Neutralino annihilation to $q\bar{q}g$

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We compute the cross section for $\chi\chi \to q\bar{q}g$ at order $\alpha_s^2/M_{\tilde{q}}^6$ arising from interference between the tree-level and loop-induced processes. This interference term is the same order in α_s as $\chi\chi \to gg$; for mass degenerate squarks $M_{\tilde{q}_R} = M_{\tilde{q}_L} = M_{\tilde{q}}$ we find $v_{\rm rel}\sigma_{\rm int} = [-2m_\chi^2/3M_{\tilde{q}}^2]v_{\rm rel}\sigma(\chi\chi \to gg)$.

I. INTRODUCTION

The presence of nonbaryonic dark matter in the universe is compelling evidence for physics beyond the standard model. Supersymmetry (SUSY) provides an especially attractive explanation with the lightest supersymmetric particle (LSP), usually a neutralino χ, as the dark matter candidate; for a recent review, see Ref. [1].

We focus in this paper on the QCD corrections to neutralino annihilation. We therefore consider only the annihilation processes involving quarks and/or gluons in the final state. We also assume that the LSP is largely gaugino, as is often the case in mSUGRA models [2]; in this case neutralino annihilation via s-channel Higgs or Z exchange is suppressed, since these particles require a Higgsino admixture to couple to the LSP. We thus focus on processes that involve an internal squark exchange, as shown in Fig. 1.

The leading contribution to neutralino annihilation via exchange of a squark of mass $M_{\tilde{q}}$, shown in Fig. 1(a), can be reduced to an effective vertex described by a dimension-six operator suppressed by $M_{\tilde{q}}^2$,

$$\mathcal{L} = (c/M_{\tilde{q}}^2)\mathcal{O}_6, \qquad \mathcal{O}_6 = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu\gamma_5 q), \quad (1)$$

where the dimensionless constant c contains the relevant couplings of the process. If the annihilation occurs at rest, the neutralino spinors reduce to a spin-singlet combination, $\bar{\chi}\gamma_\mu\gamma_5\chi \to K_\mu\bar{\chi}(\gamma_5/\sqrt{2}m_\chi)\chi$, where K is the center-of-mass momentum of the two-neutralino system. This result can be understood by considering that the Majorana nature of neutralinos makes the initial state behave as an effective pseudoscalar (details are given in Sec. II). The operator $\mathcal{O}_6$ can then be written as the divergence of the axial-vector current,

$$\mathcal{O}_6 \to [\bar{\chi}(i\gamma_5/\sqrt{2}m_\chi)\chi][\partial_\mu(\bar{q}\gamma^\mu\gamma_5 q)]. \quad (2)$$

In the massless quark limit, $m_q = 0$, the axial-vector current is conserved at tree level, $\partial_\mu(\bar{q}\gamma^\mu\gamma_5 q) = 0$, and all tree-level dimension-six amplitudes vanish in this limit. This is the well known partially-conserved axial current (PCAC) condition.

The quark mass suppression can be lifted in two ways:
(1) by going beyond leading order in α_s to include the correction to the dimension-six operator that involves the anomalous triangle diagram;
(2) by going to dimension-eight by including hard gluon radiation.

The anomalous triangle diagram is shown in Fig. 1(b) (the quark triangle appears at dimension-six when the heavy squark line is shrunk to a point). This diagram contributes to $\chi\chi \to gg$ and yields a cross section parametrically of order $\alpha_s^2 m_\chi^2/M_{\tilde{q}}^4$. The analogous process $\chi\chi \to \gamma\gamma$ was first studied using the anomaly equation in Refs. [3,4]; a modern computation of $\chi\chi \to gg$ in the $v_{\rm rel} = 0$ limit with arbitrary neutralino composition and squark mixing, and keeping $m_{q'} \neq 0$ in the loop, was performed in Ref. [5]. Ref. [6] extended the calculation to arbitrary $v_{\rm rel}$, with results presented in a set of numerical codes [7].
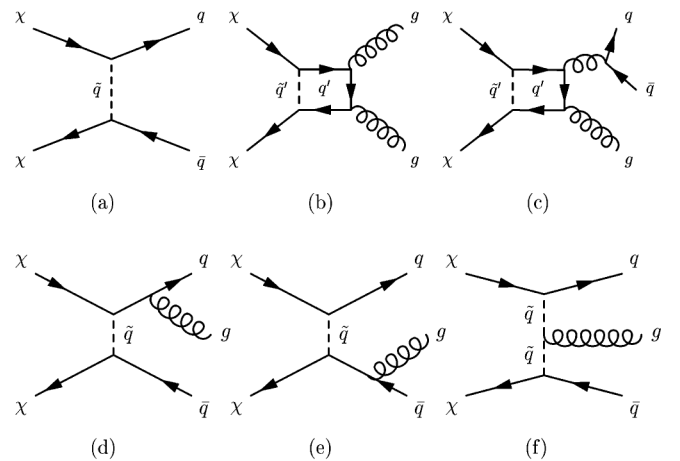

