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Towards noncommutative quantum black holes

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In this paper we study noncommutative black holes. We use a diffeomorphism between the Schwarzschild black hole and the Kantowski-Sachs cosmological model, which is generalized to noncommutative minisuperspace. Through the use of the Feynman-Hibbs procedure we are able to study the thermodynamics of the black hole, in particular, we calculate the Hawking's temperature and entropy for the noncommutative Schwarzschild black hole.

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I. INTRODUCTION

The search for a quantum theory of gravity has been a long and difficult one; a direct quantization of general relativity using the tools of quantum field theory gives a theory with an ill ultraviolet behavior, and a lack of a fundamental physical principle to construct the theory makes matters worst. One can think that the black hole plays a similar role in quantum gravity as the atom played in the development of quantum mechanics and quantum field theory. We may use it as a starting point for testing different constructions that include quantum aspects of gravity. Research on black hole physics has uncovered several mysteries: Why is the statistical black hole entropy proportional to the horizon area? What happens to the information in black hole evaporation? The answer to these questions and others have been extensively studied in the literature particularly in the two main proposals to build a quantum theory of gravity, namely, string theory [1], and loop quantum gravity [2].

Since the birth of general relativity people started searching for solutions to Einstein's field equations and very powerful and sophisticated methods have been developed. For some time it has been known that changing the causal structure of space time (i.e. interchanging the coordinates $r \leftrightarrow t$), changes a static solution for a cosmological one. The best known case is the Schwarzschild metric that under this particular diffeomorphism transforms into the Kantowski-Sachs metric [3]. This interchange of variables has been used recently as a method to generate new cosmological solutions [4–6]. Later in an independent way in [7,8], it has been suggested that by interchanging $r \leftrightarrow it$ we can get time dependent solutions from stationary solutions in string theory. In this way we may relate Dp-brane (black hole like) solutions to S-brane solutions, i.e. time-dependent backgrounds of the theory. On the other side, there are proposals to obtain S-brane solutions [6,9–11]; thus, if cosmological solutions (S-branes) can be generated from stationary ones (Dp-branes), we can expect that this procedure works the other way around.

In the past a lot of work has been done in relation with cosmology and quantum cosmology [12,13], so one can expect that the classical exchange of $t \leftrightarrow r$ can be applied at quantum level, that is, use the WDW (Wheeler-DeWitt) equation of the cosmological (time-dependent) models and from it obtain the corresponding quantum equation for the stationary ones.

In the last years, noncommutativity (NC) has attracted a lot of attention [14,15]. Although most of the work has been in the context of Yang-Mills theories, noncommutative deformations of gravity have been proposed [16,17]. In [18], the Seiberg-Witten map is used consistently to write a noncommutative theory of gravity, which only depends on the commutative fields and their derivatives. If we attempt to write down the field equations and solve them, it turns out to be technically very difficult, due to the highly nonlinear character of the theory. In [19], an alternative to incorporate noncommutativity to cosmological models has been proposed, by performing a noncommutative deformation of the minisuperspace. The authors applied this idea to the Kantowski-Sachs metric and were able to find the exact wave function for the noncommutative model.

Further, we know that from quantum mechanics we can get the thermodynamical properties of a system. This already has been used in connection with black holes [20]. In [21] the authors use the Feynman-Hibbs path integral procedure [22], to calculate the thermodynamical properties, temperature and entropy, of a black hole, in agreement with previous results [23].

In this paper we apply some of these ideas to obtain thermodynamical properties for a quantum black hole and its noncommutative counterpart. We propose a quantum equation for the Schwarzschild black hole, starting from the WDW equation for the Kantowski-Sachs cosmological model. We apply the Feynman-Hibbs method to calculate

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the thermodynamical properties. As we will show the temperature of the black hole and its entropy get corrected, one of the corrections to the entropy is a logarithmic correction that already has been obtained by other means [23]. We extend this procedure to include noncommutativity by making the same kind of ansatz as in [19], namely, imposing that the minisuperspace variables do not commute; from this we are able to define the WDW equation for the noncommutative Schwarzschild black hole and following a similar procedure as in [21], we find the noncommutative wave function, the temperature and entropy of the *"noncommutative Schwarzschild black hole."* As we will see apart of the presence of the factor $e^{-3\nu\theta}$ that multiplies the commutative entropy and temperature, the functional form of the corrections are of the same kind. We also show how the minisuperspace coordinate Ω is changed by noncommutativity, allowing us to infer the noncommutative entropy, if one uses the well-known Euclidean calculation. The result agrees with the most relevant term of the entropy calculated from the noncommutative WDW equation and the Feynman-Hibbs procedure followed here.

