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Note on the stability of axionic *D*-term *s*-strings

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We investigate the stability of a new class of BPS cosmic strings in $N = 1$ supergravity with D-terms recently proposed by Blanco-Pillado, Dvali and Redi. These have been conjectured to be the low energy manifestation of D-strings that might form from tachyon condensation after D- anti-D-brane annihilation in type IIB superstring theory. There are three one-parameter families of cylindrically symmetric one-vortex solutions to the BPS equations (tachyonic, axionic and hybrid). We find evidence that the zero mode in the axionic case, or s-strings, can be excited. Its evolution leads to the decompactification of four-dimensional spacetime at late times, with a rate that decreases with decreasing brane tension.

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Recently Blanco-Pillado, Dvali and Redi have proposed a model to describe a D-brane anti-D-brane unstable system after compactification to four dimensions [1]. In the type IIB case, the tension of the branes appears in the four-dimensional effective theory as a constant Fayet Iliopoulos (FI) term which allows for the existence of nonsingular BPS axion-dilaton strings generalising earlier work by [2–4] in heterotic scenarios.

The model contains two complex scalar fields, the axion-dilaton $S = s + ia$, and the tachyon ϕ. A vector field A_μ gauges the $U(1)$ symmetry of the tachyon and the shift symmetry of the axion-dilaton. The coupling constants for ϕ and S are q and 2δ respectively.

It was shown that this model allows for three different families of BPS cylindrically symmetric cosmic string solutions: *tachyonic* or ϕ-strings, (see also [5]), *axionic*[1] or s-strings, and a third type which is the special case where the following relation between the windings is satisfied: $q|m| = |n|$,with n and m the winding of the tachyon and the axion, respectively. Within each of the families solutions are parametrized by a real number, κ, defined by:

$$s = 2\frac{\delta}{q}(|n| - q|m|)\log r - 2\frac{\delta}{q}\log|\phi| + \kappa. \qquad (1)$$

Here r is the radial coordinate on the plane perpendicular to the string, which is centered at $r = 0$. In the first two families κ is associated to a zero mode that connects vortex solutions with different core radius ($\sim R$) and equal magnetic flux, as is seen by setting:

$$k = -2(\delta/q)(|n| - q|m|)\log R. \qquad (2)$$

[1]The name axionic is somewhat misleading for the solutions discussed here since there is no $aF\tilde{F}$ coupling, in particular, they do not share the features usually associated with axionic strings [2].

In the third case κ measures which of the fields, dilaton or tachyon, contributes more to compensate the FI term. Supergravity effects were considered in [1] and, as expected on general grounds [6,7], the zero mode survives the coupling to supergravity. The s-strings are peculiar. The authors of [1] argued that they should be associated with D anti-D bound states that are unstable in ten dimensions, and therefore only exist after compactification. (Actually the field s diverges at the core, which is interpreted as a decompactification). They also noted that s-strings share some features with semilocal strings in the Bogomolnyi limit [8]. In the semilocal case any excitation of the zero mode [9] leads invariably to the spread of the magnetic field, the growth of the core radius and the eventual disappearance of the strings [10].

We have studied numerically the dynamics of this zero mode and we find that also in the s-string case it can be excited and will lead to the dissolution of the strings. For this type of string, the coefficient in parenthesis in Eq. (2) is negative, so growing R implies growing s. s is some combination of the dilaton and the volume modulus. As the core expands the field s grows without bound, and therefore also the volume modulus, so our result would appear to imply the decompactification of spacetime from four to ten dimensions at late times.

We start by briefly reviewing the model of [1] and the three families of BPS strings. We then analyze the zero mode numerically and conclude that it can be excited. For comparison, we show the results of the same analysis on the ϕ-string, where a perturbation of the string leads to some oscillations but no runaway behavior. The behavior of the ϕ-string zeromode under collisions is less clear, we comment on this at the end of the paper.

I. THE MODEL

The authors of [1] studied a supersymmetric abelian Higgs model containing a vector superfield V, a chiral superfield Φ with charge q, (the tachyon) and an axion-dilaton superfield S, coupled to the gauge multiplet in the usual way. The lowest component of S is $s + ia$, where a is the axion. We take the Kähler potential to be $K = -M_p^2 \log(S + \bar{S})$ and the gauge kinetic function are set to be constant, $f(S) = 1/g^2$, where g is the gauge coupling.

