

# Study of exclusive semileptonic and nonleptonic decays of $B_c^-$ in a nonrelativistic quark model

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We present results for different observables measured in semileptonic and nonleptonic decays of the  $B_c^-$  meson. The calculations have been done within the framework of a nonrelativistic constituent quark model. In order to check the sensitivity of all our results against the interquark interaction we use five different quark-quark potentials. We obtain form factors, decay widths, and asymmetry parameters for semileptonic  $B_c^- \rightarrow c\bar{c}$  and  $B_c^- \rightarrow \bar{B}$  decays. In the limit of infinite heavy quark mass, our model reproduces the constraints of heavy quark spin symmetry. For the actual heavy quark masses we find nonetheless large corrections to that limiting situation for some form factors. We also analyze exclusive nonleptonic two-meson decay channels within the factorization approximation.

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## I. INTRODUCTION

Since its discovery at Fermilab by the CDF Collaboration [1], the  $B_c$  meson has drawn a lot of attention. Unlike other heavy mesons it is composed of two heavy quarks of different flavor ( $b, c$ ) and, being below the  $B$ - $D$  threshold, it can only decay through weak interactions making an ideal system to study weak decays of heavy quarks.

It is known [2] that one cannot apply heavy quark symmetry (HQS) to hadrons containing two heavy quarks: the kinetic energy term, needed in those systems to regulate infrared divergences, breaks heavy flavor symmetry. Still, there is a symmetry that survives: heavy quark spin symmetry (HQSS). This symmetry amounts to the decoupling of the two heavy quark spins since the spin-spin interaction vanishes for infinite heavy quark masses. Using HQSS, Jenkins *et al.* [2] were able to obtain, in the infinite heavy quark mass limit, relations between different form factors for semileptonic  $B_c$  decays into pseudoscalar and vector mesons. Contrary to the heavy-light meson case where standard HQS applies, no determination of corrections in inverse powers of the heavy quark masses has been worked out in this case. So one can only test any model calculation against HQSS predictions in the infinite heavy quark mass limit.

With both quarks being heavy, a nonrelativistic treatment of the  $B_c$  meson should provide reliable results. Besides a nonrelativistic model will comply with the constraints imposed by HQSS as the spin-spin interaction vanishes in the infinity heavy quark mass limit. In this paper we will study, within the framework of a nonrelativistic quark model, exclusive semileptonic and nonleptonic decays of the  $B_c^-$  meson driven by a  $b \rightarrow c$  or  $\bar{c} \rightarrow \bar{d}, \bar{s}$  transitions at the quark level. We will not consider semileptonic processes driven by the quark  $b \rightarrow u$  transition. Our experience with this kind of processes, like the analogous  $B \rightarrow \pi$  semileptonic decay [3], shows that the nonrelativistic model without any improvements under-

estimates the decay width for two reasons: first at high  $q^2$  transfers one might need to include the exchange of a  $B^*$  meson, and second the model underestimates the form factors at low  $q^2$  or high three-momentum transfers. We will concentrate thus on semileptonic  $B_c^- \rightarrow c\bar{c}$  and  $B_c^- \rightarrow \bar{B}$  transitions. As for two-meson nonleptonic decay, we will only consider channels with at least a  $c\bar{c}$  or  $B$  final meson. In the first case we will include channels with final  $D$  mesons for which there is a contribution coming from an effective  $b \rightarrow d, s$  transition. As later explained, this is not the main contribution to the decay amplitude and besides the momentum transfer in those cases is neither too high nor too low so that the problems mentioned above are avoided.

The observables studied here have been analyzed before in the context of different models like the relativistic constituent quark model [4–6], the quasipotential approach to the relativistic quark model [7,8], the instantaneous nonrelativistic approach to the Bethe-Salpeter equation [9–11], the Bethe-Salpeter equation [12,13], the three point sum rules of QCD and nonrelativistic QCD [14–17], the QCD relativistic potential model [18], the relativistic constituent quark model formulated on the light front [19], the relativistic quark-meson model [20], or in models that use the Isgur, Scora, Grinstein, and Wise wave functions [21] like the calculations in Refs. [22–24]. We will compare our results with those obtained in these latter references whenever it is possible. Besides, we will perform an exhaustive study and compile in this work all our results for exclusive semileptonic and nonleptonic  $B_c^-$  decays, paying special attention to the theoretical uncertainties affecting our predictions and providing reliable estimates for all of them.

In the present calculation, we shall use physical masses taken from Ref. [25]. For the  $B_c$  meson mass and lifetime, we shall use the central values of the recent experimental determinations by the CDF Collaboration of  $m_{B_c} = 6285.7 \pm 5.3 \pm 1.2 \text{ MeV}/c^2$  [26] and  $\tau_{B_c} = (0.463^{+0.073}_{-0.065} \pm 0.036) \times 10^{-12} \text{ s}$  [27]. This new mass value is very close to

the one we obtain with the different quark-quark potentials that we use in this work (see below) from which we get  $m_{B_c} = 6291.6^{+12}_{-33}$  MeV.

We shall also need Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and different meson decay constants. For the former we shall use the ones quoted in Ref. [4] that we reproduce in Table I. All of them are within the ranges quoted by the Particle Data Group (PDG) [25].

For the meson decay constants, the values used in this work are compiled in Table II. They correspond to central values of experimental measurements or lattice determinations. The results for  $f_\rho$  and  $f_{K^*}$  have been obtained by the authors in Ref. [4] using  $\tau$  lepton decay data. Our own theoretical calculation, obtained with the model described in Ref. [32], gives  $f_\rho = 0.189 \sim 0.227$  GeV,  $f_{K^*} = 0.180 \sim 0.220$  GeV depending on the interquark interaction used, results which agree with the determinations in Ref. [4]. We shall nevertheless use the latter for our calculations. For  $f_{\eta_c}$  we have been unable to find an experimental result or a lattice determination. There are at least two theoretical determinations that predict  $f_{\eta_c} = 0.484$  GeV [4] and  $f_{\eta_c} = 0.420 \pm 0.052$  GeV [33]. Again our own calculation gives values in the range  $f_{\eta_c} = 0.485 \sim 0.500$  GeV depending on the interquark interaction used. Here we will take  $f_{\eta_c} = 0.490$  GeV.

The rest of the paper is organized as follows. In Sec. II we introduce our meson states and the potential models used to obtain the spatial part of their wave functions. In Sec. III we study the  $B_c$  meson semileptonic decays into various  $c\bar{c}$  channels, both for a final light charged lepton ( $e, \mu$ ) and for a heavy one ( $\tau$ ). In Sec. IV we study different exclusive nonleptonic two-meson decay channels of the  $B_c^-$  meson with one of the final mesons being a  $c\bar{c}$  one. In Sec. V we study semileptonic  $B_c^- \rightarrow \bar{B}$  decays and in

Sec. VI nonleptonic two-meson decays with one of the mesons having a  $b$  quark. We briefly summarize our results in Sec. VII. The paper also contains four appendices: in Appendix A we give different sets of polarization vectors used in this paper, Appendices B and C collect the expressions for all the matrix elements needed to evaluate the different observables analyzed, finally in Appendix D we give the expressions for the helicity components of the hadron tensor to be defined below.

## II. MESON STATES AND INTERQUARK INTERACTIONS

Within a nonrelativistic constituent quark model, the state of a meson  $M$  is given by

$$|M, \lambda \vec{P}\rangle_{\text{NR}} = \int d^3p \sum_{\alpha_1, \alpha_2} \hat{\phi}_{\alpha_1, \alpha_2}^{(M, \lambda)}(\vec{p}) \times \frac{(-1)^{(1/2)-s_2}}{(2\pi)^{3/2} \sqrt{2E_{f_1}(\vec{p}_1)2E_{f_2}(\vec{p}_2)}} \times \left| q, \alpha_1 \vec{p}_1 = \frac{m_{f_1}}{m_{f_1} + m_{f_2}} \vec{P} - \vec{p} \right\rangle \times \left| \bar{q}, \alpha_2 \vec{p}_2 = \frac{m_{f_2}}{m_{f_1} + m_{f_2}} \vec{P} + \vec{p} \right\rangle, \quad (1)$$

where  $\vec{P}$  stands for the meson three momentum and  $\lambda$  represents the spin projection in the meson center of mass.  $\alpha_1$  and  $\alpha_2$  represent the quantum numbers of spin  $s$ , flavor  $f$  and color  $c$  [ $\alpha \equiv (s, f, c)$ ], of the quark and the antiquark, while  $(E_{f_1}(\vec{p}_1), \vec{p}_1)$ ,  $m_{f_1}$  and  $(E_{f_2}(\vec{p}_2), \vec{p}_2)$ ,  $m_{f_2}$  are their respective four-momenta and masses. The factor  $(-1)^{(1/2)-s_2}$  is included in order that the antiquark spin states have the correct relative phase.<sup>1</sup> The normalization of the quark and antiquark states is

$$\langle \alpha' \vec{p}' | \alpha \vec{p} \rangle = \delta_{\alpha', \alpha} (2\pi)^3 2E_f(\vec{p}) \delta(\vec{p}' - \vec{p}). \quad (2)$$

Furthermore,  $\hat{\phi}_{\alpha_1, \alpha_2}^{(M, \lambda)}(\vec{p})$  is the momentum space wave function for the relative motion of the quark-antiquark system. Its normalization is given by

$$\int d^3p \sum_{\alpha_1, \alpha_2} (\hat{\phi}_{\alpha_1, \alpha_2}^{(M, \lambda')}(\vec{p}))^* \hat{\phi}_{\alpha_1, \alpha_2}^{(M, \lambda)}(\vec{p}) = \delta_{\lambda', \lambda}, \quad (3)$$

and, thus, the normalization of our meson states is

$${}_{\text{NR}}\langle M, \lambda' \vec{P}' | M, \lambda \vec{P} \rangle_{\text{NR}} = \delta_{\lambda', \lambda} (2\pi)^3 \delta(\vec{P}' - \vec{P}). \quad (4)$$

In this calculation we will need the ground state wave function for scalar ( $0^+$ ), pseudoscalar ( $0^-$ ), vector ( $1^-$ ),

<sup>1</sup>Note that under charge conjugation ( $C$ ) quark and antiquark creation operators are related via  $C c_\alpha^\dagger(\vec{p}) C^\dagger = (-1)^{(1/2)-s} d_\alpha^\dagger(\vec{p})$ . This implies that the antiquark states with the correct spin relative phase are not  $d_\alpha^\dagger(\vec{p})|0\rangle = |\bar{q}, \alpha \vec{p}\rangle$  but are given instead by  $(-1)^{(1/2)-s} d_\alpha^\dagger(\vec{p})|0\rangle = (-1)^{(1/2)-s} |\bar{q}, \alpha \vec{p}\rangle$ .

TABLE I. Values for Cabibbo-Kobayashi-Maskawa matrix elements used in this work.

$ V_{ud} $	$ V_{us} $	$ V_{cd} $	$ V_{cs} $	$ V_{cb} $
0.975	0.224	0.224	0.974	0.0413

TABLE II. Meson decay constants in GeV used in this work.

$f_{\pi^-}$	$f_{\pi^0}$	$f_{\rho^-, \rho^0}$	$f_{K^-, K^0}$	$f_{K^{*-}, K^{*0}}$
0.1307 [25]	0.130 [25]	0.210 [4]	0.1598 [25]	0.217 [4]
$f_{\eta_c}$	$f_{J/\Psi}$			
0.490	0.405 [28]			
$f_{D^-}$	$f_{D^{*-}}$	$f_{D_s^-}$	$f_{D_s^{*-}}$	
0.2226 [29]	0.245 [30]	0.294 [31]	0.272 [30]	

axial vector ( $1^+$ ), tensor ( $2^+$ ), and pseudotensor ( $2^-$ ) mesons. Assuming always the lowest possible value for the orbital angular momentum we will have for a meson  $M$  with scalar, pseudoscalar, and vector quantum numbers:

$$\begin{aligned}\hat{\phi}_{\alpha_1, \alpha_2}^{(M(0^+))}(\vec{p}) &= \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \hat{\phi}_{(s_1, f_1), (s_2, f_2)}^{(M(0^+))}(\vec{p}) = \frac{1}{\sqrt{3}} \delta_{c_1, c_2} i \hat{\phi}_{f_1, f_2}^{(M(0^+))}(|\vec{p}|) \sum_m (1/2, 1/2, 1; s_1, s_2, -m)(1, 1, 0; m, -m, 0) Y_{1m}(\vec{p}) \\ \hat{\phi}_{\alpha_1, \alpha_2}^{(M(0^-))}(\vec{p}) &= \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \hat{\phi}_{(s_1, f_1), (s_2, f_2)}^{(M(0^-))}(\vec{p}) = \frac{1}{\sqrt{3}} \delta_{c_1, c_2} (-i) \hat{\phi}_{f_1, f_2}^{(M(0^-))}(|\vec{p}|) (1/2, 1/2, 0; s_1, s_2, 0) Y_{00}(\vec{p}) \\ \hat{\phi}_{\alpha_1, \alpha_2}^{(M(1^-), \lambda)}(\vec{p}) &= \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \hat{\phi}_{(s_1, f_1), (s_2, f_2)}^{(M(1^-), \lambda)}(\vec{p}) = \frac{1}{\sqrt{3}} \delta_{c_1, c_2} (-1) \hat{\phi}_{f_1, f_2}^{(M(1^-))}(|\vec{p}|) (1/2, 1/2, 1; s_1, s_2, \lambda) Y_{00}(\vec{p}),\end{aligned}\quad (5)$$

where  $(j_1, j_2, j_3; m_1, m_2, m_3)$  are Clebsch-Gordan coefficients,  $Y_{lm}(\vec{p})$  are spherical harmonics, and  $\hat{\phi}_{f_1, f_2}^{(M)}(|\vec{p}|)$  is the Fourier transform of the radial coordinate space wave function.

For axial mesons we need orbital angular momentum  $L = 1$ . In this case two values of the total quark-antiquark spin  $S_{q\bar{q}} = 0, 1$  are possible, giving rise to the two states:

$$\begin{aligned}\hat{\phi}_{\alpha_1, \alpha_2}^{(M(1^+, S_{q\bar{q}}=0), \lambda)}(\vec{p}) &= \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \hat{\phi}_{(s_1, f_1), (s_2, f_2)}^{(M(1^+, S_{q\bar{q}}=0), \lambda)}(\vec{p}) = \frac{1}{\sqrt{3}} \delta_{c_1, c_2} (-1) \hat{\phi}_{f_1, f_2}^{(M(1^+, S_{q\bar{q}}=0))}(|\vec{p}|) (1/2, 1/2, 0; s_1, s_2, 0) Y_{1\lambda}(\vec{p}) \\ \hat{\phi}_{\alpha_1, \alpha_2}^{(M(1^+, S_{q\bar{q}}=1), \lambda)}(\vec{p}) &= \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \hat{\phi}_{(s_1, f_1), (s_2, f_2)}^{(M(1^+, S_{q\bar{q}}=1), \lambda)}(\vec{p}) \\ &= \frac{1}{\sqrt{3}} \delta_{c_1, c_2} (-1) \hat{\phi}_{f_1, f_2}^{(M(1^+, S_{q\bar{q}}=1))}(|\vec{p}|) \sum_m (1/2, 1/2, 1; s_1, s_2, \lambda - m)(1, 1, 1; m, \lambda - m, \lambda) Y_{1m}(\vec{p}).\end{aligned}\quad (6)$$

Finally, for tensor and pseudotensor mesons we have the wave functions:

$$\begin{aligned}\hat{\phi}_{\alpha_1, \alpha_2}^{(M(2^+), \lambda)}(\vec{p}) &= \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \hat{\phi}_{(s_1, f_1), (s_2, f_2)}^{(M(2^+), \lambda)}(\vec{p}) \\ &= \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \hat{\phi}_{f_1, f_2}^{(M(2^+))}(|\vec{p}|) \sum_m (1/2, 1/2, 1; s_1, s_2, \lambda - m)(1, 1, 2; m, \lambda - m, \lambda) Y_{1m}(\vec{p}) \\ \hat{\phi}_{\alpha_1, \alpha_2}^{(M(2^-), \lambda)}(\vec{p}) &= \frac{1}{\sqrt{3}} \delta_{c_1, c_2} \hat{\phi}_{(s_1, f_1), (s_2, f_2)}^{(M(2^-), \lambda)}(\vec{p}) \\ &= \frac{1}{\sqrt{3}} \delta_{c_1, c_2} (-1) \hat{\phi}_{f_1, f_2}^{(M(2^-))}(|\vec{p}|) \sum_m (1/2, 1/2, 1; s_1, s_2, \lambda - m)(2, 1, 2; m, \lambda - m, \lambda) Y_{2m}(\vec{p}).\end{aligned}\quad (7)$$

All phases have been introduced for later convenience.

To evaluate the coordinate space wave function we use five different interquark interactions, one suggested by Bhaduri and collaborators [34], and four others suggested by Silvestre-Brac and Semay [35,36]. All of them contain a confinement term, plus Coulomb and hyperfine terms coming from one-gluon exchange, and differ from one another in the form factors used for the hyperfine terms, the power of the confining term, or the use of a form factor in the one-gluon exchange Coulomb potential. All free parameters in the potentials had been adjusted to reproduce the light ( $\pi$ ,  $\rho$ ,  $K$ ,  $K^*$ , etc.) and heavy-light ( $D$ ,  $D^*$ ,  $B$ ,  $B^*$ , etc.) meson spectra. These potentials also lead to good results for the charmed and bottom baryon ( $\Lambda_{c,b}$ ,  $\Sigma_{c,b}$ ,  $\Sigma_{c,b}^*$ ,  $\Xi_{c,b}$ ,  $\Xi_{c,b}'$ ,  $\Xi_{c,b}^*$ ,  $\Omega_{c,b}$ , and  $\Omega_{c,b}^*$ ) masses [35,37], for the semileptonic  $\Lambda_b^0 \rightarrow \Lambda_c^+ l^- \bar{\nu}_l$  and  $\Xi_b^0 \rightarrow \Xi_c^+ l^- \bar{\nu}_l$  decays [38], for the decay constants of pseudoscalar  $B$ ,  $D$  and vector  $B^*$ ,  $D^*$  mesons and the semileptonic  $B \rightarrow D$  and  $B \rightarrow D^*$  decays [32], for the  $B \rightarrow \pi$  semileptonic decay [3], and for the

strong  $\Sigma_c \rightarrow \Lambda_c \pi$ ,  $\Sigma_c^* \rightarrow \Lambda_c \pi$ , and  $\Xi_c^* \rightarrow \Xi_c \pi$  decays [39]. Preliminary results for the spectrum of doubly heavy baryons [40] also show excellent agreement with previous Faddeev calculations and lattice results. For more details on the interquark interactions see Ref. [37] or the original works [34–36].

The use of different interquark interactions will provide us with a spread in the results that we will consider, and quote, as a theoretical error added to the value obtained with the AL1 potential or Refs. [35,36] that we will use to get our central results. Another source of theoretical uncertainty, that we cannot account for, is the use of non-relativistic kinematics in the evaluation of the orbital wave functions and the construction of our states in Eq. (1) above. While this is a very good approximation for the  $B_c$  itself it is not that good for mesons with a light quark. That notwithstanding, note that any nonrelativistic quark model has free parameters in the interquark interaction that are fitted to experimental data. In that sense we think that at

least part of the ignored relativistic effects are included in an effective way in their fitted values.

### III. SEMILEPTONIC $B_c^- \rightarrow c\bar{c}$ DECAYS

In this section we will consider the semileptonic decay of the  $B_c^-$  meson into different  $c\bar{c}$  states with  $0^+$ ,  $0^-$ ,  $1^+$ ,  $1^-$ ,  $2^+$ , and  $2^-$  spin-parity quantum numbers. Those decays correspond to a  $b \rightarrow c$  transition at the quark level which is governed by the current

$$J_\mu^{cb}(0) = J_{V\mu}^{cb}(0) - J_{A\mu}^{cb}(0) = \bar{\Psi}_c(0)\gamma_\mu(I - \gamma_5)\Psi_b(0), \quad (8)$$

with  $\Psi_f$  a quark field of a definite flavor  $f$ .

