CFT/AdS correspondence, massive gravitons, and a connectivity index conjecture

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We discuss the general question of which conformal field theories have dual descriptions in terms of quantum gravity theories on anti-de Sitter space. We analyze in detail the case of a deformed product of n conformal field theories (each of which has a gravity dual), and we claim that the dual description of this is by a quantum gravity theory on a union of n anti-de Sitter spaces, connected at their boundary (by correlations between their boundary conditions). On this union of spaces, $(n - 1)$ linear combinations of gravitons obtain a mass, and we compute this mass both from the field theory and from the gravity sides of the correspondence, finding the same result in both computations. This is the first example in which a graviton mass in the bulk of anti-de Sitter space arises continuously by varying parameters. The analysis of these deformed product theories leads us to suggest that field theories may be generally classified by a "connectivity index," corresponding to the number of components (connected at the boundary) in the space-time of the dual gravitational background. In the field theory this index roughly counts the number of independent gauge groups, but we do not have a precise general formula for the index.

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I. INTRODUCTION

The AdS/CFT correspondence [1] implies that any theory of quantum gravity (such as string theory) on anti-de Sitter (AdS) space is dual to a conformal field theory (CFT). More generally, any theory of quantum gravity on a space which has an asymptotic boundary where it approaches a (possibly warped) product of anti-de Sitter space with another space is equivalent to a field theory which is conformal at high energies. This can be seen simply by computing the correlation functions of local operators in such a theory using the methods of [2,3] and noticing that they obey the usual requirements for correlation functions in a field theory. Of course, in general it is not known how to write down a Lagrangian formulation of the dual field theory (or of a theory that flows to it), and it is not even clear that such a formulation should exist. But still, the dual field theory can be implicitly (and presumably uniquely) defined through its correlation functions, which we can compute if we understand the corresponding theory of quantum gravity.

It is natural to ask whether this duality can be used also in the opposite direction—namely, whether any field theory is dual to a theory of quantum gravity (on some asymptotically-AdS space, since standard local field theories are conformal at high energies).[1] At first sight one might think that the answer must be positive, since otherwise there would be a strange division of the space of field theories into two sets—the ones which have a quantum gravity dual and the ones which do not. However, we would like to argue that the answer is negative, and that the space of quantum field theories is indeed divided into classes, such that only one class of quantum field theories is dual to quantum gravity on (asymptotically) AdS space.

Our argument will be based on considering products of n conformal field theories, each of which is dual to some quantum gravity theory on AdS space, and deforming them by products of operators from the different CFTs in a way which couples them together.[2] We will claim that the dual of such a deformed product theory is not given by a quantum gravity theory on AdS space, but rather on a union of n AdS spaces, whose boundaries are all identified together (in the sense that the boundary conditions on the different AdS spaces are correlated to each other). We will construct this picture in the limit where we can describe the quantum gravity theory semiclassically (as a theory of weakly coupled fields living on the AdS spaces), and verify in detail that it gives a consistent description of the deformed product field theory. We conjecture that the same picture is true more generally, even beyond the semiclassical gravity approximation.

It seems reasonable to assume that if some field theory has a dual description in terms of a quantum gravity theory on a sum of n AdS spaces, it cannot also have a dual description as a theory living on a single AdS space; however, it is of course difficult to be sure of such a statement, since we do not understand quantum gravity beyond the semiclassical limit, and it is possible that the

[1]Of course, in most cases this quantum gravity theory would be highly curved, with no semiclassical approximation, for instance since the field theory would not have any separation between the dimensions of operators with spin 2 or less and the dimensions of higher-spin operators. Since in general we do not have any independent definition of quantum gravity on highly curved spaces, another way to phrase the question is whether any field theory can be used as a definition of a theory of quantum gravity on asymptotically-AdS space.

[2]The generalization to deformations of nonconformal field theories is straightforward.

same theory could be described either as living on n AdS spaces or as living on a single AdS space, despite the fact that the topologies of these two descriptions are different near the boundary. If our assumption is indeed true, it suggests that quantum field theories may be characterized by a ''connectivity index'' n, which counts the number of separate components in their quantum gravity dual description; schematically this index corresponds to the number of independent gauge groups in the theory (by independent gauge groups we mean groups such that no field is charged under more than one group, and such that every group has at least one field charged under it, so that, for example, an $SU(N) \times SU(N)$ theory with a bi-fundamental field counts as having one independent gauge group; the gauge groups can be continuous groups or discrete groups as in sigma models on orbifolds). We claim that there exist theories with $n = 0$ (no nontrivial gauge symmetry) which do not have a dual gravitational description, since the energy-momentum tensor in these theories is not an independent operator, so there is no fundamental dual bulk graviton that can be associated with a diffeomorphism symmetry. Theories with $n = 1$ include all the known theories with a gravity dual. Theories with higher values of n can be constructed by (deformations of) direct products of theories with $n = 1$, and we claim that they correspond to a theory of quantum gravity on a sum of n asymptotically-AdS spaces, which are connected at their boundary. We do not know how to define n directly in the field theory; this may be due to our lack of imagination, or it may mean that our assumption is wrong and n is not really a well-defined index beyond the semiclassical limit.

We begin in Sec. II by examining in detail a product of two conformal field theories and its deformations, and how they are described in the dual gravitational picture. The generalization to a product of more field theories and to theories with no conformal invariance is straightforward. In Sec. III we briefly discuss the case with $n = 0$, and in Sec. IV we discuss further the ''connectivity index'' and its properties. We end in Sec. V with a summary of our results and conclusions.

II. THE DUAL OF A DEFORMED PRODUCT OF FIELD THEORIES

A. General description

Let us consider two conformal field theories in d dimensions, each of which is dual to a string theory on a product of AdS_{d+1} times some space $\mathcal{M}$. We consider the case where this dual string theory has a good semiclassical gravity limit (namely all radii of curvature are very large). The dual description of the product of the two theories is obviously given by the product of the string theories on the two spaces, or equivalently by string theory on the disjoint union of the two spaces (which do not talk to each other).

On the field theory side, we can deform such a product by adding to its Lagrangian a term

$$h \int d^d x \mathcal{O}_1(x)\mathcal{O}_2(x), \tag{2.1}$$

where $\mathcal{O}_1$ is an operator in the first CFT and $\mathcal{O}_2$ an operator in the second. Such a deformation could be relevant or marginal, and in some cases it can even be exactly marginal (for instance, we can consider $J_1\bar{J}_2$ deformations when $d = 2$ and J ($\bar{J}$) is a holomorphic (antiholomorphic) global $U(1)$ current, or we can consider a product of Klebanov-Witten $d = 4$ CFTs [4] and deform them by $\int d^2\theta h^{ijkl} \mathrm{tr}(A_i^{(1)}B_j^{(1)}) \mathrm{tr}(A_k^{(2)}B_l^{(2)})$, which is exactly marginal for an appropriate choice of the coefficients h^{ijkl}). For simplicity let us consider the exactly marginal case, where we have a CFT for every value of h, which we will denote by CFT_h; our results may be easily generalized also to nonmarginal deformations.

Naively, one might think that for nonzero values of h this CFT_h (which is no longer a direct product) should be dual to some theory of quantum gravity on a single AdS space. One argument for this is that the original product theory had two separate conserved energy-momentum tensors $T_{mn}^{(1)}$ and $T_{mn}^{(2)}$, while the deformed product only has one conserved energy-momentum tensor (which in simple cases is given by, to leading order in h, $T_{mn}^{(1)} + T_{mn}^{(2)} - h\eta_{mn}\mathcal{O}_1\mathcal{O}_2$). The dual of the product theory had two massless gravitons corresponding to $T^{(1)}$ and $T^{(2)}$, while in the deformed theory one would expect to remain with one massless graviton while the other graviton would obtain a mass (related to the anomalous dimension of the nonconserved combination of $T^{(1)}$ and $T^{(2)}$; we will discuss this in more detail below). So, one might think that the dual theory lives on a single space. However, if we try to write the theory on a single space we immediately run into the problem that the two spaces $\mathcal{M}_1$ and $\mathcal{M}_2$ may be different, so there is no natural ten-dimensional space on which to define CFT_h. This problem becomes even more serious if we try to couple two nonconformal field theories which are dual to asymptotically-AdS spaces, since the two asymptotically-AdS spaces corresponding to the two field theories may be completely different in the bulk, and there is no natural way to identify them.

