Precision radiative corrections to the semileptonic Dalitz plot with angular correlation between polarized decaying and emitted baryons: Effects of the four-body region

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Analytical radiative corrections of order $(\alpha/\pi)(q/M_1)$ are calculated for the four-body region of the Dalitz plot of baryon semileptonic decays when the $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ correlation is present. Once the final result is available, it is possible to exhibit it in terms of the corresponding final result of the three-body region following a set of simple changes in the latter. We cover two cases, a charged and a neutral polarized decaying baryon.

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In this report we shall improve the precision of analytical radiative corrections (RC) to the four-body region (FBR) of the Dalitz plot of baryon semileptonic decays, $A \to B l \bar{\nu}_l$, with a polarized decaying baryon by including the order $(\alpha/\pi)(q/M_1)$ contributions. The $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ angular correlation will be kept explicitly. The corresponding corrections to order $(\alpha/\pi)(q/M_1)^0$ were obtained in Ref. [1]. Here we follow the same notation and conventions, and we recall that in this region neither the neutrino nor the real photon can reach zero energy. As explained there, this region must be incorporated into the RC when bremsstrahlung photons cannot be discriminated either kinematically or by direct detection. This is the case when only the emitted baryon momentum is measured and its corresponding asymmetry coefficient is determined.

The result we want cannot be obtained from the corresponding final result for the three-body region (TBR), because the variable y_0 plays a double role in the latter case. It appears in the integrand and as the upper limit of the $y = \hat{\mathbf{p}}_2 \cdot \hat{l}$ integration variable [see Eqs. (6) and (7) of Ref. [1]]. In the FBR case y_0 is no more an upper limit of y, since now the latter has one as an upper limit, and y_0 appears only in the integrand. One is then forced to perform the complete calculation. However, once the final result is obtained, it is possible to present it in terms of the final result of the TBR by making simple changes in the latter. This allows us to make a concise presentation and avoid many unnecessary repetitions.

The calculation of RC in the FBR has the same structure as in the TBR, so there is an exact parallelism all along. To mark the difference we shall introduce a subindex F in the expressions and definitions that pertain to the FBR. In this report we shall present only new results, all others will be appropriately referenced.

The complete Dalitz plot without the restriction of kinematically discriminating real photons is

$$d\Gamma_i = d\Gamma_i^{\mathrm{TBR}} + d\Gamma_i^{\mathrm{FBR}}, \qquad i = \mathrm{C}, \mathrm{N} \tag{1}$$

where $i = \mathrm{C}$, N refers to charged and neutral decaying baryon cases, respectively. The analytical RC to order $(\alpha/\pi)(q/M_1)$ to the TBR part including the $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ correlation are found in Eqs. (24) and (44) of Ref. [2] for $i = \mathrm{C}$, N, respectively. The FBR arises only from bremsstrahlung and it can be separated into an unpolarized part ($\hat{\mathbf{s}}_1 = \mathbf{0}$) and into another one containing the $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ correlation, that is,

$$d\Gamma_i^{\mathrm{FBR}} = d\Gamma_{iB}^{\prime\mathrm{FBR}} - d\Gamma_{iB}^{(s)\mathrm{FBR}}. \tag{2}$$

Here the subindex B stresses the bremsstrahlung origin of these parts. To order $(\alpha/\pi)(q/M_1)$ the spin-independent part is found, with some changes in notation, in Eq. (32) of Ref. [3] for $i = \mathrm{C}$ and in Eq. (22) of Ref. [4] for $i = \mathrm{N}$. To order $(\alpha/\pi)(q/M_1)^0$ the spin-dependent part is found in Eq. (18) of Ref. [1] for $i = \mathrm{C}$, N.

