Muon pair creation from positronium in a circularly polarized laser field

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(Received 16 June 2006; published 23 October 2006)

We study elementary particle reactions that result from the interaction of an atomic system with a very intense laser wave of circular polarization. As a specific example, we calculate the rate for the laser-driven reaction $e^+e^- \to \mu^+\mu^-$, where the electron and positron originate from a positronium atom or, alternatively, from a nonrelativistic e^+e^- plasma. We distinguish accordingly between the coherent and incoherent channels of the process. Apart from numerical calculations, we derive by analytical means compact formulas for the corresponding reaction rates. The rate for the coherent channel in a laser field of circular polarization is shown to be damped because of the destructive interference of the partial waves that constitute the positronium ground-state wave packet. Conditions for the observation of the process via the dominant incoherent channel in a circularly polarized field are pointed out.

DOI: 10.1103/PhysRevD.74.074017 PACS numbers: 13.66.De, 36.10.Dr, 41.75.Jv

I. INTRODUCTION

The interaction of electrons and atoms with laser radiation is intensively and successfully being studied for many years now. However, due to a rapid technological progress, the high-power laser systems available today can generate peak intensities up to 10^{22} W/cm^2 in the range of near-optical infrared frequencies [1], and a further increase can be expected within the next few years [2]. Consequently, the ponderomotive energy of an e^- (or e^+) inside such a laser wave is of the order of 1 GeV, which is far beyond the typical energetic range of atomic physics but rather reaches the energy scale characteristic for elementary particle physics. If electrons or an e^- and e^+ collide at such high energies, then particle reactions like heavy lepton-pair creation or hadron production can occur. This indicates that there might be a way to merge laser physics with high-energy physics [3,4]. Similar efforts are being undertaken with respect to laser physics and nuclear physics [5–8].

The high-energy process of photon-induced e^+e^- pair creation by a projectile particle colliding with an intense laser beam has already been investigated before, both experimentally [9] and theoretically [10,11]. An essential ingredient to these studies is the ultrarelativistic energy of the incoming particle. In its rest frame, the doppler-shifted laser frequency and field strength are considerably enhanced. As a consequence, the projectile actually faces an x-ray beam of near-critical intensity.

Instead, in the present paper we study a situation where elementary particle reactions arise from the interaction of a strong laser field with a nonrelativistic atomic system. To this end, we suppose that a positronium (Ps) atom is brought into an intense laser wave. We note that the lifetimes of ortho-Ps ($\sim 10^{-7}$s) and para-Ps ($\sim 10^{-10}$ s) are much longer than the typical duration of a strong laser pulse. Because of the equal masses of its constituents, the dynamical response of the positronium to the electromagnetic forces exerted by the laser field is rather unique [12]: The laser's linearly polarized electric field leads to an antiparallel oscillatory motion of the particles in the transverse direction, while the magnetic Lorentz force causes an identical ponderomotive drift motion along the laser propagation direction. This leads to periodic e^+e^- (re)collisions (see, in particular, Fig. 1 in Ref. [12]). If the energy of the relative e^+e^- motion is large enough, then in these *coherent* collisions [13] particle reactions can occur. Thus, we shall study high-energy processes induced by e^+e^- annihilation resulting from a laser-driven Ps atom. Considering the case of a circularly polarized laser field we will find as a main result, however, that the various partial waves that constitute the Ps ground-state interfere destructively, which causes a heavy suppression of the coherent reaction rate. This quantum effect can be related to the classical trajectories of the colliding particles in the laser field. Furthermore, when the characteristic size of these trajectories (or the size of the spreading particle wave packets) exceeds the interatomic distance, then collisions between particles originating from different Ps atoms will come into play, which opens the incoherent channel of the process. Surprisingly it turns out that, in a circularly polarized laser field, the incoherent channel is dominant as the interference in the coherent channel is destructive. In order to study the incoherent process we replace the Ps atom by a nonrelativistic e^+e^- plasma. In this situation, exclusively incoherent e^+e^- collisions occur.

It should be stressed that in the described setup the e^+e^- collision energy is basically determined by the kinetic energy $\sim mc^2\xi$ contained in the transversal motion of the particles, which is considerably smaller than the ponderomotive energy $\sim mc^2\xi^2$ mentioned above. Here, mc^2 is the electron rest energy and $\xi = ea/mc^2$ denotes the so-called laser intensity parameter with the electron charge $-e$ and the laser's vector potential a. For the highest intensities attainable at present ξ is of order 10^2. In this respect, the underlying laser acceleration of the particles is considerably different from the usual laser acceleration techniques, since the latter try to extract the ponderomotive energy gain along the laser propagation direction. Nevertheless,

the energetic thresholds for muon or pion production might be within reach. We further notice, that the principal difficulties of laser acceleration implied by the Lawson-Woodward theorem (see, e.g., Ref. [14]) are completely absent here since the e^+ and e^- collide *inside* the laser wave.

Against this background, we consider the specific process $e^+e^- \to \mu^+\mu^-$ which is one of the most fundamental in high-energy physics. Its cross section sets the scale for all e^+e^- annihilation cross sections [15]. For example, at high energies one has $\sigma_{e^+e^-\to\text{hadrons}} \approx 4\sigma_{e^+e^-\to\mu^+\mu^-}$, where $\sigma_{e^+e^-\to\text{hadrons}}$ denotes the total cross section for the production of any number of strongly interacting particles [16]. The threshold energy for the reaction $e^+e^- \to \mu^+\mu^-$ amounts to $2Mc^2$ in the field-free case, where M denotes the muon mass. According to the above, a naive estimate thus suggests that a laser intensity corresponding to $\xi \approx M/m \approx 200$ is required to produce a muon pair in a laser-driven e^+e^- collision. This value is reached, e.g., for a linearly polarized laser beam of 3.8×10^{22} W/cm^2 intensity and 1 eV photon energy.

To the best of our knowledge, the process $e^+e^- \to \mu^+\mu^-$ in a laser field has not been considered before. The most closely related article treats the laser-assisted Bhabha scattering $e^+e^- \to e^+e^-$ [17]. In Ref. [17] the low-intensity case (i.e., $\xi \ll 1$) is analyzed in detail with the emphasis lying on the resonances that can occur in the scattering cross section due to the interaction of the leptons with the background laser field. We will come back to this point later. Another similar process, that has found the interest of several authors, is the Møller scattering $e^-e^- \to e^-e^-$ in a laser field (see [18–21] and references therein).

The paper is organized as follows. In Sec. II we develop a formalism that allows us to calculate the rate for the reaction $\text{Ps} \to \mu^+\mu^-$ in a strong laser field. Our treatment will be based upon the Volkov solutions to the Dirac equation. Afterwards we analyze in detail the reaction kinematics. Here we show, in particular, that the minimal laser intensity parameter required is indeed given by $\xi_{\text{min}} = M/m$ [cf. Eq. (36)]. Further, the kinematical analysis will help us to derive a compact formula that gives an approximation to the total reaction rate and displays its main dependences [cf. Eq. (51)]. In Sec. III we present our (numerical) results on the total and differential production rates and compare them with the known cross section for the field-free process $e^+e^- \to \mu^+\mu^-$. Furthermore, we briefly consider the related process of muon pair production by a superstrong laser wave interacting with a nonrelativistic e^+e^- plasma. In this situation the interference effect does not play a role. We finish with a conclusion.

II. THEORETICAL FRAMEWORK

A. Transition amplitude and reaction rate

We calculate the rate for positronium decay into muons in a strong laser field, i.e., the rate for the laser-driven process $\text{Ps} \to \mu^+\mu^-$. We assume a photon energy of about 1 eV and a laser intensity parameter of order $M/m \sim 200$ or larger [22]. For mathematical simplicity, the laser field is taken to be a monochromatic, plane wave of circular polarization with the classical four-potential [23]

$$A^\mu(x) = a_1^\mu \cos(kx) + a_2^\mu \sin(kx). \tag{1}$$

As usual, A^μ is assumed to be adiabatically switched on and off in the remote past and the distant future, respectively. In Eq. (1), $k^\mu = \omega(1, 0, 0, 1)$ is the wave four-vector and $a_{1,2}^\mu$ are constant four-vectors chosen as $a_1^\mu = (0, a, 0, 0)$ and $a_2^\mu = (0, 0, a, 0)$ with a denoting the amplitude of the vector potential. From now on we use relativistic units ($\hbar = c = 1$), except where otherwise stated. We notice that in the circularly polarized laser field (1) the e^+ and e^- are permanently colliding since, according to the classical equations of motion, they are corotating in the polarization plane.

The Ps atom is assumed to be initially at rest and in its ground state. In a usual field theoretic formalism [15], this bound initial state can be expressed as a superposition of products of free states $\psi_{p_\pm}$ for the electron and positron with definite momenta $\boldsymbol{p}_\pm = \pm\boldsymbol{p}$. The superposition is weighted by the probability amplitude $\tilde{\Phi}(\boldsymbol{p})$ for finding a particular value of $\boldsymbol{p}$. Note that this amplitude is just the Compton profile of the Ps ground state (i.e., the Fourier transform of its wave function) and $\boldsymbol{p}$ can be viewed as the relative momentum of the electron-positron two-body system (i.e., as the momentum of an effective particle of reduced mass $m/2$). When submitted to the strong laser field ($\xi \gtrsim 200$) the Ps atom will instantaneously be ionized, and the dynamics of the ionized e^- and e^+ will be governed by the laser field, which predominates over the influence of the Coulomb interaction between the particles. Therefore, in the spirit of the strong-field approximation theories [24], we may replace the free leptonic states $\psi_{p_\pm}$ by laser-dressed Volkov states [25,26]. Within this framework, the amplitude for the laser-driven process $\text{Ps} \to \mu^+\mu^-$ can be written as

$$\mathcal{S}_{\text{Ps}\to\mu^+\mu^-} = \frac{1}{\sqrt{V}} \int \frac{d^3p}{(2\pi)^3} \tilde{\Phi}(\boldsymbol{p}) \mathcal{S}_{e^+e^-\to\mu^+\mu^-} \tag{2}$$

with a normalization volume V and

$$\mathcal{S}_{e^+e^-\to\mu^+\mu^-} = -\mathrm{i}\alpha_\text{f} \int d^4x \int d^4y \bar{\Psi}_{p_+,s_+}(x)\gamma^\mu \Psi_{p_-,s_-}(x) \times D_{\mu\nu}(x-y)\bar{\Psi}_{P_-,S_-}(y)\gamma^\nu \Psi_{P_+,S_+}(y) \tag{3}$$

being the amplitude for the process $e^+e^- \to \mu^+\mu^-$ in a laser wave (cf. Fig. 1). In Eq. (3), α_f denotes the fine structure constant,

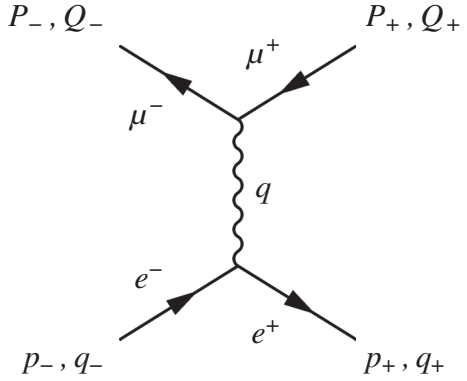

FIG. 1. Feynman diagram for the process $e^+e^- \to \mu^+\mu^-$ in a laser field. The arrows are labeled by the free and effective momenta of the corresponding particle cf. Eq. (6).