The bosonic sector of the Lagrangian, after eliminating the auxiliary field from the vector multiplet, is:

$$\mathcal{L} = -|D_\mu \phi|^2 - K_{S\bar{S}}|D_\mu S|^2 - \frac{1}{4} g^{-2} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} g^2 (\xi + 2\delta K_S - q|\phi|^2)^2. \tag{3}$$

K_S and $K_{S\bar{S}}$ represent the derivatives of the Kahler potential respect to the fields S and $\bar{S}$. ϕ is the lowest component of the chiral field, A_μ is the $U(1)$ gauge field, and $F_{\mu\nu}$ the abelian field strength. The covariant derivatives are given by $D_\mu \phi = \partial_\mu \phi - iqA_\mu \phi$ and $D_\mu S = \partial_\mu S + i2\delta A_\mu$.

The FI term ξ is related to the brane tension T by $T = g^2\xi^2/4$. We redefine:

$$\phi = \sqrt{\xi/q}\hat{\phi} \qquad s = \delta M_p^2/\xi \hat{s} \qquad a = 2\delta/q\hat{a}$$
$$A_\mu = g\sqrt{\xi/q}\hat{A}_\mu \qquad x = (g\sqrt{\xi q})^{-1}\hat{x}. \tag{4}$$

With these rescalings the axion $\hat{a}$ is defined modulo 2π, and δ is rescaled away. After dropping the hats, the bosonic sector of the Lagrangian reads:

$$\mathcal{L}(\xi g)^{-2} = -|D_\mu \phi|^2 - \frac{1}{4}(\alpha s)^{-2}(\partial_\mu s)^2 - (\alpha/s)^2(\partial_\mu a + A_\mu)^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2}(1 - s^{-1} - |\phi|^2)^2, \tag{5}$$

with $D_\mu \phi = \partial_\mu \phi - iA_\mu \phi$, and $\alpha^2 = \xi/(qM_p^2)$. Note that, since α is the symmetry breaking scale in Planck units, ignoring supergravity and superstring corrections would only be a consistent approximation for $\alpha \ll 1$. However, this limit is difficult to analyze numerically. We will present numerical results for $\alpha = 1$ and argue separately on the effect of lowering α.

To study straight vortices along, say, the z-direction, we drop the z dependence and set $A_z = 0$. For time independent configurations and defining $\tilde{S} = s + 2i\alpha^2 a$, and $\tilde{D}_\mu \tilde{S} = \partial_\mu \tilde{S} + 2i\alpha^2 A_\mu$, the energy functional can be written in the Bogomolnyi form:

$$\begin{aligned}\mathcal{E}(\xi g)^{-2} &= |(D_x \pm iD_y)\phi|^2 + \frac{1}{4}(\alpha s)^{-2}|(\tilde{D}_x \pm i\tilde{D}_y)\tilde{S}|^2 \\ &\quad + \frac{1}{2}(F_{xy} \mp (1 - s^{-1} - |\phi|^2))^2 \pm F_{xy} \\ &\quad \mp i[\partial_x(\phi^* D_y \phi) - \partial_y(\phi^* D_x \phi)] \\ &\quad \pm i\frac{1}{2}\alpha^{-2}[\partial_x(s^{-1}\tilde{D}_y\tilde{S}) - \partial_y(s^{-1}\tilde{D}_x\tilde{S})] \\ &\geq (\xi g)^2 \int d^2x F_{xy}.\end{aligned} \tag{6}$$

The bound is attained when the Bogomolnyi equations are satisfied:

$$(D_x \pm iD_y)\phi = (\tilde{D}_x \pm i\tilde{D}_y)\tilde{S} = F_{xy} \mp (1 - s^{-1} - |\phi|^2) = 0. \tag{7}$$

We will focus on cylindrically symmetric vortices, that can be described by the following ansatz:

$$\phi = f(r)e^{in\theta} \qquad s^{-1} = h^2$$
$$a = m\theta \qquad A_\theta = v(r)/r \tag{8}$$

(The dilaton ansatz makes comparison to the semilocal case easier. Also we shall see that h vanishes in various cases so we avoid having to deal with infinities). Then:

$$\mathcal{E} = A(r)^2 + B(r)^2/(\alpha h)^2 + \frac{1}{2}C(r)^2 + D(r)'/r \tag{9}$$

the Bogomolnyi equations

$$\begin{aligned}A(r) &\equiv f' - f(|n| - v)/r = 0 \\ B(r) &\equiv h' - \alpha^2 h^3(|m| - v)/r = 0 \\ C(r) &\equiv v'/r - (1 - f^2 - h^2) = 0.\end{aligned} \tag{10}$$