We would like to suggest a different interpretation. In the standard AdS/CFT correspondence, deformations of the Lagrangian are described by changing boundary conditions for the fields. ''Single-trace'' deformations (deformations by an operator which is dual to a single field in the bulk) involve changing the boundary condition on the non-normalizable mode of the corresponding field, while ''multitrace'' deformations [similar to (2.1)] involve [5,6] changes in the boundary conditions which mix the coefficients of the non-normalizable and the normalizable modes of the fields involved; for example, the deformation (2.1) in a single CFT would imply that the coefficient near the boundary of the non-normalizable mode of the field ϕ_1 dual to $\mathcal{O}_1$ must be equal to $h * (2\Delta_2 - d)$ times the coefficient of the normalizable mode of ϕ_2 (dual to $\mathcal{O}_2$ of dimension Δ_2), and vice versa.[3] Since the deformation is described purely in terms of boundary conditions, if we think of it in the two-CFT case, we do not necessarily need the two fields (corresponding to operators in the two CFTs) to live on the same space—it is enough if they share the same boundary. Note that the boundary of the gravity dual of any theory which is conformal in the UV is always the same, and looks like the $\mathbb{R}^d$ (or $S^{d-1} \times \mathbb{R}$) boundary of AdS_{d+1} (the compact space $\mathcal{M}$ shrinks to zero near the boundary in the natural field theory units), so it is always possible to identify the boundaries of the spaces dual to the two CFTs. We suggest that the proper way to think about the gravity dual of the product of two CFTs is as living on the sum of the two space-times (one for each CFT) but with the boundary identified[4]; when the product is undeformed the identification of the boundaries has no effect, but when we deform we can then implement the deformation (2.1) by an appropriate change in the boundary conditions of the fields living on the two space-times. Note that this procedure correctly implements the breaking of the symmetry from a product of two conformal groups when $h = 0$ (and the identification has no effect) to a single conformal group when h is nonzero. Note also that, as for any multitrace deformations, the changed boundary conditions are generally nonlocal on the compact spaces $\mathcal{M}$ [8,9] (and they also seem to be nonlocal on the world sheet of the corresponding string theory [8]), but this is not inconsistent with the locality and causality of CFT_h.

While this picture of two spaces connected at their boundary may seem rather arbitrary, it actually arises naturally in the AdS/CFT correspondence when we have a flow from a single CFT to a product of two CFTs. The prototypical example is the $d = 4$ $\mathcal{N} = 4$ $SU(N)$ SYM theory, at a point on its Coulomb branch at which the gauge group is broken to $SU(N_1) \times SU(N - N_1) \times U(1)$. The gravity dual of this theory is exactly known; it is asymptotically a single AdS space, but in the interior of the space there are two "throats," one for each non-Abelian gauge group factor, and the low-energy dynamics (below the mass scale of the W bosons charged under both $SU(K)$ factors) is a product of two CFTs, one in each "throat." Here the two low-energy theories are decoupled,[5] and it is not clear that the two throats share a boundary, but it seems likely that if we would deform the theory by a deformation that in the low-energy CFT would take the form (2.1), it would look like a shared-boundary interaction of the form described above.

In the special case where we are deforming a product of CFTs by an exactly marginal deformation, so that symmetry considerations imply that the two spaces remain AdS spaces also after the deformation, we could identify the two AdS spaces if we want, and obtain two decoupled theories on a single AdS space which only talk to each other through the boundary conditions related to (2.1) (if the compact spaces $\mathcal{M}_1$ and $\mathcal{M}_2$ are the same we could even identify the full ten-dimensional spaces if we want). This "folded" picture is, of course, completely equivalent to the "unfolded" picture described above involving two spaces connected at the boundary, except that in the folded picture we do not explicitly exhibit the two diffeomorphism symmetries associated with the two spaces, but rather we work in a gauge where the spaces are identified. The folded picture may be useful for performing computations (which can sometimes be reduced to computations which have already been done for theories living on a single AdS space), but we stress that it is only available in very special cases for which the spaces on the two sides are the same, and there does not seem to be an analogous picture for a coupling between two nonconformal theories which live on different spaces. Thus, we view the unfolded picture involving a union of two spaces as the more basic one.

The terminology which we were using for the gravitational description of the deformation above depended on having a semiclassical limit of the gravity theory, in which there were well-defined fields to which one could assign boundary conditions. In the absence of such a semiclassical limit it is not clear how to phrase deformations in terms of boundary conditions; for example, in a generic field theory there is no distinction between "single-trace" and "multi-trace" operators (and, indeed, even when there is a semiclassical large N limit, the two types of operators mix together at finite N). However, just as we believe that the AdS/CFT correspondence makes sense even for finite N (say, for the $SU(2)$ $\mathcal{N} = 4$ super-Yang-Mills theory, which is very far from the semiclassical large N limit), we conjecture that the general claims above are true even when there is no semiclassical limit. So, we conjecture in general that a field theory which is a deformed product of n independent CFTs is dual to a theory of quantum gravity on a space with n asymptotically-AdS regions connected at their boundary (the generalization of the discussion above to products of n CFTs rather than two is straightforward).

We begin in Sec. II B by reviewing some facts about massive gravity on AdS space, and how it can arise from a deformed product of CFTs as described above. The rest of the section is devoted to a detailed computation of the mass of the massive graviton resulting from the deformation (2.1), both on the field theory side and on the gravity side, in order to test our gravitational description of this deformation.

[3] This picture makes sense when we have a semiclassical limit of the gravity theory in which the bulk fields are weakly coupled and we know what we mean by single-particle and multiparticle states and by boundary conditions on fields.

[4] Note that this is completely different from the case of a single space-time with more than one boundary, which is relevant (for instance) for the eternal AdS black hole [7].

[5] Up to various irrelevant couplings, as discussed for instance in [10,11].

B. Massive gravity in anti-de Sitter space

It is well known that on anti-de Sitter space, unlike in flat space, there is no discontinuity separating massive spin 2 fields from massless spin 2 fields [12,13]. Thus, it is possible for the graviton to continuously acquire a mass, in a way which is completely analogous to the Higgs mechanism, see [14] for a review.

In the usual Higgs mechanism in anti-de Sitter space, a massless vector field and a massless scalar field join together to form a massive vector field. This has a simple interpretation in the dual field theory. Before Higgsing, the massless vector field maps to a conserved current J_μ of dimension $(d-1)$, $\partial^\mu J_\mu = 0$, while the massless scalar field maps to an operator $\mathcal{O}$ of dimension d. The Higgs mechanism in the bulk is reflected on the boundary by the current no longer being conserved, $\partial^\mu J_\mu \propto \mathcal{O}$, explicitly showing that the vector operator and the scalar operator join together into a single multiplet of the conformal algebra.

Similarly, a massless graviton in the bulk is dual to a conserved spin 2 operator of dimension d, $\partial^\mu T_{\mu\nu} = 0$. When the diffeomorphism symmetry generated by this spin 2 operator is broken, we have a relation of the form $\partial^\mu T_{\mu\nu} \propto K_\nu$ for some vector operator K_ν which joins together with the spin 2 operator $T_{\mu\nu}$ in a single multiplet of the conformal algebra. In the bulk this means that the vector field dual to K_ν joins together with the graviton dual to $T_{\mu\nu}$ to form a single massive spin 2 field (of course, as in the previous case, once $T_{\mu\nu}$ is not conserved, the conformal algebra implies that it must acquire an anomalous dimension which is mapped to the bulk mass). An interesting difference from the previous case is that the vector field which is swallowed is not massless. In the previous paragraph the operator $\mathcal{O}$ had to have dimension d before the Higgsing for a relation of the form $\partial^\mu J_\mu \propto \mathcal{O}$ to make sense; similarly, the relation $\partial^\mu T_{\mu\nu} \propto K_\nu$ implies that K_ν must have (before the deformation) dimension $(d+1)$, corresponding to a vector field of mass $m^2 R_{\rm AdS}^2 = 2d$ [recall that a massless vector field in AdS maps to a vector operator of dimension $(d-1)$]. Nevertheless, the conformal algebra joins together this massive vector representation with the massless spin 2 representation to form the massive spin 2 representation.[6]

Of course, it is not easy to realize this "Higgs mechanism for gravity," since this requires a breaking of the diffeomorphism symmetry associated to the conserved spin 2 operator $T_{\mu\nu}$. Up to now, the only known realizations of this phenomenon [15–17] were in the context of "locally localized gravity" [18], where an AdS_d-brane is embedded into AdS_{d+1} and there is (in some approximation) a massless bound state of the graviton living on the brane. In the dual description, there is a $(d-1)$-dimensional defect inside the d-dimensional field theory, and the energy-momentum tensor of the defect is mapped to the bound graviton. Generally, this energy-momentum tensor is not exactly conserved since there is an exchange of energy between the defect and the full CFT, and this corresponds to the localized massless graviton acquiring a mass (related to the anomalous dimension of the defect energy-momentum tensor) [19].