FIG. 1. Feynman diagrams that contribute to neutralino annihilation to quarks/gluons. Shown are the tree-level diagram for $\chi\chi \to q\bar{q}$ (a), typical one-loop diagrams for $\chi\chi \to gg$ (b) and $\chi\chi \to q\bar{q}g$ (c), and the three tree-level diagrams contributing to the dimension-eight operator for $\chi\chi \to q\bar{q}g$ (d–f).

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TABLE I. Relevant neutralino annihilation calculations and their relative suppression by powers of α_s and $m_\chi^2/M_{\tilde{q}}^2$.

Process	Order	Ref.	Approximation
$\chi\chi \to gg$, Dim-6	$\alpha_s^2 m_\chi^2/M_{\tilde{q}}^4$	[5]	exact for $v_{\rm rel} = 0$
$\chi\chi \to q\bar{q}g$, Dim-8	$\alpha_s m_\chi^6/M_{\tilde{q}}^8$	[5]	exact for $m_q = 0$
$\chi\chi \to q\bar{q}g$, Dim-6/8 interference	$\alpha_s^2 m_\chi^4/M_{\tilde{q}}^6$	[this paper]	leading $1/M_{\tilde{q}}^6$ term, $m_{q,q'} = 0$
$\chi\chi \to gg$ at NLO, Dim-6	$\alpha_s^3 m_\chi^2/M_{\tilde{q}}^4$	[8]	leading $1/M_{\tilde{q}}^4$ term, $m_{q'} = 0$

The next-to-leading order (NLO) QCD corrections to $\chi\chi \to gg$ were recently computed in Ref. [8] in the approximation $m_{q'} = 0$ and $M_{\tilde{q}} \gg m_\chi$. These corrections are of order $\alpha_s^3 m_\chi^2/M_{\tilde{q}}^4$, and provide about a 60% enhancement over the leading order (LO) $\chi\chi \to gg$ cross section for typical parameter values [8].

The tree-level process $\chi\chi \to q\bar{q}g$ corresponds in the limit $m_q = 0$ to a dimension-eight operator, with the amplitude suppressed by $1/M_{\tilde{q}}^4$. The relevant diagrams are shown in Fig. 1(d)–1(f). The similar process $\chi\chi \to f\bar{f}\gamma$ was calculated in Refs. [4,9] for a photino LSP. An exact tree-level calculation of $\chi\chi \to q\bar{q}g$ with arbitrary neutralino composition and squark mixing, for $m_q = 0$ and $v_{\rm rel} = 0$, was done in Ref. [5] and has a resulting cross section of order $\alpha_s m_\chi^6/M_{\tilde{q}}^8$.

In this paper we compute the interference term between the tree-level $\chi\chi \to q\bar{q}g$ diagrams of Figs. 1(d)–1(f) and the one-loop $\chi\chi \to q\bar{q}g$ process of Fig. 1(c). This interference term is parametrically of order $\alpha_s^2 m_\chi^4/M_{\tilde{q}}^6$. Counting α_s and $m_\chi^2/M_{\tilde{q}}^2$ as comparable suppression factors, this interference term is "the same order" as both the dimension-eight process and the NLO QCD corrections to $\chi\chi \to gg$, as summarized in Table I. Counting only the order in α_s, the interference term is the same order as $\chi\chi \to gg$.

