

Gluonic contribution to $B \rightarrow \eta^{(\prime)}$ form factorsYeo-Yie Charng,^{1,*} T. Kurimoto,^{2,†} and Hsiang-nan Li^{1,3,‡}¹*Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China*²*Department of Physics, University of Toyama, Toyama 930-8555, Japan*³*Department of Physics, National Cheng-Kung University, Tainan, Taiwan 701, Republic of China*

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We calculate the flavor-singlet contribution to the $B \rightarrow \eta^{(\prime)}$ transition form factors from the gluonic content of the $\eta^{(\prime)}$ meson in the large-recoil region using the perturbative QCD approach. The formulation for the η - η' mixing in the quark-flavor and singlet-octet schemes is compared, and employed to determine the chiral enhancement scales associated with the two-parton twist-3 $\eta^{(\prime)}$ meson distribution amplitudes. It is found that the gluonic contribution is negligible (around 5%) in the $B \rightarrow \eta$ form factors, but reaches 10%–40% in the $B \rightarrow \eta'$ ones. Its impact on the accommodation of the measured $B \rightarrow \eta^{(\prime)}K$ branching ratios in the perturbative QCD and QCD-improved factorization approaches is elaborated.

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I. INTRODUCTION

It is still uncertain whether the flavor-singlet contributions to B meson decays into $\eta^{(\prime)}$ mesons play an essential role. The flavor-singlet contributions to the $B \rightarrow \eta^{(\prime)}K$ branching ratios, including those from the $b \rightarrow sgg$ transition [1], from the spectator scattering [2,3], and from the weak annihilation, have been analyzed in the QCD-improved factorization (QCDF) approach [4]. However, at least the piece from the weak annihilation cannot be estimated unambiguously due to the presence of the end point singularities. The flavor-singlet contribution to the $B \rightarrow \eta^{(\prime)}$ transition form factors from the gluonic content of the $\eta^{(\prime)}$ meson is also involved in the annihilation amplitudes. Though this contribution seems to be most crucial among all the flavor-singlet pieces in the $B \rightarrow \eta^{(\prime)}K$ decays, it has been parametrized and varied arbitrarily [4]. The form factors associated with the decays $B \rightarrow \eta^{(\prime)}l^+l^-$ were handled in a similar way recently [5]. A sizable gluonic content in the η' meson has been indicated from a phenomenological analysis of the relevant data [6]. All these previous studies motivate us to make a more definite estimate of the gluonic contribution in the $B \rightarrow \eta^{(\prime)}$ form factors.

In this paper we shall calculate the gluonic contribution to the $B \rightarrow \eta^{(\prime)}$ form factors in the large-recoil region using the perturbative QCD (PQCD) approach [7–9]. This approach is based on k_T factorization theorem [10,11], so the end point singularities do not exist. It has been proposed to extract this gluonic contribution from the measured $B \rightarrow \eta^{(\prime)}l\nu$ decay spectra [12], which are, however, not available yet. To proceed with the calculation, we need to specify a scheme for the η - η' mixing. After comparing the quark-flavor basis [13] and the conventional singlet-octet basis,

we adopt the former, in which fewer two-parton twist-3 $\eta^{(\prime)}$ meson distribution amplitudes are introduced. To reduce the theoretical uncertainty from the distribution amplitudes, we employ the Gegenbauer coefficients constrained by the data of both exclusive processes [13–15], such as the $\eta^{(\prime)}$ transition form factors, and inclusive processes [16], such as $Y(1S) \rightarrow \eta'X$. It will be shown that the gluonic contribution is negligible (around 5%) in the $B \rightarrow \eta$ form factors, but reaches 10%–40% in the $B \rightarrow \eta'$ ones.

Whether the observed $B \rightarrow \eta K$ and $B \rightarrow \eta' K$ branching ratios [17] can be accommodated simultaneously still attracts a lot of attention [18,19]. We shall elaborate the impact of the gluonic contribution in the $B \rightarrow \eta^{(\prime)}$ form factors on this issue in the QCDF and PQCD frameworks. As noticed in [4], the gluonic contribution increases the branching ratios $B(B \rightarrow \eta' K)$, but decreases $B(B \rightarrow \eta K)$. Since the QCDF predictions for both $B(B \rightarrow \eta' K)$ and $B(B \rightarrow \eta K)$ fall short compared with the data [4], the gluonic contribution does not help. On the contrary, there is much room for this contribution to play in PQCD: the flavor-singlet amplitudes were not taken into account in the earlier PQCD analysis of the $B \rightarrow \eta^{(\prime)}K$ decays [20], whose predictions for $B(B \rightarrow \eta' K)$ [$B(B \rightarrow \eta K)$] are lower (higher) than the measured values. We shall demonstrate that the data of the $B \rightarrow \eta^{(\prime)}K$ branching ratios can be well accommodated in PQCD, if considering the maximal gluonic contribution in the $B \rightarrow \eta^{(\prime)}$ form factors.

In Sec. II we compare the quark-flavor and singlet-octet schemes for the η - η' mixing, and obtain the chiral enhancement scales associated with the two-parton twist-3 distribution amplitudes in both cases. In Sec. III we derive the factorization formulas for the quark and gluonic contributions in the $B \rightarrow \eta^{(\prime)}$ form factors, and perform the numerical evaluation, together with a detailed uncertainty analysis. Our results are then compared with those obtained in the literature. The impact of the gluonic contribution on the accommodation of the measured $B \rightarrow \eta^{(\prime)}K$ branching ratios is discussed. Section IV is the conclusion.

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II. η - η' MIXING AND DISTRIBUTION AMPLITUDES

For the η - η' mixing, the conventional singlet-octet basis and the quark-flavor basis [13] have been proposed. In the latter the $q\bar{q} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ flavor states, labeled by the η_q and η_s mesons, respectively, are defined. The physical states η and η' are related to the flavor states through a single angle ϕ ,

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = U(\phi) \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{pmatrix}, \quad (1)$$

with the matrix

$$U(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}. \quad (2)$$

It has been postulated [13] that only two decay constants f_q and f_s need to be introduced:

$$\begin{aligned} \langle 0|\bar{q}\gamma^\mu\gamma_5q|\eta_q(P)\rangle &= -\frac{i}{\sqrt{2}}f_qP^\mu, \\ \langle 0|\bar{s}\gamma^\mu\gamma_5s|\eta_s(P)\rangle &= -if_sP^\mu, \end{aligned} \quad (3)$$

for the light quark $q = u$ or d . This postulation is based on

the assumption that the intrinsic $\bar{q}q$ ($\bar{s}s$) component is absent in the η_s (η_q) meson, i.e., based on the Okubo-Zweig-Iizuka (OZI) suppression rule. The decay constants associated with the η and η' mesons:

$$\begin{aligned} \langle 0|\bar{q}\gamma^\mu\gamma_5q|\eta^{(i)}(P)\rangle &= -\frac{i}{\sqrt{2}}f_{\eta^{(i)}}^qP^\mu, \\ \langle 0|\bar{s}\gamma^\mu\gamma_5s|\eta^{(i)}(P)\rangle &= -if_{\eta^{(i)}}^sP^\mu, \end{aligned} \quad (4)$$

are then related to f_q and f_s via the same mixing matrix

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = U(\phi) \begin{pmatrix} f_q & 0 \\ 0 & f_s \end{pmatrix}. \quad (5)$$