If we want to realize this "Higgs mechanism" in the full theory on AdS_{d+1} without breaking the conformal symmetry, it seems like we need to find a conformal theory in which the energy-momentum tensor is not conserved, but this is not possible of course. Instead, we can start, as described in the previous subsection, from a theory that has more than one conserved energy-momentum tensor, namely, a product of two conformal field theories, and then one of the symmetries can be broken by coupling together the two conformal field theories by an exactly marginal deformation without breaking the overall conformal invariance. From the point of view of the conformal field theory this precisely realizes the "Higgs mechanism for gravity" described above. When we couple two CFTs together, each energy-momentum tensor separately is no longer conserved, but there is still a conserved energy-momentum tensor which is the sum of the two original energy-momentum tensors plus a contribution from the deformation (2.1),

$$T_{\mu\nu}^{\rm tot} = T_{\mu\nu}^{(1)} + T_{\mu\nu}^{(2)} - h\eta_{\mu\nu}\mathcal{O}_1\mathcal{O}_2 \tag{2.2}$$

(assuming that the scalar operators contain no derivatives). On the other hand, the difference between the two energy-momentum tensors acquires an anomalous dimension since it is no longer conserved: schematically we have (this will be made more precise below)

$$\partial^\mu (T_{\mu\nu}^{(1)} - T_{\mu\nu}^{(2)}) \sim h[(\partial_\nu\mathcal{O}_1)\mathcal{O}_2 - \mathcal{O}_1(\partial_\nu\mathcal{O}_2)]. \tag{2.3}$$

The operator on the right-hand side of (2.3) is the dimension $d+1$ vector operator which is swallowed by the dimension d spin 2 operator when it acquires an anomalous dimension; note that in this case it is a multiple-trace operator.

The naive mapping of the CFT results above to anti-de Sitter space would imply that the product CFT maps to a bulk theory with two massless gravitons, and then when we deform one of the gravitons acquires a mass by swallowing a massive vector field, which in this case is a bound state of two scalar fields [corresponding to the multitrace operator on the right-hand side of (2.3)]. This is indeed what we would find in the folded interpretation of the deformed product CFT. In the unfolded interpretation the analysis of the fields is somewhat more complicated, since each of the gravitons (the massless one and the massive one) is a linear

[6] If we start from a supersymmetric theory, the gravitino(s) would also continuously acquire a mass by swallowing massive spin 1/2 fields, but this is similar to what happens in the standard case of deformations which break supersymmetry.

combination of the gravitons on the two AdS spaces. However, since there is still a single conformal symmetry, the analysis in terms of representations of the conformal algebra is the same.

When we have a large N limit corresponding to a semiclassical theory of gravity in the bulk, the graviton mass described above arises through loop diagrams which are suppressed (at least) by $1/N^2$ (or by Newton's constant G_N). From the bulk point of view this occurs because the deformation (2.1) does not directly affect the graviton, but just modifies the boundary conditions of the scalar fields which are dual to the operators $\mathcal{O}_i$. The leading correction to the graviton propagator thus comes from a one-loop diagram with the scalar fields running in the loop. On the field theory side one can easily find the same result by noting that the first correction to two-point functions of the stress-energy tensors $T^{(1)}$ and $T^{(2)}$ arises at second order in h, and is suppressed by $1/N^2$ compared to the original two-point functions.

In the rest of this section, which is rather technical and can be skipped by readers who are not interested in the details, we will compute the graviton mass in the product of CFTs deformed by (2.1) at leading order in h, in the case of a marginal deformation ($\Delta_1 + \Delta_2 = d$). We will first compute this on the field theory side and then on the gravity side, and we will find precise agreement between the two computations.

C. Field theory computation of graviton mass

One method to compute the correction to the graviton mass [at leading order in the deformation (2.1)], which is simply related to the anomalous dimension of the nonconserved stress-energy tensor, is by using Eq. (2.3) which relates its derivative to a specific operator in the undeformed CFT (at leading order in the deformation). First, we should be more precise about which operator in the CFT is the nonconserved stress-energy tensor.

Naively the nonconserved operator which obtains a mass should be $T^{(1)} - T^{(2)}$, but in fact this operator is not orthogonal (in the sense of having a vanishing two-point function) to (2.2) and to the operator $\mathcal{O}_1\mathcal{O}_2$. After the deformation (2.1), assuming that the deformation operators $\mathcal{O}_1$ and $\mathcal{O}_2$ contain no derivatives, it is easy to check (by applying separate translations of the fields of the two CFTs) that (at leading order in h)

$$\partial^\mu T^{(1)}_{\mu\nu} = h(\partial_\nu \mathcal{O}_1)\mathcal{O}_2; \qquad \partial^\mu T^{(2)}_{\mu\nu} = h\mathcal{O}_1(\partial_\nu \mathcal{O}_2), \quad (2.4)$$

such that the full deformed stress-energy tensor (2.2) is conserved. We will work in the framework of radial quantization, where each local operator is mapped to a state. In this framework the two-point functions of operators map to numbers giving the overlaps of states, proportional to the two-point function after taking out the position dependence. For scalar operators of dimension Δ_i, with $\langle \mathcal{O}_i(0)\mathcal{O}_i(x)\rangle = N_i/|x|^{2\Delta_i}$, we simply have $\langle \mathcal{O}_i|\mathcal{O}_i\rangle = N_i$ for some arbitrary normalization factor N_i. Similarly, for traceless stress-energy tensors in a conformal field theory we have

$$\langle T_{\mu\nu}|T_{\rho\sigma}\rangle = c\left(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{d}\eta_{\mu\nu}\eta_{\rho\sigma}\right), \quad (2.5)$$

where c is (one definition of) the central charge of the conformal theory. We will denote the central charges of the two CFTs (using this definition) by c_1 and c_2.

The nonconserved operator must be some linear combination

$$\tilde{T}_{\mu\nu} = \alpha T^{(1)}_{\mu\nu} + \beta T^{(2)}_{\mu\nu} + \gamma\eta_{\mu\nu}h\mathcal{O}_1\mathcal{O}_2. \quad (2.6)$$

Requiring orthogonality to (2.2) implies that at leading order in h, $\alpha c_1 + \beta c_2 = 0$. In order to check orthogonality with $\mathcal{O}_1\mathcal{O}_2$ we need to compute the leading correction to $\langle T^{(1)}|\mathcal{O}_1\mathcal{O}_2\rangle$. Recall that the general prescription for computing corrections to correlation functions due to the deformation (2.1) in conformal perturbation theory is

$$\langle \text{correlator}\rangle_{\text{full}} = \langle \text{correlator}\rangle_{\text{undeformed}} + h\int d^d u\langle \text{correlator}\cdot\mathcal{O}_1(u)\mathcal{O}_2(u)\rangle_{\text{undeformed}} + \frac{h^2}{2}\int d^d u\int d^d v\langle \text{correlator}\cdot\mathcal{O}_1(u)\mathcal{O}_2(u)\mathcal{O}_1(v)\mathcal{O}_2(v)\rangle_{\text{undeformed}} + \cdots. \quad (2.7)$$

Thus, we find at leading order

$$\langle T^{(1)}_{\mu\nu}(x_1)\mathcal{O}_1(x_2)\mathcal{O}_2(x_3)\rangle = h\int d^d x\langle T^{(1)}_{\mu\nu}(x_1)\mathcal{O}_1(x_2)\mathcal{O}_1(x)\rangle \times\langle \mathcal{O}_2(x_3)\mathcal{O}_2(x)\rangle. \quad (2.8)$$

Using the formulas for $\langle T(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\rangle$ from [20] (which we quote below) we find that this gives an overlap $\langle T^{(i)}_{\mu\nu}|\mathcal{O}_1\mathcal{O}_2\rangle = h\frac{\Delta_i}{d}\eta_{\mu\nu}N_1N_2$, which is consistent with the orthogonality of (2.2) with $\mathcal{O}_1\mathcal{O}_2$. For the orthogonality of (2.6) with this operator we now require $\alpha\Delta_1 + \beta\Delta_2 + \gamma d = 0$, so that an appropriate choice of the nonconserved tensor $\tilde{T}$ (with an arbitrary normalization) is given by

$$\tilde{T}_{\mu\nu} = c_2T^{(1)}_{\mu\nu} - c_1T^{(2)}_{\mu\nu} + \left(c_1\frac{\Delta_2}{d} - c_2\frac{\Delta_1}{d}\right)\eta_{\mu\nu}h\mathcal{O}_1\mathcal{O}_2. \quad (2.9)$$

At leading order in the deformation, this operator satisfies

$$\partial^\mu \tilde{T}_{\mu\nu} = h(c_1 + c_2)\left(\frac{\Delta_2}{d}(\partial_\nu \mathcal{O}_1)\mathcal{O}_2 - \frac{\Delta_1}{d}\mathcal{O}_1(\partial_\nu \mathcal{O}_2)\right). \tag{2.10}$$

As a consistency check, note that if one of the operators (say $\mathcal{O}_1$) is the identity operator with $\Delta_1 = 0$, so that the two theories are not really coupled together by the deformation, we find that $\tilde{T}$ is still conserved as expected.

