Transverse momentum dependence of the quark helicity distributions and the Cahn effect in double-spin asymmetry A_{LL} in semiinclusive DIS

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Within the LO QCD parton model of Semi-Inclusive Deep Inelastic Scattering, $\ell N \to \ell h X$, with unintegrated quark distribution and fragmentation functions, we study the $\boldsymbol{P}_{hT}$ dependence of the double longitudinal-spin asymmetry A_{LL}. We include $1/Q$ kinematic corrections, which induce an azimuthal modulation of the asymmetry, analogous to the Cahn effect in unpolarized SIDIS. We show that a study of A_{LL} and of the weighted DSA $A_{LL}^{\cos\phi_h}$ allows to extract the transverse momentum dependence of the unintegrated helicity distribution function $g_{1L}^q(x, k_\perp)$ [or $\Delta q(x, k_\perp)$]. Predictions, based on some models for the unknown functions, are given for ongoing COMPASS, HERMES and JLab experiments.

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I. INTRODUCTION

After many years of intensive experimental and theoretical study the partonic origin of the nucleon spin still remains mysterious. In particular, while a lot of understanding has been achieved concerning the longitudinal structure of a fast moving proton-the x-dependence of the parton distribution functions and of the helicity distributions-very little is known about the transverse structure. Transverse refers to the direction of motion and concerns both the transverse spin distributions and the parton intrinsic motion, $\boldsymbol{k}_\perp$. These unexplored degrees of freedom cannot be considered as minor details in our modeling of the nucleon. Without a good knowledge of the total intrinsic momentum carried by the partons, and its connection with the spin, we could never understand the parton orbital motion and progress towards a more structured picture which goes beyond the simple collinear partonic representation.

Recent data on single spin azimuthal asymmetries in semi-inclusive deep inelastic scattering (SIDIS, $\ell N \to \ell h X$) obtained by HERMES [1], COMPASS [2] and CLAS-JLab [3] collaborations triggered a lot of interest towards the transverse momentum dependent (TMD) and spin dependent distribution and fragmentation functions (PDFs and FFs). In fact spin-$\boldsymbol{k}_\perp$ correlations induce new spin effects, which would be zero in the absence of intrinsic motion and are related to transverse polarizations. On the other hand, it has been known for a long time that the dependence on the transverse final hadron momentum, $\boldsymbol{P}_{hT}$, observed in unpolarized SIDIS processes, can be related, for $P_{hT} = |\boldsymbol{P}_{hT}|$ values up to about 1 GeV$/c$, to quark intrinsic motion [4–6].

We consider here polarized SIDIS processes, at twist-two in the parton model, with transverse momentum dependent distribution and fragmentation functions. Such processes can be described in terms of six time reversal even [7,8] and two (naïvely) time reversal odd PDFs. The dependence on partonic intrinsic motion induces a dependence on P_{hT}. In addition, at $\mathcal{O}(k_\perp/Q)$, kinematic corrections induce a dependence of the unpolarized cross section on the azimuthal angle ϕ_h between the leptonic and the hadron production planes-the so called Cahn effect [4]. It was shown in Ref. [5] that a careful study of the dependence of the cross section on the final hadron momentum allows to extract the average values of intrinsic momenta in unpolarized PDFs and FFs.

We expand on the work of Ref. [5] and evaluate the role of partonic intrinsic motion in polarized SIDIS;, in particular, on the double-spin asymmetry (DSA) for the scattering of longitudinally polarized leptons off a longitudinally polarized target, A_{LL}, where longitudinal refers to the incoming lepton direction, in the laboratory frame. We show that a study of A_{LL} and of the weighted asymmetry $A_{LL}^{\cos\phi_h}$ allows to learn about the $k_\perp$ dependence of the quark helicity distribution $g_{1L}^q(x, k_\perp)$ [or $\Delta q(x, k_\perp)$]. Similar results for the TMD DF g_{1T}^q have been obtained in a recent paper [9], by considering the longitudinal-transverse DSA $A_{LT}^{\cos(\phi_h - \phi_S)}$.

The article is organized as follows. In Sec. II we shortly recall the relevant formalism for polarized SIDIS. In Sec. III some predictions for the double longitudinal-spin asymmetries are presented. The results are given for different sets of kinematical cuts, according to the experimental setups of HERMES, COMPASS and JLab experiments; they indicate the best kinematical regions for the asymmetry to be sizeable. Finally, in Sec. IV we shortly discuss our results and draw some conclusions.

II. POLARIZED CROSS SECTION

Following Ref. [7], we consider the polarized SIDIS in the simple quark-parton model, with unintegrated parton distributions. We use the standard notations for DIS variables: ℓ and ℓ' are, respectively, the four-momenta of the initial and the final state leptons; $q = \ell - \ell'$ is the exchanged virtual photon momentum; P (M) is the target nucleon momentum (mass), S its polarization; P_h is the final hadron momentum; $Q^2 = -q^2$; $x = Q^2/2P \cdot q$; $y = P \cdot q/P \cdot \ell$; $z = P \cdot P_h/P \cdot q$, $Q^2 = xy(s - M^2)$, $s = (\ell + P)^2$. We work in a frame with the z-axis along the virtual photon momentum direction and the x-axis in the lepton scattering plane, with positive direction chosen along the lepton transverse momentum. The produced hadron has transverse momentum $\boldsymbol{P}_{hT}$; its azimuthal angle, ϕ_h, and the azimuthal angle of the transverse nucleon spin, ϕ_S, are measured around the z-axis (for further details see Ref. [7]).

We consider longitudinally polarized protons and leptons, where longitudinal (according to the laboratory setup) refers to the initial lepton direction. It then results that a proton with longitudinal-spin S along the incoming lepton direction, has a transverse-with respect to the γ^* direction-spin component:

$$S_T = S \sin\theta_\gamma, \tag{1}$$

where

$$\sin\theta_\gamma = \sqrt{\frac{4M^2x^2}{Q^2 + 4M^2x^2}\left(1 - y - \frac{M^2x^2y^2}{Q^2}\right)} \simeq \frac{2Mx\sqrt{1-y}}{Q}. \tag{2}$$

This component gives contributions of order M/Q.