The paper is organized as follows: In Sec. II we exhibit the WDW equation for the Schwarzschild metric and the corresponding wave function. In Sec. III we review the proposal of introducing noncommutativity in the minisuperspace and obtain the noncommutative Wheeler-DeWitt (NC-WDW) equation for the Schwarzschild black hole. In Sec. IV we use the Feynman-Hibbs method on the WDW equation to calculate the entropy of the Schwarzschild black hole, and apply the method to calculate the entropy of the noncommutative black hole. And finally Sec. V is devoted to conclusions and outlook.

II. A WHEELER-DEWITT EQUATION FOR THE SCHWARZSCHILD BLACK HOLE

Let us begin by reviewing the relationship between the cosmological Kantowski-Sachs metric and the Schwarzschild metric [3]. The Schwarzschild solution can be written as

$$ds^2 = -\left(1-\frac{2m}{r}\right)dt^2 + \left(1-\frac{2m}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{1}$$

For the case $r < 2m$, the g_{tt} and g_{rr} components of the metric change in sign and ∂_t becomes a spacelike vector. If we make the coordinate transformation $t \leftrightarrow r$, we find

$$ds^2 = -\left(\frac{2m}{t}-1\right)^{-1}dt^2 + \left(\frac{2m}{t}-1\right)dr^2 + t^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{2}$$

when compared with the parametrization by Misner of the Kantowski-Sachs metric

$$ds^2 = -N^2dt^2 + e^{(2\sqrt{3}\gamma)}dr^2 + e^{(-2\sqrt{3}\gamma)}e^{(-2\sqrt{3}\Omega)}(d\theta^2 + \sin^2\theta d\varphi^2), \tag{3}$$

we identify

$$N^2 = \left(\frac{2M}{t}-1\right)^{-1}, \qquad e^{2\sqrt{3}\gamma} = \frac{2M}{t}-1, \qquad e^{-2\sqrt{3}\gamma}e^{-2\sqrt{3}\Omega} = t^2, \tag{4}$$

where this metric with the identification of the N, γ, and Ω functions is also a classical solution for the Einstein equations. The metric (3) can be introduced into de ADM (Arnowitt-Deser-Misner) action and a consistent set of equations for N, γ, and Ω can be obtained by varying the action. These equations can be shown to be equivalent to the Einstein equations. The corresponding Wheeler-DeWitt equation for the Kantowski-Sachs metric, with some particular factor ordering, is

$$\left[-\frac{\partial^2}{\partial\Omega^2} + \frac{\partial^2}{\partial\gamma^2} + 48e^{-2\sqrt{3}\Omega}\right]\psi(\Omega,\gamma) = 0. \tag{5}$$

The solution of this equation is given by [24]

$$\psi_\nu = e^{\pm i\nu\sqrt{3}\gamma}K_{i\nu}(4e^{-\sqrt{3}\Omega}), \tag{6}$$

where ν is the separation constant and $K_{i\nu}$ are the modified Bessel functions. In [25] it has been shown that this wave function describes quantum planck size states.

III. NONCOMMUTATIVE QUANTUM COSMOLOGY AND THE QUANTUM BLACK HOLE

The noncommutative deformation for this cosmological model has been proposed in [19]. We begin by modifying the simplectic structure in minisuperspace, by assuming that the coordinates Ω and γ obey the commutation relation

$$[\Omega,\gamma] = i\theta, \tag{7}$$

in a similar fashion as in noncommutative quantum mechanics. As usual this deformation can be reformulated in terms of the Moyal product

$$f(\Omega,\gamma) * g(\Omega,\gamma) = f(\Omega,\gamma)e^{(i(\theta/2)(\overleftarrow{\partial}_\Omega\overrightarrow{\partial}_\gamma - \overleftarrow{\partial}_\gamma\overrightarrow{\partial}_\Omega))}g(\Omega,\gamma). \tag{8}$$

