

Effects of S-wave thresholds

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The opening of a new S -wave threshold is frequently accompanied by an abrupt dip in the magnitude of an amplitude for an already-open channel. One familiar example is the behavior of the $I = 0$ S -wave $\pi\pi$ scattering amplitude at $K\bar{K}$ threshold. Numerous other examples of this phenomenon in recent data are noted, and a unified description of the underlying dynamics is sought.

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I. INTRODUCTION

The rapid drop in the magnitude of the $I = 0$ S -wave $\pi\pi$ scattering amplitude near a center-of-mass energy $E_{\text{cm}} \simeq 1$ GeV has been known for many years. It appears to be associated with the rapid passage of the elastic phase shift through 180° near the center-of-mass energy at which the amplitude becomes highly inelastic as a result of the opening of the $K\bar{K}$ threshold [1]. A resonance with $J^{PC} = 0^{++}$ (J = total angular momentum, P = parity, C = charge-conjugation eigenvalue) now known as $f_0(980)$ [2], coupling both to $\pi\pi$ and $K\bar{K}$, appears to be responsible for the rapid variation of the $I = J = 0$ elastic phase shift δ_0^0 . For detailed discussions of the amplitude and phase shift, see Refs. [3,4].

A wide variety of reactions in particle physics, many of which have been observed only recently, display a similar rapid drop in the magnitude of an S -wave amplitude in one channel when another channel opens up. The present article is devoted to a discussion of this phenomenon and its possible dynamical implications, with suggestions for further experimental study.

The vanishing or rapid decrease of cross sections is familiar from fields outside particle physics. For example:

- (i) The *Ramsauer-Townsend effect* [5] corresponds to a minimum of the elastic cross section at which a phase shift for S -wave scattering goes through 180° . Writing the S -matrix as $S = \eta e^{2i\delta}$, where η is the inelasticity parameter and δ is the phase shift, one sees that for $\eta = 1$ and $\delta = \pi$, the scattering amplitude $f = (S - 1)/(2ik)$ vanishes. (Here k is the c.m. three-momentum.)
- (ii) Cusps in S -wave scattering cross sections occur at thresholds for *any* new channels [6]. This behavior has recently been discussed in Ref. [7] in the context of several processes to be mentioned here.
- (iii) *Monochromatic neutrons* may be produced by utilizing the vanishing absorption cross sections of neutrons of certain energies on specific nuclei [8].

Within particle physics, a number of recently observed dips appear to be correlated with S -wave thresholds:

- (1) The value of $R \equiv \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ drops sharply around $\sqrt{s} = 4.26$ GeV [9],

which happens to be just below the threshold for production of $D\bar{D}_1 +$ charge conjugate (c.c.), where D and D_1 are charmed mesons with $J^{PC} = 0^-$ and 1^+ , respectively.

- (2) With the advent of high-statistics Dalitz plots for heavy meson decays one often sees dips and edges correlated with thresholds [10]. The $\pi\pi$ spectrum in $D^0 \rightarrow K_S\pi^+\pi^-$ [11–13] shows discontinuities both at $M(\omega)$ (a rapid fall-off in $M(\pi\pi)$ due to ρ - ω interference) and around 1 GeV/ c^2 (a sharp rise due to rescattering from $K\bar{K}$). A recent $D^0 \rightarrow K^+K^-\pi^0$ Dalitz plot based on CLEO data [14] contains depopulated regions near $m(K^\pm\pi^0) \simeq 1$ GeV/ c^2 which may be due to the opening of the $K\pi^0 \rightarrow K\eta$ S -wave threshold or to a vanishing S -wave $K\pi$ amplitude between a low-energy $K\pi$ resonance (“ κ ”) and a higher resonance. A candidate for such a resonance exists around 1430 MeV/ c^2 [2].
- (3) Diffractive photoproduction of $3\pi^+3\pi^-$ exhibits a dip near $p\bar{p}$ threshold [15,16]. This dip also occurs in the $3\pi^+3\pi^-$ spectrum produced in radiative return in higher-energy e^+e^- collisions, i.e., in $e^+e^- \rightarrow \gamma 3\pi^+3\pi^-$, observed by the *BABAR* Collaboration at SLAC [17].
- (4) The $p\bar{p}$ spectrum produced in radiative return [17] exhibits dips at ~ 2.15 GeV/ c^2 and ~ 3.0 GeV/ c^2 , which lie just below the respective thresholds for $p\bar{\Delta}(1232)$ and $N(1520)\bar{N}(1520)$, respectively.

We shall discuss these and others in Sec. II. A possible dynamical origin of these effects is posed in Sec. III. Implications for further experiments are noted in Sec. IV, while Sec. V summarizes.

