# Extra $Z$ bosons and low-energy tests of unification

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If there is a family-independent extra $U(1)$ gauge symmetry broken at low energies, then it may be possible from the charges of the known quarks and leptons under this $U(1)$ to make inferences about how much gauge unification occurs at high scales and about the unification group. (For instance, there are certain observed properties of an extra $U(1)'$ that would be inconsistent with unification in four dimensions at high scales.) A general analysis is presented. Two criteria used in this analysis are (1) the degree to which the generator of the extra $U(1)$ mixes with hypercharge, and (2) the ratio of the extra $U(1)$ charge of the "10" and the "$\bar{5}$" of quarks and leptons.

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## I. INTRODUCTION

It will never be possible to build an accelerator that reaches energies of $10^{15}$ or $10^{16}$ GeV, so grand unified theories must be tested in more indirect ways. One way is to look for very rare processes such as proton decay [1] or $n - \bar{n}$ oscillations [2] or for cosmic relic superheavy particles such as magnetic monopoles [3]. Another is to look for unification of parameters by using the renormalization group to extrapolate to high scales, as has been done for gauge couplings [4] and may someday be possible for sparticle masses [5]. A third way is find consequences of gauge symmetry at low energy. An obvious example is the fact that the hypercharge values of the standard model are those that would come from simple $SU(5)$ or $SO(10)$ unification. Here we explore the possibility that the charges of the known quarks and leptons under a family-independent "extra" $U(1)$ group broken somewhat above the weak scale may enable us to infer something about gauge unification at high scales.

An obvious barrier to inferring anything about unification from gauge-charge values, is that the same assignments predicted by unification would in some instances also be required by anomaly cancellation even without unification. For instance, if one assumes that only the fermions of the standard model exist (without right-handed neutrinos) and that hypercharge is family-independent, the anomaly conditions fix the hypercharges of the quarks and leptons uniquely to be the same values as would be predicted by $SU(5)$. (There are four anomaly conditions [6]: $3^2 1_Y, 2^2 1_Y, 1_Y^3, 1_Y$, in an obvious notation, the last being the mixed gravity-hypercharge anomaly.) Similarly, it might not always be possible in the case of extra $U(1)$ charges to distinguish the consequences of unification from those of anomaly cancellation. (For discussions of anomaly constraints on extra $U(1)$ gauge groups see [7].) That is one question we shall study in this paper. We shall argue that one can distinguish in some circumstances, at least in principle.

To see that there can be an ambiguity, consider the case of the extra $U(1)$ contained in $SO(10)$. Let us call this $U(1)_{X_{10}}$ and its generator $X_{10}$. On the fermions of the standard model one has $X_{10}(e_L^+, Q_L, u_L^c, L_L, d_L^c, N_L^c) = (1, 1, 1, -3, -3, 5)$. On the other hand, with the same set of fermions and assuming that charge assignments are family-independent, the six anomaly conditions $3^2 1_X^2$, $2^2 1_X$, $1_Y^2 1_X$, $1_Y 1_X^2$, $1_X^3$, and $1_X$ yield the same solution without unification. However, the anomaly conditions do not yield this solution uniquely, but only up to an arbitrary mixing with hypercharge. That is, the general solution of the anomaly conditions is $X = \alpha X_{10} + \beta(Y/2)$. In fact, it is easy to see that it is always the case that the six anomaly conditions which have to satisfied by an extra $U(1)$ will allow the generator of that $U(1)$ to have an arbitrary mixing with hypercharge. We shall exploit this fact: we shall see that under certain assumptions the generator of an extra $U(1)$ *cannot* mix strongly with hypercharge if there is gauge unification at high scales, whereas it *can* and "naturally" *ought* to mix strongly if there is no unification. The degree of mixing of the generator of the extra $U(1)$ with hypercharge will be one of the tools we shall use in our analysis.

We can also learn something about the degree of gauge unification at high scales by comparing the extra-$U(1)$ charges of the "10" ($\equiv e_L^+$, $Q_L$, $u_L^c$) and the "$\bar{5}$" ($\equiv L_L$, $d_L^c$). For example, in the simplest $SO(10)$ models one has $r \equiv X(10)/X(\bar{5}) = -1/3$. Other schemes of unification give other characteristic values;, for example, if $SU(3) \times SU(2) \times U(1)_Y \times U(1)_X \subset SU(6)$, then $r = -2$. On the other hand, partially unified or nonunified models can have values of this ratio that are not achievable in any unified scheme. This will be the other tool of our analysis.

In our analysis, we do not use all the information about the extra $U(1)$ that may be obtained in principle from experiments. We are using only the charges of the known quarks and leptons under the extra $U(1)$. However, if the $Z'$ boson is actually produced in experiments, then almost certainly some of the extra fermions that must exist (to cancel the anomalies of the extra $U(1)$) will also be produced, since they are probably lighter than the $Z'$. That would give additional information that would be helpful in making inferences about the degree of unification. This fact, of course, only strengthens our main point, which is that from information about extra $U(1)$ groups near the weak scale it is in principle possible to infer something definite about physics, and, in particular, about unification, at very high scales. We also do not make use of conditions on the charges of Higgs that arise from the requirement that the light quarks and leptons be able to get realistic masses. Again, such considerations might allow even stronger inferences to be made.

It should also be emphasized that we are making ''in principle'' arguments in this paper. We are not considering how to go about measuring the charges of the known fermions under the extra $U(1)$, or concerned about the practical feasibility of it. We are considering what can be measured ''in principle'' at low energies, and what can be inferred from it about very high energies.

We make use of assumptions of ''naturalness'' in several ways: (1) If no group-theoretical consideration or anomaly-cancellation condition forces it to do so, it is an unnatural fine-tuning for ratios of fermion charges under the extra $U(1)$ to be exactly equal to simple rational numbers like $-2$ or $1/2$. (2) It is difficult to make matter multiplets in unified theories have extreme mass ''splittings'' (which is the basis of the well-known ''doublet-triplet splitting problem'' in unified theories). We assume that it is unnatural to have a large number of such split multiplets besides the usual SM or MSSM Higgs doublets. (3) It is assumed that if no symmetry or other principle makes the mixing of the extra $U(1)$ generator with hypercharge small, it will not be small.

The paper is organized as follows. In Sec. II, we shall explain our assumptions, definitions, and notation and outline our results. In Secs. III, IV, and V we shall explain the analyses that lead to those results.

## II. ASSUMPTIONS, DEFINITIONS, NOTATION, AND RESULTS

We assume that the effective low-energy theory below some scale $M_*$ has an $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'$ gauge symmetry, which we will call $3211'$ for short. The $U(1)'$ is what we mean by the extra $U(1)$, and it is assumed to be family-independent and to be broken at a scale $M'$ that is above the weak scale, but close enough to it that it can eventually be studied at accelerators. The generator of $U(1)'$ we call $X'$ and the corresponding gauge boson $Z'$.

We will say that the theory is ''fully unified'' if there is at some higher scale an effective four-dimensional theory with a simple gauge group $G$ such that $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)' \subset G$. We will say that it is ''partially unified'' there is an effective four-dimensional theory with group $G \times H \supseteq G \times U(1)_X$, such that $G$ contains $SU(3)_c \times SU(2)_L$ but does *not* contain both low-energy Abelian groups $U(1)_Y \times U(1)'$. Finally, we will say that it is ''nonunified'' if there is no four-dimensional unification of even the $SU(3)_c \times SU(2)_L$.

In both fully unified models and partially unified models we may write the low-energy group between the scales $M_*$ and $M'$ as $SU(3)_c \times SU(2)_L \times U(1)_{Y_5} \times U(1)_X$, where $SU(3)_c \times SU(2)_L \times U(1)_{Y_5} \subset SU(5) \subseteq G$, and $U(1)_X$ commutes with $SU(5)$. At $M'$ the breaking down to the standard model group can happen in two ways: (a) The generator $X$ is broken and $Y_5$ is left unbroken, in which case obviously $Y = Y_5$ and $X' = X$. This we call ''ordinary'' or ''nonflipped'' breaking. Or (b) both $X$ and $Y_5$ are broken at $M'$, leaving unbroken a hypercharge that is a linear combination of $Y_5$ and $X$. Then we have $Y/2 = aY_5/2 + bX$ and $X'$ is the orthogonal linear combination of $Y_5/2$ and $X$. This we call ''flipped breaking,'' as it happens in ''flipped $SU(5)$'' models (among others) [8].

