magnitude given in (19). In

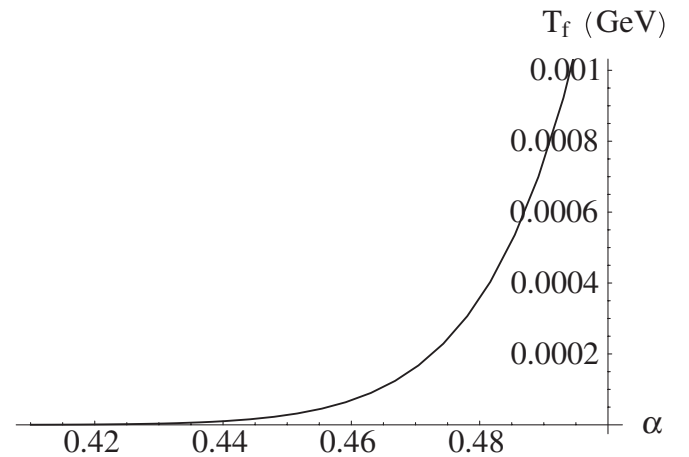


FIG. 3. T_f vs α . In this plot the parameter α assumes values ≤ 0.5 .

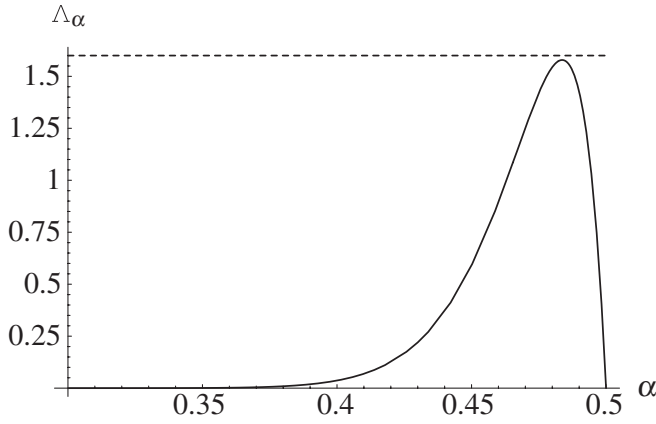


FIG. 4. Plot of Λ_α vs α . The dashed line represents the upper bound 1.6. The maximum value ~ 1.59 , corresponding to $\alpha \approx 0.4845$, agrees with the estimate (19). Moreover, in the limit $\alpha \rightarrow 1/2$, Λ_α vanishes, so no net baryon asymmetry is generated (in agreement with General Relativity).

particular, the maximum value of the function Λ_α , $\Lambda_\alpha^{(\max)} \sim 1.59$, is in excellent agreement with the estimate (19). $\Lambda_\alpha^{(\max)}$ corresponds to $\alpha \approx 0.4845$ so that $n \approx 0.97$. The gravity Lagrangian therefore reads $\mathcal{L} \sim R^{0.97}$, i.e. the

form of the function $f(R)$ is *nearly* linear in R . We also note that in the limit $\alpha \rightarrow 1/2$, the function Λ_α (hence H_α) vanishes and no net baryon asymmetry is generated. This is as expected because $\alpha = 1/2$ implies that $a(t)$ evolves as $t^{1/2}$, i.e. the conformal symmetry is restored, and therefore the Universe expands according to General Relativity field equations.

In conclusion, $f(R)$ -theories of gravity provide a natural arena in which the baryon asymmetry in the Universe may be generated through the mechanism of the gravitational baryogenesis. As we have shown, deviations from General Relativity, although very tiny (a few percent), allow to account for the observational data on the matter-antimatter asymmetry. These results rely on the exact solution of $f(R)$ -field equations (Eqs. (8) and (9)), which allows to determine (without a fine-tuning) the exponent n for the generic function $f(R) \sim R^n$. The results are also consistent with BBN.

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