To obtain the order $(\alpha/\pi)(q/M_1)$ of $d\Gamma_{CB}^{(s)\mathrm{FBR}}$ we trace a parallelism with Eq. (14) of Ref. [2] for the TBR. The final result is compactly given by

$$d\Gamma_{\mathrm{C}B}^{(s)\mathrm{FBR}} = \frac{\alpha}{\pi} d\Omega \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2 [B_2' I_{C0F} + C_{AF}^{(s)}]. \tag{3}$$

B_2' is given in Eq. (7) of Ref. [2]. The infrared convergent I_{C0F}, explicitly given in Eq. (37) of Ref. [5], corresponds to the infrared divergent I_{C0} of the TBR. $C_{AF}^{(s)}$ can be arranged as the sum

$$C_{AF}^{(s)} = C_{IF} + C_{IIF} + C_{IIIF}, \tag{4}$$

where

$$C_{IF} = \sum_{i=1}^{8} Q_{i+6}\Lambda_{iF}, \qquad C_{IIF} = \sum_{i=6}^{15} Q_i \Lambda_{(i+3)F}, \qquad C_{IIIF} = \sum_{i=16}^{25} Q_i \Lambda_{(i+3)F}. \tag{5}$$

which are the counterparts of Eqs. (21)–(23) of Ref. [2].

The Q_i $(i = 6, \ldots, 25)$ are quadratic functions of the form factors and are common to both regions. Their explicit expressions are found in Appendix A of Ref. [2]. It should be clear that for Q_6 and Q_7 we use $\tilde{Q}_6$ and $\tilde{Q}_7$ of this Appendix, since the contributions of orders $(q/M_1)^2$ and higher have been subtracted [6].

The analytical form of $C_{AF}^{(s)}$ is obtained by performing explicitly the triple integrals over the real photon variables, contained in the Λ_{iF} functions $(i = 1, \ldots, 28)$. These integrals result in a set of analytical functions θ_{jF} $(j = 0, \ldots, 22)$. The connection between the Λ_{iF} and the θ_{jF} functions requires several algebraic steps, which are exhibited in terms of intermediate functions X_{iF}, Y_{iF}, Z_{iF}, N_{iF}, I_F, η_{0F}, γ_{0F}, χ_{ijF}, and ζ_{ijF}.

Once the final results for the FBR are available it is only necessary to give explicitly the θ_{jF} functions to present it. The Λ_{iF} and the intermediate functions [7] can be obtained from the corresponding Λ_i and X_i, Y_i, Z_i, N_i, I, η_0, γ_0, χ_{ij}, and ζ_{ij}, respectively, of the TBR by making some simple changes in the latter. This is possible because of the parallelism mentioned above. These changes are: (i) a subindex F is attached to all the corresponding functions of the TBR, (ii) the terms proportional to the factor $(1 - y_0)$ are replaced by zero, and (iii) otherwise the factor y_0 is kept as such. There are three exceptions to rule (iii). In I, χ_{11}, and η_0, y_0 appears by itself only once and there it must be replaced by $y_0 = 1$ to produce I_F, χ_{11F}, and η_{0F} respectively. The Λ_i, X_i, Y_i, Z_i, N_i, I, η_0, and γ_0 are found in Appendix B of Ref. [2]. It should be stressed that $\Lambda_{2F} = 0$ as is $\Lambda_2 = 0$, and that is why it does not appear in this Appendix. The χ_{ij}, and ζ_{ij} are found in Sec. IV of Ref. [8], but ζ_{10}, χ_{10}, and χ_{20} were not directly given in terms of θ_i there and the above rules cannot be easily applied to them. It is better to give the explicit connection now, namely,

$$\zeta_{10F} = p_2 l\left[\theta_{13F} - \frac{l}{p_2}\theta_{10F} - \frac{E_\nu^0}{p_2}\theta_{5F}\right],$$

$$\chi_{10F} = l[2\eta_{0F} - E_\nu^0\theta_{4F} - l\theta_{5F}],$$

$$\chi_{20F} = 2l^2\left[p_2 y_0\theta_{4F} + E_\nu^0\theta_{5F} + 2l\theta_{10F} - p_2\theta_{13F} - \frac{1}{2}\theta_{14F} - \frac{E_\nu^0}{l}\eta_{0F}\right].$$