#### A. Form factor decomposition of hadronic matrix elements

The hadronic matrix elements involved in these processes can be parametrized in terms of a few form factors as

$$\begin{aligned} \langle c\bar{c}(0^-), \vec{P}_{c\bar{c}} | J_\mu^{cb}(0) | B_c, \vec{P}_{B_c} \rangle &= \langle c\bar{c}(0^-), \vec{P}_{c\bar{c}} | J_{V\mu}^{cb}(0) | B_c, \vec{P}_{B_c} \rangle = P_\mu F_+(q^2) + q_\mu F_-(q^2) \\ \langle c\bar{c}(1^-), \lambda \vec{P}_{c\bar{c}} | J_\mu^{cb}(0) | B_c, \vec{P}_{B_c} \rangle &= \langle c\bar{c}(1^-), \lambda \vec{P}_{c\bar{c}} | J_{V\mu}^{cb}(0) - J_{A\mu}^{cb}(0) | B_c, \vec{P}_{B_c} \rangle \\ &= \frac{-1}{m_{B_c} + m_{c\bar{c}}} \varepsilon_{\mu\nu\alpha\beta} \varepsilon_{(\lambda)}^{\nu*}(\vec{P}_{c\bar{c}}) P^\alpha q^\beta V(q^2) - i \left\{ (m_{B_c} - m_{c\bar{c}}) \varepsilon_{(\lambda)\mu}^*(\vec{P}_{c\bar{c}}) A_0(q^2) \right. \\ &\quad \left. - \frac{P \cdot \varepsilon_{(\lambda)}^*(\vec{P}_{c\bar{c}})}{m_{B_c} + m_{c\bar{c}}} (P_\mu A_+(q^2) + q_\mu A_-(q^2)) \right\} \\ \langle c\bar{c}(2^+), \lambda \vec{P}_{c\bar{c}} | J_\mu^{cb}(0) | B_c, \vec{P}_{B_c} \rangle &= \langle c\bar{c}(2^+), \lambda \vec{P}_{c\bar{c}} | J_{V\mu}^{cb}(0) - J_{A\mu}^{cb}(0) | B_c, \vec{P}_{B_c} \rangle \\ &= \varepsilon_{\mu\nu\alpha\beta} \varepsilon_{(\lambda)}^{\nu\delta*}(\vec{P}_{c\bar{c}}) P_\delta P^\alpha q^\beta T_4(q^2) - i \{ \varepsilon_{(\lambda)\mu\delta}^*(\vec{P}_{c\bar{c}}) P^\delta T_1(q^2) \\ &\quad + P^\nu P^\delta \varepsilon_{(\lambda)\nu\delta}^*(\vec{P}_{c\bar{c}}) (P_\mu T_2(q^2) + q_\mu T_3(q^2)) \}. \end{aligned} \quad (9)$$

In the above expressions  $P = P_{B_c} + P_{c\bar{c}}$ ,  $q = P_{B_c} - P_{c\bar{c}}$ , being  $P_{B_c}$  and  $P_{c\bar{c}}$  the meson four-momenta,  $m_{B_c}$  and  $m_{c\bar{c}}$  are the meson masses,  $\varepsilon^{\mu\nu\alpha\beta}$  is the fully antisymmetric tensor for which we take the convention  $\varepsilon^{0123} = +1$ , and  $\varepsilon_{(\lambda)\mu}(\vec{P})$  and  $\varepsilon_{(\lambda)\mu\nu}(\vec{P})$  are the polarization vector and tensor of vector and tensor mesons, respectively.<sup>2</sup> The latter can be evaluated in terms of the former as

$$\varepsilon_{(\lambda)}^{\mu\nu}(\vec{P}) = \sum_m (1, 1, 2; m, \lambda - m, \lambda) \varepsilon_{(m)}^\mu(\vec{P}) \varepsilon_{(\lambda-m)}^\nu(\vec{P}). \quad (10)$$

Different sets of  $\varepsilon_{(\lambda)}(\vec{P})$  used in this work appear in Appendix A.

Besides the meson states in the Lorenz decompositions of Eq. (9) are normalized such that

$$\langle M, \lambda' \vec{P}' | M, \lambda \vec{P} \rangle = \delta_{\lambda'\lambda} (2\pi)^3 2E_M(\vec{P}) \delta(\vec{P}' - \vec{P}), \quad (11)$$

where  $E_M(\vec{P})$  is the energy of the  $M$  meson with three-momentum  $\vec{P}$ . Note the factor  $2E_M$  of difference with Eq. (4).

For the  $0^+$ ,  $1^+$ , and  $2^-$  cases, the form factor decomposition is the same as for the  $0^-$ ,  $1^-$ , and  $2^+$  cases, respectively, but with  $-J_{A\mu}^{cb}(0)$  contributing where  $J_{V\mu}^{cb}(0)$  contributed before and vice versa.

<sup>2</sup>Note we have taken  $\lambda$  to be the third component of the meson spin measured in the meson center of mass.

The different form factors in Eq. (9) are all relatively real thanks to time-reversal invariance.  $F_+$ ,  $F_-$ ,  $V$ ,  $A_0$ ,  $A_+$ ,  $A_-$ , and  $T_1$  are dimensionless, whereas  $T_2$ ,  $T_3$ , and  $T_4$  have dimension of  $E^{-2}$ . They can be easily evaluated working in the center of mass of the  $B_c$  meson and taking  $\vec{q}$  in the  $z$  direction, so that  $\vec{P}_{c\bar{c}} = -\vec{q} = -|\vec{q}|\vec{k}$ , with  $\vec{k}$  representing the unit vector in the  $z$  direction.

#### 1. $B_c^- \rightarrow \eta_c l^- \bar{\nu}_l$ , $\chi_{c0} l^- \bar{\nu}_l$ decays

Let us start with the  $B_c^-$  decays into pseudoscalar  $\eta_c$  and scalar  $\chi_{c0}$   $c\bar{c}$  mesons. For  $B_c^- \rightarrow \eta_c$  transitions the form factors are given by

$$\begin{aligned} F_+(q^2) &= \frac{1}{2m_{B_c}} \left( V^0(|\vec{q}|) + \frac{V^3(|\vec{q}|)}{|\vec{q}|} (E_{\eta_c}(-\vec{q}) - m_{B_c}) \right) \\ F_-(q^2) &= \frac{1}{2m_{B_c}} \left( V^0(|\vec{q}|) + \frac{V^3(|\vec{q}|)}{|\vec{q}|} (E_{\eta_c}(-\vec{q}) + m_{B_c}) \right), \end{aligned} \quad (12)$$

whereas for  $B_c \rightarrow \chi_{c0}$  transitions we have

$$\begin{aligned} F_+(q^2) &= \frac{-1}{2m_{B_c}} \left( A^0(|\vec{q}|) + \frac{A^3(|\vec{q}|)}{|\vec{q}|} (E_{\chi_{c0}}(-\vec{q}) - m_{B_c}) \right) \\ F_-(q^2) &= \frac{-1}{2m_{B_c}} \left( A^0(|\vec{q}|) + \frac{A^3(|\vec{q}|)}{|\vec{q}|} (E_{\chi_{c0}}(-\vec{q}) + m_{B_c}) \right) \end{aligned} \quad (13)$$

with  $V^\mu(|\vec{q}|)$  and  $A^\mu(|\vec{q}|)$  ( $\mu = 0, 3$ ) calculated in our

model as

$$\begin{aligned}
 V^\mu(|\vec{q}|) &= \langle \eta_c, -|\vec{q}|\vec{k}|J_V^{cb\mu}(0)|B_c^-, \vec{0}\rangle \\
 &= \sqrt{2m_{B_c}2E_{\eta_c}(-\vec{q})}_{\text{NR}} \langle \eta_c, -|\vec{q}|\vec{k}|J_V^{cb\mu}(0)|B_c^-, \vec{0}\rangle_{\text{NR}} \\
 A^\mu(|\vec{q}|) &= \langle \chi_{c0}, -|\vec{q}|\vec{k}|J_A^{cb\mu}(0)|B_c^-, \vec{0}\rangle \\
 &= \sqrt{2m_{B_c}2E_{\chi_{c0}}(-\vec{q})}_{\text{NR}} \langle \chi_{c0}, -|\vec{q}|\vec{k}|J_A^{cb\mu}(0)|B_c^-, \vec{0}\rangle_{\text{NR}}
 \end{aligned} \tag{14}$$

where expressions are given in Appendix B.

In Fig. 1 we show our results for the  $F_+$  and  $F_-$  form factors for the semileptonic  $B_c^- \rightarrow \eta_c, \chi_{c0}$  transitions. The minimum  $q^2$  value depends on the actual final lepton and it is given, neglecting neutrino masses, by the lepton mass as  $q_{\text{min}}^2 = m_l^2$ . The form factors have been evaluated using the AL1 potential of Refs. [35,36]. For decays into  $\eta_c$ , and for the sake of comparison, we also show the results obtained with the potential developed by Bhaduri and collaborators in Ref. [34] (BHAD). As seen in the figures, the differences between the form factors evaluated with the two interquark interactions are smaller than 10%.

In Table III we show  $F_+$  and  $F_-$  evaluated at  $q_{\text{min}}^2$  and  $q_{\text{max}}^2$  for a final light lepton ( $l = e, \mu$ ) and compare them to the ones obtained by Ivanov *et al.* in Ref. [5], and, when available, by Ebert *et al.* in Ref. [7]. For the  $B_c \rightarrow \eta_c$  transition we also show the corresponding values for the  $F_0$  form factor defined as

$$F_0(q^2) = F_+(q^2) + \frac{q^2}{m_{B_c}^2 - m_{\eta_c}^2} F_-(q^2). \tag{15}$$

Our results for the  $\eta_c$  case are in excellent agreement with the ones obtained by Ebert *et al.*. Compared to the results by Ivanov *et al.* we find large discrepancies for  $F_-$ .

## 2. $B_c^- \rightarrow J/\Psi l \bar{\nu}_l, h_c l \bar{\nu}_l, \chi_{c1} l \bar{\nu}_l$ decays

Let us now see the form factors for the semileptonic  $B_c^-$  decays into vector  $J/\Psi$  and axial vector  $h_c$  ( $S_{q\bar{q}} = 0$ ) and  $\chi_{c1}$  ( $S_{q\bar{q}} = 1$ )  $c\bar{c}$  mesons. For the decay into  $J/\Psi$ , the form factors can be evaluated in terms of matrix elements as

$$\begin{aligned}
 V(q^2) &= \frac{i}{\sqrt{2}} \frac{m_{B_c} + m_{J/\Psi}}{m_{B_c}} V_{\lambda=-1}^1(|\vec{q}|) \\
 A_+(q^2) &= i \frac{m_{B_c} + m_{J/\Psi}}{2m_{B_c}} \frac{m_{J/\Psi}}{|\vec{q}|m_{B_c}} \left\{ -A_{\lambda=0}^0(|\vec{q}|) + \frac{m_{B_c} - E_{J/\Psi}(-\vec{q})}{|\vec{q}|} A_{\lambda=0}^3(|\vec{q}|) - \sqrt{2} \frac{m_{B_c} E_{J/\Psi}(-\vec{q}) - m_{J/\Psi}^2}{|\vec{q}|m_{J/\Psi}} A_{\lambda=-1}^1(|\vec{q}|) \right\} \\
 A_-(q^2) &= -i \frac{m_{B_c} + m_{J/\Psi}}{2m_{B_c}} \frac{m_{J/\Psi}}{|\vec{q}|m_{B_c}} \left\{ A_{\lambda=0}^0(|\vec{q}|) + \frac{m_{B_c} + E_{J/\Psi}(-\vec{q})}{|\vec{q}|} A_{\lambda=0}^3(|\vec{q}|) - \sqrt{2} \frac{m_{B_c} E_{J/\Psi}(-\vec{q}) + m_{J/\Psi}^2}{|\vec{q}|m_{J/\Psi}} A_{\lambda=-1}^1(|\vec{q}|) \right\} \\
 A_0(q^2) &= -i\sqrt{2} \frac{1}{m_{B_c} - m_{J/\Psi}} A_{\lambda=-1}^1(|\vec{q}|)
 \end{aligned} \tag{16}$$

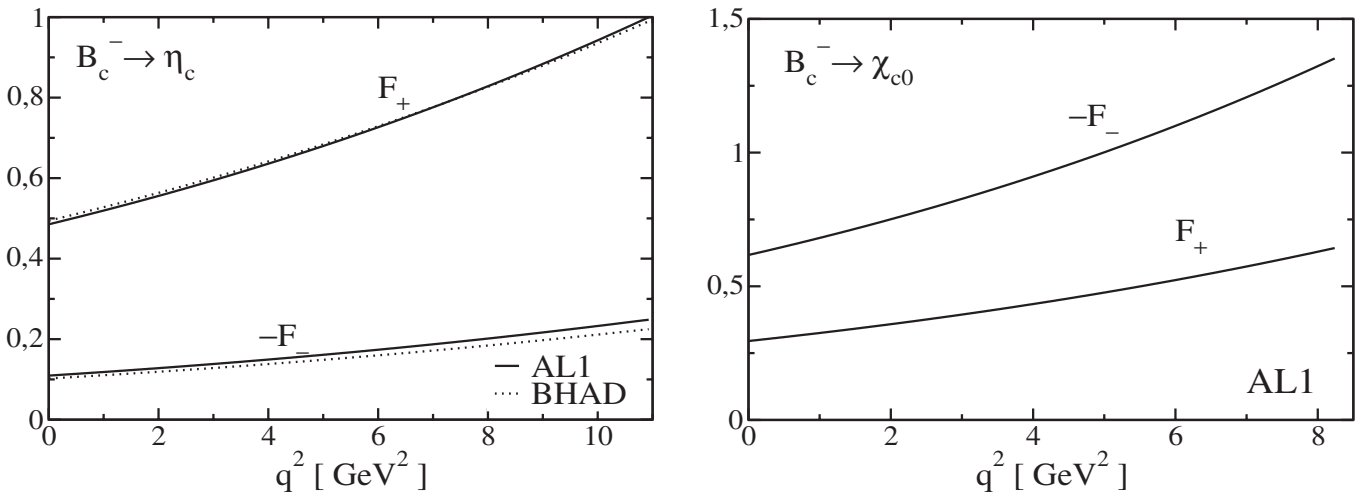


FIG. 1.  $F_+$  and  $F_-$  form factors for  $B_c^- \rightarrow \eta_c$  and  $B_c^- \rightarrow \chi_{c0}$  semileptonic decay evaluated with the AL1 potential of Refs. [35,36]. In the first case, and for comparison, we also show with dotted lines the results obtained with the Bhaduri (BHAD) potential of Ref. [34].



TABLE III.  $F_+$  and  $F_-$  evaluated at  $q_{\min}^2$  and  $q_{\max}^2$  compared to the ones obtained by Ivanov *et al.* [5] and Ebert *et al.* [7]. Our central values have been obtained with the AL1 potential. For the  $\eta_c$  channel we also show  $F_0$  (see text for definition). Here  $l$  stands for  $l = e, \mu$ .

$B_c^- \rightarrow \eta_c l^- \bar{\nu}_l$	$q_{\min}^2$	$q_{\max}^2$	$B_c^- \rightarrow \chi_{c0} l^- \bar{\nu}_l$	$q_{\min}^2$	$q_{\max}^2$
$F_+$			$F_+$		
This work	$0.49^{+0.01}$	$1.00_{-0.01}$	This work	$0.30^{+0.01}$	$0.64_{-0.01}^{+0.01}$
[5]	0.61	1.14	[5]	0.40	0.65
[7]	0.47	1.07			
$F_-$			$F_-$		
This work	$-0.11_{-0.01}^{+0.01}$	$-0.25^{+0.03}$	This work	$-0.62_{-0.03}^{+0.01}$	$-1.35^{+0.05}$
[5]	-0.32	-0.61	[5]	-1.00	-1.63
$F_0$					
This work	$0.49^{+0.01}$	$0.91^{+0.01}$			
[7]	0.47	0.92			

with  $V_\lambda^\mu(|\vec{q}|)$  and  $A_\lambda^\mu(|\vec{q}|)$  calculated in our model as

$$V_\lambda^\mu(|\vec{q}|) = \langle J/\Psi, \lambda - |\vec{q}|\vec{k}|J_V^{cb\mu}(0)|B_c^-, \vec{0}\rangle = \sqrt{2m_{B_c}2E_{J/\Psi}(-\vec{q})}_{\text{NR}} \langle J/\Psi, \lambda - |\vec{q}|\vec{k}|J_V^{cb\mu}(0)|B_c^-, \vec{0}\rangle_{\text{NR}} \quad (17)$$

$$A_\lambda^\mu(|\vec{q}|) = \langle J/\Psi, \lambda - |\vec{q}|\vec{k}|J_A^{cb\mu}(0)|B_c^-, \vec{0}\rangle = \sqrt{2m_{B_c}2E_{J/\Psi}(-\vec{q})}_{\text{NR}} \langle J/\Psi, \lambda - |\vec{q}|\vec{k}|J_A^{cb\mu}(0)|B_c^-, \vec{0}\rangle_{\text{NR}}$$

where expressions are given in Appendix B.

The form factors corresponding to transitions to the  $\chi_{c1}$  and  $h_c$  axial vector mesons are obtained from the expressions in Eq. (16) by just changing

$$V_\lambda^\mu(|\vec{q}|) \leftrightarrow -A_\lambda^\mu(|\vec{q}|) \quad (18)$$

and using the appropriate mass for the final meson. Obviously, in Eq. (17)  $J/\Psi$  has to be replaced by  $\chi_{c1}$  or  $h_c$ .

In Table IV we show the result for the different form factors evaluated at  $q_{\min}^2$  and  $q_{\max}^2$  for the case where the final lepton is light ( $l = e, \mu$ ). For the decay into  $J/\Psi$  we

TABLE IV.  $V$ ,  $A_+$ ,  $A_-$ , and  $A_0$  form factors evaluated at  $q_{\min}^2$  and  $q_{\max}^2$  compared to the ones obtained by Ivanov *et al.* [5] and Ebert *et al.* [7]. Our central values have been evaluated with the AL1 potential. For the  $J/\Psi$  channel we also show  $\tilde{A}_0$  (see text for definition). Here  $l$  stands for  $l = e, \mu$ . The asterisk to the right of a number means we have changed its sign to account for the different choice of  $\varepsilon^{0123}$  in Ref. [5].

$B_c^- \rightarrow J/\Psi l^- \bar{\nu}_l$	$q_{\min}^2$	$q_{\max}^2$	$B_c^- \rightarrow h_c l^- \bar{\nu}_l$	$q_{\min}^2$	$q_{\max}^2$	$B_c^- \rightarrow \chi_{c1} l^- \bar{\nu}_l$	$q_{\min}^2$	$q_{\max}^2$
$V$			$V$			$V$		
This work	$-0.61_{-0.03}$	$-1.26^{+0.01}$	This work	$-0.040_{-0.003}$	$-0.078_{-0.003}$	This work	$0.92_{-0.02}^{+0.04}$	$1.86_{-0.12}$
[5]	$-0.83^*$	$-1.53^*$	[5]	$-0.25^*$	$-0.365^*$	[5]	$1.18^*$	$1.81^*$
[7]	-0.49	-1.34						
$A_+$			$A_+$			$A_+$		
This work	$0.56^{+0.03}$	$1.13^{+0.01}$	This work	$-0.85_{-0.05}^{+0.01}$	$-1.90^{+0.06}$	This work	$-0.44_{-0.03}$	$-0.78^{+0.04}$
[5]	0.54	0.97	[5]	-1.08	-1.80	[5]	-0.39	-0.50
[7]	0.73	1.33						
$A_-$			$A_-$			$A_-$		
This work	$-0.60_{-0.03}$	$-1.24_{-0.01}$	This work	$0.12_{-0.02}^{+0.01}$	$0.36_{-0.06}$	This work	$0.96_{-0.01}^{+0.04}$	$1.97_{-0.13}$
[5]	-0.95	-1.76	[5]	0.52	0.89	[5]	1.52	2.36
$A_0$			$A_0$			$A_0$		
This work	$1.44^{+0.08}$	$2.58_{-0.02}^{+0.01}$	This work	$0.28^{+0.02}$	$0.52^{+0.02}$	This work	$-0.50_{-0.02}$	$-0.32_{-0.02}$
[5]	1.64	2.50	[5]	0.44	0.54	[5]	-0.064	0.46
[7]	1.47	2.59						
$\tilde{A}_0$								
This work	$0.45^{+0.03}$	$0.96_{-0.01}$						
[7]	0.40	1.06						

also show the combination of form factors<sup>3</sup>:

$$\tilde{A}_0(q^2) = \frac{m_{B_c} - m_{J/\Psi}}{2m_{J/\Psi}}(A_0(q^2) - A_+(q^2)) - \frac{q^2}{2m_{J/\Psi}(m_{B_c} + m_{J/\Psi})}A_-(q^2). \quad (19)$$

Our results for the  $B_c^- \rightarrow J/\Psi$  decay channel are in agreement with the ones obtained by Ebert *et al.*. They also agree reasonably well with the exception of  $A_-$ , with the ones obtained by Ivanov *et al.*. For the other two cases the discrepancies are in general large.