Keeping only twist-two contributions and terms up to $\mathcal{O}(M/Q)$ the cross section for SIDIS of longitudinally polarized leptons off a longitudinally polarized target can be written as:

$$\frac{d^5\overset{\rightarrow}{\sigma}{}^{\Leftarrow}}{dxdydzd^2P_{hT}} = \frac{2\alpha^2}{xy^2s}\{\mathcal{H}_{f_1} + \lambda(S_L\mathcal{H}_{g_{1L}} + S_T\mathcal{H}_{g_{1T}})\}, \tag{3}$$

where the arrows indicate the direction of the lepton ($\rightarrow$) and target nucleon ($\Leftarrow$) polarizations, with respect to the lepton momentum; λ, S_L and S_T are the magnitudes of, respectively: the longitudinal beam polarization, the longitudinal and the transverse target polarization. Notice that $\Leftarrow$ stands for a nucleon with a polarization vector, in the laboratory frame where the nucleon is at rest, *opposite* to the initial lepton momentum. For a $\Rightarrow$ polarization one reverses the signs of the S_L and S_T terms.

The three terms have a simple partonic interpretation:

$$\mathcal{H}_{f_1} = \sum_q e_q^2 \int d^2\boldsymbol{k}_\perp f_1^q(x, k_\perp)\pi y^2 \frac{\hat{s}^2 + \hat{u}^2}{Q^4} D_q^h(z, p_\perp), \tag{4}$$

$$\mathcal{H}_{g_{1L}} = \sum_q e_q^2 \int d^2\boldsymbol{k}_\perp g_{1L}^q(x, k_\perp)\pi y^2 \frac{\hat{s}^2 - \hat{u}^2}{Q^4} D_q^h(z, p_\perp), \tag{5}$$

$$\mathcal{H}_{g_{1T}} = -\sum_q e_q^2 \int d^2\boldsymbol{k}_\perp \frac{k_\perp}{M}\cos\varphi\, g_{1T}^{q\perp}(x, k_\perp)\pi y^2 \frac{\hat{s}^2 - \hat{u}^2}{Q^4} \times D_q^h(z, p_\perp), \tag{6}$$

and deserve some comments.

(i) The partonic factorized structure of the above equations is supposed to hold in the large Q^2 kinematic region where $P_{hT} \simeq \Lambda_{\rm QCD} \simeq k_\perp \ll Q$ [10]. It neglects terms of $\mathcal{O}(k_\perp/Q)^2$, in which case

$$\boldsymbol{p}_\perp = \boldsymbol{P}_{hT} - z\boldsymbol{k}_\perp,$$

where $\boldsymbol{p}_\perp$ is the intrinsic transverse momentum of the hadron h with respect to the fragmenting quark direction.

(ii) The first two contributions, Eqs. (4) and (5), give, respectively, the unpolarized cross section and the helicity asymmetry

$$\begin{aligned} \frac{d^5\sigma}{dxdydzd^2P_{hT}} &= \frac{2\alpha^2}{xy^2s}\mathcal{H}_{f_1} \\ \frac{d^5\sigma^{++}}{dxdydzd^2P_{hT}} - \frac{d^5\sigma^{+-}}{dxdydzd^2P_{hT}} &= \frac{4\alpha^2}{xy^2s}\mathcal{H}_{g_{1L}}, \end{aligned} \tag{7}$$

where $+$, $-$ stand for helicity states. The quark intrinsic motion induces a *kinematical azimuthal dependence*, via the elementary polarized cross section [7]

$$\frac{d\sigma^{lq\to lq}}{dQ^2d\varphi} \propto \frac{\hat{s}^2 + \hat{u}^2 + \lambda\lambda_q(\hat{s}^2 - \hat{u}^2)}{\hat{t}^2}, \tag{8}$$

where λ_q denotes the quark helicity. Keeping the terms up to order of $k_\perp/Q$ the Mandelstam variables for the noncoplanar $\ell q \to \ell q$ scattering are expressed as

$$\begin{aligned} \hat{s} &\simeq xs\left[1 - 2\sqrt{1-y}\frac{k_\perp}{Q}\cos\varphi\right], \\ \hat{t} &= -Q^2 = -xys, \\ \hat{u} &\simeq -xs(1-y)\left[1 - \frac{2k_\perp}{Q\sqrt{1-y}}\cos\varphi\right], \end{aligned} \tag{9}$$

where φ is the azimuthal angle of $\boldsymbol{k}_\perp$, $d^2\boldsymbol{k}_\perp = k_\perp dk_\perp d\varphi$. Equation (4) then gives the unpolarized Cahn effect [4], while Eq. (5) gives the corresponding effect for the polarized (helicity) cross section, both at $\mathcal{O}(k_\perp/Q)$.

(iii) Equation (6) contains another $\cos\varphi$ dependence, of different origin. While the distribution functions $f_1^q(x, k_\perp)$ and $g_{1L}^q(x, k_\perp)$ which appear in Eqs. (4) and (5), are just the $k_\perp$ dependent unpolarized and longitudinally polarized (helicity) PDFs, which, upon integration over $d^2\boldsymbol{k}_\perp$, give the usual $f_1^q(x)$ [or $q(x)$] and $g_1^q(x)$ [or $\Delta q(x)$] distributions, the quantity

$$-\frac{k_\perp}{M}\cos\varphi\, g_{1T}^{q\perp}(x, k_\perp) = \Delta\hat{f}_{s_z/S_T} \tag{10}$$

is related to the number of partons longitudinally polarized inside a transversely polarized proton [7,8,11]: it can only depend on the scalar product between the two corresponding polarization vectors, which gives the $\cos(\phi_{S_T} - \varphi) = -\cos\varphi$ factor explicitly shown (see, for example, Eq. (C19) of Ref. [11]). This distribution is a leading-twist one, not suppressed by $(k_\perp/Q)$ small factors. However, Eq. (6) will be multiplied by S_T, which is of $\mathcal{O}(M/Q)$, Eqs. (1)–(3); for this reason, in Eq. (6) we shall not take into account the extra $(k_\perp/Q)$ kinematical terms contained in $(\hat{s}^2 - \hat{u}^2)$ of Eq. (8).