The function ζ_{31} of Ref. [2] has also a misprint in the term proportional to θ_{12}. In order to avoid further confusions, we provide here the right expression for ζ_{31F}, which reads

$$\begin{aligned}\zeta_{31F} = {}& p_2 l y_0[2(3E^2 - l^2)\theta_{3F} - 6E^2(\theta_{4F} + \beta\theta_{5F}) + \theta_{9F}] \\ & - 30lE^2 p_2\theta_{13F} - 30l^2 E p_2\theta_{19F} - \frac{6l^3}{\beta^4}[5(l + \beta E_\nu^0) \\ & + 3\beta^2(p_2 y_0 - l)](\theta_{3F} - \theta_{4F} - \beta\theta_{5F}) \\ & - 18l^2 E E_\nu^0(\theta_{4F} - \theta_{3F}) + 6p_2 l^3 y_0\theta_{3F} \\ & + 30lE^2(l + \beta E_\nu^0)\theta_{10F} + 30El^3\theta_{20F} - \frac{1}{2}\theta_{22F} \\ & - 6p_2\left[E^2 l(\beta^2 - 5) - \frac{2lp_2^2 + 2\beta p_2 l y_0(E + E_\nu^0)}{b^+ b^-}\right]\theta_{12F},\end{aligned}$$

so that the correct ζ_{31} of the TBR can be obtained by dropping the subindex F from the θ's in the above equation. We need to point out that neither results nor conclusions are affected by that misprint.

Let us illustrate the above procedure with an example. Λ_{15F} is obtained from

$$\begin{aligned}\Lambda_{15} = {}& \frac{\beta(E + E_\nu^0)}{4M_1}\left[p_2^2\theta_7 + 2p_2^2 E(\theta_4 - \theta_3) + 2\zeta_{21} - \frac{2}{l}X_3\right] \\ & + \frac{X_4}{2M_1} - \frac{l^2 p_2}{4M_1}(y_0^2 - 1) + \frac{p_2}{4M_1 E}[4(ly_0 + p_2)X_2 \\ & + p_2\chi_{21}],\end{aligned}$$

by first putting $y_0 = 1$ into the factor $(1 - y_0)$ only and keeping y_0 as such elsewhere, and second by attaching a subindex F to the intermediate functions. The result is

$$\begin{aligned}\Lambda_{15F} = {}& \frac{\beta(E + E_\nu^0)}{4M_1}\left[p_2^2\theta_{7F} + 2p_2^2 E(\theta_{4F} - \theta_{3F}) + 2\zeta_{21F} \right.\\ & \left. - \frac{2}{l}X_{3F}\right] + \frac{X_{4F}}{2M_1} + \frac{p_2}{4M_1 E}[4(ly_0 + p_2)X_{2F} \\ & + p_2\chi_{21F}].\end{aligned}$$

With these changes all we now need is the list of the θ_{jF} functions. Their explicit forms for $j = 1, \ldots, 16$ were already calculated in Ref. [5] and they are found in Appendix B of this reference. θ_{0F} is given in Eq. (38) of this reference, $\theta_{1F} = \theta_{17F} = 0$, and $\theta_{18F} = 1$.

The functions θ_{jF} with $j = 19, \ldots, 22$ are new. They are defined as $\theta_{19F} = \int_{-1}^{1} x\xi_4^T(x)dx$, $\theta_{20F} = \int_{-1}^{1} x^3\xi_1^T(x)dx$, $\theta_{21F} = \int_{-1}^{1} x^2\xi_2^T(x)dx$, and $\theta_{22F} = \int_{-1}^{1} \xi_6^T(x)/(1 - \beta x)dx$. The functions $\xi_1^T(x)$, $\xi_2^T(x)$, and $\xi_4^T(x)$ are found in the Appendix of Ref. [3]. Here we only need the explicit form of $\xi_6^T(x)$, which reads