Employing the equation of motion

$$\partial_\mu(\bar{q}\gamma^\mu\gamma_5q) = 2im_q\bar{q}\gamma_5q + \frac{\alpha_s}{4\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}, \quad (6)$$

and the one corresponding to the s quark, where G is the field-strength tensor and \tilde{G} the dual field-strength tensor, one derives the relation [13],

$$M_{qs}^2 = U^\dagger(\phi)M^2U(\phi). \quad (7)$$

In the above expression the mass matrices are given by

$$M^2 = \begin{pmatrix} m_\eta^2 & 0 \\ 0 & m_{\eta'}^2 \end{pmatrix}, \quad M_{qs}^2 = \begin{pmatrix} m_{qq}^2 + (\sqrt{2}/f_q)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle & (1/f_s)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle \\ (\sqrt{2}/f_q)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle & m_{ss}^2 + (1/f_s)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle \end{pmatrix}, \quad (8)$$

with the abbreviations

$$\begin{aligned} m_{qq}^2 &= \frac{\sqrt{2}}{f_q}\langle 0|m_u\bar{u}i\gamma_5u + m_d\bar{d}i\gamma_5d|\eta_q\rangle, \\ m_{ss}^2 &= \frac{2}{f_s}\langle 0|m_s\bar{s}i\gamma_5s|\eta_s\rangle. \end{aligned} \quad (9)$$

The above matrix elements define the chiral enhancement scales associated with the two-parton twist-3 η_q and η_s meson distribution amplitudes. Note that the axial U(1) anomaly [21] is the only source of the nondiagonal elements of M_{qs}^2 in the quark-flavor basis, also a consequence of the postulation that leads to Eq. (3).

We repeat the above formalism for the η - η' mixing in the singlet-octet basis, where the $(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$ states, labeled by the η_1 and η_8 mesons, respectively, are considered. From Eq. (1) and the quark content of the η_1 and η_8 mesons, we have the decompositions of the η and η' meson states in the singlet-octet basis,

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = U(\theta) \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \end{pmatrix}, \quad (10)$$

with the angle $\theta = \phi - \theta_i$, θ_i being the ideal mixing angle associated with the matrix

$$U(\theta_i) = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{\sqrt{2}}{\sqrt{3}} \\ \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \quad (11)$$

The decay constants defined in terms of the SU(3) flavor-singlet and flavor-octet axial-vector currents,

$$\langle 0|J_{\mu 5}^i|\eta^{(i)}(P)\rangle = -if_{\eta^{(i)}}^iP^\mu, \quad (12)$$

for $i = 1$ or 8 , are related to f_q and f_s through

$$\begin{pmatrix} f_\eta^8 & f_\eta^1 \\ f_{\eta'}^8 & f_{\eta'}^1 \end{pmatrix} = U(\phi) \begin{pmatrix} f_q & 0 \\ 0 & f_s \end{pmatrix} U^\dagger(\theta_i). \quad (13)$$

Compared to the conventional parametrization

$$\begin{pmatrix} f_\eta^8 & f_\eta^1 \\ f_{\eta'}^8 & f_{\eta'}^1 \end{pmatrix} = U_{81} \begin{pmatrix} f_8 & 0 \\ 0 & f_1 \end{pmatrix}, \quad (14)$$

with the matrix

$$U_{81} = \begin{pmatrix} \cos\theta_8 & -\sin\theta_1 \\ \sin\theta_8 & \cos\theta_1 \end{pmatrix}, \quad (15)$$

the decay constants of the η_1 and η_8 mesons, f_1 and f_8 , respectively, then connect to f_q and f_s .

Following the similar procedure, we derive the version of Eq. (7) in the singlet-octet basis,

$$M_{81}^2 = U^\dagger(\theta)M^2U_{81}, \quad (16)$$

with the mass matrix

$$M_{81}^2 = \begin{pmatrix} m_{88}^2 & m_{18}^2 + (\sqrt{3}/f_1)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_8\rangle \\ m_{81}^2 & m_{11}^2 + (\sqrt{3}/f_1)\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_1\rangle \end{pmatrix}, \quad (17)$$

and the abbreviations

$$\begin{aligned} m_{88}^2 &= \frac{2}{\sqrt{6}f_8}\langle 0|m_u\bar{u}i\gamma_5u + m_d\bar{d}i\gamma_5d - 2m_s\bar{s}i\gamma_5s|\eta_8\rangle, \\ m_{18}^2 &= \frac{2}{\sqrt{3}f_1}\langle 0|m_u\bar{u}i\gamma_5u + m_d\bar{d}i\gamma_5d + m_s\bar{s}i\gamma_5s|\eta_8\rangle, \\ m_{81}^2 &= \frac{2}{\sqrt{6}f_8}\langle 0|m_u\bar{u}i\gamma_5u + m_d\bar{d}i\gamma_5d - 2m_s\bar{s}i\gamma_5s|\eta_1\rangle, \\ m_{11}^2 &= \frac{2}{\sqrt{3}f_1}\langle 0|m_u\bar{u}i\gamma_5u + m_d\bar{d}i\gamma_5d + m_s\bar{s}i\gamma_5s|\eta_1\rangle. \end{aligned} \quad (18)$$

The above matrix elements define the chiral enhancement scales associated with the two-parton twist-3 η_1 and η_8 meson distribution amplitudes.

Note that the four hadronic matrix elements m_{qq}^2 , m_{ss}^2 , $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle$, and $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle$ are fixed by Eq. (7) in the quark-flavor basis, given the three inputs f_q , f_s , and ϕ . However, M_{81}^2 contains six matrix elements, which cannot be fixed completely by Eq. (16). In fact, two OZI suppressed matrix elements $\langle 0|m_u\bar{u}i\gamma_5u + m_d\bar{d}i\gamma_5d|\eta_s\rangle$ and $\langle 0|m_s\bar{s}i\gamma_5s|\eta_q\rangle$ have been dropped in [13]. In the singlet-octet basis an approximation can be made to reduce the number of matrix elements [22,23]: the terms proportional to the light quark masses m_u and m_d are negligible compared with those proportional to m_s in Eq. (18), which then becomes

$$\begin{aligned} m_{88}^2 &= -\frac{4}{\sqrt{6}f_8}\langle 0|m_s\bar{s}i\gamma_5s|\eta_8\rangle, & m_{18}^2 &= -\frac{f_8}{\sqrt{2}f_1}m_{88}^2, \\ m_{11}^2 &= \frac{2}{\sqrt{3}f_1}\langle 0|m_s\bar{s}i\gamma_5s|\eta_1\rangle, & m_{81}^2 &= -\frac{\sqrt{2}f_1}{f_8}m_{11}^2. \end{aligned} \quad (19)$$

