One loop corrected mode functions for scalar QED during inflation

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We solve the one loop effective scalar field equations for spatial plane waves in massless, minimally coupled scalar quantum electrodynamics on a locally de Sitter background. The computation is done in two different gauges: a non-de Sitter invariant analogue of Feynman gauge, and in the de Sitter invariant, Lorentz gauge. In each case our result is that the finite part of the conformal counterterm can be chosen so that the mode functions experience no significant one loop corrections at late times. This is in perfect agreement with a recent, all orders stochastic prediction.

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I. INTRODUCTION

Gravitons and massless, minimally coupled (MMC) scalars are unique in possessing zero mass without classical conformal invariance. This allows them to mediate vastly enhanced quantum effects during inflation [1]. Many examples have been studied over the history of inflation, starting with the first work on scalar [2] and tensor [3] density perturbations. Those were tree order analyses. More recently, there have been analyses of loop corrections to density perturbations [4–7] and to similar fixed-momentum correlators [10].

A reasonable paradigm for much of inflation is de Sitter background and a wide variety of enhanced quantum loop effects have been studied on this background in many different theories. In pure quantum gravity the one loop self-energy [11] and the two loop expectation value of the metric [12,13] have been calculated. For a MMC scalar with a quartic self interaction the expectation value of the stress tensor [14,15] and the self-mass-squared [16] have both been computed at two loop order. In scalar quantum electrodynamics (SQED) the vacuum polarization [17,18] and the scalar self-mass-squared [19] were evaluated at one loop order, and the expectation value of two scalar bilinears have been obtained at two loop order [20]. In Yukawa theory the one loop fermion self-energy [21,22] and the scalar self-mass-squared [23] have both been evaluated at one loop order. A leading logarithm resummation has also been obtained for Yukawa, and checked against an explicit two loop computation [24]. And the one loop fermion self-energy has been calculated in Dirac + Einstein [25].

Of course density perturbations are directly observable, as are the expectation values of operators such as the stress tensor. However, an additional step is necessary in order to infer physics from a one particle irreducible (1PI) function such as the vacuum polarization $[{}^{\mu}\Pi^{\nu}](x;x')$, the fermion self-energy $[{}_{i}\Sigma_{j}](x;x')$, or the scalar self-mass-squared $M^2(x;x')$. These 1PI functions correct the classical linearized field equations as follows,

$$\partial_{\nu}(\sqrt{-g}g^{\nu\rho}g^{\mu\sigma}\mathcal{F}_{\rho\sigma}(x)) + \int d^4x'[{}^{\mu}\Pi^{\nu}](x;x')\mathcal{A}_{\nu}(x') = 0, \qquad (1)$$

$$\sqrt{-g}e^{\mu}_{a}\gamma^{a}_{ij}\Big(\partial_{\mu} + \frac{i}{2}A_{\mu bc}J^{bc}\Big)_{jk}\Psi_k(x) - \int d^4x'[{}_{i}\Sigma_{j}](x;x')\Psi_j(x') = 0, \qquad (2)$$

$$\partial_{\mu}(\sqrt{-g}g^{\mu\nu}\partial_{\nu}\Phi(x)) - \int d^4x' M^2(x;x')\Phi(x') = 0. \qquad (3)$$

One can infer physics from the quantum-corrected mode functions.

Doing this for photons in SQED [13], and for fermions in Yukawa [11], leads to the surprising result that the inflationary production of scalars endows these particles with mass. For fermions in Dirac + Einstein one finds that the field strength grows by an amount which eventually becomes nonperturbatively large [26]. On the other hand, the result for the scalars of Yukawa theory is that the finite part of the conformal counterterm can be chosen so that there are no significant late time corrections at one loop order [23]. The purpose of this paper is to demonstrate that the same result applies for the scalar mode functions of SQED.

We begin in Sec. II by defining the effective mode equation and the sense in which we solve it. We also explain the relation between the ℂ-number solutions we shall find and the quantum operators of SQED. Of course the self-mass-squared of SQED is gauge dependent, and has been computed in two different gauges. Section III solves for the mode functions in a non-de Sitter invariant version of Feynman gauge [19,27]; Sec. IV does the same for the de Sitter invariant Lorentz gauge [28,29]. In both cases our result is that the finite part of the conformal counterterm can be chosen so that there are no significant one loop corrections at late times. Section V summarizes our work and discusses its implications.

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II. THE EFFECTIVE MODE EQUATION

It turns out that the same operator formalism gives rise to many different effective field equations in quantum field theory. The purpose of this section is to specify both the particular ones we are solving and the sense in which we shall solve them. We begin by recalling the operator formalism of SQED [19,20]. We then parameterize general quantum-corrected mode functions and contrast the parameter selections appropriate to flat space scattering problems with the choices appropriate to cosmology. A brief review is given of the Schwinger-Keldysh formalism whose linearized effective field equations give the quantum-corrected mode functions for cosmology. We close with a discussion of the limitations on our knowledge and what they imply about the sense in which we should solve the effective mode equation.

A. Relation to fundamental operators

We work on the nondynamical metric background of de Sitter in conformal coordinates with spacelike signature,

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = a^2(\eta)(-d\eta^2 + d\vec{x}\cdot d\vec{x}) \quad \text{where } a(\eta) = -\frac{1}{H\eta}. \tag{4}$$

In other words, $g_{\mu\nu} = a^2\eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric. When expressed in terms of renormalized fields and couplings, the Lagrangian of SQED takes the form,

$$\mathcal{L} = \frac{1}{4}F_{\rho\sigma}F_{\mu\nu}g^{\rho\mu}g^{\sigma\nu}\sqrt{-g} - (\partial_\mu - ieA_\mu)\varphi^*(\partial_\nu + ieA_\nu)\varphi g^{\mu\nu}\sqrt{-g} - \delta Z_2(\partial_\mu - ieA_\mu)\varphi^*(\partial_\nu + ieA_\nu)\varphi g^{\mu\nu}\sqrt{-g} - \delta\xi\varphi^*\varphi R\sqrt{-g} - \frac{1}{4}\delta Z_3 F_{\rho\sigma}F_{\mu\nu}g^{\rho\mu}g^{\sigma\nu}\sqrt{-g} - \frac{1}{4}\delta\lambda(\varphi^*\varphi)^2\sqrt{-g}. \tag{5}$$

We have chosen to study the exactly massless version of the theory in which the renormalized values of the conformal and quartic couplings vanish. Hence the parameters have the following expansions in terms of the charge e,

$$\delta\xi = e^2\delta\xi_2 + e^4\delta\xi_4 + \ldots, \qquad \delta\lambda = e^4\delta\lambda_4 + \ldots, \tag{6}$$

$$\delta Z_2 = e^2\delta Z_{2,2} + e^4\delta Z_{2,4} + \ldots, \qquad \delta Z_3 = e^2\delta Z_{3,2} + e^4\delta Z_{3,4}. \tag{7}$$

The corrected mode functions for which we will solve are related to the fundamental operators through a number of choices which require extensive explanations. To fix notation for these explanations we begin by stating the relations,

$$\Phi(x;\vec{k}) = \langle\Psi_f|[\varphi(x), \alpha^\dagger(\vec{k})]|\Psi_i\rangle, \tag{8}$$

$$\mathcal{A}_\mu(x;\vec{k},\lambda) = \langle\Psi_f|[A_\mu(x), \gamma^\dagger(\vec{k},\lambda)]|\Psi_i\rangle. \tag{9}$$

The quantities to be explained are the states $|\Psi_i\rangle$ and $|\Psi_f\rangle$, and the free creation operators $\alpha^\dagger(\vec{k})$ and $\gamma^\dagger(\vec{k},\lambda)$.

Flat space scattering problems correspond to taking $|\Psi_i\rangle$ to be the state whose wave functional is free vacuum at asymptotically early times. One also chooses $|\Psi_f\rangle$ to be the state whose wave functional is free vacuum at asymptotically late times. Although these choices have great physical interest for flat space scattering problems, they have little relevance for cosmology. In cosmology the universe often begins at a finite time, and it evolves to some unknown state in the asymptotic future. Persisting with the choices of flat space scattering theory would result in acausal effective field equations which are dominated by the imperative of forcing the universe to approach free vacuum. The fact that we are looking at matrix elements, rather than expectation values, would also have the curious consequence of giving complex results for the matrix elements of Hermitian operators.

In cosmology it is better to imagine releasing the universe from a prepared state at some finite time, and then watching it evolve as it will. This corresponds to the choice $|\Psi_f\rangle = |\Psi_i\rangle$. We additionally assume that both are free vacuum at $\eta = \eta_i$.