$$\begin{aligned}\xi_6^T(x) = {}& h_1(x)(1 - \beta x) + \frac{h_2(x)}{(x + a^+)(x + a^-)} + \frac{h_3^+(x)}{(x + a^+)^2} \\ & + \frac{h_3^-(x)}{(x + a^-)^2} + \frac{h_4^+(x)}{(x + a^+)} + \frac{h_4^-(x)}{(x + a^-)},\end{aligned} \tag{6}$$

where

$$h_1(x) = 24p_2l^2\left[\frac{a^+y_0^+}{b^+(x+a^+)} + \frac{a^-y_0^-}{b^-(x+a^-)}\right]x, \tag{7a}$$

$$h_2(x) = 8l\{6x[2a_1^2 + p_2^2 - y_0p_2xa_1] - p_2^2x(3+x^2) - 3y_0p_2a_1(1+x^2) + 6xa_1^2 + 6p_2[2y_0a_1 - (y_0^2+1)p_2x]\}, \tag{7b}$$

$$h_3^\pm(x) = 2l\{2p_2^2x(3+x^2) + 6y_0p_2a_1(1+x^2) - 12xa_1^2 \mp 2p_2^2y_0(y_0^2+3) \pm 6a_1[2y_0a_1 - (y_0^2+1)p_2x]\}, \tag{7c}$$

$$h_4^\pm(x) = \mp\left[\frac{4l^2}{p_2}\right][3p_2(p_2x + y_0a_1) - a^\pm(p_2^2x^2 + 3y_0p_2a_1x - 6a_1^2)]. \tag{7d}$$

We have used the definitions

$$a^\pm = \frac{E_\nu^0 \pm p_2}{l}, \qquad b^\pm = 1 + \beta a^\pm,$$
$$y_0^\pm = y_0 \pm a^\pm, \qquad a_1 = E_\nu^0 + lx.$$

The analytical results of θ_{19F}, θ_{20F}, θ_{21F} and θ_{22F} are

$$\theta_{19F} = \frac{4}{3p_2}, \qquad \theta_{20F} = \frac{1}{p_2}(T_{20F}^+ + T_{20F}^-),$$
$$\theta_{21F} = T_{21F}^+ + T_{21F}^-, \qquad \theta_{22F} = T_{22F}^+ + T_{22F}^-,$$

where

$$T_{20F}^\pm = \pm\frac{1}{4}[1-(a^\pm)^4]I_2^\pm \pm \frac{a^\pm}{2}\left[(a^\pm)^2 + \frac{1}{3}\right],$$
$$T_{21F}^\pm = \frac{4}{3} \mp 2a^\pm(y_0 \pm a^\pm)[a^\pm I_2^\pm - 2],$$
$$T_{22F}^\pm = \frac{1}{2}L_0 + \frac{1}{2}L_1I_1 + L_2^\pm I_2^\pm + L_3^\pm I_3^\pm.$$

The I_1, $I_2^\pm$, and $I_3^\pm$ functions are found at the end of Appendix C of Ref. [8] and the L_i are defined as

$$L_0 = 48l^2p_2\left(\frac{a^+y_0^+}{b^+} + \frac{a^-y_0^-}{b^-}\right) - 16E[p_2^2 + 3l(2l - p_2y_0)],$$
$$L_1 = \frac{\beta^2h_2(1/\beta)}{b^+b^-} + \frac{\beta[\beta h_3^+(1/\beta) + b^+h_4^+(1/\beta)]}{(b^+)^2} + \frac{\beta[\beta h_3^-(1/\beta) + b^-h_4^-(1/\beta)]}{(b^-)^2},$$
$$L_2^\pm = \frac{-24p_2l^2(a^\pm)^2y_0^\pm + h_4^\pm(-a^\pm)}{b^\pm} + \frac{b^\pm h_3^\pm(0) - h_3^\pm(-a^\pm)}{b^\pm a^\pm} - 4Ea^\pm[3l(2l - y_0p_2) - p_2^2]$$
$$- \frac{\beta^2a^\pm h_3^\pm(1/\beta)}{(b^\pm)^2} \mp \frac{lh_2(-a^\pm)}{2p_2b^\pm},$$
$$L_3^\pm = \frac{h_3^\pm(-a^\pm)}{b^\pm},$$

where the functions $h_i(x)$ are the same of Eq. (7), but evaluated at the indicated arguments.