The three inputs f_q , f_s , and ϕ in Eq. (7) have been extracted from the data of the relevant exclusive processes [13]

$$\begin{aligned} f_q &= (1.07 \pm 0.02)f_\pi, & f_s &= (1.34 \pm 0.06)f_\pi, \\ \phi &= 39.3^\circ \pm 1.0^\circ, \end{aligned} \quad (20)$$

which correspond, via Eqs. (13) and (14), to [24]

$$\begin{aligned} f_8 &\approx 1.26f_\pi, & f_1 &\approx 1.17f_\pi, \\ \theta_8 &\approx -21.2^\circ, & \theta_1 &\approx -9.2^\circ, \end{aligned} \quad (21)$$

in the singlet-octet basis. The above values are well consistent with those in [23], which were also extracted from the relevant data but based on the approximation in Eq. (19). We stress that the approximations in the quark-flavor basis [13] and in the singlet-octet basis [23] for decreasing the number of hadronic matrix elements are very different. Therefore, the above agreement implies that these approximations make sense, and that the two bases will be equivalent to each other, if one chooses the parameters obeying the constraints in Eqs. (7), (13), (14), (16), and (19) for a calculation.

Viewing Eqs. (9) and (18), it is obvious that fewer two-parton twist-3 distribution amplitudes are introduced in the quark-flavor scheme [25]. Hence, we adopt this scheme for the η - η' mixing, and specify the η_q and η_s meson distribution amplitudes. Their two-parton quark components are defined via the nonlocal matrix elements

$$\begin{aligned} \langle \eta_q(P)|\bar{q}_\gamma^a(z)q_\beta^b(0)|0\rangle &= -\frac{i}{2\sqrt{N_c}}\delta^{ab}\int_0^1 dx e^{ixP\cdot z}\{[\gamma_5\not{P}]_{\beta\gamma}\phi_q^A(x) + [\gamma_5]_{\beta\gamma}m_0^q\phi_q^P(x) + m_0^q[\gamma_5(\not{n}_+\not{n}_- - 1)]_{\beta\gamma}\phi_q^T(x)\}, \\ \langle \eta_s(P)|\bar{s}_\gamma^a(z)s_\beta^b(0)|0\rangle &= -\frac{i}{\sqrt{2N_c}}\delta^{ab}\int_0^1 dx e^{ixP\cdot z}\{[\gamma_5\not{P}]_{\beta\gamma}\phi_s^A(x) + [\gamma_5]_{\beta\gamma}m_0^s\phi_s^P(x) + m_0^s[\gamma_5(\not{n}_+\not{n}_- - 1)]_{\beta\gamma}\phi_s^T(x)\}, \end{aligned} \quad (22)$$

where $P = (P^+, 0, \mathbf{0}_T)$ is the $\eta_{q,s}$ meson momentum, the lightlike vector $z = (0, z^-, \mathbf{0}_T)$ the coordinate of the q and s quarks, the dimensionless vector $n_+ = (1, 0, \mathbf{0}_T)$ parallel to P , $n_- = (0, 1, \mathbf{0}_T)$ parallel to z , the superscripts a and b the color indices, the subscripts γ and β the Dirac indices, $N_c = 3$ the number of colors, and x the momentum fraction carried by the q and s quarks. The chiral enhancement scales m_0^q and m_0^s have been fixed by Eq. (7), whose explicit expressions are

$$\begin{aligned} m_0^q &\equiv \frac{m_{qq}^2}{2m_q} = \frac{1}{2m_q}\left[m_\eta^2\cos^2\phi + m_{\eta'}^2\sin^2\phi - \frac{\sqrt{2}f_s}{f_q}(m_{\eta'}^2 - m_\eta^2)\cos\phi\sin\phi\right], \\ m_0^s &\equiv \frac{m_{ss}^2}{2m_s} = \frac{1}{2m_s}\left[m_\eta^2\cos^2\phi + m_{\eta'}^2\sin^2\phi - \frac{f_q}{\sqrt{2}f_s}(m_{\eta'}^2 - m_\eta^2)\cos\phi\sin\phi\right], \end{aligned} \quad (23)$$

respectively, assuming the exact isospin symmetry $m_q \equiv m_u = m_d$ (For the inclusion of the isospin symmetry breaking effect, refer to [26]).

The distribution amplitudes $\phi_{q,s}^A$ are twist-2, and $\phi_{q,s}^P$ and $\phi_{q,s}^T$ twist-3. As explained in [27], both the twist-2 and twist-3 two-parton distribution amplitudes contribute at leading power in the analysis of exclusive B meson decays. We follow the parametrization for the pion distribution amplitudes proposed in [28]

$$\begin{aligned}\phi_{q(s)}^A(x) &= \frac{f_{q(s)}}{2\sqrt{2N_c}} 6x(1-x)[1 + a_2^{q(s)} C_2^{3/2}(2x-1)], \\ \phi_{q(s)}^P(x) &= \frac{f_{q(s)}}{2\sqrt{2N_c}} \left[1 + \left(30\eta_3 - \frac{5}{2}\rho_{q(s)}^2 \right) C_2^{1/2}(2x-1) - 3 \left\{ \eta_3\omega_3 + \frac{9}{20}\rho_{q(s)}^2(1 + 6a_2^{q(s)}) \right\} C_4^{1/2}(2x-1) \right], \\ \phi_{q(s)}^T(x) &= \frac{f_{q(s)}}{2\sqrt{2N_c}} (1-2x) \left[1 + 6 \left(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_{q(s)}^2 - \frac{3}{5}\rho_{q(s)}^2 a_2^{q(s)} \right) (1-10x+10x^2) \right],\end{aligned}\quad (24)$$

with the mass ratios $\rho_q = 2m_q/m_{qq}$ and $\rho_s = 2m_s/m_{ss}$, and the Gegenbauer polynomials

$$C_2^{1/2}(t) = \frac{1}{2}(3t^2 - 1), \quad C_2^{3/2}(t) = \frac{3}{2}(5t^2 - 1), \quad C_4^{1/2}(t) = \frac{1}{8}(3 - 30t^2 + 35t^4). \quad (25)$$

The values of the constant parameters a_2^q , η_3 , and ω_3 in Eq. (24) will be given in the next section.