Throughout this paper we assume that the LSP is lighter than the top quark, so that $\chi\chi \to t\bar{t}$ is kinematically inaccessible. (The QCD corrections to $\chi\chi \to Z^*, h^* \to t\bar{t}$ near threshold were recently computed in Ref. [10].) We take the five light quark species to be massless. As noted earlier, we also assume that the neutralinos annihilate at rest and that the LSP is gauginolike (the coupling of the Higgsino component to quark-squark pairs is proportional to the quark mass, taken here to be zero). We also assume that the squarks are heavy compared to the LSP, $M_{\tilde{q}} \gg m_\chi$, and work to leading order in the expansion in $m_\chi^2/M_{\tilde{q}}^2$. In the calculation of the quark/squark loop in Fig. 1(c) we sum over five massless internal quark species, and ignore the top quark contribution; the massive quark loop decouples very quickly with decreasing m_χ/m_t.

II. CALCULATION

In the limit of zero relative neutralino velocity, $v_{\rm rel} = 0$, a neutralino pair behaves as a pseudoscalar due to the Majorana nature of the particles. In particular, the antisymmetrized wave function of the initial-state identical particles $\chi\chi$, of total momentum K, can be reduced to a projection operator for each of the four spin combinations of the initial-state neutralinos:

$$
\begin{aligned}
u_1(K/2)\bar{v}_2(K/2) - u_2(K/2)\bar{v}_1(K/2) &= (m_\chi + \slashed{K}/2)\gamma_5 \quad \text{for } |\rightarrow\rangle|\leftarrow\rangle \quad \text{or} \quad -|\leftarrow\rangle|\rightarrow\rangle, \\
u_1(K/2)\bar{v}_2(K/2) - u_2(K/2)\bar{v}_1(K/2) &= 0 \quad \text{for } |\rightarrow\rangle|\rightarrow\rangle \quad \text{or} \quad |\leftarrow\rangle|\leftarrow\rangle.
\end{aligned}
\tag{3}
$$

The cross section is then obtained by averaging over the squares of the amplitudes of the two contributing spin states.

One can alternatively use the spin-singlet initial state,

$$
(|\rightarrow\rangle|\leftarrow\rangle - |\leftarrow\rangle|\rightarrow\rangle)/\sqrt{2},
\tag{4}
$$

which produces an additional $\sqrt{2}$ in the antisymmetrized spinor combination and in the resulting matrix elements:

$$
u_1(K/2)\bar{v}_2(K/2) - u_2(K/2)\bar{v}_1(K/2) = \sqrt{2}(m_\chi + \slashed{K}/2)\gamma_5
$$
$$
\text{(spin singlet).}
\tag{5}
$$

Either approach leads to the same overall result for the annihilation cross section; we use Eq. (5) with the additional $\sqrt{2}$ in what follows in order to obtain the matrix element for the spin-singlet state. (After squaring the matrix element, one must still average over the four initial neutralino spin states, yielding a factor of $1/4$ in the cross section.)

A. Dimension-six amplitude

In the $m_q = 0$ limit, the leading contribution to the dimension-six amplitude arises at order α_s^2 from the loop-induced process $\chi\chi \to gg$. In the language of the partially-conserved axial current, this can be expressed as the anomaly equation [11],

$$\partial_\mu(\bar{q}\gamma^\mu\gamma_5 q) = 2m_q\bar{q}i\gamma_5 q + \frac{\alpha_s}{4\pi}G^{(a)}_{\mu\nu}\tilde{G}^{(a)\mu\nu}, \quad (6)$$

where $\frac{1}{2}\tilde{G}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$ is the dual of the color field strength tensor. For $m_q \ll m_\chi$, the first term on the right-hand side of Eq. (6) can be neglected; the dimension-six amplitude can then be expressed in the form

$$\mathcal{M}_{\text{loop}}(\chi\chi \to gg) = \left(\frac{c/m_\chi}{2M_{\tilde{q}}^2}\right)(\bar{\chi}i\gamma_5\chi)\frac{\alpha_s}{4\pi}G^{(a)}_{\mu\nu}\tilde{G}^{(a)\mu\nu}.$$

$$(7)$$

The Feynman diagrams for $\chi\chi \to gg$ consist of Fig. 1(b), the analogous diagram with one neutralino and one gluon line interchanged, and the corresponding diagrams with the gluons or the neutralinos crossed. The amplitude for $\chi\chi \to gg$ in the massless quark limit is given by