The integrals in Eqs. (4)–(6) can be analytically performed, if one assumes a simple factorized and gaussian behavior of the involved TMD PDFs and FFs

$$f_1^q(x, k_\perp) = f_1^q(x)\frac{1}{\pi\mu_0^2}\exp\left(-\frac{k_\perp^2}{\mu_0^2}\right), \tag{11}$$

$$D_q^h(z, p_\perp) = D_q^h(z)\frac{1}{\pi\mu_D^2}\exp\left(-\frac{p_\perp^2}{\mu_D^2}\right), \tag{12}$$

$$g_{1T}^{q\perp}(x, k_\perp) = g_{1T}^q(x)\frac{1}{\pi\mu_1^2}\exp\left(-\frac{k_\perp^2}{\mu_1^2}\right), \tag{13}$$

$$g_{1L}^q(x, k_\perp) = g_1^q(x)\frac{1}{\pi\mu_2^2}\exp\left(-\frac{k_\perp^2}{\mu_2^2}\right), \tag{14}$$

yielding, at $\mathcal{O}(P_{hT}/Q)$:

$$\mathcal{H}_{f_1} = \left[1 + (1-y)^2 - 4(2-y)\sqrt{1-y}\,\frac{z\mu_0^2 P_{hT}}{Q(\mu_D^2 + z^2\mu_0^2)} \times \cos\phi_h\right]\frac{\exp(-\frac{P_{hT}^2}{\mu_D^2+z^2\mu_0^2})}{\mu_D^2 + z^2\mu_0^2}\sum_q e_q^2 f_1^q(x) D_q^h(z), \tag{15}$$

$$\mathcal{H}_{g_{1L}} = y\left[2 - y - 4\sqrt{1-y}\,\frac{z\mu_2^2 P_{hT}}{Q(\mu_D^2 + z^2\mu_2^2)}\cos\phi_h\right] \times \frac{\exp(-\frac{P_{hT}^2}{\mu_D^2+z^2\mu_2^2})}{\mu_D^2 + z^2\mu_2^2}\sum_q e_q^2 g_1^q(x) D_q^h(z), \tag{16}$$

$$\mathcal{H}_{g_{1T}} = -y(2-y)\frac{z\mu_1^2 P_{hT}}{M(\mu_D^2 + z^2\mu_1^2)} \times \cos\phi_h \frac{\exp(-\frac{P_{hT}^2}{\mu_D^2+z^2\mu_1^2})}{\mu_D^2 + z^2\mu_1^2}\sum_q e_q^2 g_{1T}^q(x) D_q^h(z). \tag{17}$$

III. PREDICTIONS FOR A_{LL}

We use Eqs. (3) and (15)–(17) to compute observables which depend on partonic intrinsic motions. Notice that we have allowed different average values of $\langle k_\perp^2\rangle$ for the different distribution functions: $\langle k_\perp^2\rangle = \mu_0^2$ for the unpolarized distributions, $\langle k_\perp^2\rangle = \mu_2^2$ for the helicity distributions, and $\langle k_\perp^2\rangle = \mu_1^2$ for $g_{1T}^{q\perp}(x, k_\perp)$; each of these value is taken to be constant and flavour independent. For the fragmentation functions we have $\langle p_\perp^2\rangle = \mu_D^2$. Following Ref. [5] we use

$$\mu_0^2 = 0.25(\mathrm{GeV}/c)^2 \qquad \mu_D^2 = 0.20(\mathrm{GeV}/c)^2, \tag{18}$$

while we consider μ_1^2 and μ_2^2 as free parameters, which can give interesting information on the quark transverse motion in polarized protons; the naïve positivity bounds imply that we should have

$$\mu_1^2 \le \mu_0^2 \qquad \mu_2^2 \le \mu_0^2. \tag{19}$$

Our approach is supposed to hold up to $P_{hT} \simeq 1$ (GeV/c) [6]. Above that higher order pQCD corrections must be taken into account, and lead to tiny variations of the values given in Eq. (18) [6]; however, we shall consider experiments which are expected to produce data mainly in the low P_{hT} region, and both our approach and $\mu_{0,D}^2$ values are well adequate.

We consider the P_{hT} dependence of the double longitudinal-spin asymmetry

$$A_{LL}(x, y, z, P_{hT}) = \frac{\int_0^{2\pi} d\phi_h [d\sigma^{\stackrel{\rightarrow}{\Leftarrow}} - d\sigma^{\stackrel{\rightarrow}{\Rightarrow}}]}{\lambda S \int_0^{2\pi} d\phi_h [d\sigma^{\stackrel{\rightarrow}{\Leftarrow}} + d\sigma^{\stackrel{\rightarrow}{\Rightarrow}}]}, \tag{20}$$

and the $\cos\phi_h$ weighted asymmetry, defined as

$$A_{LL}^{\cos\phi_h}(x, y, z, P_{hT}) = \frac{2\int_0^{2\pi} d\phi_h [d\sigma^{\stackrel{\rightarrow}{\Leftarrow}} - d\sigma^{\stackrel{\rightarrow}{\Rightarrow}}]\cos\phi_h}{\lambda S \int_0^{2\pi} d\phi_h [d\sigma^{\stackrel{\rightarrow}{\Leftarrow}} + d\sigma^{\stackrel{\rightarrow}{\Rightarrow}}]}. \tag{21}$$

