Implications of holographic QCD in chiral perturbation theory with hidden local symmetry

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Based on the chiral perturbation theory with the hidden local symmetry, we propose a methodology to calculate a part of the large N_c corrections in the holographic QCD (HQCD). As an example, we apply the method to an HQCD model recently proposed by Sakai and Sugimoto. We show that the ρ-π-π coupling is in good agreement with the experiment due to the $1/N_c$-subleading corrections.

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I. INTRODUCTION

Recently the duality in string/gauge theory [1] has provided us with a new perspective for solving the problem of strongly coupled gauge theories: Strongly coupled gauge theory can be reformulated from the weakly coupled string theory based on the anti-de Sitter/conformal-field-theory (AdS/CFT) correspondence [2]. Some important qualitative features of the dynamics of QCD such as the confinement and chiral symmetry breaking have been reproduced from this holographic point of view, so-called holographic QCD (HQCD), although the theory in the UV region is substantially different from QCD, i.e., lack of asymptotic freedom. Several authors [3,4] proposed a model of HQCD where the chiral symmetry breaking is realized. In particular, starting with a stringy setting, Sakai and Sugimoto (SS) [3] have succeeded in producing the realistic chiral symmetry breaking $U(N_f)_L \times U(N_f)_R$ down to $U(N_f)_V$ and also a natural emergence of the hidden local symmetry (HLS) [5] for vector/axial-vector mesons. Moreover, most of them [3,4] analyze observables of QCD related to the pion and the vector mesons in the large N_c limit such as $m_\rho^2/(g_{\rho\pi\pi}^2 F_\pi^2) \simeq 3.0$, where $g_{\rho\pi\pi}$ denotes the ρ-π-π coupling, and F_π the pion decay constant. This, however, substantially deviates from one of the celebrated Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relations (KSRF II), $m_\rho^2/(g_{\rho\pi\pi}^2 F_\pi^2) = 2$, which agrees with the experiment. Since the holographic result is only the one at the leading order in the $1/N_c$ expansion, the deviation may be cured by subleading effects in the $1/N_c$ expansion. So far, however, no holographic models succeed in including the effect of the $1/N_c$ corrections. On the other hand, it is well known that meson loops yield the next order in the $1/N_c$ expansion.

In this paper, we propose a methodology for calculating a part of the $1/N_c$ corrections to the HQCD through the meson loop, based on the "HLS chiral perturbation theory (ChPT)" [6–9] which incorporates the vector meson loops into the ChPT [10] through the HLS model [5]. It is important to note [11] that the *HLS is crucial for the systematic power counting* when the vector meson mass is light (see, for a review, Ref. [9]). As an example, we apply our method to an HQCD model proposed by SS [3]. We show that the $1/N_c$ corrections make the ratio $m_\rho^2/(g_{\rho\pi\pi}^2 F_\pi^2)$ in good agreement with the experimental value or the KSRF II relation. Our formalism proposed in this paper is applicable to other models holographically dual to strongly coupled gauge theories, which will give us implications of HQCD.

II. REVIEW OF A HOLOGRAPHIC MODEL

Let us start with the low-energy effective action on the 5-dimensional space-time induced from a holographic model, based on the N_f D8-$\overline{\rm D8}$ branes transverse to the N_c D4-branes, proposed by the authors in Ref. [3]:

$$S_{D8} = N_c G \int d^4x dz \Big(-\frac{1}{2} K^{-1/3}(z)\, \mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] + K(z) M_{\rm KK}^2\, \mathrm{tr}[F_{\mu z}F^{\mu z}] + \mathcal{O}(F^3)\Big), \tag{1}$$

where $K(z)$ is the induced measure of 5-dimensional space-time given by

$$K(z) = 1 + z^2. \tag{2}$$

The coupling G is the rescaled 't Hooft coupling expressed as

$$G = \frac{N_c g_{\rm YM}^2}{108\pi^3}, \tag{3}$$

where $g_{\rm YM}$ is the gauge coupling of the $U(N_c)$ gauge symmetry on the N_c D4-branes. It should be noted that the mass scale $M_{\rm KK}$ in Eq. (1) is related to the scale of the compactification of the N_c D4-branes onto the S^1.