$$\mathcal{M}_{\text{loop}} = \frac{\alpha_s}{2\sqrt{2}\pi}\sum_{q'}\left[\frac{|g_r|^2}{M_{\tilde{q}'_R}^2} + \frac{|g_\ell|^2}{M_{\tilde{q}'_L}^2}\right]$$
$$\times i\epsilon^{\mu\nu\alpha\beta}\epsilon^{*a}_\mu(k_1)\epsilon^{*a}_\nu(k_2)k_{1\alpha}k_{2\beta}, \quad (8)$$

where the sum runs over the five light (massless) quark flavors q'. An expression for the expansion of $\mathcal{M}_{\text{loop}}$ in powers of $m_\chi^2/M_{\tilde{q}}^2$ is given in the appendix. The $\chi q'\tilde{q}'_{R,L}$ couplings $g_{r,\ell}$ for right and left quark helicities, respectively, are

$$g_r = -\sqrt{2}N_{11}g'Q,$$
$$g_\ell = -\sqrt{2}N_{11}g'(T_3 - Q) + \sqrt{2}N_{12}gT_3. \quad (9)$$

Here T_3 is the quark isospin, Q is the quark electric charge, g and g' are the weak couplings under $SU(2)_L$ and $U(1)_Y$, respectively, and N_{11} and N_{12} are the bino and wino components of the neutralino as defined in Ref. [12]. The matrix element in Eq. (8) results in an annihilation cross section for $\chi\chi \to gg$ of

$$\upsilon_{\text{rel}}\sigma_{\text{dim6}} = \frac{\alpha_s^2}{32\pi^3}m_\chi^2\left\{\sum_{q'}\left[\frac{|g_r|^2}{M_{\tilde{q}'_R}^2} + \frac{|g_\ell|^2}{M_{\tilde{q}'_L}^2}\right]\right\}^2. \quad (10)$$

For massless quarks in the loop, taking one of the final-state gluons off-shell by an amount $q^2 \neq 0$ yields a relative shift in the loop integral by $q^2/M_{\tilde{q}}^2$; in particular, this shift is higher order in the $1/M_{\tilde{q}}^2$ expansion than the leading term. To leading $1/M_{\tilde{q}}^2$ order it is then straightforward to extend the loop amplitude to include one off-shell gluon splitting into a quark-antiquark pair, shown schematically in Fig. 1(c). The matrix element is

$$\mathcal{M}_{\text{split}} = -\frac{g_s}{\sqrt{2}}\sum_{q'}\left[\frac{|g_r|^2}{M_{\tilde{q}'_R}^2} + \frac{|g_\ell|^2}{M_{\tilde{q}'_L}^2}\right]i\epsilon^{\mu\nu\alpha\beta}\epsilon^{*c}_\mu q_{3\alpha}(q_1 + q_2)_\beta$$

$$\times \frac{\alpha_s}{2\pi(q_1 + q_2)^2}\bar{u}(q_1)T^c\gamma_\nu v(q_2), \quad (11)$$

where q_1, q_2, and q_3 are the momenta of the final-state

quark, antiquark, and gluon, respectively, and the sum runs over the five light (massless) quarks in the loop. Note that the gluon-splitting couples the box diagram to a *vectorlike* quark current.

The genuine anomaly loop is a closed fermion loop which has a negative sign. The loop in the present case is basically a box with one side as the scalar quark line, so it is not a closed fermion loop. However, it does have a relative negative sign with respect to the dimension-eight amplitude (see the next subsection) because the two fermion lines are topologically twisted—replacing the neutralino spinor pair with the zero-velocity result from Eq. (5) requires contracting the two spinors together, yielding an extra overall minus sign. The situation is similar to that of the two amplitudes in Bhabha scattering.

Using Eq. (11) we can compute the zero-velocity neutralino annihilation cross section from the gluon-splitting process. Squaring the matrix element, summing over final-state polarizations and colors and averaging over initial-state polarizations, we obtain

$$\frac{1}{4}\sum_{\text{pols}}|\mathcal{M}_{\text{split}}|^2 = \frac{\alpha_s^3}{\pi}\left\{\sum_{q'}\left[\frac{|g_r|^2}{M_{\tilde{q}'_R}^2} + \frac{|g_\ell^2|}{M_{\tilde{q}'_L}^2}\right]\right\}^2$$
$$\times \frac{(q_1\cdot q_3)^2 + (q_2\cdot q_3)^2}{q_1\cdot q_2}, \quad (12)$$

for a single (massless) final-state quark flavor.