$$D_{\mu\nu}(x-y) = \int \frac{d^4q}{(2\pi)^4} \frac{\mathrm{e}^{\mathrm{i}q\cdot(x-y)}}{q^2} g^{\mu\nu} \tag{4}$$

is the free photon propagator [27,28], and the laser-dressed states for the electron and positron are given by [25,26]

$$\Psi_{p_\pm,s_\pm}(x) = \sqrt{\frac{m}{p^0_\pm}}\left(1 \pm \frac{e\not{k}\not{A}}{2(kp_\pm)}\right) u_{p_\pm,s_\pm} \mathrm{e}^{\mathrm{i}f^{(\pm)}} \tag{5}$$

with

$$f^{(\pm)} = \pm(q_\pm x) + \frac{e(p_\pm a_1)}{(kp_\pm)} \sin(kx) - \frac{e(p_\pm a_2)}{(kp_\pm)} \cos(kx).$$

In Eq. (5), $p_\pm$ are the initial free four-momenta of the electron and positron (outside the laser field), $s_\pm$ denote the particle spin states, the $u_{p_\pm,s_\pm}$ are free Dirac spinors [29], and

$$q^\mu_\pm = p^\mu_\pm + \frac{e^2a^2}{2(kp_\pm)} k^\mu \tag{6}$$

are the effective four-momenta of the particles in the laser field [26]. Note that Eq. (6) implies $\boldsymbol{q}^\perp_\pm = \boldsymbol{p}^\perp_\pm$ and thus $\boldsymbol{q}^\perp_+ + \boldsymbol{q}^\perp_- = 0$, where the label $\perp$ denotes the momentum component that is perpendicular to the laser propagation direction. The corresponding effective mass reads $m^2_* = q^2_\pm = (1+\xi^2)m^2$ with the dimensionless laser intensity parameter

$$\xi = \frac{ea}{m}. \tag{7}$$

Like free states, the Volkov states in Eq. (5) are normalized to a δ-function in $p_\pm$ space [26,30]. Analogous expressions hold for the Volkov states $\Psi_{P_\pm,S_\pm}$, the free momenta $P_\pm$, the spin states $S_\pm$, the effective momenta $Q^\mu_\pm$, the effective mass $M_* = M(1+\Xi^2)^{1/2}$, and the intensity parameter $\Xi = ea/M$ of the muons. Note that the amplitude (3) fully accounts for the interaction of the leptons with the laser field, while their interaction with the QED vacuum is taken into account to lowest order. Similar approaches have been used for the theoretical description of laser-assisted e^+e^- [17] and e^-e^- [18–21] scattering.

By the standard procedure of using the generating function of the Bessel functions [31], one can perform the space-time integrations in Eq. (3) to get

$$\mathcal{S}_{e^+e^-\to\mu^+\mu^-} = -\mathrm{i}(2\pi)^4\alpha_\mathrm{f} \frac{m}{\sqrt{p^0_+p^0_-}} \frac{M}{\sqrt{P^0_+P^0_-}} \int \frac{d^4q}{q^2} \sum_{n,N} \mathcal{M}^\mu(p_+, p_-|n)\mathcal{M}_\mu(P_+, P_-|N)\delta(q_+ + q_- - q - nk) \times \delta(Q_+ + Q_- - q - Nk) \tag{8}$$

with the electronic spinor-matrix product

$$\mathcal{M}^\mu(p_+, p_-|n) = \bar{u}_{p_+,s_+}\left\{\left(\gamma^\mu - \frac{e^2a^2k^\mu\not{k}}{2(kp_+)(kp_-)}\right)b^0_n + \left(\frac{e\gamma^\mu\not{k}\not{a}_1}{2(kp_+)} - \frac{e\not{a}_1\not{k}\gamma^\mu}{2(kp_-)}\right)b^+_n + \left(\frac{e\gamma^\mu\not{k}\not{a}_2}{2(kp_+)} - \frac{e\not{a}_2\not{k}\gamma^\mu}{2(kp_-)}\right)b^-_n\right\}u_{p_-,s_-} \tag{9}$$

and a corresponding expression $\mathcal{M}_\mu(P_+, P_-|N)$ for the muons. The coefficients in Eq. (9) are given by

$$\begin{aligned} b^0_n &= J_n(\alpha)\mathrm{e}^{-\mathrm{i}n\varphi_0}, \\ b^+_n &= \frac{1}{2}[J_{n-1}(\alpha)\mathrm{e}^{-\mathrm{i}(n-1)\varphi_0} + J_{n+1}(\alpha)\mathrm{e}^{-\mathrm{i}(n+1)\varphi_0}], \\ b^-_n &= \frac{1}{2\mathrm{i}}[J_{n-1}(\alpha)\mathrm{e}^{-\mathrm{i}(n-1)\varphi_0} - J_{n+1}(\alpha)\mathrm{e}^{-\mathrm{i}(n+1)\varphi_0}] \end{aligned} \tag{10}$$

with $\alpha = \sqrt{\alpha_1^2 + \alpha_2^2}$, $\varphi_0 = \arccos(\alpha_1/\alpha) = \arcsin(\alpha_2/\alpha)$, and

$$\alpha_j = \frac{e(a_jp_-)}{(kp_-)} - \frac{e(a_jp_+)}{(kp_+)} \tag{11}$$

for $j = 1, 2$. As is expressed by the energy-momentum conserving δ-function at the first vertex, the integer number n in Eq. (8) counts the laser photons that are emitted (if $n > 0$) or absorbed (if $n < 0$) by the electron and positron. Similarly, N is the number of laser photons emitted (if $N < 0$) or absorbed (if $N > 0$) by the muons. Denoting the total number of absorbed laser photons by $r := N - n$ and integrating over the virtual photon momentum yields

$$\mathcal{S}_{e^+e^-\to\mu^+\mu^-} = -\mathrm{i}(2\pi)^4\alpha_\mathrm{f}\frac{m}{\sqrt{p_+^0 p_-^0}}\frac{M}{\sqrt{P_+^0 P_-^0}}\sum_{n,r}\mathcal{M}^\mu(p_+, p_-|n)\mathcal{M}_\mu(P_+, P_-|n+r)\frac{\delta(q_+ + q_- - Q_+ - Q_- + rk)}{(q_+ + q_- - nk)^2}. \tag{12}$$

In general, the denominator $(q_+ + q_- - nk)^2$ in Eq. (12) could become zero. By way of a renormalization procedure, such mathematical singularities can be transformed into physical resonances that appear in the production process [17–20]. One can easily see, however, that in the present situation, due to the large value of the laser intensity parameter and the nonrelativistic electron and positron momenta $p_\pm$, one is always far off resonance [32]. Namely, on the one hand we have

$$(q_+ + q_- - nk)^2 = 2m_*^2 + 2(q_+q_-) - 2n(kp_+) - 2n(kp_-) \approx 4m_*^2 - 4n\omega m$$

which becomes zero for

$$n_\mathrm{res} \approx \xi^2\frac{m}{\omega} \sim 10^{10}. \tag{13}$$

On the other hand, the Bessel functions $J_n(\alpha)$, that enter the production amplitude through the coefficients in Eq. (10), practically vanish unless $\alpha \gtrsim n$. Since $q_+^\mu \approx q_-^\mu$ and m_*, $q_\perp \ll q_z$, the argument approximately equals

$$\alpha \approx 2\frac{ea}{\omega}\frac{|\boldsymbol{q}_\perp|}{q_0 - q_z} \approx 4\xi\frac{m}{\omega}\frac{|\boldsymbol{q}_\perp|q_z}{m_*^2},$$

where we have dropped the particle labels $\pm$. Now, $|\boldsymbol{q}_\perp| = |\boldsymbol{p}_\perp| \sim m\alpha_\mathrm{f}$ and $q_z \approx m\xi^2/2$. Hence,

$$\alpha \sim \alpha_\mathrm{f}\xi\frac{m}{\omega} \sim 10^6 \tag{14}$$

which, according to Eq. (13), is orders of magnitude smaller than would be required for a resonance to occur.

The above argument can be further exploited. From Eq. (14) we know that the main contribution to the production amplitude comes from photon numbers n with $|n| \lesssim 10^6$. But for those numbers we have to a very good approximation

$$(q_+ + q_- - nk)^2 \approx (q_+ + q_-)^2$$

which, thus, can be pulled out of the sum in Eq. (12):

$$\mathcal{S}_{e^+e^-\to\mu^+\mu^-} \approx -\mathrm{i}(2\pi)^4\alpha_\mathrm{f}\frac{m}{\sqrt{p_+^0 p_-^0}}\frac{M}{\sqrt{P_+^0 P_-^0}}\frac{1}{(q_+ + q_-)^2}\sum_{n,r}\mathcal{M}^\mu(p_+, p_-|n)\mathcal{M}_\mu(P_+, P_-|n+r) \times \delta(q_+ + q_- - Q_+ - Q_- + rk). \tag{15}$$

The summation over n can now be performed analytically by virtue of Graf's addition theorem [31] with the result

$$\sum_n \mathcal{M}^\mu(p_+, p_-|n)\mathcal{M}_\mu(P_+, P_-|n+r) = J_r u^\mu U_\mu + K_r^+(u^\mu V_\mu + v^\mu U_\mu) + K_r^-(u^\mu W_\mu + w^\mu U_\mu) + L_r^+ v^\mu V_\mu + M_r(v^\mu W_\mu + w^\mu V_\mu) + L_r^- w^\mu W_\mu. \tag{16}$$

Here we have used the abbreviations

$$\begin{aligned} u^\mu &= \bar{u}_{p_+,s_+}\left(\gamma^\mu - \frac{e^2a^2k^\mu \not{k}}{2(kp_+)(kp_-)}\right)u_{p_-,s_-} \\ v^\mu &= \bar{u}_{p_+,s_+}\left(\frac{e\gamma^\mu \not{k}\not{a}_1}{2(kp_+)} - \frac{e\not{a}_1\not{k}\gamma^\mu}{2(kp_-)}\right)u_{p_-,s_-} \\ w^\mu &= \bar{u}_{p_+,s_+}\left(\frac{e\gamma^\mu \not{k}\not{a}_2}{2(kp_+)} - \frac{e\not{a}_2\not{k}\gamma^\mu}{2(kp_-)}\right)u_{p_-,s_-} \end{aligned} \tag{17}$$

and similarly U_μ, V_μ, and W_μ for the muons. The coefficients in Eq. (16) read

$$\begin{aligned} J_r &= J_r(\delta)\epsilon^r, \\ K_r^+ &= \frac{1}{2}[J_{r-1}(\delta)\epsilon^{r-1} + J_{r+1}(\delta)\epsilon^{r+1}], \\ K_r^- &= \frac{\mathrm{i}}{2}[J_{r-1}(\delta)\epsilon^{r-1} - J_{r+1}(\delta)\epsilon^{r+1}], \\ L_r^\pm &= \frac{1}{4}[2J_r(\delta)\epsilon^r \pm J_{r-2}(\delta)\epsilon^{r-2} \pm J_{r+2}(\delta)\epsilon^{r+2}], \\ M_r &= \frac{\mathrm{i}}{4}[J_{r-2}(\delta)\epsilon^{r-2} - J_{r+2}(\delta)\epsilon^{r+2}], \end{aligned} \tag{18}$$

with

$$\gamma = \beta - \alpha e^{i(\varphi_0 - \eta_0)}, \qquad \delta = |\gamma|, \quad \text{and} \quad \epsilon = \frac{\gamma}{\delta} e^{i\eta_0},$$

where β and η_0 are the muonic quantities that correspond to α and φ_0. We will see later that $\gamma \approx \beta$ since $\alpha \ll \beta$ for the typical parameters. An insignificant overall phase factor of $e^{ir\eta_0}$ can be dropped in Eq. (18).