So much for the states in relations (8) and (9). To understand the free creation and annihilation operators one integrates the invariant field equations of SQED,

$$D_\mu(\sqrt{-g}g^{\mu\nu}D_\nu\varphi) - \frac{\delta\xi\sqrt{-g}R\varphi}{1+\delta Z_2} - \frac{\delta\lambda\sqrt{-g}\varphi^*\varphi^2}{2(1+\delta Z_2)} = 0, \tag{10}$$

$$\partial_\nu(\sqrt{-g}g^{\nu\rho}g^{\mu\sigma}F_{\rho\sigma}) + \frac{ie\sqrt{-g}g^{\mu\nu}}{1+\delta Z_3}(\varphi^* D_\mu\varphi - (D_\nu\varphi)^*\varphi) = 0. \tag{11}$$

Here the covariant derivative operator is $D_\mu \equiv \partial_\mu + ieA_\mu$. The result of integrating (10) and (11) is the Yang-Feldman equations [30],

$$\varphi(x) = \varphi_0(x) + \int_{\eta_i}^0 d\eta' \int d^{D-1}x' G(x;x') I[\varphi^*, \varphi, A](x'), \tag{12}$$

$$A_\mu(x) = A_{0\mu}(x) + \int_{\eta_i}^0 \int d^{D-1}x' [{}_\mu G_\nu](x;x') \times J^\nu[\varphi^*, \varphi, A](x'). \tag{13}$$

The two interactions are,

$$I[\varphi^*, \varphi, A] = -ieA_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu \varphi - ie\partial_\mu(\sqrt{-g} g^{\mu\nu} A_\nu \varphi) + \sqrt{-g} g^{\mu\nu} e^2 A_\mu A_\nu \varphi + \frac{\delta\xi \sqrt{-g} R\varphi}{1 + \delta Z_2} + \frac{\delta\lambda \sqrt{-g} \varphi^* \varphi^2}{2(1 + \delta Z_2)}, \tag{14}$$

$$J^\mu[\varphi^*, \varphi, A] = \frac{-ie\sqrt{-g} g^{\mu\nu}}{1 + \delta Z_3} (\varphi^* D_\nu \varphi - (D_\nu \varphi)^* \varphi). \tag{15}$$

$G(x;x')$ is any solution to the free scalar Green's function equation,

$$\partial_\mu(\sqrt{-g} g^{\mu\nu} \partial_\nu G(x;x')) = \delta^D(x - x'), \tag{16}$$

and $[{}_\mu G_\nu](x;x')$ is any solution to the analogous photon Green's function equation in whatever gauge is employed. For flat space scattering problems the Green's functions would obey Feynman boundary conditions—and hence amount to $-i$ times the Feynman propagators. In cosmology it is more natural to use retarded boundary conditions.

We stress that the fundamental operators $\varphi(x)$ and $A_\mu(x)$ are unique and unaffected by the choices of η_i and the boundary conditions for the Greens functions. What changes as we vary η_i and the Green's functions is the free fields, $\varphi_0(x)$ and $A_{0\mu}(x)$. The fact that they obey the linearized equations of motion and agree with the full fields at $\eta = \eta_i$ implies that they can be expanded in terms of free creation and annihilation operators,

$$\varphi_0(x) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \{u(\eta, k) e^{i\vec{k}\cdot\vec{x}} \alpha(\vec{k}) + u^*(\eta, k) e^{-i\vec{k}\cdot\vec{x}} \beta^\dagger(\vec{k})\}, \tag{17}$$

$$A_{0\mu}(x) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \sum_\lambda \{\epsilon_\mu(\eta, k, \lambda) e^{i\vec{k}\cdot\vec{x}} \gamma(\vec{k}, \lambda) + \epsilon_\mu^*(\eta, k, \lambda) e^{-i\vec{k}\cdot\vec{x}} \gamma^\dagger(\vec{k}, \lambda)\}. \tag{18}$$

Here $u(\eta, k)$ and $\epsilon_\mu(\eta, k, \lambda)$ are the Bunch-Davies mode functions [31] in whatever gauge is being used. They also have canonically normalized Wronskians, which for the scalar is,

$$u(\eta, k)\partial_0 u^*(\eta, k) - \partial_0 u(\eta, k) u^*(\eta, k) = \frac{i}{a^{D-2}}. \tag{19}$$

Although the various creation and annihilation operators change as different Green's functions are employed in (12) and (13), their nonzero commutation relations remain fixed,

$$[\alpha(\vec{k}), \alpha^\dagger(\vec{k}')] = (2\pi)^{D-1} \delta^{D-1}(\vec{k} - \vec{k}') = [\beta(\vec{k}), \beta^\dagger(\vec{k}')], \tag{20}$$

$$[\gamma(\vec{k}, \lambda), \gamma^\dagger(\vec{k}', \lambda')] = (2\pi)^{D-1} \delta^{D-1}(\vec{k} - \vec{k}') \delta_{\lambda\lambda'}. \tag{21}$$

One develops perturbation theory by iterating (12) and (13) to obtain expansions for the full fields, $\varphi(x)$ and $A_\mu(x)$, in terms of the free fields, $\varphi_0(x)$ and $A_{0\mu}(x)$. With our choice of retarded boundary conditions the expansion for $\varphi(x)$ takes the form,

$$\varphi(x) = \varphi_0(x) + \int_{\eta_i}^0 d\eta' \int d^{D-1}x' G_{\rm ret}(x;x') I[\varphi^*, \varphi, A](x'), \tag{22}$$

$$= \varphi_0(x) + \int_{\eta_i}^0 d\eta' \int d^{D-1}x' G_{\rm ret}(x;x') I[\varphi_0^*, \varphi_0, A_0](x') + \dots . \tag{23}$$

With our choice of $|\Psi_f\rangle = |\Psi_i\rangle$ as free vacuum at η_i, the quantum-corrected scalar mode function is,

$$\Phi(x;\vec{k}) = \langle\Omega|[\varphi(x), \alpha^\dagger(\vec{k})]|\Omega\rangle = u(\eta, k) e^{i\vec{k}\cdot\vec{x}} + O(e^2). \tag{24}$$

B. The Schwinger-Keldysh formalism

It is straightforward to demonstrate [26] that the quantum-corrected mode function (24) obeys the linearized effective field equations of the Schwinger-Keldysh formalism [32–35]. This is a covariant extension of Feynman diagrams which produces true expectation values rather than the in-out matrix elements of conventional Feynman diagrams. Because many excellent reviews of this subject exist [4,36–39] we will simply state that part of the formalism which is necessary for this work.

The basic idea is that the endpoints of propagators acquire a $\pm$ polarity, so every propagator $i\Delta(x;x')$ of the in-out formalism generalizes to four Schwinger-Keldysh propagators: $i\Delta_{++}(x;x')$, $i\Delta_{+-}(x;x')$, $i\Delta_{-+}(x;x')$ and $i\Delta_{--}(x;x')$. The usual vertices attach to $+$ endpoints, whereas $-$ endpoints attach to the conjugate vertices. Had the state been some perturbative correction of free vacuum, this correction would give rise to additional vertices on the initial value surface—$+$ ones from $\Psi_i[\varphi_+^*(\eta_i), \varphi_+(\eta_i)]$ and $-$ ones from $\Psi_f[\varphi_-^*(\eta_i), \varphi_-(\eta_i)]$.

Because each external line can be either + or − in the Schwinger-Keldysh formalism, each N-point 1PI function of the in-out formalism corresponds to 2^N Schwinger-Keldysh N-point functions. The Schwinger-Keldysh effective action is the generating functional of these 1PI functions, so it depends upon + fields and − fields. The effective field equations come from varying with respect to either polarity and then setting the two polarities equal,

$$\frac{\delta\Gamma[\varphi^*_\pm, \varphi_\pm]}{\delta\varphi^*_+(x)}\Bigg|_{\substack{\varphi_\pm=\varphi\\ \varphi^*_\pm=\varphi^*}} = \partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi(x)) - \int_{\eta_i}^0 d\eta' \int d^3x' \{M^2_{++}(x;x') + M^2_{+-}(x;x')\}\varphi(x') + O(\varphi^*\varphi^2). \tag{25}$$

Note that we have taken the regularization parameter D to its unregulated value of $D = 4$, in view of the fact that the self-mass-squared is assumed to be fully renormalized. It is the *linearized* effective field equation which $\Phi(x;\vec{k})$ obeys,

$$\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi(x;\vec{k})) - \int_{\eta_i}^0 d\eta' \int d^3x' \{M_{++}(x;x') + M_{+-}(x;x')\}\Phi(x';\vec{k}) = 0. \tag{26}$$

Converting the single, in-out self-mass-squared—$M^2(x;x')$—into the four polarities of the Schwinger-Keldysh formalism is simple. The ++ polarization is the usual in-out self-mass-squared and the −− polarization is minus its conjugate,

$$M^2_{++}(x;x') = M^2(x;x') \quad \text{and} \quad M^2_{--}(x;x') = -(M^2(x;x'))^*. \tag{27}$$

To obtain the mixed polarizations we observe that, in de Sitter conformal coordinates, the in-out result depends mostly upon the conformal coordinate interval [26],

$$\Delta x^2_{++}(x;x') \equiv \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\delta)^2. \tag{28}$$

To obtain the +− and −+ polarities at one loop order we must do three things [26]:

(i) Replace $\Delta x^2_{++}(x;x')$ with the appropriate coordinate interval,

$$\Delta x^2_{+-}(x;x') \equiv \|\vec{x} - \vec{x}'\|^2 - (\eta - \eta' + i\delta)^2, \tag{29}$$

$$\Delta x^2_{-+}(x;x') \equiv \|\vec{x} - \vec{x}'\|^2 - (\eta - \eta' - i\delta)^2; \tag{30}$$

(ii) Drop all delta function terms; and

(iii) Multiply the result by −1 to account for the − vertex.