Note the divergence in Eq. (12) as the gluon propagator goes on shell, $(q_1 + q_2)^2 = 2q_1 \cdot q_2 \to 0$. This is the usual divergence that appears in NLO calculations when a final-state parton is soft or collinear; in this case the divergence is cut off by the quark mass and gives rise to a $\log(m_\chi^2/m_q^2)$ enhanced term in the total cross section [9]. This logarithmic term is precisely canceled by the renormalization of the strong coupling due to the quark bubble that appears in the virtual part of the NLO correction to $\chi\chi \to gg$ [8]. This is the familiar cancellation of logarithmic divergences guaranteed by the Kinoshita-Lee-Nauenberg theorem [13].

B. Dimension-eight amplitude

The dimension-eight amplitude comes from computing the tree-level process $\chi\chi \to q\bar{q}g$ shown in Fig. 1(d)–1(f) in the massless quark limit, $m_q = 0$. Because the $\chi\chi \to q\bar{q}$ amplitude [Fig. 1(a)] is zero in this limit, the gluon radiation diagram contains no soft or collinear divergences. The tree-level matrix element has the general form,

$$i\mathcal{M}_{\text{tree}} = (i)^5 g_s|g_{r,\ell}|^2\bar{u}(q_1)T^cA^cP_{R,L}v(q_2), \quad (13)$$

where the amplitude A^c is given below. The i in front of $\mathcal{M}_{\text{tree}}$ follows the usual Feynman rule convention. The factor $(i)^5$ counts three vertices and two propagators. The $SU(3)$ matrix T^c links the outgoing gluon with color label c (contained inside A^c) to the $q\bar{q}$ system. The chirality projection operators $P_{R,L} = (1 \pm \gamma_5)/2$ reflect the fact that the squarks couple to quarks of specific helicity. The

pieces of the amplitude are $A^c = A_1^c + A_2^c + A_3^c$, with

$$A_1^c = \frac{-\slashed{\epsilon}^{*c}\frac{1}{\slashed{q}_1+\slashed{q}_3}\frac{\slashed{K}}{\sqrt{2}}\gamma_5}{(K/2 - q_2)^2 - M_{\tilde{q}_{R,L}}^2} \rightarrow \frac{-\slashed{\epsilon}^{*c}\gamma_5/\sqrt{2}}{(K/2 - q_2)^2 - M_{\tilde{q}_{R,L}}^2},$$

$$A_2^c = \frac{-\frac{\slashed{K}}{\sqrt{2}}\gamma_5\frac{1}{-\slashed{q}_2-\slashed{q}_3}\slashed{\epsilon}^{*c}}{(-K/2 + q_1)^2 - M_{\tilde{q}_{R,L}}^2} \rightarrow \frac{+\slashed{\epsilon}^{*c}\gamma_5/\sqrt{2}}{(-K/2 + q_1)^2 - M_{\tilde{q}_{R,L}}^2},$$

$$A_3^c = \frac{-\frac{\slashed{K}}{\sqrt{2}}\gamma_5(q_1 - q_2)\cdot\epsilon^{*c}}{[(-K/2 + q_1)^2 - M_{\tilde{q}_{R,L}}^2][(K/2 - q_2)^2 - M_{\tilde{q}_{R,L}}^2]},$$

$$(14)$$

where A_1^c, A_2^c, and A_3^c correspond to cases that the gluon is radiated from the quark line, the antiquark line and the squark line, as in Figs. 1(d)–1(f), respectively. The outgoing four-momenta of the quark, antiquark and gluon are denoted q_1, q_2 and q_3, respectively, and K is the incoming four-momentum of the initial two-neutralino system. The Dirac equation was applied to A_1^c and A_2^c in the massless quark limit. Combining these three contributions, we get

$$A^c = -\frac{1}{\sqrt{2}}\frac{(q_2 - q_1)\cdot K\slashed{\epsilon}^{*c} + (q_1 - q_2)\cdot\epsilon^{*c}\slashed{K}}{[(-K/2 + q_1)^2 - M_{\tilde{q}_{R,L}}^2][(K/2 - q_2)^2 - M_{\tilde{q}_{R,L}}^2]}\gamma_5.\tag{15}$$