Now the Wheleer-DeWitt equation for the noncommutative theory will be

$$[-P_\Omega^2 + P_\gamma^2 - 48e^{-2\sqrt{3}\Omega}] * \psi(\Omega,\gamma) = 0. \tag{9}$$

As is known in noncommutative quantum mechanics, the original phase space is modified. It is possible to reformulate in terms of the commutative variables and the ordinary product of functions, if the new variables $\Omega \to \Omega + \frac{1}{2}\theta P_\gamma$ and $\gamma \to \gamma - \frac{1}{2}\theta P_\Omega$ are introduced. The momenta remain

the same. As a consequence, the original WDW equation changes, with a modified potential $V(\Omega, \gamma)$ [26],

$$V(\Omega, \gamma) * \psi(\Omega, \gamma) = V\left(\Omega - \frac{\theta}{2}P_\gamma, \gamma + \frac{\theta}{2}P_\Omega\right)\psi(\Omega, \gamma), \tag{10}$$

so the NC-WDW equation takes the form

$$\left[-\frac{\partial^2}{\partial\Omega^2} + \frac{\partial^2}{\partial\gamma^2} + 48e^{(-2\sqrt{3}\Omega+\sqrt{3}\theta P_\gamma)}\right]\psi(\Omega, \gamma) = 0. \tag{11}$$

We solve this equation by separation of variables with the ansatz

$$\psi(\Omega, \gamma) = e^{i\sqrt{3}\nu\gamma}\chi(\Omega), \tag{12}$$

where $\sqrt{3}\nu$ is the eigenvalue of P_γ. Thus $\chi(\Omega)$ satisfies the equation [19],

$$\left[-\frac{d^2}{d\Omega^2} + 48e^{(-2\sqrt{3}\Omega+3\nu\theta)} - 3\nu^2\right]\chi(\Omega) = 0. \tag{13}$$

The solution of the NC-WDW equation is

$$\psi_\nu(\Omega, \gamma) = e^{(i\sqrt{3}\nu\gamma)}K_{i\nu}[4e^{-\sqrt{3}(\Omega-\sqrt{3}\nu\theta/2)}]. \tag{14}$$

In [19] the consequences of this wave function have been analyzed. As a result of noncommutativity the probability density has several maxima, which correspond to new stable states of the Universe, opposite to the commutative case where only one stable state exists. Following the previous section, this wave function could describe noncommutative quantum black holes. We can find the noncommutative temporal evolution of Ω and γ through a WKB type method; this yields the classical noncommutative solutions of the Kantowski-Sachs universe [27,28].

IV. NONCOMMUTATIVE BLACK HOLE ENTROPY

In this section we compute the temperature and entropy for the black hole using the Feynman-Hibbs procedure for statistical mechanics, and then we apply it to the noncommutative black hole.

Following Sec. II, we consider Eq. (5). This equation depends on two variables (Ω, γ), but after the separation of variables we have $\psi(\Omega, \gamma) = e^{i\sqrt{3}\nu\gamma}\chi(\Omega)$. Thus, the dependence on the variable γ is the one of a plane wave and is eliminated when computing the thermodynamical observables. Therefore, we could consider this as a suitable approach for the black hole, described by the following equation:

$$\left[-\frac{d^2}{d\Omega^2} + 48e^{-2\sqrt{3}\Omega}\right]\chi(\Omega) = 3\nu^2\chi(\Omega). \tag{15}$$

Now that we have this quantum equation, we can use the Feynman-Hibbs procedure to compute the partition function of the black hole. This procedure has the advantage that the relevant information is contained in the potential function

$$V(\Omega) = 48e^{-2\sqrt{3}\Omega}. \tag{16}$$

This exponential potential can always be expanded, in particular, the calculations are simplified for small Ω. After expanding to second order in Ω, we make the change of variable $\alpha = \sqrt{6}\Omega - 1/\sqrt{2}$, multiply Eq. (15) by $\frac{E_p}{12}$, and finally rename $\alpha = \frac{x}{l_p}$. We get the familiar form,

$$\left[-\frac{1}{2}l_p^2E_p\frac{d^2}{dx^2} + 4\frac{E_p}{l_p^2}x^2\right]\chi(x) = E_p\left[\frac{\nu^2}{4} - 2\right]\chi(x). \tag{17}$$

When comparing with the usual harmonic oscillator we identify $\hbar\omega = \sqrt{\frac{3}{2\pi}}E_p$ in order to obtain the correct Hawking temperature for the black hole [29].