II. ILLUSTRATIONS OF S-WAVE THRESHOLDS**A. Cusp in $\pi^0\pi^0$ spectrum at $\pi^+\pi^-$ threshold**

The $\pi^0\pi^0$ S -wave scattering amplitude is expected to have a cusp at $\pi^+\pi^-$ threshold [18,19]. This behavior can be studied in the decay $K^+ \rightarrow \pi^+\pi^0\pi^0$, where the contribution from the $\pi^+\pi^+\pi^-$ intermediate state allows one to study the charge-exchange reaction $\pi^+\pi^- \rightarrow \pi^0\pi^0$ and thus to measure the $\pi\pi$ S -wave scattering length differ-

ence $a_0 - a_2$ [20]. The CERN NA48 Collaboration has performed such a measurement, finding $(a_0 - a_2)m_{\pi^+} = 0.264 \pm 0.006(\text{stat}) \pm 0.004(\text{sys}) \pm 0.013(\text{ext})$ [21] in remarkable agreement with the prediction [20] 0.265 ± 0.004 . One can also study this effect in $\pi^+ \pi^-$ atoms. In this manner the DIRAC Collaboration measured $|a_0 - a_2| = (0.264_{-0.020}^{+0.033})/m_{\pi^+}$ [22].

B. Cusp in $\pi^0 p$ spectrum at $\pi^+ n$ threshold

In the photoproduction reaction $\gamma p \rightarrow \pi^0 p$ the $\pi^+ n$ threshold lies a few MeV above the $\pi^0 p$ threshold as a result of the $\pi^+ - \pi^0$ and $n - p$ mass differences. The real part of the π^0 photoproduction electric dipole amplitude E_{0+} shows a pronounced cusp at $\pi^+ n$ threshold [23], in accord with predictions of chiral perturbation theory [19,24].

C. Behavior of R in $e^+ e^-$ annihilations

The parameter R describing the cross section for $e^+ e^- \rightarrow \text{hadrons}$ normalized by $\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)$ describes, *on the average*, the sum of squares of charges Q_i of those quarks which can be produced at a given energy:

$$R = \sum_i Q_i^2 \left(1 + \frac{\alpha_S}{\pi} + \dots \right), \quad (1)$$

where the α_S/π term is the leading QCD correction. Thus, below charm threshold one expects the average value of R to be slightly more than 2, as verified by a recent measurement [25] averaged between $E_{\text{cm}} = 3.650$ and 3.6648 GeV: $\bar{R}_{uds} = 2.224 \pm 0.019 \pm 0.089$.

The change in R due to the opening of the charmed quark threshold should be

$$\Delta R_c = 3(2/3)^2 \left(1 + \frac{\alpha_S}{\pi} + \dots \right), \quad (2)$$

or slightly more than $4/3$. As one sees from Fig. 1, this may be true on the average, but there are strong fluctuations which are usually ascribed to resonances around $E_{\text{cm}} \simeq 4040, 4160, \text{ and } 4415$ MeV [2]. However, equally striking is the sharp dip in R , which drops from $4.01 \pm 0.14 \pm 0.14$ at 4190 MeV to $2.71 \pm 0.12 \pm 0.13$ at 4250 MeV. This decrease is just about the amount that would correspond to total suppression of charm production.

The reaction $e^+ e^- \rightarrow c\bar{c}$ produces a $c\bar{c}$ system with $J^{PC} = 1^{--}$ which one expects corresponds mainly to a 3S_1 state. Production of the 3D_1 state, while allowed by $J, P,$ and C conservation, is expected to be suppressed because the D -wave quarkonium wave function vanishes at the origin [26]. The $c\bar{c}$ system now must fragment into hadronic final states.

The lowest available channels are $D\bar{D}$ (threshold 3.73 GeV), $D\bar{D}^* + \text{c.c.}$ (threshold 3.87 GeV), and $D^* \bar{D}^*$ (threshold 4.02 GeV). Each of these corresponds to production of a charmed meson-antimeson pair in a P -wave. (In principle an F -wave is also possible, but highly sup-