Now we come back to the reaction $\mathrm{Ps} \to \mu^+\mu^-$. In order to obtain the corresponding amplitude we have, according to Eq. (2), to multiply Eq. (15) by the Compton profile $\tilde{\Phi}(\boldsymbol{p})$ of the positronium ground state and integrate over the relative momentum $\boldsymbol{p}$. It turns out that this integration is a very difficult task that can only be done in an approximate way:

First, within the momentum range given by $\tilde{\Phi}(\boldsymbol{p})$ the electronic spinor-matrix products in Eq. (17) are practically constant (on the 1% level since $|\boldsymbol{p}|/m \sim \alpha_f$) and can therefore be pulled out of the integration. The same holds for the kinematic factors $p_\pm^0 \approx m$, $(q_+ + q_-)^2 \approx 4m_*^2$, and the energy-momentum conserving δ-function [33]. Hence, we are left with integrals of the form

$$\bar{J}_r = \frac{1}{\sqrt{V}} \int \frac{d^3p}{(2\pi)^3} \tilde{\Phi}(\boldsymbol{p}) J_r(\delta) e^{ir\chi}, \tag{19}$$

where $\chi = \arctan[\alpha \sin(\varphi_0 - \eta_0)/(\alpha \cos(\varphi_0 - \eta_0) - \beta)]$ such that $\exp(i\chi) = \gamma/\delta$. The highly oscillating factor $\exp(ir\chi)$ leads to a very small value of $\bar{J}_r$. The oscillatory damping of the amplitude is due to a destructive interference of the various partial waves within the Ps wave packet. Classically, this interference effect can be related to the extended motion of the e^+ and e^- in the polarization plane of the laser. Therefore, the mean impact parameter of the e^+e^- collisions is much larger than the initial Ps size and the resulting $\mu^+\mu^-$ production amplitude is suppressed. The laborious evaluation of the integral (19) is performed in the appendix. The result is

$$\bar{J}_r \approx -\frac{\sqrt{2}}{\pi^{3/2} a_0^{3/2} \sqrt{V}} \left(\frac{\omega}{m\alpha_f \xi}\right)^2 \beta^{1/3} J_r(\beta) \tag{20}$$

with the Ps radius $a_0 = 2/\alpha_f m$. The damping factor can also be written as $(\omega/m\alpha_f\xi)^2 = (\pi a_0/\lambda\xi)^2 \sim 10^{-12}$. Note that $2\lambda\xi$ gives the average impact parameter of the e^+e^- collisions since, according to their classical trajectories, the particles corotate in the polarization plane on opposite sides of a circle of radius $\lambda\xi$. However, the classical picture suggests that the process probability is proportional to $(a_0/\lambda\xi)^2$. Instead, this factor is contained in the process amplitude such that the probability scales as $(a_0/\lambda\xi)^4$. This indicates that the damping factor is truly of quantum mechanical origin.

The square of the amplitude reads

$$|\mathcal{S}_{\mathrm{Ps}\to\mu^+\mu^-}|^2 = (2\pi)^4 \alpha_f^2 \frac{m^2}{p_+^0 p_-^0} \frac{M^2}{P_+^0 P_-^0} \frac{1}{(q_+ + q_-)^4} \sum_r \left| \overline{\sum_n} \mathcal{M}^\mu(p_+, p_-|n) \mathcal{M}_\mu(P_+, P_-|n + r) \right|^2 \times \delta(q_+ + q_- - Q_+ - Q_- + rk) VT, \tag{21}$$

where $\overline{\sum}_n$ indicates the sum over n in Eq. (16) averaged over the Ps ground state, as described above, $p_\pm$ ($q_\pm$) are to be understood as some typical values of the electron and positron (effective) momenta, and the factors of volume V and time T come, as usual, from the square of the δ-function. Note that the energy-momentum conserving δ-function, in particular, implies $\boldsymbol{Q}_+^\perp + \boldsymbol{Q}_-^\perp = 0$. From Eq. (21) we get the total reaction rate by averaging over the initial spin states, summing over the final spin states, and integrating over the final momenta:

$$R_{\mathrm{Ps}\to\mu^+\mu^-} = \frac{1}{T} \int \frac{d^3P_+}{(2\pi)^3} \int \frac{d^3P_-}{(2\pi)^3} \frac{1}{4} \sum_{s_\pm, S_\pm} |\mathcal{S}_{\mathrm{Ps}\to\mu^+\mu^-}|^2. \tag{22}$$

In the next but one subsection we derive a compact analytical formula that gives an estimate for the muon production rate (22). But before, we analyze in some detail the reaction kinematics.

B. Kinematical considerations

In the following we provide estimates for the minimal ($r_{\min}$) and the typical ($\bar{r}$) photon numbers that are net absorbed during the production process. From the latter we also find the typical momenta of the created muons.

A lower bound on r can be derived from the equation

$$(q_+ + q_- + rk)^2 = 2(q_+ + q_- + rk) \cdot Q_\pm \tag{23}$$

that follows from the energy-momentum conservation condition expressed by the δ function in Eq. (21). Setting $Q_r \equiv q_0^+ + q_0^- + r\omega$ and $q_r \equiv q_z^+ + q_z^- + r\omega$, this can be rewritten as

$$\cos\theta_{Q_\pm} = \frac{2Q_r Q_\pm^0 - (Q_r^2 - q_r^2)}{2q_r |\boldsymbol{Q}_\pm|} \tag{24}$$

with the polar angle $\theta_{Q_\pm} = \angle(\boldsymbol{k}, \boldsymbol{Q}_\pm)$. Demanding $\cos^2\theta_{Q_\pm} \le 1$, we get

$$\left| Q_\pm^0 - \frac{Q_r}{2} \right| \le \frac{q_r}{2} \left(1 - \frac{4M_*^2}{Q_r^2 - q_r^2}\right)^{1/2}. \tag{25}$$

Hence, it is required that $4M_*^2 \leq Q_r^2 - q_r^2 = (q_+ + q_- + rk)^2$, i.e., the laser-dressed collision energy has to exceed twice the laser-dressed muon mass. Using $(q_+ + q_- + rk)^2 \approx 4m_*^2 + 4r\omega m$, we find

$$r \gtrsim r_{\min} \equiv \frac{M^2 - m^2}{\omega m} \approx \frac{M^2}{\omega m}. \tag{26}$$

This means that, e.g., for $\omega = 1$ eV at least 2×10^{10} photons have to be absorbed from the laser wave for muon production to take place from the initially low-energy e^+e^- pair. This number is independent of the laser intensity.

Assuming a symmetric situation with $Q_+^0 \approx Q_-^0$, Eq. (26) implies that the minimal muon energy is approximately given by

$$Q_{\min}^0 \approx \frac{1}{2} Q_{r_{\min}} \approx \frac{m}{2}\left(\xi^2 + \frac{M^2}{m^2}\right) \approx \frac{M^2}{m}. \tag{27}$$

Hence, the muons are typically produced with highly relativistic momenta such that their dispersion relation approximately reads $Q_\pm^0 \approx |\boldsymbol{Q}_\pm|$. Furthermore, they are emitted roughly along the laser propagation direction (note that $Q_z \gg Q_\perp$ since $q_r \approx m\xi^2 + r\omega$, $Q_r \approx 2m + m\xi^2 + r\omega$ so that $q_r \approx Q_r$). More precisely, by solving Eq. (24) for $Q_\pm^0$ we find that the polar emission angle satisfies the relation

$$\cos\theta_{Q_\pm} \geq \frac{Q_r}{q_r}\left[1 - \frac{(Q_r^2 - q_r^2)^2}{4Q_r^2 M_*^2}\right]^{1/2} \approx \left(1 - \frac{4m^2}{M_*^2}\right)^{1/2}. \tag{28}$$

For these reasons one can say that the muon kinematics is similar to that of the laser photons. This "photonlike" nature of the muons results from the fact that they are essentially produced by a huge number of laser photons whose total energy, according to Eq. (27), exceeds the initial nonrelativistic energy of the e^- and e^+ by orders of magnitude.

The typical number of absorbed laser photons can be estimated by exploiting the properties of the Bessel function $J_r(\beta)$ in Eq. (20). To this end, let us again assume a symmetric situation, which allows us to drop the particle indices $\pm$ in what follows. The energy-conservation condition then can approximately be written as $2Q_0 \approx 2q_0 + r\omega$. Because of the photonlike muon momenta this can be expressed as

$$2Q_z\left(1 + \frac{Q_\perp^2 + M_*^2}{2Q_z^2}\right) \approx 2q_z\left(1 + \frac{q_\perp^2 + m_*^2}{2q_z^2}\right) + r\omega, \tag{29}$$

where $q_z \approx q_0 \approx m\xi^2/2$. Applying the momentum conservation condition $2Q_z \approx 2q_z + r\omega$, we thus get

$$\frac{Q_\perp^2 + M_*^2}{Q_z} \approx \frac{q_\perp^2 + m_*^2}{q_z}. \tag{30}$$

Now, let $r_0 \equiv q_0/\omega$ and $\ell \equiv r/r_0$. Then, again by the momentum conservation condition, $Q_z \approx (1 + \ell/2)q_z$. Hence, Eq. (30) implies

$$Q_\perp \approx \left[\left(1 + \frac{\ell}{2}\right)m_*^2 - M_*^2\right]^{1/2}, \tag{31}$$

where $m_* \gg q_\perp$ was used. Thus, the argument of the Bessel functions in Eq. (20) approximately equals

$$\begin{aligned}\beta &\approx 2\frac{ea}{\omega}\frac{Q_\perp}{Q_0 - Q_z} \approx 4\xi\frac{m}{\omega}\frac{Q_\perp Q_z}{Q_\perp^2 + M_*^2} \approx 2\xi\frac{Q_\perp}{\omega} \\ &\approx 2\xi\frac{m}{\omega}\left(\frac{r\omega}{m} - \frac{M^2}{m^2}\right)^{1/2}.\end{aligned} \tag{32}$$