For convenience we will denote the set of multiplets $(e_L^+, Q_L, u_L^c)$ by 10 and $(L_L, d_L^c)$ by ''$\bar{5}$'' (with quotation marks) whether or not there is actually any $SU(5)$ unification. By the notation $\bar{X}$ we mean any generator that has equal values for all the multiplets in ''10'' and equal values for the multiplets in ''$\bar{5}$.'' We will call the ratio of these values $r$. That is, $r \equiv \bar{X}(\text{“10”})/\bar{X}(\text{“}\bar{5}\text{”})$.

Both unification (full or partial) and anomaly cancellation without unification can lead to the result that $X'$ has the form $X' = \alpha\bar{X} + \beta Y/2$. If $\beta/\alpha \neq 0$ and is not small, we will say that $X'$ ''mixes strongly with hypercharge.'' If $\beta/\alpha \ll 1$, we will say that there is small mixing. The degree of mixing with hypercharge is crucial to our analysis.

We will generally not assume anything about whether there is supersymmetry (SUSY). SUSY will not affect most of our analysis if we make certain reasonable assumptions. SUSY would, of course, mean that there would be Higgsinos that could be charged under the extra $U(1)$ and contribute to anomalies. However, these contributions would typically cancel for the following reasons. Consider the case of unification. The Higgs fields that get vacuum expectation values (VEVs) at the weak scale, namely $H_u$ and $H_d$, must then have color-triplet partners. These partners must have masses much larger than $M'$ to avoid proton decay, and that would require them to ''mate'' with other triplets of opposite $X'$. On the other hand, those Higgs fields that get VEVs of order $M'$ or larger must come paired with Higgs fields that have opposite $X'$, generally, in order to avoid $D$-term breaking of SUSY at large scales.

### A. Why mixing with hypercharge is significant

In a model with no unification, there is no symmetry or other principle that prevents $X'$, the generator of the extra $U(1)$, from mixing with hypercharge. Anomaly-cancellation constraints never prevent this, and neither can the form of the Yukawa terms, since those terms must be invariant under $U(1)_Y$ anyway. Therefore, one expects $X'$ to be of the form $X' = \alpha\bar{X} + \beta Y/2$, with $\beta/\alpha$ of order one.

However, the situation is quite different in fully or partially unified models. As noted above, in such models one can write the low-energy group between the scales $M_*$ and $M'$ as $SU(3)_c \times SU(2)_L \times U(1)_{Y_5} \times U(1)_X$, where there is an $SU(5)$ that contains $SU(3)_c \times SU(2)_L \times U(1)_{Y_5}$ and $U(1)_X$ commutes with that $SU(5)$. If there is ordinary breaking of the extra $U(1)$ at $M'$, i.e. if only $U(1)_X$ breaks, then as noted before $X' = X$ and $Y = Y_5$, and so $U(1)'$ commutes with $SU(5) \supset SU(3)_c \times SU(2)_L \times U(1)_Y$. That would mean that $X'$ does not mix with $Y$.

In other words, the unified group protects $X'$ from mixing with $Y$. However, because the unified group is broken at $M_{\rm GUT}$, radiative effects induce a slight mixing below $M_{\rm GUT}$. In particular, a small effective mixing will in general be produced by renormalization of the gauge kinetic terms [9]: one-loop diagrams produce a term of the form $\epsilon F^{\mu\nu}_{(Y_5)} F_{(X)\mu\nu}$, which upon bringing the gauge kinetic terms to canonical form shifts the gauge fields to produces an effective mixing of $X'$ and $Y$ with $\beta/\alpha \sim \epsilon$. If the particles going around the loop form complete, degenerate $SU(5)$ multiplets, then these diagrams vanish by the tracelessness of $Y_5$. However, split $SU(5)$ matter or Higgs multiplets that contain both Weak-scale and superheavy components (like the Higgs multiplets that break the Weak interactions) will give a contribution to $\epsilon$ that is of order $\frac{g_1 g_X}{16\pi^2} \times \ln(M^2_{\rm GUT}/M^2_W) \sim \frac{\alpha}{4\pi} \ln(M^2_{\rm GUT}/M^2_W)$. For a Higgs doublet in a typical grand unified model, like $SO(10)$, this will be about 0.02. Thus, typically, in fully unified or partially unified models there is small mixing with hypercharge (of order a few percent). Of course, in principle, large numbers of split multiplets all contributing to $\epsilon$ with the same sign could exist and produce strong mixing of $X'$ with hypercharge. However, it is notoriously difficult to produce split multiplets naturally in unified models (hence the "doublet-triplet splitting problem"). The naturalness problem is compounded the more such split multiplets there are. It therefore seems quite unlikely that there would be large numbers of such multiplets. Even if there were, one might expect that their light components would have mass near or below $M'$, where they could be observed and their effect on mixing could be calculated and thus taken into account. Nevertheless, one cannot rule out the possibility that many split multiplets exist whose lighter components are at some inaccessible intermediate scale. However, this seems a highly artificial possibility.

The basic pattern, then, is simple: in nonunified models $X'$ is expected to mix strongly with $Y$, whereas in partially unified or fully unified models with ordinary breaking of the extra $U(1)$ the mixing should be small (of order a few percent).

Matters are made slightly more complicated by the possibility in certain cases of breaking at $M'$ that is not ordinary, as we shall now see. Suppose that the groups $U(1)_{Y_5}$ and $U(1)_X$ both break at $M'$ leaving unbroken $U(1)_Y$, with $Y/2 = aY_5/2 + bX$, where $a$, $b \neq 0$. This implies that the broken generator $X'$ also is a linear combination of $Y_5/2$ and $X$, and therefore of $Y/2$ and $X$, i.e. just what we mean by "mixing with hypercharge." Now consider what follows from the requirement that the quark doublet $Q_L$ and the lepton doublet $L_L$ come out with the correct hypercharges. Since $Q_L$ has to be in the **10** of $SU(5)$, it has $Y_5/2 = 1/6$. Call its $X$ value $x$. Then one has $1/6 = a(1/6) + bx$. The lepton doublet must be in the $\bar{\mathbf{5}}$ of $SU(5)$ and thus have $Y_5/2 = -1/2$. Call its $X$ value $x/r$. Then one has $-1/2 = a(-1/2) + bx/r$. Combining these two equations gives $0 = b(3 + 1/r)x$. Since $b \neq 0$ there are only two possibilities. The first possibility is that $r = -1/3$, which corresponds to the $X$ charges of **10** and $\bar{\mathbf{5}}$ being in the ratio 1 to $-3$, as in $SO(10)$ models (but, as we shall see, not only in $SO(10)$ models). This leads to the well-known "flipped" breaking. This $r = -1/3$ case is very special and has to be treated separately. We shall see that it still only produces "small mixing with hypercharge" in fully unified models, but can produce "strong mixing with hypercharge" in partially unified models.

The second possibility is that $x = 0$, i.e. the $X$ charge vanishes on both the quark doublet and lepton doublet. In a fully unified or partially unified model this means that $X$ vanishes on all the known quarks and leptons. This also is a special case, which turns out to be possible in partially unified models but not fully unified ones. It leads to mixing (i.e. $X' = \alpha X + \beta Y/2$), but since $X = 0$ on the known fermions, $X' = \beta Y/2$ on those fermions.

We will now simply state the results of our analysis and give that analysis later.

### B. Results of the analysis

We classify models with family-independent extra $U(1)$ groups into seven types, based on whether the generator $X'$ (corresponding to the massive $Z'$ boson) has the form $X' = \alpha\bar{X} + \beta Y/2$, and the values of the parameters $\beta/\alpha$ and $r \equiv \bar{X}(\text{"10"})/\bar{X}(\text{"}\bar{5}\text{"})$. The classes are listed in an order that moves generally from more unification to less.

Class 1 $X' = \alpha\bar{X} + \beta Y/2$, with $\beta/\alpha \ll 1$ ("small mixing with hypercharge"), and $r = -2$, $1/2$, $4/3$ or (perhaps) certain other simple rational values.
Such models are fully unified. If $r = -2$, then the full-unification group is $SU(6)$ or some group containing it, either a larger unitary group or $E_6$. If $r = 1/2$ or $4/3$, then the full-unification group is either $SO(10)$ or $E_6$. (In partially unified models, these values of $r$ would only result from tuning.)

Class 2 $X' = \alpha\bar{X} + \beta Y/2$, with $\beta/\alpha \ll 1$ ("small mixing with hypercharge"), and $r = +1$ or $-1/2$.
Such models are either fully unified in $E_6$ or partially unified in $G \times U(1)$, where $G \supseteq SO(10)$.

Class 3 $X' = \alpha\bar{X} + \beta Y/2$, with $\beta/\alpha \ll 1$ ("small mixing with hypercharge"), and $r$ is not equal to $-2$, $1/2$, $4/3$, 1, $-1/2$, $-1/3$ (i.e. the values characteristic of Classes 1, 2, and 4).
Such models are partially unified.