The leading-twist gluonic distribution amplitudes of the η_q and η_s mesons are defined by [16]

$$\begin{aligned}\langle \eta_q(P) | A_{[\mu}^a(z) A_{\nu]}^b(0) | 0 \rangle &= \frac{\sqrt{2}f_q}{\sqrt{3}} \frac{C_F}{4\sqrt{3}} \frac{\delta^{ab}}{N_c^2 - 1} \epsilon_{\mu\nu\rho\sigma} \frac{n_-^\rho P^\sigma}{n_- \cdot P} \int_0^1 dx e^{ixP \cdot z} \frac{\phi_q^G(x)}{x(1-x)}, \\ \langle \eta_s(P) | A_{[\mu}^a(z) A_{\nu]}^b(0) | 0 \rangle &= \frac{f_s}{\sqrt{3}} \frac{C_F}{4\sqrt{3}} \frac{\delta^{ab}}{N_c^2 - 1} \epsilon_{\mu\nu\rho\sigma} \frac{n_-^\rho P^\sigma}{n_- \cdot P} \int_0^1 dx e^{ixP \cdot z} \frac{\phi_s^G(x)}{x(1-x)},\end{aligned}\quad (26)$$

with the notation $A_{[\mu}^a(z) A_{\nu]}^b(w) \equiv [A_{\mu}^a(z) A_{\nu}^b(w) - A_{\nu}^a(z) A_{\mu}^b(w)]/2$ and the function [14, 29–32],

$$\begin{aligned}\phi_{q(s)}^G(x) &= x^2(1-x)^2 B_2^{q(s)} C_1^{5/2}(2x-1), \\ C_1^{5/2}(t) &= 5t.\end{aligned}\quad (27)$$

According to the above definition, the gluon labeled by the subscript μ carries the fractional momentum xP . The two Gegenbauer coefficients B_2^q and B_2^s could be different in principle. However, due to the large uncertainty in their values, it is acceptable to assume $B_2^q = B_2^s \equiv B_2$. As shown later, the contribution from the above gluonic distribution amplitudes is smaller than that from the quark distribution amplitudes. Therefore, the subleading-twist gluonic distribution amplitudes of the $\eta_{q,s}$ meson will be dropped below.

The gluonic distribution amplitude of the η' meson defined in [33] is related to those in Eq. (26) through Eq. (1),

$$\begin{pmatrix} \langle \eta | A_{[\mu}^a(z) A_{\nu]}^b(0) | 0 \rangle \\ \langle \eta' | A_{[\mu}^a(z) A_{\nu]}^b(0) | 0 \rangle \end{pmatrix} = U(\phi) \begin{pmatrix} \langle \eta_q | A_{[\mu}^a(z) A_{\nu]}^b(0) | 0 \rangle \\ \langle \eta_s | A_{[\mu}^a(z) A_{\nu]}^b(0) | 0 \rangle \end{pmatrix}, \quad (28)$$

which also defines the gluonic distribution amplitude of the η meson. Besides, our parametrization of the gluonic distribution amplitudes are identical to those in [4] except

a different definition of B_2 : their Gegenbauer coefficient is 2/9 times of ours.

III. $B \rightarrow \eta^{(\prime)}$ FORM FACTORS

After defining the quark and gluonic distribution amplitudes of the η_q and η_s mesons, we are ready to calculate the $B \rightarrow \eta^{(\prime)}$ transition form factors at leading order of the strong coupling constant α_s . In the B meson rest frame, we choose the B meson momentum P_1 and the $\eta^{(\prime)}$ meson momentum P_2 in the light-cone coordinates:

$$P_1 = \frac{m_B}{\sqrt{2}} (1, 1, \mathbf{0}_T), \quad P_2 = \frac{m_B}{\sqrt{2}} (\rho, 0, \mathbf{0}_T), \quad (29)$$

where the energy fraction ρ carried by the $\eta^{(\prime)}$ meson is related to the lepton-pair momentum $q = P_1 - P_2$ via $q^2 = (1 - \rho)m_B^2$, m_B being the B meson mass. The $\eta^{(\prime)}$ meson mass, appearing only in power-suppressed terms, has been neglected. The spectator momenta k_1 on the B meson side and k_2 on the $\eta^{(\prime)}$ meson side are parametrized as

$$k_1 = \left(0, x_1 \frac{m_B}{\sqrt{2}}, \mathbf{k}_{1T} \right), \quad k_2 = \left(x_2 \rho \frac{m_B}{\sqrt{2}}, 0, \mathbf{k}_{2T} \right), \quad (30)$$

x_1 and x_2 being the parton momentum fractions, and \mathbf{k}_{1T} and \mathbf{k}_{2T} the parton transverse momenta.

We first compute the $B \rightarrow \eta_{q(s)}$ form factors defined by the local matrix elements,

$$\langle \eta_{q(s)}(P_2) | \bar{b}(0) \gamma_\mu u(0) | B(P_1) \rangle = F_+^{B\eta_{q(s)}}(q^2) \left[(P_1 + P_2)_\mu - \frac{m_B^2 - m_{\eta_{q(s)}}^2}{q^2} q_\mu \right] + F_0^{B\eta_{q(s)}}(q^2) \frac{m_B^2 - m_{\eta_{q(s)}}^2}{q^2} q_\mu, \quad (31)$$

$$\langle \eta_{q(s)}(P_2) | \bar{b}(0) i \sigma^{\mu\nu} q_\nu d(0) | B(P_1) \rangle = \frac{F_T^{B\eta_{q(s)}}(q^2)}{m_B + m_{\eta_{q(s)}}} [(m_B^2 - m_{\eta_{q(s)}}^2) q^\mu - q^2 (P_1^\mu + P_2^\mu)],$$

where the $\eta_{q(s)}$ meson mass $m_{\eta_{q(s)}}$ will be set to zero eventually. The form factors $F_{+,0}$ are associated with the semileptonic decay $B \rightarrow \eta^{(\prime)} l \nu$, and F_T with $B \rightarrow \eta^{(\prime)} l^+ l^-$. For the involved $\bar{b} \rightarrow \bar{u}, \bar{d}$ transitions, the above form factors are decomposed into

$$F_{+(0,T)}^{B\eta_q} = F_{q+(0,T)}^{B\eta_q} + F_{g+(0,T)}^{B\eta_q}, \quad F_{+(0,T)}^{B\eta_s} = F_{g+(0,T)}^{B\eta_s}. \quad (32)$$

That is, the η_s meson state contributes only through the flavor-singlet pieces $F_{g+,g0,gT}^{B\eta_s}$. The $B \rightarrow \eta^{(\prime)}$ form factors are then obtained from the mixing,

$$\begin{pmatrix} F_{+(0,T)}^{B\eta} \\ F_{+(0,T)}^{B\eta'} \end{pmatrix} = U(\phi) \begin{pmatrix} F_{+(0,T)}^{B\eta_q} \\ F_{+(0,T)}^{B\eta_s} \end{pmatrix}. \quad (33)$$

It is then expected that the gluonic contribution is more significant in the $B \rightarrow \eta'$ form factors than in the $B \rightarrow \eta$ ones, since those from the η_q and η_s mesons add up in the former, but partially cancel in the latter.