The 5-dimensional gauge field A_M transforms as

$$A_M(x^\mu, z) \to g(x^\mu, z) \cdot A_M(x^\mu, z) \cdot g^\dagger(x^\mu, z) - i\partial_M g(x^\mu, z) \cdot g^\dagger(x^\mu, z), \tag{4}$$

where $g(x^\mu, z)$ is the transformation matrix of the 5-dimensional gauge symmetry. We choose the same boundary condition of the 5-dimensional gauge field A_M as in Ref. [3]:

$$A_M(x^\mu, z = \pm\infty) = 0, \tag{5}$$

which makes the local chiral symmetry a global one

$g_{R,L} \in \mathrm{U}_{R,L}(N_f)$. The chiral field U defined in Ref. [3],

$$U(x^\mu) = \mathrm{P}\exp\left[i \int_{-\infty}^{\infty} dz' A_z(x^\mu, z') \right], \tag{6}$$

is parametrized by the Nambu-Goldstone (NG) boson field π as

$$U(x^\mu) = e^{[2i\pi(x^\mu)]/F_\pi}, \tag{7}$$

where F_π denotes the decay constant of π. U is divided as

$$U(x^\mu) = \xi_L^\dagger(x^\mu) \cdot \xi_R(x^\mu), \tag{8}$$

where

$$\xi_{R,L}(x^\mu) = \mathrm{P}\exp\left[i \int_0^{\pm\infty} dz' A_z(x^\mu, z') \right]. \tag{9}$$

$\xi_{R,L}$ transform as [5]

$$\xi'_{R,L} = h(x^\mu) \cdot \xi_{R,L} \cdot g_{R,L}^\dagger, \tag{10}$$

where $h(x^\mu) = g(x^\mu, z = 0)$ is a gauge transformation. We further parametrize $\xi_{R,L}$ as [5]

$$\xi_{R,L}(x^\mu) = e^{i\sigma(x^\mu)/F_\sigma} \cdot e^{\pm i\pi(x^\mu)/F_\pi}, \tag{11}$$

where σ denote the NG bosons associated with the spontaneous breaking of the HLS, and F_σ the decay constant of σ. The σ are absorbed into the gauge bosons of the HLS which acquire the mass through the Higgs mechanism.

It is convenient to work in the $A_z(x^\mu, z) \equiv 0$ gauge [3]. There still exists a residual gauge symmetry, $h(x^\mu) = g(x^\mu, z = 0)$, which was identified with the hidden local symmetry (HLS) in Ref. [3]. The 5-dimensional gauge field A_μ transforms under the residual gauge symmetry (HLS) as

$$\begin{aligned} A_\mu(x^\mu, z) \to h(x^\mu) \cdot A_\mu(x^\mu, z) \cdot h^\dagger(x^\mu) \\ - i\partial_\mu h(x^\mu) \cdot h^\dagger(x^\mu). \end{aligned} \tag{12}$$

In this gauge, the NG boson fields are included in the boundary condition for the 5-dimensional gauge field A_μ as

$$A_\mu(x^\mu, z = \pm\infty) = \alpha_\mu^{R,L}(x^\mu), \tag{13}$$

where

$$\alpha_\mu^{R,L}(x^\mu) = i\xi_{R,L}(x^\mu)\partial_\mu \xi_{R,L}^\dagger(x^\mu), \tag{14}$$

which transform under the HLS as in the same way as in Eq. (12).

III. RELATION TO HLS IN THE LARGE N_C LIMIT

In contrast to Ref. [3] where vector meson fields are identified with the Callan-Coleman-Wess-Zumino (CCWZ) matter fields transforming *homogeneously* under HLS, we here introduce vector meson fields as an infinite tower of the HLS *gauge fields* $V_\mu^{(k)}$ ($k = 1, 2, \ldots$), which transform *inhomogeneously* under the HLS as in Eq. (12) [9]. Using $V_\mu^{(k)}$ together with $\alpha_\mu^{R,L}$, we expand the 5-dimensional gauge field A_μ as

$$\begin{aligned} A_\mu(x^\mu, z) = \alpha_\mu^R(x^\mu)\phi^r(z) + \alpha_\mu^L(x^\mu)\phi^l(z) \\ + \sum_{k\geq 1} V_\mu^{(k)}(x^\mu)\phi_k(z), \end{aligned} \tag{15}$$