It will be seen that the ++ and +− terms in (26) exactly cancel for $\eta' > \eta$ and also, in the limit $\delta \to 0$, for x'^μ outside the light-cone of x^μ. This is how the Schwinger-Keldysh formalism gives causal effective field equations.

C. The meaning of "Solve"

Two important limitations on our current knowledge restrict the sense in which we can usefully solve the effective mode equation. The first of these is that we do not know the exact scalar self-mass-squared. The full result can be expressed as a series in powers of the loop counting parameter e^2,

$$M_{++}(x;x') + M_{+-}(x;x') = \sum_{\ell=1}^{\infty} e^{2\ell}\mathcal{M}^2_\ell(x;x'). \tag{31}$$

Because we possess only the $\ell = 1$ term, we can only solve the effective mode Eq. (26) perturbatively to order e^2. That is, we substitute a series solution,

$$\Phi(x;\vec{k}) \equiv u(\eta,k)e^{i\vec{k}\cdot\vec{x}} + \sum_{\ell=1}^{\infty} e^{2\ell}\Phi_\ell(\eta,k)e^{i\vec{k}\cdot\vec{x}}, \tag{32}$$

and then segregate according to powers of e^2. The 0th order solution is well known,

$$u(\eta,k) = \frac{H}{\sqrt{2k^3}}\left[1 - \frac{ik}{Ha}\right]\exp\left[\frac{ik}{Ha}\right],. \tag{33}$$

The 1st order solution $\Phi_1(\eta,k)$ obeys the equation,

$$a^2[\partial_0^2 + 2Ha\partial_0 + k^2]\Phi_1(\eta,k) = -\int_{\eta_i}^0 d\eta' \int d^3x' \mathcal{M}^2_1(x;x')u(\eta',k)e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}. \tag{34}$$

The second limitation is indicated by the subscript "η_i" on the temporal integration in (34): we release the universe in free vacuum at time $\eta = \eta_i$. Not much is known about the wave functionals of interacting quantum field theories in curved space, but it can hardly be that free vacuum is very realistic. All the finite energy states of interacting flat space quantum field theories possess important corrections. It is inconceivable that similar corrections are not present in curved space, at least in the far ultraviolet regime for which the geometry is effectively flat.

Although one could perturbatively correct the free states to a few orders, just as in nonrelativistic quantum mechanics, the standard procedure of flat space quantum field theory is to instead release the system in free vacuum at asymptotically early times. In the weak operator sense, infinite time evolution resolves the difference between free vacuum and true vacuum into shifts of the mass, field strength and background field [40]. In cosmology we cannot typically employ this procedure, however, it is still possible to perturbatively correct the state wave functionals.

Corrections to the initial state would show up as new interaction vertices on the initial value surface. One would expect them to have a large effect on the expectation values of operators near the initial value. One would also expect their effects to decay in the expectation values of late time operators. We have not worked out these surface vertices but the need for them has been apparent from the very first fully regulated computations of this type [14,18]. For example, consider the expectation value of the stress energy tensor of a massless, minimally coupled scalar with a quartic self interaction on a locally de Sitter background. If the state is released in free vacuum at the instant when the de Sitter scale factor takes the value $a = 1$, then one can choose the renormalization parameters so that the energy density and pressure induced at two loop order are [15],

$$\rho = \frac{\lambda H^4}{(2\pi)^4}\Big\{\frac{1}{8}\ln^2(a) + \frac{a^{-3}}{18} - \frac{1}{8}\sum_{n=1}^{\infty}\frac{(n+2)a^{-n-1}}{(n+1)^2}\Big\} + O(\lambda^2), \tag{35}$$

$$p = \frac{\lambda H^4}{(2\pi)^4}\Big\{-\frac{1}{8}\ln^2(a) - \frac{1}{12}\ln(a) - \frac{1}{24}\sum_{n=1}^{\infty}\frac{(n^2-4)a^{-n-1}}{(n+1)^2}\Big\} + O(\lambda^2). \tag{36}$$

It is the terms which fall off like powers of $1/a$ that we suspect can be absorbed into an order λ correction of the initial state,

$$\Delta\rho = \frac{\lambda H^4}{(2\pi)^4}\Big\{\frac{a^{-3}}{18} - \frac{1}{8}\sum_{n=1}^{\infty}\frac{(n+2)a^{-n-1}}{(n+1)^2}\Big\} + O(\lambda^2), \tag{37}$$

$$\Delta p = \frac{\lambda H^4}{(2\pi)^4}\Big\{-\frac{1}{24}\sum_{n=1}^{\infty}\frac{(n^2-4)a^{-n-1}}{(n+1)^2}\Big\} + O(\lambda^2). \tag{38}$$

Of course the fact that they fall off as one evolves away from the initial value surface suggests that they can be absorbed into some kind of local interaction there. Note also that they are separately conserved, which is exactly what would be the case if they could be canceled by a new interaction vertex.

Note particularly that expressions (37) and (38) *diverge* on the initial value surface. Divergences on the initial value surface have also be found in the effective mode equations for photons in SQED [18,41], for fermions in Yukawa theory [21], for scalars in Yukawa theory [23], and for fermions in Dirac + Einstein [26]. These initial value divergences reflect the fact that free vacuum is very far away from any physically accessible state. Equation (34) accurately determines the one loop correction to the mode function (24) appropriate to free vacuum, however, that mode function has little physical relevance because free vacuum cannot be assembled.

If one desires expressions for physically relevant mode functions which are valid even for times near the initial value surface, there is no alternative to including corrections to the state wave functional in the self-mass-squared. This seems quite practicable because one need only do it perturbatively to the same order as the ordinary, "volume" contributions are known. Although there are no stationary states for SQED on de Sitter background, it would probably be an excellent approximation—becoming exact in the ultraviolet—to simply solve for the relevant corrections in flat space.

Unfortunately, we do not now possess the order λ correction to the state wave functional. That means there is absolutely no point in trying to solve (34) for all times. However, it does not mean (34) is devoid of physical information. In particular, the effects of the state corrections must fall off at late times—and quite rapidly, like inverse powers of the scale factor. This fall off is evident in expressions (37) and (38). It simply reflects the same process which is employed in flat space quantum field theory [40] whereby time evolution washes away the difference between free vacuum and true vacuum. We can therefore extract valid information from (34) by solving it in the late time limit.