Next, we need to compute the relation between the norm of (2.10) and the anomalous dimension of $\tilde{T}$. Recall that using the conformal algebra, a standard manipulation gives for a scalar operator $\mathcal{O}$ of dimension Δ (with no summation over μ)

$$|\partial_\mu \mathcal{O}|^2 = |P_\mu \mathcal{O}|^2 = \langle \mathcal{O}|K^\mu P_\mu|\mathcal{O}\rangle = \langle \mathcal{O}|2i\delta^\mu_\mu D|\mathcal{O}\rangle = 2\Delta\langle \mathcal{O}|\mathcal{O}\rangle, \tag{2.11}$$

leading to the standard unitarity bound $\Delta \geq 0$. The same manipulation for the derivative of a traceless spin 2 operator gives (with no sum over ν)

$$|\partial^\mu \tilde{T}_{\mu\nu}|^2 = 2c(\Delta_{\tilde{T}} - d)\frac{(d+2)(d-1)}{d}, \tag{2.12}$$

where c is the constant appearing in (2.5) for the 2-point function of $\tilde{T}$ with itself, which for $\tilde{T}$ defined above is equal to $c = c_1c_2(c_1 + c_2)$. This equation is consistent with the known unitarity condition saying that $\Delta_{\tilde{T}} \geq d$, with equality if and only if $\tilde{T}$ is conserved.

Now, we can compare Eq. (2.12) to the norm of the operator on the right-hand side of (2.10). We find

$$2c(\Delta_{\tilde{T}} - d)\frac{(d+2)(d-1)}{d} = 2h^2(c_1 + c_2)^2 N_1 N_2 \frac{1}{d^2} \times (\Delta_2^2\Delta_1 + \Delta_1^2\Delta_2), \tag{2.13}$$

leading to a mass squared of the graviton given by (at leading order in h, and in units of the AdS radius)

$$M^2_{\mathrm{grav}} = d(\Delta_{\tilde{T}} - d) = h^2 N_1 N_2\left(\frac{1}{c_1} + \frac{1}{c_2}\right)\frac{\Delta_1\Delta_2 d}{(d+2)(d-1)}. \tag{2.14}$$

Note that despite the appearance of N_1 and N_2 this is independent of how we normalize the operators, since h also changes when we change the normalization of the two-scalar operators. Note also that this scales as the inverse central charges of the CFTs, namely, as $1/N^2$ in the large N limit of an $SU(N)$ gauge theory, as expected since it is related to a one-loop diagram in the bulk.

An alternative way to compute the correction to the graviton mass is by a direct computation of the logarithmic corrections to $\langle \tilde{T}(x)\tilde{T}(y)\rangle$, using the techniques of conformal perturbation theory. We will not do this computation in complete generality, but we will show that for the case of equal central charges of the two CFTs it gives results which are consistent with the previous computation.

The two-point function of $\tilde{T}$ contains various terms. One term involves $\langle T^{(1)}T^{(2)}\rangle$; the leading order correction to this two-point function is given by

$$\langle T^{(1)}_{\mu\nu}(x)T^{(2)}_{\sigma\tau}(y)\rangle_{\mathrm{full}} = \frac{h^2}{2}\int d^d u d^d v \langle T^{(1)}_{\mu\nu}(x)T^{(2)}_{\sigma\tau}(y)\mathcal{O}_1(u) \times \mathcal{O}_2(u)\mathcal{O}_1(v)\mathcal{O}_2(v)\rangle_{\mathrm{undeformed}} + \cdots \tag{2.15}$$

Since the two CFTs only interact via the deformation (2.1) this correlator factorizes into

$$\int d^d u \int d^d v \langle T^{(1)}_{\mu\nu}(x)\mathcal{O}_1(u)\mathcal{O}_1(v)\rangle^{(1)} \times \langle T^{(2)}_{\sigma\tau}(y)\mathcal{O}_2(u)\mathcal{O}_2(v)\rangle^{(2)}, \tag{2.16}$$

where these correlation functions are computed in the respective undeformed theories. The general form of these correlation functions is determined by conformal invariance and is given in [20]:

$$\langle T^{(i)}_{\mu\nu}(x)\mathcal{O}_i(u)\mathcal{O}_i(v)\rangle^{(i)} = \frac{A_i}{(x-u)^d(u-v)^{2\Delta_i - d + 2}(x-v)^d} \times \left((u-x)_\mu(u-x)_\nu\frac{(x-v)^2}{(x-u)^2} - (u-x)_\mu(v-x)_\nu - (v-x)_\mu(u-x)_\nu + (x-v)_\mu(x-v)_\nu\frac{(x-u)^2}{(x-v)^2} - \frac{1}{d}\delta_{\mu\nu}(u-v)^2\right) \tag{2.17}$$

where Δ_i is the conformal dimension of $\mathcal{O}_i$, d is the space-time dimension of the CFT, and A_i is a constant given by $A_i = -d\Delta_i N_i \Gamma(d/2)/2(d-1)\pi^{d/2}$. Inserting (2.17) into the expression (2.15) for the stress-energy tensor two-point function of interest, and introducing the separation variable $D^\mu \equiv x^\mu - y^\mu$, we can rewrite the leading order correction as a sum of 4 independent integrals:

$$\langle T^{(1)}_{\mu\nu}(x)T^{(2)}_{\sigma\tau}(y)\rangle_{h^2} = \frac{h^2A_1A_2}{2}\int\frac{d^du}{u^d}\int\frac{d^dv}{v^d}\frac{1}{(D-u)^d}\frac{1}{(D-v)^d}\frac{1}{(u-v)^4} \times\bigg\{\bigg[\bigg[(D-u)_\mu(D-u)_\nu u_\sigma u_\tau\frac{(D-v)^2}{(D-u)^2}\frac{v^2}{u^2}+(D-u)_\mu(D-u)_\nu v_\sigma v_\tau\frac{(D-v)^2}{(D-u)^2}\frac{u^2}{v^2} + ((D-u)_\mu(D-v)_\nu u_\sigma v_\tau+(\sigma\leftrightarrow\tau)) - \bigg((D-u)_\mu(D-u)_\nu u_\sigma v_\tau\frac{(D-v)^2}{(D-u)^2}+(\sigma\leftrightarrow\tau) + ((\sigma,\tau)\leftrightarrow(\mu,\nu))+((\sigma,\tau)\leftrightarrow(\nu,\mu))\bigg)\bigg]+(\mu\leftrightarrow\nu)-\text{traces}\bigg\}. \tag{2.18}$$

In general, it is quite complicated to regularize and evaluate these integrals. Using dimensional regularization, for example, requires the introduction of four Feynman parameter integrals. However, it is easy to see that at least some of the terms in (2.18) diverge logarithmically, leading to a nonzero anomalous dimension.

The computation of $\langle\tilde{T}\tilde{T}\rangle$ contains also various other terms; in particular, it contains terms of the form $\langle T^{(1)}T^{(1)}\rangle$, and these are difficult to compute at order h^2 since it requires knowing the precise form of $\langle T^{(1)}T^{(1)}\mathcal{O}_1\mathcal{O}_1\rangle$ which is not determined purely by conformal invariance. So, in general we do not know how to compute the anomalous dimension directly by this method. However, there is a trick we can use in the special case of $c_1 = c_2$. In this case $\tilde{T} = c_1(T^{(1)} - T^{(2)} + \frac{\Delta_2-\Delta_1}{d}\eta h\mathcal{O}_1\mathcal{O}_2)$, and when we compute the difference $\langle\tilde{T}(x)\tilde{T}(y) - c_1^2T^{\mathrm{tot}}(x)T^{\mathrm{tot}}(y)\rangle$ the terms proportional to $\langle T^{(1)}(x)T^{(1)}(y)\rangle$ and to $\langle T^{(2)}(x)T^{(2)}(y)\rangle$ drop out, and the traceless part of the resulting expression is simply given by $-4c_1^2\langle T^{(1)}(x)T^{(2)}(y)\rangle$. Since T^{tot} has no anomalous dimension, the logarithmic terms in this expression should be the same as the ones appearing in $\langle\tilde{T}\tilde{T}\rangle$. Even without computing these logarithmic terms exactly, we can see how they depend on the operators $\mathcal{O}_1$ and $\mathcal{O}_2$, since the expression (2.18) depends on these operators only through the combination $A_1A_2 \propto \Delta_1\Delta_2N_1N_2$. Thus, the anomalous dimension arising from the computation above is given by some function of d times $\Delta_1\Delta_2N_1N_2$, in full agreement with the result (2.14) which we found above.

D. Graviton mass from a bulk gravity calculation

Since we know how to describe the deformation (2.1) in terms of deformed boundary conditions for fields in the bulk, as described in Sec. II A, we can calculate the mass of the graviton in the bulk directly, using the methods of [16,17], by a scalar loop correction to the graviton propagator (with the scalars obeying the deformed boundary conditions). Schematically, one computes the graviton self-energy in the bulk, and then to find the mass one extracts the coefficient of the term arising from a massive spin-1 state by matching the long-distance behavior and the tensor structure. In [16,17] this technique was used to analyze the mass of the massive graviton that arises in "locally localized gravity" as a bound state on an AdS-sliced brane. Here, we will use it in the bulk.