All the form factors are depicted in Figs. 2 and 3.

### 3. $B_c^- \rightarrow \Psi(3836)l\bar{\nu}_l$ , $\chi_{c2}l\bar{\nu}_l$ , decays

Finally, let us see the form factors for the  $B_c^-$  decays into tensor  $\chi_{c2}$  and pseudotensor  $\Psi(3836)$ <sup>4</sup> mesons. For the decay into  $\chi_{c2}$  the form factors can be evaluated in terms of matrix elements as

$$\begin{aligned} T_1(q^2) &= -i \frac{2m_{\chi_{c2}}}{m_{B_c} |\vec{q}|} A_{T\lambda=+1}^1(|\vec{q}|) \\ T_2(q^2) &= i \frac{1}{2m_{B_c}^3} \left\{ -\sqrt{\frac{3}{2}} \frac{m_{\chi_{c2}}^2}{|\vec{q}|^2} A_{T\lambda=0}^0(|\vec{q}|) - \sqrt{\frac{3}{2}} \frac{m_{\chi_{c2}}^2}{|\vec{q}|^3} (E_{\chi_{c2}}(-\vec{q}) - m_{B_c}) A_{T\lambda=0}^3(|\vec{q}|) \right. \\ &\quad \left. + \frac{2m_{\chi_{c2}}}{|\vec{q}|} \left( 1 - \frac{E_{\chi_{c2}}(-\vec{q})}{|\vec{q}|^2} (E_{\chi_{c2}}(-\vec{q}) - m_{B_c}) \right) A_{T\lambda=+1}^1(|\vec{q}|) \right\} \\ T_3(q^2) &= i \frac{1}{2m_{B_c}^3} \left\{ -\sqrt{\frac{3}{2}} \frac{m_{\chi_{c2}}^2}{|\vec{q}|^2} A_{T\lambda=0}^0(|\vec{q}|) - \sqrt{\frac{3}{2}} \frac{m_{\chi_{c2}}^2}{|\vec{q}|^3} (E_{\chi_{c2}}(-\vec{q}) + m_{B_c}) A_{T\lambda=0}^3(|\vec{q}|) \right. \\ &\quad \left. + \frac{2m_{\chi_{c2}}}{|\vec{q}|} \left( 1 - \frac{E_{\chi_{c2}}(-\vec{q})}{|\vec{q}|^2} (E_{\chi_{c2}}(-\vec{q}) + m_{B_c}) \right) A_{T\lambda=+1}^1(|\vec{q}|) \right\} \\ T_4(q^2) &= i \frac{m_{\chi_{c2}}}{m_{B_c}^2 |\vec{q}|^2} V_{T\lambda=+1}^1(|\vec{q}|) \end{aligned} \quad (20)$$

with  $V_{T\lambda}^\mu(|\vec{q}|)$  and  $A_{T\lambda}^\mu(|\vec{q}|)$  calculated in our model as

$$\begin{aligned} V_{T\lambda}^\mu(|\vec{q}|) &= \langle \chi_{c2}, \lambda - |\vec{q}|\vec{k}|J_V^{cb\mu}(0)|B_c^-, \vec{0}\rangle = \sqrt{2m_{B_c} 2E_{\chi_{c2}}(-\vec{q})}_{\text{NR}} \langle \chi_{c2}, \lambda - |\vec{q}|\vec{k}|J_V^{cb\mu}(0)|B_c^-, \vec{0}\rangle_{\text{NR}} \\ A_{T\lambda}^\mu(|\vec{q}|) &= \langle \chi_{c2}, \lambda - |\vec{q}|\vec{k}|J_A^{cb\mu}(0)|B_c^-, \vec{0}\rangle = \sqrt{2m_{B_c} 2E_{\chi_{c2}}(-\vec{q})}_{\text{NR}} \langle \chi_{c2}, \lambda - |\vec{q}|\vec{k}|J_A^{cb\mu}(0)|B_c^-, \vec{0}\rangle_{\text{NR}} \end{aligned} \quad (21)$$

where expressions are given in Appendix B.

The form factors corresponding to transitions to  $\Psi(3836)$  are obtained from the expressions in Eq. (20) by just changing

$$V_{T\lambda}^\mu(|\vec{q}|) \leftrightarrow -A_{T\lambda}^\mu(|\vec{q}|) \quad (22)$$

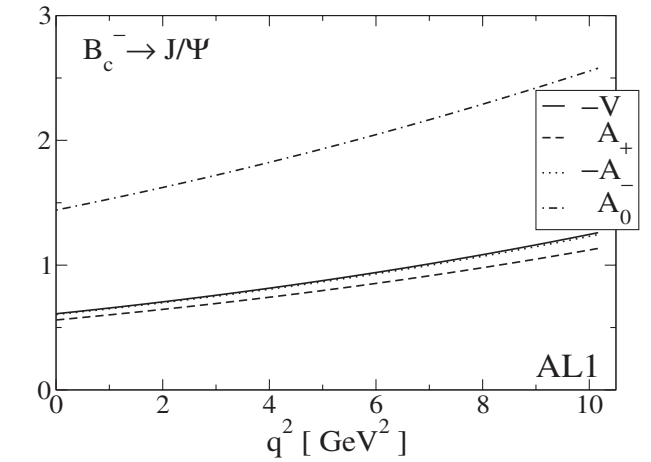


FIG. 2.  $V$  (solid line),  $A_+$  (dashed line),  $A_-$  (dotted line), and  $A_0$  (dash-dotted line) form factors for the  $B_c^- \rightarrow J/\Psi$  semi-leptonic decay evaluated with the AL1 potential.

<sup>3</sup>This combination is called  $A_0$  by the authors of Ref. [7].

<sup>4</sup>Note that while the  $\Psi(3836)$  was still quoted in the particle listings of the former Review of Particle Physics [25], it has been excluded from the more recent one [31]. We shall nevertheless keep it in our study to illustrate the results to be expected for a ground state pseudotensor particle.

and using the appropriate mass for the final meson. Besides in Eq. (21)  $\chi_{c2}$  has to be replaced by  $\Psi(3836)$ .

The results for the different form factors appear in Fig. 4. In Table V we show  $T_1$ ,  $T_2$ ,  $T_3$ , and  $T_4$  evaluated at  $q_{\min}^2$  and  $q_{\max}^2$  for the case of a final light lepton, and compare them to the values obtained by Ivanov *et al.* [5]. For the  $B_c^- \rightarrow \chi_{c2}$  transition we find a reasonable agreement between the two calculations. For  $B_c^- \rightarrow \Psi(3836)$  there is also a rea-

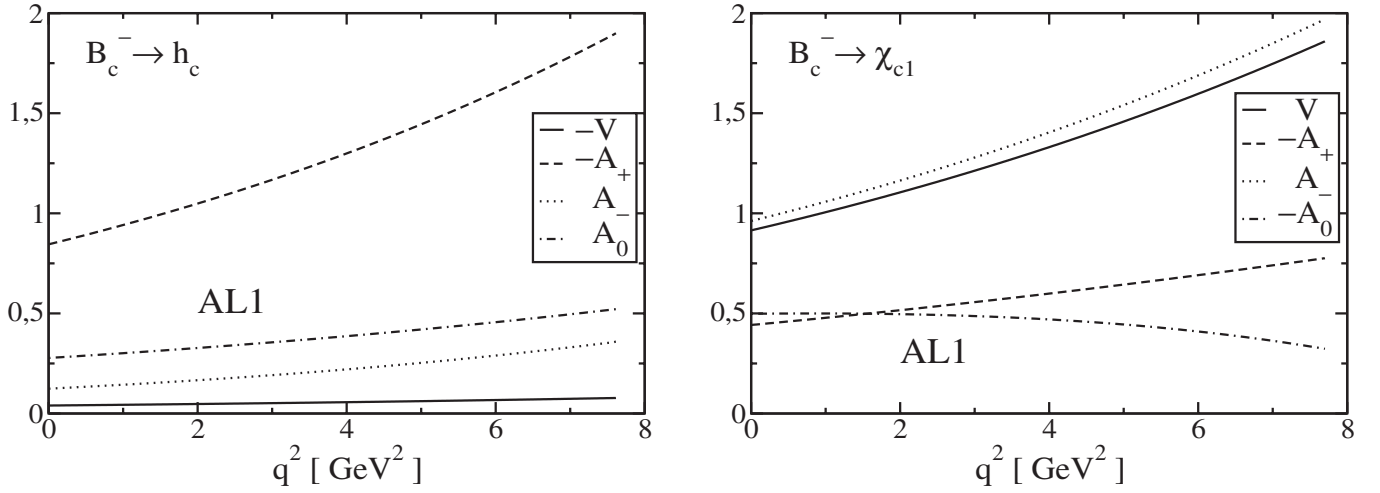


FIG. 3.  $V$  (solid line),  $A_+$  (dashed line),  $A_-$  (dotted line), and  $A_0$  (dash-dotted line) form factors for  $B_c^- \rightarrow h_c$  and  $B_c^- \rightarrow \chi_{c1}$  semileptonic decay evaluated with the AL1 potential.

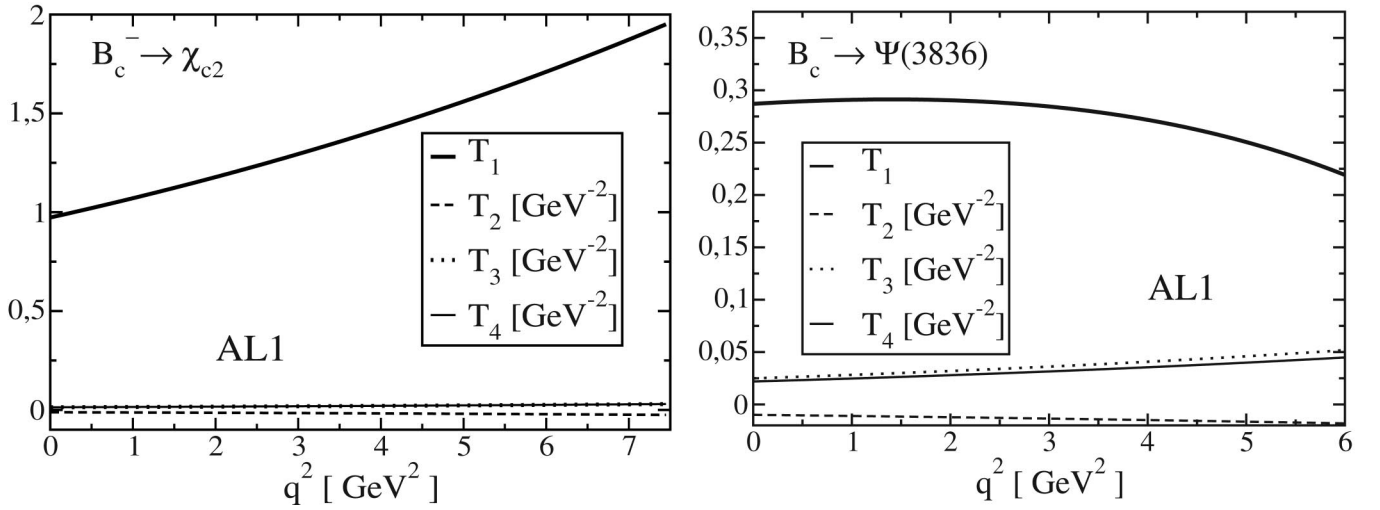


FIG. 4.  $T_1$  (bold solid line),  $T_2$  (dashed line),  $T_3$  (dotted line), and  $T_4$  (thin solid line) form factors for  $B_c^- \rightarrow \chi_{c2}$  and  $B_c^- \rightarrow \Psi(3836)$  semileptonic decay evaluated with the AL1 potential.

TABLE V. Values for  $T_1$ ,  $T_2$ ,  $T_3$ , and  $T_4$  evaluated at  $q_{\min}^2$  and  $q_{\max}^2$  compared to the ones obtained by Ivanov *et al.* [5]. Here  $l$  stands for  $l = e, \mu$ . Asterisk as in Table IV.

$B_c^- \rightarrow \chi_{c2} l^- \bar{\nu}_l$	$q_{\min}^2$	$q_{\max}^2$	$B_c^- \rightarrow \Psi(3836) l^- \bar{\nu}_l$	$q_{\min}^2$	$q_{\max}^2$
$T_1$			$T_1$		
This work	$0.97^{+0.08}_{-0.01}$	$1.95_{-0.06}$	This work	$0.29_{-0.02}$	$0.22_{-0.01}$
[5]	1.22	1.69	[5]	0.052	0.35
$T_2$ [GeV $^{-2}$ ]			$T_2$ [GeV $^{-2}$ ]		
This work	$-0.012_{-0.001}$	$-0.025^{+0.001}$	This work	$-0.010^{+0.006}$	$-0.018^{+0.001}$
[5]	-0.011	-0.018	[5]	0.0071	0.0090
$T_3$ [GeV $^{-2}$ ]			$T_3$ [GeV $^{-2}$ ]		
This work	$0.013^{+0.001}$	$0.030_{-0.001}$	This work	$0.025_{-0.002}$	$0.052_{-0.004}$
[5]	0.025	0.040	[5]	-0.036	-0.052
$T_4$ [GeV $^{-2}$ ]			$T_4$ [GeV $^{-2}$ ]		
This work	$0.013^{+0.001}$	$0.030_{-0.001}$	This work	$0.022_{-0.001}$	$0.045_{-0.003}$
[5]	0.021*	0.033*	[5]	-0.026*	-0.038*



sonable agreement for the absolute values of the form factors but we disagree for some of the signs.

### B. Decay width

For a  $B_c$  at rest, the double differential decay width with respect to  $q^2$  and  $x_l$ , being  $x_l$  the cosine of the angle between the final meson momentum and the momentum of the final charged lepton measured in the lepton-neutrino center of mass frame (CMF), is given by<sup>5</sup>

$$\frac{d^2\Gamma}{dq^2 dx_l} = \frac{G_F^2}{64m_{B_c}^2} \frac{|V_{bc}|^2}{8\pi^3} \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{B_c}} \times \frac{q^2 - m_l^2}{q^2} \mathcal{H}_{\alpha\beta}(P_{B_c}, P_{c\bar{c}}) \mathcal{L}^{\alpha\beta}(p_l, p_\nu), \quad (23)$$

where  $G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$  [25] is the Fermi constant,  $\lambda(a, b, c) = (a + b - c)^2 - 4ab$ ,  $m_l$  is the mass of the charged lepton,  $\mathcal{H}_{\alpha\beta}$  and  $\mathcal{L}^{\alpha\beta}$  are the hadron and lepton tensors, and  $P_{B_c}$ ,  $P_{c\bar{c}}$ ,  $p_l$ ,  $p_\nu$  are the meson and lepton four-momenta. The lepton tensor is<sup>6</sup>

$$\mathcal{L}^{\alpha\beta}(p_l, p_\nu) = 8(p_l^\alpha p_\nu^\beta + p_l^\beta p_\nu^\alpha - g^{\alpha\beta} p_l \cdot p_\nu \mp i\varepsilon^{\alpha\beta\sigma\rho} p_{l\sigma} p_{\nu\rho}). \quad (24)$$

As for the hadron tensor it is given by

$$\mathcal{H}_{\alpha\beta}(P_{B_c}, P_{c\bar{c}}) = \sum_\lambda h_{(\lambda)\alpha}(P_{B_c}, P_{c\bar{c}}) h_{(\lambda)\beta}^*(P_{B_c}, P_{c\bar{c}}) \quad (25)$$

with

$$h_{(\lambda)\alpha}(P_{B_c}, P_{c\bar{c}}) = \langle c\bar{c} | \lambda \vec{P}_{c\bar{c}} | J_\alpha^{cb}(0) | B_c, \vec{P}_{B_c} \rangle. \quad (26)$$

The quantity

$$\mathcal{H}_{\alpha\beta}(P_{B_c}, P_{c\bar{c}}) \mathcal{L}^{\alpha\beta}(p_l, p_\nu) \quad (27)$$

is a scalar and to evaluate it we can choose  $\vec{P}_{c\bar{c}}$  along the negative  $z$ -axis. This implies also that the CMF of the final leptons moves in the positive  $z$ -direction. Furthermore we shall follow Ref. [5] and introduce helicity components for the hadron and lepton tensors. For that purpose we rewrite

$$\mathcal{H}_{\alpha\beta}(P_{B_c}, P_{c\bar{c}}) \mathcal{L}^{\alpha\beta}(p_l, p_\nu) = \mathcal{H}^{\sigma\rho}(P_{B_c}, P_{c\bar{c}}) g_{\sigma\alpha} g_{\rho\beta} \times \mathcal{L}^{\alpha\beta}(p_l, p_\nu) \quad (28)$$

and use [41]

$$g_{\mu\nu} = \sum_{r=t,\pm 1,0} g_{rr} \varepsilon_{(r)\mu}(q) \varepsilon_{(r)\nu}^*(q); \quad g_{tt} = 1, \quad g_{\pm 1,0} = -1 \quad (29)$$

with  $\varepsilon_{(r)}^\mu(q) = q^\mu / q^2$  and where the  $\varepsilon_{(r)}(q)$ ,  $r = \pm 1, 0$  are the polarization vectors for an on-shell vector particle with four-momentum  $q$  and helicity  $r$ . Defining helicity compo-

nents for the hadron and lepton tensors as

$$\mathcal{H}_{rs}(P_{B_c}, P_{c\bar{c}}) = \varepsilon_{(r)\sigma}^*(q) \mathcal{H}^{\sigma\rho}(P_{B_c}, P_{c\bar{c}}) \varepsilon_{(s)\rho}(q) \quad (30)$$

$$\mathcal{L}_{rs}(p_l, p_\nu) = \varepsilon_{(r)\alpha}(q) \mathcal{L}^{\alpha\beta}(p_l, p_\nu) \varepsilon_{(s)\beta}^*(q),$$

we have that

$$\mathcal{H}_{\alpha\beta}(P_{B_c}, P_{c\bar{c}}) \mathcal{L}^{\alpha\beta}(p_l, p_\nu) = \sum_{r=t,\pm 1,0} \sum_{s=t,\pm 1,0} g_{rr} g_{ss} \times \mathcal{H}_{rs}(P_{B_c}, P_{c\bar{c}}) \mathcal{L}_{rs}(p_l, p_\nu). \quad (31)$$

Let us start with the lepton tensor. We can take advantage of the fact that the Wigner rotation relating the original frame and the CMF of the final leptons is the identity to evaluate the lepton tensor helicity components in this latter reference system

$$\mathcal{L}_{rs}(p_l, p_\nu) = \varepsilon_{(r)\alpha}(q) \mathcal{L}^{\alpha\beta}(p_l, p_\nu) \varepsilon_{(s)\beta}^*(q) = \varepsilon_{(r)\alpha}(\tilde{q}) \mathcal{L}^{\alpha\beta}(\tilde{p}_l, \tilde{p}_\nu) \varepsilon_{(s)\beta}^*(\tilde{q}), \quad (32)$$

where the tilde stands for momenta measured in the final leptons CMF. For the purpose of evaluation, we can use<sup>7</sup>

$$\tilde{p}_l^\alpha = (E_l(|\tilde{p}_l|), -|\tilde{p}_l| \sqrt{1 - x_l^2}, 0, -|\tilde{p}_l| x_l) \quad (33)$$

$$\tilde{p}_\nu^\alpha = (|\tilde{p}_l|, |\tilde{p}_l| \sqrt{1 - x_l^2}, 0, |\tilde{p}_l| x_l)$$

with  $|\tilde{p}_l|$  the modulus of the lepton three-momentum measured in the leptons CMF.