III. NON-DE SITTER INVARIANT GAUGE

Reliable results for the one loop scalar self-mass-squared have been obtained in two gauges. The first is a non-de Sitter invariant analogue of Feynman gauge which corresponds to adding the gauge-fixing term [19],

$$\mathcal{L}_{\rm GF} = -\frac{1}{2}a^{D-4}(\eta^{\mu\nu}A_{\mu,\nu})^2. \tag{39}$$

In this gauge the renormalized in-out (and hence also, the $++$) self-mass-squared is,

$$\begin{aligned} M^2_{\substack{\rm non\\ ++}}(x;x') = {} & -e^2\delta Z^{\rm non}_{\rm fin}\sqrt{-g}\square\delta^4(x-x') + 12e^2H^2\delta\xi^{\rm inv}_{\rm fin}\sqrt{-g}\delta^4(x-x') + \frac{e^2aa'}{8\pi^2}\ln(aa')\partial^2\delta^4(x-x') \\ & - \frac{e^2H^2}{4\pi^2}a^4\ln(a)\delta^4(x-x') - \frac{ie^2aa'}{2^8\pi^4}\partial^6\Big\{\ln^2\Big(\frac{1}{4}H^2\Delta x^2_{++}\Big) - 2\ln\Big(\frac{1}{4}H^2\Delta x^2_{++}\Big)\Big\} \\ & - \frac{ie^2H^4}{2^6\pi^4}(aa')^3\Big\{\nabla^2\Big[\ln^2\Big(\frac{1}{4}H^2\Delta x^2_{++}\Big) - \ln\Big(\frac{1}{4}H^2\Delta x^2_{++}\Big)\Big] + \frac{\partial^2}{2}\Big[\ln^2\Big(\frac{1}{4}H^2\Delta x^2_{++}\Big) - 3\ln\Big(\frac{1}{4}H^2\Delta x^2_{++}\Big)\Big]\Big\} \\ & + O(e^4). \end{aligned} \tag{40}$$

Here $e^2\delta Z^{\rm non}_{\rm fin}$ and $e^2\delta\xi^{\rm non}_{\rm fin}$ are the arbitrary finite parts of δZ_2 and $\delta\xi$, in this gauge and at order e^2. The various derivative operators are,

$$\square \equiv \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu), \qquad \partial^2 \equiv \eta^{\mu\nu}\partial_\mu\partial_\nu \quad \text{and} \quad \nabla^2 \equiv \partial_i\partial_i. \tag{41}$$

Applying the rules of the previous section allows us to recognize the $+-$ self-mass-squared as,

$$M^2_{\substack{\rm non\\ +-}}(x;x') = \frac{ie^2aa'}{2^8\pi^4}\partial^6\Big\{\ln^2\Big(\frac{1}{4}H^2\Delta x^2_{+-}\Big) - 2\ln\Big(\frac{1}{4}H^2\Delta x^2_{+-}\Big)\Big\} + \frac{ie^2H^4}{2^6\pi^4}(aa')^3\Big\{\nabla^2\Big[\ln^2\Big(\frac{1}{4}H^2\Delta x^2_{+-}\Big) - \ln\Big(\frac{1}{4}H^2\Delta x^2_{+-}\Big)\Big] + \frac{\partial^2}{2}\Big[\ln^2\Big(\frac{1}{4}H^2\Delta x^2_{+-}\Big) - 3\ln\Big(\frac{1}{4}H^2\Delta x^2_{+-}\Big)\Big]\Big\} + O(e^4). \tag{42}$$

The next step is to combine (40) with (42) to read off $\mathcal{M}^2_1(x;x')$ for the right hand side of (34). To simplify the notation we first define the spatial and temporal intervals,

$$\Delta r \equiv \|\vec{x} - \vec{x}'\| \quad \text{and} \quad \Delta\eta \equiv \eta - \eta'. \tag{43}$$

We also take note of the differences of powers of $++$ and $+-$ logarithms,

$$\ln\Big(\frac{1}{4}H^2\Delta x^2_{++}\Big) - \ln\Big(\frac{1}{4}H^2\Delta x^2_{+-}\Big) = 2\pi i\theta(\Delta\eta - \Delta r), \tag{44}$$

$$\ln^2\Big(\frac{1}{4}H^2\Delta x^2_{++}\Big) - \ln^2\Big(\frac{1}{4}H^2\Delta x^2_{+-}\Big) = 4\pi i\theta(\Delta\eta - \Delta r)\ln\Big[\frac{H^2}{4}(\Delta\eta^2 - \Delta r^2)\Big]. \tag{45}$$

With this notation we find,

$$\begin{aligned}\mathcal{M}^2_{\rm non1}(x;x') = {}& -\delta Z^{\rm non}_{\rm fin}\sqrt{-g}\square\delta^4(x-x') + 12H^2\delta\xi^{\rm non}_{\rm fin}\sqrt{-g}\delta^4(x-x') + \frac{aa'}{8\pi^2}\ln(aa')\partial^2\delta^4(x-x') \\ & - \frac{H^2}{4\pi^2}a^4\ln(a)\delta^4(x-x') + \frac{aa'}{2^6\pi^3}\partial^6\Big\{\theta(\Delta\eta - \Delta r)\Big(\ln\Big[\frac{H^2}{4}(\Delta\eta^2 - \Delta r^2)\Big] - 1\Big)\Big\} \\ & + \frac{H^4}{2^4\pi^3}(aa')^3\Big\{\nabla^2\Big[\theta(\Delta\eta - \Delta r)\Big(\ln\Big[\frac{H^2}{4}(\Delta\eta^2 - \Delta r^2)\Big] - \frac{1}{2}\Big)\Big] \\ & + \frac{\partial^2}{2}\Big[\theta(\Delta\eta - \Delta r)\Big(\ln\Big[\frac{H^2}{4}(\Delta\eta^2 - \Delta r^2)\Big] - \frac{3}{2}\Big)\Big]\Big\}.\end{aligned} \tag{46}$$

Note that the result is manifestly real and that each of the nonlocal factors is shielded by a causality-preserving factor of $\theta(\Delta\eta - \Delta r)$. These are features of the Schwinger-Keldysh formalism which are absent from the usual (in-out) effective field equations.

We now evaluate the contribution each term in (46) makes to the right hand side of the one loop effective mode Eq. (34). The local contributions are simple,

$$-\int_{\eta_i}^0 d\eta'\int d^3x'\{-\delta Z^{\rm non}_{\rm fin}a^4\square\delta^4(x-x')u(\eta',k)e^{i\vec{k}\cdot(\vec{x}'-\vec{x})}\} = \delta Z^{\rm non}_{\rm fin}a^4\square[u(\eta,k)e^{i\vec{k}\cdot\vec{x}}]e^{-i\vec{k}\cdot\vec{x}} = 0, \tag{47}$$

$$-\int_{\eta_i}^0 d\eta'\int d^3x'\{12H^2\delta\xi^{\rm non}_{\rm fin}a^4\delta^4(x-x')u(\eta',k)e^{ii\vec{k}\cdot(\vec{x}'-\vec{x})}\} = \frac{H^2a^4}{4\pi^2}\{-48\pi^2\delta\xi^{\rm non}_{\rm fin}u(\eta,k)\}, \tag{48}$$

$$\begin{aligned}& -\int_{\eta_i}^0 d\eta'\int d^3x'\Big\{\frac{aa'}{8\pi^2}\ln(aa')\partial^2\delta^4(x-x')u(\eta',k)e^{i\vec{k}\cdot(\vec{x}'-\vec{x})}\Big\} \\ & = \frac{a}{8\pi^2}\{\ln(a)(\partial_0^2 + k^2)(au(\eta,k)) + (\partial_0^2 + k^2)(a\ln(a)u(\eta,k))\},\end{aligned} \tag{49}$$

$$= \frac{H^2 a^4}{4\pi^2}\left\{2\ln(a)u(\eta,k) + \frac{3}{2}u(\eta,k) + \frac{1}{Ha}\partial_0 u(\eta,k)\right\}, \tag{50}$$

$$-\int_{\eta_i}^{0} d\eta' \int d^3x' \left\{-\frac{H^2}{4\pi^2} a^4 \ln(a)\delta^4(x-x')u(\eta',k)e^{i\vec{k}\cdot(\vec{x}'-\vec{x})}\right\} = \frac{H^2 a^4}{4\pi^2}\{\ln(a)u(\eta,k)\}. \tag{51}$$

The contribution of the first nonlocal term in (46) has been evaluated in a previous study of one loop corrections to the scalar mode functions of Yukawa theory [23]. By making the replacements,

$$f \to 1, \qquad \mu \to \frac{H}{2} \quad \text{and} \quad g(\eta',k) \to u(\eta',k), \tag{52}$$

we can read off the result from equation (68) of that paper, and some of the subsequent asymptotic expansions,

$$-\frac{a}{2^6\pi^3}e^{-i\vec{k}\cdot\vec{x}}\partial^6 \int_{\eta_i}^{\eta} d\eta' a' u(\eta',k)\int_{\Delta r\le\Delta\eta} d^3x' e^{i\vec{k}\cdot\vec{x}'}\left\{\ln\left[\frac{1}{4}H^2(\Delta\eta^2 - \Delta r^2)\right] - 1\right\}$$
$$= \frac{a}{4\pi^2}(\partial_0^2 + k^2)\left\{\begin{array}{c} -a\ln(a)u(\eta,k) \\ +\partial_0\int_{\eta_i}^{\eta} d\eta' a' u(\eta',k)\cos(k\Delta\eta)\ln(1-\frac{a'}{a}) \\ +k\int_{\eta_i}^{\eta} d\eta' a' u(\eta',k)\sin(k\Delta\eta)\ln(1-\frac{a'}{a}) \end{array}\right\}, \tag{53}$$

$$= \frac{H^2 a^4}{4\pi^2}\left\{-2\ln(a)u(\eta,k) - 3u(\eta,k) + O\left(\frac{1}{a^2}\right)\right\}. \tag{54}$$