From Eqs. (15)–(17) one has

$$A_{LL}(x, y, z, P_{hT}) = \frac{\Delta\sigma_{LL}}{\sigma_0}, \tag{22}$$

with

$$\Delta\sigma_{LL} = \frac{y(2-y)}{xy^2}\frac{1}{\mu_D^2 + z^2\mu_2^2} \times \exp\left(-\frac{P_{hT}^2}{\mu_D^2 + z^2\mu_2^2}\right)\sum_q e_q^2 g_1^q(x) D_q^h(z). \quad (23)$$

and

$$\sigma_0 = \frac{1+(1-y)^2}{xy^2}\frac{1}{\mu_D^2 + z^2\mu_0^2} \times \exp\left(-\frac{P_{hT}^2}{\mu_D^2 + z^2\mu_0^2}\right)\sum_q e_q^2 f_1^q(x) D_q^h(z). \quad (24)$$

Analogously, Eqs. (2) and (15)–(17) give

$$A_{LL}^{\cos\phi_h}(x, y, z, P_{hT}) = \frac{\Delta\sigma_{LL}^{\cos\phi_h} + \Delta\sigma_{LT}^{\cos\phi_h}}{\sigma_0}, \quad (25)$$

where the contribution from the longitudinal part of the target polarization is given by

$$\Delta\sigma_{LL}^{\cos\phi_h} = -4\frac{\sqrt{1-y}}{xy}\frac{z\mu_2^2 P_{hT}}{Q(\mu_D^2 + z^2\mu_2^2)^2} \times \exp\left(-\frac{P_{hT}^2}{\mu_D^2 + z^2\mu_2^2}\right)\sum_q e_q^2 g_1^q(x) D_q^h(z), \quad (26)$$

and the contribution from the transverse part of the target polarization by

$$\Delta\sigma_{LT}^{\cos\phi_h} = \frac{-2(2-y)\sqrt{1-y}}{y}\frac{z\mu_1^2 P_{hT}}{Q(\mu_D^2 + z^2\mu_1^2)^2} \times \exp\left(-\frac{P_{hT}^2}{\mu_D^2 + z^2\mu_1^2}\right)\sum_q e_q^2 g_{1T}^q(x) D_q^h(z). \quad (27)$$

Of course, both the numerator and denominator of Eqs. (20) and (21) can be integrated over some of the variables, according to the range covered by the setups of the experiments we shall consider:

(i) COMPASS: positive (h^+), all (h) and negative (h^-) hadron production, $Q^2 > 1.0\ (\mathrm{GeV}/c)^2$, $W^2 > 25\ \mathrm{GeV}^2$, $0.1 < x < 0.6$, $0.5 < y < 0.9$ and $0.4 < z < 0.9$

(ii) HERMES: π^+, π^0 and π^- production, $Q^2 > 1.0\ (\mathrm{GeV}/c)^2$, $W^2 > 10\ \mathrm{GeV}^2$, $0.1 < x < 0.6$, $0.45 < y < 0.85$ and $0.4 < z < 0.7$

(iii) JLab at 6 GeV: π^+, π^0 and π^- production, $Q^2 > 1.0\ (\mathrm{GeV}/c)^2$, $W^2 > 4\ \mathrm{GeV}^2$, $0.2 < x < 0.6$, $0.4 < y < 0.85$ and $0.4 < z < 0.7$.

We start by considering Eqs. (22)–(24). Notice that they are leading-twist quantities, not suppressed by any inverse power of Q. Concerning the usual integrated distribution and fragmentation functions we use the LO GRV98 [12] unpolarized and the corresponding GRSV2000 [13] polarized (standard scenario) DFs, and Kretzer [14] FFs. We can then compute the P_{hT} dependence of A_{LL}, depending on the only unknown quantity μ_2^2. We plot the results of our computations in Figs. 1 and 2, for a proton and deuteron (+ neutron, for JLab) target, respectively.

The results depend clearly on the relative values of $\langle k_\perp^2\rangle$ for the unpolarized and helicity distribution, μ_0^2 and μ_2^2 respectively: $A_{LL}(P_{hT})$ is approximately constant if $\mu_2^2 = \mu_0^2 = 0.25\ (\mathrm{GeV}/c)^2$, whereas it sharply decreases with P_{hT} if $\mu_2^2 < \mu_0^2$. The trend of $A_{LL}(P_{hT})$ is thus a significant indication of the average quark transverse motion inside unpolarized versus longitudinally polarized nucleons. Although our numerical estimates are based on the gaussian factorization ansatz, Eqs. (11)–(14), we expect them to have a more general interpretation and information content. The P_{hT} dependence of A_{LL} reflects, essentially, the difference between the $k_\perp$ dependence of $f_1^q(x, k_\perp)$ and $g_{1L}^q(x, k_\perp)$, independently of their functional forms; the trend of $A_{LL}(P_{hT})$, whether constant or decreasing, reveals the behavior of $g_{1L}^q(x, k_\perp)/f_1^q(x, k_\perp)$ and their relative $k_\perp$ dependence.

Similarly, we can use Eqs. (24)–(27) in order to give some estimates of $A_{LL}^{\cos\phi_h}$. Notice that $\Delta\sigma_{LL}^{\cos\phi_h}$ and $\Delta\sigma_{LT}^{\cos\phi_h}$ are (kinematical) higher-twist quantities, proportional to P_{hT}/Q; in addition, $\Delta\sigma_{LT}^{\cos\phi_h}$ contains one unknown function, namely $g_{1T}^q(x)$, related to the helicity distribution of partons inside a transversely polarized proton. In the absence of any better guidance, we adopt the same strategy as in Ref. [9]. We start by noticing that, from Eq. (13):

$$g_{1T}^{q(1)}(x) \equiv \int d^2k_\perp \frac{k_\perp^2}{2M^2} g_{1T}^{q\perp}(x, k_T^2) = \frac{\mu_1^2}{2M^2} g_{1T}^q(x). \quad (28)$$

According to Refs. [8,15] $g_{1T}^{q(1)}(x)$ is directly related to the DF $g_2^q(x)$, which has both twist-two and higher-twist contributions,

$$g_2^q(x) = \frac{d}{dx} g_{1T}^{q(1)}(x). \quad (29)$$

This relation, although much debated, arises from constraints imposed by Lorentz invariance on the antiquark-target forward scattering amplitude and the use of QCD equations of motion for quark fields [8]. If, in addition, one uses the Wandzura and Wilczek [16] approximation for the twist-two part of $g_2^q(x)$,

$$g_2^q(x) \simeq -g_1^q(x) + \int_x^1 dx' \frac{g_1^q(x')}{x'}, \quad (30)$$

the following relation can be derived [17],

$$g_{1T}^{q(1)}(x) \simeq x \int_x^1 dx' \frac{g_1^q(x')}{x'}, \quad (31)$$

which, via Eq. (28), allows to express $g_{1T}^q(x)$ through the well known integrated helicity distributions.