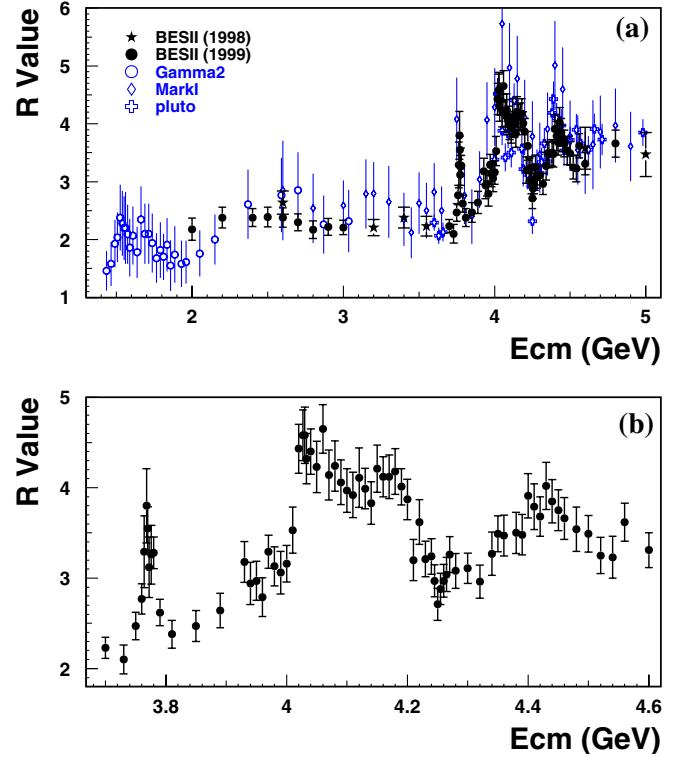


FIG. 1 (color online). (a) Compilation of R values from $E_{\text{cm}} = 1.4$ to 5 GeV. (b) BES results from 3.7 to 4.6 GeV. From Ref. [9].

pressed, for $D^* \bar{D}^*$.) The cross section for production in a state of orbital angular momentum ℓ grows as $(p^*)^{2\ell+1}$, where p^* is the magnitude of either particle's three-momentum in the center-of-mass. Thus unless a resonance is present [as is the case for $\psi(4040)$], one does not expect abrupt effects at any of these thresholds.

The lowest-lying meson-antimeson channel with $J^{PC} = 1^{--}$ into which $c\bar{c}$ can fragment with zero relative orbital angular momentum ℓ of the meson-antimeson pair is $D\bar{D}_1 - \text{c.c.}$ [27]. Here D_1 is a P -wave bound state of a charmed quark and a light (\bar{u} or \bar{d}) antiquark with $J^P = 1^+$. The minus sign corresponds to the negative C eigenvalue. The lightest established candidate for D_1 has a mass of about 2.42 GeV/ c^2 , corresponding to a threshold of 4.285 GeV. It is this threshold that we associate with the dip in R between 4.19 and 4.25 GeV.

In principle there can be a lighter $J^P = 1^+$ nonstrange D meson D_1^* , as two are expected as mixtures of 3P_1 and 1P_1 states. If so, the relevant threshold would lie below 4.285 GeV. It might be smeared out, as the D_1^* is expected to be broader than the (observed) D_1 [28]. Moreover, a 3P_0 state D_0 could be sufficiently light that the lowest S -wave meson pair with $J^{PC} = 1^{--}$ is $D^* \bar{D}_0 - \text{c.c.}$ Then its mass would have to be lower than about 2.28 GeV/ c^2 . Again, since a 3P_0 state is expected to be relatively broad, the threshold might be broadened correspondingly.

Just above the dip in R there is a newly discovered state, the $Y(4260)$, with $J^{PC} = 1^{--}$ identified by its production

in radiative return [29] and in a direct e^+e^- scan [30]. We shall discuss its possible interpretation in the next Section, but its mass and quantum numbers suggest some relation to the dip mentioned here.

D. Dip in $3\pi^+3\pi^-$ spectra at $p\bar{p}$ threshold

The diffractive photoproduction of $3\pi^+3\pi^-$ leads to a spectrum with a pronounced dip near $1.9 \text{ GeV}/c^2$ [15,16], as shown in Fig. 2. This feature can be reproduced by a 1^{--} resonance with $M = 1.91 \pm 0.01 \text{ GeV}/c^2$ and width $\Gamma = 37 \pm 13 \text{ MeV}$ interfering destructively with a broader 1^{--} resonance at lower mass. In Ref. [16] this resonance is considered unlikely to be a nucleon-antinucleon bound state because it is not seen in the reaction $\bar{n}p \rightarrow 3\pi^+2\pi^-\pi^0$ studied by the OBELIX Collaboration [31]. However, this interpretation needs to be re-examined if, as noted here, the dip is really due to the opening of a new channel, in which case a resonance pole, if present at all, may not be located in the expected place. This aspect will be examined in the next Section.

The $3\pi^+3\pi^-$ spectrum also has been studied in the radiative return reaction $e^+e^- \rightarrow \gamma 3\pi^+3\pi^-$ by the BABAR Collaboration [17] in the course of a systematic examination of all final states contributing to R below about 3 GeV . In contrast to the photoproduction reaction, where the diffractive nature of $3\pi^+3\pi^-$ production suggests but does not firmly imply that the multipion system has $J^{PC} = 1^{--}$, here the identification is unambiguous. A dip is again seen at $1.9 \text{ GeV}/c^2$.

