Lorentz symmetry derived from Lorentz violation in the bulk

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We consider bulk fields coupled to the graviton in a Lorentz violating fashion. We expect that the overly tested Lorentz symmetry might set constraints on the induced Lorentz violation in the brane, and hence on the dynamics of the interaction of bulk fields on the brane. We also use the requirement for Lorentz symmetry to constrain the cosmological constant observed on the brane.

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I. INTRODUCTION

Lorentz invariance is one of the most well-tested symmetries of physics. Nevertheless, the possibility of violation of this invariance has been widely discussed in the recent literature (see e.g. [1]). Indeed, the spontaneous breaking of Lorentz symmetry may arise in the context of string/M theory due to the existence of nontrivial solutions in string field theory [2], in loop quantum gravity [3], in noncommutative field theories [4,5], or via the spacetime variation of fundamental couplings [6]. This putative breaking also has implications in ultrahigh energy cosmic ray physics [7,8]. Lorentz violation modifications of the dispersion relations via five-dimensional operators for fermions have also been considered and constrained [9]. It has also been speculated that Lorentz symmetry is connected with the cosmological constant problem [10]. However, the main conclusion of these studies is that Lorentz symmetry holds up to about two parts in 2×10^{-25} [1,8].

Efforts to examine a putative breaking of Lorentz invariance have been concerned mainly with the phenomenological aspects of the spontaneous breaking of Lorentz symmetry in particle physics and only recently have the implications for gravity been more closely studied [11,12]. The idea is to consider a vector field coupled to gravity that undergoes spontaneous Lorentz symmetry breaking by acquiring a vacuum expectation value in a potential.

Moreover, recent developments in string theory suggest that we may live in a braneworld embedded in a higher dimensional universe. In the context of the Randall-Sundrum cosmological models, the warped geometry of the bulk along the extra spatial dimension suggests an anisotropy which could be associated with the breaking of the bulk Lorentz symmetry.

In this paper we study how spontaneous Lorentz violation in the bulk repercusses on the brane and how it can be constrained. We consider a vector field in the bulk which acquires a nonvanishing expectation value in the vacuum and introduces spacetime anisotropies in the gravitational field equations through the coupling with the graviton. For this purpose, we derive the field equations and project them parallel and orthogonal to the brane. We then establish how to derive brane quantities from bulk quantities by adopting Fermi normal coordinates with respect to the directions on the brane and continuing into the bulk using the Gauss normal prescription.

We parametrize the world sheet in terms of coordinates $x^A = (t_b, \mathbf{x}_b)$ intrinsic to the brane. Using the chain rule, we may express the brane tangent and normal unit vectors in terms of the bulk basis as follows:

$$\hat{e}_A = \frac{\partial}{\partial x^A} = X^\mu_A \frac{\partial}{\partial x^\mu} = X^\mu_A \hat{e}_\mu, \qquad \hat{e}_N = \frac{\partial}{\partial n} = N^\mu \frac{\partial}{\partial x^\mu} = N^\mu \hat{e}_\mu, \tag{1}$$

with

$$g_{\mu\nu} N^\mu N^\nu = 1, \qquad g_{\mu\nu} N^\mu X^\nu_A = 0, \tag{2}$$

where $\mathbf{g}$ is the bulk metric,

$$\mathbf{g} = g_{\mu\nu}\hat{e}_\mu \otimes \hat{e}_\nu = g_{AB}\hat{e}_A \otimes \hat{e}_B + g_{AN}\hat{e}_A \otimes \hat{e}_N + g_{NB}\hat{e}_N \otimes \hat{e}_B + g_{NN}\hat{e}_N \otimes \hat{e}_N. \tag{3}$$