Although such a procedure is appealing and convenient, we should stress there are strong arguments [18–20] against the validity of the relation (29). Therefore, we

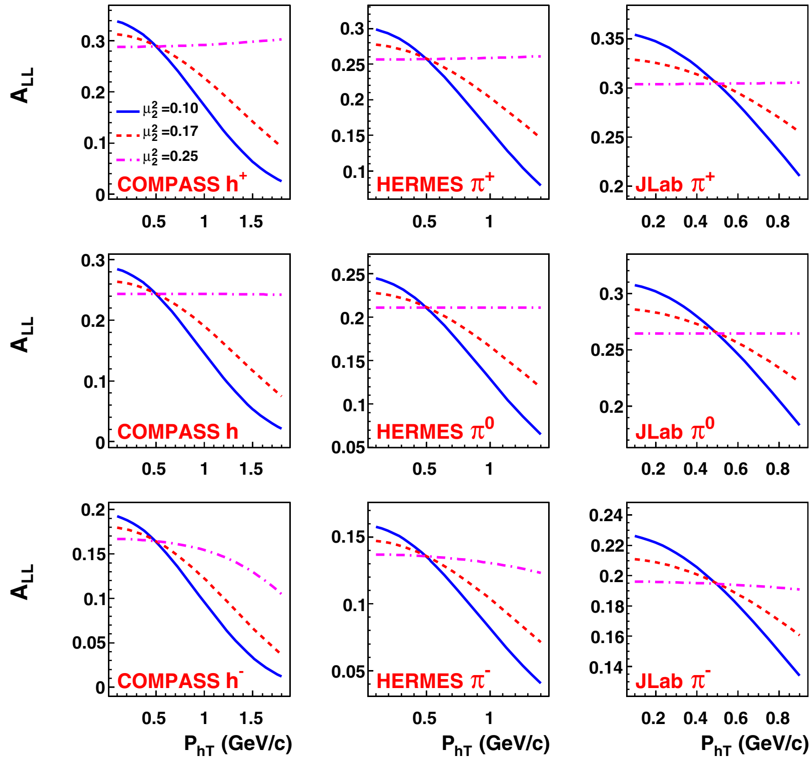

FIG. 1 (color online). Predicted dependence of A_{LL} on P_{hT}, for scattering off a proton target, with different choices of μ_2^2: 0.1 $(\mathrm{GeV}/c)^2$-continuous, 0.17 $(\mathrm{GeV}/c)^2$-dashed and 0.25 $(\mathrm{GeV}/c)^2$-dot-dashed lines.

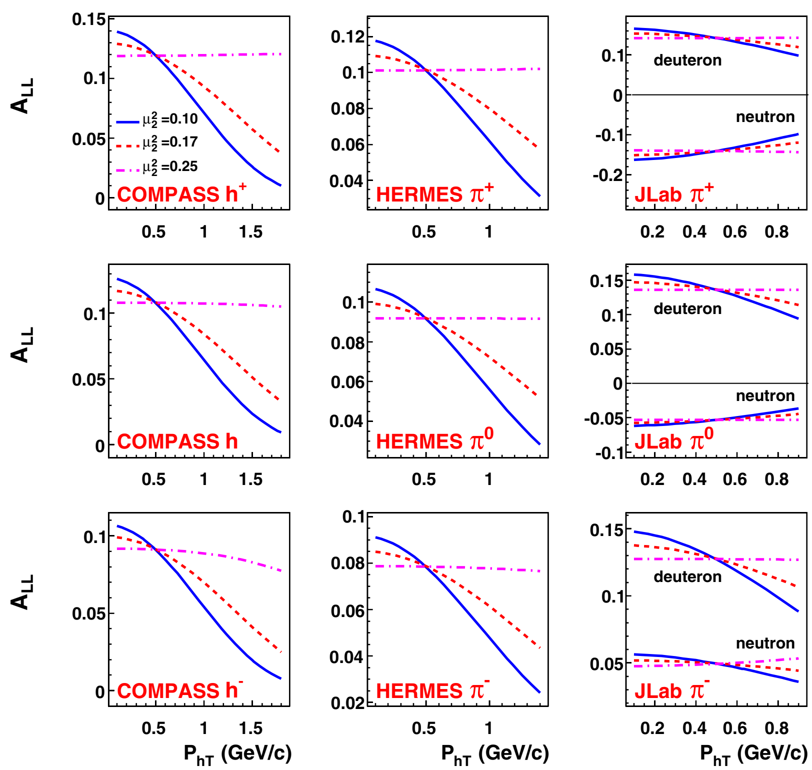

FIG. 2 (color online). Predicted dependence of A_{LL} on P_{hT}, for scattering off a deuteron (and neutron for JLab) target, with different choices of μ_2^2: 0.1 $(\mathrm{GeV}/c)^2$-continuous, 0.17 $(\mathrm{GeV}/c)^2$-dashed and 0.25 $(\mathrm{GeV}/c)^2$-dot-dashed lines.