We note that, according to Eq. (26), the expression under the square root on the right-hand side of Eq. (32) is positive. By the properties of the Bessel functions [31], the typical number of absorbed laser photons is expected to be determined by the condition $\beta \approx r$. This yields

$$r \sim \bar{r} \equiv 2\xi^2\frac{m}{\omega}(1 + \sqrt{1 - \kappa^2}) \tag{33}$$

with $\kappa \equiv M/m\xi$ [34]. The corresponding typical muon momenta read

$$\begin{aligned}\bar{Q}_\perp &\approx m\xi(1 + \sqrt{1 - \kappa^2}), \\ \bar{Q}_z &\approx \frac{m}{2}\xi^2[1 + 2(1 + \sqrt{1 - \kappa^2})].\end{aligned} \tag{34}$$

Using the Eq. (6) between the effective and the free four-momenta and the relations (45) below, we find for the typical values of the muon momenta after the interaction with the laser field

$$\begin{aligned}\bar{P}_\perp &= \bar{Q}_\perp \approx m\xi(1 + \sqrt{1 - \kappa^2}), \\ \bar{P}_z &\approx \bar{Q}_z - \frac{m}{2}\xi^2 \approx m\xi^2(1 + \sqrt{1 - \kappa^2}).\end{aligned} \tag{35}$$

For example, for $\xi = 250$ and $\omega = 1$ eV we have $\bar{r} \approx 10^{11}$, $\bar{Q}_\perp = \bar{P}_\perp \approx 2M$, $\bar{Q}_z \approx 620M$, and $\bar{P}_z \approx 470M$. From Eqs. (33) and (35) we see that the typical final energy of the muon pair satisfies the relation $2\bar{P}_0 \approx \bar{r}\omega$, which reflects the law of energy conservation after the laser has been switched off. Equation (33) also implies that the minimal intensity parameter required for the process to have a significant probability (i.e., to be able to fulfill $r \approx \beta(r)$) amounts to

$$\xi_{\min} = \frac{M}{m} \tag{36}$$

which agrees with our earlier naive estimate.

We notice that the partial rate for muon production by the absorption of $r = r_{\min}$ photons is zero. Namely, according to Eqs. (31) and (32), for $\ell = r_{\min}/r_0 \approx 2M^2/m^2\xi^2$ the

transverse muon momentum and with it the argument of the Bessel functions practically vanish. According to the above, the partial rate reaches a maximum at $r \sim \bar{r}$, which at $\xi = \xi_{\min}$ is twice as large as the minimal number of photons: $\bar{r} = 2r_{\min}$.

It is interesting to observe that the typical muon momenta in Eq. (35) can be interpreted by employing a classical simple man's model of the creation process. In the classical picture, the threshold value of the laser intensity (36) corresponds to the situation when, in the center-of-mass frame of the e^+e^- system, the kinetic energy is large enough to create muons at rest; i.e., denoting the laser phase by $\tau \equiv \omega(t - z)$, we have

$$P'_\perp(\tau_0) = P'_z(\tau_0) = 0 \tag{37}$$

at the creation phase τ_0, where the prime indicates the center-of-mass frame. The classical equation of motion for a muon in a laser field with the initial condition (37) has the solution

$$\begin{aligned} P'_\perp(\tau) &= eA(\tau) - eA(\tau_0), \\ P'_z(\tau) &= \frac{e^2}{2M}[A(\tau) - A(\tau_0)]^2. \end{aligned} \tag{38}$$

Consequently, after the interaction with the laser field the muon momenta equal

$$P'_\perp = m\xi, \qquad P'_z = \frac{m^2}{2M}\xi^2. \tag{39}$$

Because of the e^+e^- longitudinal drift motion in the laser field, the relative velocity between the center-of-mass frame and the lab frame amounts to $v_{\rm rel} = q_z/q_0 = \xi^2/(2 + \xi^2)$ [see Eq. (6)]. The Lorentz transformation to the lab frame thus yields

$$P_\perp = m\xi, \qquad P_z = \frac{M}{2}\frac{\xi^2}{\sqrt{1+\xi^2}} + \frac{m^2}{2M}\xi^2\sqrt{1+\xi^2}. \tag{40}$$

The latter coincides with the typical muon momenta at the threshold $\xi = \xi_{\min}$ given by the quantum theory: $\bar{P}_z \approx M^2/m$ and $\bar{P}_\perp \approx M$ [see Eq. (35)]. The typical number of absorbed photons is determined by the muon final energy: $\bar{r}\omega = 2(P_0 - m) \approx 2M^2/m$, which is in agreement with Eq. (33).

Our simple man's model can also explain a peculiarity in the angular distribution of the muons (see Fig. 4 in Sec. III A). Since $P_z \gg P_\perp$, the muons move in a narrow cone with the axis parallel to the laser propagation direction, but at very small angles the angular spectrum has a dip. The dark region in the angular distribution occurs because the muons, although having been created with zero transverse momentum in the laser field, acquire a nonvanishing transverse momentum after switching off the laser field [see Eqs. (37) and (38)].

C. An approximative formula for the total rate

Equation (22) for the total rate of the reaction $\mathrm{Ps} \to \mu^-\mu^+$, although looking rather innocent, is actually quite involved and can be evaluated only numerically. Therefore it is desirable to find, by analytical means, an approximation to Eq. (22) that displays its main physical content. To this end, we consider the contribution to the total rate stemming from the first term on the right-hand side of Eq. (16). From our numerical calculations we learn that this term gives by far the main contribution ($\sim 90\%$). Thus, we need to calculate

$$\tilde{R}_{\mathrm{Ps}\to\mu^+\mu^-} = \frac{\alpha_{\rm f}^2 a_0}{2^7\pi^5}\left(\frac{\omega}{\xi}\right)^4 \frac{m^2}{p_+^0 p_-^0}\frac{M^2}{(q_+ + q_-)^4}\int\frac{d^3P_+}{P_+^0}\int\frac{d^3P_-}{P_-^0}\sum_r [J_r(\beta)]^2\beta^{2/3}\sum_{s_\pm,S_\pm}|u^\mu U_\mu|^2\delta(q_+ + q_- - Q_+ - Q_- + rk). \tag{41}$$

With the help of the δ-function and the relations [26]

$$\frac{d^3P_\pm}{P_\pm^0} = \frac{d^3Q_\pm}{Q_\pm^0}, \qquad d^3Q_- = |\boldsymbol{Q}_-|Q_-^0\, dQ_-^0\, d\cos\theta_{Q_-}\, d\phi_{Q_-}$$

we can integrate over d^3Q_+ and dQ_-^0 to find

$$\tilde{R}_{\mathrm{Ps}\to\mu^+\mu^-} \approx \frac{\alpha_{\rm f}^2 a_0}{2^{11}\pi^5}\left(\frac{\omega}{\xi}\right)^4\frac{M^2}{m_*^4}\int d\phi_{Q_-}\int d\cos\theta_{Q_-}\sum_r [J_r(\beta)]^2\beta^{2/3}\sum_{s_\pm,S_\pm}|u^\mu U_\mu|^2, \tag{42}$$

where the relations $p_\pm^0 \approx m$, $(q_+ + q_-)^2 \approx 4m_*^2$, and $|\boldsymbol{Q}_-| \approx Q_+^0$ have been used. The spin sum in Eq. (42) can be converted in the usual way into a product of two traces:

$$\mathcal{T}_{uU,uU} := \sum_{s_\pm,S_\pm} |u^\mu U_\mu|^2$$
$$= \mathrm{Tr}\left\{\left(\gamma^\mu - \frac{e^2a^2k^\mu \not{k}}{2(kp_+)(kp_-)}\right)\frac{\not{p}_- + m}{2m}\left(\gamma^\nu - \frac{e^2a^2k^\nu \not{k}}{2(kp_+)(kp_-)}\right)\frac{\not{p}_+ - m}{2m}\right\}$$
$$\times \mathrm{Tr}\left\{\left(\gamma_\mu - \frac{e^2a^2k_\mu \not{k}}{2(kP_+)(kP_-)}\right)\frac{\not{P}_- + M}{2M}\left(\gamma_\nu - \frac{e^2a^2k_\nu \not{k}}{2(kP_+)(kP_-)}\right)\frac{\not{P}_+ - M}{2M}\right\}. \quad (43)$$

We want to find some typical value of $\mathcal{T}_{uU,uU}$. The standard trace technology yields

$$\mathcal{T}_{uU,uU} \approx \frac{2}{m^2M^2}[(p_-P_-)(p_+P_+) + (p_-P_+)(p_+P_-)] + \frac{2(P_+P_-)}{M^2} + \frac{2(p_+p_-)}{m^2}$$
$$- \frac{2\xi\Xi}{mM}[(p_-P_-) + (p_-P_+) + (p_+P_+) + (p_+P_-) - 2(P_+P_-) - 2(p_+p_-)] + 4\xi^2 + 4\Xi^2 + 4\xi^2\Xi^2 + 4. \quad (44)$$

Here we have used the (remarkable) relations

$$\omega m \approx (kp_+) \approx (kp_-) \approx (kP_+) \approx (kP_-) \quad (45)$$

that hold to a good approximation since

$$(kP) = (kQ) \approx \omega\frac{Q_\perp^2 + M_*^2}{2Q_z} \approx \omega\frac{q_\perp^2 + m_*^2}{2q_z} \approx (kq) = (kp) \quad (46)$$

by Eq. (30). With $(p_+p_-) \approx m^2$, $(p_\pm P_\pm) \approx mP_0$, and $(P_+P_-) \approx M^2 + 2P_\perp^2$ the expression in Eq. (44) becomes

$$\mathcal{T}_{uU,uU} \approx \frac{4}{M^2}(P_0 - m\xi^2)^2 + 8\xi^2\frac{P_\perp^2}{M^2} \quad (47)$$

with P_0 and $P_\perp$ denoting some characteristic values of the muonic energies and transversal momenta that, by Eq. (35), amount to $P_0 \approx 2m\xi^2$ and $P_\perp \approx 2m\xi$ at $\xi \gg \xi_{\min}$. This leads to the desired typical value of

$$\mathcal{T}_{uU,uU} \approx 36\xi^4\frac{m^2}{M^2}, \quad (48)$$

which can be pulled out of the integration in Eq. (42). We proceed by performing the further approximations

$$\int d\phi_{Q_-} \approx 2\pi, \qquad \int d\cos\theta_{Q_-} \approx \frac{\theta_{\max}^2}{2},$$
$$\beta^{2/3} \approx \bar{r}^{2/3}, \qquad \sum_r [J_r(\beta(r))]^2 \approx 1, \quad (49)$$

where, according to Eq. (28), the maximum polar emission angle is given by

$$\theta_{\max} \approx \frac{2m}{M_*}. \quad (50)$$

Putting all pieces together, we arrive at the handy formula

$$\tilde{R}_{\mathrm{Ps}\to\mu^+\mu^-} \approx \frac{3^2}{2^6\pi^4}\frac{\alpha_f^2}{\xi^2}\left(\frac{\omega^2}{m_*M_*}\right)^2\left(\frac{4m\xi^2}{\omega}\right)^{2/3}\frac{1}{r_e}, \quad (51)$$

where r_e denotes the classical electron radius [35]. Equation (51) is the desired analytical estimate for the rate of laser-driven Ps decay into muons.