The only helicity components that we shall need are the following:

$$\mathcal{L}_{tt}(p_l, p_\nu) = 4 \frac{m_l^2(q^2 - m_l^2)}{q^2}$$

$$\mathcal{L}_{t0}(p_l, p_\nu) = \mathcal{L}_{0t}(p_l, p_\nu) = -4x_l \frac{m_l^2(q^2 - m_l^2)}{q^2}$$

$$\mathcal{L}_{+1+1}(p_l, p_\nu) = (q^2 - m_l^2) \left( 4(1 \pm x_l) - 2(1 - x_l^2) \frac{q^2 - m_l^2}{q^2} \right)$$

$$\mathcal{L}_{-1-1}(p_l, p_\nu) = (q^2 - m_l^2) \left( 4(1 \mp x_l) - 2(1 - x_l^2) \frac{q^2 - m_l^2}{q^2} \right)$$

$$\mathcal{L}_{00}(p_l, p_\nu) = 4(q^2 - m_l^2) \left( 1 - x_l^2 \frac{q^2 - m_l^2}{q^2} \right). \quad (34)$$

As for the hadron tensor, it is convenient to introduce helicity amplitudes defined as

$$h_{(\lambda)r}(P_{B_c}, P_{c\bar{c}}) = \varepsilon_{(r)\alpha}^*(q) h_{(\lambda)}^\alpha(P_{B_c}, P_{c\bar{c}}), \quad r = t, \pm 1, 0 \quad (35)$$

in terms of which

<sup>7</sup>Note this is in accordance with the definition of  $x_l$  and the fact that we have taken  $\vec{P}_{c\bar{c}}$  in the negative  $z$  direction. Furthermore there can be no dependence on the  $\varphi_l$  azimuthal angle so that we can take  $\tilde{p}_l$ , and then  $\tilde{p}_\nu$ , in the  $OXZ$  plane.

<sup>5</sup>We shall neglect neutrino masses in the calculation.

<sup>6</sup>The  $\mp$  signs correspond, respectively, to decays into  $l^- \bar{\nu}_l$  (for  $B_c^-$  decays) and  $l^+ \nu_l$  (for  $B_c^+$  decays).

$$\mathcal{H}_{rs}(P_{B_c}, P_{c\bar{c}}) = \sum_{\lambda} h_{(\lambda)r}(P_{B_c}, P_{c\bar{c}}) h_{(\lambda)s}^*(P_{B_c}, P_{c\bar{c}}). \quad (36)$$

We now give the expressions for the helicity amplitudes evaluated in the original frame:

– Case  $0^- \rightarrow 0^-, 0^+$ :

$$\begin{aligned} h_t(P_{B_c}, P_{c\bar{c}}) &= \frac{m_{B_c}^2 - m_{c\bar{c}}^2}{\sqrt{q^2}} F_+(q^2) + \sqrt{q^2} F_-(q^2) \\ h_0(P_{B_c}, P_{c\bar{c}}) &= \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{\sqrt{q^2}} F_+(q^2) \\ h_{+1}(P_{B_c}, P_{c\bar{c}}) &= h_{-1}(P_{B_c}, P_{c\bar{c}}) = 0. \end{aligned} \quad (37)$$

– Case  $0^- \rightarrow 1^-, 1^+$ :

$$\begin{aligned} h_{(\lambda)t}(P_{B_c}, P_{c\bar{c}}) &= i\delta_{\lambda 0} \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{c\bar{c}}\sqrt{q^2}} \left( (m_{B_c} - m_{c\bar{c}})(A_0(q^2) - A_+(q^2)) - \frac{q^2}{m_{B_c} + m_{c\bar{c}}} A_-(q^2) \right) \\ h_{(\lambda)+1}(P_{B_c}, P_{c\bar{c}}) &= -i\delta_{\lambda-1} \left( \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{m_{B_c} + m_{c\bar{c}}} V(q^2) + (m_{B_c} - m_{c\bar{c}}) A_0(q^2) \right) \\ h_{(\lambda)-1}(P_{B_c}, P_{c\bar{c}}) &= -i\delta_{\lambda+1} \left( -\frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{m_{B_c} + m_{c\bar{c}}} V(q^2) + (m_{B_c} - m_{c\bar{c}}) A_0(q^2) \right) \\ h_{(\lambda)0}(P_{B_c}, P_{c\bar{c}}) &= i\delta_{\lambda 0} \left( (m_{B_c} - m_{c\bar{c}}) \frac{m_{B_c}^2 - q^2 - m_{c\bar{c}}^2}{2m_{c\bar{c}}\sqrt{q^2}} A_0(q^2) - \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{c\bar{c}}\sqrt{q^2}} \frac{A_+(q^2)}{m_{B_c} + m_{c\bar{c}}} \right). \end{aligned} \quad (38)$$

– Case  $0^- \rightarrow 2^-, 2^+$ :

$$\begin{aligned} h_{(\lambda)t}(P_{B_c}, P_{c\bar{c}}) &= -i\delta_{\lambda 0} \sqrt{\frac{2}{3}} \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{4m_{c\bar{c}}^2\sqrt{q^2}} (T_1(q^2) + (m_{B_c}^2 - m_{c\bar{c}}^2)T_2(q^2) + q^2 T_3(q^2)) \\ h_{(\lambda)+1}(P_{B_c}, P_{c\bar{c}}) &= i\delta_{\lambda-1} \frac{1}{\sqrt{2}} \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{c\bar{c}}} (T_1(q^2) - \lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)T_4(q^2)) \\ h_{(\lambda)-1}(P_{B_c}, P_{c\bar{c}}) &= i\delta_{\lambda+1} \frac{1}{\sqrt{2}} \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{c\bar{c}}} (T_1(q^2) + \lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)T_4(q^2)) \\ h_{(\lambda)0}(P_{B_c}, P_{c\bar{c}}) &= -i\delta_{\lambda 0} \sqrt{\frac{2}{3}} \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{4m_{c\bar{c}}^2\sqrt{q^2}} ((m_{B_c}^2 - q^2 - m_{c\bar{c}}^2)T_1(q^2) + \lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)T_2(q^2)). \end{aligned} \quad (39)$$

We see that the helicity amplitudes, and thus the helicity components of the hadron tensor, only depend on  $q^2$ . The expressions for the latter are collected in Appendix D.

We can now define the combinations [5]

$$\begin{aligned} H_U &= \mathcal{H}_{+1+1} + \mathcal{H}_{-1-1} & H_L &= \mathcal{H}_{00} \\ H_P &= \mathcal{H}_{+1+1} - \mathcal{H}_{-1-1} & H_S &= 3\mathcal{H}_{tt} \\ H_{SL} &= \mathcal{H}_{t0} & \tilde{H}_J &= \frac{m_l^2}{2q^2} H_J; \quad J = U, L, S, SL \end{aligned} \quad (40)$$

with  $U$ ,  $L$ ,  $P$ ,  $S$ , and  $SL$  representing, respectively, unpolarized-transverse, longitudinal, parity-odd, scalar, and scalar-longitudinal interference.

Finally, the double differential decay width is written in terms of the above defined combinations as

$$\begin{aligned} \frac{d^2\Gamma}{dq^2 dx_l} &= \frac{G_F^2}{8\pi^3} |V_{bc}|^2 \frac{(q^2 - m_l^2)^2}{12m_{B_c}^2 q^2} \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{B_c}} \\ &\times \left\{ \frac{3}{8} (1 + x_l^2) H_U + \frac{3}{4} (1 - x_l^2) H_L \pm \frac{3}{4} x_l H_P \right. \\ &\left. + \frac{3}{4} (1 - x_l^2) \tilde{H}_U + \frac{3}{2} x_l^2 \tilde{H}_L + \frac{1}{2} \tilde{H}_S + 3x_l \tilde{H}_{SL} \right\}. \end{aligned} \quad (41)$$

Note that for antiparticle decay  $H_P$  has the opposite sign to

the case of particle decay while all other hadron tensor helicity component combinations defined in Eq. (40) do not change (see Appendix C for details). The sign change of  $H_P$  compensates the extra sign coming from the lepton tensor. This means that in fact the double differential decay width is the same for  $B_c^-$  or  $B_c^+$  decay.

Integrating over  $x_l$ , we obtain the differential decay width

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{8\pi^3} |V_{bc}|^2 \frac{(q^2 - m_l^2)^2}{12m_{B_c}^2 q^2} \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{B_c}} \times \{H_U + H_L + \tilde{H}_U + \tilde{H}_L + \tilde{H}_S\} \quad (42)$$

from where, integrating over  $q^2$ , we obtain the total decay width that we write, following Ref. [5], as

$$\Gamma = \Gamma_U + \Gamma_L + \tilde{\Gamma}_U + \tilde{\Gamma}_L + \tilde{\Gamma}_S \quad (43)$$

with  $\Gamma_J$  and  $\tilde{\Gamma}_J$  partial helicity widths defined as

$$\Gamma_J = \int dq^2 \frac{G_F^2}{8\pi^3} |V_{bc}|^2 \frac{(q^2 - m_l^2)^2}{12m_{B_c}^2 q^2} \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{B_c}} H_J \quad (44)$$

and similarly for  $\tilde{\Gamma}_J$  in terms of  $\tilde{H}_J$ .

$$\begin{aligned} \frac{d\Gamma_{B_c \rightarrow \mu^- \mu^+ (J/\Psi) l \bar{\nu}_l}}{dx_\mu} = & \left( 1 + \frac{\sqrt{1 - 4m_\mu^2/m_{J/\Psi}^2} (\Gamma_U + \tilde{\Gamma}_U - 2(\Gamma_L + \tilde{\Gamma}_L + \tilde{\Gamma}_S))}{(2 - \sqrt{1 - 4m_\mu^2/m_{J/\Psi}^2}) (\Gamma_U + \tilde{\Gamma}_U) + 2(\Gamma_L + \tilde{\Gamma}_L + \tilde{\Gamma}_S)} x_\mu^2 \right) \frac{\Gamma_{J/\Psi \rightarrow \mu^- \mu^+}}{\Gamma_{J/\Psi}} \frac{1}{1 + 2m_\mu^2/m_{J/\Psi}^2} \\ & \times \left[ \frac{3}{4} (\Gamma_L + \tilde{\Gamma}_L + \tilde{\Gamma}_S) + \frac{3}{8} (2 - \sqrt{1 - 4m_\mu^2/m_{J/\Psi}^2}) (\Gamma_U + \tilde{\Gamma}_U) \right], \end{aligned} \quad (47)$$

where  $x_\mu$  is the cosine of the polar angle for the final  $\mu^- \mu^+$  pair, relative to the momentum of the decaying  $J/\Psi$ , measured in the  $\mu^- \mu^+$  CMF,  $\Gamma_{J/\Psi \rightarrow \mu^- \mu^+}$  is the  $J/\Psi$  decay width into the  $\mu^- \mu^+$  channel, and  $\Gamma_{J/\Psi}$  is the total  $J/\Psi$  decay width. The asymmetry parameter

$$\alpha^* = \frac{\sqrt{1 - 4m_\mu^2/m_{J/\Psi}^2} (\Gamma_U + \tilde{\Gamma}_U - 2(\Gamma_L + \tilde{\Gamma}_L + \tilde{\Gamma}_S))}{(2 - \sqrt{1 - 4m_\mu^2/m_{J/\Psi}^2}) (\Gamma_U + \tilde{\Gamma}_U) + 2(\Gamma_L + \tilde{\Gamma}_L + \tilde{\Gamma}_S)} \quad (48)$$

governs the muon angular distribution in their CMF.

### 1. Results

In Table VI we give our results for the partial helicity widths corresponding to  $B_c^-$  decays. For  $B_c^+$  decay the “P” column changes sign while all others remain the same. The central values have been evaluated with the AL1 potential and the theoretical errors quoted reflect the dependence of the results on the interquark potential.

In Table VII we show the asymmetry parameters. Our values for  $\alpha^*$  compare well with the results obtained in

Another quantity of interest is the forward-backward asymmetry of the charged lepton measured in the leptons CMF. This asymmetry is defined as<sup>8</sup>

$$A_{FB} = \frac{\Gamma_{x_l > 0} - \Gamma_{x_l < 0}}{\Gamma_{x_l > 0} + \Gamma_{x_l < 0}} \quad (45)$$

and it is given in terms of partial helicity widths as

$$= \frac{3}{4} \frac{\pm \Gamma_P + 4\tilde{\Gamma}_{SL}}{\Gamma_U + \Gamma_L + \tilde{\Gamma}_U + \tilde{\Gamma}_L + \tilde{\Gamma}_S} \quad (46)$$

being the same for a negative charged lepton  $l^-$  ( $B_c^-$  decay) as for a positive charged one  $l^+$  ( $B_c^+$  decay), as  $\Gamma_P$  for antiparticle decay has the opposite sign as for particle decay.

Finally, for the decay channel  $B_c \rightarrow J/\Psi l \bar{\nu}_l$  with the  $J/\Psi$  decaying into  $\mu^- \mu^+$  we can evaluate the differential cross section

Ref. [5]. The same is true for the forward-backward asymmetry with some exceptions: most notably we get opposite signs for  $B_c \rightarrow \chi_{c1}$  and  $B_c \rightarrow \Psi(3836)$ .

In Fig. 5 we show the differential decay width  $d\Gamma/dq^2$  for the decay channels  $\eta_c l^- \bar{\nu}_l$  and  $\chi_{c0} l^- \bar{\nu}_l$  for the case where the final lepton is a light one  $l = e, \mu^9$  or a heavy one  $l = \tau$ . We show the results obtained with the AL1 and BHAD potentials, finding no significant difference for the  $\tau$  case, while for the light final lepton case the differences are around 10% at low  $q^2$ .

In Fig. 6 we show now the results for vector and tensor mesons. As before, only for the case where the final lepton is light we see up to 10% differences between the calculation with the AL1 and the BHAD potentials.

Finally, in Tables VIII and IX we give the total decay widths and corresponding branching ratios for the different transitions. The branching ratios evaluated by Ivanov *et al.*

<sup>8</sup>The forward direction is determined by the momentum of the final meson that we have chosen in the negative  $z$ -direction.

<sup>9</sup>We show the distribution corresponding to a final electron. The distribution for a final muon differs from the former only for  $q^2$  around  $m_\mu^2$ .

TABLE VI. Partial helicity widths in units of  $10^{-15}$  GeV for  $B_c^-$  decays. Central values have been evaluated with the AL1 potential.

$B_c^- \rightarrow$	$\Gamma_U$	$\tilde{\Gamma}_U$	$\Gamma_L$	$\tilde{\Gamma}_L$	$\Gamma_P$	$\tilde{\Gamma}_S$	$\tilde{\Gamma}_{SL}$
$\eta_c e^- \bar{\nu}_e$	0	0	$6.95^{+0.31}$	$0.13^{+0.01} 10^{-5}$	0	$0.44^{+0.03} 10^{-5}$	$0.14^{+0.01} 10^{-5}$
$\eta_c \mu^- \bar{\nu}_\mu$	0	0	$6.80^{+0.31}$	$0.28^{+0.02} 10^{-1}$	0	$0.10^{+0.01}$	$0.31^{+0.01} 10^{-1}$
$\eta_c \tau^- \bar{\nu}_\tau$	0	0	$0.71^{+0.02}$	$0.17^{+0.01}$	0	$1.58^{+0.04}$	$0.29^{+0.01}$
$\chi_{c0} e^- \bar{\nu}_e$	0	0	$1.55^{+0.14}_{-0.02}$	$0.37^{+0.05} 10^{-6}$	0	$0.11^{+0.01} 10^{-5}$	$0.37^{+0.05} 10^{-6}$
$\chi_{c0} \mu^- \bar{\nu}_\mu$	0	0	$1.51^{+0.13}_{-0.02}$	$0.75^{+0.09} 10^{-2}$	0	$0.23^{+0.02} 10^{-1}$	$0.75^{+0.09} 10^{-2}$
$\chi_{c0} \tau^- \bar{\nu}_\tau$	0	0	$0.80^{+0.04}_{-0.02} 10^{-1}$	$0.23^{+0.01}_{-0.01} 10^{-1}$	0	$0.84^{+0.07} 10^{-1}$	$0.25^{+0.02} 10^{-1}$
$J/\Psi e^- \bar{\nu}_e$	$11.5^{+0.6}$	$0.32^{+0.02} 10^{-6}$	$10.4^{+0.6}$	$0.12^{+0.01} 10^{-5}$	$-5.48_{-0.24}$	$0.32^{+0.03} 10^{-5}$	$0.11^{+0.01} 10^{-5}$
$J/\Psi \mu^- \bar{\nu}_\mu$	$11.4^{+0.6}$	$0.13^{+0.01} 10^{-1}$	$10.2^{+0.7}$	$0.28^{+0.03} 10^{-1}$	$-5.45_{-0.24}$	$0.68^{+0.07} 10^{-1}$	$0.25^{+0.02} 10^{-1}$
$J/\Psi \tau^- \bar{\nu}_\tau$	$2.78^{+0.10}_{-0.01}$	$0.59^{+0.02}$	$1.74^{+0.07}_{-0.01}$	$0.39^{+0.02}$	$-1.10_{-0.03}$	$0.36^{+0.02}$	$0.21^{+0.01}$
$\chi_{c1} e^- \bar{\nu}_e$	$0.90^{+0.05}_{-0.03}$	$0.43^{+0.03}_{-0.01} 10^{-7}$	$0.35^{+0.03} 10^{-1}$	$0.28^{+0.03} 10^{-8}$	$-0.75^{+0.02}_{-0.04}$	$0.57^{+0.07} 10^{-8}$	$0.22^{+0.02} 10^{-8}$
$\chi_{c1} \mu^- \bar{\nu}_\mu$	$0.89^{+0.05}_{-0.03}$	$0.18^{+0.01}_{-0.01} 10^{-2}$	$0.35^{+0.03} 10^{-1}$	$0.77^{+0.08} 10^{-4}$	$-0.75^{+0.02}_{-0.04}$	$0.11^{+0.01} 10^{-3}$	$0.49^{+0.05} 10^{-4}$
$\chi_{c1} \tau^- \bar{\nu}_\tau$	$0.75^{+0.02} 10^{-1}$	$0.21^{+0.01} 10^{-1}$	$0.46^{+0.04} 10^{-2}$	$0.12^{+0.01} 10^{-2}$	$-0.64_{-0.03} 10^{-1}$	$0.23^{+0.02} 10^{-3}$	$0.28^{+0.02} 10^{-3}$
$h_c e^- \bar{\nu}_e$	$0.16^{+0.02}$	$0.57^{+0.07} 10^{-8}$	$2.23^{+0.12}$	$0.72^{+0.09} 10^{-6}$	$-0.26_{-0.03} 10^{-1}$	$0.23^{+0.03} 10^{-5}$	$0.74^{+0.09} 10^{-6}$
$h_c \mu^- \bar{\nu}_\mu$	$0.16^{+0.02}$	$0.24^{+0.03} 10^{-3}$	$2.16^{+0.21}_{-0.01}$	$0.14^{+0.01} 10^{-1}$	$-0.26_{-0.03} 10^{-1}$	$0.45^{+0.05}_{-0.01} 10^{-1}$	$0.14^{+0.02} 10^{-1}$
$h_c \tau^- \bar{\nu}_\tau$	$0.23^{+0.02} 10^{-1}$	$0.60^{+0.06} 10^{-2}$	$0.67^{+0.05} 10^{-1}$	$0.20^{+0.01} 10^{-1}$	$-0.27_{-0.03} 10^{-2}$	$0.97^{+0.05}_{-0.03} 10^{-1}$	$0.26^{+0.02}_{-0.01} 10^{-1}$
$\chi_{c2} e^- \bar{\nu}_e$	$0.71^{+0.03}_{-0.03}$	$0.37^{+0.02}_{-0.02} 10^{-7}$	$1.17^{+0.08}_{-0.05}$	$0.31^{+0.03}_{-0.01} 10^{-6}$	$-0.35^{+0.01}_{-0.02}$	$0.88^{+0.09}_{-0.03} 10^{-6}$	$0.30^{+0.03}_{-0.01} 10^{-6}$
$\chi_{c2} \mu^- \bar{\nu}_\mu$	$0.71^{+0.02}_{-0.03}$	$0.15^{+0.01} 10^{-2}$	$1.14^{+0.07}_{-0.05}$	$0.62^{+0.06}_{-0.02} 10^{-2}$	$-0.34^{+0.01}_{-0.02}$	$0.16^{+0.02} 10^{-1}$	$0.57^{+0.06}_{-0.02} 10^{-2}$
$\chi_{c2} \tau^- \bar{\nu}_\tau$	$0.49^{+0.01}_{-0.03} 10^{-1}$	$0.15_{-0.01} 10^{-1}$	$0.43^{+0.01}_{-0.02} 10^{-1}$	$0.13_{-0.01} 10^{-1}$	$-0.18^{+0.01} 10^{-1}$	$0.12_{-0.01} 10^{-1}$	$0.70^{+0.02}_{-0.04} 10^{-2}$
$\Psi(3836) e^- \bar{\nu}_e$	$0.58_{-0.07} 10^{-1}$	$0.47_{-0.06} 10^{-8}$	$0.33^{+0.01}_{-0.02} 10^{-2}$	$0.68^{+0.04}_{-0.04} 10^{-9}$	$-0.48^{+0.05} 10^{-1}$	$0.17^{+0.01}_{-0.01} 10^{-8}$	$0.60^{+0.03}_{-0.04} 10^{-9}$
$\Psi(3836) \mu^- \bar{\nu}_\mu$	$0.57_{-0.06} 10^{-1}$	$0.19_{-0.02} 10^{-3}$	$0.32^{+0.01}_{-0.02} 10^{-2}$	$0.15_{-0.01} 10^{-4}$	$-0.48^{+0.06} 10^{-1}$	$0.28^{+0.02}_{-0.02} 10^{-4}$	$0.11^{+0.01} 10^{-4}$
$\Psi(3836) \tau^- \bar{\nu}_\tau$	$0.78_{-0.09} 10^{-3}$	$0.28_{-0.03} 10^{-3}$	$0.74_{-0.06} 10^{-4}$	$0.25_{-0.02} 10^{-4}$	$-0.69^{+0.08} 10^{-3}$	$0.54_{-0.04} 10^{-5}$	$0.65_{-0.05} 10^{-5}$

TABLE VII. Asymmetry parameters in semileptonic  $B_c \rightarrow c\bar{c}$  decays. Our central values have been evaluated with the AL1 potential. We also show the results obtained by Ivanov *et al.* [5].