To obtain the metric induced on the brane we expand the bulk basis vectors in terms of the coordinates intrinsic to the brane and keep the doubly brane tangent components only. It follows that

$$g_{AB} = X^\mu_A X^\nu_B g_{\mu\nu} \tag{4}$$

is the (3 + 1)-dimensional metric induced on the brane by the (4 + 1)-dimensional bulk metric $g_{\mu\nu}$. The induced metric with upper indices is defined by the relation

$$g_{AB} g^{BC} = \delta_A{}^C. \tag{5}$$

It follows that we can write any bulk tensor field as a linear combination of mutually orthogonal vectors on the brane, $\hat{e}_A$, and a vector normal to the brane, $\hat{e}_N$. We illustrate the example of vector B_μ and tensor $T_{\mu\nu}$ bulk fields as follows:

$$\mathbf{B} = B_A \hat{e}_A + B_N \hat{e}_N, \tag{6}$$

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$$\mathbf{T} = T_{AB}\hat{e}_A \otimes \hat{e}_B + T_{AN}\hat{e}_A \otimes \hat{e}_N + T_{NB}\hat{e}_N \otimes \hat{e}_B + T_{NN}\hat{e}_N \otimes \hat{e}_N. \tag{7}$$

Derivative operators decompose similarly. We write the derivative operator ∇ as

$$\nabla = (X^\mu_A + N^\mu)\nabla_\mu = \nabla_A + \nabla_N. \tag{8}$$

Fixing a point on the boundary, we introduce coordinates for the neighborhood, choosing them to be Fermi normal. All the Christoffel symbols of the metric on the boundary are thus set to zero, although the partial derivatives do not, in general, vanish. The nonvanishing connection coefficients are

$$\begin{aligned} \nabla_A \hat{e}_B &= -K_{AB}\hat{e}_N, & \nabla_A \hat{e}_N &= +K_{AB}\hat{e}_B, \\ \nabla_N \hat{e}_A &= +K_{AB}\hat{e}_B, & \nabla_N \hat{e}_N &= 0, \end{aligned} \tag{9}$$

as determined by the Gaussian normal prescription for the continuation of the coordinates off the boundary. For the derivative operator $\nabla\nabla$ we find that

$$\begin{aligned} \nabla\nabla &= g^{\mu\nu}\nabla_\mu\nabla_\nu \\ &= g^{AB}[(X^\mu_A\nabla_\mu)(X^\nu_B\nabla_\nu) - X^\mu_A(\nabla_\mu X^\nu_B)\nabla_\nu] \\ &\quad + g^{NN}[(N^\mu\nabla_\mu)(N^\nu\nabla_\nu) - N^\mu(\nabla_\mu N^\nu)\nabla_\nu] \\ &= g^{AB}[\nabla_A\nabla_B + K_{AB}\nabla_N] + \nabla_N\nabla_N. \end{aligned} \tag{10}$$

We can now decompose the Riemann tensor, $R_{\mu\nu\rho\sigma}$, along the tangent and the normal directions to the surface of the brane as follows:

$$R_{ABCD} = R^{(\mathrm{ind})}_{ABCD} + K_{AD}K_{BD} - K_{AC}K_{BD}, \tag{11}$$

$$R_{NBCD} = K_{BC;D} - K_{BD;C}, \tag{12}$$

$$R_{NBND} = K_{BC}K_{DC} - K_{BC,N}, \tag{13}$$

from which we find the decomposition of the Einstein tensor, $G_{\mu\nu}$, obtaining the Gauss-Codacci relations

$$\begin{aligned} G_{AB} &= G^{(\mathrm{ind})}_{AB} + 2K_{AC}K_{CB} - K_{AB}K - K_{AB,N} \\ &\quad - \tfrac{1}{2}g_{AB}(3K_{CD}K_{DC} - K^2 - 2K_{,N}), \end{aligned} \tag{14}$$

$$G_{AN} = K_{AB;B} - K_{;A}, \tag{15}$$

$$G_{NN} = \tfrac{1}{2}(-R^{(\mathrm{ind})} - K_{CD}K_{DC} + K^2). \tag{16}$$

II. BULK VECTOR FIELD COUPLED TO GRAVITY

We consider a bulk vector field **B** with a nontrivial coupling to the graviton in a five-dimensional anti-de Sitter space. The Lagrangian density consists of the Hilbert term, the cosmological constant term, the kinetic and potential terms for **B**, and the **B**-graviton interaction term, as follows:

$$\begin{aligned} \mathcal{L} &= \frac{1}{\kappa^2_{(5)}}R - 2\Lambda + \lambda B^\mu B^\nu R_{\mu\nu} \\ &\quad - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - V(B^\mu B_\mu \pm b^2), \end{aligned} \tag{17}$$

where $B_{\mu\nu} = \nabla_\mu B_\nu - \nabla_\nu B_\mu$ is the tensor field associated with B_μ and V is the potential which induces the breaking of Lorentz symmetry once the **B** field is driven to the minimum at $B^\mu B_\mu \pm b^2 = 0$, b^2 being a real positive constant. As discussed in the Introduction, this model has been proposed in order to analyze the impact on the gravitational sector of the breaking of Lorentz symmetry [11,12]. Furthermore, in the present model $\kappa^2_{(5)} = 8\pi G_N = M^3_{Pl}$, M_{Pl} is the five-dimensional Planck mass and λ is a dimensionless coupling constant that we have inserted to track the effect of the interaction. In the cosmological constant term $\Lambda = \Lambda_{(5)} + \Lambda_{(4)}$, we have included both the bulk vacuum value $\Lambda_{(5)}$ and that of the brane $\Lambda_{(4)}$, described by a brane tension σ localized on the locus of the brane, $\Lambda_{(4)} = \sigma\delta(N)$.

By varying the action with respect to the metric, we obtain the Einstein equation

$$\frac{1}{\kappa^2_{(5)}}G_{\mu\nu} + \Lambda g_{\mu\nu} - \lambda L_{\mu\nu} - \lambda\Sigma_{\mu\nu} = \frac{1}{2}T_{\mu\nu}, \tag{18}$$

where

$$L_{\mu\nu} = \tfrac{1}{2}g_{\mu\nu}B^\rho B^\sigma R_{\rho\sigma} - (B_\mu B^\rho R_{\rho\nu} + R_{\mu\rho}B^\rho B_\nu), \tag{19}$$

$$\begin{aligned} \Sigma_{\mu\nu} &= \tfrac{1}{2}[\nabla_\mu\nabla_\rho(B_\nu B^\rho) + \nabla_\nu\nabla_\rho(B_\mu B^\rho) \\ &\quad - \nabla^2(B_\mu B_\nu) - g_{\mu\nu}\nabla_\rho\nabla_\sigma(B^\rho B^\sigma)] \end{aligned} \tag{20}$$

are the contributions from the interaction term and

$$T_{\mu\nu} = B_{\mu\rho}B_\nu{}^\rho + 4V' B_\mu B_\nu + g_{\mu\nu}[-\tfrac{1}{4}B_{\rho\sigma}B^{\rho\sigma} - V] \tag{21}$$

is the contribution from the vector field for the stress-energy tensor. For the equation of motion for the vector field **B**, obtained by varying the action with respect to B_μ, we find that

$$\nabla^\nu(\nabla_\nu B_\mu - \nabla_\mu B_\nu) - 2V' B_\mu + 2\lambda B^\nu R_{\mu\nu} = 0, \tag{22}$$

where $V' = dV/dB^2$.

We now proceed to project the equations parallel and orthogonal to the surface of the brane. Following the prescription used in the derivation of the Gauss-Codacci relations, we derive the components of the stress-energy tensor and of the interaction terms. Similarly, the equation of motion for the vector field **B** decomposes as follows:

$$\nabla_C(\nabla_C B_A - \nabla_A B_C) + \nabla_N(\nabla_N B_A - \nabla_A B_N) + 2K_{AC}(\nabla_C B_N - \nabla_N B_C) + K(\nabla_N B_A - \nabla_A B_N) - 2V' B_A + 2\lambda[B_C(R^{(\mathrm{ind})}_{AC} + 2K_{AD}K_{DC} - K_{AC}K - K_{AC,N}) + B_N(K_{AC;C} - K_{;A})] = 0, \quad (23)$$

$$\nabla_C(\nabla_C B_N - \nabla_N B_C) - 2V' B_N + 2\lambda[B_C(K_{CD;D} - K_{;C}) + B_N(K_{CD}K_{CD} - K_{;N})] = 0, \quad (24)$$

parallel and orthogonal to the brane, respectively, which we include here for the purpose of illustration.