We notice that Eq. (51) can also be represented in the form

$$\tilde{R}_{\mathrm{Ps}\to\mu^+\mu^-} \approx \frac{\sigma}{a_0^3}\left(\frac{a_0}{\lambda\xi}\right)^4\left(\frac{4m\xi^2}{\omega}\right)^{2/3}. \quad (52)$$

Here $\sigma = (9/8)(\alpha_f^2/M_*^2)$ stands for the process cross section, which for $\xi \gg \xi_{\min}$ becomes

$$\sigma \approx \frac{9}{8}\frac{r_e^2}{\xi^2}. \quad (53)$$

Recalling the simple man's model and using the electron energy in the center-of-mass frame $p_0' \approx m\xi$, we can infer that Eq. (52) is based on the process cross section

$$\sigma \sim \frac{r_e^2}{\gamma^2}, \quad (54)$$

with $\gamma = p_0'/m \approx \xi$ being the electron gamma-factor in the center-of-mass frame. Equation (54) is in accordance with the known field-free cross section for muon production in e^+e^- collisions [see Eq. (56)].

If the interaction volume contains N positronium atoms, then the rate will increase correspondingly:

$$R^{(N)}_{\mathrm{Ps}\to\mu^+\mu^-} = NR_{\mathrm{Ps}\to\mu^+\mu^-}. \quad (55)$$

Here we have assumed that each Ps atom independently creates a muon pair, i.e. there is no interference between electrons (positrons) stemming from different Ps atoms. The latter is the case when the electron wave packets from different atoms do not overlap, i.e. when $\lambda\xi \ll n^{-1/3}$, where n is the Ps density. Otherwise, when the spatial extension of the electron wave packet is large, then the gas of Ps atoms transforms into an e^+e^- plasma. Against this background, in the following we will denote the rate in Eq. (55), respectively (22) as the *coherent* rate for muon production since the colliding e^+ and e^- originate from

one and the same Ps atom. In contrast to that, the rate for muon creation from an e^+e^- plasma, that might have been formed from an initial Ps gas, will be referred to as *incoherent* rate. In this situation, electrons and positrons from different Ps atoms can collide which gives a total number of N^2 incoherent collisions. In the next section we will present our results both on the coherent and the incoherent channel of muon production.

III. RESULTS AND DISCUSSION

A. Muon pair creation by a laser-driven Ps atom: The coherent process

Based on Eq. (22), we have numerically calculated the coherent rate for $\mu^+\mu^-$ pair creation from a single Ps atom submitted to a strong laser field of circular polarization. The laser frequency has been taken to be $\omega = 1$ eV, throughout. For the laser intensity parameter we have chosen the three different values $\xi = 250$, 500, and 1000. The corresponding laser intensities amount to 1.1×10^{23} W/cm^2, 4.5×10^{23} W/cm^2, and 1.8×10^{24} W/cm^2, respectively.

In Fig. 2, the dependence of the total muon creation rate on the laser intensity parameter is shown. For the intensity parameters under consideration we find production rates of 1.0×10^{-15} s^{-1} ($\xi = 250$), 1.6×10^{-16} s^{-1} ($\xi = 500$), and 1.1×10^{-17} s^{-1} ($\xi = 1000$). The analytical approximation (51) overestimates these numbers, but is still in rather good agreement with them (cf. Fig. 2). The reason for the overestimation, in particular, for $\xi \approx \xi_{\min}$, are the rather large values of P_0 and $P_\perp$ used in Eq. (47). A total production rate of 10^{-15} s^{-1} means that in a finite laser pulse of femtosecond duration the probability to create a muon pair from a single Ps atom is of order 10^{-30}. We notice that a typical laser focal volume will contain only one Ps atom on average since the highest positronium densities achievable at present are on the order of 10^8 cm^{-3} [36]. However, proposals to reach a Ps density of 10^{14} cm^{-3} [37] or even a Ps Bose-Einstein condensate of 10^{18} cm^{-3} [38] are being considered. The above numbers of created muons seem too small to be experimentally accessible. Clearly, the main reason for the smallness of the coherent reaction rate lies in the damping factor $(a_0/\lambda\xi)^4 \sim 10^{-26}$ [cf. Eq. (20)]. The latter results from the destructive interference of the partial waves constituting the Ps wave packet in the laser field, which makes the recollision of an e^+ and e^- from the same Ps atom highly unlikely. From a simplified, classical point of view, the reason for the damping lies in the large collisional impact parameter of order $\lambda\xi$ that is due to the equal handed rotations of the particles in the laser polarization plane [39].

In Fig. 3, the partial production rates with respect to the number r of absorbed laser photons are shown (i.e., the contributions to the total rate stemming from a net absorption of r laser photons in the production process). The photon number is given in units of $r_0 \approx m\xi^2/2\omega$, which amounts to 1.6×10^{10} ($\xi = 250$), 6.4×10^{10} ($\xi = 500$), and 2.6×10^{11} ($\xi = 1000$), respectively. The shape of the curves in Fig. 3 can be understood with the help of the kinematical analysis in Sec. II B. First, according to Eq. (26), the minimal number of laser photons required from kinematical constraints amounts to $r_{\min} = 2.2 \times 10^{10}$, independent of the laser intensity. If we express this number with respect to the respective values of r_0, then we get $r_{\min}/r_0 = 1.4$ ($\xi = 250$), 0.3 ($\xi = 500$), and 0.1 ($\xi = 1000$). The partial reaction rate for $r = r_{\min}$ is always zero, as can be seen in Fig. 3. Further, in agreement with

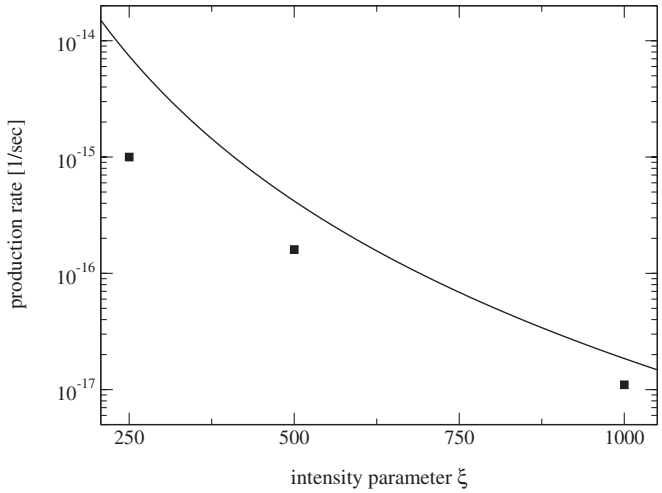

FIG. 2. Total rates for the laser-driven reaction Ps $\rightarrow \mu^+\mu^-$ as a function of the intensity parameter ξ of the applied laser field. The black squares show the results of our numerical calculations based on Eq. (22); the solid line shows the analytical estimate via Eq. (51).

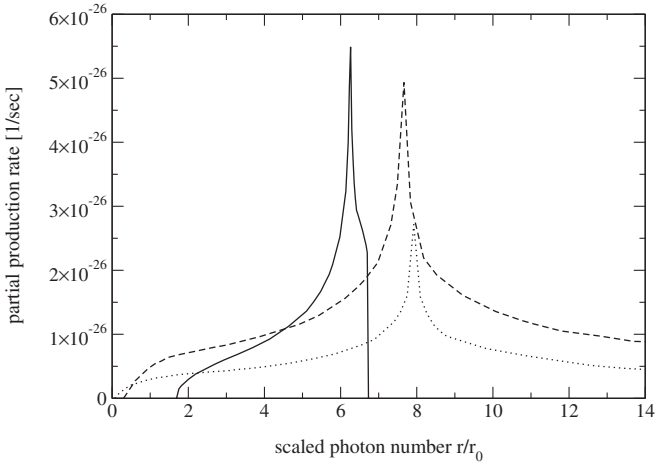

FIG. 3. Partial rates for the laser-driven reaction Ps $\rightarrow \mu^+\mu^-$ as a function of the number of absorbed laser photons r for an intensity parameter of $\xi = 250$ (solid line), 500 (dashed line), and 1000 (dotted line). The latter two curves are enhanced by factors of 10^2 and 5×10^3, respectively.

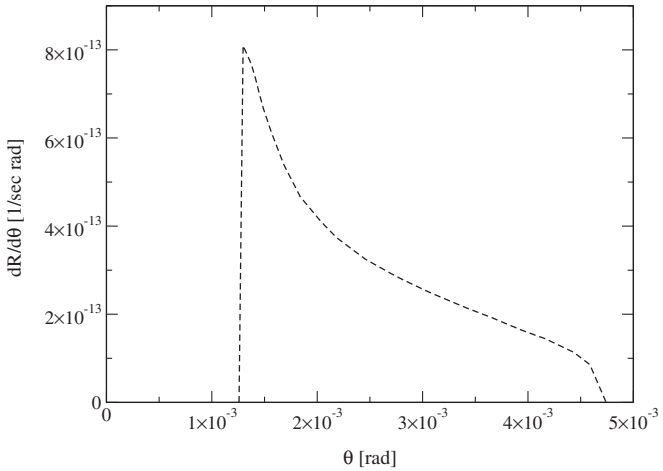

FIG. 4. Angular spectrum with respect to the polar angle of the effective momentum for one of the produced muons at $\xi = 250$.

Eq. (33), the curves exhibit maxima at $r \approx \bar{r}$; for $\xi = 250$ the maximum is located at $\bar{r}/r_0 \approx 6$, while for $\xi = 500$ and 1000 we have $\bar{r}/r_0 \approx 8$. This feature reflects the mathematical properties of the Bessel functions and is correctly predicted by our simple man's model.

Figure 4 shows the angular distribution $dR/d\theta_Q$ for one of the produced muons. For symmetry reasons, the spectra for the muon μ^- and the antimuon μ^+ are identical. The differential rate is expressed with respect to the polar angle $\theta_Q = \angle(\boldsymbol{k}, \boldsymbol{Q})$ of the effective muon momentum. The value of the laser intensity parameter is chosen to be $\xi = 250$. One can see that the muon is emitted into a very narrow angular range starting from 1.3×10^{-3} rad $\approx 0.07°$ and extending to 4.7×10^{-3} rad $\approx 0.27°$, which is in agreement with Eq. (50). In other words, as has already been mentioned before, the muon moves practically parallel to the propagation direction of the laser beam. The occurrence of a minimal emission angle arises from the fact that, according to Eq. (32), the argument of the Bessel function is proportional to the transverse momentum component $Q_\perp$, which itself is proportional to $\sin\theta_Q \approx \theta_Q$. Thus, the emission angle cannot be too small because otherwise the Bessel function will vanish. Since $P_\perp = Q_\perp$, a dark angular region also exists in the angular spectrum $dR/d\theta_P$ with respect to the muon momentum outside the laser beam. An alternative explanation of this phenomenon in terms of a simple man's model has been given at the end of Sec. II B.