The factorization formulas for $F_{q+,q0,qT}^{B\eta_q}$ are similar to those for the $B \rightarrow \pi$ form factors [27,34]:

$$\begin{aligned} F_{q+}^{B\eta_q}(q^2) &= \frac{8}{\sqrt{2}} \pi m_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \left\{ \left[(1 + x_2 \rho) \phi_q^A(x_2) + r_q \left(\frac{2}{\rho} - 1 - 2x_2 \right) \phi_q^T(x_2) \right. \right. \\ &\quad \left. \left. + r_q (1 - 2x_2) \phi_q^P(x_2) \right] E(t^{(1)}) h(x_1, x_2, b_1, b_2) + 2r_q \phi_q^P E(t^{(2)}) h(x_2, x_1, b_2, b_1) \right\}, \\ F_{q0}^{B\eta_q}(q^2) &= \frac{8}{\sqrt{2}} \pi m_B^2 C_F \rho \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \left\{ \left[(1 + x_2 \rho) \phi_q^A(x_2) + r_q (1 - 2x_2) \phi_q^T(x_2) \right. \right. \\ &\quad \left. \left. + r_q \left(\frac{2}{\rho} - 1 - 2x_2 \right) \phi_q^P(x_2) \right] E(t^{(1)}) h(x_1, x_2, b_1, b_2) + 2r_q \phi_q^P E(t^{(2)}) h(x_2, x_1, b_2, b_1) \right\}, \\ F_{qT}^{B\eta_q}(q^2) &= \frac{8}{\sqrt{2}} \pi m_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \left\{ \left[\phi_q^A(x_2) + r_q \left(\frac{2}{\rho} + x_2 \right) \phi_q^T(x_2) - r_q x_2 \phi_q^P(x_2) \right] \right. \\ &\quad \left. \times E(t^{(1)}) h(x_1, x_2, b_1, b_2) + 2r_q \phi_q^P E(t^{(2)}) h(x_2, x_1, b_2, b_1) \right\}, \end{aligned} \quad (34)$$

with the color factor $C_F = 4/3$, the B meson wave function ϕ_B , the impact parameter b_1 (b_2) conjugate to the parton transverse momentum k_{1T} (k_{2T}), the mass ratio $r_q = m_0^q/m_B$, the hard function h , and the evolution factor,

$$E(t) = \alpha_s(t) e^{-S_B(t) - S_{\eta_q}(t)}. \quad (35)$$

The choice of the hard scale t , and the explicit expressions for h , for the Sudakov exponent S_B associated with the B meson, and for the Sudakov exponent S_{η_q} associated with a light meson are referred to [27]. The threshold resummation factor associated with the hard function is the same as in the $B \rightarrow \pi$ form factors [35]. The coefficient $1/\sqrt{2}$ appears, because only the u or d quark component of the η_q meson is involved. We point out that the term proportional to $2/\rho$ in $F_{qT}^{B\eta_q}$ is missed in [36].

For the gluonic contribution, it has been argued [4] that the diagram in Fig. 1 with the two gluons emitted from the

light spectator quark of the B meson is leading. Another diagram with the two gluons crossed, giving the identical contribution, is not displayed. The third diagram vanishes, in which the virtual gluon from the u and \bar{u} quark annihilation couples the two valence gluons in the $\eta^{(\prime)}$ meson. The flavor-singlet pieces in the $B \rightarrow \eta_q$ form factors are written as

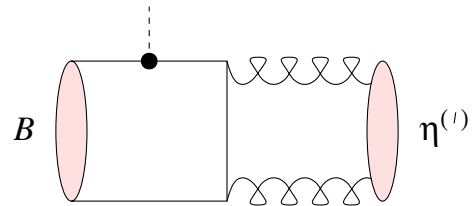


FIG. 1 (color online). Gluonic contribution to the $B \rightarrow \eta^{(\prime)}$ form factors. Another diagram with the two gluons crossed is suppressed.

$$\begin{aligned}
 F_{g^+}^{B\eta_q}(q^2) &= -\frac{32}{3}\pi m_B^2 f_q \frac{C_F^2 N_c \sqrt{N_c}}{N_c^2 - 1} \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \frac{\phi^G(x_2)}{x_2(1-x_2)} x_1 [1 + (1-\rho)x_2] \\
 &\quad \times E(t^2) h(x_2, x_1, b_2, b_1), \\
 F_{g^0}^{B\eta_q}(q^2) &= -\frac{32}{3}\pi m_B^2 f_q \frac{C_F^2 N_c \sqrt{N_c}}{N_c^2 - 1} \rho \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \frac{\phi^G(x_2)}{x_2(1-x_2)} x_1 [1 - (1-\rho)x_2] \\
 &\quad \times E(t^2) h(x_2, x_1, b_2, b_1), \\
 F_{g^T}^{B\eta_q}(q^2) &= -\frac{32}{3}\pi m_B^2 f_q \frac{C_F^2 N_c \sqrt{N_c}}{N_c^2 - 1} \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \frac{\phi^G(x_2)}{x_2(1-x_2)} x_1 (1+x_2) E(t^2) h(x_2, x_1, b_2, b_1).
 \end{aligned} \tag{36}$$

A factor 2 has been included, which is attributed to the identical contribution from the second diagram mentioned above. The calculation is similar to that of the $\eta' g^* g^{(*)}$ vertex in [37,38]. The expressions for $F_{g^+,g^0,g^T}^{B\eta_s}$ are the same as in Eq. (36), but with f_q being replaced by $f_s/\sqrt{2}$. It is observed that Eq. (36) is proportional to the small momentum fraction $x_1 \sim \Lambda/m_B$ [39], Λ being a hadronic scale, compared to the quark contribution. However, the overall coefficient of the former is larger by a factor $16C_F N_c^2/[3(N_c^2 - 1)] = 8$, which might compensate the smallness of x_1 , such that the gluonic contribution is sizable. Because the above factorization formulas do not develop the end point singularities, if removing k_T , the gluonic contribution can in fact be computed in collinear factorization theorem.

The evolution factor in Eq. (36) is given by

$$E(t) = \alpha_s(t) e^{-S_B(t) - S_G(t)}, \tag{37}$$

where the Sudakov exponent S_G is associated with the gluonic distribution amplitudes of the $\eta_{q,s}$ mesons. Following the studies in [40], its expression is, up to the leading-logarithm accuracy, similar to S_{η_q} , but with the anomalous dimension $\alpha_s C_F/\pi$ being replaced by $\alpha_s C_A/\pi$, $C_A = 3$ being a color factor:

$$\begin{aligned}
 S_G(t) &= s_G(x_2 P_2^+, b_2) + s_G((1-x_2)P_2^+, b_2), \\
 s_G(Q, b) &= \int_{1/b}^Q \frac{d\mu}{\mu} \left[\ln\left(\frac{Q}{\mu}\right) A(\alpha_s(\mu)) \right], \quad A = \frac{\alpha_s}{\pi} C_A.
 \end{aligned} \tag{38}$$

That is, the Sudakov suppression is stronger in the gluonic distribution amplitudes than in the quark ones. There is no point to include the next-to-leading-logarithm resummation, since the gluonic distribution amplitudes are not very certain yet. For consistency, we neglect the single-logarithm renormalization-group summation governed by the anomalous dimension of the gluon wave function. We adopt the one-loop expression of the running coupling constant α_s , when evaluating the above Sudakov factors.