		$A_{FB}(e)$	$A_{FB}(\mu)$	$A_{FB}(\tau)$	$\alpha^*(e)$	$\alpha^*(\mu)$	$\alpha^*(\tau)$
$B_c \rightarrow \eta_c$	This work [5]	$0.60^{+0.01} 10^{-6}$ $0.953 10^{-6}$	$0.13^{+0.01} 10^{-1}$	0.35 0.36			
$B_c \rightarrow \chi_{c0}$	This work [5]	$0.72^{+0.02} 10^{-6}$ $1.31 10^{-6}$	$0.15 10^{-1}$	0.40 0.39			
$B_c \rightarrow J/\Psi$	This work [5]	-0.19 -0.21	$-0.18_{-0.01}$	$-0.35^{+0.02} 10^{-1}$ $-0.48 10^{-1}$	$-0.29_{-0.01}$ -0.34	-0.29	-0.19 -0.24
$B_c \rightarrow \chi_{c1}$	This work [5]	$-0.60_{-0.01}$ 0.19	$-0.60_{-0.01}$	-0.46 0.34			
$B_c \rightarrow h_c$	This work [5]	$-0.83_{-0.05} 10^{-2}$ $-3.6 10^{-2}$	$0.97^{+0.01}_{-0.05} 10^{-2}$	0.35 0.31			
$B_c \rightarrow \chi_{c2}$	This work [5]	-0.14 -0.16	-0.13	$0.55^{+0.02} 10^{-1}$ $0.44 10^{-1}$			
$B_c \rightarrow \Psi(3836)$	This work [5]	-0.59 0.21	-0.59	-0.42 0.41			

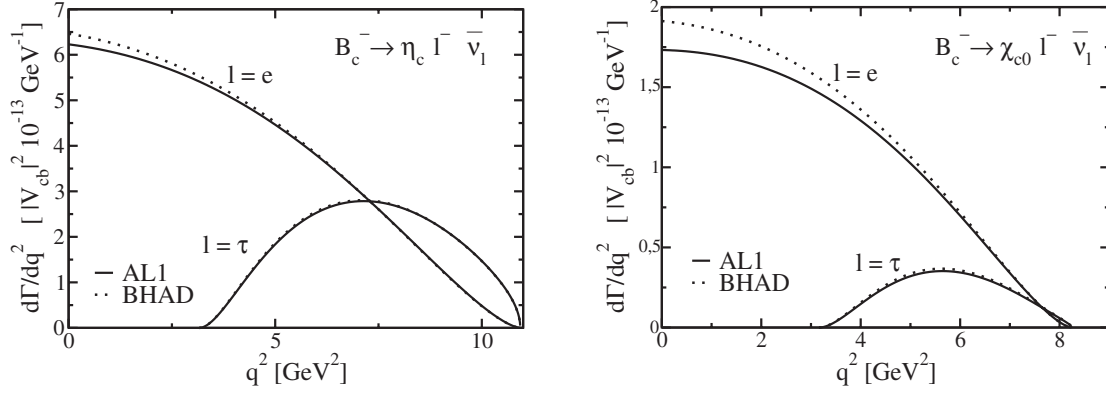


FIG. 5. Differential decay width for the  $B_c^- \rightarrow \eta_c l^- \bar{\nu}_l$  and  $B_c^- \rightarrow \chi_{c0} l^- \bar{\nu}_l$  processes obtained with the AL1 potential. For comparison, we also show with dotted lines the results obtained with the Bhaduri (BHAD) potential. The distribution for a final muon is not explicitly shown. It differs appreciably from the corresponding to a final electron only for  $q^2$  around  $m_\mu^2$ .

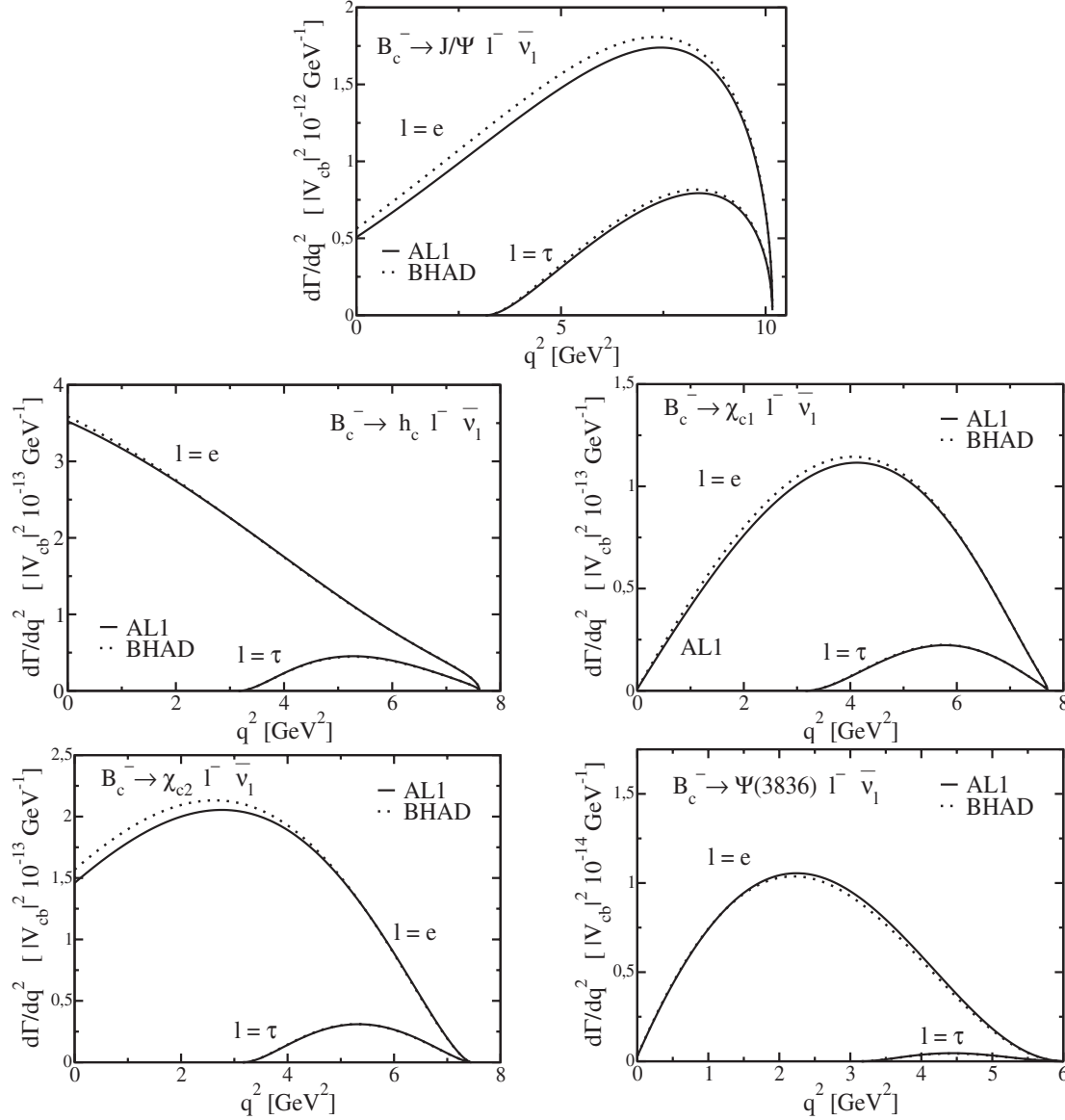


FIG. 6. Differential decay width for the  $B_c^- \rightarrow J/\Psi l^- \bar{\nu}_l$ ,  $B_c^- \rightarrow h_c l^- \bar{\nu}_l$ ,  $B_c^- \rightarrow \chi_{c1} l^- \bar{\nu}_l$ ,  $B_c^- \rightarrow \chi_{c2} l^- \bar{\nu}_l$ , and  $B_c^- \rightarrow \Psi(3836) l^- \bar{\nu}_l$  decay channels obtained with the AL1 potential (solid lines) and the Bhaduri (BHAD) potential (dotted lines). The distribution for a final muon is not explicitly shown. It differs appreciably from the corresponding to a final electron only for  $q^2$  around  $m_\mu^2$ .

TABLE VIII. Decay widths in units of  $10^{-15}$  GeV for semileptonic  $B_c^- \rightarrow c\bar{c}$  decays. Our central values have been evaluated with the AL1 potential. Here  $l$  stands for  $l = e, \mu$ .

	$\Gamma$ [ $10^{-15}$ GeV]									
	This work	[5]	[7]	[9,10]	[12]	[13]	[14]	[18]	[19]	[22]
$B_c^- \rightarrow \eta_c l^- \bar{\nu}_l$	$6.95^{+0.29}_{-0.01}$	10.7	5.9	14.2	11.1	8.31	$11 \pm 1$	2.1 (6.9)	8.6	10
$B_c^- \rightarrow \eta_c \tau^- \bar{\nu}_\tau$	$2.46^{+0.07}_{-0.01}$	3.52							$3.3 \pm 0.9$	
$B_c^- \rightarrow \chi_{c0} l^- \bar{\nu}_l$	$1.55^{+0.14}_{-0.02}$	2.52		1.69						
$B_c^- \rightarrow \chi_{c0} \tau^- \bar{\nu}_\tau$	$0.19^{+0.01}_{-0.01}$	0.26		0.25						
$B_c^- \rightarrow J/\Psi l^- \bar{\nu}_l$	$21.9^{+1.2}_{-0.03}$	28.2	17.7	34.4	30.2	20.3	$28 \pm 5$	21.6 (48.3)	17.2	42
$B_c^- \rightarrow J/\Psi \tau^- \bar{\nu}_\tau$	$5.86^{+0.23}_{-0.03}$	7.82							$7 \pm 2$	
$B_c^- \rightarrow \chi_{c1} l^- \bar{\nu}_l$		1.40		2.21						
$B_c^- \rightarrow \chi_{c1} \tau^- \bar{\nu}_\tau$	0.10	0.17		0.35						
$B_c^- \rightarrow h_c l^- \bar{\nu}_l$	$2.40^{+0.23}_{-0.01}$	4.42		2.51						
$B_c^- \rightarrow h_c \tau^- \bar{\nu}_\tau$	$0.21^{+0.01}_{-0.01}$	0.38		0.36						
$B_c^- \rightarrow \chi_{c2} l^- \bar{\nu}_l$	$1.89^{+0.11}_{-0.08}$	2.92		2.73						
$B_c^- \rightarrow \chi_{c2} \tau^- \bar{\nu}_\tau$	$0.13^{+0.01}_{-0.01}$	0.20		0.42						
$B_c^- \rightarrow \Psi(3836) l^- \bar{\nu}_l$	$0.062_{-0.008}$	0.13								
$B_c^- \rightarrow \Psi(3836) \tau^- \bar{\nu}_\tau$	$0.0012_{-0.0002}$	0.0031								

[4], where they have used the new  $B_c$  mass determination by the CDF Collaboration [26], are in reasonable agreement with our results. Discrepancies are larger for the decay widths in Table VIII where they use the larger mass value  $m_{B_c} = 6400$  MeV quoted by the PDG [25].

### C. Heavy quark spin symmetry

As mentioned in the Introduction, one cannot apply HQS to systems with two heavy quarks due to flavor symmetry breaking by the kinetic energy terms. The symmetry that survives for such systems is HQSS amounting to the decoupling of the two heavy quark spins. Using HQSS, Jenkins *et al.* [2] obtained relations between different form

factors for semileptonic  $B_c$  decays into ground state vector and pseudoscalar mesons. Let us check the agreement of our calculations with their results. For that purpose let us rewrite the general form factor decompositions in Eq. (9) introducing the four vectors  $v$  and  $k$  such that

$$P_{B_c} = m_{B_c} v; \quad P_{c\bar{c}} = m_{c\bar{c}} v + k. \quad (49)$$

$v$  is the four-velocity of the initial  $B_c$  meson whereas  $k$  is a residual momentum. In terms of those we have

$$\begin{aligned} P &= P_{B_c} + P_{c\bar{c}} = (m_{B_c} + m_{c\bar{c}})v + k; \\ q &= P_{B_c} - P_{c\bar{c}} = (m_{B_c} - m_{c\bar{c}})v - k \end{aligned} \quad (50)$$

TABLE IX. Branching ratios in percent for semileptonic  $B_c^- \rightarrow c\bar{c}$  decays. Our central values have been evaluated with the AL1 potential. Here  $l$  stands for  $l = e, \mu$ .

	Branching ratio (%)								
	This work	[4]	[7]	[9–11]	[12]	[15,16]	[18]	[19]	[20]
$B_c^- \rightarrow \eta_c l^- \bar{\nu}_l$	$0.48^{+0.02}_{-0.01}$	0.81	0.42	0.97	0.76	0.75	0.15	0.59	0.51
$B_c^- \rightarrow \eta_c \tau^- \bar{\nu}_\tau$	$0.17^{+0.01}_{-0.01}$	0.22				0.23		0.20	
$B_c^- \rightarrow \chi_{c0} l^- \bar{\nu}_l$	$0.11^{+0.01}_{-0.01}$	0.17		0.12					
$B_c^- \rightarrow \chi_{c0} \tau^- \bar{\nu}_\tau$	$0.013^{+0.001}_{-0.001}$	0.013		0.017					
$B_c^- \rightarrow J/\Psi l^- \bar{\nu}_l$	$1.54^{+0.06}_{-0.01}$	2.07	1.23	2.35	2.01	1.9	1.47	1.20	1.44
$B_c^- \rightarrow J/\Psi \tau^- \bar{\nu}_\tau$	$0.41^{+0.02}_{-0.01}$	0.49				0.48		0.34	
$B_c^- \rightarrow \chi_{c1} l^- \bar{\nu}_l$	$0.066^{+0.003}_{-0.002}$	0.092		0.15					
$B_c^- \rightarrow \chi_{c1} \tau^- \bar{\nu}_\tau$	$0.0072^{+0.0002}_{-0.0003}$	0.0089		0.024					
$B_c^- \rightarrow h_c l^- \bar{\nu}_l$	$0.17^{+0.02}_{-0.01}$	0.27		0.17					
$B_c^- \rightarrow h_c \tau^- \bar{\nu}_\tau$	$0.015^{+0.001}_{-0.001}$	0.017		0.024					
$B_c^- \rightarrow \chi_{c2} l^- \bar{\nu}_l$	$0.13^{+0.01}_{-0.01}$	0.17		0.19					
$B_c^- \rightarrow \chi_{c2} \tau^- \bar{\nu}_\tau$	$0.0093^{+0.0002}_{-0.0005}$	0.0082		0.029					
$B_c^- \rightarrow \Psi(3836) l^- \bar{\nu}_l$	$0.0043^{+0.0005}_{-0.0005}$	0.0066							
$B_c^- \rightarrow \Psi(3836) \tau^- \bar{\nu}_\tau$	$0.000\,083^{+0.000\,010}_{-0.000\,010}$	0.000\,099							



and we can write for the  $\eta_c(c\bar{c}(0^-))$  final state case

$$\begin{aligned}\langle \eta_c, \vec{P}_{\eta_c} | J_{\mu}^{cb}(0) | B_c^-, \vec{P}_{B_c} \rangle &= \langle \eta_c, \vec{P}_{\eta_c} | J_{V\mu}^{cb}(0) | B_c^-, \vec{P}_{B_c} \rangle \\ &= ((m_{B_c} + m_{\eta_c})F_+(q^2) + (m_{B_c} - m_{\eta_c})F_-(q^2))v_{\mu} + (F_+(q^2) - F_-(q^2))k_{\mu} \\ &= \sqrt{2m_{B_c}2m_{\eta_c}}(\Sigma_1^{(0-)}(q^2)v_{\mu} + \bar{\Sigma}_2^{(0-)}(q^2)k_{\mu}),\end{aligned}\quad (51)$$

where we have introduced the new form factors

$$\Sigma_1^{(0-)}(q^2) = \frac{1}{\sqrt{2m_{B_c}2m_{\eta_c}}}((m_{B_c} + m_{\eta_c})F_+(q^2) + (m_{B_c} - m_{\eta_c})F_-(q^2)) \quad \bar{\Sigma}_2^{(0-)}(q^2) = \frac{1}{\sqrt{2m_{B_c}2m_{\eta_c}}}(F_+(q^2) - F_-(q^2)). \quad (52)$$

Similarly, for the  $J/\Psi(c\bar{c}(1^-))$  final state case we have

$$\begin{aligned}\langle J/\Psi, \lambda \vec{P}_{J/\Psi} | J_{\mu}^{cb}(0) | B_c^-, \vec{P}_{B_c} \rangle &= \langle J/\Psi, \lambda \vec{P}_{J/\Psi} | J_{V\mu}^{cb}(0) - J_{A\mu}^{cb}(0) | B_c^-, \vec{P}_{B_c} \rangle \\ &= \frac{2m_{B_c}}{m_{B_c} + m_{J/\Psi}} \varepsilon_{\mu\nu\alpha\beta} \varepsilon_{(\lambda)}^{\nu*}(\vec{P}_{J/\Psi}) v^{\alpha} k^{\beta} V(q^2) - i \left\{ (m_{B_c} - m_{J/\Psi}) \varepsilon_{(\lambda)\mu}^*(\vec{P}_{J/\Psi}) A_0(q^2) \right. \\ &\quad \left. + \frac{m_{B_c}}{m_{J/\Psi}} k \cdot \varepsilon_{(\lambda)}^*(\vec{P}_{J/\Psi}) \left( \frac{A_+(q^2)}{m_{B_c} + m_{J/\Psi}} ((m_{B_c} + m_{J/\Psi})v_{\mu} + k_{\mu}) \right. \right. \\ &\quad \left. \left. + \frac{A_-(q^2)}{m_{B_c} + m_{J/\Psi}} ((m_{B_c} - m_{J/\Psi})v_{\mu} - k_{\mu}) \right) \right\} \\ &= -\sqrt{2m_{B_c}2m_{J/\Psi}} \varepsilon_{\mu\nu\alpha\beta} \varepsilon_{(\lambda)}^{\nu*}(\vec{P}_{J/\Psi}) v^{\alpha} k^{\beta} \bar{\Sigma}_2^{(1-)}(q^2) - i \sqrt{2m_{B_c}2m_{J/\Psi}} \varepsilon_{(\lambda)\mu}^*(\vec{P}_{J/\Psi}) \Sigma_1^{(1-)}(q^2) \\ &\quad + (k \cdot \varepsilon_{(\lambda)}^*(\vec{P}_{J/\Psi})) \bar{\Sigma}_2^{(1-)}(q^2) v_{\mu} + (k \cdot \varepsilon_{(\lambda)}^*(\vec{P}_{J/\Psi})) \bar{\Sigma}_3^{(1-)}(q^2) k_{\mu}\end{aligned}\quad (53)$$

with

$$\begin{aligned}\Sigma_1^{(1-)}(q^2) &= \frac{1}{\sqrt{2m_{B_c}2m_{J/\Psi}}}(m_{B_c} - m_{J/\Psi})A_0(q^2) & \bar{\Sigma}_2^{(1-)}(q^2) &= \frac{1}{\sqrt{2m_{B_c}2m_{J/\Psi}}} \frac{m_{B_c}}{m_{J/\Psi}} \left( A_+(q^2) + \frac{m_{B_c} - m_{J/\Psi}}{m_{B_c} + m_{J/\Psi}} A_-(q^2) \right) \\ \bar{\Sigma}_2^{(1-)}(q^2) &= \frac{1}{\sqrt{2m_{B_c}2m_{J/\Psi}}} \frac{-2m_{B_c}}{m_{B_c} + m_{J/\Psi}} V(q^2) & \bar{\Sigma}_3^{(1-)}(q^2) &= \frac{1}{\sqrt{2m_{B_c}2m_{J/\Psi}}} \frac{m_{B_c}}{m_{J/\Psi}} \frac{1}{m_{B_c} + m_{J/\Psi}} (A_+(q^2) - A_-(q^2)).\end{aligned}\quad (54)$$

$\Sigma_1^{(0-)}$  and  $\Sigma_1^{(1-)}$  are dimensionless,  $\bar{\Sigma}_2^{(0-)}$ ,  $\bar{\Sigma}_2^{(1-)}$ ,  $\bar{\Sigma}_2^{(1-)}$  have dimensions of  $E^{-1}$ , and  $\bar{\Sigma}_3^{(1-)}$  has dimensions of  $E^{-2}$ .