Next we proceed to derive the induced equations of motion for both the metric and the vector field in terms of quantities measured on the brane. The induced equations on the brane are the (AB) projected components after the singular terms across the brane are subtracted out by the substitution of the matching conditions. Considering the brane as a Z_2-symmetric shell of thickness 2δ in the limit $\delta \to 0$, derivatives of quantities discontinuous across the brane generate singular distributions on the brane. Integration of these terms in the coordinate normal to the brane relates the induced geometry with the localization of the induced stress energy in the form of matching conditions. First we consider the Einstein equations and then the equations of motion for **B** which, due to the coupling of **B** to gravity, will also be used as complementary conditions for the dynamics of the metric on the brane.

Combining the Gauss-Codacci relations with the projections of the stress-energy tensor and the interaction source terms, we integrate the (AB) component of the Einstein equation in the coordinate normal to the brane to obtain the matching conditions for the extrinsic curvature across the brane, i.e. the Israel matching conditions. From the Z_2 symmetry it follows that $B_A(-\delta) = +B_A(+\delta)$ but that $B_N(-\delta) = -B_N(+\delta)$ and, consequently, that $(\nabla_N B_A)(-\delta) = -(\nabla_N B_A)(+\delta)$ and $(\nabla_N B_N)(-\delta) = +(\nabla_N B_N)(+\delta)$. Moreover, $g_{AB}(N = -\delta) = +g_{AB}(N = +\delta)$ implies that $K_{AB}(N = -\delta) = -K_{AB}(N = +\delta)$. Hence, we find for the (AB) matching conditions that

$$\frac{1}{\kappa^2_{(5)}}[-K_{AB} + g_{AB}K] = \frac{1}{2}\int_{-\delta}^{+\delta} dN[-g_{AB}\Lambda_{(4)}] + \frac{\lambda}{2}[\nabla_A(B_B B_N) + \nabla_B(B_A B_N) - \nabla_N(B_A B_B) + 4(B_A B_C K_{CB} + K_{AC}B_C B_B) - 2K_{AB}B_N B_N + g_{AB}(-2\nabla_C(B_C B_N) - \nabla_N(B_N B_N) + K_{CD}B_C B_D - KB_N B_N)]. \quad (25)$$

These provide boundary conditions for 10 of the 15 degrees of freedom. Five additional boundary conditions are provided by the junction conditions for the (AN) and (NN) components of the projection of the Einstein equations. From inspection of the (AN) component, we note that

$$G_{AN} = K_{AB;B} - K_{;A} = -\nabla_B\left(\int_{-\delta}^{+\delta} dN G_{AB}\right) = -\kappa^2_{(5)}\nabla_B \mathcal{T}_{AB} = 0 \quad (26)$$

which vanishes due to conservation of the induced stress-energy tensor $\mathcal{T}_{AB}$ on the brane. From integration of the (NN) component in the normal direction to the brane, we find the following junction condition:

$$\nabla_C(B_C B_N) + 3KB_N B_N - K_{CD}B_C B_D = \sigma, \quad (27)$$

which we substitute back in, obtaining

$$\frac{1}{\kappa^2_{(5)}}\frac{1}{2}(-R^{(\mathrm{ind})} - K_{CD}K_{CD} + K^2) = \frac{1}{2}\left[-\frac{1}{4}B_{CD}B_{CD} - V\right] + \frac{1}{2}[-\nabla_C\nabla_D(B_C B_D) - \nabla_C\nabla_C(B_N B_N) + 12B_N\nabla_C\nabla_C B_N - 20V' B_N B_N + 2(K_{CD} - g_{CD}K)\nabla_C(B_D B_N) + 2K_{CD}B_D(\nabla_C B_N) + K_{CD;C}B_D B_N + (7K_{CD}K_{CD} - K^2)B_N B_N + (7K_{CE}K_{ED} + KK_{CD})B_C B_D]. \quad (28)$$