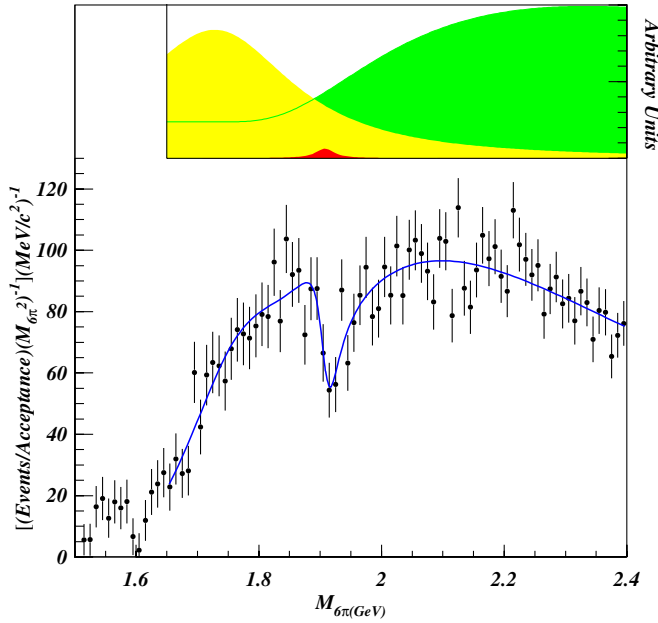


FIG. 2 (color online). Spectrum of diffractively produced $3\pi^+3\pi^-$ [16], together with results of a fit with two resonances and continuum. The inset shows the relative fraction of each amplitude without interference.

E. Dips in $p\bar{p}$ spectra at higher thresholds

The $p\bar{p}$ spectrum studied in radiative return [17] shows two dips: an appreciable one at $2.15 \text{ GeV}/c^2$ and a less-significant one at $3 \text{ GeV}/c^2$. These both can be correlated with important S -wave thresholds.

The first S -wave quasi-two-body threshold in the $J^{PC} = 1^{--}$ channel consisting of a nucleon-antinucleon state corresponds to $N\bar{\Delta}$ - c.c., and occurs around $2.17 \text{ GeV}/c^2$. By necessity it occurs in the isovector channel, since $I_N = 1/2$ while $I_\Delta = 3/2$. This implies that the threshold effect should be equally strong in the $p\bar{p}$ and $n\bar{n}$ channels, a prediction which will be difficult to check. However, it also implies that the production ratios for $p\bar{\Delta}^-$ and $n\bar{\Delta}^0$ should be equal, which may be verifiable using missing-mass techniques.

The next-highest nucleon resonance above the Δ is the Roper resonance, with a mass around $M_R \approx 1.44 \text{ GeV}/c^2$. There does not seem to be a prominent dip around $M_R + M_N \approx 2.38 \text{ GeV}/c^2$. The first negative-parity nucleon resonance is $N(1520)$, with $J^P = 3/2^-$. If produced in a relative S -wave with another resonance in a state with $J^{PC} = 1^{--}$, that resonance must also have negative parity, so the corresponding threshold is $2(1.52) = 3.04 \text{ GeV}/c^2$. Here there does seem to be a slight dip in the $p\bar{p}$ spectrum. In principle both the isovector and isoscalar channels can be affected. The production of $N^0(1520)\bar{N}^0(1520)$ is easily detected through the $p\pi^-\bar{p}\pi^+$ final state, but detection of $N^+(1520)\bar{N}^-(1520)$ requires observation of $p\pi^0\bar{p}\pi^0$, somewhat more challenging.

III. MOLECULES AND BOUND STATES

A. Nonelementary nature of light scalar mesons

The lightest scalar mesons may be denoted $f_0(600) = \sigma$, $K_0^*(800) = \kappa$, $f_0(980)$, and $a_0(980)$ [2]. Although they have been variously assigned to $q\bar{q}$, $qq\bar{q}\bar{q}$, and meson-meson dynamical resonances, it seems difficult to avoid the conclusion that mesonic (not just quark) degrees of freedom play a key role in their properties. (See, e.g., [4,32–35], and references therein.) The σ and κ may be dynamically generated using just current algebra, unitarity, and crossing symmetry [34,36–38], as discussed in Ref. [10]. The $K\bar{K}$ thresholds play a key role in the properties of the $f_0(980)$ and $a_0(980)$, whose masses may be strongly affected by coupling to the $K\bar{K}$ channels [39]. The resonance $f_0(980)$ below $K\bar{K}$ threshold has a pole not far from the real axis; its width is 40 – 100 MeV [2]. The $f_0(980)$ drives the elastic $\pi\pi$ phase rapidly through 180° when its effects are combined with the more-slowly-varying $\pi\pi$ S -wave behavior of the σ [40–42].