The last term is a boundary term with:

$$D(r) \equiv v - f^2(v - |n|) - h^2(v - |m|). \tag{11}$$

The condition $A(r) = B(r) = 0$ implies the following relation between the tachyon and the dilaton:

$$1/(\alpha h)^2 = 2(|n| - |m|)\log r - 2\log f + \kappa. \tag{12}$$

According to the asymptotic value of the profile functions for large values of r, f_∞, h_∞ and v_∞, solutions to the Bogomolnyi equations are classified in three families. Each of them is parametrized by the integration constant κ. In the first two families the value of κ is associated to a particular width of the strings.

(i) In the first case, $f_\infty = 1$, $h_\infty = 0$, $v_\infty = n$ the tachyon develops a vacuum expectation value far from the center of the string. The magnetic flux of these vortices is induced by the winding of the tachyon, n. The profile function $h(r)$ tends very slowly, (logarithmically), to zero at large r. The details about the asymptotics of the fields can be found in [1], where these vortices are called ϕ-strings.

(ii) In the second case, the dilaton alone is responsible for compensating the D-term. The function $h(r)$, approaches a non vanishing constant for $r \to \infty$, while the tachyon expectation value tends to zero. The magnetic flux is induced by the winding of the axion, m: $f_\infty = 0$, $h_\infty = 1$, $v_\infty = m$. These vortices have been denominated s-strings.

(iii) For strings of the third family both the tachyon and the dilaton contribute to compensate the D-term. This happens when the axion and dilaton have the same winding, $f_\infty^2 + h_\infty^2 = 1$ and $v_\infty = n = m$. Each particular f_∞, can be associated to a single κ, which means that this parameter cannot be related any more to the width of the strings, so we will not discuss it.

As can be seen in (10) the derivatives of h scale as α^2. As a consequence for very small α the ϕ-strings are similar to a Nielsen-Olesen (NO) string. In the case of s-strings as α decreases the width of the string increases. But we shall see that the main effect of decreasing α for s-strings is a slowing down of the dynamics.

II. DISCRETIZED EQUATIONS OF MOTION

The functions $f(r)$, $h(r)$ and $v(r)$ are replaced by their values f_k, h_k and v_k at the lattice points $r_k = (k + \frac{1}{2})\Delta$, where Δ is the lattice spacing. To analyze the response of the solutions of the Bogomolnyi equations under perturbations we must make sure that the configurations we perturb are stationary solutions of the equations of motion. This is automatic in the continuous case but not in an arbitrary discretization. Following [10] we construct a discrete version of the energy functional for static configurations given by:

$$\mathcal{E}_k \Delta^2 = A_k^2 + \frac{4B_k^2}{\alpha^2 (h_{k+1} + h_k)^2} + \frac{C_k^2}{2\Delta^2} + \frac{(D_{k+1} - D_k)}{k + \frac{1}{2}} \tag{13}$$

where

$$\begin{aligned} A_k &= f_{k+1} - f_k - (n - v_k)\frac{f_{k+1} + f_k}{2k + 1}, \\ B_k &= h_{k+1} - h_k - \alpha^2 (m - v_k)\frac{(h_{k+1} + h_k)^3}{2k + 1}, \\ C_k &= \frac{v_{k+1} - v_k}{k + \frac{1}{2}} - \Delta^2 (1 - h_{k+1}^2 - f_{k+1}^2), \\ D_k &= v_k - f_k^2 (v_k - n) - h_k^2 (v_k - m). \end{aligned} \tag{14}$$

The profile functions are obtained minimizing the total energy for static configurations:

$$\mathcal{E} = 2\pi \sum_{k=0}^{\infty} \Delta^2 \left(k + \frac{1}{2}\right) E_k, \tag{15}$$

for which the discretized Bogomolnyi equations have to be satisfied: $A_k = B_k = C_k = 0$. The boundary conditions are set at r_0, r_1. At r_0 we impose $v_0 = 0$, and at r_1 we fix the values of f_1 and h_1. One of these is tuned to obtain the correct asymptotic behavior, and each value of the other one corresponds to a different solution within a family. The following discretized version of the action is naturally associated to the energy functional (15):

$$\begin{aligned} \mathcal{S} &= -2\pi\tau\Delta^2 \sum_{l,k=0}^{\infty} \left(k + \frac{1}{2}\right)(E_k^l - T_k^l)\tau^2 \\ \mathcal{T}_k^l &= (f_k^{l+1} - f_k^l)^2 + \frac{(h_k^{l+1} - h_k^l)^2}{\alpha^2 h_k^2} - \frac{(v_k^{l+1} - v_k^l)^2}{(2k + 1)\Delta^4}. \end{aligned} \tag{16}$$