Further, the Feynman-Hibbs procedure allows us to incorporate the quantum corrections to the partition function through the *corrected* potential, which results in [22]

$$U(x) = \frac{3E_p}{4\pi l_p^2}\left[x^2 + \frac{\beta l_p^2E_p}{12}\right]. \tag{18}$$

Thus, the corrected partition function is given by

$$Z_Q = \sqrt{\frac{3}{2\pi}}\frac{e^{-(\beta^2E_p^2/16\pi)}}{\beta E_p}, \tag{19}$$

from which we can proceed to calculate the thermodynamics of interest. The internal energy of the black hole is

$$\bar{E} = -\frac{\partial}{\partial\beta}\ln Z_Q = \frac{1}{8\pi}\beta E_p^2 + \frac{1}{\beta} = Mc^2. \tag{20}$$

Solving for β in terms of the Hawking temperature $\beta_H = \frac{8\pi Mc^2}{E_p^2}$, the corrected temperature of the black hole is

$$\beta = \frac{8\pi Mc^2}{E_p^2}\left[1 - \frac{1}{8\pi}\left(\frac{E_p}{Mc^2}\right)^2\right] = \beta_H\left[1 - \frac{1}{\beta_H}\frac{1}{Mc^2}\right]. \tag{21}$$

In order to calculate the entropy we use

$$\frac{S}{k} = \ln Z_Q + \beta\bar{E}, \tag{22}$$

from which we arrive to the corrected entropy. In terms of the Bekenstein-Hawking entropy $\frac{S_{\rm BH}}{k} = 4\pi(\frac{Mc^2}{E_p})^2 = \frac{A_s}{4l_p^2}$, the black hole entropy takes the simple form,

$$\frac{S}{k} = \frac{S_{\rm BH}}{k} - \frac{1}{2}\ln\left[\frac{S_{\rm BH}}{k}\right] + \mathcal{O}(S_{\rm BH}^{-1}). \tag{23}$$

This result has the interesting feature that the coefficient of the first correction, the logarithmic one, agrees with the one obtained in string theory [30] as well as in loop quantum gravity [31]. The form of this correction had been known already from other works [23], where it was obtained by other considerations.

In order to calculate the thermodynamics of the noncommutative black hole, we follow the same method as before, and we start by considering Eq. (13). A straightforward calculation, using the same steps as in the commutative case, shows that Eq. (13) can be cast to the simple expression,

$$\left(-\frac{1}{2}l_p^2 E_p \frac{d^2}{dx^2} + 4\frac{E_p}{l_p^2}e^{3\nu\theta}x^2\right)\chi(x) = E_p\left(\frac{\nu^2}{4} - 2e^{3\nu\theta}\right)\chi(x). \tag{24}$$

Thus noncommutativity gives a modified version of Eq. (17), with modified potential

$$V_{\mathrm{NC}}(x) = 4\frac{E_p}{l_p^2}e^{3\nu\theta}x^2, \tag{25}$$

and a ''frequency,''

$$\hbar\,\omega_{\mathrm{NC}} = \sqrt{\frac{3}{2\pi}}E_p e^{3\nu\theta/2}, \tag{26}$$

which coincides with the commutative case for $\theta = 0$. Now we can directly apply the Feynman-Hibbs method to Eq. (24) and obtain the corrected partition function,

$$Z_{\mathrm{NC}} = \sqrt{\frac{2\pi}{3}}\frac{e^{-(3\nu\theta/2)}}{\beta E_p}\exp\left(-\frac{\beta^2 E_p^2 e^{3\nu\theta}}{16\pi}\right), \tag{27}$$

from which we calculate the temperature of the noncommutative black hole in terms of the commutative Hawking temperature,

$$\beta = \beta_H e^{-3\nu\theta}\left[1 - \frac{1}{\beta_H}\frac{e^{3\nu\theta}}{Mc^2}\right]. \tag{28}$$