where the functions ϕ^r, ϕ^l, and ϕ_k ($k = 1, 2, \ldots$) form a complete set in the z-coordinate space. These functions $\{\phi^r, \phi^l, \phi_k\}$ are different from the eigenfunctions ψ_n in [3] which satisfy the eigenvalue equation

$$- K^{1/3}\partial_z(K\partial_z\psi_n) = \lambda_n\psi_n, \tag{16}$$

with the eigenvalues λ_n. Then, the functions $\{\phi^r, \phi^l, \phi_k\}$ are not separately the solutions of the eigenvalue equation but are expressed by linear combinations of the solutions, as we will see later.

Substituting Eq. (15) into the action (1), we obtain the 4-dimensional theory with an infinite tower of the massive vector and axial-vector mesons and the NG bosons associated with the chiral symmetry breaking. We would like to stress that, since the 5-dimensional gauge field A_μ is expanded in terms of the HLS gauge fields $V_\mu^{(k)}$, the action (1) is expressed as the form *manifestly gauge invariant* under the HLS, which enables us to calculate the $1/N_c$-subleading correction in a systematic way.

Let us concentrate on the lightest vector meson together with the NG bosons by integrating out the heavy vector and axial-vector meson fields.[1] As a result, the HLS gauge field V_μ corresponding to the lightest vector meson is embedded into A_μ as

$$A_\mu(x^\mu, z) = \alpha_\mu^R(x^\mu)\varphi^r(z) + \alpha_\mu^L(x^\mu)\varphi^l(z) + V_\mu(x^\mu)\varphi(z), \tag{17}$$

where φ^r, φ^l, and φ denote the wave functions modified by integrating out the heavier mesons. Note that they satisfy the following constraint:

$$\varphi^r(z) + \varphi^l(z) + \varphi(z) = 1, \tag{18}$$

which follows from the consistency condition between the transformation properties (*inhomogeneous term*) of the left- and right-hand sides of Eq. (17). The relations between $\{\varphi^r, \varphi^l, \varphi\}$ and the eigenfunctions of the eigenvalue equation are obtained in the following way: We introduce the 1-forms $\hat\alpha_{\mu||}$ and $\hat\alpha_{\mu\perp}$ defined as

$$\hat\alpha_{\mu||}(x^\mu) = \frac{\alpha_\mu^R(x^\mu) + \alpha_\mu^L(x^\mu)}{2} - V_\mu(x^\mu), \tag{19}$$

[1] This is contrasted with simply putting the heavy fields $V_\mu^{(k)}$ ($k \geq 2$) $= 0$ in Eq. (15). The wave functions $\phi_k(z)$ are thus modified, when we integrate out the heavier fields [12].

$$\hat{\alpha}_{\mu\perp}(x^\mu) = \frac{\alpha_\mu^R(x^\mu) - \alpha_\mu^L(x^\mu)}{2}. \tag{20}$$

Then Eq. (17) is rewritten in the following form:

$$\begin{aligned} A_\mu(x^\mu, z) = {} & \hat{\alpha}_{\mu\perp}(x^\mu)(\varphi^r(z) - \varphi^l(z)) \\ & + (\hat{\alpha}_{\mu\|}(x^\mu) + V_\mu(x^\mu))(\varphi^r(z) + \varphi^l(z)) \\ & + V_\mu(x^\mu)\varphi(z). \end{aligned} \tag{21}$$

Since the 1-form $\hat{\alpha}_{\mu\perp}$ includes the NG boson field as $\hat{\alpha}_{\mu\perp} = \frac{1}{F_\pi}\partial_\mu\pi + \cdots$, we identify the combination $\varphi^r - \varphi^l$ with the eigenfunction ψ_0 for the zero eigenvalue as

$$\varphi^r(z) - \varphi^l(z) = \psi_0(z) = \frac{2}{\pi}\tan^{-1}z. \tag{22}$$

On the other hand, since the HLS gauge field V_μ corresponds to the lightest vector meson, we identify the wave function φ with the eigenfunction of the first excited Kaluza-Klein (KK) mode,

$$\varphi(z) = -\psi_1(z). \tag{23}$$

Then, by using Eq. (18), the wave functions φ^r and φ^l are expressed in terms of the eigenfunctions ψ_0 and ψ_1 as

$$\varphi^{r,l}(z) = \tfrac{1}{2} \pm \tfrac{1}{2}\psi_0(z) + \tfrac{1}{2}\psi_1(z). \tag{24}$$