The superscript l labels the time slices, separated by an interval τ. $\mathcal{T}_k^l$ represents energy density associated to the time derivatives. The equations of motion can be derived from (16) by setting to zero the partial derivatives of the action with respect to f_k^l, h_k^l, v_k^l. As the solutions of the Bogomolnyi equations are static and minimize the discretized energy functional they must be stationary solutions of the discretized time dependent equations of motion. The boundary conditions have been implemented using the method of [10]. All measured quantities take information from a region of radius $r_{\rm cal} = 4.5$. The simulation is stopped at $t_{\rm max} = 2(r_{\rm max} - r_{\rm cal})$. In this way the region from which we take data is not affected by the presence of the boundary. A typical value for the size of the lattice is $r_{\rm max} = 21.5$, but it varies. During the simulation we keep track of the following quantities, where $k_{\rm cal}$ is defined by $r_{\rm cal} = (\frac{1}{2} + k_{\rm cal})\Delta$:

$$E(l, k_{\rm cal}) = 2\pi \sum_{k=0}^{k_{\rm cal}} \Delta^2 \left(k + \frac{1}{2}\right) E_k^l \tag{17}$$

$$T(l, k_{\rm cal}) = 2\pi\Delta^2 \sum_{k=0}^{k_{\rm cal}} \left(k + \frac{1}{2}\right) T_k^l \tag{18}$$

$$E_T(l, k_{\rm cal}) = \frac{E(l, k_{\rm cal}) + T(l, k_{\rm cal})}{E(0, k_{\rm cal}) + T(0, k_{\rm cal})} \tag{19}$$

$$W = 2\pi\Delta^3 \frac{\sum_{k=0}^{k_{\rm cal}} (k + \frac{1}{2})^2 E_k^l}{E(l, k_{\rm cal})} \tag{20}$$

$$F(l, k_{\rm cal}) = 2\pi (D_{k_{\rm cal}}^l - D_0^l) \tag{21}$$

E is the static energy, and T the energy due to the time derivatives, both measured in the interval $r < r_{\rm cal}$. E_T is the total energy normalized to the initial value. W is a measure of the width of the string. F gives the magnetic flux confined in $r < r_{\rm cal}$. To implement the perturbation, we set a solution of the discrete Bogomolnyi equations in the first time slice $l = 0$, and in the second, $l = 1$, we put the same solution slightly deformed. We characterize the strength of the perturbation by the fractional change of width between the first, ($l = 0$), and second, ($l = 1$), time slices:

$$\dot{W}_0 \equiv (W(l = 1) - W(l = 0))/W(l = 0). \tag{22}$$

During the simulation we have set $\Delta = 0.1$ and $\tau = 0.05$, with τ/Δ smaller than the Courant number $1/\sqrt{2}$.

A. Tachyonic strings

In this case to implement the perturbation we take the second time slice to be:

$$f_k^1 = (1 + p(r))f_k^0, \qquad v_k^1 = (1 + p(r))v_k^0$$
$$h_k^1 = (1/(h_k^0)^2 + 2\log(1 - p(r)))^{-1/2} \tag{23}$$

with the perturbation $p(r) = p_0[1 - 3(r/r_0)^2 + 2(r/r_0)^3]$ for $r < r_0$ and zero otherwise. The perturbation has been chosen in order to maximize the fraction of energy absorbed by the zero mode. Notice the relation between the perturbations of the tachyon field and the dilaton, and the Eq. (12) that gives h in terms of f, in fact for small values of the parameter p_0 the perturbed profile approximately satisfies the Bogomolnyi equations. p_0 is chosen so that the perturbation initially reduces the width of the string, but the same results are obtained in the opposite case. The parameter r_0 takes values close to $r_0 = 2$. Figure 1 shows the evolution of an isolated ϕ-string, with $n = 1$, $m = 0$ and a core size $h_1 = 0.51$. We show the case $\alpha = 1$, which is also the choice made in [1]. The perturbation applied has a strength $\dot{W}_0 = -0.261$, which corresponds to a 0.4% perturbation in the energy. We have plotted the observables defined in the last section as a function of time. Modulo to rescalings upper solid line represents E, the dashed line just below is the magnetic flux in $r < r_{cal}$, which remains almost constant. The kinetic energy, T, is represented by the bottom solid line. The dashed line at the center is the total energy. After $t = 12$ the system reaches a stationary state where all quantities oscillate except the magnetic flux and the total energy. The solid line in the center is the string width. As the energy contained in the region $r \leq r_{cal}$ remains constant and the width of the vortex oscillates only around the initial value we conclude that ϕ-strings are stable under this perturbation. The experiment has been repeated in a wide range of the parameters, p_0, r_0, and for different initial widths (parametrized by h_1), and windings obtaining similar results. Evolutions with different values of α do not show any qualitative change. As was mentioned before, the smaller the value of α, the more similar the ϕ-string is to a NO string, which is known to be stable.