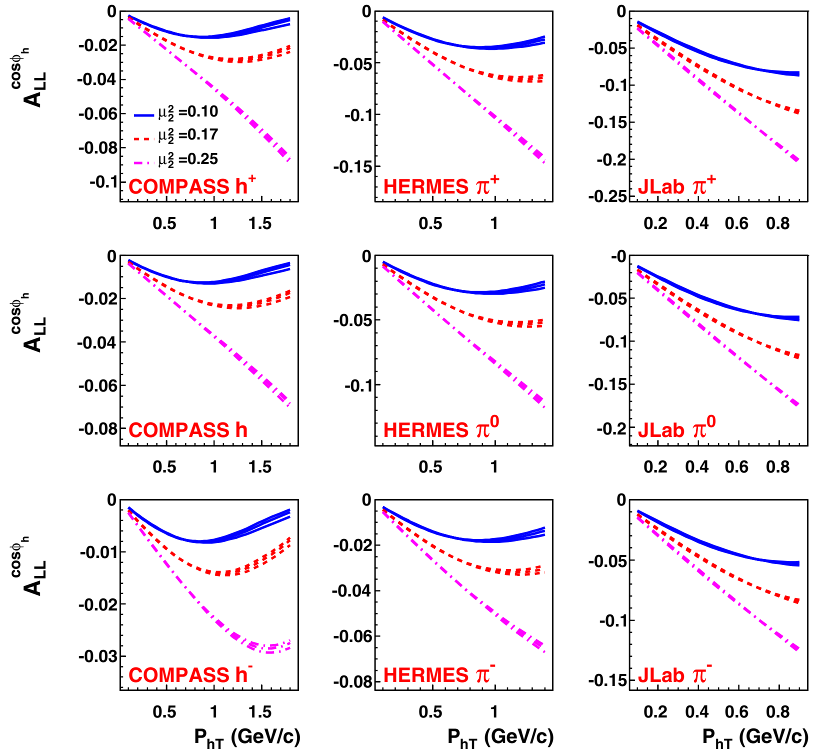

FIG. 3 (color online). Predicted dependence of $A_{LL}^{\cos\phi_h}$ on P_{hT} for scattering off a proton target with different choices of μ_2^2: 0.1 $(\mathrm{GeV}/c)^2$-continuous, 0.17 $(\mathrm{GeV}/c)^2$-dashed and 0.25 $(\mathrm{GeV}/c)^2$-dot-dashed lines. Each line splits into three almost overlapping lines corresponding, for each value of μ_2^2, to three different values of $\mu_1^2 =$ (up-down) 0.1, 0.15 and 0.2 $(\mathrm{GeV}/c)^2$.

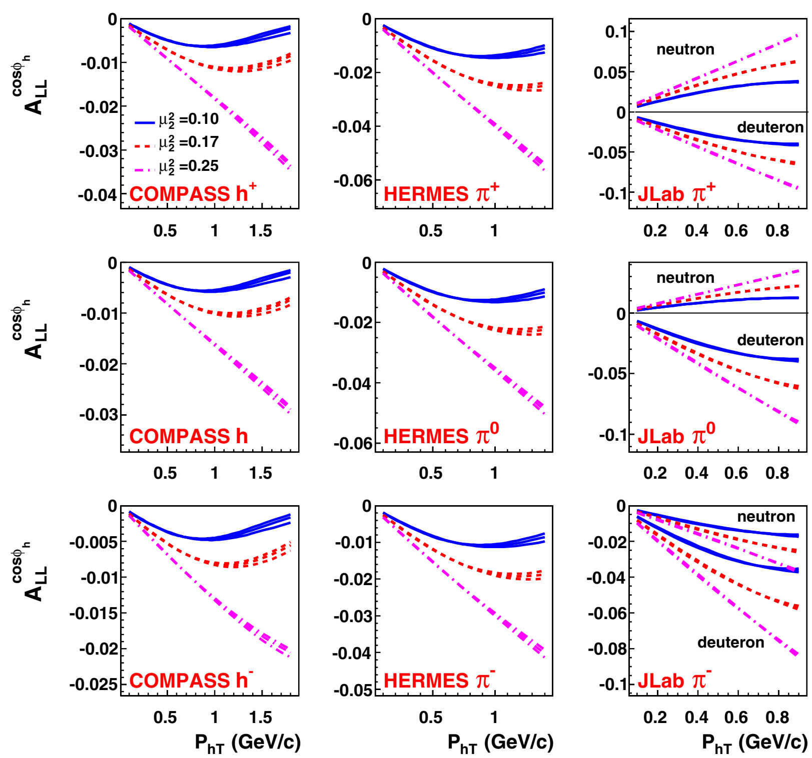

FIG. 4 (color online). Predicted dependence of $A_{LL}^{\cos\phi_h}$ on P_{hT} for scattering off a deuteron (and neutron for JLab) target with different choices of μ_2^2: 0.1 $(\mathrm{GeV}/c)^2$-continuous, 0.17 $(\mathrm{GeV}/c)^2$-dashed and 0.25 $(\mathrm{GeV}/c)^2$-dot-dashed lines. Each line splits into three almost overlapping lines corresponding, for each value of μ_2^2, to three different values of $\mu_1^2 =$ (up-down) 0.1, 0.15 and 0.2 $(\mathrm{GeV}/c)^2$.