Class 4 $X' = \alpha\bar{X} + \beta Y/2$, with $\beta/\alpha \ll 1$ ("small mixing with hypercharge"), and $r = -1/3$.
Such models are either fully unified (with gauge group $SO(10)$ or $E_6$) or partially unified with ordinary (i.e. nonflipped) breaking. (The partial unification groups do not have to contain $SO(10)$ or $E_6$: they may also be unitary groups.)

Class 5 $X' = \alpha\bar{X} + \beta Y/2$, $\beta/\alpha \sim 1$ ("strong mixing with hypercharge"), and $r = -1/3$.
Such models are either partially unified with flipped breaking of $3211'$ to the standard model at $M'$, or else they are nonunified, with specific extra fermions (i.e. fermions that do not exist in the standard model).

Class 6 $X' = \beta Y/2$.
Such models are either partially unified or nonunified, but cannot be fully unified.

Class 7 $X' \neq \alpha\bar{X} + \beta Y/2$.
Such models are nonunified.

## III. FULLY UNIFIED MODELS

As was shown in the previous section, in unified models there will only be radiatively-induced (and typically small) mixing of $X'$ with hypercharge except in two special cases: the case where $r = -1/3$ and the case where $X$ vanishes on all the known quarks and leptons. If $r = -1/3$ in a fully unified model, then the gauge group must be $SO(10)$ or $E_6$. This very special case will be treated at length in the next subsection. In the subsection after that it will be shown that the case where $X$ vanishes on all the known quarks and leptons (leading to $X' \propto Y/2$) cannot be realized in fully unified models. In the present subsection the general case where $r \neq -1/3$ will be treated.

First, let us consider the simplest example of a fully unified group: $SU(6)$. The simplest anomaly-free set of $SU(6)$ fermion representations that gives one family consists of $\mathbf{15} + 2 \times (\bar{\mathbf{6}})$. (We shall also denote a $p$-index totally antisymmetric tensor representation as $[p]$. So we could also write the anomaly-free set as $[2] + 2 \times [\bar{1}]$.) The generator of $U(1)_X$ (in the fundamental representation) is $X = \mathrm{diag}(1, 1, 1, 1, 1, -5)$. Under the subgroup $SU(5) \times U(1)_X$ the fermions of a family decompose into $\mathbf{10}^2 + \mathbf{5}^{-4} + 2 \times (\bar{\mathbf{5}}^{-1} + \mathbf{1}^5)$. The effective theory below $M_*$ would have group $SU(3)_c \times SU(2)_L \times U(1)_{Y_5} \times U(1)_X$.

Since $X$ does not vanish on the known quarks and leptons, and $r \neq -1/3$, the analysis in the previous section tells us that there is only the (typically small) radiatively-induced mixing of $X'$ with hypercharge, and therefore $X' = \alpha X + \beta Y/2$ with $\beta/\alpha \ll 1$, and $Y/2 = Y_5/2$. Thus, the standard model group is contained in the $SU(5)$; the "10" $\equiv (e_L^+, Q_L, u_L^c)$ is the $\mathbf{10}$ of $SU(5)$; and the "$\bar{5}$" $\equiv (L_L, d_L^c)$ is the $\bar{\mathbf{5}}$ of $SU(5)$. Consequently, the generator $X$ has equal values for all the multiplets in the 10 and similarly for "$\bar{5}$," and so we may put a bar over it and write $X' = \alpha\bar{X} + \beta Y/2$. Moreover, from the $X$ values of the $SU(5)$ multiplets we see that $r \equiv \bar{X}(\text{"10"})/\bar{X}(\text{"}\bar{5}\text{"}) = X(\mathbf{10})/X(\bar{\mathbf{5}}) = 2/(-1) = -2$. This model falls into Class 1.

It seems to be the case, as discussed in the appendix, that $r = -2$ is the only value obtainable in realistic fully unified models based on the unitary groups, i.e. $SU(N)$. Thus $SU(N)$ full unification, as far as we can tell, leads always to models of Class 1. The value $r = -2$ can also arise in fully unified models based on $E_6$, since $E_6 \supset SU(6)$. However, $r = -2$ does not seem to arise in fully unified models based on $SO(10)$. Also, as we shall see in Sec. IV, the value $r = -2$ does not seem to arise (except by artificial tuning of charge assignments) in partially unified models.

There are some values of $r$, such as $1/2$ and $4/3$ that seem to arise only in fully unified models based on $SO(10)$. Consider $SO(10) \supset SU(5) \times U(1)_X$, with each family containing a $\mathbf{16} + \mathbf{10} = (\mathbf{10}^1 + \bar{\mathbf{5}}^{-3} + \mathbf{1}^5) + (\bar{\mathbf{5}}^2 + \mathbf{5}^{-2})$. If the known quarks and leptons are in the $\mathbf{10}^1 + \bar{\mathbf{5}}^2$, then $r = 1/2$ results. (It should be noted that in this case flipped breaking is not possible, and so there is not strong mixing with hypercharge as there can be in the $r = -1/3$ case.) If each family consists of a $\mathbf{16} + \mathbf{45}$ of $SO(10)$, then the known quarks and leptons could be in a $\mathbf{10}^{-4} + \bar{\mathbf{5}}^{-3}$ of $SU(5) \times U(1)_X$, yielding $r = 4/3$. (This model has so many light multiplets that it can only narrowly escape a Landau pole at scales below the unification scale.) Models with $r = 1/2$ and $r = 4/3$ also fall into Class 1.

There are values of $r$, such as 1 and $-1/2$ that can result from either full unification or partial unification. These values arise from full unification in $E_6$ if $E_6 \supset SO(10) \times U(1)_X \supset SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$. Then, if a family is a $\mathbf{27}$, it decomposes under $SU(5) \times U(1)_X$ as $\mathbf{10}^1 + \bar{\mathbf{5}}^1 + \mathbf{1}^1 + \bar{\mathbf{5}}^{-2} + \mathbf{5}^{-2} + \mathbf{1}^4$ (where we use $SU(5)$ multiplets as shorthand for the standard model multiplets). If the known quarks and leptons are in $\mathbf{10}^1 + \bar{\mathbf{5}}^1$ then $r = +1$, and if they are in $\mathbf{10}^1 + \bar{\mathbf{5}}^{-2}$ then $r = -1/2$. However, these same values of $r$ can also arise in partial unification based on $SO(10) \times U(1)_X \supset SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$, since anomaly cancellation and family-independence alone are enough to fix the $U(1)_X$ charges to be the "$E_6$ values" if there are only $\mathbf{16} + \mathbf{10} + \mathbf{1}$ in each family. On the other hand, the values $r = +1$ and $-1/2$ do not arise in nonunified models (without artificial tuning of charge assignments). Models with $r = +1$ or $r = -1/2$ fall into Class 2.

The value $r = -1/3$, as noted before, is very special. It arises in full unification based on $SO(10)$, but also, as we shall see in later sections, it can arise naturally in both partially unified models and nonunified models.

Depending on how much mixing there is of the extra $U(1)$ charge $X'$ with hypercharge these models fall into Class 4 or 5.

A general conclusion about fully unified models is that there is not strong mixing of the extra $U(1)$ charge with hypercharge (except in the somewhat artificial case that there are many highly split multiplets that induce it radiatively).

### A. An important special case: $r = -1/3$

In fully unified models, the case $r = -1/3$ arises only in $SO(10)$ or $E_6$. Let us look at this special case more closely. (The present analysis will carry over almost completely also to the case $r = -1/3$ in partially unified models.)