The only diagrams that can contribute to the graviton mass at leading order are those involving the scalars ϕ_1 and ϕ_2, dual to the operators $\mathcal{O}_1$ and $\mathcal{O}_2$. In the undeformed bulk theory, if we denote the boundary conditions for the scalars (in the coordinate system $ds^2_{\mathrm{AdS}} = z^{-2}(dz^2 + dx^\mu dx_\mu)$) by $\phi_i(z\to 0) \sim \alpha_i(x)z^{d-\Delta_i} + \beta_i(x)z^{\Delta_i}$ where Δ_i is the conformal dimension of the dual operator $\mathcal{O}_i$ (we assume $\Delta_i \neq d/2$), the boundary conditions were given by $\alpha_i(x) = 0$. After the deformation we are considering, the boundary conditions change to $\alpha_1(x) = h(2\Delta_2 - d)\beta_2(x)$ and $\alpha_2(x) = h(2\Delta_1 - d)\beta_1(x)$: the "non-normalizable mode"[7] of the scalar in one of the AdS spaces is related via the deformation to the normalizable mode of the scalar in the other AdS space. In the scalar loop diagram contributing to the graviton mass, these altered boundary conditions create a situation in which the scalar in the loop can propagate across the boundary from one AdS space to the other and back.

In principle we could compute the correction to the graviton mass directly in this two-AdS-space picture, but in practice it will be simpler to do the computation thinking about the two gravitons and the two scalars as living on the same AdS space (of course, this does not change the physics, but just simplifies the language of the computation). We will limit ourselves to the case of $c_1 = c_2$. Working in this folded picture, we may directly apply the results of [9] to find the corrections to the scalar propagator arising from the deformation (2.1). The corrected propagator is a 2×2 matrix since the boundary conditions mix the two-scalar fields, and it is given by [9]:

$$G^{ij}_\phi = \frac{1}{1+\tilde{h}^2}\begin{pmatrix} G^1_\phi + \tilde{h}^2G^2_\phi & \tilde{h}G^1_\phi - \tilde{h}G^2_\phi \\ \tilde{h}G^1_\phi - \tilde{h}G^2_\phi & G^2_\phi + \tilde{h}^2G^1_\phi\end{pmatrix}, \tag{2.19}$$

where $\tilde{h} \equiv (2\Delta_1 - d)h$ and G^i_ϕ is the propagator of the ith scalar field in the undeformed theory, namely, the propagator corresponding to a field dual to an operator of dimension Δ_i. The contribution of these scalar propagators to the graviton propagator (which is now also a 2×2 matrix),

[7]Since we are interested in the case of $\Delta < (d+1)/2$, the mode proportional to α is actually normalizable, but it is still nonfluctuating in the undeformed theory because of the boundary condition.

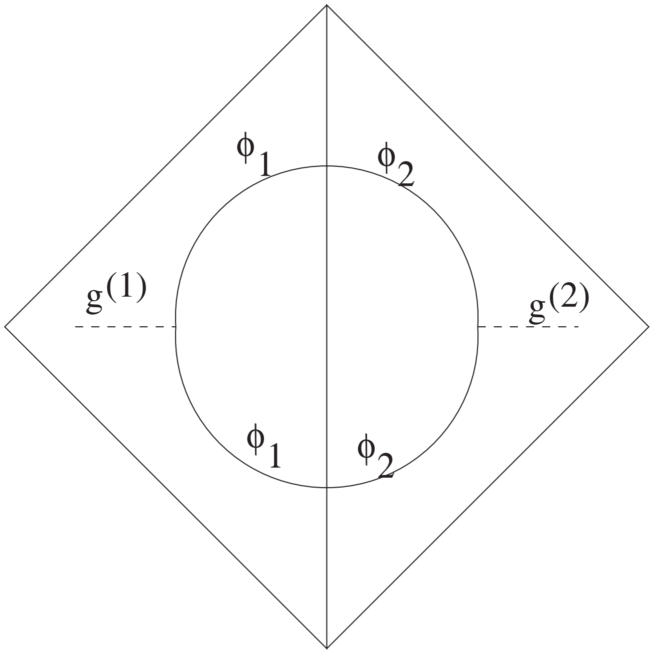

FIG. 1. The scalar loop graph contributing to the graviton mass in the two-AdS-space picture. The dashed lines are graviton propagators, and the solid lines are scalar propagators. The graviton mixes with a two-scalar state and becomes massive.

through the one-loop diagram shown in Fig. 1, in which $g^{(1)}_{\mu\nu}$ couples only to ϕ_1 and not to ϕ_2, and similarly for $g^{(2)}_{\mu\nu}$, are proportional to:

$$G^{ij}_{\rm grav} \sim \frac{1}{(1+\tilde{h}^2)^2}\begin{pmatrix} (G^1_\phi + \tilde{h}^2 G^2_\phi)^2 & \tilde{h}^2(G^1_\phi - G^2_\phi)^2 \\ \tilde{h}^2(G^1_\phi - G^2_\phi)^2 & (G^2_\phi + \tilde{h}^2 G^1_\phi)^2 \end{pmatrix}. \tag{2.20}$$

As expected, this results in no correction for one linear combination of the graviton propagators (the sum) and a correction starting at order $\tilde{h}^2$ to the other (the difference). In the formalism of [17], the two-point function of stress tensors must now be promoted to a 2×2 matrix and evaluated carefully with the corrected scalar propagators.

To extract a graviton mass to match against the field theory result of the previous subsection we need to compute the correction to the mass of the "off-diagonal" graviton. For the case of $c_1 = c_2$ we need to look at the one-loop correction to the propagator of the graviton dual to $\tilde{T} = (T_1 - T_2)/\sqrt{2}$ (at leading order in h), where for convenience we normalized $\tilde{T}$ to have the same 2-point function as T_1 and T_2. This normalization differs from how we defined $\tilde{T}$ in the field theory, but it is trivial to see that the normalization of $\tilde{T}$ does not affect the graviton mass (which, in the field theory language, is the scaling dimension of the operator). The graviton dual to $\tilde{T}$ couples to ϕ_1 with a positive sign (times $1/\sqrt{2}$) and to ϕ_2 with the opposite sign. Plugging the scalar propagators into the 1-loop diagram correcting the mass of this specific graviton, one finds that the diagram is the same as in the unperturbed theory, minus $\frac{2\tilde{h}^2}{(1+\tilde{h}^2)^2}(G^1_\phi - G^2_\phi)^2$. Thus, the answer we are looking for is the correction to the graviton mass coming from a scalar running in the loop with propagator $G^1_\phi - G^2_\phi$, multiplied (at leading order in h) by $[-2h^2(\Delta_1 - \Delta_2)^2]$. While our field theory analysis of the previous subsection was valid only to leading order in h, the gravity calculation is done at leading order in $1/N$ (we only calculate the 1-loop correction), but it is exact to all orders in h as long as we keep the full $\frac{\tilde{h}^2}{(1+\tilde{h}^2)^2}$ prefactor [Eq. (2.19) is exact at leading order in $1/N$]. For comparison we restrict ourselves to the leading term in h, but gravity gives us a prediction for the field theory answer at large N for any h, while the field theory predicts the leading h behavior of the gravity answer to all orders in $1/N$.

In fact, we will now do a more general computation; we will consider the graviton mass induced by a one-loop diagram of a scalar whose propagator is a_{Δ_1} times the propagator of a scalar dual to an operator with dimension Δ_1, plus a_{Δ_2} times the propagator with dimension $\Delta_2 = d - \Delta_1$. This is a generalization of the calculation of [17] to arbitrary d and Δ_1. As we just argued, the case we are interested in corresponds to $a_{\Delta_1} = -a_{\Delta_2} = 1$ and has an additional overall factor of $[-2h^2(\Delta_1 - \Delta_2)^2]$ in the graviton mass.