We evaluate the contribution of the second nonlocal term in (46) by first making the change of variable $\vec{r} = \vec{x}' - \vec{x}$, then performing the angular integrations, and finally changing the radial variable to $r = \Delta\eta z$,

$$-\frac{H^4 a^3}{2^4\pi^3}e^{-i\vec{k}\cdot\vec{x}}\nabla^2 \int_{\eta_i}^{\eta} d\eta' a'^3 u(\eta',k)\int_{\Delta r\le\Delta\eta} d^3x' e^{i\vec{k}\cdot\vec{x}'}\left\{\ln\left[\frac{1}{4}H^2(\Delta\eta^2-\Delta r^2)\right] - \frac{1}{2}\right\}$$
$$= \frac{H^4 a^3}{4\pi^2}k\int_{\eta_i}^{\eta} d\eta' a'^3 u(\eta',k)\Delta\eta^2\int_0^1 dz z\sin(k\Delta\eta z)\left\{2\ln(H\Delta\eta) + \ln\left(\frac{1-z^2}{4}\right) - \frac{1}{2}\right\}. \tag{55}$$

The z integration can be performed in terms of sine and cosine integrals but there is no need to do this. We simply observe the behavior of the z integral for small $\Delta\eta$,

$$\int_0^1 dz z\sin(k\Delta\eta z)\left\{2\ln(H\Delta\eta) + \ln\left(\frac{1-z^2}{4}\right) - \frac{1}{2}\right\} \to \frac{2}{3}k\Delta\eta\ln(H\Delta\eta). \tag{56}$$

This implies that the η' integration in (56) converges at arbitrarily late times ($\eta \to 0^-$). Hence the second nonlocal term can be expanded as follows,

$$-\frac{H^4 a^3}{2^4\pi^3}e^{-i\vec{k}\cdot\vec{x}}\nabla^2\int_{\eta_i}^{\eta} d\eta' a'^3 u(\eta',k)\int_{\Delta r\le\Delta\eta} d^3x' e^{i\vec{k}\cdot\vec{x}'}\left\{\ln\left[\frac{1}{4}H^2(\Delta\eta^2-\Delta r^2)\right] - \frac{1}{2}\right\} = \frac{H^2 a^3}{4\pi^2}\left\{K + O\left(\frac{\ln(a)}{a}\right)\right\}, \tag{57}$$

where the constant K is,

$$K = \frac{k}{H^2}\int_{a_i}^{\infty} da' \frac{u(\eta',k)}{a'}\int_0^1 dz z\sin\left(\frac{kz}{Ha'}\right)\left[-2\ln(a') + \ln\left(\frac{1-z^2}{4}\right) - \frac{1}{2}\right\}. \tag{58}$$

We will see that only terms which grow as fast as a^4 can give significant corrections at late times.

The contribution from the final nonlocal term in (46) is the most difficult to evaluate. The initial steps are the same as for the second term. However, one must then act the temporal derivatives and express the z integration in terms of sine and cosine integrals,

$$-\frac{H^4 a^3}{2^5\pi^3} e^{-i\vec{k}\cdot\vec{x}}\partial^2 \int_{\eta_i}^{\eta} d\eta' a'^3 u(\eta',k) \int_{\Delta r\le\Delta\eta} d^3x' e^{i\vec{k}\cdot\vec{x}'}\Big\{\ln\Big[\frac{1}{4}H^2(\Delta\eta^2-\Delta r^2)\Big]-\frac{3}{2}\Big\}$$
$$=\frac{H^4a^3}{8\pi^2 k}(\partial_0^2+k^2)\int_{\eta_i}^{\eta} d\eta' a'^3 u(\eta',k)\Delta\eta^2\times\int_0^1 dz z\sin(k\Delta\eta z)\Big\{2\ln(H\Delta\eta)+\ln\Big(\frac{1-z^2}{4}\Big)-\frac{3}{2}\Big\}, \tag{59}$$

$$=\frac{H^4a^3}{8\pi^2k}\int_{\eta_i}^{\eta} d\eta' a'^3 u(\eta',k)\Big\{-2\cos(\alpha)\int_0^{2\alpha} dt\frac{\sin(t)}{t}+2\sin(\alpha)\Big[\int_0^{2\alpha}dt\frac{\cos(t)-1}{t}+2\ln\Big(\frac{H\alpha}{k}\Big)-\frac{1}{2}\Big]\Big\}. \tag{60}$$

Here we define $\alpha\equiv k\Delta\eta$.

To extract the leading late time behavior of expression (60) we first note that the term in curly brackets vanishes like $4\alpha\ln(\alpha)$ for $\eta'=\eta$. This means that the upper limit makes no contribution if we integrate by parts on a'^3,

$$\int_{\eta_i}^{\eta} d\eta' a'^3 u(\eta',k)\{\quad\}=\frac{a'^2}{2H}u(\eta',k)\{\quad\}\Big|_{\eta_i}^{\eta}-\frac{1}{2H}\int_{\eta_i}^{\eta}d\eta' a'^2\partial_0'[u(\eta',k)\{\quad\}], \tag{61}$$

$$=-\frac{1}{2H}a_i^2u_i\{\quad\}_i-\frac{1}{2H}\int_{\eta_i}^{\eta}d\eta' a'^2[\partial_0' u(\eta',k)\{\quad\}+u(\eta',k)\partial_0'\{\quad\}]. \tag{62}$$

Of course the term from the lower limit approaches a constant at late times, and it need not concern us further. The same is true of the integral in (62) which involves the derivative of $u(\eta',k)$,

$$\partial_0' u(\eta',k)=\frac{H}{\sqrt{2k^3}}\Big[-\frac{k^2}{Ha'}\Big]\exp\Big[\frac{ik}{Ha'}\Big]. \tag{63}$$

That leaves only the integral which involves the derivative of the curly bracketed term of (60),

$$\partial_0'\{\quad\}=-2k\sin(\alpha)\int_0^{2\alpha}dt\frac{\sin(t)}{t}-2k\cos(\alpha)\Big[\int_0^{2\alpha}dt\frac{\cos(t)-1}{t}+2\ln(H\Delta\eta)-\frac{1}{2}\Big]. \tag{64}$$

The term we need comes from substituting (64) into the final integral of expression (62). The contribution from the two t integrals approaches a constant at late times,

$$\frac{k}{H}\int_{\eta_i}^{\eta}d\eta' a'^2 u(\eta',k)\int_0^{2\alpha}dt\Big[\frac{\cos(t-\alpha)-\cos(\alpha)}{t}\Big]\longrightarrow\text{constant}. \tag{65}$$

Each of the two remaining terms in (64) requires a further partial integration to extract the leading late time behavior. For the term $k\cos(\alpha)$ we integrate by parts on a'^2,

$$\frac{k}{2H}\int_{\eta_i}^{\eta}d\eta' a'^2u(\eta',k)\cos(\alpha)=\frac{k}{2H^2}a'u(\eta',k)\cos(\alpha)\Big|_{\eta_i}^{\eta}-\frac{k}{2H^2}\int_{\eta_i}^{\eta}d\eta' a'\partial_0'[u(\eta',k)\cos(\alpha)], \tag{66}$$

$$=\frac{k}{2H^2}au(\eta,k)+\text{constant}. \tag{67}$$

The logarithm requires that we partially integrate on $a'^2\ln(H\Delta\eta)$ using the relation,

$$\int d\eta' a'^2\ln(H\Delta\eta)=-\frac{1}{H}\Big[(a-a')\ln\Big(1-\frac{a'}{a}\Big)+a'\ln(a')\Big]. \tag{68}$$

The result is,

$$-\frac{2k}{H}\int_{\eta_i}^{\eta}d\eta' a'^2\ln(H\Delta\eta)u(\eta',k)\cos(\alpha)=-\frac{2k}{H^2}a\ln(a)u(\eta,k)+\text{constant}. \tag{69}$$

It follows that the late time expansion of the final nonlocal term in (46) has the form,

$$-\frac{H^4a^3}{2^5\pi^3}e^{-i\vec{k}\cdot\vec{x}}\partial^2\int_{\eta_i}^{\eta}d\eta' a'^3u(\eta',k)\times\int_{\Delta r\le\Delta\eta}d^3x' e^{i\vec{k}\cdot\vec{x}'}\Big\{\ln\Big[\frac{1}{4}H^2(\Delta\eta^2-\Delta r^2)\Big]-\frac{3}{2}\Big\}=\frac{H^2a^4}{4\pi^2}\Big\{-\ln(a)u(\eta,k)+\frac{1}{4}u(\eta,k)+O\Big(\frac{1}{a}\Big)\Big\}. \tag{70}$$