B. Comparison with the field-free process $e^+e^- \to \mu^+\mu^-$

In this subsection we want to draw a comparison between the coherent muon creation from a laser-driven Ps atom and the corresponding field-free process $e^+e^- \to \mu^+\mu^-$. In the high-energy limit, the cross section for this reaction reads [15]

$$\sigma_{\text{free}} = \frac{4\pi}{3}\frac{m^2}{s} r_e^2, \tag{56}$$

where $\sqrt{s} \gg 2M$ denotes the collision energy. In the case with laser field, the square root of the quantity

$$(q_+ + q_- + \bar{r}k)^2 \approx 4m_*^2 + 4\bar{r}\omega m \approx 20m^2\xi^2 \tag{57}$$

can be regarded as some average collision energy. For the ξ-values considered in this paper, the laser-dressed collision energy is thus about 1 GeV. Hence, the reference cross section in Eq. (56) to compare with should be taken at $\sqrt{s} \approx 1$ GeV, where its value is about 100 nbarn. To transform this cross section into a reaction rate, we have to multiply by the incident particle flux. In collider experiments, instead of the incoming flux, the luminosity is more commonly used. When a beam of N_+ positrons collides at high energy with a beam of N_- electrons, then the luminosity is given by

$$L = \frac{N_+ N_-}{UA}, \tag{58}$$

where U is the circumference of the collider ring and A is the beam cross sectional area at the collision point. To make the comparison with a single laser-driven Ps atom, we use $N_\pm = 1$ along with the typical values $U \approx 10^3$ m and $A \approx 10^{-5}$ cm^2. Note that the corresponding mean impact parameter $\sim\sqrt{A}$ of the field-free collision is of the same order of magnitude as the electron-positron spatial separation $\sim\lambda\xi$ in the laser wave. The resulting luminosity $L \sim 10^{11}$ cm^{-2} s^{-1} leads to a muon creation rate of 10^{-20} s^{-1}. This number is considerably smaller than the production rates we found in Sec. III A. However, in a real collider experiment one has bunches of $N_\pm \sim 10^{10}$ particles leading to much higher luminosities and reaction rates, of course.

C. Muon production from laser-plasma interaction: The incoherent process

We have seen in Sec. III A that the large electron-positron wave packet size during the motion in the laser field suppresses the coherent reaction rate dramatically. To reduce this size and achieve e^+e^- collisions at microscopic impact parameters, one can think of employing different, more complicated field configurations [13]. Otherwise, the muon creation from Ps atoms will be dominated by the incoherent production channel introduced at the end of Sec. II C. The latter coincides with the process of muon creation, when a low-energy e^+e^- plasma interacts with a strong laser beam. We have redone our calculation for this situation, i.e., for a free, initially nonrelativistic e^+ and e^- that collide in a strong laser field and create a $\mu^+\mu^-$ pair by annihilation. Our results on the partial production rates for reasonable experimental parameters

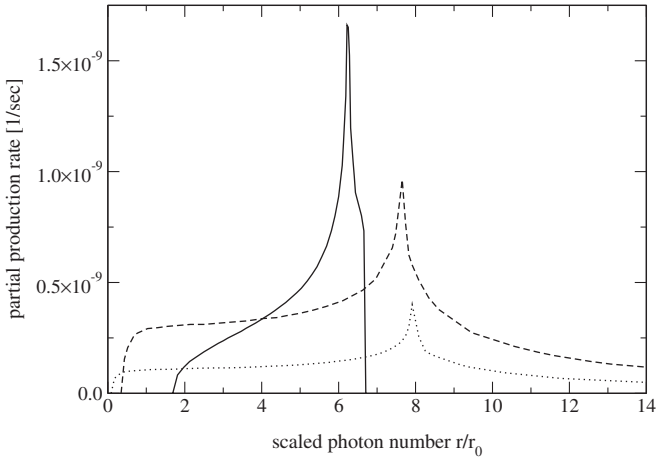

FIG. 5. Partial rates for the reaction $e^+e^- \to \mu^+\mu^-$ from a laser-driven nonrelativistic plasma. It is assumed that $N_\pm = 10^7$ particles are contained in the interaction volume $V_{\rm int} = 10^{-9}$ cm^3. The laser intensity parameter is $\xi = 250$ (solid line), 500 (dashed line), and 1000 (dotted line). The latter two curves are enhanced by factors of 2×10^2 and 2×10^4, respectively.

(see below) are shown in Fig. 5. The shape of the curves is similar to those in Fig. 3.

From Eqs. (20) and (51) one can infer that the total rate for muon creation from laser-plasma interaction approximately reads

$$R \approx \frac{9}{16\pi} \frac{m^2 r_e^2}{M_*^2} \frac{N_+ N_-}{V_{\rm int}}, \tag{59}$$

where $N_\pm$ denotes the number of electrons and positrons in the interaction volume $V_{\rm int}$, which is given by the laser focal volume. From Eq. (59) we can estimate the total number of produced muons $N_\mu = R\tau N_s$ during the interaction with N_s laser shots, each single shot having a pulse duration of τ. When plugging in some reasonable numbers: $\tau = 100$ fs, $V_{\rm int} \approx (10\lambda)^3 \approx 10^{-9}$ cm^3, and assuming that the presently achievable number of positrons $N_+ \approx 10^7 \approx N_-$ [40] can be compressed into the interaction volume or, alternatively, is created via a newly emerging laser-based technique [41], then we get $N_\mu \approx 1$ muon production event during $N_s = 10^{10}$ shots. This number, being based on rather optimistic experimental parameters (especially concerning the positron compression), indicates that the realization of the incoherent muon creation process might be not inhibitory difficult, but still it will be very hard with modern experimental techniques.

IV. CONCLUSION

In this paper we have studied $\mu^+\mu^-$ production by e^+e^- annihilation from a laser-driven Ps atom. To this end, a calculational framework has been developed where the initial bound state is described as a superposition of Volkov states weighted by the Compton profile of the Ps ground state. By virtue of the interaction with the QED vacuum, which is treated in the first order of perturbation theory, this initial state can decay into a laser-dressed $\mu^+\mu^-$ pair. Also, the related process of muon creation by the interaction of a strong laser field with a low-energy e^+e^- plasma has been examined.

We have found that the minimal laser intensity required for the process to occur corresponds to an intensity parameter of $\xi \approx M/m \approx 200$. In the case of a near-optical laser wave of circular polarization, e.g., this value is reached for an intensity of 7×10^{22} W/cm^2. This means that, starting from a nonrelativistic Ps atom or e^+e^- plasma, fundamental particle reactions can be ignited by a superstrong laser field of an intensity that is just 1 order of magnitude larger than the highest values available today.

However, in the Ps case, the total production rate resulting from the coherent recollisions is extremely small and amounts to about 10^{-15} per second only. The strong suppression is caused by a destructive interference of the different partial waves constituting the bound initial state in the superintense laser field. This phenomenon is also expressed by a compact formula for the total rate that we derived by analytical means. As a consequence, in the considered setup the production rate will be dominated by the muon creation via incoherent e^+e^- scattering, for which the system of Ps atoms has no advantage compared to an e^+e^- plasma. For the incoherent collisions, the interference plays no role and the resulting muon creation rate is significantly larger than the corresponding rate from the coherent production channel. Nevertheless, very demanding experimental conditions are required in order to achieve observable muon yields.

Finally, we note that similar reaction rates can be expected for laser-driven $\pi^+\pi^-$ production from e^+e^- or $\mu^+\mu^-$. A more promising alternative within circularly polarized field configurations might be the process $e^+e^- \to e^+e^- + e^+e^-$, the field-free cross section of which being several orders of magnitude larger than the one for $\mu^+\mu^-$ creation in Eq. (56) [42].

APPENDIX

In this appendix we calculate the integral in Eq. (19). Using cylindrical coordinates it reads

$$\bar{J}_r = \frac{1}{(2\pi)^3\sqrt{V}} \times \int_{-\infty}^{+\infty} dp_z \int_0^{\infty} p_\perp dp_\perp \int_{-\pi}^{+\pi} d\varphi \tilde{\Phi}(\boldsymbol{p}) J_r(\delta) \mathrm{e}^{\mathrm{i}r\chi} \tag{A1}$$

with

$$\delta = [\alpha^2 + \beta^2 - 2\alpha\beta\cos(\varphi_0 - \eta_0)]^{1/2},$$
$$\alpha = \frac{2m\xi}{\omega}\frac{p_0 p_\perp}{p_\perp^2 + m^2},$$
$$\chi = \arctan\left\{\frac{\alpha\sin(\varphi_0 - \eta_0)}{\alpha\cos(\varphi_0 - \eta_0) - \beta}\right\},$$
$$\tilde{\Phi}(\boldsymbol{p}) = \frac{8\sqrt{\pi}a_0^{3/2}}{[1 + (a_0 p_\perp)^2 + (a_0 p_z)^2]^2},$$

and $\beta \approx r$ being of order 10^{11} for the laser parameters under consideration [see Eq. (33)]. Note that φ_0 coincides with the azimuthal angle φ. Going over to the variables $a_0 p_\perp \to x$, $a_0 p_z \to z$, and $(\varphi_0 - \eta_0) \to \varphi$, Eq. (A1) reads

$$\bar{J}_r = \frac{2}{\pi^{5/2}a_0^{3/2}\sqrt{V}} \times \int_0^\infty dz \int_0^\infty x\,dx \int_{-\pi}^{+\pi} d\varphi \frac{J_r(\delta)e^{ir\chi}}{(1 + x^2 + z^2)^2}. \quad \text{(A2)}$$

For brevity, the overall factor $2/(\pi^5 a_0^3 V)^{1/2}$ will be dropped in what follows and only restored in the final result [see Eq. (A14)]. Since $\alpha \ll \beta$ we can expand the functions δ and χ:

$$\delta = \beta - \alpha\cos\varphi + \mathcal{O}\left(\frac{\alpha^2}{\beta}\right),$$
$$\chi = -\frac{\alpha}{\beta}\sin\varphi + \mathcal{O}\left(\frac{\alpha^2}{\beta^2}\cos^2\varphi\right).$$

Note here that, since $\alpha^2/\beta \lesssim 1$ [see Eq. (14)], we may drop terms of this order in the expansion of δ. Further, since the Bessel function $J_r(\delta)$ is exponentially or oscillatorily damped unless $|r - \delta| \lesssim r^{1/3}$, the main contribution to the integral comes from the region with $|\alpha\cos\varphi| \lesssim r^{1/3} \sim \beta^{1/3}$. Accordingly, terms of order $r(\alpha\cos\varphi/\beta)^2 \lesssim \beta^{-1/3}$ may be neglected in the phase $r\chi$. Hence, we obtain

$$\bar{J}_r \approx \int_0^\infty dz \int_0^\infty \frac{x\,dx}{(1 + x^2 + z^2)^2} \times \int_{-\pi}^{+\pi} d\varphi J_r(\beta - \alpha\cos\varphi)e^{-i\alpha\sin\varphi}.$$