Note that the pieces with only one squark propagator in the denominator have canceled in the massless quark limit. The resulting matrix element is

$$\mathcal{M}_{\text{tree}} = -\frac{g_s}{\sqrt{2}}|g_{r,\ell}|^2\bar{u}(q_1)T^c\frac{(q_2 - q_1)\cdot q_3\slashed{\epsilon}^{*c} + (q_1 - q_2)\cdot\epsilon^{*c}\slashed{q}_3}{[(-K/2 + q_1)^2 - M_{\tilde{q}_{R,L}}^2][(K/2 - q_2)^2 - M_{\tilde{q}_{R,L}}^2]}\gamma_5 P_{R,L}v(q_2),\tag{16}$$

for the diagrams involving $\tilde{q}_R$ and $\tilde{q}_L$, respectively.

Using the Chisholm identity,

$$g^{\alpha\beta}\gamma^\mu - g^{\alpha\mu}\gamma^\beta = \gamma^\alpha\gamma^\beta\gamma^\mu - g^{\beta\mu}\gamma^\alpha - i\epsilon^{\mu\nu\alpha\beta}\gamma_\nu\gamma_5, \quad \text{with} \quad \epsilon^{0123} = +1,\tag{17}$$

together with the Dirac equation and the polarization condition $\epsilon_\mu^* q_3^\mu = 0$ for the external gluon, we obtain an amplitude which has a similar tensor structure to that of the dimension-six amplitude in Eq. (11):

$$\mathcal{M}_{\text{tree}} = +\frac{g_s}{\sqrt{2}}|g_{r,\ell}|^2\bar{u}(q_1)T^c\gamma_\nu P_{R,L}v(q_2)\frac{i\epsilon^{\mu\nu\alpha\beta}\epsilon_\mu^{*c}q_{3\alpha}(q_1 + q_2)_\beta}{[(-K/2 + q_1)^2 - M_{\tilde{q}_{R,L}}^2][(K/2 - q_2)^2 - M_{\tilde{q}_{R,L}}^2]}.\tag{18}$$

Such a form will allow us to easily read off the interference term.

The matrix element can be expanded in powers of $m_\chi^2/M_{\tilde{q}}^2$ for $M_{\tilde{q}} \gg m_\chi$, yielding a leading $(1/M_{\tilde{q}})^4$ behavior. The leading term comes from neglecting the $(-K/2 + q_1)^2$ and $(K/2 - q_2)^2$ terms in the propagators compared to $M_{\tilde{q}}^2$:

$$\mathcal{M}_{\text{tree}} \simeq +\frac{g_s}{\sqrt{2}}|g_{r,\ell}|^2\bar{u}(q_1)T^c\gamma_\nu P_{R,L}v(q_2)$$

$$\times \frac{i\epsilon^{\mu\nu\alpha\beta}\epsilon_\mu^{*c}q_{3\alpha}(q_1 + q_2)_\beta}{M_{\tilde{q}_{R,L}}^4}.\tag{19}$$

As a check, we can now compute the zero-velocity neutralino annihilation cross section from the tree-level $\chi\chi \rightarrow q\bar{q}g$ process, neglecting the final-state quark mass. Squaring the matrix element, summing over final-state polarizations and colors and averaging over initial-state polarizations, we obtain

$$\frac{1}{4}\sum_{\text{pols}}|\mathcal{M}_{\text{tree}}|^2 = 2g_s^2\left[\frac{|g_r|^4}{M_{\tilde{q}_R}^8} + \frac{|g_\ell|^4}{M_{\tilde{q}_L}^8}\right]$$

$$\times q_1\cdot q_2[(q_1\cdot q_3)^2 + (q_2\cdot q_3)^2],\tag{20}$$

for a single final-state quark flavor, where we include the contributions from exchange of both $\tilde{q}_L$ and $\tilde{q}_R$. Integrating over the phase space, we find

$$v_{\text{rel}}\sigma(\chi\chi \rightarrow q\bar{q}g) = \frac{4}{15}\frac{m_\chi^6}{(4\pi)^2}\left[\frac{|g_r|^4}{M_{\tilde{q}_R}^8} + \frac{|g_\ell|^4}{M_{\tilde{q}_L}^8}\right]\alpha_s\tag{21}$$

(tree level).