Summing the various local and nonlocal contributions, and recalling that $\partial_0 u(\eta,k)\sim -k^2/Ha\times u(0,k)$, we find that the one loop effective mode equation in this gauge takes the form,

$$a^2[\partial_0^2+2Ha\partial_0+k^2]\Phi_1^{\rm non}(\eta,k)=\frac{H^2a^4}{4\pi^2}\Big\{-\Big[\frac{5}{4}+48\pi^2\delta\xi_{\rm fin}^{\rm non}\Big]u(\eta,k)+O\Big(\frac{1}{a}\Big)\Big\}. \tag{71}$$

To infer the asymptotic solution it is best to change the derivatives from conformal time η to comoving time $t = -\ln(-H\eta)/H$,

$$a^2[\partial_0^2 + 2Ha\partial_0 + k^2] = a^4\Big[\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\Big]. \qquad (72)$$

By choosing the finite part of the conformal counterterm appropriately,

$$\delta\xi^{\rm non}_{\rm fin} = -\frac{5}{12}\frac{1}{(4\pi)^2}, \qquad (73)$$

we reach an equation of the form,

$$\Big[\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\Big]\Phi_1^{\rm non}(\eta,k) = \frac{H^2}{4\pi^2}\Big\{\frac{C}{a} + O\Big(\frac{\ln(a)}{a^2}\Big)\Big\}, \qquad (74)$$

where C is a constant. Except for possible homogeneous terms—which can be absorbed into the finite part of the field strength renormalization at late times—the one loop correction rapidly redshifts to zero,

$$\Phi_1^{\rm non}(\eta,k) = -\frac{C}{8\pi^2}\frac{1}{a} + O\Big(\frac{\ln(a)}{a^2}\Big). \qquad (75)$$

IV. DE SITTER-LORENTZ GAUGE

Reliable results also exist for de Sitter-Lorentz gauge [28,29],

$$\partial_\mu(\sqrt{-g}g^{\mu\nu}A_\nu) = 0. \qquad (76)$$

Because this gauge preserves manifest de sitter invariance it is best to express results in terms of the following function of the invariant length $\ell(x;x')$,

$$y(x;x') = 4\sin^2\Big[\frac{1}{2}H\ell(x;x')\Big] = aa'H^2(x-x')^\mu(x-x')^\nu\eta_{\mu\nu}. \qquad (77)$$

The imaginary parts appropriate for the polarizations of the Schwinger-Keldysh formalism are,

$$y_{++}(x;x') \equiv aa'H^2\Delta x^2_{++}(x;x') \quad \text{and} \quad y_{+-}(x;x') \equiv aa'H^2\Delta x^2_{+-}(x;x'). \qquad (78)$$

In this notation the scalar self-mass-squared is [20],

$$M^2_{\substack{\rm inv\\++}}(x;x') = -e^2\delta Z^{\rm inv}_{\rm fin}\sqrt{-g}\Box\delta^4(x-x') + 12e^2H^2\delta\xi^{\rm inv}_{\rm fin}\sqrt{-g}\delta^4(x-x') - \frac{i3e^2H^2}{(4\pi)^4}\sqrt{-g}\sqrt{-g'}\Box\Big\{\frac{4}{y_{++}}\ln\Big(\frac{y_{++}}{4}\Big)\Big\} + \frac{i3e^2H^4}{(4\pi)^4}\sqrt{-g}\sqrt{-g'}\Box\Big\{7\Big(\frac{4}{y_{++}}\Big) + \Big[4\Big(\frac{4}{y_{++}}\Big) - 4\ln\Big(\frac{y_{++}}{4}\Big) + 8\ln\Big(1-\frac{y_{++}}{4}\Big) - \frac{6}{1-\frac{y_{++}}{4}}\Big]\ln\Big(\frac{y_{++}}{4}\Big) + 8\sum_{n=1}^{\infty}\frac{1}{n^2}\Big(\frac{y_{++}}{4}\Big)^n\Big\} + O(e^4). \qquad (79)$$

Before giving the $+-$ term and combining to work out $\mathcal{M}_1^2(x;x')$, it is best to remove the factors of $1/y$ using the identities,

$$\frac{4}{y} = \frac{\Box}{H^2}\Big\{\ln\Big(\frac{y}{4}\Big)\Big\} + 3, \qquad (80)$$

$$\frac{4}{y}\ln\Big(\frac{y}{4}\Big) = \frac{\Box}{H^2}\Big\{\frac{1}{2}\ln^2\Big(\frac{y}{4}\Big) - \ln\Big(\frac{y}{4}\Big)\Big\} + 3\ln\Big(\frac{y}{4}\Big) - 2. \qquad (81)$$

Because any analytic function of $y(x;x')$ will cancel in the sum of $++$ and $+-$ polarizations, we do not write out such terms explicitly,

$$M^2_{\substack{\rm inv\\++}}(x;x') = -e^2\delta Z^{\rm inv}_{\rm fin}\sqrt{-g}\Box\delta^4(x-x') + 12e^2H^2\delta\xi^{\rm inv}_{\rm fin}\sqrt{-g}\delta^4(x-x') + \frac{i3e^2H^6}{(4\pi)^4}\sqrt{-g}\sqrt{-g'}\Big\{\frac{\Box^3}{H^6}\Big[-\frac{1}{2}\ln^2\Big(\frac{y_{++}}{4}\Big) + \ln\Big(\frac{y_{++}}{4}\Big)\Big] + \frac{\Box^2}{H^4}\Big[2\ln^2\Big(\frac{y_{++}}{4}\Big)\Big] + \frac{\Box}{H^2}\Big[-4\ln^2\Big(\frac{y_{++}}{4}\Big) + 8\ln\Big(1-\frac{y_{++}}{4}\Big)\ln\Big(\frac{y_{++}}{4}\Big) + 12\ln\Big(\frac{y_{++}}{4}\Big) - \frac{6\ln(\frac{y_{++}}{4})}{1-\frac{y_{++}}{4}}\Big] + \text{Analytic}\Big\} + O(e^4). \qquad (82)$$

The procedure of Sec. II gives the corresponding $+-$ self-mass-squared,

$$M^2_{\substack{\rm inv\\ +-}}(x;x') = -\frac{i3e^2H^6}{(4\pi)^4}\sqrt{-g}\sqrt{-g'}\Big\{\frac{\Box^3}{H^6}\Big[-\frac{1}{2}\ln^2\Big(\frac{y_{+-}}{4}\Big) + \ln\Big(\frac{y_{+-}}{4}\Big)\Big] + \frac{\Box^2}{H^4}\Big[2\ln^2\Big(\frac{y_{+-}}{4}\Big)\Big] + \frac{\Box}{H^2}\Big[-4\ln^2\Big(\frac{y_{+-}}{4}\Big) + 8\ln\Big(1-\frac{y_{+-}}{4}\Big)\ln\Big(\frac{y_{+-}}{4}\Big) + 12\ln\Big(\frac{y_{+-}}{4}\Big) - \frac{6\ln(\frac{y_{+-}}{4})}{1-\frac{y_{+-}}{4}}\Big] + \text{Analytic}\Big\} + O(e^4). \quad (83)$$

Of course the delta functions terms in $\mathcal{M}^2_1(x;x')$ make the same contributions as they did for the noninvariant gauge of the previous section,

$$-\int_{\eta_i}^0 d\eta'\int d^3x'\{-\delta Z^{\rm inv}_{\rm fin}a^4\Box\delta^4(x-x') + 12H^2\delta\xi^{\rm inv}_{\rm fin}a^4\delta^4(x-x')\}u(\eta',k)e^{i\vec{k}\cdot(\vec{x}'-\vec{x})} = -12H^2\delta\xi_{\rm fin}a^4u(\eta,k). \quad (84)$$

We can avoid a lengthy computation of the contributions from the nonlocal terms by taking note of four points:

(i) As explained at the end of Sec. II, the only sensible and physically interesting regime in which we can solve the effective mode equation is for late times, long after the state was released;

(ii) In this regime we can replace the 0th order mode function by a constant,[1]

$$-\int_{\eta_i}^0 d\eta'\int d^3x'\mathcal{M}^2_1(x;x')u(\eta',k)e^{i\vec{k}\cdot(\vec{x}'-\vec{x})} \to -u(0,k)\int_{\eta_i}^0 d\eta'\int d^3x'\mathcal{M}^2_1(x;x'); \quad (85)$$

(iii) This reduces the nonlocal contributions to a sum of terms of the form,

$$-u(0,k)\frac{i3H^6a^4}{(4\pi)^4}\Big(\frac{\Box}{H^2}\Big)^N\int_{\eta_i}^0 d\eta' a'^4 \times\int d^3x'\Big\{f\Big(\frac{y_{++}}{4}\Big) - f\Big(\frac{y_{+-}}{4}\Big)\Big\}; \text{and} \quad (86)$$

(iv) The four integrals of the form (86) which we require have been worked out in a previous computation of the vacuum expectation value of two coincident scalar bilinears at two loop order [20].