By using this, Eq. (21) is rewritten in the following form:

$$\begin{aligned} A_\mu(x^\mu, z) = {} & \hat{\alpha}_{\mu\perp}(x^\mu)\psi_0(z) + (\hat{\alpha}_{\mu\|}(x^\mu) + V_\mu(x^\mu)) \\ & + \hat{\alpha}_{\mu\|}(x^\mu)\psi_1(z). \end{aligned} \tag{25}$$

It should be noticed that neither the wave function φ^r nor φ^l is the eigenfunction for the zero eigenvalue. This is the reflection of the well-known fact that the massless photon field is given by a linear combination of the HLS gauge field and the gauge field corresponding to the chiral symmetry [5,9].

Now, since we introduce the vector meson field as the gauge field of the HLS, the derivative expansion of the Lagrangian becomes possible. This is an important difference compared with the formulation done in Ref. [3]. Then, the leading Lagrangian counted as $\mathcal{O}(p^2)$ in the derivative expansion is constructed by the terms generated from the $F_{\mu z}F^{\mu z}$ term in the action (1) together with the kinetic term of the HLS gauge field V_μ from the $F_{\mu\nu}F^{\mu\nu}$ term. On the other hand, the $\mathcal{O}(p^4)$ terms come from the remainder of the $F_{\mu\nu}F^{\mu\nu}$ term in the action (1). The resultant Lagrangian takes the form of the HLS model [5,9]:

$$\begin{aligned} \mathcal{L} = {} & F_\pi^2 \,\mathrm{tr}[\hat{\alpha}_{\mu\perp}\hat{\alpha}_\perp^\mu] + F_\sigma^2 \,\mathrm{tr}[\hat{\alpha}_{\mu\|}\hat{\alpha}_\|^\mu] \\ & - \frac{1}{2g^2}\,\mathrm{tr}[V_{\mu\nu}V^{\mu\nu}] + \mathcal{L}_{(4)}, \end{aligned} \tag{26}$$

where $\mathcal{L}_{(4)}$ is constructed by the $\mathcal{O}(p^4)$ terms [7,9]:

$$\begin{aligned} \mathcal{L}_{(4)} = {} & y_1\,\mathrm{tr}[\hat{\alpha}_{\mu\perp}\hat{\alpha}_\perp^\mu\hat{\alpha}_{\nu\perp}\hat{\alpha}_\perp^\nu] + y_2\,\mathrm{tr}[\hat{\alpha}_{\mu\perp}\hat{\alpha}_{\nu\perp}\hat{\alpha}_\perp^\mu\hat{\alpha}_\perp^\nu] + y_3\,\mathrm{tr}[\hat{\alpha}_{\mu\|}\hat{\alpha}_\|^\mu\hat{\alpha}_{\nu\|}\hat{\alpha}_\|^\nu] + y_4\,\mathrm{tr}[\hat{\alpha}_{\mu\|}\hat{\alpha}_{\nu\|}\hat{\alpha}_\|^\mu\hat{\alpha}_\|^\nu] \\ & + y_5\,\mathrm{tr}[\hat{\alpha}_{\mu\perp}\hat{\alpha}_\perp^\mu\hat{\alpha}_{\nu\|}\hat{\alpha}_\|^\nu] + y_6\,\mathrm{tr}[\hat{\alpha}_{\mu\perp}\hat{\alpha}_{\nu\perp}\hat{\alpha}_\|^\mu\hat{\alpha}_\|^\nu] + y_7\,\mathrm{tr}[\hat{\alpha}_{\mu\perp}\hat{\alpha}_{\nu\perp}\hat{\alpha}_\|^\nu\hat{\alpha}_\|^\mu] + y_8\{\mathrm{tr}[\hat{\alpha}_{\mu\perp}\hat{\alpha}_\|^\mu\hat{\alpha}_{\nu\perp}\hat{\alpha}_\|^\nu] \\ & + \mathrm{tr}[\hat{\alpha}_{\mu\perp}\hat{\alpha}_\|^\nu\hat{\alpha}_{\nu\perp}\hat{\alpha}_\|^\mu]\} + y_9\,\mathrm{tr}[\hat{\alpha}_{\mu\perp}\hat{\alpha}_\|^\nu\hat{\alpha}_{\mu\perp}\hat{\alpha}_\|^\nu] + iz_4\,\mathrm{tr}[V_{\mu\nu}\hat{\alpha}_\perp^\mu\hat{\alpha}_\perp^\nu] + iz_5\,\mathrm{tr}[V_{\mu\nu}\hat{\alpha}_\|^\mu\hat{\alpha}_\|^\nu]. \end{aligned} \tag{27}$$