We need to calculate corrections to the graviton self-energy or the two-point function, $\langle \hat{T}_{\mu\nu}(x)\hat{T}_{\rho'\sigma'}(y)\rangle$, of stress-energy tensors in the bulk. We follow the conventions of [21–23] where unprimed indices indicate tensor indices evaluated at x while primed indices are evaluated at y. Since we are deforming the boundary conditions of scalar fields, it will suffice to consider only the scalar field contribution to the bulk stress tensor $\hat{T}$. Since AdS is a maximally symmetric space-time, the stress tensor two-point function may be decomposed in the following basis of 5 linearly independent bi-tensors [17,21,23]:

$$\begin{aligned} \mathcal{O}_1 &\equiv g_{\mu\nu} g_{\rho'\sigma'}, \\ \mathcal{O}_2 &\equiv \hat{n}_\mu \hat{n}_\nu \hat{n}_{\rho'} \hat{n}_{\sigma'}, \\ \tilde{\mathcal{O}}_3 &\equiv g_{\mu\rho'} g_{\nu\sigma'} + g_{\mu\sigma'} g_{\nu\rho'}, \\ \mathcal{O}_4 &\equiv g_{\mu\nu} \hat{n}_{\rho'} \hat{n}_{\sigma'} + g_{\rho'\sigma'} \hat{n}_\mu \hat{n}_\nu, \\ \tilde{\mathcal{O}}_5 &\equiv g_{\mu\rho'} \hat{n}_\nu \hat{n}_{\sigma'} + g_{\mu\sigma'} \hat{n}_\nu \hat{n}_{\rho'} + g_{\nu\rho'} \hat{n}_\mu \hat{n}_{\sigma'} + g_{\nu\sigma'} \hat{n}_\mu \hat{n}_{\rho'}, \end{aligned} \tag{2.21}$$

where $\tilde{\mu}$ is the geodesic distance between x and y, $\hat{n}_{\alpha(')} \equiv \nabla_{\alpha(')}\tilde{\mu}$ is the unit tangent vector to the geodesic from x to y, evaluated at one end point or the other according to whether the index is primed, and $g_{\alpha\beta'}$ is the "parallel propagator" which transports unit vectors between the two points. Rules for manipulating these objects are summarized in Table 1 of [21]. To extract the graviton mass following [17], one must obtain the constant piece of the coefficient of the transverse-traceless part of the graviton propagator. Since this must arise as the effect of a (bound state) massive spin-1 particle, it suffices to focus on that tensor structure in the long range behavior of the graviton self-energy. First, one computes the full graviton self-energy, $\Sigma_{\mu\nu\rho'\sigma'} = 8\pi G_N\langle\hat{T}_{\mu\nu}(x)\hat{T}_{\rho'\sigma'}(y)\rangle$, by evaluating the stress tensor two-point function of free scalars and expanding in the bitensor basis in the limit of large separation of points (large $\tilde{\mu}$ or large negative $Z \equiv -\cosh(\tilde{\mu})$). This allows one to neglect contact terms that may arise through use of the equations of motion to simplify expressions. Second, one separately computes the transverse-traceless piece which arises due to the exchange of a massive spin-1 particle (with the appropriate mass for being swallowed by the graviton, as discussed in Sec. II B), $\Pi^{\text{spin-1}}_{\mu\nu\rho'\sigma'}$. This may be found by taking covariant derivatives of the massive spin-1 propagator, $D_{\mu\rho'}$:

$$\Pi^{\text{spin-1}}_{\mu\nu\rho'\sigma'} = -2\nabla_\mu\nabla_{\rho'}D_{\nu\sigma'}(\mu). \tag{2.22}$$

Matching the coefficient and tensor structure of the leading large $|Z|$ behavior allows one to identify the mass of the graviton in this framework, as was done in [17] for the special case of a conformally coupled scalar in AdS_4.

We need to perform this calculation for a free scalar field X in $d+1$ dimensions with Lagrangian

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{2}(\partial X)^2 - \frac{m^2}{2}X^2 - \alpha\frac{d-1}{8d}RX^2\right), \tag{2.23}$$

where we allowed contributions to the mass coming either from explicit masses or from couplings to the background curvature; the case of $m=0$, $\alpha=1$ corresponds to a conformally coupled scalar. We normalize the radius of curvature of AdS space to one. The two possible dimensions of the dual operator are given by $\Delta_{1,2} = \frac{d}{2}\pm\frac{1}{2}\times\sqrt{d^2+4m^2-\alpha(d^2-1)}$. The case considered in [17] corresponds to $d=3$, $m=0$, $\alpha=1$, $\Delta_1=2$. From the dual field theory point of view it is clear that the final answer should not depend on α and m individually, but only on the combination Δ_1, and we will find that this is indeed the case. The stress tensor derived from this Lagrangian (after using the scalar equation of motion) is

$$\begin{aligned}\hat{T}_{\mu\nu} = {}&\left(1-\alpha\frac{d-1}{2d}\right)\partial_\mu X\partial_\nu X - \alpha\frac{d-1}{2d}X\nabla_\mu\partial_\nu X\\ &- g_{\mu\nu}\left(\left(\frac{1}{2}-\alpha\frac{d-1}{2d}\right)(\partial X)^2 + \left(\frac{m^2}{2}-\alpha m^2\frac{d-1}{2d}\right.\right.\\ &\left.\left.+\frac{\alpha(\alpha d+\alpha-d)}{8}\frac{(d-1)^2}{d}\right)X^2\right).\end{aligned} \tag{2.24}$$

Calculating the $\langle\hat{T}\hat{T}\rangle$ two-point function is straightforward using Wick contractions and taking up to four covariant derivatives of the scalar propagator. Because of the sheer number of terms to keep track of, we implemented the rules of Table 1 of [21] for manipulations in intrinsic coordinates in MATHEMATICA, and performed the calculation of the correlator, as well as the subsequent decomposition into tensor structures, on the computer. For the scalar propagator we use a superposition of the dimension Δ_1 and $\Delta_2 = (d-\Delta_1)$ propagators [24], written using the hypergeometric function F:

$$\begin{aligned}G_X(Z) = {}& a_{\Delta_1}2^{-\Delta_1}\frac{\Gamma(\Delta_1)}{\pi^{d/2}(2\Delta_1-d)\Gamma(\Delta_1-\frac{d}{2})}\\ &\times(-Z)^{-\Delta_1}F\left(\frac{\Delta_1}{2},\frac{\Delta_1+1}{2},\Delta_1-\frac{d}{2}+1;\frac{1}{Z^2}\right)\\ &+a_{\Delta_2}2^{-\Delta_2}\frac{\Gamma(\Delta_2)}{\pi^{d/2}(2\Delta_2-d)\Gamma(\Delta_2-\frac{d}{2})}\\ &\times(-Z)^{-\Delta_2}F\left(\frac{\Delta_2}{2},\frac{\Delta_2+1}{2},\Delta_2-\frac{d}{2}+1;\frac{1}{Z^2}\right).\end{aligned} \tag{2.25}$$

For the decomposition into tensor structures we only need the leading terms in a power series expansion of $\langle\hat{T}\hat{T}\rangle$ in large $|Z|$; in order to simplify these expansions in MATHEMATICA we only kept the cross-terms proportional to $a_{\Delta_1}a_{\Delta_2}$ in the $\langle\hat{T}\hat{T}\rangle$ correlator, since we expect the mass to vanish in the case where either a_{Δ_1} or a_{Δ_2} vanishes. It is also easy to see that unless Δ_1 is a half-integer, only cross-terms lead to integer powers of Z which can match the massive spin-1 structure (2.22) we are looking for. For comparison with [17] one needs to substitute the notations $a_{\Delta_1} = \frac{1}{2}(a_- - a_+)$, $a_{\Delta_2} = -\frac{1}{2}(a_- + a_+)$.

The extraction of the massive spin-1 piece in principle works along the same lines as in [17]. One new complication in our case is that while for the conformally coupled scalar considered in [17], the $\langle\hat{T}\hat{T}\rangle$ correlator and hence the self-energy were transverse and traceless automatically, for the general massive scalar one first needs to isolate the transverse-traceless part. The self-energy can easily be made traceless by subtracting out the μ, ν and the ρ', σ' traces. The remainder then can be decomposed as

$$\begin{aligned}\Sigma^{\text{traceless}}_{\mu\nu\rho'\sigma'} = {}&\Sigma^{\perp}_{\mu\nu\rho'\sigma'} + \left(\nabla_\mu\nabla_\nu - \frac{1}{d+1}g_{\mu\nu}\nabla^2\right)\\ &\times\left(\nabla_{\rho'}\nabla_{\sigma'} - \frac{1}{d+1}g_{\rho'\sigma'}(\nabla')^2\right)B(Z)\end{aligned} \tag{2.26}$$

for some function $B(Z)$, where $\Sigma^{\perp}_{\mu\nu\rho'\sigma'}$ is the desired transverse-traceless piece that contains the graviton mass. The decomposition of a general linear metric fluctuation could also contain a vector piece $\nabla_\mu A_\nu + \nabla_\nu A_\mu$. A corresponding structure is not present in our expression for $\Sigma_{\mu\nu\rho'\sigma'}$; presumably this is due to the fact that the vector piece is not sourced by the conserved stress tensors. The decomposition (2.26) was once more performed using MATHEMATICA. The result obtained for the graviton mass using the (generalized) formalism of [17] is

$$M_{\rm grav}^2 = -G_N \frac{2^{4-d}\pi^{3/2-d/2}}{(d+2)\Gamma(\frac{d+3}{2})} \frac{a_{\Delta_1}a_{\Delta_2}\Delta_1\Delta_2\Gamma(\Delta_1)\Gamma(\Delta_2)}{\Gamma(\Delta_1-\frac{d}{2})\Gamma(\Delta_2-\frac{d}{2})}, \tag{2.27}$$

which for $d = 3$, $\Delta_1 = 2$ was previously obtained in [17].