Next we observe that $\bar{J}_r = 2\,\mathrm{Re}I_r$ with

$$I_r \equiv \int_0^\infty dz \int_0^\infty \frac{x\,dx}{(1 + x^2 + z^2)^2} \times \int_0^\pi d\varphi J_r(\beta - \alpha\cos\varphi)e^{-i\alpha\sin\varphi}$$
$$= \int_0^\infty dz \int_0^\infty \frac{x\,dx}{(1 + x^2 + z^2)^2} \int_0^{\pi/2} d\varphi[J_r(\beta - \alpha\cos\varphi) + J_r(\beta + \alpha\cos\varphi)]e^{-i\alpha\sin\varphi}$$
$$\approx 2\int_0^\infty dz \int_0^\infty \frac{x\,dx}{(1 + x^2 + z^2)^2} \times \int_0^{\pi/2} d\varphi J_r(\beta - \alpha\cos\varphi)e^{-i\alpha\sin\varphi}. \quad \text{(A3)}$$

In the last step we exploited the fact that the contributing ranges within the exponential and oscillatory regions of the Bessel function have a similar size. Let now

$$\zeta = \frac{x\sqrt{x^2 + z^2 + \epsilon^{-2}}}{x^2 + \epsilon^{-2}}, \qquad \rho = \zeta\cos\varphi$$

with $\epsilon \equiv \alpha_f/2$, such that $\alpha = \alpha_0\zeta$ with $\alpha_0 \equiv 2m\xi/\omega$. Performing the substitution of variables $(x, z, \varphi) \to (x, \zeta, \rho)$ we obtain

$$I_r = 2\int_0^\infty \frac{x^4 dx}{(x^2 + \epsilon^{-2})^3} \int_{\zeta_{\min}}^\infty \frac{\zeta\,d\zeta}{\sqrt{\zeta^2 - \frac{x^2}{x^2+\epsilon^{-2}}[\zeta^2 - \frac{x^2(\epsilon^{-2}-1)}{(x^2+\epsilon^{-2})^2}]^2}} \times \int_0^\zeta \frac{d\rho}{\sqrt{\zeta^2 - \rho^2}} J_r(\beta - \alpha_0\rho)e^{-i\alpha_0\sqrt{\zeta^2-\rho^2}},$$

with $\zeta_{\min} = x/\sqrt{x^2 + \epsilon^{-2}}$. As mentioned above, the value of $J_r(\beta - \alpha_0\rho)$ will be exponentially small unless $\rho \lesssim \rho_0 \equiv \beta^{1/3}/\alpha_0$. Therefore, we can approximately write

$$I_r \approx 2J_r(\beta)\int_0^\infty \frac{x^4 dx}{(x^2 + \epsilon^{-2})^3} \times \int_{\zeta_{\min}}^\infty \frac{\zeta\,d\zeta}{\sqrt{\zeta^2 - \frac{x^2}{x^2+\epsilon^{-2}}[\zeta^2 - \frac{x^2(\epsilon^{-2}-1)}{(x^2+\epsilon^{-2})^2}]^2}} \times \int_0^{\rho_{\max}} \frac{d\rho}{\sqrt{\zeta^2 - \rho^2}} e^{-i\alpha_0\sqrt{\zeta^2-\rho^2}} \quad \text{(A4)}$$

with $\rho_{\max} \equiv \min\{\zeta, \rho_0\}$.

In the following, the integral in Eq. (A4) will be evaluated in several steps. Let us first consider the integral over ρ. We show:

$$I(\zeta) \equiv \int_0^{\rho_{\max}} d\rho \frac{e^{-i\alpha_0\sqrt{\zeta^2-\rho^2}}}{\sqrt{\zeta^2 - \rho^2}} \approx \begin{cases} \frac{\pi}{2} & (\zeta \ll \alpha_0^{-1}), \\ \sqrt{\frac{\pi}{2}}\frac{e^{i(\pi/4)}}{\sqrt{\zeta\alpha_0}}e^{-i\alpha_0\zeta} & (\alpha_0^{-1} \ll \zeta \ll \alpha_0\rho_0^2), \\ \frac{\rho_0}{\zeta}e^{-i\alpha_0\zeta} & (\zeta \gg \alpha_0\rho_0^2). \end{cases} \quad \text{(A5)}$$

Note that, for the parameters of interest, we have $\alpha_0^{-1} \sim$

10^{-8} and $\rho_0 \sim 10^{-5}$ such that $\alpha_0\rho_0^2 \sim 10^{-2}$. In the first range ($\zeta \ll \alpha_0^{-1}$) we get

$$I(\zeta) \approx \int_0^\zeta \frac{d\rho}{\sqrt{\zeta^2-\rho^2}} = \frac{\pi}{2}.$$

The second range we split into two cases. For $\alpha_0^{-1} \ll \zeta < \rho_0$ the integrand is highly oscillating and the main contribution comes from the boundary terms:

$$I(\zeta) = \int_0^\zeta dy \frac{e^{-i\alpha_0 y}}{\sqrt{\zeta^2-y^2}} \approx \frac{1}{i\alpha_0\zeta} + \int^\zeta dy \frac{e^{-i\alpha_0 y}}{\sqrt{\zeta^2-y^2}}$$
$$\approx \frac{e^{-i\alpha_0\zeta}}{\sqrt{2\zeta}} \int_0 dt \frac{e^{i\alpha_0 t}}{\sqrt{t}} \approx \sqrt{\frac{\pi}{2}} \frac{e^{i(\pi/4)}}{\sqrt{\zeta\alpha_0}} e^{-i\alpha_0\zeta},$$

where in the first and second steps the substitutions $y = \sqrt{\zeta^2-\rho^2}$ and $t = \zeta - y$ were made. Note here that the contribution from the lower bound can be neglected since $\alpha_0\zeta \gg 1$. Similarly, for $\rho_0 < \zeta \ll \alpha_0\rho_0^2$ we can write

$$I(\zeta) = \int_{\sqrt{\zeta^2-\rho_0^2}}^\zeta dy \frac{e^{-i\alpha_0 y}}{\sqrt{\zeta^2-y^2}}$$
$$\approx \frac{e^{-i\alpha_0\sqrt{\zeta^2-\rho_0^2}}}{i\alpha_0\rho_0} + \int^\zeta dy \frac{e^{-i\alpha_0 y}}{\sqrt{\zeta^2-y^2}}$$
$$\approx \sqrt{\frac{\pi}{2}} \frac{e^{i(\pi/4)}}{\sqrt{\zeta\alpha_0}} e^{-i\alpha_0\zeta}.$$

Finally, in the third range ($\zeta \gg \alpha_0\rho_0^2$) we obtain

$$I(\zeta) \approx \int_0^{\rho_0} \frac{d\rho}{\zeta} e^{-i\alpha_0\zeta} = \frac{\rho_0}{\zeta} e^{-i\alpha_0\zeta}.$$

This shows Eq. (A5).

Now we continue with the integrations over $dxd\zeta$. To this end, let us divide the integration range into five regions:

I: $0 \le x \le x_1$,	$\zeta_{\min} \le \zeta \le \zeta_1$,
II: $0 \le x \le x_1$,	$\zeta_1 \le \zeta \le \zeta_2$,
III: $x_1 \le x \le x_2$,	$\zeta_{\min} \le \zeta \le \zeta_2$,
IV: $0 \le x \le x_2$,	$\zeta \ge \zeta_2$,
V: $x \ge x_2$,	$\zeta \ge \zeta_{\min}$,

with

$$x_1 = \frac{1}{\alpha_0\epsilon} \sim 10^{-6}, \qquad x_2 = \frac{\alpha_0\rho_0^2}{\epsilon} \sim 1,$$
$$\zeta_1 = \frac{1}{\alpha_0} \sim 10^{-8}, \qquad \zeta_2 = \alpha_0\rho_0^2 \sim 10^{-2}.$$

Note that ζ_1 and ζ_2 coincide with the values of $\zeta_{\min}$ taken at positions x_1 and x_2, respectively. Hence, it is easily seen, that this division covers the whole range of integration, i.e.:

$$I_r = I_r^{(\mathrm{I})} + I_r^{(\mathrm{II})} + I_r^{(\mathrm{III})} + I_r^{(\mathrm{IV})} + I_r^{(\mathrm{V})}.$$

Taking into account Eq. (A5), we find in the first range

$$I_r^{(\mathrm{I})} \approx J_r(\beta)\pi \int_0^{(\alpha_0\epsilon)^{-1}} \frac{x^4 dx}{\epsilon^{-6}} \times \int_{x\epsilon}^{\alpha_0^{-1}} \frac{\zeta d\zeta}{\sqrt{\zeta^2-x^2\epsilon^2}[\zeta^2-x^2\epsilon^4(\epsilon^{-2}-1)]^2}$$
$$= J_r(\beta)\pi \frac{\epsilon}{\alpha_0^2} \times \int_0^1 x^4 dx \int_x^1 \frac{\zeta d\zeta}{\sqrt{\zeta^2-x^2}(\zeta^2-x^2+x^2\epsilon^2)^2},$$

where in the second step the transformations $\alpha_0\epsilon x \to x$, $\alpha_0\zeta \to \zeta$ have been made. Introducing the variable $t = \zeta^2 - x^2$, this becomes

$$I_r^{(\mathrm{I})} = J_r(\beta)\frac{\pi}{2}\frac{\epsilon}{\alpha_0^2}\int_0^1 x^4 dx \int_0^{1-x^2} \frac{dt}{\sqrt{t}(t+x^2\epsilon^2)^2}$$
$$= J_r(\beta)\frac{\pi}{2}\frac{1}{\alpha_0^2\epsilon^2}\left\{\epsilon\int_0^1 \frac{x^2\sqrt{1-x^2}dx}{(1-x^2+x^2\epsilon^2)} + \int_0^1 x \arctan\left(\frac{\sqrt{1-x^2}}{x\epsilon}\right)dx\right\}, \quad \text{(A6)}$$

where in the second step formula 1.2.15.13 from Ref. [43] was used. Since both integrals in Eq. (A6) are of order unity, we conclude that

$$I_r^{(\mathrm{I})} \approx J_r(\beta)\frac{\pi}{2}\frac{1}{\alpha_0^2\epsilon^2}. \quad \text{(A7)}$$

Turning to the second range, we get similar as before

$$I_r^{(\mathrm{II})} = J_r(\beta)\sqrt{\frac{2\pi}{\alpha_0}} e^{i(\pi/4)} \int_0^{(\alpha_0\epsilon)^{-1}} \frac{x^4 dx}{\epsilon^{-6}} \int_{\alpha_0^{-1}}^{\alpha_0\rho_0^2} \frac{\sqrt{\zeta} e^{-i\alpha_0\zeta} d\zeta}{\sqrt{\zeta^2-x^2\epsilon^2}[\zeta^2-x^2\epsilon^4(\epsilon^{-2}-1)]^2}$$
$$= J_r(\beta)\sqrt{2\pi}\frac{\epsilon}{\alpha_0^2} e^{i(\pi/4)} \int_0^1 x^4 dx \int_1^{\alpha_0^2\rho_0^2} \frac{\sqrt{\zeta} e^{-i\zeta} d\zeta}{\sqrt{\zeta^2-x^2}(\zeta^2-x^2+x^2\epsilon^2)^2}.$$