Note that all the parameters in the Lagrangian are expressed in terms of the parameters of the 5-dimensional gauge theory as

$$F_\pi^2 = N_c G M_{\mathrm{KK}}^2 \int dz K(z)[\dot{\psi}_0(z)]^2, \tag{28}$$

$$F_\sigma^2 = N_c G M_{\mathrm{KK}}^2 \lambda_1 \langle\psi_1^2\rangle, \tag{29}$$

$$\frac{1}{g^2} = N_c G\langle\psi_1^2\rangle, \tag{30}$$

$$y_1 = -y_2 = -N_c G\cdot\langle 1 + \psi_1 - \psi_0^2\rangle, \tag{31}$$

$$y_3 = -y_4 = -N_c G\cdot\langle\psi_1^2(1 + \psi_1)^2\rangle, \tag{32}$$

$$y_5 = 2y_8 = -y_9 = -2N_c G\cdot\langle\psi_1^2\psi_0^2\rangle, \tag{33}$$

$$y_6 = -y_5 - y_7, \tag{34}$$

$$y_7 = 2N_c G\cdot\langle\psi_1(1 + \psi_1)(1 + \psi_1 - \psi_0^2)\rangle, \tag{35}$$

$$z_4 = -2N_c G\cdot\langle\psi_1(1 + \psi_1 - \psi_0^2)\rangle, \tag{36}$$

$$z_5 = -2N_c G\cdot\langle\psi_1^2(1 + \psi_1)\rangle, \tag{37}$$

with λ_1 being the eigenvalue determined by solving the eigenvalue equation, and

$$\langle A\rangle \equiv \int dz K^{-1/3}(z)A(z) \tag{38}$$

for a function $A(z)$. In Eq. (29), we used an identity

$$\int dz K(z)\dot{\psi}_1^2(z) = \lambda_1\int dz K^{-1/3}(z)\psi_1^2(z). \tag{39}$$

We should note that the normalization of the eigenfunction ψ_1 is not solely determined from the eigenvalue equation and the boundary condition $\psi_1(\pm\infty) = 0$. In addition, the values of the 't Hooft coupling G and the mass scale M_{KK} are not fixed in the model. As a result, none of three parameters of the HLS at the leading order, (F_π, F_σ, g), are fixed in the present model: We need three phenomenological inputs to fix their values. However, this implies that several physical predictions can be made from only three phenomenological inputs.

It should be also noticed that the Lagrangian (26) has all the parameters consistently with the large N_c counting rule, although several are absent since the external gauge fields are not incorporated in the model: As is well known, terms including two or more traces are suppressed by $1/N_c$ compared with terms of just one trace in the large N_c limit [10]. We note that all the terms in Eq. (26) are of $\mathcal{O}(N_c)$ as one can easily see in Eqs. (22)–(31), which are constructed by just one trace of the product.