We then perform a detailed numerical analysis, including that of theoretical uncertainties [41]. The B meson wave function is the same as in [42] with the shape

parameter varied between $\omega_B = (0.40 \pm 0.04)$ GeV. There is another B meson wave function in the heavy-quark limit [43], whose contribution to transition form factors may be finite. However, it has been shown that its effect can be well mimicked by a single B meson wave function, if a suitable ω_B is chosen [41]. Therefore, we adopt this approximation, and vary ω_B in the above range. To obtain the chiral enhancement scale m_0^q , we need the masses $m_\eta = 0.548$ GeV and $m_{\eta'} = 0.958$ GeV, and the inputs in Eq. (20). Because meson distribution amplitudes are defined at 1 GeV, we take the light quark mass $m_q(1 \text{ GeV}) = (5.6 \pm 1.6)$ MeV [44]. We have confirmed that the twist-2 and twist-3 quark distribution amplitudes of the η' meson in [33] are consistent with Eqs. (16) and (22), if their η' meson decay constant $f_{\eta'}$ is regarded as $f_{\eta'}^1$ in the singlet-octet basis. Hence, it is legitimate to adopt $a_2^q = -0.008 \pm 0.054$ extracted from the relevant data [16] for the twist-2 distribution amplitude. The twist-3 distribution amplitudes have not yet been constrained experimentally, so we choose $\eta_3 = 0.015$ and $\omega_3 = -3$ the same as for the pion distribution amplitudes [28]. It is not necessary to specify the parameters involved in the quark distribution amplitudes of the η_s meson here. The overall coefficients in Eq. (26) have been arranged in the way that the decay constants in [33] and in Eq. (26) satisfy Eq. (13). Following Eq. (28), the range of $B_2 = 4.6 \pm 2.5$ extracted in [16] can also be adopted directly. The theoretical uncertainty arising from the variation of the above parameters will be investigated.

The following parametrization for the $B \rightarrow \eta^{(\prime)}$ form factors was proposed in [4],

$$F_0^{B\eta^{(\prime)}}(0) = F_1 \frac{f_{\eta^{(\prime)}}^q}{\sqrt{2}f_\pi} + F_2 \frac{\sqrt{2}f_{\eta^{(\prime)}}^q + f_{\eta^{(\prime)}}^s}{\sqrt{6}f_\pi}, \tag{39}$$

where F_1 (F_2) corresponds to the quark (gluonic) contribution, and a factor $1/\sqrt{2}$ is included to match our convention. F_1 has been set to the $B \rightarrow \pi$ form factor, and the unknown F_2 varied arbitrarily between [0, 0.1] [4]. The above parametrization also applies to the other form factors F_+ and F_T . It is easy to identify, from Eq. (33), the relations of $F_{1,2}$ to our form factors

TABLE I. Form-factor values at maximal recoil, the ratios of their gluonic contributions, and values of $F_{1,2}$ defined in Eq. (39).

	$F_{+,0}^{B\eta}$	$F_T^{B\eta}$	$F_{+,0}^{B\eta'}$	$F_T^{B\eta'}$
$F(0)$	0.153	0.143	0.155	0.142
Ratio	0.036	0.032	0.22	0.20
F_1	0.252	0.236	0.252	0.236
F_2	0.044	0.035	0.044	0.035

$$F_1 = \frac{\sqrt{2}f_\pi}{f_q} F_q^{B\eta_q}(0), \quad F_2 = \frac{\sqrt{3}f_\pi}{f_q} F_g^{B\eta_q}(0). \quad (40)$$

To have a picture of the magnitude of the gluonic contribution, we present in Table I the central values of the form factors at maximal recoil, the ratios of their gluonic contributions, and the values of $F_{1,2}$.

It is indicated that the gluonic contribution is below 5% in the $B \rightarrow \eta$ form factors, and about 20% in the $B \rightarrow \eta'$ ones. The central value of F_1 for the form factors $F_{+,0}^{B\eta^{(\prime)}}$ is indeed close to the $B \rightarrow \pi$ form factors $F_{+,0}^{B\pi^{(\prime)}}$ [27]. The central value of $F_2 = 0.044$ for $F_{+,0}^{B\eta^{(\prime)}}$ is located within [0, 0.1] [4] (to be precise, F_2 runs between [0.018, 0.075]), considering the ranges of the parameters ω_B and B_2). However, we stress that $F_2 = 0.044$ corresponds to the Gegenbauer coefficient $B_2 = 4.6$ of the gluonic distribution amplitudes. If using $B_2 = 9 \pm 12$ as in [4], the central value of F_2 will be lifted to 0.09. Hence, for self-consistency, either F_2 should be varied in a wider range

[0, 0.2], or a lower B_2 should be adopted to evaluate the other flavor-singlet contributions to the $B \rightarrow \eta^{(\prime)}K$ decays in [4]. We notice the difference between F_2 for $F_{+,0}^{B\eta^{(\prime)}}$ and F_2 for $F_T^{B\eta^{(\prime)}}$ (0), which is obvious from the factorization formulas in Eq. (36): $F_{gT}^{B\eta_q}(0)$ contains an additional term proportional to x_2 compared with $F_{g+,g0}^{B\eta_q}(0)$. Our observation is thus contrary to the assumption $F_{g+,g0}^{B\eta_q}(0) = F_{gT}^{B\eta_q}(0)$ postulated in the analysis of the $B \rightarrow \eta^{(\prime)}l^+l^-$ decays [5]. Nevertheless, this difference is not crucial for the estimate in [5], because the gluonic contribution is not dominant.

The q^2 dependence of the $B \rightarrow \eta$ and $B \rightarrow \eta'$ form factors corresponding to the central values of the inputs are displayed in Figs. 2 and 3, respectively. It is found that the gluonic contribution remains negligible in the $B \rightarrow \eta$ form factors, but sizable in the $B \rightarrow \eta'$ ones in the whole large-recoil region. The form factors $F_0^{B\eta^{(\prime)}}(q^2)$ exhibit a smaller slope with q^2 , because the overall coefficients of $F_{q0,g0}^{B\eta_q}$ contain the energy fraction ρ as shown in Eqs. (34) and (36), consistent with the large-energy form-factor relation in [45]. The $B \rightarrow \eta$ and $B \rightarrow \eta'$ form factors are approximately equal, since the former are proportional to $\cos\phi F_q^{B\eta_q}$, and the latter to $\sin\phi F_q^{B\eta_q}$ plus some amount of gluonic contributions. Without the gluonic contribution, we shall have $F^{B\eta}(q^2) > F^{B\eta'}(q^2)$, simply due to $\cos\phi > \sin\phi$ for $\phi \approx 39.3^\circ$. Therefore, the measurement of the semileptonic branching ratios alone can provide the information on the importance of the gluonic contribution.