Suppose one has $SU(3)_c \times SU(2)_L \times U(1)_{Y_5} \times U(1)_X \subset SU(5) \times U(1)_X \subset SO(10)$. A family consists of the $SU(5) \times U(1)_X$ representations $\mathbf{10}^1 + \bar{\mathbf{5}}^{-3} + \mathbf{1}^5$. Let the covariant derivative contain the following combination of $U(1)$ gauge fields

$$iD_\mu = i\partial_\mu + \left(g_1 \frac{\hat{Y}_5}{2} B_{1\mu} + g_X \hat{X} B_{X\mu}\right) + \ldots, \qquad (1)$$

where the subscripts 1 and $X$ refer, respectively, to $U(1)_{Y_5}$ and $U(1)_X$, and we denote by hats generators normalized consistently in $SO(10)$, so that $\mathrm{tr}_{16}\hat{\lambda}^2 = 2$. Then $\frac{\hat{Y}_5}{2} = \sqrt{\frac{3}{5}}\frac{Y_5}{2}$ and $\hat{X} = \frac{1}{\sqrt{40}} X$ The flipped breaking at $M'$ can be achieved by the VEV of an $SU(3)_c \times SU(2)_L$-singlet field having $Y_5/2 = X = 1$ (such as exists in the spinor of $SO(10)$). This leaves unbroken $Y/2 = \frac{1}{5}(-Y_5/2 + X)$. Therefore, in terms of the normalized generators, we may write

$$\frac{\hat{Y}}{2} = -\frac{1}{5}\frac{\hat{Y}_5}{2} + \frac{\sqrt{24}}{5}\hat{X}. \qquad (2)$$

The $U(1)$ charge that is orthogonal to this in $SO(10)$ is given by

$$\hat{\bar{X}} = \frac{\sqrt{24}}{5}\frac{\hat{Y}_5}{2} + \frac{1}{5}\hat{X}. \qquad (3)$$

Inspection of Eq. (1) shows that the massive gauge boson is

$$Z'_\mu = \frac{\sqrt{24} g_1 B_{1\mu} + g_X B_{X\mu}}{\sqrt{24 g_1^2 + g_X^2}}, \qquad (4)$$

and the gauge field $B_\mu$ of $U(1)_Y$ is the orthogonal combination

$$B_\mu = \frac{-g_X B_{1\mu} + \sqrt{24} g_1 B_{X\mu}}{\sqrt{24 g_1^2 + g_X^2}}, \qquad (5)$$

Inverting Eqs. (4) and (5), Eq. (1) can be rewritten as

$$iD_\mu = i\partial_\mu + \frac{g_1 g_X}{\sqrt{24 g_1^2 + g_X^2}}\left[-\frac{\hat{Y}_5}{2} + \sqrt{24}\hat{X}\right]B_\mu + \left[\frac{\sqrt{24} g_1^2}{\sqrt{24 g_1^2 + g_X^2}}\frac{\hat{Y}_5}{2} + \frac{g_X^2}{\sqrt{24 g_1^2 + g_X^2}}\hat{X}\right]Z'_\mu + \ldots. \qquad (6)$$

Then inverting Eqs. (2) and (3), this can be reexpressed as

$$iD_\mu = i\partial_\mu + \left(\frac{5 g_1 g_X}{\sqrt{24 g_1^2 + g_X^2}}\right)\frac{\hat{Y}}{2}B_\mu + \left[\frac{1}{5}\sqrt{24 g_1^2 + g_X^2}\,\hat{\bar{X}} + \frac{\sqrt{24}}{5}\frac{g_X^2 - g_1^2}{\sqrt{24 g_1^2 + g_X^2}}\frac{\hat{Y}}{2}\right]Z'_\mu \ldots. \qquad (7)$$

Note that $B$ couples to hypercharge, as it should, and $Z'$ couples to a combination of $\bar{X}$ and hypercharge. Let us see what $\bar{X}$ is. It is convenient to normalize it as $\bar{X} = \sqrt{40}\hat{\bar{X}} = \frac{1}{5}(24\frac{Y_5}{2} + X)$. The charges of the known quarks and leptons under $Y_5$, $X$, $Y/2$, and $\bar{X}$ are given in Table I.

One sees that the "10" $\equiv (e^+, Q, u^c)$ does not coincide with the $\mathbf{10}^1$ of $SU(5) \times U(1)_X$, and the "$\bar{5}$" $\equiv (L, d^c)$ does not coincide with the $\bar{\mathbf{5}}^{-3}$ of $SU(5) \times U(1)_X$ (though there is another $SU(5) \times U(1)$ subgroup of $SO(10)$ of which they *are* multiplets). This is just the well-known phenomenon of flipping. However, note that the generator $\bar{X}$ does have equal values for all the multiplets in the "10" and equal values for all the multiplets in the "$\bar{5}$." which is why we have denoted it with a bar, consistent with the notation we explained in the previous section. Thus, the generator $X'$ to which $Z'$ couples can be written

$$X' = \alpha\hat{\bar{X}} + \beta\frac{\hat{Y}}{2} = \frac{1}{\sqrt{40}}\alpha\bar{X} + \sqrt{\frac{3}{5}}\beta\frac{Y}{2}, \qquad (8)$$

where from Eq. (7) one has

$$\beta/\alpha = \frac{\sqrt{24}(g_X^2 - g_1^2)}{24 g_1^2 + g_X^2}. \qquad (9)$$

In other words, there is "mixing with hypercharge." If the couplings $g_1$ and $g_X$ were equal at $M'$, the expression in Eq. (9) would vanish. Of course, these couplings are equal

TABLE I. The charges are related by $Y/2 = \frac{1}{5}[-(Y_5/2) + X]$ and $\bar{X} = \frac{1}{5}[24(Y_5/2) + X]$.

| field | $SU(5)$ | $Y_5/2$ | $X$ | $Y/2$ | $\bar{X}$ | |
|---|---|---|---|---|---|---|
| $N^c$ | **10** | 1 | 1 | 0 | 5 | "1" |
| $Q$ | **10** | $\frac{1}{6}$ | 1 | $\frac{1}{6}$ | 1 | 10 |
| $d^c$ | **10** | $-\frac{2}{3}$ | 1 | $\frac{1}{3}$ | $-3$ | "5" |
| $L$ | $\bar{\mathbf{5}}$ | $-\frac{1}{2}$ | $-3$ | $-\frac{1}{2}$ | $-3$ | 5 |
| $u^c$ | $\bar{\mathbf{5}}$ | $\frac{1}{3}$ | $-3$ | $-\frac{2}{3}$ | 1 | 10 |
| $e^+$ | **1** | 0 | 5 | 1 | 1 | 10 |

at the scale where $SO(10)$ breaks; however, they will in general run slightly differently between $M_{SO(10)}$ and $M'$ due primarily to the Higgs contributions to the beta functions. The known quarks and leptons do *not* make $g_1$ and $g_X$ run differently at one loop, because they form complete $SO(10)$ multiplets. (Remember that the $N^c$ have masses of order $M'$ since they are protected by $U(1)_X$. It is possible for some other, $SO(10)$-singlet fields to play the role of superheavy right-handed neutrinos for the seesaw mechanism.) The Higgs-boson multiplets' contribution being relatively small, one expects that the contribution to $\beta/\alpha$ from Eq. (9) will be rather small. Indeed, in typical cases it turns out to be a few percent. Of course, there is also the contribution to $\beta/\alpha$ coming from the radiatively-induced gauge kinetic mixing discussed earlier, which is also a few percent typically. So in fully unified models, the mixing with hypercharge is small whether the breaking is flipped or ordinary. We shall see in the next section that this is not the case in partially unified models.

### B. Another special case: $X' \propto Y/2$ and why it is impossible

As noted in section II, mixing of $X'$ with hypercharge is possible in unified models if $X$ vanishes on the known quarks and leptons. Then $Y/2 = \alpha Y_5/2 + \beta X \propto Y_5/2$, which, of course, would give the right hypercharge assignments. This would be interesting as it would mean that $X'$ (which is also a linear combination of $Y_5/2$ and $X$) would have values for the known quarks and leptons proportional to their hypercharges—the defining characteristic of models in Class 6. However, we will now show that in a *fully* unified model this possibility cannot be realized (though it can be realized in partially unified models). The reason is that in a fully unified model there are in general extra nonsinglet fermions in each family whose existence is compelled by the fact that the multiplets of the full-unification groups are large. It turns out that if $X$ vanishes on the known fermions then the extra fermions end up being chiral under hypercharge and electric charge and thus cannot obtain mass. We will illustrate this with some examples, and it will be obvious that it generalizes.

Example 1—Suppose that $SU(3)_c \times SU(2)_L \times U(1)_{Y_5} \times U(1)_X \subset SU(5) \times U(1)_X \subset SU(7)$, with the generator $X$ being $X = \mathrm{diag}(0, 0, 0, 0, 0, \frac{1}{2}, -\frac{1}{2})$. An anomaly-free set that gives one family is $[2] + 3 \times [\bar{1}] = \mathbf{21} + 3 \times (\bar{\mathbf{7}})$. Under $SU(5) \times U(1)$ this decomposes to $\mathbf{10}^0 + \mathbf{5}^{1/2} + \mathbf{5}^{-1/2} + \mathbf{1}^0 + 3 \times (\bar{\mathbf{5}}^0 + \mathbf{1}^{1/2} + \mathbf{1}^{-1/2})$. By assumption, $X$ vanishes on the known quarks and leptons, which therefore consist of $(\mathbf{10}^0 + \bar{\mathbf{5}}^0)$, and the remaining fermions $\mathbf{5}^{1/2} + \mathbf{5}^{-1/2} + \bar{\mathbf{5}}^0 + \bar{\mathbf{5}}^0$ etc. must mate to obtain masses large enough that they have not been observed. However, the hypercharge of the standard model is, by assumption, a nontrivial linear combination of $Y_5/2$ and $X$. Therefore it is clear that the fields in $\mathbf{5}^{1/2} + \mathbf{5}^{-1/2}$ do not have hypercharges that are opposite to the hypercharges of the fields in $\bar{\mathbf{5}}^0 + \bar{\mathbf{5}}^0$, and consequently they do not have opposite electric charges either. They are prevented from acquiring mass unless electric charge breaks. Moreover, the residual light fermions in $\mathbf{5}^{1/2} + \mathbf{5}^{-1/2}$ will have exotic hypercharges.