IV. ρ-π-π COUPLING AND THE KSRF II RELATION AT THE LARGE N_c LIMIT

As usual in the HLS model [5,9], from the Lagrangian (26), we can easily read off the ρ mass square and the ρ-π-π coupling:

$$m_\rho^2 = a g^2 F_\pi^2, \qquad g_{\rho\pi\pi} = \tfrac{1}{2} a g (1 + \tfrac{1}{2} g^2 z_4), \tag{40}$$

where phenomenologically important parameter a is defined by

$$a \equiv \frac{F_\sigma^2}{F_\pi^2}. \tag{41}$$

It should be noted that these quantities are expressed in terms of the parameters of the 5-dimensional gauge theory, by using Eqs. (28)–(30) and (36), as

$$m_\rho^2 = \lambda_1 M_{\mathrm{KK}}^2, \tag{42}$$

$$g_{\rho\pi\pi} = \frac{\pi}{4} \frac{\lambda_1}{\sqrt{N_c G}} \sqrt{\frac{\langle \psi_1 (1 - \psi_0^2) \rangle^2}{\langle \psi_1^2 \rangle}}. \tag{43}$$

Since they are independent of the normalization of the eigenfunction ψ_1, m_ρ^2 and $g_{\rho\pi\pi}$ are completely determined, once the values of G and M_{KK} are fixed. Moreover, the following ratio related to the KSRF II relation is calculable even independently of these inputs:

$$\frac{m_\rho^2}{g_{\rho\pi\pi}^2 F_\pi^2} = \frac{4}{a(1 + \frac{1}{2} g^2 z_4)^2} = \frac{4}{\pi} \frac{\langle \psi_1^2 \rangle}{\lambda_1 \langle \psi_1 (1 - \psi_0^2) \rangle^2} \simeq 3.0, \tag{44}$$

which is roughly 50% larger than the value of the KSRF II relation,

$$\frac{m_\rho^2}{g_{\rho\pi\pi}^2 F_\pi^2} = 2, \tag{45}$$

or the experimental value estimated as

$$\left. \frac{m_\rho^2}{g_{\rho\pi\pi}^2 F_\pi^2} \right|_{\mathrm{exp}} = 1.96, \tag{46}$$

where use has been made of $F_\pi = 92.4$ MeV, $m_\rho = 775.8$ MeV, and $g_{\rho\pi\pi} = 5.99$. Alternatively, when we use $F_\pi(0) = 86.4$ MeV in the chiral limit [9],

$$\left. \frac{m_\rho^2}{g_{\rho\pi\pi}^2 F_\pi^2(0)} \right|_{\mathrm{chi}} = 2.24. \tag{47}$$

The result coincides with that in Ref. [3]. This must be so, since different identifications of the ρ meson field, whether the gauge field or the CCWZ matter field, cannot lead to different results *as far as the tree-level amplitude is concerned* [9].

V. $1/N_c$-SUBLEADING CORRECTIONS

Now we propose a way to include a part of the $1/N_c$ corrections through meson loops as follows: Let us consider the Lagrangian (26), which has the parameters determined in the large N_c limit, as the *bare Lagrangian* defined at a scale Λ: $\mathcal{L} = \mathcal{L}(\Lambda)$ [8,9]. Then the parameters in the bare Lagrangian are defined as the *bare parameters* such as $F_\pi = F_\pi(\Lambda)$, $a = a(\Lambda)$, $g = g(\Lambda)$, and so on. The bare theory is matched to the HQCD at the scale Λ which we call the matching scale. Then, the $1/N_c$ corrections are incorporated into physical quantities in such a way that we consider the quantum correction generated from the ρ and π loops in HLS ChPT.

For $m_\rho \le \mu \le \Lambda$ the quantum corrections are incorporated through the renormalization group equations (RGEs) for $F_\pi(\mu)$, $a(\mu)$, $g(\mu)$, and $z_4(\mu)$ in the HLS theory *including the quadratic divergence* in the Wilsonian sense [8,9]:[2]

$$\mu \frac{dF_\pi^2}{d\mu} = \frac{N_f}{2(4\pi)^2} [3a^2 g^2 F_\pi^2 + 2(2 - a)\mu^2], \tag{48}$$

$$\mu \frac{da}{d\mu} = -\frac{N_f}{2(4\pi)^2} (a - 1) \left[3a(a+1) g^2 - (3a - 1) \frac{\mu^2}{F_\pi^2} \right], \tag{49}$$

$$\mu \frac{dg^2}{d\mu} = -\frac{N_f}{2(4\pi)^2} \frac{87 - a^2}{6} g^4, \tag{50}$$

$$\mu \frac{dz_4}{d\mu} = \frac{N_f}{2(4\pi)^2} \frac{2 + 3a - a^2}{6}. \tag{51}$$