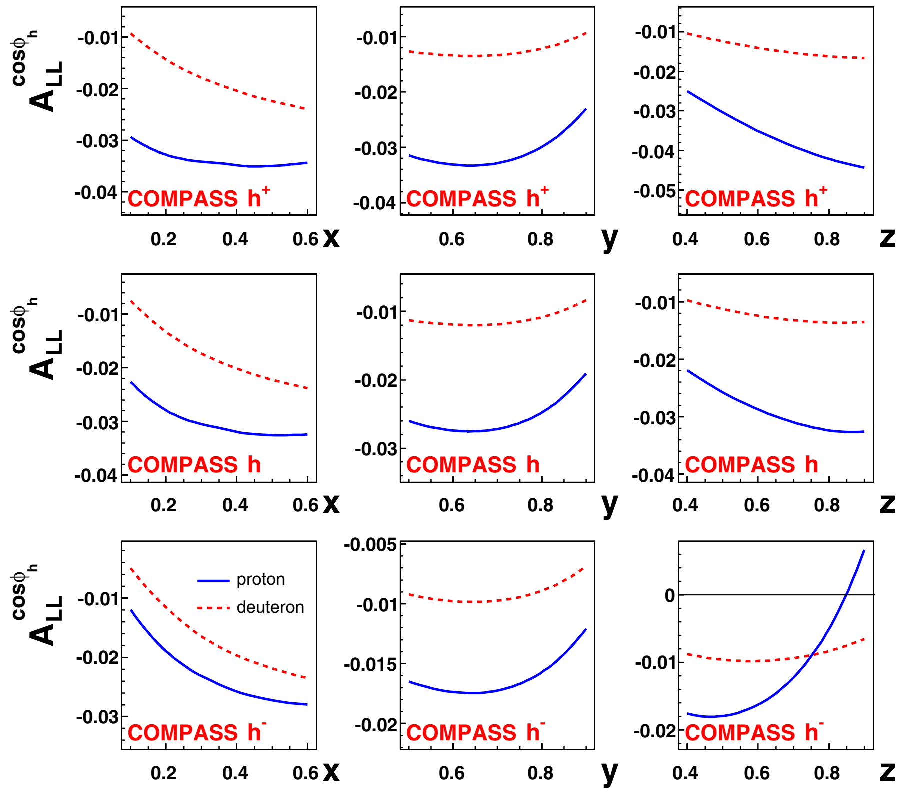

FIG. 5 (color online). Predicted dependence of $A_{LL}^{\cos\phi_h}$ on x, y and z, for proton and deuteron targets, for COMPASS.

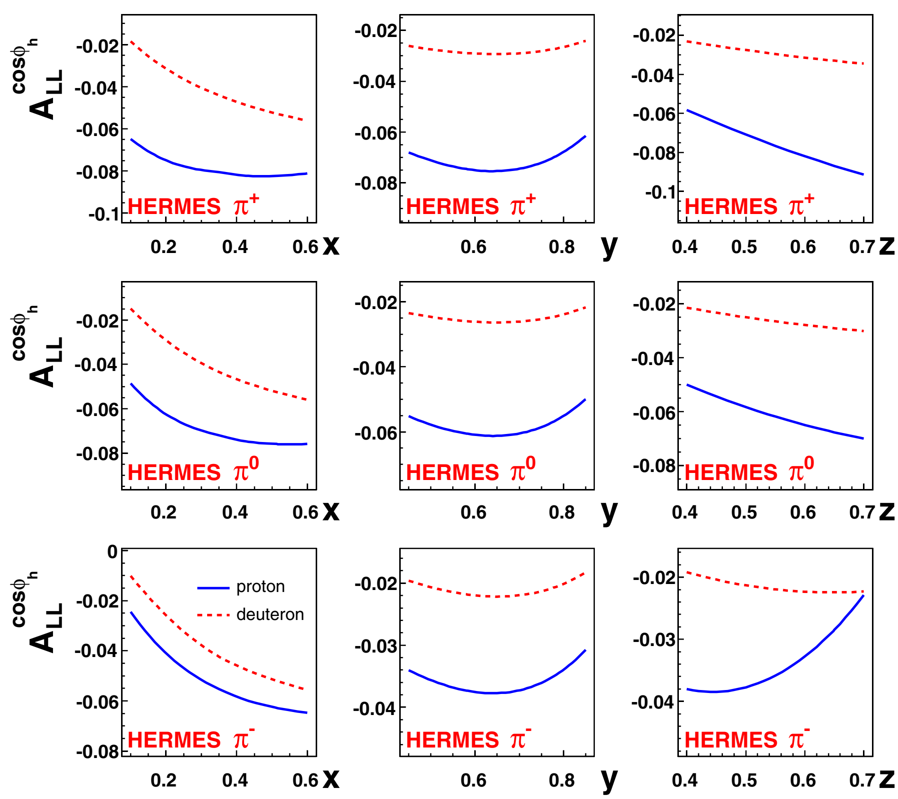

FIG. 6 (color online). Predicted dependence of $A_{LL}^{\cos\phi_h}$ on x, y and z, for proton and deuteron targets, for HERMES.

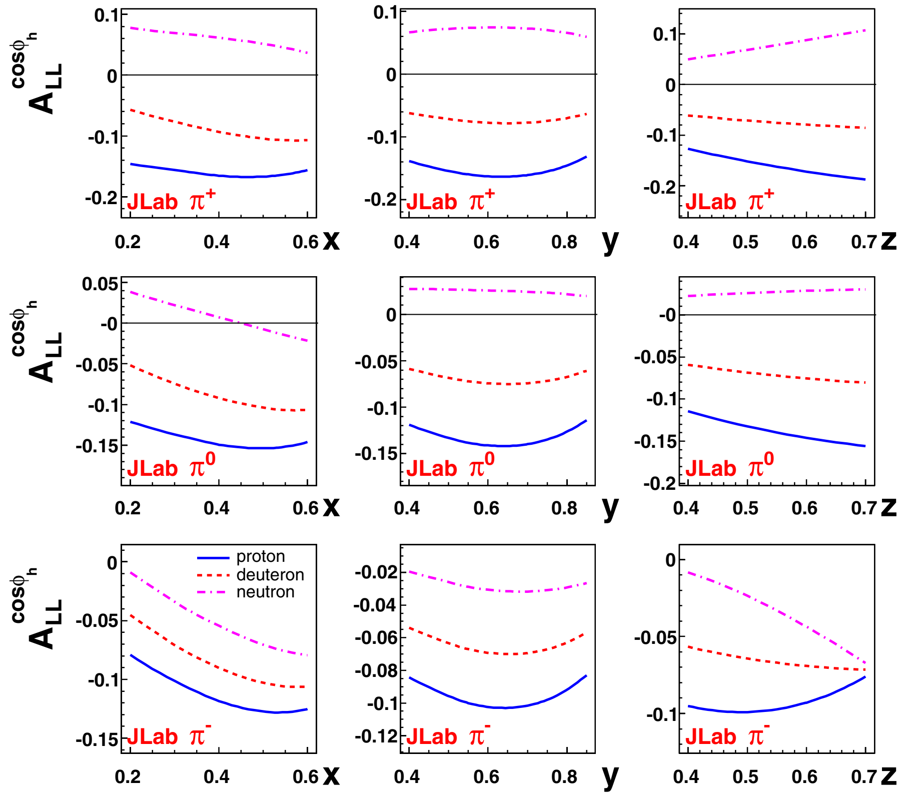

FIG. 7 (color online). Predicted dependence of $A_{LL}^{\cos\phi_h}$ on x, y and z, for proton, neutron and deuteron targets, for JLab.

should consider the above expression, Eq. (31), only as a rough model for the otherwise unknown function $g_{1T}^q(x)$.