E. Comparison between field theory and gravity

We are now in a position to put all the bits and pieces together to compare our result (2.27) for the mass of the graviton with the field theory result for the anomalous dimension of the nonconserved stress tensor. As we argued earlier, the case of two CFTs coupled together via a double trace deformation corresponds to $a_{\Delta_1} = -a_{\Delta_2} = 1$ and has an additional overall factor of $-2h^2(\Delta_1 - \Delta_2)^2$ compared to the case of a single scalar with mixed propagator we worked out in the previous subsection. So, the final gravity prediction for the graviton mass is

$$M_{\rm grav}^2 = -h^2(\Delta_1-\Delta_2)^2 G_N \frac{2^{5-d}\pi^{3/2-d/2}}{(d+2)\Gamma(\frac{d+3}{2})} \times \frac{\Delta_1\Delta_2\Gamma(\Delta_1)\Gamma(\Delta_2)}{\Gamma(\Delta_1-\frac{d}{2})\Gamma(\Delta_2-\frac{d}{2})}. \tag{2.28}$$

Note that this expression is positive (even though the expression (2.27) is not necessarily positive), consistent with our theory being unitary.

To compare this with our field theory answer (2.14) we need to plug in the appropriate values of N_1, N_2, and c, that is the normalization of the two-point functions of the operators dual to canonically normalized bulk scalar fields (as we assumed in the computation above), as well as the central charge of the field theory in terms of the bulk Newton's constant. For a canonically normalized scalar the two-point function of the dual operator with dimension Δ_i is [24]

$$\frac{N_i}{|x|^{2\Delta_i}} = \frac{1}{\pi^{d/2}} \frac{(2\Delta_i-d)\Gamma(\Delta_i)}{\Gamma(\Delta_i-\frac{d}{2})} \frac{1}{|x|^{2\Delta_i}}, \tag{2.29}$$

so that for our case with $\Delta_2 = d - \Delta_1$ one gets

$$N_1N_2 = -(\Delta_1-\Delta_2)^2 \frac{\Gamma(\Delta_1)\Gamma(\Delta_2)}{\pi^d\Gamma(\Delta_1-\frac{d}{2})\Gamma(\Delta_2-\frac{d}{2})}. \tag{2.30}$$

The value of the central charge may be read from the results of [25],

$$c = \frac{1}{16\pi G_N}\frac{d(d+1)\Gamma(d)}{(d-1)\pi^{d/2}\Gamma(\frac{d}{2})} = \frac{d}{d-1}\frac{\Gamma(\frac{d+3}{2})}{G_N 2^{4-d}\pi^{(3/2)+(d/2)}} \tag{2.31}$$

(in [25] the prefactor of $1/16\pi G_N$ was set to one, but we are reinstating it here). Plugging these values into (2.14), we find an exact agreement with (2.28), confirming our description of the deformation coupling the two conformal field theories.

III. NON-DUALITY FOR NON-GAUGE THEORIES

In the previous section we argued that some conformal field theories are not dual to quantum gravity on AdS space (but rather to a sum of AdS spaces); this naturally raises the general question of classifying which conformal field theories are dual to quantum gravity on AdS space, which are dual to a sum of AdS spaces, and which (perhaps) have no quantum gravity dual at all. In this section we will discuss the case of free field theories with no gauge symmetry and the theories which one can flow to from these, and in the next section we will discuss this question more generally.

Consider a free field theory including some number of free scalars, fermions and $U(1)$ gauge fields. Can this theory have a quantum gravity dual ? We claim that the answer is no. A free theory has many conserved spin 2 operators, which are the energy-momentum tensors of each free field in the theory, so if it had a dual it would have to involve many massless gravitons (from the field theory point of view there is nothing special about the "total energy-momentum tensor"). However, each of these spin 2 operators is not an independent operator, but rather a product (or a sum of products) of other (gauge-invariant) operators; for a scalar field we have $T_{\mu\nu} \sim (\partial_\mu\phi)(\partial_\nu\phi)$, and for a free vector field $T_{\mu\nu} \sim \eta^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma}$. In the AdS/CFT correspondence, we generally map operators to fields in the bulk, but this is really only true for a special class of operators ("single-trace" operators when the duality involves a large N gauge theory); the states created by these operators map to single-particle states of the corresponding fields in the bulk, while the states created by products of these operators map to multiparticle states in the bulk.[8] If we use this rule for free field theories, we would conclude that the dual bulk theories (for each free field) involve a single field in the bulk, which would be dual to a basic operator (a free scalar field, a free fermion field, or $F_{\mu\nu}$ for free gauge fields); all other operators in the theory are products of (descendants of) this basic operator, so they would map to multiparticle states of this single field in the bulk. However, this implies that the energy-momentum tensor of the free field theory would map to a two-particle bound state of this basic field, and it does not seem likely that this bound state can really be identified as a massless graviton in the bulk (at least, we do not know of any consistent examples in which a massless graviton arises as a bound state, and there are arguments against it in flat space [26]).

Thus, we claim that free field theories do not map to bulk quantum gravity theories. This is supported by a well-

[8]The distinction between these two types of states is not sharp when the bulk theory is interacting, since they mix together, but we assume that it still exists.

known example which arises (for instance) in the near-horizon limit of N D3-branes in string theory. The low-energy field theory living on N D3-branes is a $U(N)$ $\mathcal{N} = 4$ super-Yang-Mills theory. This decomposes into a direct sum of a free $U(1)$ $\mathcal{N} = 4$ theory (including a vector field, six scalar fields and four fermions) and an interacting $SU(N)$ theory (at least in the absence of any external sources in the fundamental representation). Obviously, type IIB string theory on $AdS_5 \times S^5$ does not decompose into a product of two decoupled theories; and indeed, the spectrum of propagating fields in this theory seems to map just to the spectrum of operators in the $SU(N)$ theory, without the $U(1)$ part.[9] The supergravity spectrum in the bulk [27] does contain so-called "singleton" fields (which are sometimes called "doubleton" fields in this case), which are in a one-to-one correspondence with the free fields of the $U(1)$ multiplet, but these fields can always be gauged away in the bulk. So, one can think of these fields as living on the boundary; for most purposes one can just set them to zero and think of the theory as a pure $SU(N)$ theory, but for other purposes it is sometimes more convenient to include them (see, for example, [28]). Clearly, the $U(1)$ theory itself does not have any bulk gravitational dual (at least not with a propagating graviton).

Based on this example and on the general arguments above, we suggest that in general free field theories have no gravitational dual, and that if one couples a free field theory to a gauge theory with a gravity dual, one should think of this free field theory as living on the boundary. Once we accept this for free field theories, it seems that it must be true also for any deformations of free field theories, since it is hard to imagine how a bulk theory would suddenly emerge when we deform. So, we claim that any deformation of a free field theory, such as the infrared fixed point of the ϕ^4 field theory of a single scalar field in $2 + 1$ dimensions, also has no gravity dual. Note that we are discussing here deformations by adding gauge-invariant operators to the Lagrangian; by such deformations we cannot obtain a theory with nontrivial gauge interactions. It is easy to see that the key feature of the stress tensor failing to be an independent operator survives such a deformation. Turning on nongauge interactions will add terms to the stress tensor, but the new terms will also be products of gauge-invariant operators, not independent operators.

Which theories can have a gravity dual? In all known examples which have an explicit Lagrangian description, these theories have a gauge group which acts nontrivially on *all* the fields. Then, the energy-momentum tensor can no longer be written as a product of gauge-invariant operators (it is generally a sum of products of non-gauge-invariant operators), and it makes sense to identify it with a massless bulk graviton. The gauge group can be continuous (as in $SU(N)^k$ gauge theories with bifundamental and adjoint fields) or discrete (as in the $1 + 1$ dimensional sigma model on the T^{4N}/S_N orbifold). In some cases the gauge theory could have a free field limit where it does not contain any interactions (for instance, in the $\mathcal{N} = 4$ $SU(N)$ super-Yang-Mills theory one could take the $g_{\rm YM} \to 0$ limit), but even in this limit it is not identical to a free field theory of the type discussed above, since the path integral still involves a division by the gauge symmetry, so (at least on a compact space) the spectrum of a free $SU(N)$ theory is very different from the spectrum of a free $U(1)^{N^2-1}$ theory.

Does every gauge theory have a quantum gravity dual? We do not know the answer to this question, but it seems likely that the answer is positive; many theories can be reached by flows from known examples, and there is no natural separation of gauge theories into two classes (one which would have a gravity dual and one which would not). Then, when we couple additional singlet fields to a gauge theory, these fields would live on the boundary rather than in the bulk (this has a simple description in the AdS/CFT correspondence, by just making the coupling constants of the CFT, which have a known description as boundary conditions in AdS space, dynamical). However, our discussion of the previous section suggests that the story is more complicated, and that some gauge theories are actually dual to a quantum gravity theory on a sum of several spaces; we will discuss this in the next section.

IV. THE "CONNECTIVITY INDEX"

The discussion of Sec. II suggests that the space of conformal field theories can be classified according to a "connectivity index" n, labeling the number of components in the gravity dual space-time (which are only connected at their boundary). As discussed in the introduction, it is not clear that this index is well defined (in the sense that theories with different n's could not be identified in some way), but it certainly seems to be well-defined in the semiclassical approximation, and it is hard to see how quantum corrections could change the number of components of space-time (near the boundary), so we will assume here that it is well-defined, and see if we can understand this from the field theory point of view.