Example 2—The previous example can easily be generalized to $SU(N)$. Consider $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X \subset SU(5) \times U(1)_X \subset SU(N)$. Let $X$ (in the fundamental representation) be given by $\mathrm{diag}(0, 0, 0, 0, 0, \frac{1}{2}, -\frac{1}{2}, 0, \ldots, 0)$, where the first five entries correspond to the $SU(5)$ that contains $SU(3)_c \times SU(2)_L$. Let the fermions be in totally antisymmetric tensor representations: $n_1 \times [1] + n_2 \times [2] + n_3 \times [3] + \ldots$. An antisymmetric tensor representation decomposes under the $SU(5) \times U(1)$ subgroup as follows.

$$\begin{aligned}[p] \to & \left[\binom{N-7}{p-4} + \binom{N-7}{p-6}\right] \times \bar{\mathbf{5}}^0 \\ & + \binom{N-7}{p-5} \times (\bar{\mathbf{5}}^{1/2} + \bar{\mathbf{5}}^{-1/2}) \\ & + \left[\binom{N-7}{p-3} + \binom{N-7}{p-5}\right] \times \overline{\mathbf{10}}^0 \\ & + \binom{N-7}{p-4} \times (\overline{\mathbf{10}}^{1/2} + \overline{\mathbf{10}}^{-1/2}) \\ & + \left[\binom{N-7}{p-2} + \binom{N-7}{p-4}\right] \times \mathbf{10}^0 \\ & + \binom{N-7}{p-3} \times (\mathbf{10}^{1/2} + \mathbf{10}^{-1/2}) \\ & + \left[\binom{N-7}{p-1} + \binom{N-7}{p-3}\right] \times \mathbf{5}^0 \\ & + \binom{N-7}{p-2} \times (\mathbf{5}^{1/2} + \mathbf{5}^{-1/2}) \\ & + \text{singlets}. \end{aligned} \tag{10}$$

The known standard model families must consist, by assumption, of $3 \times (\mathbf{10}^0 + \bar{\mathbf{5}}^0)$. The remaining fermions, if they are to get mass, must be vectorlike under $U(1)_{Y_5} \times U(1)_X$. (Otherwise, their masses would break electric charge, as we have seen.) That means that there must be equal numbers of $(\mathbf{10}^{1/2} + \mathbf{10}^{-1/2})$ and of $(\overline{\mathbf{10}}^{1/2} + \overline{\mathbf{10}}^{-1/2})$, and similarly of $(\mathbf{5}^{1/2} + \mathbf{5}^{-1/2})$ and of $(\bar{\mathbf{5}}^{1/2} + \bar{\mathbf{5}}^{-1/2})$. These two conditions give, respectively,

$$\begin{aligned} \sum_p n_p \binom{N-7}{p-3} &= \sum_p n_p \binom{N-7}{p-4}, \\ \sum_p n_p \binom{N-7}{p-2} &= \sum_p n_p \binom{N-7}{p-5}. \end{aligned} \tag{11}$$

However, these imply that the number of $\mathbf{10}^0$ minus the number of $\overline{\mathbf{10}}^0$, i.e. the number of families, must vanish:

$$n_{\rm fam} = \sum_p n_p\left[\binom{N-7}{p-2} + \binom{N-7}{p-4} - \binom{N-7}{p-3} - \binom{N-7}{p-5}\right] = 0. \qquad (12)$$

We believe that this generalizes to all other types of representations, other full-unification groups, and other $U(1)_X$ subgroups.

## IV. PARTIALLY UNIFIED MODELS

We have defined a partially unified model to be one where the group $3211'$ describing physics below $M'$ is embedded as follows: $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X \subset SU(5) \times U(1)_X \subseteq G \times U(1)_X \subseteq G \times H$, where $G$ is a simple group. The same reasoning as for fully unified groups shows that $X$ does mix strongly with hypercharge except in two special cases: (a) $r = -1/3$ and the breaking at $M'$ happens in a flipped way, or (b) $X$ vanishes on the known quarks and leptons. The reason, again, is that except for these two special cases strong mixing of $X'$ with hypercharge will cause the hypercharges of the known quarks and leptons to come out wrong. The value $r = -1/3$ arises in the simplest $SO(10)$ models, and so we will call models with $r = -1/3$ "$SO(10)$-like," even though, as we shall see, they may be based on other groups (including unitary ones), both partially unified and nonunified.

### A. The "$SO(10)$-like" and flipped special case

Consider a model with group $SU(5) \times U(1)_X$ and fermion multiplets (per family) of $\mathbf{10}^a + \bar{\mathbf{5}}^b + \mathbf{1}^c$. Then there are three anomalies that must be satisfied by the $X$ charges: $5^2 1_X$, $1_X^3$ and $1_X$. These give the unique solution (up to overall normalization) $(a, b, c) = (1, -3, 5)$. (As always, we assume that $X$ is family-independent.) These are the same charges that would be obtained if $SU(5) \times U(1)_X$ were embedded in $SO(10)$. We will therefore call such models "$SO(10)$-like." The analysis given in Eqs. (1)–(9) of what happens if the $U(1)_X$ is broken in a flipped manner applies here as well, except that the gauge coupling of $U(1)_{Y_5}$ is not unified with that of $U(1)_X$. Consequently, what we called $g_1$ and $g_X$ are not related, and there is no reason for the parameter $\beta/\alpha$ given in Eq. (9) to be small. Rather, one expects it to be of order one, typically. This gives models of Class 5, then, rather than Class 4.

It is worth noting that one can get $SO(10)$-like models with other choices of fermion content and other partial unification groups. For example, in $SU(5) \times U(1)_X$, if there are (per family) $\mathbf{10}^a + \bar{\mathbf{5}}^b + \mathbf{1}^c + \mathbf{1}^d$, the unique solution (up to overall normalization) is $(a, b, c, d) = (1, -3, 5, 0)$. Note that the $\mathbf{1}^0$ could play the role of right-handed neutrino with superheavy mass, giving realistic seesaw masses for the light neutrinos. later we shall see an $SO(10)$-like model resulting from unitary groups like $SU(6) \times U(1)$.

### B. The $X' \propto Y/2$ special case

This special case can be realized in partial unification without producing massless fermions with exotic charges—in fact quite, trivially. For example, let the only quarks and leptons be in $3 \times (\mathbf{10}^0 + \bar{\mathbf{5}}^0 + \mathbf{1}^0)$ of $SU(5) \times U(1)_X$, and let some Higgs field (for example $\mathbf{10}_H^q$) break both $Y_5$ and $X$ at $M'$, leaving unbroken $SU(3) \times SU(2) \times U(1)$. There is no problem here with extra quark and lepton multiplets that have chiral values of hypercharge and electric charge which prevent them from obtaining mass, since unlike the fully unified case there is here no larger simple group containing $SU(5) \times U(1)$ that implies their existence. Thus models of Class 6 can arise from partial unification.

### C. The general case of no mixing

Turning now to the more generic cases where $X'$ does *not* mix strongly with hypercharge, we will show that the partially unified models can be distinguished from the fully unified ones by the fact that they generally give different values of $r$. It is simplest to consider a few examples.

Consider, first, a model with group $SU(5) \times U(1)_X$ and fermions content (per family) consisting of $\mathbf{10}^a + \bar{\mathbf{5}}^b + \bar{\mathbf{5}}^c + \mathbf{5}^d + \mathbf{1}^e$. There is a unique solution of the anomaly conditions (up to interchange of the two $\bar{\mathbf{5}}$'s and overall normalization): $(a, b, c, d, e) = (1, -3, x, -x, 5)$, with $x$ undetermined. If one takes a family to consist of $\mathbf{10}^1 + \bar{\mathbf{5}}^{-3} + \mathbf{1}^5$, one has an $SO(10)$-like model. However, it is possible that a family consists of $\mathbf{10}^1 + \bar{\mathbf{5}}^x + \mathbf{1}^5$. In this case, one has $r = \bar{X}(\mathbf{10})/\bar{X}(\bar{\mathbf{5}}) = X(\mathbf{10})/X(\bar{\mathbf{5}}) = 1/x$. This can be any number; anomaly cancellation leaves it completely undetermined. It therefore has no reason to be equal to one of the characteristic values (like $-2$ and $-1/3$) that occur in fully unified models. Therefore, such a model would be in Class 3.