In Fig. 3 we show our predictions for $A_{LL}^{\cos\phi_h}(P_{hT})$ as measurable by COMPASS, HERMES and JLab collaboration experiments on a proton target. The analogous results, for scattering off a deuteron target (and a neutron target as well, for JLab) are shown in Fig. 4. Again, we present the results for three different choices of $\mu_2^2 = 0.1$, 0.17 and 0.25 $(\mathrm{GeV}/c)^2$, which turn out to be well different from each other. Instead, when varying the values of μ_1^2 our results hardly change: each line, obtained at a fixed μ_2^2 value, simply splits in three almost overlapping lines (corresponding, from up down, to $\mu_1^2 = 0.1$, 0.15 and 0.2 $(\mathrm{GeV}/c)^2$). This is not surprising, as, when adopting the expression (28), there remains little dependence on μ_1^2 in Eq. (27). Our computations show instead a clear strong dependence on μ_2^2.

It is interesting also to compute the dependence of $A_{LL}^{\cos\phi_h}$ on each of the other single variables;, for example, the x-dependence is computed as

$$A_{LL}^{\cos\phi_h}(x) = \frac{\int_{P^2_{hT,\min}}^{P^2_{hT,\max}} dP^2_{hT} \int dy \int dz (\Delta\sigma_{LL}^{\cos\phi_h} + \Delta\sigma_{LT}^{\cos\phi_h})}{\int_{P^2_{hT,\min}}^{P^2_{hT,\max}} dP^2_{hT} \int dy \int dz \sigma_0}, \tag{32}$$

while the y- and z-dependences are calculated in a similar way. In Figs. 5–7 we present the x-, y- and z-dependences of $A_{LL}^{\cos\phi_h}$ integrated over P_{hT} with $P_{hT,\min} = 0.5$ GeV/c and $\mu_1^2 = 0.15$ $(\mathrm{GeV}/c)^2$, $\mu_2^2 = 0.25$ $(\mathrm{GeV}/c)^2$ for COMPASS ($P_{hT,\max} = 2$ GeV/c), HERMES ($P_{hT,\max} = 1.5$ GeV/c) and JLab ($P_{hT,\max} = 1$ GeV/c) kinematics.

IV. DISCUSSION AND CONCLUSIONS

We have studied the P_{hT} dependence of A_{LL} and $A_{LL}^{\cos\phi_h}$, measurable in SIDIS processes by COMPASS, HERMES and JLab collaborations. For P_{hT} values up to ~1 GeV/c this dependence is entirely generated by intrinsic motion, both of partons inside the nucleons, and of hadrons in the parton fragmentation process [5,6].

Within a simple factorized gaussian model for the $k_\perp$ and $p_\perp$ dependence of the distribution and fragmentation functions, it turns out that $A_{LL}(P_{hT})$ is strongly sensitive to the relative value of $\langle k_\perp^2 \rangle$ in unpolarized (μ_0^2) and helicity (μ_2^2) quark distributions: similar values, $\mu_0^2 \simeq \mu_2^2$, would reflect into an approximately constant $A_{LL}(P_{hT})$, while $\mu_2^2 < \mu_0^2$, would lead to a decreasing trend. Such different behaviors are expected in general, independently of the factorized gaussian assumption, as the shape of $A_{LL}(P_{hT})$ is essentially related to the ratio of the $k_\perp$ dependence of g_1^q and f_1^q. Notice, however, that we have assumed the same constant values of $\langle k_\perp^2 \rangle$ and $\langle p_\perp^2 \rangle$ for all quark flavors; more involved choices might lead to different behaviors. A comparison of the quark intrinsic transverse momentum in unpolarized and longitudinally polarized protons might give new important information concerning the spin and orbital motion of quarks. For example, one expects that parton transverse motion contributes to the longitudinal component of the angular momentum, differently inside unpolarized and longitudinally polarized nucleons.

The P_{hT} dependence of $A_{LL}^{\cos\phi_h}$ is not only related to kinematical noncollinear contributions, but also to a new TMD and spin dependent function, which gives the number density of longitudinally polarized quarks inside a transversely polarized nucleon. This function, $g_{1T}^{q\perp}$, induces a $\cos\phi_h$ dependence, but it is unknown; we adopted a much debated relationship, together with the twist-two part the Wandzura-Wilczek sum rule (and the usual Gaussian factorization), in order to link the x-dependent part of $g_{1T}^{q\perp}$ to the integrated helicity distributions. Within such an approach, it turns out that also $A_{LL}^{\cos\phi_h}(P_{hT})$ has a strong dependence on μ_2^2 alone, thus giving further information on the average transverse motion of quarks inside a longitudinally polarized proton.

We conclude by noticing, as it was done in Ref. [9], that the exact $k_\perp$ dependence of the distribution functions $f_1^q(x, k_\perp)$, $g_{1L}^q(x, k_\perp)$ and $g_{1T}^{q\perp}(x, k_\perp)$ is crucial when considering the general positivity bounds of Ref. [21], which involve in one inequality the three previous functions and the Sivers function. The $k_\perp$ dependence might play an essential role in fulfilling the inequality, and a check of its validity is a fundamental test for the self consistency of the LO QCD description of SIDIS processes.

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