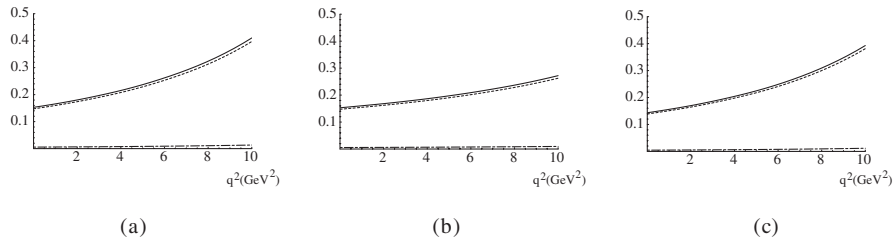
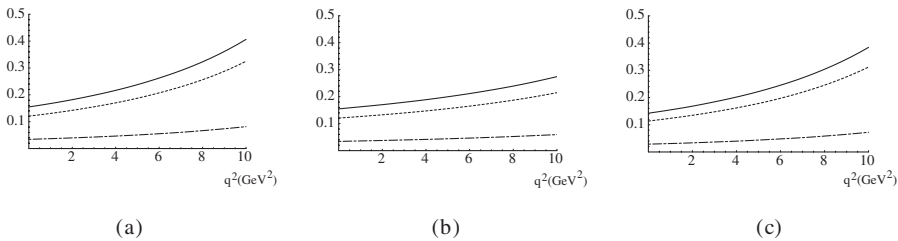

 FIG. 2. q^2 dependence of (a) $F_{+,0}^{B\eta}$, (b) $F_0^{B\eta}$, and (c) $F_T^{B\eta}$ corresponding to the central values of the inputs with the dotted, dash-dotted, and solid lines representing the quark, gluonic, and total contributions, respectively.

 FIG. 3. q^2 dependence of (a) $F_{+,0}^{B\eta'}$, (b) $F_0^{B\eta'}$, and (c) $F_T^{B\eta'}$ corresponding to the central values of the inputs with the dotted, dash-dotted, and solid lines representing the quark, gluonic, and total contributions, respectively.

TABLE II. Theoretical uncertainty of the form factors at maximal recoil from each of the parameters.

	$F_{+,0}^{B\eta}(0)$	$F_T^{B\eta}(0)$	$F_{+,0}^{B\eta'}(0)$	$F_T^{B\eta'}(0)$
$\omega_B = (0.36-0.44)$	0.179–0.131	0.168–0.123	0.179–0.135	0.165–0.124
$f_q = (1.05-1.09)f_\pi$	0.062–0.244	0.053–0.234	0.080–0.229	0.068–0.216
$f_s = (1.28-1.40)f_\pi$	0.364–(–0.059)	0.354–(–0.066)	0.327–(–0.017)	0.313–(–0.028)
$\phi = (38.3-40.3)$	0.098–0.212	0.089–0.203	0.107–0.210	0.095–0.197
$m_q = (4.0-7.2)$	0.196–0.129	0.186–0.120	0.190–0.135	0.177–0.123
$a_2^q = (-0.062)-0.046$	0.149–0.157	0.140–0.147	0.152–0.158	0.140–0.145
$B_2 = 2.1-7.1$	0.150–0.156	0.141–0.146	0.136–0.173	0.127–0.158

However, the current data [46],

$$\begin{aligned}
 B(B^+ \rightarrow \eta l^+ \nu_l) &= (0.84 \pm 0.27 \pm 0.21) \times 10^{-4} \\
 &< 1.4 \times 10^{-4} (90\% \text{C.L.}), \\
 B(B^+ \rightarrow \eta' l^+ \nu_l) &= (0.33 \pm 0.60 \pm 0.30) \times 10^{-4} \\
 &< 1.3 \times 10^{-4} (90\% \text{C.L.}),
 \end{aligned} \tag{41}$$

are not yet precise enough for the purpose. Figure 3 also indicates that the q^2 dependence of the gluonic contribution is weaker than that of the quark contribution. Hence, the assumption [5,12] that both pieces show the same q^2 dependence is not appropriate.

We then investigate the theoretical uncertainty for the $B \rightarrow \eta^{(\prime)}$ form factors from each of the input parameters, and the results are presented in Table II. The form factors $F_{q^+,q^0,q^T}^{B\eta_q}$ in Eq. (34) receive substantial contributions from the twist-3 distribution amplitudes ϕ_q^P and ϕ_q^T , which

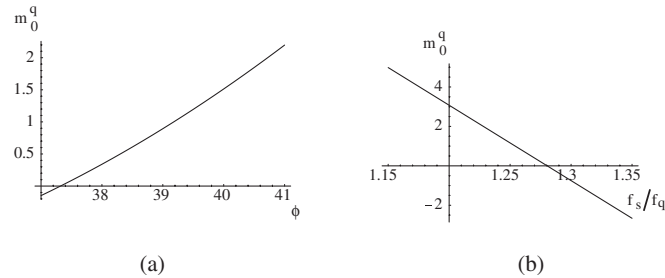


FIG. 4. Dependence of m_0^q (a) on ϕ in units of degrees, and (b) on f_s/f_q .

appear together with the mass ratio $r_q = m_0^q/m_B$. The chiral enhancement scale m_0^q changes rapidly with the mixing angle ϕ and with the decay constants $f_{q,s}$ as illustrated in Fig. 4. This explains the sensitivity of the form factors to these inputs. We notice that m_0^q runs into the unrealistic negative region easily, as increasing f_s , leading to the negative form factor values in Table II. Therefore, we prefer not to vary f_s . The dependence of the form factors on the Gegenbauer coefficients a_2^q and B_2 is weak, because a_2^q is close to zero and the gluonic contribution is subdominant. The variation of the ratios of the gluonic contributions in the $B \rightarrow \eta^{(\prime)}$ form factors with each of the parameters is listed in Table III. Ignoring the effect from changing f_s for the reason stated above, we conclude that the gluonic contribution is negligible, always below 10%, in the $B \rightarrow \eta$ transitions, and sizable, ranging between 10%–40%, in $B \rightarrow \eta'$.

At last, we discuss the impact of the gluonic contribution in the $B \rightarrow \eta^{(\prime)}$ form factors on the predictions for the $B \rightarrow \eta^{(\prime)}K$ branching ratios in QCDF and in PQCD. The current data of the branching ratios are summarized below [17]:

$$\begin{aligned}
 B(B^\pm \rightarrow \eta' K^\pm) &= (69.7_{-2.8}^{+2.8}) \times 10^{-6}, \\
 B(B^0 \rightarrow \eta' K^0) &= (64.9 \pm 3.5) \times 10^{-6}, \\
 B(B^\pm \rightarrow \eta K^\pm) &= (2.2 \pm 0.3) \times 10^{-6}, \\
 B(B^0 \rightarrow \eta K^0) &< 1.9 \times 10^{-6}.
 \end{aligned} \tag{42}$$

Table 4 in the QCDF analysis [4] shows the dependence of the $B \rightarrow \eta^{(\prime)}K$ branching ratios on the gluonic contribution F_2 : $B(B \rightarrow \eta'K)$ increases with F_2 , but $B(B \rightarrow \eta K)$ de-

TABLE III. Theoretical uncertainty of the ratios of the gluonic contributions in the form factors at maximal recoil from each of the parameters.