B. The $Y(4260)$ as a $D\bar{D}_1$ - c.c. bound state

The observation of the $Y(4260)$ has sparked many interpretations. It has variously been identified as a conventional $4S$ quarkonium level [43], displacing the $\psi(4415)$

[44] in this role; a two-quark-two-antiquark state [45]; or a hybrid [27,46], corresponding to excitation of gluonic degrees of freedom. The favored decay in this last scenario is precisely to one S -wave and one P -wave meson, for example $D\bar{D}_1 - \text{c.c.}$ If this channel is closed, one may still be able to observe its effects through the off-shell decay of D_1 to $D^*\pi$.

If the closed $D\bar{D}_1 - \text{c.c.}$ channel is responsible for the $Y(4260)$, one might expect a closed $D_s\bar{D}_{s1} - \text{c.c.}$ channel to generate similar behavior at higher energy [47]. The threshold for this channel is about $4430 \text{ MeV}/c^2$, so the $\psi(4415)$ might have enhanced coupling to it or to $D_s^*\bar{D}_{s0}$, which has a similar threshold [44].

C. Effect of a 3S_1 $p\bar{p}$ bound state on $\gamma^{(*)} \rightarrow 3\pi^+3\pi^-$

The six-pion channel with isospin $I = I_3 = 1$ does not display resonant activity above $\bar{n}p$ threshold [31]. However, if there is a state with strong coupling to $3\pi^+3\pi^-$ and $p\bar{p}$ below $p\bar{p}$ threshold, one may expect behavior similar to what is observed in the $\pi\pi$ S -wave channel as described above. (A satisfactory fit to the behavior illustrated in Fig. 2 was obtained in Ref. [7] from a cusp effect alone, without recourse to a resonance. One objection to a $\bar{p}p$ resonance is the absence of a peak near threshold in the cross section for $\bar{p}p \rightarrow \bar{n}n$.)

The 1S_0 $p\bar{p}$ system exhibits behavior characteristic of a shallowly-bound state with mass around $1835 \text{ MeV}/c^2$. Such a state is consistent with what is observed in the radiative decays $J/\psi \rightarrow \gamma p\bar{p}$ [48] and $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$ [49]. Here, if the state is produced in the expected way via conversion of two gluons emitted in the decay $J/\psi \rightarrow \gamma gg$, it is likely to be isoscalar. (A satisfactory fit to the behavior illustrated in Fig. 2 was obtained in Ref. [7] from a cusp effect alone, without recourse to a resonance. One objection to a $\bar{p}p$ resonance is the absence of a peak near threshold in the cross section for $\bar{p}p \rightarrow \bar{n}n$.)

D. A state near $\omega\phi$ and $K^*\bar{K}^*$ threshold

The BES II Collaboration has observed an enhancement in $M(\omega\phi)$ near threshold in the decay $J/\psi \rightarrow \gamma\omega\phi$ [50]. The state emerges from a partial wave analysis as a candidate for $J^P = 0^+$ with mass $M = 1812^{+19}_{-26} \pm 18 \text{ MeV}/c^2$ and width $\Gamma = 105 \pm 20 \pm 28 \text{ MeV}/c^2$. The spin and parity are consistent with being an S -wave state of $\omega\phi$ just about $10 \text{ MeV}/c^2$ above threshold. It is interesting that the $K^*\bar{K}^*$ and $\omega\phi$ thresholds are only about $10\text{--}20 \text{ MeV}/c^2$ apart depending on the K^* charges. We suspect the reaction $\omega\phi \leftrightarrow K^*\bar{K}^*$ plays an important part in stabilizing this resonance. One interpretation of this state [51] implies a larger partial width to $K^*\bar{K}^*$ than to $\omega\phi$.

E. $p\bar{\Delta} - \text{c.c.}$ and $N(1520)\bar{N}(1520)$ bound states

The relative parities of a p and a $\bar{\Delta}$ are negative. The S -wave bound states of a suitably antisymmetrized state

will have $P = C = -$. The allowed J values for $[p\bar{\Delta}]_{L=0}$ are 1 and 2, but only $J = 1$ couples to the virtual photon.

The relative parities of $N(1520)$ and $\bar{N}(1520)$ also are negative, so all their S -wave bound states have negative parity. The total spins S can take on all values between 0 and 3, but one expects (as for spin-1/2 particles) that the charge-conjugation eigenvalue of the pair is $C = (-1)^{L+S}$. The $J = 1$ state thus is permitted by C to couple to the virtual photon.

The dynamics which would give rise to bound states in the $J = 1$ channels are unclear. By analogy with the $p\bar{p}$ system mentioned above one might expect bound states with more than one J value.