However, the Israel matching conditions also contain terms which depend on the prescription for the continuation of **B** out of the brane and into the bulk, namely $\nabla_N B_A$ and $\nabla_N B_N$. The five additional boundary conditions required are those for the vector field **B**. In Ref. [13] the boundary conditions for bulk fields were derived subject to the condition that modes emitted by the brane into the bulk do not violate the gauge defined in the bulk. Here, however, we integrate the (A) and (N) components of the equation of motion for **B** [Eqs. (23) and (24), respectively] to find the corresponding junction condition for B_A and for B_N across the brane. From Eq. (23) we have that

$$\int_{-\delta}^{+\delta} dN[\nabla_N(\nabla_N B_A - \nabla_A B_N) - 2K_{AC}(\nabla_N B_C) - 2\lambda B_C K_{AC,N}] = 0. \quad (29)$$

If δ is sufficiently small, the difference between $K_{AB;N}$ and $K_{AB,N}$ is negligibly small. It follows that, in the limit where $\delta \to 0$, we can assume that $\nabla_N \approx \partial_N$. It then follows that

$$\nabla_N B_A - \nabla_A B_N - 2K_{AC}B_C = 0. \quad (30)$$

Similarly, from Eq. (24) we find that

$$\int_{-\delta}^{+\delta} dN[-\nabla_N \nabla_C B_C - \lambda \nabla_N (K B_N)] = 0, \quad (31)$$

which becomes

$$\nabla_C B_C + \lambda K B_N = 0. \quad (32)$$

The junction conditions Eqs. (30) and (32) offer the required $(4 + 1)$ boundary conditions, respectively, for B_A and B_N on the brane. Substituting the junction condition for B_A back in Eq. (23) and using the result from $G_{AN} = 0$, we find for the induced equation of motion for B_A on the brane that

$$\nabla_C(\nabla_C B_A - \nabla_A B_C) + 2K_{AC}(\nabla_C B_N) - 2V' B_A + 2\lambda B_C(R^{(\mathrm{ind})}_{AC} + 2K_{AD}K_{DC}) = 0. \quad (33)$$

Similarly, substituting the junction condition for B_N back in Eq. (24), we obtain

$$\nabla_C \nabla_C B_N - 2V' B_N + \lambda[K(\nabla_N B_N) + B_N K_{CD} K_{CD}] = 0. \quad (34)$$

Thus, Eq. (30) provides the value at the boundary for $\nabla_N B_A$ and Eq. (34) provides that for $\nabla_N B_N$. Using the results derived above in the Israel matching conditions, we find that

$$\begin{aligned}\frac{1}{\kappa^2_{(5)}}[-K_{AB} + g_{AB}K] = {} & \frac{1}{2}(-g_{AB}\sigma) + \frac{1}{2}[(\nabla_A B_B)B_N + (\nabla_B B_A)B_N] + B_A B_C K_{CB} + K_{AC} B_C B_B - K_{AB} B_N B_N \\ & + g_{AB}\bigg[-\nabla_C(B_C B_N) + \frac{1}{2}K_{CD}B_C B_D - \frac{1}{2}K B_N B_N \\ & + \frac{1}{K}(B_N \nabla_C \nabla_C B_N - 2V' B_N B_N + B_N B_N K_{CD} K_{CD})\bigg]. \end{aligned} \quad (35)$$