We can take the infinite heavy quark mass limit  $m_b \gg m_c \gg \Lambda_{\text{QCD}}$  with the result that near zero recoil

$$\Sigma_1^{(0-)} = \Sigma_1^{(1-)} \quad \bar{\Sigma}_2^{(0-)} = \bar{\Sigma}_2^{(1-)} = \bar{\Sigma}_2^{(1-)} = 0 \quad \bar{\Sigma}_3^{(1-)} = 0. \quad (55)$$

This agrees perfectly with the result obtained in Ref. [2] using HQSS.<sup>10</sup>

In Fig. 7 we give our results for the above quantities for the semileptonic  $B_c^- \rightarrow \eta_c$  and  $B_c^- \rightarrow J/\Psi$  decays for the actual heavy quark masses. Even though we are not in the infinite heavy quark mass limit, we find that  $\Sigma_1^{(0-)}$  and  $\Sigma_1^{(1-)}$  dominate over the whole  $q^2$  interval. This dominant behavior would be more so near the zero-recoil point where  $k \approx 0$  and thus the contributions from the terms in  $\bar{\Sigma}_2^{(0-)}$ ,

$\bar{\Sigma}_2^{(1-)}$ ,  $\bar{\Sigma}_2^{(1-)}$ , and  $\bar{\Sigma}_3^{(1-)}$  are even more suppressed. Thus, even for the actual heavy quark masses we find that near zero recoil

$$\begin{aligned}\langle \eta_c, \vec{P}_{\eta_c} | J_{\mu}^{cb}(0) | B_c^-, \vec{P}_{B_c} \rangle &\approx \sqrt{2m_{B_c}2m_{\eta_c}} \Sigma_1^{(0-)}(q^2) v_{\mu} \\ \langle J/\Psi, \lambda \vec{P}_{J/\Psi} | J_{\mu}^{cb}(0) | B_c^-, \vec{P}_{B_c} \rangle &\approx -i \sqrt{2m_{B_c}2m_{J/\Psi}} \varepsilon_{(\lambda)\mu}^*(\vec{P}_{J/\Psi}) \\ &\quad \times \Sigma_1^{(1-)}(q^2),\end{aligned}\quad (56)$$

Besides, as seen in Fig. 8,  $\Sigma_1^{(0-)}$  of the  $B_c^- \rightarrow \eta_c$ , and  $\Sigma_1^{(1-)}$  of the  $B_c^- \rightarrow J/\Psi$  semileptonic decays are very close to each other over the whole  $q^2$  interval. This implies that the result obtained in Ref. [2] near zero recoil using HQSS seems to be valid, to a very good approximation, outside the infinite heavy quark mass limit.

#### IV. NONLEPTONIC $B_c^- \rightarrow c\bar{c}M_F^-$ TWO-MESON DECAYS

In this section we will evaluate decay widths for nonleptonic  $B_c^- \rightarrow c\bar{c}M_F^-$  two-meson decays where  $M_F^-$  is a

<sup>10</sup>Note, however, the different global phases and notation used in Ref. [2].

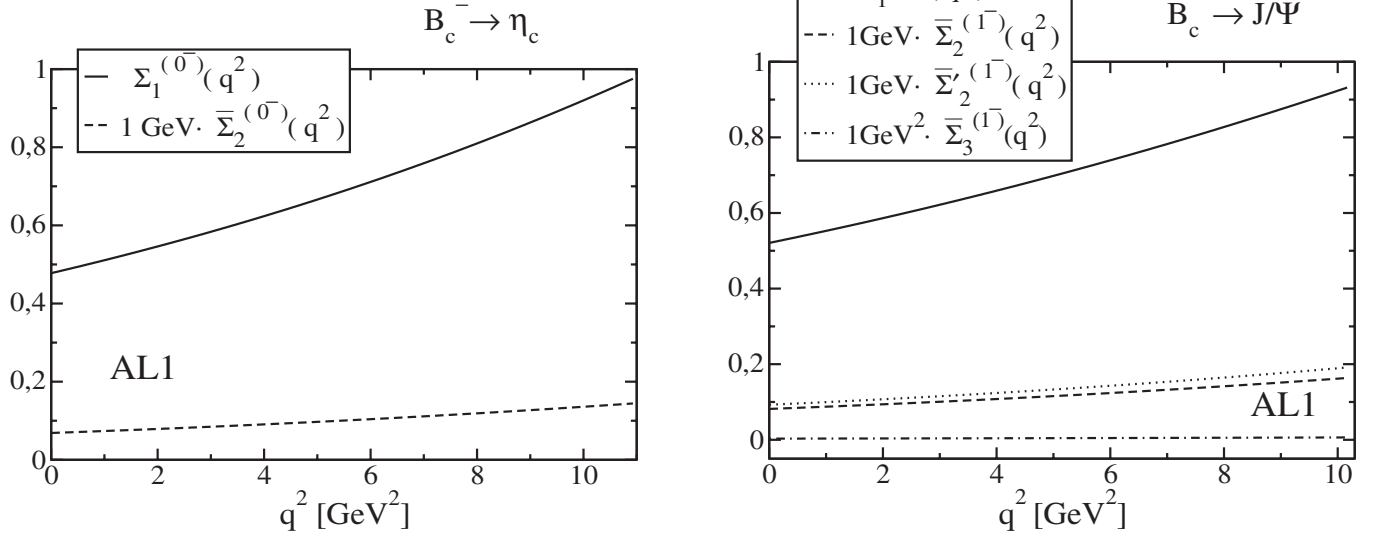


FIG. 7.  $\Sigma_1^{(0-)}$  (solid line) and  $\bar{\Sigma}_2^{(0-)}$  (dashed line) of the  $B_c^- \rightarrow \eta_c$ , and  $\Sigma_1^{(1-)}$  (solid line),  $\bar{\Sigma}_2^{(1-)}$  (dashed line),  $\bar{\Sigma}_2'^{(1-)}$  (dotted line), and  $\bar{\Sigma}_3^{(1-)}$  (dash-dotted line) of the  $B_c^- \rightarrow J/\Psi$  semileptonic decays evaluated with the AL1 potential.

pseudoscalar or vector meson. These decay modes involve a  $b \rightarrow c$  transition at the quark level and they are governed, neglecting penguin operators, by the effective Hamiltonian [4,7,18]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \{V_{cb}[c_1(\mu)Q_1^{cb} + c_2(\mu)Q_2^{cb}] + \text{H.c.}\}, \quad (57)$$

where  $c_1, c_2$  are scale-dependent Wilson coefficients, and  $Q_1^{cb}, Q_2^{cb}$  are local four-quark operators given by

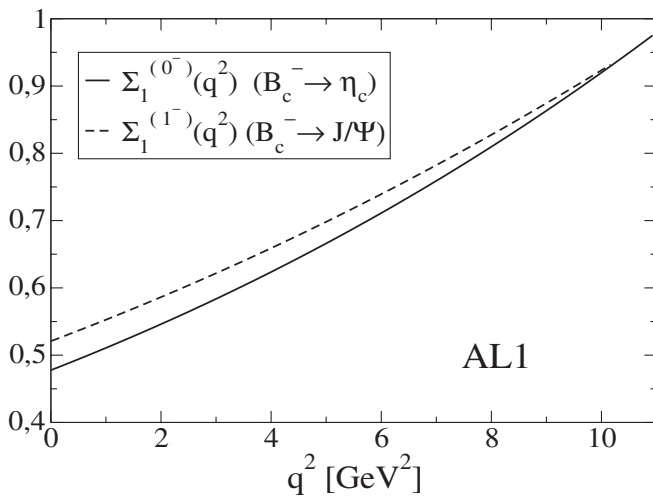


FIG. 8.  $\Sigma_1^{(0-)}$  (solid line) of the  $B_c^- \rightarrow \eta_c$ , and  $\Sigma_1^{(1-)}$  (dashed line) of the  $B_c^- \rightarrow J/\Psi$  semileptonic decays evaluated with the AL1 potential.

$$\begin{aligned} Q_1^{cb} &= \bar{\Psi}_c(0)\gamma_\mu(I - \gamma_5)\Psi_b(0)[V_{ud}^*\bar{\Psi}_d(0)\gamma^\mu(I - \gamma_5)\Psi_u(0) \\ &\quad + V_{us}^*\bar{\Psi}_s(0)\gamma^\mu(I - \gamma_5)\Psi_u(0) \\ &\quad + V_{cd}^*\bar{\Psi}_d(0)\gamma^\mu(I - \gamma_5)\Psi_c(0) \\ &\quad + V_{cs}^*\bar{\Psi}_s(0)\gamma^\mu(I - \gamma_5)\Psi_c(0)] \\ Q_2^{cb} &= \bar{\Psi}_d(0)\gamma_\mu(I - \gamma_5)\Psi_b(0)[V_{ud}^*\bar{\Psi}_c(0)\gamma^\mu(I - \gamma_5)\Psi_u(0) \\ &\quad + V_{cd}^*\bar{\Psi}_c(0)\gamma^\mu(I - \gamma_5)\Psi_c(0)] \\ &\quad + \bar{\Psi}_s(0)\gamma_\mu(I - \gamma_5)\Psi_b(0)[V_{us}^*\bar{\Psi}_c(0)\gamma^\mu(I - \gamma_5)\Psi_u(0) \\ &\quad + V_{cs}^*\bar{\Psi}_c(0)\gamma^\mu(I - \gamma_5)\Psi_c(0)], \end{aligned} \quad (58)$$

where the different  $V_{jk}$  are CKM matrix elements.

We shall work in the factorization approximation which amounts to evaluating the hadron matrix elements of the effective Hamiltonian as a product of quark-current matrix elements: one of these is the matrix element for the  $B_c^-$  transition to one of the final mesons, while the other matrix element corresponds to the transition from the vacuum to the other final meson. The latter is given by the corresponding meson decay constant. This factorization approximation is schematically represented in Fig. 9.

When writing the factorization amplitude one has to take into account the Fierz reordered contribution so that the relevant coefficients are not  $c_1$  and  $c_2$  but the combinations

$$\begin{aligned} a_1(\mu) &= c_1(\mu) + \frac{1}{N_C}c_2(\mu); \\ a_2(\mu) &= c_2(\mu) + \frac{1}{N_C}c_1(\mu) \end{aligned} \quad (59)$$

with  $N_C = 3$  the number of colors. The energy scale  $\mu$  appropriate in this case is  $\mu \simeq m_b$  and the values for  $a_1$  and

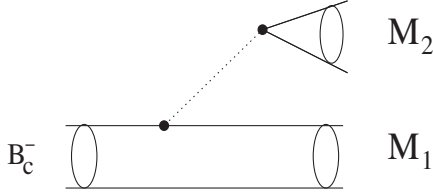


FIG. 9. Diagrammatic representation of  $B_c^-$  two-meson decay in the factorization approximation.

$a_2$  that we use are [4]

$$a_1 = 1.14; \quad a_2 = -0.20. \quad (60)$$

### A. $M_F^- = \pi^-, \rho^-, K^-, K^{*-}$

This is the simplest case. The decay width is given by

$$\Gamma = \frac{G_F^2}{16\pi m_{B_c}^2} |V_{cb}|^2 |V_F|^2 a_1^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{c\bar{c}}^2, m_F^2)}{2m_{B_c}} \times \mathcal{H}_{\alpha\beta}(P_{B_c}, P_{c\bar{c}}) \hat{\mathcal{H}}^{\alpha\beta}(P_F) \quad (61)$$

with  $m_F$  the mass of the  $M_F^-$  final meson, and  $V_F = V_{ud}$  or  $V_F = V_{us}$  depending on whether  $M_F^- = \pi^-, \rho^-,$  or  $M_F^- = K^-, K^{*-}$ .  $\mathcal{H}_{\alpha\beta}(P_{B_c}, P_{c\bar{c}})$  is the hadron tensor for the  $B_c^- \rightarrow c\bar{c}$  transition and  $\hat{\mathcal{H}}^{\alpha\beta}(P_F)$  is the hadron tensor for the vacuum  $\rightarrow M_F^-$  transition. The latter is

$$\begin{aligned} \hat{\mathcal{H}}^{\alpha\beta}(P_F) &= P_F^\alpha P_F^\beta f_F^2 & M_F^- \equiv 0^- \text{ case} \\ \hat{\mathcal{H}}^{\alpha\beta}(P_F) &= (P_F^\alpha P_F^\beta - m_F^2 g^{\alpha\beta}) f_F^2 & M_F^- \equiv 1^- \text{ case} \end{aligned} \quad (62)$$

with  $f_F$  being the  $M_F^-$  decay constant.

Similarly to the semileptonic case, the product  $\mathcal{H}_{\alpha\beta}(P_{B_c}, P_{c\bar{c}}) \hat{\mathcal{H}}^{\alpha\beta}(P_F)$  can now be easily written in terms of helicity amplitudes for the  $B_c^- \rightarrow c\bar{c}$  transition so that the width is given as [4]

$M_F^- \equiv 0^-$  case

$$\Gamma = \frac{G_F^2}{16\pi m_{B_c}^2} |V_{cb}|^2 |V_F|^2 a_1^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{c\bar{c}}^2, m_F^2)}{2m_{B_c}} m_F^2 f_F^2 \times \mathcal{H}_{tt}^{B_c^- \rightarrow c\bar{c}}(m_F^2)$$

$M_F^- \equiv 1^-$  case

$$\begin{aligned} \Gamma &= \frac{G_F^2}{16\pi m_{B_c}^2} |V_{cb}|^2 |V_F|^2 a_1^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{c\bar{c}}^2, m_F^2)}{2m_{B_c}} \\ &\times m_F^2 f_F^2 (\mathcal{H}_{+1+1}^{B_c^- \rightarrow c\bar{c}}(m_F^2) \\ &+ \mathcal{H}_{-1-1}^{B_c^- \rightarrow c\bar{c}}(m_F^2) + \mathcal{H}_{00}^{B_c^- \rightarrow c\bar{c}}(m_F^2)) \end{aligned} \quad (63)$$

with the different  $\mathcal{H}_{rr}$  evaluated at  $q^2 = m_F^2$ . In Table X we show the decay widths for a general value of the Wilson coefficient  $a_1$ , whereas in Table XI we give the corresponding branching ratios evaluated with  $a_1 = 1.14$ . Our results

TABLE X. Decay widths in units of  $10^{-15}$  GeV, and for general values of the Wilson coefficient  $a_1$ , for exclusive non-leptonic decays of the  $B_c^-$  meson. Our central values have been obtained with the AL1 potential.

	$\Gamma [10^{-15} \text{ GeV}]$
$B_c^- \rightarrow \eta_c \pi^-$	$1.02^{+0.07}_{-0.07} a_1^2$
$B_c^- \rightarrow \eta_c \rho^-$	$2.60^{+0.16}_{-0.16} a_1^2$
$B_c^- \rightarrow \eta_c K^-$	$0.082^{+0.004}_{-0.004} a_1^2$
$B_c^- \rightarrow \eta_c K^{*-}$	$0.15^{+0.01}_{-0.01} a_1^2$
$B_c^- \rightarrow J/\psi \pi^-$	$0.83^{+0.09}_{-0.09} a_1^2$
$B_c^- \rightarrow J/\psi \rho^-$	$2.61^{+0.27}_{-0.27} a_1^2$
$B_c^- \rightarrow J/\psi K^-$	$0.065^{+0.007}_{-0.007} a_1^2$
$B_c^- \rightarrow J/\psi K^{*-}$	$0.16^{+0.01}_{-0.01} a_1^2$
$B_c^- \rightarrow \chi_{c0} \pi^-$	$0.28^{+0.03}_{-0.03} a_1^2$
$B_c^- \rightarrow \chi_{c0} \rho^-$	$0.73^{+0.07}_{-0.07} a_1^2$
$B_c^- \rightarrow \chi_{c0} K^-$	$0.022^{+0.003}_{-0.003} a_1^2$
$B_c^- \rightarrow \chi_{c0} K^{*-}$	$0.041^{+0.005}_{-0.005} a_1^2$
$B_c^- \rightarrow \chi_{c1} \pi^-$	$0.0015^{+0.0002}_{-0.0002} a_1^2$
$B_c^- \rightarrow \chi_{c1} \rho^-$	$0.11^{+0.01}_{-0.01} a_1^2$
$B_c^- \rightarrow \chi_{c1} K^-$	$0.00012^{+0.00001}_{-0.00001} a_1^2$
$B_c^- \rightarrow \chi_{c1} K^{*-}$	$0.0080^{+0.0007}_{-0.0002} a_1^2$
$B_c^- \rightarrow h_c \pi^-$	$0.58^{+0.07}_{-0.07} a_1^2$
$B_c^- \rightarrow h_c \rho^-$	$1.41^{+0.17}_{-0.17} a_1^2$
$B_c^- \rightarrow h_c K^-$	$0.045^{+0.006}_{-0.006} a_1^2$
$B_c^- \rightarrow h_c K^{*-}$	$0.078^{+0.009}_{-0.009} a_1^2$
$B_c^- \rightarrow \chi_{c2} \pi^-$	$0.24^{+0.02}_{-0.02} a_1^2$
$B_c^- \rightarrow \chi_{c2} \rho^-$	$0.71^{+0.07}_{-0.03} a_1^2$
$B_c^- \rightarrow \chi_{c2} K^-$	$0.018^{+0.002}_{-0.002} a_1^2$
$B_c^- \rightarrow \chi_{c2} K^{*-}$	$0.041^{+0.004}_{-0.001} a_1^2$
$B_c^- \rightarrow \Psi(3836) \pi^-$	$0.00045^{+0.00003}_{-0.00003} a_1^2$
$B_c^- \rightarrow \Psi(3836) \rho^-$	$0.021^{+0.001}_{-0.002} a_1^2$
$B_c^- \rightarrow \Psi(3836) K^-$	$0.000034^{+0.000002}_{-0.000002} a_1^2$
$B_c^- \rightarrow \Psi(3836) K^{*-}$	$0.0015^{+0.0002}_{-0.0002} a_1^2$

for a final  $\eta_c$  or  $J/\psi$  are in good agreement with the ones obtained by Ebert *et al.* [7], El-Hady *et al.* [12] and Anisimov *et al.* [19], but they are a factor 2 smaller than the results by Ivanov *et al.* [4] and Kiselev [16]. Large discrepancies with Ivanov's results show up for the other transitions.