This agrees with the result of Drees *et al.* [5] when their Eq. (2.18) is expanded to extract the leading $1/M_{\tilde{q}}^8$ dependence.

The cross section for the similar tree-level process $\chi\chi \rightarrow f\bar{f}\gamma$ was computed in Ref. [9] for the case of pure photino LSPs and degenerate left- and right-handed

squarks, $M_{\tilde{q}_L} = M_{\tilde{q}_R} \equiv M_{\tilde{q}}$. Replacing the color factor with the appropriate electric charge entails $\alpha_s \to \alpha Q_f^2/4$. The photino couplings are $g_\ell = -g_r = \sqrt{2}eQ_f$, so that the couplings $|g_{r,\ell}|^4$ become $|g_{r,\ell}|^4 \to 4e^4Q_f^4$. Thus we obtain, for the tree-level (dimension-eight) $\chi\chi \to f\bar{f}\gamma$ with $m_f = 0$ and $v_{\rm rel} \to 0$,

$$v_{\rm rel}\sigma(\chi\chi \to f\bar{f}\gamma) = \frac{8}{15}\frac{m_\chi^6}{M_{\tilde{q}}^8}\alpha^3 Q_f^6 \qquad \text{(tree level). (22)}$$

Note that this result is a factor of 2 smaller than that of Ref. [9].

C. Interference term

We now combine the dimension-six and dimension-eight contributions. While the dimension-six gluon-splitting amplitude in Eq. (11) contains a purely vectorlike final-state quark current, the dimension-eight amplitude contains both vectorlike and axial-vectorlike quark currents:

$$\mathcal{M}_{\rm tree} = \frac{g_s}{\sqrt{2}}i\epsilon^{\mu\nu\alpha\beta}\epsilon_\mu^{*c}q_{3\alpha}(q_1+q_2)_\beta\left\{\frac{1}{2}\left[\frac{|g_r|^2}{M_{\tilde{q}_R}^4} + \frac{|g_\ell|^2}{M_{\tilde{q}_L}^4}\right]\right.$$
$$\times \bar{u}(q_1)T^c\gamma_\nu v(q_2)$$
$$\left.+ \frac{1}{2}\left[\frac{|g_r|^2}{M_{\tilde{q}_R}^4} - \frac{|g_\ell|^2}{M_{\tilde{g}_L}^4}\right]\bar{u}(q_1)T^c\gamma_\nu\gamma_5 v(q_2)\right\}. \quad (23)$$

Only the vectorlike part of $\mathcal{M}_{\rm tree}$ will interfere with the dimension-six amplitude. Summing over final-state polarizations and colors and averaging over initial-state polarizations, we obtain the interference term,

$$\frac{1}{4}\sum_{\rm pols}2\,{\rm Re}[\mathcal{M}_{\rm tree}\mathcal{M}_{\rm split}^*] = -4\alpha_s^2\sum_{q'}\left[\frac{|g_r|^2}{M_{\tilde{q}_R'}^2} + \frac{|g_\ell|^2}{M_{\tilde{q}_L'}^2}\right]$$
$$\times \sum_q\left[\frac{|g_r|^2}{M_{\tilde{q}_R}^4} + \frac{|g_\ell|^2}{M_{\tilde{q}_L}^4}\right]$$
$$\times [(q_1 \cdot q_3)^2 + (q_2 \cdot q_3)^2], \quad (24)$$

where we have summed over the five light internal quarks q' in $\mathcal{M}_{\rm split}$ of Eq. (11) and over the five light external quarks q. Integrating over the phase space, we find the contribution to the cross section from the interference term,

$$v_{\rm rel}\sigma_{\rm int} = -\frac{\alpha_s^2}{32\pi^3}m_\chi^2\frac{2m_\chi^2}{3}\sum_{q'}\left[\frac{|g_r|^2}{M_{\tilde{q}_R'}^2} + \frac{|g_\ell|^2}{M_{\tilde{q}_L'}^2}\right]$$
$$\times \sum_q\left[\frac{|g_r|^2}{M_{\tilde{q}_R}^4} + \frac{|g_\ell|^2}{M_{\tilde{q}_L}^4}\right]. \quad (25)$$