Since $z_4(\Lambda)$ is related to $(a(\Lambda), g(\Lambda))$ as

$$a(\Lambda)(1 + \tfrac{1}{2} g^2(\Lambda) z_4(\Lambda))^2 \simeq \tfrac{4}{3}, \tag{52}$$

through the HQCD result in Eq. (44), all four parameters in the low-energy region are determined from just three bare parameters $F_\pi(\Lambda)$, $g(\Lambda)$, and $a(\Lambda)$ through the above RGEs. Note that the ρ meson mass m_ρ is determined by

[2]Coefficients of RGEs for all the $\mathcal{O}(p^4)$ terms including z_4 are given in Appendix D, Table 20, of Ref. [9].

the on-shell condition:

$$m_\rho^2 = a(m_\rho) F_\pi^2(m_\rho) g^2(m_\rho). \tag{53}$$

For $0 \le \mu \le m_\rho$, on the other hand, the couplings other than F_π do not run, while F_π does by the quantum corrections from the π loop alone. As a result, the physical decay constant $F_\pi = F_\pi(0)$ is related to $F_\pi(m_\rho)$ via the RGEs [9]:

$$F_\pi^2(0) = F_\pi^2(m_\rho)\left[1 - \frac{N_f}{(4\pi)^2}\frac{m_\rho^2}{F_\pi^2(m_\rho)}\left(1 - \frac{a(m_\rho)}{2}\right)\right]. \tag{54}$$

Following Ref. [9], we take as inputs $N_f = 3$, $F_\pi(0) = 86.4 \pm 9.7$ MeV (value at the chiral limit), and $m_\rho = 775.8$ MeV [13], and a particular parameter choice[3]

$$z_4(\Lambda) = 0, \qquad \text{i.e., } a(\Lambda) \simeq \tfrac{4}{3} \simeq 1.33, \tag{55}$$

among those satisfying Eq. (52), so that $z_4(m_\rho)$ is solely induced by the loop corrections ($1/N_c$ corrections). From these, we determine the values of $F_\pi(\Lambda)$ and $g(\Lambda)$ as done in Ref. [9]. We choose the matching scale Λ as $\Lambda = 1.0$, 1.1, and 1.2 GeV since the effect from the a_1 meson is not included.

We should carefully define the physical ρ-π-π coupling $g_{\rho\pi\pi}$. One would naively regard the physical ρ-π-π coupling as

$$g_{\rho\pi\pi} = \tfrac{1}{2}a(m_\rho)g(m_\rho)[1 + \tfrac{1}{2}g^2(m_\rho)z_4(m_\rho)], \tag{56}$$

where $a(m_\rho) = F_\sigma^2(m_\rho)/F_\pi^2(m_\rho)$. However, $g_{\rho\pi\pi}$ should be defined for the rho meson and the pion both on the mass shell. While F_σ^2 and g as well as z_4 do not run for $\mu < m_\rho$, F_π^2 does run. Since the on-shell pion decay constant is given by $F_\pi(0)$, we have to use $F_\pi(0)$ to define the on-shell ρ-π-π coupling constant [9]. The resultant expression is given by

$$g_{\rho\pi\pi} = \tfrac{1}{2}a(0)g(m_\rho)(1 + \tfrac{1}{2}g^2(m_\rho)z_4(m_\rho)), \tag{57}$$

where $a(0) \equiv F_\sigma^2(m_\rho)/F_\pi^2(0)$ is related to $a(m_\rho)$ through Eq. (54) as

$$\frac{1}{a(0)} = \frac{1}{a(m_\rho)}\left[1 - \frac{3}{(4\pi)^2}\frac{m_\rho^2}{F_\pi^2(m_\rho)}\left(1 - \frac{a(m_\rho)}{2}\right)\right]. \tag{58}$$

By using the above $g_{\rho\pi\pi}$, the physical quantity related to the KSRF II relation is given by

$$\frac{m_\rho^2}{g_{\rho\pi\pi}^2 F_\pi^2(0)} = \frac{4}{a(0)(1 + \frac{1}{2}g^2(m_\rho)z_4(m_\rho))^2} \simeq 2.0, \tag{59}$$

in good agreement with the experiment, where we have computed

$$a(0) \simeq 2.0, \qquad \tfrac{1}{2}g^2(m_\rho)z_4(m_\rho) \simeq -8.0 \times 10^{-3} \tag{60}$$

through RGE analysis for $\Lambda = 1.1$ GeV, which are compared with the bare values $a(\Lambda) \simeq 4/3$ and $\frac{1}{2}g^2(\Lambda)z_4(\Lambda) = 0$. Equation (59) is our main result, which is compared with the holographic result Eq. (44).