Table I reproduces the necessary integrals from previous work [20]. Because the integrals can only depend upon a, the following identity is useful for acting the d'Alembertians,

$$\frac{\Box}{H^2}\Big(\alpha\ln^2(a) + \beta\ln(a) + \gamma + \delta\frac{\ln(a)}{a}\Big) = -\alpha(6\ln(a)+2) - 3\beta + \delta\Big[\frac{2\ln(a)-1}{a}\Big]. \quad (87)$$

The triple d'Alembertian nonlocal contribution is,

$$-u(0,k)\frac{i3H^6a^4}{(4\pi)^4}\frac{\Box^3}{H^6}\int_{\eta_i}^0 d\eta' a'^4\int d^3x'\Big\{-\frac{1}{2}\ln^2\Big(\frac{y_{++}}{4}\Big) + \ln\Big(\frac{y_{++}}{4}\Big) - (+-)\Big\} = u(0,k)\frac{3H^2a^4}{16\pi^2}\frac{\Box^3}{H^6}\left\{\begin{array}{l} -\frac{1}{12}\ln^2(a) + \frac{4}{9}\ln(a) - \frac{7}{8} + \frac{\pi^2}{18} \\ +\frac{1}{6}\ln(a) - \frac{11}{36} + O(\frac{\ln(a)}{a})\end{array}\right\}, \quad (88)$$

$$= u(0,k)\frac{3H^2a^4}{16\pi^2}\Big\{0 + O\Big(\frac{\ln(a)}{a}\Big)\Big\}. \quad (89)$$

The double d'Alembertian term gives,

$$-u(0,k)\frac{i3H^6a^4}{(4\pi)^4}\frac{\Box^2}{H^4}\int_{\eta_i}^0 d\eta' a'^4\int d^3x'\Big\{2\ln^2\Big(\frac{y_{++}}{4}\Big) - (+-)\Big\} = u(0,k)\frac{3H^2a^4}{16\pi^2}\frac{\Box^2}{H^4}\Big\{\frac{1}{3}\ln^2(a) - \frac{16}{9}\ln(a) + \frac{7}{2} - \frac{2\pi^2}{9} + O\Big(\frac{\ln(a)}{a}\Big)\Big\}, \quad (90)$$

$$= u(0,k)\frac{3H^2a^4}{16\pi^2}\Big\{6 + O\Big(\frac{\ln(a)}{a}\Big)\Big\}. \quad (91)$$

[1]For $u(\eta',k)$ this is obvious from the fact that expression (33) approaches a nonzero constant at late times. That the spatial plane wave factor of $e^{i\vec{k}\cdot(\vec{x}'-\vec{x})}$ can also be dropped follows from the causality of the Schwinger-Keldysh formalism. The factors of $\theta(\Delta\eta-\Delta r)$ which arise whenever $++$ and $+-$ terms are added—for example, expressions (44) and (45)—require $\|\vec{x}'-\vec{x}\| \leq \eta-\eta'$, which is small in the late time regime.

And the term with only a single d'Alembertian is,

$$-u(0,k)\frac{i3H^6a^4}{(4\pi)^4}\frac{\square}{H^2}\int_{\eta_i}^0 d\eta' a'^4\int d^3x'\left\{\begin{matrix}-4\ln^2(\frac{y_{++}}{4})+8\ln(1-\frac{y_{++}}{4})\ln(\frac{y_{++}}{4})-(+-)\\ +12\ln(\frac{y_{++}}{4})-6\ln(\frac{y_{++}}{4})/(1-\frac{y_{++}}{4})-(+-)\end{matrix}\right\}$$

$$=u(0,k)\frac{3H^2a^4}{16\pi^2}\frac{\square}{H^2}\left\{\begin{matrix}-\frac{2}{3}+\frac{4\pi^2}{9}+O(\frac{\ln(a)}{a})\\ +2\ln(a)-\frac{11}{3}-3\end{matrix}\right\}, \tag{92}$$

$$=u(0,k)\frac{3H^2a^4}{16\pi^2}\left\{-6+O\left(\frac{\ln(a)}{a}\right)\right\}. \tag{93}$$

Summing the local contribution (84), and the three nonlocal ones (89), (91), and (93), gives the following effective mode equation in Lorentz gauge,

$$a^2[\partial_0^2+2Ha\partial_0+k^2]\Phi_1^{\rm inv}=\frac{H^2a^4}{16\pi^2}\left\{-12\pi^2\delta\xi_{\rm fin}^{\rm inv}u(\eta,k)+O\left(\frac{\ln(a)}{a}\right)\right\}. \tag{94}$$

This is quite similar to the result of the previous section, and the subsequent analysis is the same. By choosing $\delta\xi_{\rm fin}^{\rm inv}=0$, and converting to comoving time, we obtain the form,

$$\left[\partial_t^2+3H\partial_t+\frac{k^2}{a^2}\right]\Phi_1^{\rm inv}(\eta,k)=\frac{H^2}{16\pi^2}\left\{C\frac{\ln(a)}{a}+O\left(\frac{1}{a}\right)\right\}, \tag{95}$$

where C is a constant. Except for possible homogeneous terms—which can be absorbed into the finite part of the field strength renormalization at late times—the one loop correction rapidly redshifts to zero,

$$\Phi_1^{\rm inv}(\eta,k)=-\frac{C}{32\pi^2}\frac{\ln(a)}{a}+O\left(\frac{1}{a}\right). \tag{96}$$

V. DISCUSSION

The Schwinger-Keldysh formalism gives effective field equations which are causal, and which allow real solutions for Hermitian fields. They are well adapted to cosmological settings in which the physically sensible experiment is to release the universe in a prepared state at some finite time. If this state is chosen to be free vacuum, the formalism will be simple in the sense of lacking interaction vertices on the initial value surface. However, the unphysical initial state results in effective field equations which diverge on the initial value surface. Even at late times, there will be some contamination from the unphysical initial state in the form of terms which decay. This could be avoided by perturbatively correcting the initial state—a worthy project which we have, unfortunately, not undertaken. However, one can still employ the simple equations reliably at late times where there are no divergences and the contamination is dying away.

We have solved the free-vacuum equations for one loop corrections to the scalar mode functions of SQED on a locally de Sitter background. Because the mode functions are gauge dependent, the computation was made in two different gauges. In each case it was possible to choose the finite part of the conformal counterterm so as to prevent the occurrence of significant corrections at late times.

This might seem a very different outcome from similar studies of one loop corrections to the mode functions of photons in SQED [18,41], fermions in Yukawa theory [21,22] and fermions in Dirac + Einstein [26]. In each of those cases the mode functions suffer one loop corrections which *grow* at late times, instead of falling off. In fact our result for the scalar mode functions of SQED does fit this pattern for a generic value of the conformal counterterm. Had we not chosen $\delta\xi_{\rm fin}^{\rm inv}=0$ in (94) the one loop correction would obey the equation,

$$\left[\partial_t^2+3H\partial_t+\frac{k^2}{a^2}\right]\Phi_1=-\frac{3}{4}H^2\delta\xi_{\rm fin}u(0,k)+O\left(\frac{\ln(a)}{a}\right). \tag{97}$$

In that case the resulting solution would indeed grow at late times,

$$\Phi_1=-\frac{1}{4}\delta\xi_{\rm fin}u(0,k)\ln(a)+O\left(\frac{\ln(a)}{a}\right). \tag{98}$$

So the difference between the scalar of SQED and the other cases is just that SQED possesses a free parameter which can be tuned to suppress the large one loop corrections that would otherwise occur.

It is worth pursuing this point a little further. Based upon (98), the form one expects for the largest late time corrections to the full mode function is,

TABLE I. Integrals of de Sitter Invariants. Multiply each term by $\frac{i(4\pi)^2}{H^4}$.