The Israel matching conditions provide an equation for the trace of the extrinsic curvature, K. Finally, using Eq. (28) in the (AB) Einstein equation, we find for the Einstein equation induced on the brane

$$\begin{aligned} & \frac{1}{\kappa^2_{(5)}}\bigg[G^{(\mathrm{ind})}_{AB} + 2K_{AC}K_{BC} - K_{AB}K + \frac{1}{2}g_{AB}(-R^{(\mathrm{ind})} - 4K_{CD}K_{CD} + 2K^2)\bigg] + g_{AB}\Lambda_{(5)} \\ & + \frac{1}{2}\bigg[-B_{AC}B_{BC} - 4V' B_A B_B + \frac{1}{2}g_{AB}(B_{CD}B_{CD} + 6V)\bigg] \\ & = \frac{1}{2}g_{AB}[-2\nabla_C\nabla_D(B_C B_D) - \nabla_C\nabla_C(B_N B_N) + 12 B_N \nabla_C \nabla_C B_N - 20V' B_N B_N \\ & + 4(K_{CD} - g_{CD}K)\nabla_D(B_C B_N) + 6K_{CD}B_D(\nabla_C B_N) + K B_C(\nabla_C B_N) \\ & + B_C B_D R^{(\mathrm{ind})}_{CD} + 9K_{CD}K_{CD}B_N B_N + 14 K_{CE}K_{DE}B_C B_D - K\sigma] \\ & + \frac{1}{2}[\nabla_A \nabla_C(B_B B_C) + \nabla_B\nabla_C(B_A B_C) - \nabla_C\nabla_C(B_A B_B) - 2K_{AC}\nabla_C(B_B B_N) \\ & - 2K_{AC}(B_B \nabla_C B_N + B_C \nabla_B B_N) - 2K_{BC}\nabla_C(B_A B_N) - 2K_{BC}(B_A \nabla_C B_N + B_C \nabla_A B_N) \\ & + K B_N(\nabla_A B_B + \nabla_B B_A) - 2K_{AB}B_N(\nabla_C B_C) - \frac{8}{K}K_{AB}(\nabla_C\nabla_C B_N - 2V' B_N + B_N K_{CD}K_{CD}) \\ & - 2B_A B_C(R^{(\mathrm{ind})}_{CB} + 2K_{CD}K_{BD}) - 2B_B B_C(R^{(\mathrm{ind})}_{CA} + 2K_{AD}K_{CD}) + (K_{AC;B} + K_{BC;A} - 2K_{AB;C})B_N B_N \\ & + (K_{AC}B_B + K_{BC}B_A)(-5K_{DC}B_D + K B_C) - 6K_{AC}K_{BD}B_C B_D - 2K_{AB}(3K B_N B_N - \sigma)]. \end{aligned} \quad (36)$$

The results obtained above show both the coupling of the bulk to the brane and the coupling of the vector field **B** to the geometry of the spacetime. The first is manifested in the dependence on normal components in the induced equations; the latter is manifested in the presence of terms of the form $(R_{AB}B_C B_D)$. Terms of the form $(K_{AB}B_N)$ illustrate both couplings, where B_N relates with K and B_A by Eq. (32). The directional dependence on the N direction is encapsulated in the extrinsic curvature. In the fourth line we can substitute the Israel matching condition found above. However, the derivatives of the extrinsic curvature along directions parallel to the brane which appear in the ninth line are not reducible to quantities intrinsic to the brane.

III. BULK VECTOR FIELD WITH A NONVANISHING VACUUM EXPECTATION VALUE

In this section we particularize the formalism developed above for the case when the bulk vector field **B** acquires a nonvanishing vacuum value by spontaneous symmetry breaking akin to the Higgs mechanism. The vacuum value generates the breaking of the Lorentz symmetry by selecting the direction orthogonal to the plane of the brane. The tangential part of the vector field with respect to the brane will acquire an expectation value, $\langle B_A\rangle \neq 0$, whereas the expectation value of the normal component is chosen, for simplicity, to vanish on the brane, $\langle B_N\rangle = 0$, as we are interested in the effect that Lorentz symmetry breaking in the bulk has on the brane. Choosing instead $\langle B_A\rangle = 0$ and $\langle B_N\rangle \neq 0$ would also violate Lorentz symmetry on the brane. However, the implied matching conditions would be incompatible with the condition for the covariant conservation of the vacuum expectation value of the field **B**, $\nabla_A\langle B_C\rangle = 0$ [11,12]. Moreover, the vacuum value is at a zero of both the potential V and its derivative V'. The junction conditions from the equations for B_A, B_N, G_{NN}, and G_{AB} reduce, respectively, to