If we define the noncommutative Hawking temperature $\beta_H^{\mathrm{NC}} = \beta_H e^{-3\nu\theta}$, the black hole temperature takes the same form as in the commutative case,

$$\beta = \beta_H^{\mathrm{NC}}\left[1 - \frac{1}{\beta_H^{\mathrm{NC}}}\frac{1}{Mc^2}\right]. \tag{29}$$

The entropy is calculated following the previous steps. If we define the noncommutative Hawking-Bekenstein entropy as $S_{\mathrm{NCBH}} = S_{\mathrm{BH}}e^{-3\nu\theta}$, we get

$$\frac{S_{\mathrm{NC}}}{k} = \frac{S_{\mathrm{NCBH}}}{k} - \frac{1}{2}\ln\left[\frac{S_{\mathrm{NCBH}}}{k}\right] + \mathcal{O}(S_{\mathrm{NCBH}}^{-1}). \tag{30}$$

It has the same form as the commutative case; again the logarithmic correction to the entropy appears with a $-\frac{1}{2}$ factor. Also it is clear that we get the commutative entropy in the limit $\theta \to 0$.

We may infer some properties for the temperature and entropy, using Eq. (4), we arrive to

$$(1 - e^{-2\sqrt{3}\gamma})e^{-\sqrt{3}\gamma}e^{-\sqrt{3}\Omega} = 2M. \tag{31}$$

We can construct a similar expression for the noncommutative metric by replacing the functions γ, Ω, and the mass parameter M by their noncommutative counterparts [28] γ_{NC}, Ω_{NC}, and M_{NC}. To relate the noncommutative theory with the commutative one, we substitute the classical solutions from [27,28] in Eq. (31) and in its noncommutative version. From this we can find a simple relationship between the masses $M_{\mathrm{NC}} = Me^{\sqrt{3}\theta P_{\gamma_0}}$. From the relation between the temperature and entropy and the mass of the black hole [32], we arrive to

$$\beta_{\mathrm{NCBH}} = \beta_{\mathrm{BH}}e^{-\sqrt{3}\theta P_{\gamma_0}}, \qquad S_{\mathrm{NCBH}} = S_{\mathrm{BH}}e^{-\sqrt{3}\theta P_{\gamma_0}}. \tag{32}$$

Thus, we have the same behavior for the noncommutative temperature and entropy as in (29) and (30).

V. CONCLUSIONS AND OUTLOOK

Black holes are natural candidates to probe aspects of quantum gravity, in particular, noncommutative effects.

In principle, in order to study noncommutative black holes, we would require a noncommutative version of general relativity, and then solve its field equations to find the noncommutative metric. This is a very difficult task.

However, it is possible to circumvent these difficulties as we show in this paper. With this idea in mind, we have extended the proposal of noncommutativity in [19] to the Schwarzschild black hole by using the diffeomorphism between the Kantowski-Sachs and Schwarzschild metrics. The corresponding NC-WDW equation is used as the quantum equation for the noncommutative black hole.

We use the Feynman-Hibbs formalism to calculate the thermodynamics of the Schwarzschild black hole and obtain the Hawking-Bekenstein temperature and then the entropy which has the logarithmic correction with the coefficient $-1/2$. This value has been recently predicted by string theory [30] and loop quantum gravity [31].

Finally, these ideas are applied to the noncommutative black hole, and the corresponding entropy and temperature are obtained. As a result, these thermodynamic quantities are modified due to the presence of the noncommutative parameter. In particular, noncommutativity decreases the value of the entropy, which can be understood from the fact that noncommutativity decreases the available physical states.

These ideas could be applied to other singular static solutions, i.e. black hole type solutions.