The discussion of the previous sections suggests that the "connectivity index" n is related to the number of independent gauge groups, and that it can be defined in the following way: given a Lagrangian formulation of a field theory in terms of a gauge group G (continuous or discrete), then n is the maximal number such that we can write $G = G_1 \times G_2 \times \cdots \times G_n$, and such that no field is charged under more than one G_i factor.

The definition above raises several immediate questions. One disturbing issue is that it involves not only having a Lagrangian formulation for the theory, but also a counting of gauge groups, even though gauge groups are not really

[9] So far this has only been checked in the supergravity approximation, but it is believed to be true more generally.

physically well-defined objects but just redundancies in our description of a theory. Obviously, if n is really well-defined, there should be a way to define it which does not refer to a counting of gauge groups or to a Lagrangian formulation, but just to abstract properties of the theory. So far we have not been able to find such a more general definition, but we believe that it exists. Clearly, a theory with connectivity index n should have at least n spin 2 operators which cannot be written as (sums of) products of other operators, and which can be identified with (linear combinations of) the gravitons on the n components of space-time. However, generally all but one of these spin 2 operators have dimensions bigger than d, and generic theories have many such operators, so that it is difficult to identify which of the spin 2 operators correspond to gravitons and which do not (the question may not even make sense beyond the semiclassical gravity limit). In examples of the type we discussed in Sec. II, we can deform the parameters of the field theory with index n continuously to a point where it is a product of n independent theories (with n independent spin 2 operators of dimension d), and then it is clear that the index must be at least n; but it is not clear if it is always possible to do this for any theory with $n > 1$.

Note that the value of n can change when we take a low-energy (IR) limit of a nonconformal field theory. Clearly, n can decrease in the IR if we have a product of some gauge groups, and some of them confine and develop a mass gap. In the gravity dual this would mean that some of the n spaces do not host any low-energy fields, so that the IR limit involves only the other spaces. It is also possible for n to increase as we flow; a simple example of this which we mentioned above is the point on the moduli space of the $\mathcal{N} = 4$ SYM theory where $SU(N)$ is spontaneously broken to $SU(N_1) \times SU(N - N_1) \times U(1)$. Despite the product structure, this theory has $n = 1$ (since there are bifundamental fields), but as we flow to the IR it decouples into a product of three theories, two which have a gravity dual (which is just a smaller $AdS_5 \times S^5$) and one (the $U(1)$ theory) which does not. Again, this has a simple picture in the gravity dual, as we discussed above; we have a flow which in the UV is given by a single $AdS_5 \times S^5$ space, but there are two throat regions in this space where low-energy fields live. Each of these regions locally looks like $AdS_5 \times S^5$, and as we go to energies below the scale of the string stretching between the two "throats," we get two decoupled theories (which in some sense share the same boundary where the throats connect).

Let us consider two more examples. The theory of $SU(N_c)$ SQCD with N_f flavors and $N_c \leq N_f < 3N_c/2$ is believed to flow to a free theory in the IR. For $N_f \leq N_c + 1$ this is a free theory of scalars and fermions, so we have a flow from $n = 1$ to $n = 0$, while for $N_f > N_c + 1$ the IR theory is a free $SU(N_f - N_c)$ gauge theory, so we have a flow from $n = 1$ to $n = 1$. For $3N_c/2 < N_f < 3N_c$ the theory flows to an interacting superconformal field theory, which is believed to also be the end-point of the flow from the dual "magnetic" $SU(N_f - N_c)$ theory [29]. In this case it seems that on both sides of the duality we have a flow from $n = 1$ to $n = 1$, since it seems unlikely that the energy-momentum tensor in the IR SCFT would be a composite operator. Note that the magnetic theory has many singlet fields, and in the dual gravitational description we argued that they should be interpreted as living on the boundary of space-time.

Another example is the $d = 3$ $\mathcal{N} = 2$ supersymmetric $U(1)$ gauge theory with one positively charged and one negatively charged chiral multiplet ($N_f = 1$). This theory is believed [30] to flow to the same nontrivial IR fixed point as the theory of three chiral multiplets with a superpotential $W = XYZ$. So, in this case we have flows to the same fixed point from theories with $n = 0$ and with $n = 1$. It seems likely that the IR theory in this case has $n = 0$, since it includes X, Y, and Z as primary operators, and the energy-momentum tensor is a composite in these variables.

According to our conjectures the $O(N)$ vector model in $d = 3$ should not have any quantum gravity dual (it has $n = 0$), even in the large N limit; however, when the $O(N)$ symmetry is gauged (even very weakly), such that only the singlet sector of this model is physical, the model (with $n = 1$) could have a quantum gravity dual (as suggested in [31], see [32] for a recent discussion).

Unfortunately, it seems that even with all the caveats above the definition that we gave is too naive. This is because[10] there is a counter-example where we can connect a theory with $n = 0$ to a theory with $n = 1$ by a marginal deformation (which should not change n according to our arguments); this is the example of the $c = 1$ conformal field theory of a free scalar field on a circle, which is connected by marginal deformations to the theory of a scalar on S^1/Z_2. According to our general arguments, the first theory should have $n = 0$ and the second (involving gauging a discrete group) should have $n = 1$. Thus, it seems that our definition above is too naive (at least when low dimensions and discrete gauge groups are involved), and that it should be made more precise. We hope that there exists some direct and precise field theory definition of n, but we have not yet been able to find it.

V. SUMMARY AND CONCLUSIONS

In this paper we constructed the natural generalization of the AdS/CFT correspondence to product CFTs deformed by multitrace operators. The natural gravitational dual to a product of n CFTs is quantum gravity on n AdS spaces, with their boundaries identified in the sense that the boundary conditions of fields on one AdS space are related to the boundary conditions of fields in the other AdS spaces. These altered boundary conditions will generically create

[10] We thank D. Kutasov for reminding us of this fact.

a situation where one linear combination of the bulk gravitons remains massless while all other linearly independent combinations acquire some mass proportional (at leading order in the deformation, but valid for any conformal field theory, independent of the large N limit) to the square of the deformation parameter, h, via the AdS Higgs mechanism for gravitons. For special cases when the CFTs are identical, the bulk picture can be folded and thought of as a single AdS-space with one massless and several massive gravitons. For more general theories, the unfolded picture is necessary to accommodate theories whose gravitational duals have different compact spaces or which are different in the IR (for instance, one could consider a product of a conformal gauge theory with a confining gauge theory).

We have made this construction explicit in the semiclassical limit (which, for gauge theories, is the same as the 't Hooft large N limit), in the case of a product of field theories which admit semiclassical gravitational duals. Our construction suggests the existence of a "connectivity index" characterizing field theories, which on the gravity side counts the number of components of space-time, and on the field theory side roughly counts the number of independent gauge groups. We have only explored this in detail in the semiclassical limit. However, since it is difficult to imagine that quantum effects could change the integer number of components of space-time which share a common boundary, we conjecture that this integer is in fact a true index, and that a general definition of this index can be found for field theories, which does not rely on a Lagrangian or gauge theory formulation. We conjecture that this index could then be used to classify field theories according to whether they have a gravitational dual description in the following way: field theories with $n = 0$ *will not* have a dual description as quantum gravity on some asymptotically anti-de Sitter space, while field theories with $n \geq 1$ *will* have a dual description as quantum gravity on n asymptotically anti-de Sitter spaces with a common boundary, in the manner discussed above.

We have discussed in detail only the case of coupling together conformal theories in a way which preserves conformal invariance, but the generalizations to many other cases are straightforward. For instance, it is easy to discuss the case of coupling together two theories at finite energy density or temperature, by replacing the AdS backgrounds by AdS black holes. When the two theories have the same temperature the system is in thermal equilibrium, while otherwise there will be a flow of energy from one theory to the other across the boundary (which will be very slow in the large N limit). It may also be interesting to put in a finite UV cutoff (integrating out the high-energy modes), in which case our model becomes a version of the two-throat Randall-Sundrum [33] model (involving two field theories coupled to gravity). In the presence of a finite cutoff, one could also couple directly the stress-energy tensors of the two CFTs, and obtain a graviton mass of order the curvature scale (rather than a one-loop mass suppressed by G_N as in our discussion), as discussed in a similar context in [34].

We did not discuss here the string theory construction of the theories dual to deformed product CFTs, but the procedure to obtain this is a straightforward generalization of the discussion in [8]. The string description of a product of CFTs is by a world sheet theory which is a sum of two sigma models (on a product of AdS space with some compact space), such that each connected component of the world sheet maps to one of the two space-times. The deformation couples the two sigma models together, but this coupling is nonlocal on the world sheet, and given by the translation to the world sheet of (2.1) (with each space-time operator mapping to an integrated vertex operator on the world sheet).

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Note added.—We are aware that E. Kiritsis has independently been working on similar ideas [36].

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