$$\nabla_N\langle B_A\rangle - 2K_{AC}\langle B_C\rangle = 0, \tag{37}$$

$$\nabla_C\langle B_C\rangle = 0, \tag{38}$$

$$-K_{CD}\langle B_C\rangle\langle B_D\rangle = \sigma, \tag{39}$$

$$\frac{1}{\kappa_{(5)}^2}[-K_{AB} + g_{AB}K] = -g_{AB}\sigma + \langle B_A\rangle\langle B_C\rangle K_{CB} + \langle B_B\rangle\langle B_C\rangle K_{AC}, \tag{40}$$

and the induced equations of motion become

$$\nabla_C(\nabla_C\langle B_A\rangle - \nabla_A\langle B_C\rangle) + 2\langle B_C\rangle(R_{AC}^{(\text{ind})} + 2K_{AD}K_{DC}) = 0 \tag{41}$$

for B_A,

$$\frac{1}{\kappa_{(5)}^2}\frac{1}{2}(-R^{(\text{ind})} - K_{CD}K_{CD} + K^2) = \frac{1}{2}\Big[-\frac{1}{4}\langle B_{CD}\rangle\langle B_{CD}\rangle - \nabla_C\nabla_D(\langle B_C\rangle\langle B_D\rangle) + 7K_{CE}K_{ED}\langle B_C\rangle\langle B_D\rangle - K\sigma\Big] \tag{42}$$

for G_{NN}, and finally

$$\frac{1}{\kappa_{(5)}^2}\Big[G_{AB}^{(\text{ind})} + 2K_{AC}K_{BC} - \frac{1}{2}K_{AB}K + \frac{1}{2}g_{AB}(R^{(\text{ind})} - K_{CD}K_{CD} - K^2)\Big] + g_{AB}\Lambda_{(5)} - \frac{1}{2}\langle B_{AC}\rangle\langle B_{BC}\rangle$$
$$= \frac{1}{2}\Big[\frac{1}{4}\langle B_A\rangle\nabla_C(5\nabla_C\langle B_B\rangle - 9\nabla_B\langle B_C\rangle) + \frac{1}{4}\langle B_B\rangle\nabla_C(5\nabla_C\langle B_A\rangle - 9\nabla_A\langle B_C\rangle) + \nabla_A\nabla_C(\langle B_B\rangle\langle B_C\rangle) + \nabla_B\nabla_C(\langle B_A\rangle\langle B_C\rangle)$$
$$- 2(\nabla_C\langle B_A\rangle)(\nabla_C\langle B_B\rangle) + \frac{5}{2}\langle B_A\rangle\langle B_C\rangle R_{CB}^{(\text{ind})} + \frac{5}{2}\langle B_B\rangle\langle B_C\rangle R_{AC}^{(\text{ind})} - 6K_{AC}K_{BD}\langle B_C\rangle\langle B_D\rangle + 2K_{AB}\sigma\Big]$$
$$+ \frac{1}{2}g_{AB}[\langle B_C\rangle\langle B_D\rangle R_{CD}^{(\text{ind})} + 2K\sigma] \tag{43}$$

from G_{AB}, where we also used the previous results, namely, the G_{NN} equation, the Israel matching condition, and the B_A equation.