F. The $\Lambda(1405)$ as a $\bar{K}N$ bound state

Another system which has been known for nearly 50 years to have features in common with the bound states mentioned above is the $I = 0$ S -wave kaon-nucleon system, which has a $J^P = 1/2^-$ resonance at $1405 \text{ MeV}/c^2$, about $26 \pm 4 \text{ MeV}/c^2$ below K^-p threshold [52]. This state, the $\Lambda(1405)$, decays essentially 100% of the time to $\Sigma\pi$. Although it may be viewed in the quark model as a uds state with orbital angular momentum $\ell = 1$, the detailed properties of the $\Lambda(1405)$ can be viewed equally successfully in terms of the final-state channels to which it couples [53–55]. This is true of a number of other members of the negative-parity baryon 70-dimensional multiplet of $SU(6)$. For example, the $N(1535)$, also with $J^P = 1/2^-$, can be viewed both as an $L = 1$ three-quark state and as a dynamical effect with large coupling to ηN [56].

G. Excited Ξ as a $\Sigma\bar{K}$ threshold effect

The *BABAR* Collaboration has recently analyzed the mass, width, and spin of a baryon with strangeness -2 near $1685 \text{ MeV}/c^2$ [57], observed in the decay $\Lambda_c \rightarrow \Lambda\bar{K}^0K^+$. The existence of this state, and its correlation with $\Sigma\bar{K}$ threshold, has been known for a number of years (see the references in [2]), but the *BABAR* analysis finds its spin to be consistent with $1/2$, which would correspond to an S -wave $\bar{K}\Sigma$ state. The mass, found in Ref. [57] to be $1684.7 \pm 1.3^{+2.2}_{-1.6} \text{ MeV}/c^2$, probably lies above the $K^-\Sigma^+$ threshold of $1683.0 \text{ MeV}/c^2$ but below the $\bar{K}^0\Sigma^0$ threshold of $1690.3 \text{ MeV}/c^2$. Thus in some sense one could regard it as a $\bar{K}^0\Sigma^0$ S -wave bound state.

H. Peak in $M(\Lambda p)$ at ΣN threshold

For a number of years, it has been known that the Λp mass spectrum in $K^-d \rightarrow \Lambda p\pi^-$ exhibits a sharp peak around ΣN threshold [58]. Early references with the largest data samples include Refs. [59] ($p_K \simeq 0$) and [60] ($p_K = 0.7 \text{ GeV}/c$); others may be found in [61]. The mass of this state is consistent with being equal to $M(\Sigma^+n) = 2128.93 \text{ MeV}/c^2$; the $\Sigma^0 p$ threshold at

2130.91 MeV/ c^2 is 2 MeV/ c^2 higher. This state has been interpreted as the 3S_1 $S = -1$, $I = 1/2$ partner of the deuteron in the $\overline{10}$ -dimensional representation of $SU(3)$ [62,63]. A recent discussion [7], however, finds that it is difficult to determine whether the data demand an actual pole in addition to a threshold cusp. This ambiguity is common to a number of examples we have discussed here.

I. The $X(3872)$ as a $D^0\bar{D}^{*0} + \text{c.c.}$ bound state

The suggestion that charmed mesons might form molecules or bound states with one another was made shortly after their discovery [64]. It now appears that one has a good candidate for such a state. The $X(3872)$, discovered by Belle in B decays [65] and confirmed by BABAR [66] and in hadronic production [67], decays predominantly into $J/\psi\pi^+\pi^-$. It has many features in common with an S -wave bound state of $(D^0\bar{D}^{*0} + \bar{D}^0D^{*0})/\sqrt{2} \sim c\bar{c}u\bar{u}$ with $J^{PC} = 1^{++}$ [68]. Its $J^{PC} = 1^{++}$ assignment is supported by its recently reported observation in the $D^0\bar{D}^0\pi^0$ channel [69]. It has recently been described more generally as being associated with a large scattering length in $D^0\bar{D}^{*0}$ scattering [7,70].

J. The $D_{s0}(2317)$ and $D_{s1}^*(2460)$ as $D^{(*)}K$ bound states

The lowest-lying $J^P = 0^+$ and 1^+ $c\bar{s}$ mesons [71] have turned out to be lighter than predicted in most [72,73] (but not all [74]) quarkonium calculations. In fact, they lie below their respective DK and D^*K thresholds, and thus must decay via emission of a photon or an isospin-violating π^0 to a lower $c\bar{s}$ state. While low masses are predicted [75] by viewing these states as parity-doublets of the $D_s(0^-)$ and $D_s^*(1^-)$ $c\bar{s}$ ground states, one can also view them as bound states of $D^{(*)}K$ [76–78] (the binding energy in each case would be 41 MeV), or as $c\bar{s}$ states with masses lowered by coupling to $D^{(*)}K$ channels [79].