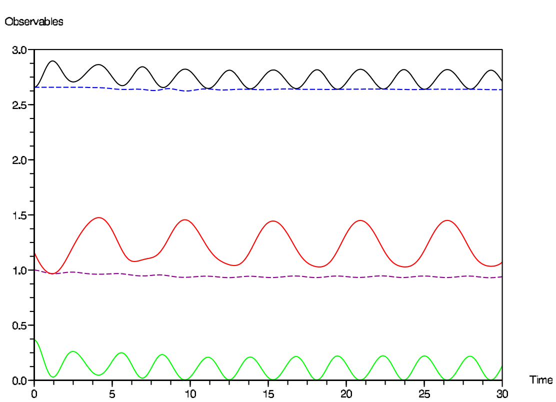

FIG. 1 (color online). Response of a ϕ-string ($n = 1$, $m = 0$), with $\alpha = 1$ and condensate size $h_1 = 0.5$, to a perturbation with strength $\dot{W}_0 = -0.261$. The plotted lines are, from top to bottom, $E/2$, $F/2$, W, E_T and $T/2$ (some plots have been rescaled to fit in the window). The core width, W, oscillates but is constant on average showing that the zero mode is not excited.

B. Axionic strings

The perturbation that excites the zero mode is:

$$f_k^1 = (1 - p(r))f_k^0, \qquad v_k^1 = (1 - p(r))v_k^0$$
$$h_k^1 = (1/(h_k^0)^2 + 2\log(1 + p(r)))^{-1/2}. \tag{24}$$

Figure 2 shows the result of applying this perturbation with a strength of $\dot{W}_0 = -0.095$ to an s-string, with $\alpha = 1$. In this case the perturbation in energy is 2.4%. The string has windings $n = 0$ and $m = 1$, and core size $f_1 = 0.51$. The functions plotted are the same ones that appear in Fig. 1. The magnetic flux and the total energy decrease with time, meaning that the energy flows out from the region $r \leq r_{cal}$, and the magnetic flux spreads. The width of the string, ignoring the oscillatory behavior, increases at a constant rate. Although the perturbation was chosen to reduce the core width, the time interval when the core is contracting cannot be seen clearly in the figures because it ends before the initial burst of radiation comes out from the observed region. We show this case because the expanding regime can be seen more clearly. Figure 3 shows the effect of applying perturbations of different strengths to a $n = 0$, $m = 2$ vortex. In this case the vortex has also a condensate size of $f_1 = 0.51$. The strengths applied are: $\dot{W}_0 = -0.004$, -0.014, -0.024. The fraction of magnetic flux lost through the boundary $r = r_{cal}$ and the rate of growth of the radius increase with the strength of the perturbation.

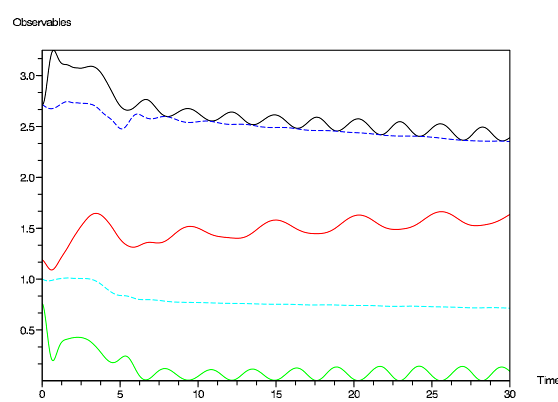

FIG. 2 (color online). Response of an s-string ($n = 0$, $m = 1$), with $\alpha = 1$ and condensate size $f_1 = 0.49$, to a perturbation with strength $\dot{W}_0 = -0.095$. From top to bottom: $E/2$, $F/2$, W, E_T and $T/2$. The core width, W, after a transient, oscillates and increases at a constant rate, thus the zero mode is excited.