For a neutral process we proceed again in parallel to the calculation of $d\Gamma_{NB}^{(s)}$ given in Sec. IIIB of Ref. [2]. Thus, we can express $d\Gamma_{NB}^{(s)\mathrm{FBR}}$ as in Eq. (31) of this reference as

$$d\Gamma_{NB}^{(s)\mathrm{FBR}} = \frac{\alpha}{\pi}d\Omega\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2[B_2'I_{N0F} + C_{AF}^{(s)} + C_{NAF}^{(s)}], \tag{8}$$

where B_2' and $C_{AF}^{(s)}$ are those of Eq. (3). The function I_{N0F} corresponds to the infrared divergent I_{N0} of Eq. (35) of Ref. [9]. As in the charged case, I_{N0F} is no longer infrared divergent. It can be calculated up to order (q/M_1) by using the approximation $1/p_2 \cdot k \simeq 1/p_1 \cdot k + q \cdot k/(p_1 \cdot k)^2$. The result is

$$I_{N0F} = I_{C0F} + \delta I_0,$$

where I_{C0F} is the same of Eq. (3) and

$$\delta I_0 = \frac{4p_2}{M_1\beta}\left[\frac{y_0}{2}\ln\left(\frac{y_0+1}{y_0-1}\right) - 1\right]\left[\left(\frac{1-\beta^2}{\beta}\right)\mathrm{arctanh}\beta - 1\right].$$

It should be remarked that this expression is equal to zero when $\beta \to 0$ and $y_0 \to \infty$.

The other new integrals appear only in $C_{NAF}^{(s)}$. They are

$$C_{NAF}^{(s)} = D_3\rho_{N3F} + D_4\rho_{N4F}.$$

Here $D_3 = 2(f_1g_1 - g_1^2)$ and $D_4 = 2(f_1g_1 + g_1^2)$, ρ_{N3F} and ρ_{N4F} are

$$\rho_{N3F} = \rho_{IF} + \rho_{IIF} + \rho_{IIIF},$$
$$\rho_{N4F} = \rho_{IF}' + \rho_{IIF}' + \rho_{IIIF}'.$$

The analytical forms of the integrals ρ_{iF} and ρ_{iF}' ($i = I, II, III$) can be obtained from Eqs. (34)–(39) of Ref. [2] again by changing y_0 to one as the upper limit in the y integrals. After the integration one can see that the analytical form of the ρ_{iF} and ρ_{iF}' can be obtained from the analytical expressions of ρ_i and ρ_i' for TBR, which are found in Appendix C of Ref. [2], using the same rules (i), (ii), and (iii) applied to Λ_i to obtain the Λ_{iF} of the FBR for charged decaying baryons.

We have made crosschecks between numerical integrals and analytical results of the Λ_{iF}, ρ_{iF}, and ρ'_{iF}, and they were satisfactory. Our results are model-independent and include terms up to order $(\alpha/\pi)(q/M_1)$, this is as far one can improve the precision of RC before introducing model-dependence into them. They are useful when $l = e^{\pm}$, $\mu^{\pm}$, and $\tau^{\pm}$. The two cases discussed here, charged and neutral decays, allow one to cover the other four charge assignments predicted by heavy quarks in baryons [10]. When such quarks are involved, the results of Ref. [1] are useful in low statistics experiments (several hundreds of events) and the improved results presented here are useful in medium statistics experiments (several thousands of events).

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