If one requires in this latter model (where a family is in $\mathbf{10}^1 + \bar{\mathbf{5}}^x + \mathbf{1}^5$) that the light Higgs doublets $H_d$ and $H_u$ have opposite $X$, and further that the quark and lepton masses all come from dimension-four Yukawa terms, then it would force $X(H_u) = -2$, $X(H_d) = +2$, and $x = -3$, giving an $SO(10)$-like model. However, it is also possible that $H_u$, $H_d$ have $X = -2$, $+2$, but that the Yukawa terms have the form: $(\mathbf{10}^1\mathbf{10}^1)\mathbf{5}_H^{-2}$ and $(\mathbf{10}^1\bar{\mathbf{5}}^x)\bar{\mathbf{5}}_H^2\mathbf{1}_H^{-3-x}/M'$. Note that there must, in any event, be a Higgs field $\mathbf{1}_H^{3+x}$ with VEV of order $M'$ if the extra fermions $\bar{\mathbf{5}}^{-3} + \mathbf{5}^{-x}$ are to get large mass together. In fact, by integrating out these extra fermions, it is possible to get just the dimension-five Yukawa term that we have written. Thus, the Yukawa terms do not impose any *a priori* constraint on $x$ or $r$. (This integrating out of the extra fermions to produce effective dimension-five Yukawa terms for the known quarks and leptons will produce some mixing between the $\bar{\mathbf{5}}^{-3}$ and the $\bar{\mathbf{5}}^{x}$. This mixing can be small, in which case $r$ would be given very nearly by $1/x$ for all families. If, however, this mixing is large, one would get a complicated pattern of couplings to $Z'$ that would not in general be family-independent.)

The next example is one where anomalies actually force a unique solution for $r$, but it does not come out to be one of the characteristic values that arise from full unification. Consider a model with gauge group $SU(5) \times U(1)$ and fermion content (per family) of $\mathbf{10}^a + \bar{\mathbf{5}}^b + \mathbf{15}^c + \overline{\mathbf{15}}^d$. This gives the solutions $(a, b, c, d) = (4, -5, -1, 0)$ or $(4, -5, 0, -1)$. In either case, $r = -4/5$. This model is also in Class 3.

These examples show that one can get values of $r$ in partially unified models that do not arise in fully unified models. Turning the question around, one can ask whether partially unified models can give all the values of $r$ that do arise in fully unified models. Of course, one can get them in some models by simply choosing the charges arbitrarily to have the right values, which is a kind of fine-tuning. The question is whether anomaly cancellation without full unification can *force* $r$ to have one of those values characteristic of full unification. We have already seen that the answer is yes for the special cases $r = +1$ and $r = -1/2$ (which can come from $SO(10) \times U(1)_X$ as well as $E_6$) and $r = -1/3$ (which can arise in many ways). However, these $E_6$-like and $SO(10)$-like values are special in this regard. The value $r = -2$ characteristic of full-unification based on $SU(N)$ does not seem ever to be forced by anomaly cancellation in partially unified models.

We will now see why this the case by looking at a simple example. We saw that the value $r = -2$ can arise in fully unified models based on $SU(6)$. Can the value $r = -2$ be forced by anomaly cancellation (plus the assumption of family-independence) in a partially unified model? In the simplest $SU(6)$ model, a family consists of $\mathbf{21} + 2 \times \bar{\mathbf{7}}$, which decomposes under the $SU(5) \times U(1)_X$ subgroup into $\mathbf{10}^2 + \mathbf{5}^{-4} + 2 \times (\bar{\mathbf{5}}^{-1} + \mathbf{1}^5)$, as noted before. One might take this set of $SU(5)$ representations and ask whether anomaly cancellation alone would force the same solution for the $U(1)_X$ charges. The answer is no. For the set $\mathbf{10}^a + \mathbf{5}^b + \bar{\mathbf{5}}^c + \bar{\mathbf{5}}^d + \mathbf{1}^e + \mathbf{1}^f$, the most general solution to the anomaly-cancellation conditions has two parameters (not counting overall normalization). A simple one-parameter subset of this solution is $(a, b, c, d, e, f) = (2, -4, -1 + x, -1 - x, 5 + x, 5 - x)$, with $x$ arbitrary; so that $r = 2/(-1 \pm x)$ and can be anything. (Note that $x = \mp 5$ would give $SO(10)$-like charges, and $x = 0$ gives $SU(6)$-like charges.) Anomaly cancellation does not force a particular value of $x$. The reason for this ambiguity lies in $E_6$. $E_6$ has the chain of subgroups $E_6 \supset SU(6) \times SU(2) \supset SU(5) \times U(1)_6 \times SU(2) \supset SU(5) \times U(1)_6 \times U(1)_2$. The $\mathbf{27}$ of $E_6$ decomposes into these $5 1_6 1_2$ multiplets: $\mathbf{10}^{(2,0)} + \mathbf{5}^{(-4,0)} + (\bar{\mathbf{5}}^{(-1,+1)} + \bar{\mathbf{1}}^{(-1,-1)}) + (\mathbf{1}^{(5,+1)} + \mathbf{1}^{(5,-1)})$. Clearly, any $U(1)$ generator that is a linear combination of the generators of $U(1)_6$ and $U(1)_2$ will satisfy the anomaly-cancellation conditions since $E_6$ is an anomaly-free group. The undetermined parameter $x$ that we found in the $SU(5) \times U(1)$ solution just reflects this ambiguity.

If one reduces the number of multiplets per family, one reduces the number of unknowns and may get a unique solution to the anomaly-cancellation conditions; however, the unique solution will not be $r = -2$. In fact, if we remove one of the singlets one has the first example in this subsection, for which anomaly cancellation gives $\mathbf{10}^1 + \bar{\mathbf{5}}^{-x} + \bar{\mathbf{5}}^{-3} + \mathbf{5}^x + \mathbf{1}^5$, so that either $r = -1/3$ or $r = 1/x$ (i.e. undetermined). If we remove a pair of fundamental plus antifundamental, we have already seen that one gets uniquely the $SO(10)$-like solution $\mathbf{10}^1 + \bar{\mathbf{5}}^{-3} + \mathbf{1}^5 + \mathbf{1}^0$.

On the other hand, if one adds more $SU(5)$ multiplets per family, it just increases the number of undetermined parameters, so that again $r$ is not forced to be $-2$. Nor does going to larger partial unification groups allow situations where anomalies force $r = -2$. Consider, for instance, $SU(6) \times U(1)_1$ with fermions $\mathbf{15}^a + \bar{\mathbf{6}}^b + \bar{\mathbf{6}}^c + \mathbf{1}^d$. The three anomaly conditions $(6^2 1, 1^3, 1)$ force the values $(a, b, c, d) = (1, 1, -5, 9)$ (up to an overall normalization). Under the subgroup $SU(5) \times U(1)_6 \times U(1)_1$ the fermions of a family decompose into $\mathbf{10}^{(2,1)} + \mathbf{5}^{(-4,1)} + \bar{\mathbf{5}}^{(-1,1)} + \mathbf{1}^{(5,1)} + \bar{\mathbf{5}}^{(-1,-5)} + \mathbf{1}^{(5,-5)} + \mathbf{1}^{(0,9)}$. The extra $U(1)_X$ is some linear combination of $U(1)_6$ and $U(1)_1$. Which linear combination it is depends on how the groups break at the scale $M_*$, and that in turn depends on what kinds of standard model-singlet Higgs fields exist in the model.