Imposing that $\nabla_A\langle B_C\rangle = 0$, it follows that $\langle B_{AC}\rangle = \nabla_A\langle B_C\rangle - \nabla_C\langle B_A\rangle = 0$, which enables us to further simplify Eq. (43):

$$\frac{1}{\kappa_{(5)}^2}\Big[G_{AB}^{(\text{ind})} + 2K_{AC}K_{BC} - \frac{1}{2}K_{AB}K + \frac{1}{2}g_{AB}(R^{(\text{ind})} - 2K_{CD}K_{CD} - K^2)\Big] + g_{AB}\Lambda_{(5)}$$
$$= \frac{1}{2}\Big[\frac{5}{2}\langle B_A\rangle\langle B_C\rangle R_{CB}^{(\text{ind})} + \frac{5}{2}\langle B_B\rangle\langle B_C\rangle R_{AC}^{(\text{ind})} - 6K_{AC}K_{BD}\langle B_C\rangle\langle B_D\rangle + 2K_{AB}\sigma\Big]$$
$$+ \frac{1}{2}g_{AB}[\langle B_C\rangle\langle B_D\rangle R_{CD}^{(\text{ind})} + 2K\sigma]. \tag{44}$$

Hence, in order to obtain a vanishing cosmological constant and ensure that Lorentz invariance holds on the brane, we must impose, respectively, that

$$\Lambda_{(5)} = K\sigma \tag{45}$$

and that

$$2K_{AC}K_{BC} - \tfrac{1}{2}K_{AB}K + \tfrac{1}{2}g_{AB}(R^{(\text{ind})} - 2K_{CD}K_{CD} - K^2)$$
$$= \kappa_{(5)}^2[\tfrac{5}{4}\langle B_A\rangle\langle B_C\rangle R_{CB}^{(\text{ind})} + \tfrac{5}{4}\langle B_B\rangle\langle B_C\rangle R_{AC}^{(\text{ind})} - 3K_{AC}K_{BD}\langle B_C\rangle\langle B_D\rangle + K_{AB}\sigma + \tfrac{1}{2}g_{AB}\langle B_C\rangle\langle B_D\rangle R_{CD}^{(\text{ind})}]. \tag{46}$$

We observe that there is a close relation between the vanishing of the cosmological constant and the keeping of the Lorentz invariance on the brane. These conditions are enforced so that the higher dimensional signatures encapsulated in the induced geometry of the brane cancel the Lorentz symmetry breaking inevitably induced on the brane, thus reproducing the observed geometry. The first condition, Eq. (45), can be modified to account for any nonvanishing value for the cosmological constant, as appears to be suggested by the recent Wilkinson Microwave Anisotropy Probe data, by defining the observed cosmological constant Λ such that $\Lambda_{(5)} = \Lambda + \tilde{\Lambda}_{(5)}$. A much more elaborate fine-tuning, however, is required for the Lorentz symmetry to be observed on the brane, as described by the condition in Eq. (46). To our knowledge this is a new feature in braneworld models, as in most models Lorentz invariance is a symmetry shared by both the bulk and the brane. We shall examine further implications of this mechanism elsewhere [14]. In this future study the inclusion of a scalar field will also be discussed.

IV. DISCUSSION AND CONCLUSIONS

In this paper we analyzed the spontaneous symmetry breaking of Lorentz invariance in the bulk and its consequent effect on the brane. For this purpose, we considered a bulk vector field subject to a potential which endows the field with a nonvanishing vacuum expectation value, thus allowing for the spontaneous breaking of the Lorentz symmetry. This bulk vector field is directly coupled to the Ricci tensor, so that after the breaking of Lorentz invariance the loss of this symmetry is transmitted to the gravitational sector of the model. For simplicity, we assumed that the vacuum expectation value of the component of the vector field normal to the brane vanishes. The complex interplay between matching conditions and the Lorentz symmetry breaking terms was examined. We found that Lorentz invariance on the brane can be made exact via the dynamics of the graviton, vector field and the geometry of the extrinsic curvature of the surface of the brane. As a consequence of the exact reproduction of Lorentz symmetry on the brane, we found a condition for the matching of the observed cosmological constant in four dimensions. This tuning does not follow from a dynamical mechanism but is imposed by phenomenological reasons only. From this point of view, both the value of the cosmological constant and the Lorentz symmetry seem to be a consequence of a complex fine-tuning. We aim to further study the implications of our mechanism by also considering the inclusion of a scalar field in a forthcoming publication [14].

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