K. Is there a rule for bound state formation?

The feature which the above examples have in common is the coexistence of at least two channels, one closed and one open, near the energy at which either a dip or a peak is observed. This is a necessary but probably not sufficient condition for what is termed a *Feshbach resonance* [80], which has been used to great advantage in the study of Bose-Einstein condensates [81]. Such a resonance is characterized by a phase shift increasing rapidly by 180° as the energy rises through the resonance. This is how the $I = 0$ S -wave $\pi\pi$ phase shift behaves [3], but it is not known whether the phase shifts in the other channels we have mentioned behave similarly. However, all of these channels are dominated by a single partial wave (e.g., 3S_1 for the $c\bar{c}$ pair produced directly or via radiative return in e^+e^- collisions), so the possibility of a rapidly decreasing or vanishing amplitude exists. The importance of hadronic as

well as quark degrees of freedom in processes such as those we have discussed has been stressed in Refs. [4,7,32–35,79,82,83].

A regularity governing resonance formation was noted some time ago [84]. If two mesons are allowed by the quark model to resonate, they do so for $p^* < p_0^{MM} = 350$ MeV/ c , where p^* is the c.m. momentum. The corresponding value for meson-baryon systems is $p_0^{MB} = 250$ MeV/ c . In order to form nonexotic resonances, an antiquark in a meson must annihilate with the corresponding quark in the other meson or baryon.

The $\omega\phi$ threshold of Ref. [50] is one possible counterexample, because the quark flavors in the two decay products are distinct from one another: $(u\bar{u} + d\bar{d})/\sqrt{2}$ for the ω and $s\bar{s}$ for the ϕ . However, if this resonance couples strongly not only to $\omega\phi$ but also to $K^*\bar{K}^*$, the latter component may be chiefly responsible through the proposed quark-antiquark annihilation mechanism for the resonant behavior.

It is possible that a stronger form of the above rule holds when neither resonating particle is a pion. The pion may be considered as anomalously light in view of its role as a Nambu-Goldstone boson of spontaneously broken chiral $SU(2) \times SU(2)$. Thus, π^+ and π^- form a ρ meson (or possibly a σ) well above threshold, but K and \bar{K} seem to exhibit their first resonances in the $I = 0$ state $f_0(980)$ and $I = 1$ state $a_0(980)$, both of which lie below threshold. The K^- and p form the $\Lambda(1405)$, also below threshold. This behavior is frequent enough that a more general dynamical principle may be at work.

IV. IMPLICATIONS FOR EXPERIMENTS

A. Dip in R : hadronic makeup of final states

If the dip in R is due to a new threshold in the hadronization of $c\bar{c}$, it should be confined to final states consisting of charmed mesons and charmed antimesons (with possible additional pions). The cross section for e^+e^- production of noncharmed final states should not be affected.

B. Isovector states produced via $B \rightarrow \bar{D}^{(*)}W^{*+}$ decay

The dip in the $p\bar{p}$ spectrum at $p\bar{\Delta}^-$ threshold, if indeed due to this threshold, must be occurring in the isovector channel. Then production of $p\bar{n}$ by the charged weak current should exhibit a similar dip at $p\bar{\Delta}^0$ threshold. One should be able to observe this effect in the decay (e.g.) $B^0 \rightarrow D^{*-}p\bar{n}$, where the \bar{n} is reconstructed via missing mass. Similarly, the decay $B^0 \rightarrow D^{*-}(6\pi)^+$ should show a dip in the six-pion effective mass spectrum at $p\bar{n}$ threshold, or around 1.9 GeV/ c^2 .

C. Elastic $\Sigma\pi$ scattering at $\Lambda(1405)$

The SELEX Collaboration has completed a program of studies with a Σ^- beam [85]. Although the main focus of

this experiment was charm production, it was capable of studying $\Sigma^- \pi^+$ scattering through the peripheral process $\Sigma^- p \rightarrow Xn$ with a cut on small momentum transfer to isolate pion exchange. If the $K^- p$ threshold plays a role similar to that noted in the above examples, one would expect a sharp dip in the cross sections for $\Sigma^- \pi^+ \rightarrow \Sigma^\pm \pi^\pm$ in the vicinity of $\Lambda(1405)$.