We see that the form of this interference term is rather similar to that of the dimension-six cross section given in Eq. (10). In the special case of degenerate right- and left-

handed squarks, $M_{\tilde{q}_R} = M_{\tilde{q}_L} \equiv M_{\tilde{q}}$, we find that the interference term is related to the dimension-six cross section by a multiplicative constant:[1]

$$v_{\rm rel}\sigma_{\rm int} = v_{\rm rel}\sigma_{\rm dim\,6}\left[-\frac{2}{3}\frac{m_\chi^2}{M_{\tilde{q}}^2}\right]. \quad (26)$$

This result can be compared to the NLO corrections to $\chi\chi \to gg$ [8]; for $N_f = 5$ light flavors and taking the renormalization scale $\mu = 2m_\chi$ we obtain

$$v_{\rm rel}\sigma = v_{\rm rel}\sigma_{\rm dim6}\left[1 + \frac{221}{12}\frac{\alpha_s^{(5)}(2m_\chi)}{\pi} - \frac{2}{3}\frac{m_\chi^2}{M_{\tilde{q}}^2}\right]. \quad (27)$$

The second term in the square brackets is the NLO correction from Ref. [8] and the third term is our result for the interference term. Note that for $m_\chi \sim 100$ GeV the NLO correction is roughly 60%; for, e.g., $M_{\tilde{q}} \sim 2m_\chi$, the interference term cancels off about one quarter of the NLO correction.

Finally, we note that we have *not* included a term of order $\alpha_s^2 m_\chi^4/M_{\tilde{q}}^6$ from the expansion of the squark propagator in $\chi\chi \to gg$ in powers of $1/M_{\tilde{q}}^2$. This is because the loop integral is odd in the argument $(m_\chi^2/M_{\tilde{q}}^2)$ with the next term in the series contributing to the cross section at order $\alpha_s^2 m_\chi^6/M_{\tilde{q}}^8$, which is beyond our present interest.

III. CONCLUSIONS

We have computed the cross section for neutralino annihilation $\chi\chi \to q\bar{q}g$ at order $\alpha_s^2/M_{\tilde{q}}^6$ arising from interference between the tree-level and loop-induced processes, in the limit of zero relative neutralino velocity and ignoring the masses of the five light quark flavors. Our result for the total cross section from this interference term is given in Eq. (25). We also provided the spin-averaged squares of matrix elements for the dimension-six, dimension-eight, and interference term contributions to $\chi\chi \to q\bar{q}g$ in terms of the final-state particle momenta in order to facilitate their implementation into Monte Carlos for the computation of indirect dark matter detection rates.

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[1]We disagree with the statement of Ref. [9] that the interference is zero for photinolike neutralinos when $M_{\tilde{q}_R} = M_{\tilde{q}_L}$. This conclusion was due to a spurious γ_5 in the loop amplitude matrix element of Ref. [9], which resulted in it interfering with the axial-vector part of the dimension-eight amplitude.

APPENDIX A

The matrix element for $\chi\chi \to gg$ with on-shell final-state gluons and massless quarks in the loop can be expressed as an expansion in powers of $m_\chi^2/M_{\tilde{q}}^2$ as follows:

$$
\mathcal{M}_{\text{loop}} = \frac{\alpha_s}{2\sqrt{2}\pi} \sum_{q'} \left[\frac{|g_r|^2}{M_{\tilde{q}'_R}^2} F(m_\chi^2/M_{\tilde{q}'_R}^2) + \frac{|g_\ell|^2}{M_{\tilde{q}'_L}^2} F(m_\chi^2/M_{\tilde{q}'_L}^2) \right]
$$
$$
\times i\epsilon^{\mu\nu\alpha\beta}\epsilon_\mu^{*a}(k_1)\epsilon_\nu^{*a}(k_2)k_{1\alpha}k_{2\beta}, \tag{A1}
$$

where

$$
F(a) = 1 + \frac{a^2}{9} + \frac{a^4}{25} + \cdots + \frac{a^{2n}}{(2n+1)^2} + \cdots \tag{A2}
$$

with $a = m_\chi^2/M_{\tilde{q}}^2$, in agreement with Ref. [5].

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