We note that those corrections are of $\mathcal{O}(1/N_c)$. Actually, we may set $a(m_\rho) \simeq a(\Lambda)$ in Eq. (58), since $a(\mu)$ does hardly run for $m_\rho < \mu < \Lambda$ due to the fact that the bare value $a(\Lambda) \simeq 1.33$ is close to the fixed point value $a = 1$ of the RGE (49) (see also Fig. 17 of Ref. [9]). Then

$$\frac{1}{a(0)} \simeq \frac{1}{a(\Lambda)}\left[1 - \frac{3}{(4\pi)^2}\frac{m_\rho^2}{F_\pi^2(m_\rho)}\left(1 - \frac{a(\Lambda)}{2}\right)\right] \tag{61}$$

whose second term in the bracket with m_ρ^2/F_π^2 ($\sim 1/N_c$) is nothing but the $\mathcal{O}(1/N_c)$ correction essentially coming from the pion loop contributions for $0 < \mu < m_\rho$.

In Table I, we show the predicted values of $m_\rho^2/(g_{\rho\pi\pi}^2 F_\pi^2(0))$ and of $g_{\rho\pi\pi}$ for $\Lambda = 1.0$, 1.1, 1.2 GeV in good agreement with the experiment within the errors coming from the input value $F_\pi(0)$ evaluated at the chiral limit [9]. The result is fairly insensitive to the choice of the matching scale Λ. This implies that $1/N_c$ corrections actually improve the HQCD prediction, Eq. (44), $m_\rho^2/(g_{\rho\pi\pi}^2 F_\pi^2)|_\Lambda \simeq 3.0$, into the realistic value $\simeq 2.0$. It should be emphasized that the $1/N_c$ corrections make the value always closer to the experimental value for a wide range of the value of the parameter $a(\Lambda)$ not restricted to the present one $a(\Lambda) \simeq 4/3$.

By introducing the external field, SS [3] obtained "vector meson dominance" for the pion electromagnetic form factor, though not the celebrated "ρ dominance" due to significant contributions from higher resonances, particularly the ρ'. The above peculiarity is closely related to its prediction of g_ρ, the ρ-γ mixing strength, or the pion form factor just on the ρ pole in the *timelike region*, namely, a wrong KSRF I relation, $g_\rho/(g_{\rho\pi\pi}F_\pi^2) \simeq 4$ [3], which is a factor 2 larger than the correct one. These problems will be dealt with in the forthcoming paper [12].

TABLE I. Predicted values for the KSRF II relation and $g_{\rho\pi\pi}$ including the $1/N_c$ corrections with $F_\pi(0)$ and m_ρ used as inputs. Value of the ratio $m_\rho^2/(g_{\rho\pi\pi}^2 F_\pi^2)$ indicated by "Exp." is obtained with the experimental value $F_\pi = 92.4$ MeV, while the one by "Chi." is with the value $F_\pi(0) = 86.4 \pm 9.7$ MeV (at the chiral limit). All errors of the predictions arise from the input value of $F_\pi(0)$.

Λ[GeV]	$m_\rho^2/(g_{\rho\pi\pi}^2 F_\pi^2)$	$g_{\rho\pi\pi}$
1.0	1.98 ± 1.01	6.38 ± 1.46
1.1	2.01 ± 1.02	6.34 ± 1.45
1.2	2.04 ± 1.04	6.28 ± 1.44
Exp.	1.96 ± 0.00	5.99 ± 0.03
Chi.	2.24 ± 0.50	

[3] $a = 4/3$ implies the ρ dominance of the π-π scattering [9].

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