D. Threshold $K\eta$ or subthreshold $K\eta'$ resonance

The depletion of the $K\pi$ spectrum just above $1 \text{ GeV}/c^2$ in the Dalitz plot for $D^0 \rightarrow K^+ K^- \pi^0$ [14] occurs just around the $K\eta$ threshold. This depletion should be confirmed by the analysis of a larger $D^0 \rightarrow K^+ K^- \pi^0$ data sample [86] and the $D^0 \rightarrow K^+ K^- \eta$ Dalitz plot studied to determine if there is an enhancement at $K\eta$ threshold. However, the coupling of a 0^+ resonance to $K\eta$ is expected to be suppressed [87]; coupling to $K\eta'$ is favored. An alternative interpretation is that the dip above $1 \text{ GeV}/c^2$ is due to the vanishing of the S -wave amplitude between a low-energy $J^P = 0^+ K\pi$ resonance known as the κ (for a discussion, see Refs. [4,88]) and a higher $0^+ K\pi$ resonance (e.g., the $K_0^*(1430)$ [2]), presumably with large $K\eta'$ coupling [87].

E. Bound states of $\bar{B}^{(*)}K$

If the $D_{s0}(2317)$ and $D_{s1}^*(2460)$ are, respectively, states of DK and D^*K each bound by 41 MeV, perhaps by annihilation of a u or d quark in the K with the corresponding antiquark in the $D^{(*)}$, one might expect a $J^P = 0^+ b\bar{s}$ state with a mass of about $M(B) + M(K) - 41 \approx 5733 \text{ MeV}/c^2$ and a $J^P = 1^+ b\bar{s}$ state with a mass of about $M(B^*) + M(K) - 41 \approx 5778 \text{ MeV}/c^2$. (Here we have taken the average of charged and neutral kaon masses, and ignored changes in hyperfine energies when replacing c by b . Calculations in Ref. [78] find a $\bar{B}K$ bound state at $5725 \pm 39 \text{ MeV}/c^2$ and a \bar{B}^*K bound state at $5778 \pm 7 \text{ MeV}/c^2$, while the most recent of Refs. [74] finds a $\bar{B}K$ bound state at $5627 \text{ MeV}/c^2$ and two \bar{B}^*K bound states at 5660 and $5781 \text{ MeV}/c^2$.) In analogy with the charmed-strange system, these states might be expected to decay via photon or π^0 emission to B_s and/or B_s^* , subject to the usual spin-parity selection rules.

F. Effects in exotic baryon-antibaryon channels

The regularity noted above for meson-meson and meson-baryon resonance formation, that when an antiquark in a meson can annihilate a quark in a meson or baryon then one expects a resonance to be formed below a certain value of $p^* < p_0$, was generalized to baryon-antibaryon resonances [84] to predict a corresponding value $p_0^{B\bar{B}} = 200 \text{ MeV}/c$. This rule, if correct, would predict the existence of exotic baryon-antibaryon resonances [89], which have never been seen, not far above baryon-antibaryon threshold. Examples of manifestly exotic chan-

nels are $\Delta^{++}\bar{n}$ and $\bar{\Lambda}p\pi^+$, with minimal quark content $uud\bar{d}$ and $uud\bar{s}$, respectively. Some recent suggestions for observing such resonances in B decays were made in Ref. [90].

A stronger version of the above resonance formation model is suggested in the present article: If neither incident particle is a pion, at least one resonance may be formed *below threshold*. Thus, the search for exotic mesons may require one to study spectra well below baryon-antibaryon threshold, where ground-state exotic mesons are indeed expected [91].

V. SUMMARY

The rapid decrease of the S -wave $I = 0 \pi\pi$ scattering amplitude near $K\bar{K}$ threshold is seen to be mirrored in a host of other phenomena, whose origin may reflect similar physics. In the case of the coupled $\pi\pi$ and $K\bar{K}$ channels, a key role is played by the $f_0(980)$ resonance, whose presence is responsible for a rapid increase of the elastic $I = J = 0$ phase shift δ_0^0 through 180° , where the S -wave $\pi\pi$ amplitude vanishes just before becoming highly inelastic. Other effects which may be regarded within the same framework include the dip in R for $e^+e^- \rightarrow$ (hadrons) around a center-of-mass energy of 4.25 GeV and several dips in the cross section for photoproduction or e^+e^- production of specific final states near the thresholds for others.

It is suggested that all these effects may be associated with the formation of bound states in the channels which are about to open, with a possible relation to Feshbach resonances which similarly occur in coupled-channel problems. Tests of this proposal include the observation of a dip in $I = 0 S$ -wave $\Sigma\pi$ scattering near $K^- p$ threshold and the presence of exotic baryon-antibaryon resonances below threshold. This phenomenon does not appear to be universal, but widespread. Further dynamical information remains to be found in order to predict its occurrence reliably.

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Note added.—In central $\pi^+\pi^-$ production in $pp \rightarrow pp\pi^+\pi^-$ at $\sqrt{s} = 63 \text{ GeV}$, the $M(\pi^+\pi^-)$ spectrum falls off steeply both at $1 \text{ GeV}/c^2$ and around $1.5 \text{ GeV}/c^2$: See Fig. 5 of Ref. [92]. I thank M. Albrow for calling this behavior to my attention.

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