### B. $M_F^- = D^-, D^{*-}, D_s^-, D_s^{*-}$

In this subsection we shall evaluate the nonleptonic two-meson  $B_c^- \rightarrow \eta_c D^-, \eta_c D^{*-}, J/\psi D^-, J/\psi D^{*-}$  and  $B_c^- \rightarrow \eta_c D_s^-, \eta_c D_s^{*-}, J/\psi D_s^-, J/\psi D_s^{*-}$  decay widths. In these cases there are two different contributions in the factorization approximation. Following the same steps that led to Eq. (63), we shall get

$$\Gamma = \frac{G_F^2}{16\pi m_{B_c}^2} |V_{cb}|^2 |V_F|^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{c\bar{c}}^2, m_F^2)}{2m_{B_c}} \mathcal{H} \mathcal{H}, \quad (64)$$

where now  $V_F = V_{cd}$  for  $M_F^- = D^-, D^{*-}$  and  $V_F = V_{cs}$

TABLE XI. Branching ratios in percent for exclusive nonleptonic decays of the  $B_c^-$  meson. Our central values have been obtained with the AL1 potential.

	Branching ratio (%)								
	This work	[4]	[7]	[9–11]	[12]	[16,17]	[18]	[19]	[23]
$B_c^- \rightarrow \eta_c \pi^-$	$0.094^{+0.006}$	0.19	0.083	0.18	0.14	0.20	0.025	0.13	
$B_c^- \rightarrow \eta_c \rho^-$	$0.24^{+0.01}$	0.45	0.20	0.49	0.33	0.42	0.067	0.30	
$B_c^- \rightarrow \eta_c K^-$	$0.0075^{+0.0005}$	0.015	0.006	0.014	0.011	0.013	0.002	0.013	
$B_c^- \rightarrow \eta_c K^{*-}$	$0.013^{+0.001}$	0.025	0.011	0.025	0.018	0.020	0.004	0.021	
$B_c^- \rightarrow J/\Psi \pi^-$	$0.076^{+0.008}$	0.17	0.060	0.18	0.11	0.13	0.13	0.073	
$B_c^- \rightarrow J/\Psi \rho^-$	$0.24^{+0.02}$	0.49	0.16	0.53	0.31	0.40	0.37	0.21	
$B_c^- \rightarrow J/\Psi K^-$	$0.0060^{+0.0006}$	0.013	0.005	0.014	0.008	0.011	0.007	0.007	
$B_c^- \rightarrow J/\Psi K^{*-}$	$0.014^{+0.002}$	0.028	0.010	0.029	0.018	0.022	0.020	0.016	
$B_c^- \rightarrow \chi_{c0} \pi^-$	$0.026^{+0.003}$	0.055		0.028		0.98			
$B_c^- \rightarrow \chi_{c0} \rho^-$	$0.067^{+0.006}_{-0.001}$	0.13		0.072		3.29			
$B_c^- \rightarrow \chi_{c0} K^-$	$0.0020^{+0.0002}$	0.0042		0.000 21					
$B_c^- \rightarrow \chi_{c0} K^{*-}$	$0.0037^{+0.0005}$	0.0070		0.000 39					
$B_c^- \rightarrow \chi_{c1} \pi^-$	$0.000 14^{+0.000 01}$	0.0068		0.007		0.0089			
$B_c^- \rightarrow \chi_{c1} \rho^-$	$0.010^{+0.001}_{-0.001}$	0.029		0.029		0.46			
$B_c^- \rightarrow \chi_{c1} K^-$	$1.1^{+0.1} 10^{-5}$	$5.1 10^{-4}$		$5.2 10^{-5}$					
$B_c^- \rightarrow \chi_{c1} K^{*-}$	$0.000 73^{+0.000 07}_{-0.000 02}$	0.0018		0.000 18					
$B_c^- \rightarrow h_c \pi^-$	$0.053^{+0.007}$	0.11		0.05		1.60			
$B_c^- \rightarrow h_c \rho^-$	$0.13^{+0.01}$	0.25		0.12		5.33			
$B_c^- \rightarrow h_c K^-$	$0.0041^{+0.0006}$	0.0083		0.000 38					
$B_c^- \rightarrow h_c K^{*-}$	$0.0071^{+0.0008}$	0.013		0.000 68					
$B_c^- \rightarrow \chi_{c2} \pi^-$	$0.022^{+0.002}$	0.046		0.025		0.79			0.0076
$B_c^- \rightarrow \chi_{c2} \rho^-$	$0.065^{+0.006}_{-0.002}$	0.12		0.051		3.20			0.023
$B_c^- \rightarrow \chi_{c2} K^-$	$0.0017^{+0.0001}$	0.0034		0.000 18					0.000 56
$B_c^- \rightarrow \chi_{c2} K^{*-}$	$0.0038^{+0.0003}_{-0.0002}$	0.0065		0.000 31					0.0013
$B_c^- \rightarrow \Psi(3836) \pi^-$	$4.1^{+0.03}_{-0.02} 10^{-5}$	0.0017				0.030			
$B_c^- \rightarrow \Psi(3836) \rho^-$	$0.0020^{+0.0003}_{-0.0003}$	0.0055				0.98			
$B_c^- \rightarrow \Psi(3836) K^-$	$3.1^{+0.2}_{-0.2} 10^{-6}$	0.000 12							
$B_c^- \rightarrow \Psi(3836) K^{*-}$	$0.000 14^{+0.000 02}_{-0.000 02}$	0.000 32							

for  $M_F^- = D_s^-, D_s^{*-}$ . The quantity  $\mathcal{H}\mathcal{H}$  incorporates all information on the hadron matrix elements and depends on the transition as [4]<sup>11</sup>

$$\begin{aligned}
\mathcal{H}\mathcal{H}^{B_c^- \rightarrow \eta_c D^-} &= |a_1 h_t^{B_c^- \rightarrow \eta_c} (m_{D^-}^2) m_{D^-} f_{D^-} \\
&\quad + a_2 h_t^{B_c^- \rightarrow D^-} (m_{\eta_c}^2) m_{\eta_c} f_{\eta_c}|^2 \\
\mathcal{H}\mathcal{H}^{B_c^- \rightarrow \eta_c D^{*-}} &= |-a_1 h_0^{B_c^- \rightarrow \eta_c} (m_{D^{*-}}^2) m_{D^{*-}} f_{D^{*-}} \\
&\quad + a_2 i h_{(0)t}^{B_c^- \rightarrow D^{*-}} (m_{\eta_c}^2) m_{\eta_c} f_{\eta_c}|^2 \\
\mathcal{H}\mathcal{H}^{B_c^- \rightarrow J/\Psi D^-} &= |a_1 i h_{(0)t}^{B_c^- \rightarrow J/\Psi} (m_{D^-}^2) m_{D^-} f_{D^-} \\
&\quad - a_2 h_0^{B_c^- \rightarrow D^-} (m_{J/\Psi}^2) m_{J/\Psi} f_{J/\Psi}|^2 \\
\mathcal{H}\mathcal{H}^{B_c^- \rightarrow J/\Psi D^{*-}} &= \sum_{r=+1,-1,0} |a_1 h_{(r)r}^{B_c^- \rightarrow J/\Psi} (m_{D^{*-}}^2) m_{D^{*-}} f_{D^{*-}} \\
&\quad + a_2 h_{(r)r}^{B_c^- \rightarrow D^{*-}} (m_{J/\Psi}^2) m_{J/\Psi} f_{J/\Psi}|^2 \quad (65)
\end{aligned}$$

<sup>11</sup>Note the different phases used in Ref. [4].

and similarly for  $D_s^-, D_s^{*-}$ . Note that the helicity amplitudes corresponding to  $B_c^- \rightarrow D^-, D^{*-}, D_s^-, D_s^{*-}$  have been evaluated from the matrix elements for the effective current operators  $\bar{\Psi}_{d,s}(0) \gamma^\mu (I - \gamma_5) \Psi_b(0)$  in Eq. (58). While in practice this is a  $b \rightarrow d, s$  transition, the momentum transfer ( $m_{\eta_c}^2$  or  $m_{J/\Psi}^2$ ) is neither too high, so that one has to include a  $B_c^*$  resonance, nor too low, so as to have too high three-momentum transfers.<sup>12</sup> Besides the contribution is weighed by the much smaller  $a_2$  Wilson coefficient. In Table XII we give the decay widths for general values of the Wilson coefficients  $a_1$  and  $a_2$ , and in Table XIII we show the branching ratios. We are in reasonable agreement with the results by Ivanov *et al.* [4], El-Hady *et al.* [12], and Kiselev [16]. For decays with a final  $D_s^-, D_s^{*-}$  the agreement is also reasonable with the results by Colangelo *et al.* [18] and Anisimov *et al.* [19].

<sup>12</sup>Our experience with the  $B \rightarrow \pi$  decay [3], where we have a similar  $b \rightarrow u$  quark transition, shows that the naive nonrelativistic quark model gives reliable results for  $q^2 \approx 9 \text{ GeV}^2$ .

TABLE XII. Decay widths in units of  $10^{-15}$  GeV, and for general values of the Wilson coefficients  $a_1$  and  $a_2$ , for exclusive nonleptonic decays of the  $B_c^-$  meson. Our central values have been obtained with the AL1 potential. For the vector-vector final state we show the three different contributions corresponding to  $r = +1, -1, 0$  [see Eq. (65)].

	$\Gamma [10^{-15} \text{ GeV}]$
$B_c^- \rightarrow \eta_c D^-$	$(0.438^{+0.010}_{-0.009} a_1 + 0.236^{+0.030}_{-0.023} a_2)^2$
$B_c^- \rightarrow \eta_c D^{*-}$	$(-0.390^{+0.015}_{-0.009} a_1 - 0.136^{+0.015}_{-0.022} a_2)^2$
$B_c^- \rightarrow J/\Psi D^-$	$(-0.328^{+0.016}_{-0.012} a_1 - 0.156^{+0.016}_{-0.019} a_2)^2$
$B_c^- \rightarrow J/\Psi D^{*-}$	$(-0.195^{+0.006}_{-0.008} a_1 - 0.066^{+0.006}_{-0.011} a_2)^2$ $+ (-0.390^{+0.019}_{-0.018} a_1 - 0.209^{+0.019}_{-0.032} a_2)^2$ $+ (0.447^{+0.016}_{-0.016} a_1 + 0.167^{+0.016}_{-0.027} a_2)^2$
$B_c^- \rightarrow \eta_c D_s^-$	$(2.54^{+0.05}_{-0.04} a_1 + 1.93^{+0.10}_{-0.14} a_2)^2$
$B_c^- \rightarrow \eta_c D_s^{*-}$	$(-1.84^{+0.02}_{-0.04} a_1 - 1.17^{+0.02}_{-0.14} a_2)^2$
$B_c^- \rightarrow J/\Psi D_s^-$	$(-1.85^{+0.01}_{-0.06} a_1 - 1.23^{+0.01}_{-0.06} a_2)^2$
$B_c^- \rightarrow J/\Psi D_s^{*-}$	$(-1.01^{+0.02}_{-0.04} a_1 - 0.60^{+0.02}_{-0.07} a_2)^2$ $+ (-2.00^{+0.03}_{-0.06} a_1 - 1.71^{+0.03}_{-0.18} a_2)^2$ $+ (2.17^{+0.02}_{-0.08} a_1 + 1.42^{+0.02}_{-0.16} a_2)^2$

TABLE XIII. Branching ratios in percent for exclusive nonleptonic decays of the  $B_c^-$  meson. Our central values have been obtained with the AL1 potential.

	Branching ratio (%)						
	This work	[4]	[9]	[12]	[16]	[18]	[19]
$B_c^- \rightarrow \eta_c D^-$	$0.014^{+0.001}_{-0.001}$	0.019	0.0012	0.014	0.015	0.005	0.010
$B_c^- \rightarrow \eta_c D^{*-}$	$0.012^{+0.001}_{-0.001}$	0.019	0.0010	0.013	0.010	0.002	0.0055
$B_c^- \rightarrow J/\Psi D^-$	$0.0083^{+0.0005}_{-0.0005}$	0.015	0.0009	0.009	0.009	0.013	0.0044
$B_c^- \rightarrow J/\Psi D^{*-}$	$0.031^{+0.001}_{-0.001}$	0.045		0.028	0.028	0.019	0.010
$B_c^- \rightarrow \eta_c D_s^-$	$0.44^{+0.02}_{-0.02}$	0.44	0.054	0.26	0.28	0.50	0.35
$B_c^- \rightarrow \eta_c D_s^{*-}$	$0.24^{+0.02}_{-0.02}$	0.37	0.044	0.24	0.27	0.038	0.36
$B_c^- \rightarrow J/\Psi D_s^-$	$0.24^{+0.02}_{-0.02}$	0.34	0.041	0.15	0.17	0.34	0.12
$B_c^- \rightarrow J/\Psi D_s^{*-}$	$0.68^{+0.03}_{-0.03}$	0.97		0.55	0.67	0.59	0.62

## V. SEMILEPTONIC $B_c^- \rightarrow \bar{B}^0, \bar{B}^{*0}, \bar{B}_s^0, \bar{B}_s^{*0}$ DECAYS

In this section we shall study the semileptonic  $B_c^- \rightarrow \bar{B}^0, \bar{B}^{*0}, \bar{B}_s^0, \bar{B}_s^{*0}$  decays. With obvious changes, the calculations are done as before, with the only novel thing being that now it is the antiquark that suffers the transition (we have  $\bar{c} \rightarrow \bar{d}, \bar{s}$ ), and thus we have to take into account the changes in the form factors according to the results in Appendix C.

### A. Form factors

In Fig. 10 we show the form factors for the above transitions evaluated with the AL1 potential. For the  $\bar{B}^0$  and  $\bar{B}_s^0$  cases, we also show the results obtained with the BHAD potential. Although they are less visible in the figures, the larger differences, of up to 25%, occur for the  $F_-$  form factor.

In Table XIV we show  $F_+, F_-,$  and  $F_0$  [defined as in Eq. (15) changing the mass of the final meson] of the  $B_c^- \rightarrow \bar{B}^0, \bar{B}_s^0$  transitions evaluated at  $q_{\min}^2$  and  $q_{\max}^2$  and compare them with the results by Ivanov *et al.* [6] and

Ebert *et al.* [8]. Notice that, to favor comparison, we have changed the signs of the form factors by Ebert *et al.* (they evaluate  $B_c^+$  decay) in accordance with the results in Appendix C. The agreement with the results by Ebert *et al.* is good. We also agree with Ivanov *et al.* for  $F_+$ , but get very different results for  $F_-$ . As fermion masses are very small, the disagreement in the  $F_-$  form factor will have a negligible effect on the decay width.

In Table XV we show  $V, A_+, A_-, A_0,$  and  $\tilde{A}_0$  [defined as in Eq. (19) changing the mass of the final meson] of the  $B_c^- \rightarrow \bar{B}^{*0}, \bar{B}_s^{*0}$  evaluated at  $q_{\min}^2$  and  $q_{\max}^2$  and compare them with the results by Ivanov *et al.* [6] and Ebert *et al.* [8]. With some exceptions the agreement with Ebert's results is bad in this case. We are also in clear disagreement with Ivanov's results.

### B. Decay width

In Tables XVI and XVII we give, respectively, our results for the partial helicity widths and forward-backward asymmetries.

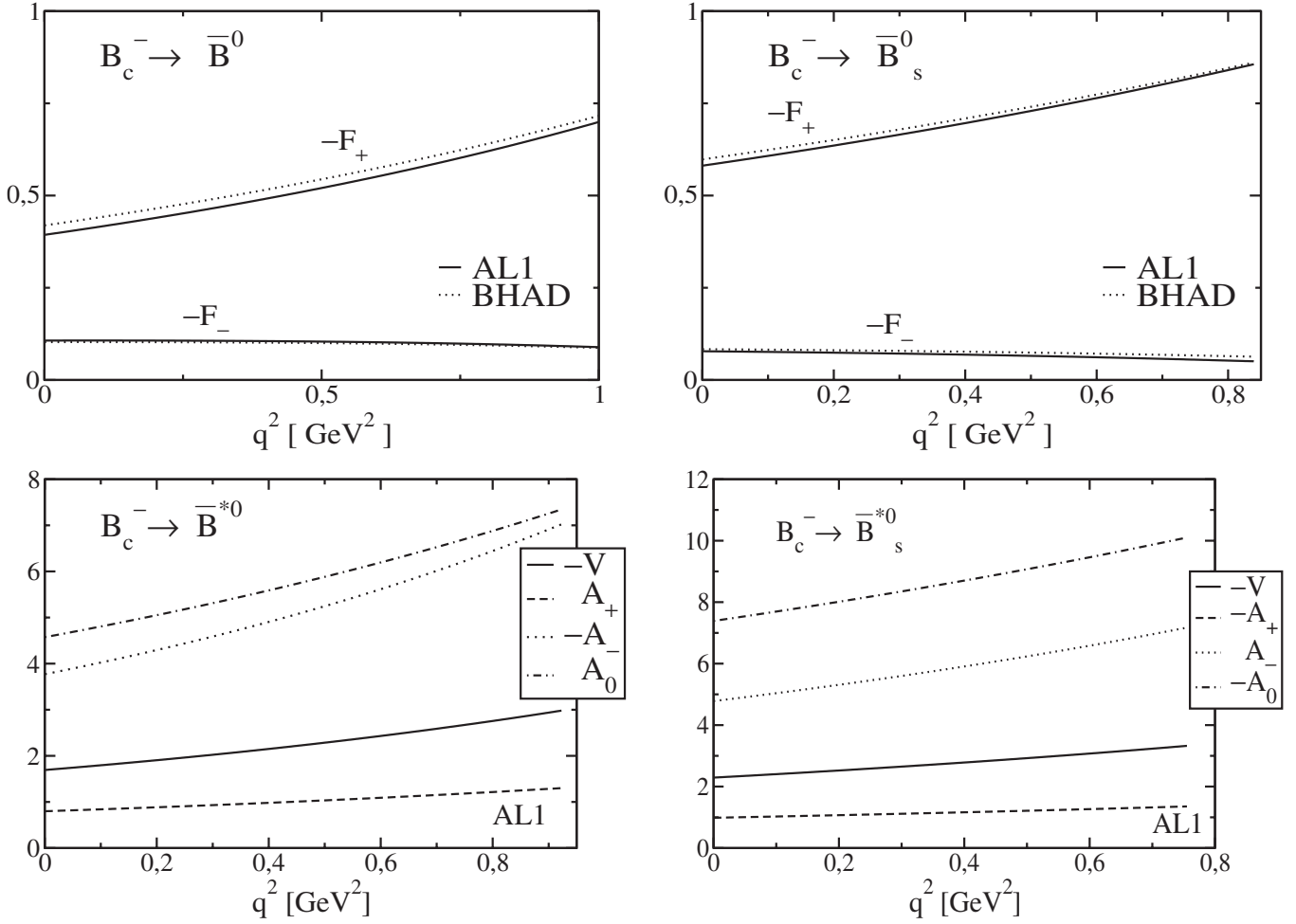


FIG. 10.  $F_+$  and  $F_-$  form factors (solid lines) for  $B_c^- \rightarrow \bar{B}^0$  and  $B_c^- \rightarrow \bar{B}_s^0$  semileptonic decay, and  $V$  (solid line),  $A_+$  (dashed line),  $A_-$  (dotted line), and  $A_0$  (dash-dotted line) form factors for  $B_c^- \rightarrow \bar{B}^{*0}$  and  $B_c^- \rightarrow \bar{B}_s^{*0}$  semileptonic decay evaluated with the AL1 potential. For the first two cases, and for comparison, we also show with dotted lines the results obtained with the Bhaduri (BHAD) potential.

TABLE XIV.  $F_+$ ,  $F_-$ , and  $F_0$  evaluated at  $q_{\min}^2$  and  $q_{\max}^2$  compared to the ones obtained by Ivanov *et al.* [6] and Ebert *et al.* [8]. Our central values have been obtained with the AL1 potential. Here  $l$  stands for  $l = e, \mu$ .