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[1] G. T. Horowitz, New J. Phys. **7**, 201 (2005).
[2] A. Ashtekar, New J. Phys. **7**, 198 (2005).
[3] R. Kantowski and R. K. Sachs, J. Math. Phys. (N.Y.) **7**, 443 (1966).
[4] O. Obregon, H. Quevedo, and M. P. Ryan, Phys. Rev. D **65**, 024022 (2001).
[5] H. Quevedo and M. P. Ryan, *Mathematical and Quantum Aspects of Relativity and Cosmology*, edited by S. Cotsakis and G. W. Gibbons (Springer Verlag, Berlin, 2000), pp. 191–213.
[6] O. Obregón, H. Quevedo, and M. P. Ryan, J. High Energy Phys. 07 (2004) 005.
[7] F. Quevedo, G. Tasinato, and I. Zavala, hep-th/0211031.
[8] G. Tasinato, I. Zavala, C. P. Burgess, and F. Quevedo, J. High Energy Phys. 04 (2004) 038.
[9] F. Leblond and A. W. Peet, J. High Energy Phys. 04 (2003) 048.
[10] H. García-Compeán, G. García-Jimenez, O. Obregón, and C. Ramírez, Phys. Rev. D **71**, 063517 (2005).
[11] O. Obregón, H. Quevedo, and M. P. Ryan, Phys. Rev. D **70**, 064035 (2004).
[12] M. Ryan, Jr., *Hamiltonian Cosmology* (Springer Verlag, Berlin, 1972).
[13] J. B. Hartle and S. W. Hawking, Phys. Rev. D **28**, 2960 (1983); A. Vilenkin, Phys. Rev. D **37**, 888 (1988).
[14] A. Connes, M. R. Douglas, and A. Schwarz, J. High Energy Phys. 02 (1998) 003.
[15] N. Seiberg and E. Witten, J. High Energy Phys. 09 (1999) 032.
[16] J. W. Moffat, Phys. Lett. B **491**, 345 (2000).
[17] A. H. Chamseddine, J. Math. Phys. (N.Y.) **44**, 2534 (2003); A. H. Chamseddine, Phys. Rev. D **69**, 024015 (2004).
[18] H. García-Compeán, O. Obregón, C. Ramírez, and M. Sabido, Phys. Rev. D **68**, 044015 (2003); **68**, 045010 (2003).
[19] H. García-Compeán, O. Obregón, and C. Ramírez, Phys. Rev. Lett. **88**, 161301 (2002).
[20] J. W. York, Phys. Rev. D **33**, 2092 (1986).
[21] O. Obregon, M. Sabido, and V. I. Tkach, Gen. Relativ. Gravit. **33**, 913 (2001).
[22] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).
[23] S. Hod, Phys. Rev. Lett. **81**, 4293 (1998); C. Vaz, Phys. Rev. D **61**, 064017 (2000); G. Scharf, Nuovo Cimento Soc. Ital. Fis. B **113**, 821 (1998); B. Harms and Y. Leblanc, Phys. Rev. D **46**, 2334 (1992); G. Gour, Phys. Rev. D **61**, 021501 (2000).
[24] C. Misner, in *Magic without Magic: John Archibald Wheeler* (Freeman, San Francisco, 1972).
[25] O. Obregón and M. P. Ryan, Mod. Phys. Lett. A **13**, 3251 (1998).
[26] L. Mezincescu, hep-th/0007046.
[27] E. Mena, O. Obregón, and M. Sabido (unpublished).
[28] G. D. Barbosa and N. Pinto-Neto, Phys. Rev. D **70**, 103512 (2004).
[29] The factor corresponds to an ambiguity in the value of the lowest eigenvalue of the spectrum [33]. In loop quantum gravity the Barbero-Immirizi parameter [34] is fixed in a similar way.
[30] S. Mukherji and S. S. Pal, J. High Energy Phys. 05 (2002) 026.
[31] M. Domagala and J. Lewandowski, Classical Quantum Gravity **21**, 5233 (2004); K. A. Meissner, Classical Quantum Gravity **21**, 5245 (2004); A. Ghosh and P. Mitra, Phys. Rev. D **71**, 027502 (2005); A. Corichi, J. Diaz-Polo, and Fernandez-Borja, gr-qc/0605014.
[32] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973); J. D. Bekenstein, Phys. Rev. D **9**, 3292 (1974); S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975); **46**, 206(E) (1976).
[33] H. A. Kastrup, Phys. Lett. B **385**, 75 (1996); **413**, 267 (1997).
[34] A. Ashtekar, J. Baez, A. Corichi, and K. Krasnov, Phys. Rev. Lett. **80**, 904 (1998).