	$F_{+,0}^{B\eta}(0)$	$F_T^{B\eta}(0)$	$F_{+,0}^{B\eta'}(0)$	$F_T^{B\eta'}(0)$
$\omega_B = (0.36-0.44)$	0.033–0.039	0.029–0.035	0.21–0.23	0.19–0.22
$f_q = (1.05-1.09)f_\pi$	0.083–0.024	0.081–0.021	0.42–0.15	0.41–0.13
$f_s = (1.28-1.40)f_\pi$	0.017–(–0.083)	0.014–(–0.06)	0.10–(–2.1)	0.089–(–1.0)
$\phi = (38.3-40.3)$	0.062–0.023	0.058–0.020	0.32–0.16	0.30–0.15
$m_q = (4.0-7.2)$	0.028–0.043	0.025–0.039	0.18–0.25	0.16–0.23
$a_2^q = (-0.062)-0.046$	0.037–0.035	0.033–0.032	0.224–0.217	0.21–0.20
$B_2 = 2.1-7.1$	0.017–0.055	0.015–0.049	0.11–0.30	0.10–0.28

creases. The reason is as follows. The gluonic contribution enhances the $B \rightarrow \eta_q K$ amplitude (containing the $B \rightarrow \eta_q$ transition), such that its cancellation with the $B \rightarrow K \eta_s$ amplitude (containing the $B \rightarrow K$ transition) becomes more exact in the $B \rightarrow \eta K$ decays. However, the above two amplitudes are constructive in the $B \rightarrow \eta' K$ decays, whose branching ratios then exhibit an opposite behavior with the gluonic contribution. Table 4 in [4] also shows the predictions from the default scenario of inputs (with the strange quark mass $m_s = 100$ MeV), $B(B^\pm \rightarrow \eta' K^\pm) \approx 42 \times 10^{-6}$, and $B(B^\pm \rightarrow \eta K^\pm) \approx 1.7 \times 10^{-6}$ for $F_2 = 0$, both of which fall short compared to the data. Enlarging F_2 , the predicted $B(B^\pm \rightarrow \eta' K^\pm)$ increases, but $B(B^\pm \rightarrow \eta K^\pm)$ decreases and deviates more from the measured value. That is, there is not much room for the gluonic contribution to play in QCDF. Nevertheless, a smaller $m_s = 80$ MeV does help, since it lifts both $B(B^\pm \rightarrow \eta' K^\pm)$ and $B(B^\pm \rightarrow \eta K^\pm)$ as demonstrated in Table 4 of [4].

We have stated that the flavor-singlet amplitudes were not taken into account in the PQCD study of the $B \rightarrow \eta^{(\prime)} K$ decays [20]. The predictions $B(B^0 \rightarrow \eta' K^0) \approx 45 \times 10^{-6}$ and $B(B^0 \rightarrow \eta K^0) \approx 4.6 \times 10^{-6}$ were obtained for $m_s = 100$ MeV and for the chiral enhancement scale $m_0^q = 1.4$ GeV (see Table 2 of [20]), which is within our parameter range for m_0^q as displayed in Fig. 4. Because of the dynamical enhancement of penguin contributions [8,47], both the above branching ratios are larger than those in QCDF from the default scenario [4]. Comparing the PQCD predictions with the data in Eq. (42), and knowing their dependence on F_2 , there is more room for the gluonic contribution to play apparently. Since the other flavor-singlet contributions are negligible [4], we obtain the naive PQCD estimate for the $B \rightarrow \eta^{(\prime)} K$ branching ratios by adding the gluonic contribution into the amplitudes F_e^l in [20], that involve the $B \rightarrow \eta^{(\prime)}$ form factors. The maximal gluonic contribution in Table II, i.e., 40% (8%) in the $B \rightarrow \eta^{(\prime)}$ form factor, lead to the improved branching ratios

$$\begin{aligned} B(B^0 \rightarrow \eta' K^0) &\approx 61 \times 10^{-6}, \\ B(B^0 \rightarrow \eta K^0) &\approx 2 \times 10^{-6}. \end{aligned} \quad (43)$$

Note that we have input a smaller $B \rightarrow K$ form factor $F_0^{BK} = 0.43$ [48] corresponding to the updated kaon distribution amplitudes (vs. $F_0^{BK} = 0.47$ in [20]). The complete PQCD analysis of the $B \rightarrow \eta^{(\prime)} K$ decays including all flavor-singlet amplitudes will be published elsewhere.

Before concluding, we mention that the form factor $F_T^{B\eta}(0) = 0.16 \pm 0.03$ has been derived from light-cone QCD sum rules [49], which, however, did not include the two-parton twist-3 and gluonic contributions. Instead, the three-parton η meson distribution amplitudes were taken

into account. Even so, their result is in agreement with ours in Tables I and II. The form factors $F_+^{B\eta_q}(0) = -0.023 \pm 0.048(0.045 \pm 0.086)$ and $F_+^{B\eta_s}(0) = -0.099 \pm 0.024 (-0.066 \pm 0.043)$ [50] have been extracted by fitting parametrization in the soft-collinear effective theory [51] to the data of two-body nonleptonic B meson decays, where the numbers in the parentheses represent the second solution of the fitting. These values, after considering the large uncertainties, still differ from our observations dramatically, $F_+^{B\eta_q}(0) = 0.217$ and $F_+^{B\eta_s}(0) = 0.024$ (notice the opposite sign). Our $F_+^{B\eta_s}(0)$, containing only the gluonic contribution at leading order of α_s , is much smaller than $F_+^{B\eta_q}(0)$.

IV. CONCLUSION

In this paper we have calculated the gluonic contribution to the $B \rightarrow \eta^{(\prime)}$ transition form factors in the large-recoil region using the PQCD approach. The quark-flavor and singlet-octet schemes for the η - η' mixing were compared, and it was found that fewer two-parton twist-3 distribution amplitudes could be introduced in the former. The leading-twist quark and gluonic distribution amplitudes of the η_q and η_s mesons in the quark-flavor basis were constrained experimentally. The parameters involved in the two-parton twist-3 quark distribution amplitudes were determined either by equations of motion associated with the η - η' mixing, or taken the same as in the pion distribution amplitudes from QCD sum rules. Therefore, we are able to predict the unknown gluonic contribution with less theoretical uncertainty. It has been shown that this contribution is negligible (around 5%) in the $B \rightarrow \eta$ form factors, but reaches 10%–40% in the $B \rightarrow \eta'$ ones. These predictions can be confronted with the future measurement of the $B \rightarrow \eta^{(\prime)} l \nu$ decay spectra. We have elaborated the impact of the gluonic contribution on the $B \rightarrow \eta^{(\prime)} K$ branching ratios obtained in QCDF and in PQCD. The observation is that the gluonic contribution does not help accommodating the measured $B \rightarrow \eta^{(\prime)} K$ branching ratios in QCDF, but does in PQCD. With the maximal gluonic contribution in the $B \rightarrow \eta^{(\prime)}$ form factors, the naive PQCD estimate in Eq. (43) is consistent with the data.

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