$f(x)$	$\int d^4x' a'^4\{f(\frac{y_{++}}{4})-f(\frac{y_{+-}}{4})\}$
$\frac{\ln(x)}{1-x}$	$\frac{1}{2}+O(\frac{\ln(a)}{a})$
$\ln^2(x)-2\ln(1-x)\ln(x)$	$\frac{1}{6}-\frac{\pi^2}{9}+O(\frac{\ln(a)}{a})$
$\ln(x)$	$\frac{1}{6}\ln(a)-\frac{11}{36}+O(\frac{1}{a})$
$\ln^2(x)$	$\frac{1}{6}\ln^2(a)-\frac{8}{9}\ln(a)+\frac{7}{4}-\frac{\pi^2}{9}+O(\frac{\ln(a)}{a})$

$$\Phi(\eta, k) = u(\eta, k) + \sum_{\ell=1}^{\infty} e^{2\ell}\Phi_\ell(\eta, k) \longrightarrow \frac{H}{\sqrt{2k^3}}\Big\{1 + \sum_{\ell=1}^{\infty} c_\ell (e^2 \ln(a))^\ell\Big\}. \qquad (99)$$

Secular corrections which involve powers of the infrared logarithm $\ln(a)$ are ubiquitous in quantum field theories that include MMC scalars and/or gravitons [4,10,12–15,20,26]. The continued growth of $\ln(a)$ offers the fascinating prospect of compensating for the small loop counting parameters—e^2 in this case—which usually suppress quantum loop corrections. However, the valid conclusion from a series of the form (99) is that theory breaks down at $\ln(a) \sim 1/e^2$, not that quantum loop effects necessarily become strong. A nonperturbative analysis is required to determine what actually transpires.

Starobinskiĭ has developed a stochastic formalism [42] which has been proven to reproduce the leading infrared logarithms at all orders in scalar potential models [43,44]. Starobinskiĭ and Yokoyama have shown how this formalism can be used to obtain explicit, nonperturbative results for general expectation values in these models [45]. Starobinskiĭ's formalism has recently been extended to Yukawa theory [25] and, of special interest to us, to SQED [46,47]. This nonperturbative formalism allows us to view the results we have obtained in a larger context.

First, note that the choice of $\delta\xi^{\rm inv}_{\rm fin} = 0$, which enforces the absence of significant late time corrections for de Sitter-Lorentz gauge, coincides precisely with the choice which cancels the leading infrared logarithm corrections to the expectation value of $\varphi^*(x)\varphi(x)$ at order e^2 in the same gauge [20]. This same choice cancels the $\varphi^*\varphi$ contribution to the effective potential [46,47]. The concurrence of three distinct results seems to confirm the stochastic prediction [46,47] that significant late time effects on the scalar derive solely from the effective potential and not, for example, from corrections to the effective action which carry derivatives.

Our result means that SQED can be renormalized so that the inflationary production of scalars is not affected at one loop order. That is all we can conclude from the analysis of this paper, but the stochastic formalism can of course see to any order. It informs us that the quartic counterterm $\delta\lambda$ can be chosen to cancel any $(e^2\varphi^*\varphi)^2$ contribution to the effective potential [46,47]. This presumably implies that $c_2 = 0$ in (99). However, there are unavoidable corrections to the effective potential at order $(e^2\varphi^*\varphi)^3$ and higher, which implies that the c_ℓ's do not vanish for $\ell \geq 3$. So our result that $c_1 = 0$ is an artifact of being at one loop order which also happens to be repeated at two loop order. The nonperturbative outcome for SQED is that inflationary particle production engenders a nonzero photon mass, and the associated vacuum energy prevents the average scalar field strength from growing past $\varphi^*\varphi \sim H^2/e^2$ [46,47]. The secular corrections to the scalar mode function (99) represent the scalar's initial response to the positive scalar mass associated with the vacuum energy of massive photons.

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[1] R. P. Woodard, in *Norman 2003, Quantum Field Theory Under the Influence of External Conditions* (Rinton Press, Princeton, 2004) edited by K. A Milton, pp. 325–330.
[2] V. F. Mukhanov and G. V. Chibisov, JETP Lett. **33**, 532 (1981).
[3] A. A. Starobinskiĭ, JETP Lett. **30**, 682 (1979).
[4] S. Weinberg, Phys. Rev. D **72**, 043514 (2005).
[5] D. Boyanovsky, H. J. de Vega, and N. Sanchez, Phys. Rev. D **72**, 103006 (2005).
[6] D. Boyanovsky and H. J. de Vega, and N. Sanchez Nucl. Phys. **B747**, 25 (2006).
[7] M. Sloth, Nucl. Phys. **B748**, 149 (2006).
[8] D. Boyanovsky, H. J. de Vega, and N. G. Sanchez, Phys. Rev. D **72**, 103006 (2005).
[9] Boyanovsky, H. J. de Vega, and N. G. Sanchez,Nucl. Phys. **B747**, 25 (2006).
[10] S. Weinberg, Phys. Rev. D **74**, 023508 (2006).
[11] N. C. Tsamis and R. P. Woodard, Phys. Rev. D **54**, 2621 (1996).
[12] N. C. Tsamis and R. P. Woodard, Nucl. Phys. **B474**, 235 (1996).
[13] N. C. Tsamis and R. P. Woodard, Ann. Phys. (N.Y.) **253**, 1 (1997).
[14] V. K. Onmeli and R. P. Woodard, Class. Quant. Grav. **19**, 4607 (2002).
[15] V. K. Onmeli and R. P. Woodard, Phys. Rev. D **70**, 107301 (2004).
[16] T. Brunier, V. K. Onemli, and R. P. Woodard, Class. Quant. Grav. **22**, 59 (2005).
[17] T. Prokopec, O. Törnkvist, and R. P. Woodard, Phys. Rev. Lett. **89**, 101301 (2002).
[18] T. Prokopec, O. Törnkvist, and R. P. Woodard, Ann. Phys. (N.Y.) **303**, 251 (2003).
[19] E. O. Kahya and R. P. Woodard, Phys. Rev. D **72**, 104001 (2005).
[20] T. Prokopec, N. C. Tsamis, and R. P. Woodard, gr-qc/0607094.
[21] T. Prokopec and R. P. Woodard, J. High Energy Phys. 10

(**2003**) 059.
[22] B. Garbrecht and T. Prokopec, Phys. Rev. D **73**, 064036 (2006).
[23] L. D. Duffy and R. P. Woodard, Phys. Rev. D **72**, 024023 (2005).
[24] S. P. Miao and R. P. Woodard, Class. Quant. Grav. **23**, 1721 (2006).
[25] S. P. Miao and R. P. Woodard, Phys. Rev. D **74**, 044019 (2006).
[26] S. P. Miao and R. P. Woodard, Phys. Rev. D **74**, 024021 (2006).
[27] R. P. Woodard, in *DESERFEST: A Celebration of the Life and Works of Stanley Deser*, edited by J. T. Liu, M. J. Duff, K. S. Stelle, and R. P. Woodard (World Scientific, Singapore, 2006) pp. 339-351.
[28] B. Allen and T. Jacobson, Commun. Math. Phys. **103**, 669 (1986).
[29] N. C. Tsamis and R. P. Woodard, ''A Maximally Symmetric Vector Propagator'' (to be published).
[30] C. N. Yang and D. Feldman, Phys. Rev. **79**, 972 (1950).
[31] N. D. Birrel and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
[32] J. Schwinger, J. Math. Phys. (N.Y.) **2**, 407 (1961).
[33] K. T. Mahanthappa, Phys. Rev. **126**, 329 (1962).
[34] P. M. Bakshi and K. T. Mahanthappa, J. Math. Phys. (N.Y.) **4**, 1 (1963); J. Math. Phys. (N.Y.) **4**, 12 (1963).
[35] L. V. Keldysh, Sov. Phys. JETP **20**, 1018 (1965).
[36] K. C. Chou, Z. B. Su, B. L. Hao, and L. Yu, Phys. Rep. **118**, 1 (1985).
[37] R. D. Jordan, Phys. Rev. D **33**, 444 (1986).
[38] E. Calzetta and B. L. Hu, Phys. Rev. D **35**, 495 (1987).
[39] L. H. Ford and R. P. Woodard, Class. Quant. Grav. **22**, 1637 (2005).
[40] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).
[41] T. Prokopec and R. P. Woodard, Ann. Phys. (N.Y.) **312**, 1 (2004).
[42] A. A. Starobinskiĭ, in *Field Theory, Quantum Gravity and Strings*, edited by H. J. de Vega and N. Sanchez (Springer-Verlag, Berlin, 1986), pp. 107–126.
[43] R. P. Woodard, Nucl. Phys. B, Proc. Suppl. **148**, 108 (2005).
[44] N. C. Tsamis and R. P. Woodard, Nucl. Phys. **B724**, 295 (2005).
[45] A. A. Starobinskiĭ and J. Yokoyama, Phys. Rev. D **50**, 6357 (1994).
[46] T. Prokopec, N. C. Tsamis, and R. P. Woodard, ''Stochastic Inflationary Scalar Electrodynamics'' (to be published).
[47] R. P. Woodard, gr-qc/0608037.