Generally, the $U(1)_1$ charges of the Higgs fields are not constrained by anomaly cancellation. (Even in supersymmetric models where the Higgs fields have fermionic partners, these generally come in conjugate pairs, for reasons explained above, and so their anomalies cancel.) There is at least one standard model-singlet Higgs fields that must appear in such a model, namely, the one required to give mass to the extra $\bar{\mathbf{5}} + \mathbf{5}$ of quarks and leptons. There are two possibilities: the $\mathbf{5}^{(-4,1)}$ can either get mass with the $\bar{\mathbf{5}}^{(-1,1)}$ or with the $\bar{\mathbf{5}}^{(-1,5)}$. In the former case, the required singlet Higgs is $\mathbf{1}_H^{(5,-2)}$, in the latter it is $\mathbf{1}_H^{(5,4)}$. In neither case does the singlet Higgs have to be the one responsible for breaking down to $3211'$ at $M_*$. (These singlets could get VEVs much smaller than the scale $M_*$, with some other singlet doing the breaking at $M_*$.) However, if either of them is the one responsible for the breaking to $3211'$ at $M_*$, one easily sees that an $SO(10)$-like extra $U(1)$ is left unbroken. For example, if $\langle \mathbf{1}_H^{(5,-2)} \rangle \sim M_*$, then the unbroken generator is $X = (2X_6 + 5X_1)/9$, where we have used a convenient normalization. This leads to the known quarks and leptons having $X(\mathbf{10}^{(2,1)}) = 1$ and $X(\bar{\mathbf{5}}^{(-1,-5)}) = -3$, giving $r = -1/3$. The other case is similar. The reason that an $SO(10)$-like model results is simple: the theory below $M_*$ has the fermion content per family $\mathbf{10} + \bar{\mathbf{5}} + \mathbf{1} + \mathbf{1}$, which we have already seen to be forced by anomaly cancellation to give a $U(1)_X$ that is $SO(10)$-like.

One sees that partial unification can lead to $SO(10)$-like models with $r = -1/3$, in which case the models are Class 4 or Class 5, depending on whether the breaking of $3211'$ at $M'$ happens in the ordinary or flipped manner. It can also lead to $E_6$-like models $r = +1$ or $-1/2$, which fall into Class 2. Finally, it can lead to models with arbitrary values of $r$ not equal to any of the special values characteristic of full unification; such models fall into Class 3.

## V. NON-UNIFIED MODELS

In nonunified models, there is no unification of the groups $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'$ above the scale $M_*$, and therefore no group-theory constraints on the charge assignments of *either* $U(1)$ group. If the only constraints on these charges come from anomaly cancellation, there is no guarantee in general that the hypercharge assignments will come out correct or even that a $U(1)$ group will be left unbroken when the extra fermions acquire mass at $M'$. However, if after the breaking at $M'$ only the standard model quark and lepton multiplets remain light and a $U(1)$ is indeed left unbroken, then anomaly cancellation and family-independence guarantee that the charge assignments of the light quarks and leptons under the unbroken $U(1)$ will correspond to the known hypercharges [10].

### A. Obtaining the hypercharge group

Let us illustrate some of these points with a simple example. Consider first a model in which there are two extra singlets per family, so that each family consists of $(Q, u^c, d^c, L, e^c, N, N')$. Let the gauge group be $SU(3)_c \times SU(2)_L \times U(1)_1 \times U(1)_2$, where we label the Abelian factors as we do because we do not yet know whether hypercharge will emerge from the anomaly conditions. (Of course, we assume as always that the gauge groups couple in a family-independent way.)

There are ten anomaly conditions that constrain the Abelian charge assignments: $3^2 1_1$, $2^2 1_1$, $1_1^3$, $1_1$, $3^2 1_2$, $2^2 1_2$, $1_2^3$, $1_2$, $1_1^2 1_2$, and $1_1 1_2^2$. The first four anomaly conditions, which constrain only the $U(1)_1$ charge assignments, force them to be of the form $X_1 = (1, -4 + x_1, 2 - x_1, -3, 6 - e_1 - f_1, e_1, f_1)$, where we list them in the same order as we listed the multiplets above. The cubic anomaly condition gives the relation $0 = 6x_1(x_1 - 6) + (e_1 + f_1)(e_1 - 6)(f_1 - 6)$. We have chosen to normalize these charges so that $X_1(Q) = 1$. The next four anomaly conditions, which constrain only the $U(1)_2$ charge assignments, force them to be $X_2 = (1, -4 + x_2, 2 - x_2, -3, 6 - e_2 - f_2, e_2, f_2)$, where $0 = 6x_2(x_2 - 6) + (e_2 + f_2)(e_2 - 6)(f_2 - 6)$. Again, we have normalized these to make $X_2(Q) = 1$. Finally, the remaining two anomaly conditions ($1_1^2 1_2$ and $1_1 1_2^2$) give a pair of cubic equations that must be satisfied by the parameters $x_1$, $e_1$, $f_1$, $x_2$, $e_2$, and $f_2$. Altogether, then, there are six parameters that must satisfy four nonlinear equations. That means that there are two-parameter families of solutions. We may take those parameters to be $e_1$ and $f_1$, which are the charges under $U(1)_1$ of the extra singlets $N$ and $N'$, and these may take values in a finite range.

At first glance, it is not obvious that in the general case any linear comination of $X_1$ and $X_2$ will give the standard model hypercharges for the known quarks and leptons. However, it is not difficult to see that one linear combination does and that it is easy to break $U(1)_1 \times U(1)_2$ down to it. For suppose that there is a singlet Higgs field $S$ that has Dirac couplings to the extra singlet fermions: $h_{ij}(N_i N_j')S$, where $i$, $j$ are family indices. If $S$ obtains a VEV of order $M'$ it leaves one linear combination of $X_1$ and $X_2$ unbroken. Since it also leaves only the quarks and leptons of the standard model light, we know from anomaly cancellation that the unbroken $U(1)$ must act on the known quarks and leptons as the standard model hypercharge (up to an overall nomalization, of course, that can be absorbed into the gauge coupling.)

On the other hand, suppose that the extra fermions got mass at the scale $M'$ from Majorana terms like $(NN) \times \langle S \rangle + (N'N')\langle S' \rangle$. Then, unless the charge assignments were tuned to special values, no $U(1)$ group would be left unbroken below $M'$, so that the standard model would not be reproduced.

### B. Models that reproduce the standard model

If one is dealing with a model that reproduces the standard model, then we can write $U(1)_1 \times U(1)_2 = U(1)_{Y'} \times U(1)'$, where $Y'$ equals the standard hypercharges on the known quarks and leptons. (However, $Y'$ need not have the "standard" values on extra fermions: on them it can have any values consistent with their mass terms. The extra fermions, whose masses are of order $M'$ are obviously vectorlike under $Y'$.) Thus, $Y'$ satisfies several anomaly conditions automatically (namely $3^2 1_{Y'}$, $2^2 1_{Y'}$, $1_{Y'}^3$ and $1_{Y'}$), and we need only consider six anomaly-cancellation conditions for the extra $U(1)$: $3^2 1_{X'}$, $2^2 1_{X'}$, $1_{Y'}^2 1_{X'}$, $1_{Y'} 1_{X'}^2$, $1_{X'}^3$, $1_{X'}$.

If the only fermions are those of the standard model, and their charges are assumed to be family-independent, then there are only four unknowns, namely, the ratios of the $X'$ charges of $Q$, $u^c$, $d^c$, $L$, $e^+$. The only solution is hypercharge itself, i.e. $X' = \beta Y/2$. Of course, the Higgs fields or other scalars that might exist in the low-energy theory can have arbitrary $X'$. Such models would fall into Class 6.

If there are additional fermion multiplets per family, then several possibilities exist, depending on what those fermions are. In some cases, the solutions still give $X' = \beta Y/2$ on the known fermions and fall into Class 6. In other cases, the solutions are $SO(10)$-like, in that case $X' = \alpha X_{10} + \beta Y/2$, on the known fermions, where $X_{10}(e^+, Q, u^c, L, d^c) = (1, 1, 1, -3, -3)$. Such models fall into Class 5, since the parameter $\beta$ has no reason to be small. (The mixing with hypercharge in this case does not have to arise from a simultaneous breaking of $Y_5$ and $X$ at $M'$, as in partially unified or fully unified models, where $X$ is a generator that commutes with an $SU(5)$ and is unmixed with hypercharge to begin with. In nonunified models, the anomaly conditions permit arbitrary mixing with hypercharge, and there is *nothing* to make that mixing small—not even assumptions about which Yukawa couplings and Higgs multiplets exist, since all Yukawa couplings must conserve hypercharge.) Finally, there are cases, where the solutions are messy and not of the form $X' = \alpha\bar{X} + \beta$. These fall into Class 7.

Let us look at some simple examples. (A) If there exists in addition to the known light fermions just one standard model singlet per family (call it $N$), then the most general solution is $SO(10)$-like: $X' = \alpha X_{10} + \beta Y/2$, and falls into Class 5. (In this case there are five unknowns satisfying six equations, since the overall normalization of the charges does not matter. The fact that a nontrivial solution exists is explained, of course, by the fact that the fermions in this case are able to fit into the spinor of $SO(10)$, even though no $SO(10)$ actually exists in the model.)