The main contribution to the highly oscillating integral over ζ comes from the lower boundary. In this way we find

$$I_r^{(\mathrm{II})} \approx J_r(\beta)\sqrt{2\pi}\frac{\epsilon}{\alpha_0^2} e^{i(\pi/4)} \int_0^1 \frac{x^4 dx e^{-i}}{i\sqrt{1-x^2}(1-x^2+x^2\epsilon^2)^2}. \quad \text{(A8)}$$

The remaining integral can be done analytically. Its value is $\frac{\pi}{4}\epsilon^{-3}$ such that we get

$$I_r^{(\mathrm{II})} \approx J_r(\beta)\left(\frac{\pi}{2}\right)^{3/2}\frac{e^{-i(\pi/4)}e^{-i}}{\alpha_0^2\epsilon^2}. \tag{A9}$$

In the third region the integral reads

$$\begin{aligned} I_r^{(\mathrm{III})} &= J_r(\beta)\sqrt{\frac{2\pi}{\alpha_0}}e^{i(\pi/4)}\int_{(\alpha_0\epsilon)^{-1}}^{\alpha_0\rho_0^2\epsilon^{-1}}\frac{x^4dx}{\epsilon^{-6}}\int_{x\epsilon}^{\alpha_0\rho_0^2}\frac{\sqrt{\zeta}e^{-i\alpha_0\zeta}d\zeta}{\sqrt{\zeta^2-x^2\epsilon^2}[\zeta^2-x^2\epsilon^4(\epsilon^{-2}-1)]^2} \\ &= J_r(\beta)\sqrt{2\pi}\frac{\epsilon}{\alpha_0^2}e^{i(\pi/4)}\int_1^{\alpha_0^2\rho_0^2}x^4e^{-ix}dx\int_0^{\alpha_0^2\rho_0^2-x}\frac{\sqrt{t+x}e^{-it}dt}{\sqrt{t(t+2x)}[t(t+2x)+x^2\epsilon^2]^2}, \end{aligned}$$

where first the same transformations as before were made ($\alpha_0\epsilon x \to x$, $\alpha_0\zeta \to \zeta$) and then the variable $t = \zeta - x$ was introduced. Because of the highly oscillating integrand and the singularity, the main contribution to the t integration comes from the region around $t = 0$. Thus, we arrive at

$$I_r^{(\mathrm{III})} \approx J_r(\beta)\sqrt{2\pi}\frac{\epsilon}{\alpha_0^2}e^{i(\pi/4)}\int_1^{\alpha_0^2\rho_0^2}\frac{x^4e^{-ix}\sqrt{x}}{\sqrt{2x}(x\epsilon)^4}dx\int_0^\infty\frac{e^{-it}}{\sqrt{t}}dt = J_r(\beta)\frac{i\pi}{\alpha_0^2\epsilon^3}(e^{-i\alpha_0^2\rho_0^2}-e^{-i}). \tag{A10}$$

According to Eq. (A5), in the fourth range we have

$$\begin{aligned} I_r^{(\mathrm{IV})} &= 2J_r(\beta)\rho_0\int_0^{\alpha_0\rho_0^2\epsilon^{-1}}\frac{x^4dx}{\epsilon^{-6}}\int_{\alpha_0\rho_0^2}^\infty\frac{e^{-i\alpha_0\zeta}d\zeta}{\sqrt{\zeta^2-x^2\epsilon^2}[\zeta^2-x^2\epsilon^4(\epsilon^{-2}-1)]^2} \\ &= 2J_r(\beta)\epsilon\alpha_0\rho_0^3\int_0^1x^4dx\int_1^\infty\frac{e^{-i\alpha_0^2\rho_0^2\zeta}d\zeta}{\sqrt{\zeta^2-x^2}(\zeta^2-x^2+x^2\epsilon^2)^2}, \end{aligned}$$

where we have substituted $(\alpha_0\rho_0^2\epsilon^{-1})^{-1}x \to x$ and $(\alpha_0\rho_0^2)^{-1}\zeta \to \zeta$. The highly oscillating integral over ζ can be evaluated with the same methods as before to give

$$I_r^{(\mathrm{IV})} \approx 2J_r(\beta)\frac{\epsilon\rho_0}{\alpha_0}\int_0^1\frac{x^4dxe^{-i\alpha_0^2\rho_0^2}}{i\sqrt{1-x^2}(1-x^2+x^2\epsilon^2)^2}. \tag{A11}$$

The integral over x we have already encountered in Eq. (A8). Hence:

$$I_r^{(\mathrm{IV})} \approx J_r(\beta)\frac{\pi}{2i}\frac{\rho_0}{\alpha_0\epsilon^2}e^{-i\alpha_0^2\rho_0^2}. \tag{A12}$$

In the fifth range, after the substitution $\epsilon x \to x$, the integral can be written as

$$\begin{aligned} I_r^{(\mathrm{V})} &\approx 2J_r(\beta)\epsilon\rho_0\int_{\alpha_0\rho_0^2}^\infty\frac{x^4dx}{(1+x^2)^3} \\ &\quad\times\int_{x/\sqrt{1+x^2}}^\infty\frac{e^{-i\alpha_0\zeta}d\zeta}{\sqrt{\zeta^2-\frac{x^2}{1+x^2}}[\zeta^2-\frac{x^2(1-\epsilon^2)}{(1+x^2)^2}]^2}. \end{aligned}$$

With the definitions $a = x/\sqrt{1+x^2}$ and $b = x^2(\epsilon^2 + x^2)/(1+x^2)^2$, the ζ integral can be cast into the form

$$\begin{aligned} &\int_a^\infty\frac{e^{-i\alpha_0\zeta}d\zeta}{\sqrt{\zeta^2-a^2}(\zeta^2-a^2+b)^2} \approx \frac{e^{-i\alpha_0 a}}{\sqrt{2a}b^2}\int_0^\infty\frac{e^{-i\alpha_0 t}dt}{\sqrt{t}} \\ &= \sqrt{\frac{\pi}{2\alpha_0}}\frac{e^{-i(\pi/4)}}{\sqrt{a}b^2}e^{-i\alpha_0 a}, \end{aligned}$$

where the substitution $t = \zeta - a$ was made and the oscillatory nature of the integrand exploited. This yields

$$\begin{aligned} I_r^{(\mathrm{V})} &\approx J_r(\beta)\sqrt{\frac{2\pi}{\alpha_0}}\epsilon\rho_0e^{-i(\pi/4)}\int_{\alpha_0\rho_0^2}^\infty\frac{(1+x^2)^{5/4}}{\sqrt{x}(x^2+\epsilon^2)^2} \\ &\quad\times e^{-i\alpha_0(x/\sqrt{1+x^2})}dx. \end{aligned}$$

The main contribution to this integral stems from the lower boundary since for $x \gg 1$, where the oscillations fade away, the integrand is damped by the power $x^{-13/4}$. For this reason we obtain

$$I_r^{(\mathrm{V})} \approx J_r(\beta)\sqrt{2\pi}\frac{e^{-i(3\pi/4)}e^{-i\alpha_0^2\rho_0^2}}{\alpha_0^2\epsilon^3}. \tag{A13}$$

In summary we have shown that

$$I_r^{(\mathrm{I})} \sim I_r^{(\mathrm{II})} \sim 10^{-12}J_r(\beta), \qquad I_r^{(\mathrm{III})} \sim I_r^{(\mathrm{V})} \sim 10^{-10}J_r(\beta),$$
$$I_r^{(\mathrm{IV})} \sim 10^{-9}J_r(\beta),$$

i.e., the main contribution to the integral (A4) comes from region IV. Consequently,

$$\bar{J}_r \approx 2\,\mathrm{Re}I_r^{(\mathrm{IV})} = -\frac{2}{\pi^{3/2}a_0^{3/2}\sqrt{V}}\frac{\beta^{1/3}}{\alpha_0^2\epsilon^2}J_r(\beta)\sin(\beta^{2/3}) \tag{A14}$$

where we have restored the factor $2/(\pi^5a_0^3V)^{1/2}$ that was dropped from Eq. (A2).

We note that Eq. (A14) introduces a fast oscillating factor $\sin^2(\beta^{2/3})$ to the differential reaction rate. Since $\beta \sim 2m\xi^2/\omega$ these oscillations depend on the laser intensity and frequency [see also Eq. (32)]. In reality, however, neither of these parameters has a definite value in a short laser pulse but spreads over a certain range. In an experiment the oscillations are therefore averaged out, and we shall do the same:

$$\bar{J}_r \approx -\frac{\sqrt{2}}{\pi^{3/2} a_0^{3/2}\sqrt{V}} \frac{\beta^{1/3}}{\alpha_0^2 \epsilon^2} J_r(\beta). \tag{A15}$$

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[23] We employ the metric tensor $g^{\mu\nu} = \mathrm{diag}(+---)$, such that the scalar product of two four-vectors $a = (a^0, \boldsymbol{a})$ and $b = (b^0, \boldsymbol{b})$ reads $(ab) = a^0 b^0 - \boldsymbol{ab}$, which sometimes is also denoted by $a \cdot b$ in order to avoid too many parentheses. Furthermore, Feynman slash notation is used, i.e., $\not{a} = (\gamma a)$.

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[33] The approximate treatment of the δ function can be justified as follows. We can write the effective momentum as $q^\mu = q^\mu(0) + \Delta q^\mu(p)$ with a p-independent part $q^\mu(0)$ and

$$\Delta q^\mu(p) \approx -\xi^2 \frac{\boldsymbol{p}^2}{4m} \frac{k^\mu}{\omega}.$$

Since $\Delta q^0(p) \sim \alpha_f^2 \xi^2 m \ll q^0(0)$, we can perform a Taylor expansion of the δ function, the next-to-leading-order term of which being proportional to $\Delta q^0(p)$. Taking this additional term into account in the square of the reaction amplitude, results in a correction term which is suppressed by a factor of $\Delta q^0(\bar{p})/\bar{Q}_0 \sim \alpha_f^2 \ll 1$, where $\bar{p}$, $\bar{Q}_0$ denote some typical values of p and Q_0 [cf. Eq. (34)].
[34] The equation $r = \beta(r)$ has two solutions: $r_\pm = 2\xi^2(m/\omega)(1 \pm \sqrt{1-\kappa^2})$. At large r, the function $J_r(\beta)$ is of significant value only in the region $\delta \equiv |r - \beta(r)| \sim r^{1/3}$ around $r_\pm$. The region is much larger around r_+ than around r_-. In fact, $\delta_+ \equiv |r - r_+| \approx 2\delta$ while $\delta_- \equiv |r - r_-| \approx \delta \cdot (M^2/2m^2\xi^2)$ for $\xi \gg M/m$. Therefore, the main contribution to the process probability comes from $r \approx r_+$.
[35] With factors of $\hbar$ and c restored, formula (51) reads

$$\tilde{R}_{\mathrm{Ps}\to\mu^+\mu^-} \approx \frac{3^2}{2^6\pi^4} \frac{\alpha_f^2}{\xi^2} \left(\frac{\hbar\omega}{m_* c^2}\right)^2 \left(\frac{\hbar\omega}{M_* c^2}\right)^2 \left(\frac{4mc^2\xi^2}{\hbar\omega}\right)^{2/3} \frac{c}{r_e}.$$

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