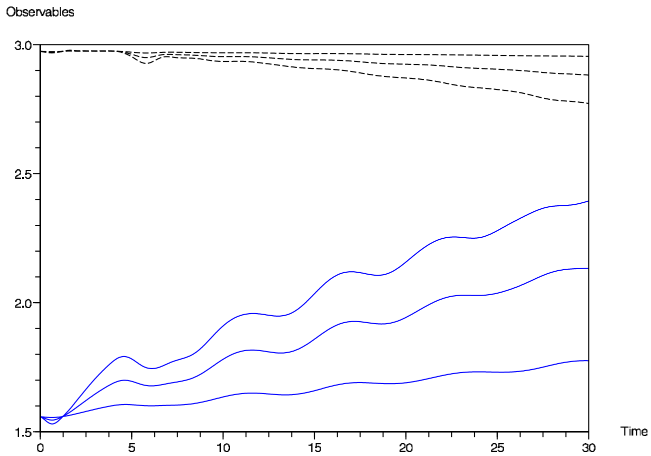

FIG. 3 (color online). Response of an $n = 0$, $m = 2$, $\alpha = 1$, $f_1 = 0.51$, s-string to perturbations with $\dot{W}_0 = -0.004$, -0.014 and -0.024. Dashed lines correspond to $F/4$, the strength is lowest for the top one. Solid lines represent W, the strength is highest for the top one.

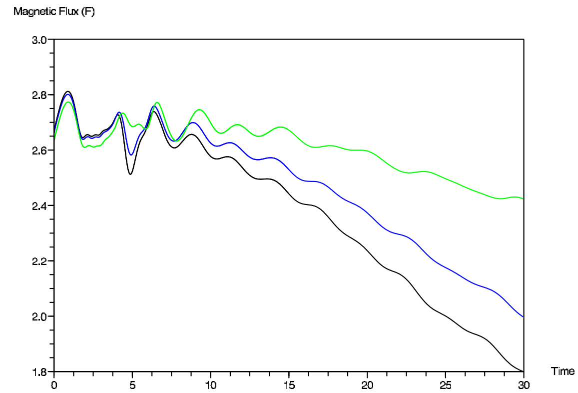

FIG. 4 (color online). Response of an $n = 0$, $m = 1$, $f_1 = 0.51$, s-string to a perturbation with $\dot{W}_0 = 0.233$. The curves represent $F/4$. From bottom to top $\alpha = 0.95$, 0.94 and 0.90.

The effect of varying α on the rate of expansion of a s-string, ($m = 1$, $n = 0$), is displayed in Fig. 4. The perturbation has been chosen to initially increase the core size $\dot{W}_0 = 0.233 > 0$. As we decrease α the rate of expansion of the string decreases. This can be understood from Eq. (16). The energy associated to the field h scales as the inverse of α^2. Deviations from the solution to the Bogomolnyi equations cost more energy for smaller values of α, thus for a fixed perturbation strength the evolution rates should decrease with α. The values of alpha are: $\alpha = 0.95$, 0.94, 0.90. The technique used here does not allow to obtain reliable data for values of α lower than 0.7, where the evolution is too slow to be appreciated during the time of the simulation. However, time is measured in units of the inverse of the Higgs mass, thus even for lower values of α decompactification is still possible on cosmological time scales. The results of the last two sections apply to the case of isolated ϕ- and s-strings. In order to be conclusive in the stability analysis, multiple vortex configurations should also be studied. We have looked at the excitability of the zero mode in two-vortex collisions using the moduli space approximation. In the s-strings case our results are unchanged, the zero mode is excited in collisions. More importantly, we have preliminary results suggesting that the zero mode of the ϕ-strings is also excited in collisions. This work is in progress and the results will be presented elsewhere. Careful studies of intercommutation would also help to elucidate the validity of this conjecture. We would like to thank the referee for pointing us to this issue. To summarize, we have investigated the stability of a new class of axionic D-term strings proposed by [1]. We found that finite energy perturbations can make the string radius and the field s grow without bound. Since the field s involves the volume modulus of the compact dimensions, the growth of s is interpreted as a decompactification of spacetime at late times. The rate of growth of s increases with the parameter α which is related to the brane tension T as $\alpha^4 \sim \xi^2/M_p^4 \sim T/(g^2 M_p^4)$.

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