(B) If the extra fermions per family are just two standard model singlets, $N$ and $N'$, then there are *two* solutions to the anomaly conditions. One solution has $X' = \alpha X_N + \beta Y/2$, where $X_N$ is $+1$ and $-1$ on the two singlets and vanishes on all other fermions. On the known fermions this gives $X' = \beta Y/2$, and therefore falls into Class 6. The other solution has $X' = \alpha X_{10} + \beta Y/2$, where $X_{10}$ has the values given above for the known fermions and is $+5$ and 0 on the two singlets $N$, $N'$. This is $SO(10)$-like and falls into Class 5.

(C) If the extra fermions per family are just three singlets, $N$, $N'$ and $N'''$, then the general solution is $X' = \alpha X_{10} + \beta Y/2 + \gamma X_N$, where $X_{10}$ has the values given above for the known fermions and is $+5$, 0, and 0 on the three singlets $N$, $N'$, $N'''$. The generator $X_N$ is $+1$ and $-1$ on the two singlets that have $X_{10} = 0$ and vanishes on the other. This solution will fall into Class 5. (We assume that coefficients such as $\alpha$ are not tuned to zero if group theory or anomaly cancellation do not require it.) If there are more than three singlets per family one gets solutions similar to those above.

(E) If the extra fermions per family are just a conjugate pair $R + \bar{R}$ of nonsinglet irreducible representations, then the solution is $X' = \alpha X_R + \beta Y/2$, where $X_R$ is $+1$ and $-1$ on the conjugate pair and vanishes on the known fermions. This falls into Class 6. (If $R$ has the same standard model charges as a known fermion, then there is a solution trivially obtained from the previous by interchanging the two multiplets. This would fall into Class 7.)

(F) If the extra fermions per family are a conjugate pair and a singlet, $R + \bar{R} + N$, then there are two distinct solutions. One is $SO(10)$-like: $X' = \alpha X_{10} + \beta Y/2 + \gamma X_R$, in a notation that is obvious. This falls into Class 5. The other solution is a messy two-parameter solution that falls into Class 7.

These simple examples show what kinds of possibilities exist. Of course, generally speaking, the more extra fermions that exist, the more undetermined parameters will exist in the solution. Complicated cases will usually fall into Class 7.

## VI. CONCLUSIONS

We have argued that a discovery of an extra $Z$ boson can provide information that allows inferences about the degree of gauge unification at high scales. For example, if there is strong mixing of the generator of the $U(1)'$ with hypercharge and $r \neq -1/3$ (and if $X'$ is not proportional to $Y$), it would strongly disfavor conventional four-dimensional unification of the standard model in a simple group. (However, it would not rigorously disprove it, given the possibility, which seems artificial and hard to obtain naturally, that there might be numerous highly split multiplets that induce $O(1)$ radiative mixing of $X'$ with $Y$.). As another example, if $X' \propto Y$ it disproves "full unification," i.e. the unification of the standard model group *and* the $U(1)'$ in a single simple group.

On the other hand, the discovery of certain patterns of $U(1)'$ charge assignments would be strong evidence in favor of certain specific kinds of gauge unification. For instance, finding $r = -2$ and small mixing of the $U(1)'$ charge with hypercharge would strongly favor full unification in a group that contained $SU(6)$, i.e. either a unitary group of $E_6$. However, as far as such positive inferences go, there is always the possibility that certain charges can take special values purely by accident—by fine-tuning, as it were.

There remains much more to be done. First, we have not yet succeeded in rigorously proving some of our conclusions even though we have strong evidence for them, based on both partial proofs and the working out of examples. Second, there is the question of how much stronger the conclusions would be if one also used information about the spectrum of extra light quarks and leptons and their charges under both the standard model group and the $U(1)'$. Third, it may be that extra $U(1)'$ charges in the original gauge basis may be family-independent, but that family-dependence arises as a result of symmetry breaking and mixing in the fermion mass matrices. We have not addressed this case, but only cases where the observed charge assignments are family-independent.

Of course, it is likely that there is no extra gauge symmetry at low energies. However, if there is, it would prove to be a potent tool in unraveling the mystery of what is happening at superhigh scales.

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## APPENDIX

In this appendix we consider values of $r$ that can arise in fully unified models based on unitary groups. Consider a model based on $SU(7)$, with each family consisting of the multiplets $2\times[\bar{3}]+[2]+[1]=2\times\overline{\mathbf{35}}+\mathbf{21}+\mathbf{7}$. The group $SU(7)$ contains the subgroup $SU(5)\times U(1)_1\times U(1)_2$, where the generators of the two $U(1)$ groups are $X_1=\mathrm{diag}(1,1,1,1,1,-\frac{5}{2},-\frac{5}{2})$ and $X_2=\mathrm{diag}(0,0,0,0,0,\frac{1}{2},-\frac{1}{2})$. Each family thus decomposes into $2\times(\overline{\mathbf{10}}^{3,0}+\mathbf{10}^{-1/2,1/2}+\mathbf{10}^{-1/2,-1/2}+\mathbf{5}^{-4,0})+(\overline{\mathbf{10}}^{-2,0}+\bar{\mathbf{5}}^{3/2,-1/2}+\bar{\mathbf{5}}^{3/2,1/2}+\mathbf{1}^{5,0})+(\bar{\mathbf{5}}^{-1,0}+\mathbf{1}^{5/2,-1/2}+\mathbf{1}^{5/2,1/2})$. We assume that $SU(5)$ is broken to the standard model group at a very high scale, but we use $SU(5)$ notation to describe the quark and lepton content for simplicity. We will consider two breaking schemes in which singlet VEVs first break the group down to $G_{\rm SM}\times U(1)'$ at a scale $M_*$, and then at a scale $M'\ll M_*$ other singlets break the $U(1)'$.

Case A Let Higgs fields in the representations $\mathbf{1}^{5/2,1/2}+\mathbf{1}^{-5/2,-1/2}$ obtain VEVs of order $M_*$. This breaks $U(1)_1\times U(1)_2$ down to $U(1)'$, where $X'=X_1-5X_2$. The singlet VEVs also give mass to the pairs of fermions $(\mathbf{10}^{-1/2,-1/2}+\overline{\mathbf{10}}^{-2,0})$, $2\times(\mathbf{10}^{-1/2,1/2}+\overline{\mathbf{10}}^{3,0})$, and $(\bar{\mathbf{5}}^{3/2,-1/2}+\mathbf{5}^{-4,0})$. That leaves light the following multiplets in each family: $\mathbf{10}^{-1/2,-1/2}+\bar{\mathbf{5}}^{-1.0}+\bar{\mathbf{5}}^{3/2,1/2}+\mathbf{5}^{-4,0}$. Or in terms of the $U(1)'$ charges, these are $\mathbf{10}^{2}+\bar{\mathbf{5}}^{-1}+\bar{\mathbf{5}}^{-1}+\mathbf{5}^{-4}$. This is, in fact, just the same set of multiplets that arise in $SU(6)$ models, as can be seen by comparing with the discussion of $SU(6)$ at the beginning of section II. One sees that $r=-2$.

Case B Let Higgs fields in the representations $\mathbf{1}^{5,0}+\mathbf{1}^{-5,0}$ obtain VEVs of order $M_*$. This breaks $U(1)_1\times U(1)_2$ down to $U(1)'=U(1)_2$. The singlet VEVs also give mass to the pair of fermions $(\bar{\mathbf{5}}^{-1,0}+\mathbf{5}^{-4,0})$. At a lower scale, $M'$ the singlets $\mathbf{1}^{5/2,1/2}+\mathbf{1}^{-5/2,-1/2}$ obtain VEVs and give mass to the pairs $(\mathbf{10}^{-1/2,-1/2}+\overline{\mathbf{10}}^{-2,0})$, $2\times(\mathbf{10}^{-1/2,1/2}+\overline{\mathbf{10}}^{3,0})$, and $(\bar{\mathbf{5}}^{3/2,-1/2}+\mathbf{5}^{-4,0})$. That leaves light the following multiplets in each family: $\mathbf{10}^{-1/2,-1/2}+\bar{\mathbf{5}}^{3/2,1/2}$. Or in terms of the $U(1)'$ charges, these are $\mathbf{10}^{-1/2}+\bar{\mathbf{5}}^{1/2}$, implying that $r=-1$. However, it is clear that Case B is completely unrealistic as a model with a low-energy $Z'$ boson, since so many multiplets of quarks and leptons have mass of order $M'$ that unless $M'$ is near the unification scale the gauge couplings will blow up below the unification scale.

This seems to be a general feature in fully unified models based on unitary groups: either one breaks to an $SU(6)$-like low-energy model, giving $r=-2$, or there end up being too many light quarks and leptons for unification of gauge coupling, i.e. there is a Landau pole below the unification scale.

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