$B_c^- \rightarrow \bar{B}^0 l^- \bar{\nu}_l$	$q_{\min}^2$	$q_{\max}^2$	$B_c^- \rightarrow \bar{B}_s^0 l^- \bar{\nu}_l$	$q_{\min}^2$	$q_{\max}^2$
$F_+$			$F_+$		
This work	$-0.39^{+0.03}_{-0.03}$	$-0.70^{+0.004}_{-0.02}$	This work	$-0.58^{+0.01}_{-0.02}$	$-0.86^{+0.01}_{-0.01}$
[6]	-0.58	-0.96	[6]	-0.61	-0.92
[8]	-0.39	-0.96	[8]	-0.50	-0.99
$F_-$			$F_-$		
This work	$-0.11^{+0.02}_{-0.03}$	$-0.09^{+0.01}_{-0.06}$	This work	$-0.08^{+0.01}_{-0.02}$	$-0.05^{+0.01}_{-0.03}$
[6]	2.14	2.98	[6]	1.83	2.35
$F_0$			$F_0$		
This work	$-0.39^{+0.03}_{-0.03}$	$-0.71^{+0.05}_{-0.02}$	This work	$-0.58^{+0.01}_{-0.02}$	$-0.86^{+0.01}_{-0.01}$
[8]	-0.39	-0.80	[8]	-0.50	-0.86



TABLE XV.  $V$ ,  $A_+$ ,  $A_-$ ,  $A_0$ , and  $\tilde{A}_0$  evaluated at  $q_{\min}^2$  and  $q_{\max}^2$  compared to the ones obtained by Ivanov *et al.* [6] and Ebert *et al.* [8]. Our central values have been obtained with the AL1 potential. Here  $l$  stands for  $l = e, \mu$ . Asterisk as in Table IV.

$B_c^- \rightarrow \bar{B}^{*0} l^- \bar{\nu}_l$	$q_{\min}^2$	$q_{\max}^2$	$B_c^- \rightarrow \bar{B}_s^{*0} l^- \bar{\nu}_l$	$q_{\min}^2$	$q_{\max}^2$
$V$			$V$		
This work	$-1.69^{+0.11}_{-0.09}$	$-2.98^{+0.17}_{-0.03}$	This work	$-2.29^{+0.02}_{-0.09}$	$-3.32^{+0.04}_{-0.01}$
[6]		$-5.32^*$	[6]		$-4.91^*$
[8]	$-3.94$	$-8.91$	[8]	$-3.44$	$-6.25$
$A_+$			$A_+$		
This work	$-0.80^{+0.02}_{-0.02}$	$-1.30^{+0.02}_{-0.02}$	This work	$-0.98^{+0.01}_{-0.03}$	$-1.35^{+0.03}_{-0.02}$
[6]		$0.49$	[6]		$0.21$
[8]	$-2.89$	$-2.83$	[8]	$-2.19$	$-2.62$
$A_-$			$A_-$		
This work	$3.77^{+0.15}_{-0.17}$	$7.02^{+0.02}_{-0.35}$	This work	$4.78^{+0.23}_{-0.01}$	$7.17^{+0.06}_{-0.18}$
[6]		$18.0$	[6]		$15.9$
$A_0$			$A_0$		
This work	$-4.57^{+0.27}_{-0.33}$	$-7.34^{+0.28}_{-0.49}$	This work	$-7.39^{+0.14}_{-0.30}$	$-10.10^{+0.22}_{-0.16}$
[6]		$-5.07$	[6]		$-6.60$
[8]	$-5.08$	$-8.70$	[8]	$-6.60$	$-10.23$
$\tilde{A}_0$			$\tilde{A}_0$		
This work	$-0.34^{+0.03}_{-0.03}$	$-0.60^{+0.05}_{-0.02}$	This work	$-0.51^{+0.01}_{-0.03}$	$-0.74^{+0.01}_{-0.01}$
[8]	$-0.20$	$-1.06$	[8]	$-0.35$	$-0.91$

In Fig. 11 we show the differential decay width for the  $B_c^- \rightarrow \bar{B}^0 l^- \bar{\nu}_l$ ,  $B_c^- \rightarrow \bar{B}_s^0 l^- \bar{\nu}_l$ ,  $B_c^- \rightarrow \bar{B}^{*0} l^- \bar{\nu}_l$ , and  $B_c^- \rightarrow \bar{B}_s^{*0} l^- \bar{\nu}_l$  transitions ( $l = e, \mu$ ). In Tables XVIII and XIX we give the total decay widths and branching ratios and compare them with determinations by other groups. Our results are in better agreement with the ones obtained by Ebert *et al.* [8], Colangelo *et al.* [18], Anisimov *et al.* [19], and Lu *et al.* [24].

### C. Heavy quark spin symmetry

In Fig. 12 we give  $\Sigma_1^{(0-)}$  and  $\Sigma_2^{(0-)}$  of the  $B_c^- \rightarrow \bar{B}^0$  and  $B_c^- \rightarrow \bar{B}_s^0$  transitions, and  $\Sigma_1^{(1-)}$ ,  $\tilde{\Sigma}_2^{(1-)}$ ,  $\tilde{\Sigma}_2^{(1-)}$ , and  $\tilde{\Sigma}_3^{(1-)}$  of the  $B_c^- \rightarrow \bar{B}^{*0}$  and  $B_c^- \rightarrow \bar{B}_s^{*0}$  transitions.

We can take the infinite heavy quark mass limit on our analytic expressions with the result that near zero recoil

TABLE XVI. Partial helicity widths in units of  $10^{-15}$  GeV for  $B_c^-$  decay. Central values have been evaluated with the AL1 potential.

$B_c^- \rightarrow$	$\Gamma_U$	$\tilde{\Gamma}_U$	$\Gamma_L$	$\tilde{\Gamma}_L$	$\Gamma_P$	$\tilde{\Gamma}_S$	$\tilde{\Gamma}_{SL}$
$\bar{B}^0 e^- \bar{\nu}_e$	0	0	$0.65^{+0.07}_{-0.09}$	$0.14^{+0.02}_{-0.02} 10^{-5}$	0	$0.47^{+0.05}_{-0.07} 10^{-5}$	$0.15^{+0.01}_{-0.02} 10^{-5}$
$\bar{B}^0 \mu^- \bar{\nu}_\mu$	0	0	$0.55^{+0.06}_{-0.07}$	$0.15^{+0.01}_{-0.02} 10^{-1}$	0	$0.64^{+0.16}_{-0.09} 10^{-1}$	$0.17^{+0.02}_{-0.02} 10^{-1}$
$\bar{B}^{*0} e^- \bar{\nu}_e$	0	0	$15.1^{+0.7}_{-0.3}$	$0.43^{+0.02}_{-0.01} 10^{-4}$	0	$0.14^{+0.01}_{-0.01} 10^{-3}$	$0.45^{+0.03}_{-0.01} 10^{-4}$
$\bar{B}^{*0} \mu^- \bar{\nu}_\mu$	0	0	$12.4^{+0.5}_{-0.3}$	$0.40^{+0.02}_{-0.01} 10^{-5}$	0	$1.69^{+0.08}_{-0.03} 10^{-5}$	$0.47^{+0.02}_{-0.01} 10^{-6}$
$\bar{B}^{*0} e^- \bar{\nu}_e$	$0.83^{+0.08}_{-0.11}$	$0.26^{+0.03}_{-0.04} 10^{-6}$	$0.76^{+0.09}_{-0.11}$	$0.10^{+0.02}_{-0.01} 10^{-5}$	$0.36^{+0.03}_{-0.05}$	$0.27^{+0.04}_{-0.05} 10^{-5}$	$0.96^{+0.15}_{-0.15} 10^{-6}$
$\bar{B}^{*0} \mu^- \bar{\nu}_\mu$	$0.79^{+0.10}_{-0.11}$	$0.97^{+0.11}_{-0.13} 10^{-2}$	$0.68^{+0.08}_{-0.10}$	$0.14^{+0.01}_{-0.03} 10^{-1}$	$0.34^{+0.02}_{-0.05}$	$0.28^{+0.04}_{-0.05} 10^{-1}$	$0.11^{+0.02}_{-0.02} 10^{-1}$
$\bar{B}_s^{*0} e^- \bar{\nu}_e$	$16.7^{+0.8}_{-0.7}$	$0.66^{+0.04}_{-0.03} 10^{-5}$	$16.8^{+1.1}_{-0.8}$	$0.33^{+0.02}_{-0.02} 10^{-4}$	$6.11^{+0.21}_{-0.20}$	$0.88^{+0.08}_{-0.05} 10^{-4}$	$0.30^{+0.03}_{-0.01} 10^{-4}$
$\bar{B}_s^{*0} \mu^- \bar{\nu}_\mu$	$15.6^{+0.8}_{-0.6}$	$0.24^{+0.02}_{-0.01}$	$14.5^{+1.0}_{-0.6}$	$0.37^{+0.02}_{-0.02}$	$5.66^{+0.18}_{-0.19}$	$0.77^{+0.06}_{-0.04}$	$0.30^{+0.02}_{-0.01}$

TABLE XVII. Forward-backward asymmetry. Our central values have been evaluated with the AL1 potential. We would obtain the same results for  $B_c^+ \rightarrow B^0$  decays.

	$A_{FB}(e)$	$A_{FB}(\mu)$
$B_c^- \rightarrow \bar{B}^0$	$0.67^{+0.02} 10^{-5}$	$0.82^{+0.01} 10^{-1}$
$B_c^- \rightarrow \bar{B}_s^0$	$0.89^{+0.01} 10^{-5}$	$0.96^{+0.01} 10^{-1}$
$B_c^- \rightarrow \bar{B}^{*0}$	$0.17_{-0.01}$	$0.19_{-0.01}$
$B_c^- \rightarrow \bar{B}_s^{*0}$	$0.14_{-0.01}$	$0.16^{+0.01}$

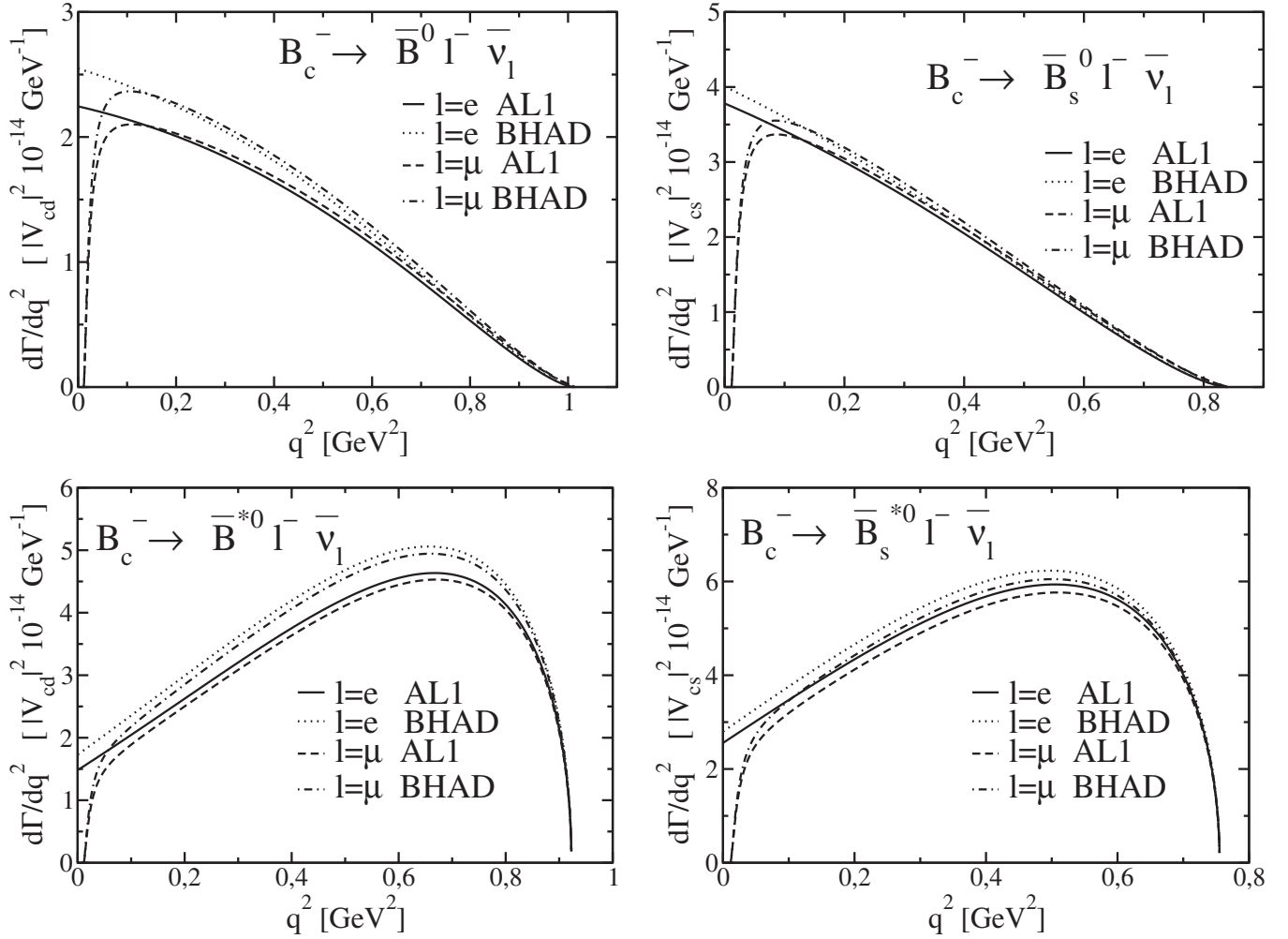


FIG. 11. Differential decay width for the  $B_c^- \rightarrow \bar{B}^0 l^- \bar{\nu}_l$  and  $B_c^- \rightarrow \bar{B}_s^0 l^- \bar{\nu}_l$ ,  $B_c^- \rightarrow \bar{B}^{*0} l^- \bar{\nu}_l$ , and  $B_c^- \rightarrow \bar{B}_s^{*0} l^- \bar{\nu}_l$  ( $l = e, \mu$ ) transitions. Solid line: results for a final  $e$  evaluated with the AL1 potential; dotted line: results for a final  $e$  evaluated with the Bhaduri (BHAD) potential; dashed line: results for a final  $\mu$  evaluated with the AL1 potential; dash-dotted line: results for a final  $\mu$  evaluated with the Bhaduri (BHAD) potential.

TABLE XVIII. Decay widths in units of  $10^{-15}$  GeV. Our central values have been evaluated with the AL1 potential.

	$\Gamma$ [ $10^{-15}$ GeV]										
	This work	[6]	[8]	[9]	[12]	[13]	[16]	[18]	[19]	[20]	[24]
$B_c^- \rightarrow \bar{B}^0 e \bar{\nu}_e$	$0.65^{+0.07}_{-0.09}$	2.1	0.6	2.30	1.14	1.90	4.9	0.9(1.0)			0.59
$B_c^- \rightarrow \bar{B}^0 \mu \bar{\nu}_\mu$	$0.63^{+0.07}_{-0.09}$										
$B_c^- \rightarrow \bar{B}_s^0 e \bar{\nu}_e$	$15.1^{+0.7}_{-0.3}$	29	12	26.6	14.3	26.8	59	11.1(12.9)	15	12.3	11.75
$B_c^- \rightarrow \bar{B}_s^0 \mu \bar{\nu}_\mu$	$14.5^{+0.6}_{-0.3}$										
$B_c^- \rightarrow \bar{B}^{*0} e \bar{\nu}_e$	$1.59^{+0.17}_{-0.22}$	2.3	1.7	3.32	3.53	2.34	8.5	2.8(3.2)			2.44
$B_c^- \rightarrow \bar{B}^{*0} \mu \bar{\nu}_\mu$	$1.52^{+0.17}_{-0.21}$										
$B_c^- \rightarrow \bar{B}_s^{*0} e \bar{\nu}_e$	$33.5^{+1.9}_{-1.5}$	37	25	44.0	50.4	34.6	65	33.5(37.0)	34	19.0	32.56
$B_c^- \rightarrow \bar{B}_s^{*0} \mu \bar{\nu}_\mu$	$31.5^{+1.8}_{-1.4}$										

$$\Sigma_1^{(1-)} = \Sigma_1^{(0-)} \quad \bar{\Sigma}_2^{(1-)} = \bar{\Sigma}_2^{(1-)} = -\bar{\Sigma}_2^{(0-)} \quad \bar{\Sigma}_3^{(1-)} = 0. \quad (66)$$

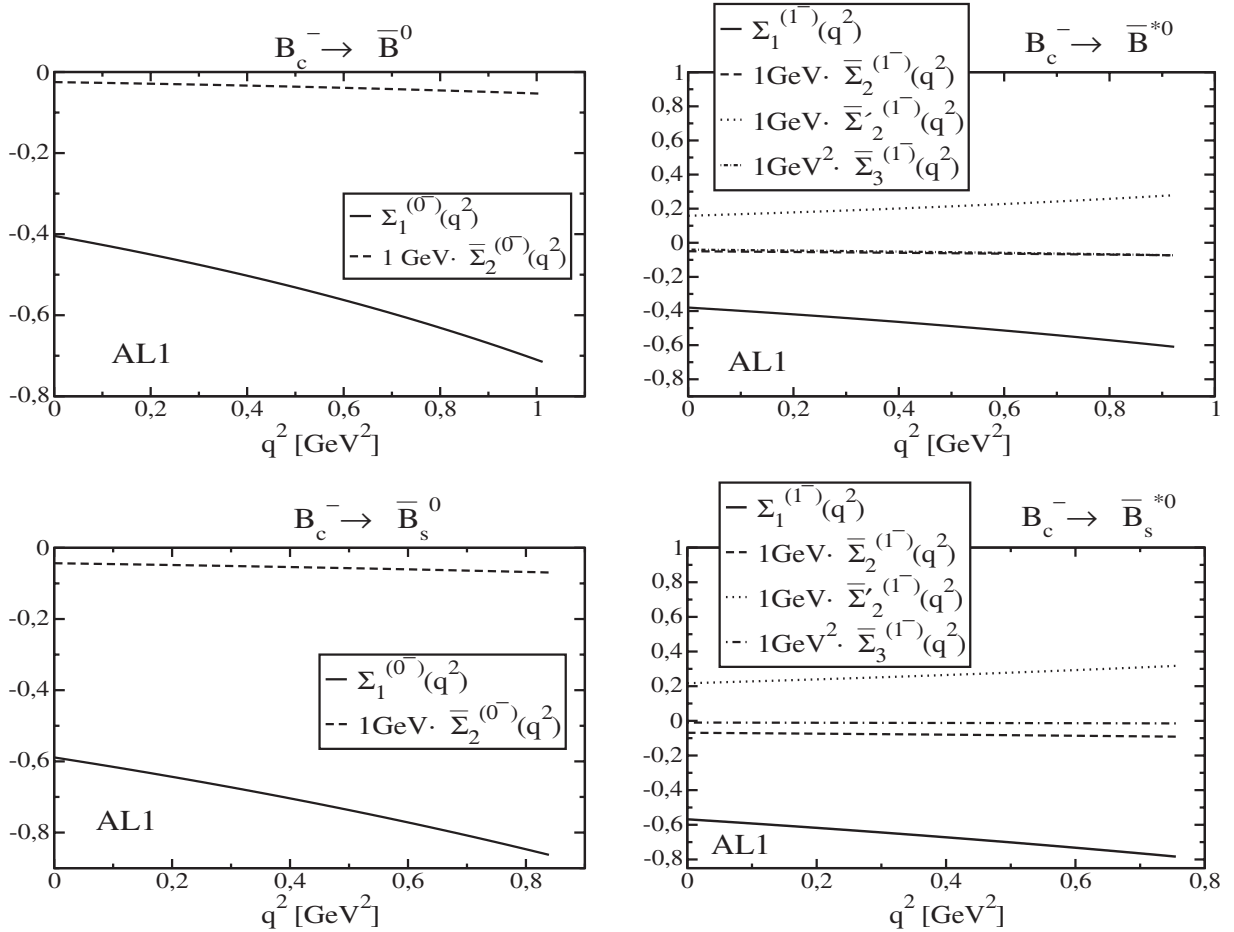
we see differences. In Ref. [2] we find<sup>13</sup>  $\bar{\Sigma}_2^{(1-)} = \bar{\Sigma}_2^{(0-)}$  instead. This is wrong as there is a misprint in Ref. [2]

When compared to the results of HQSS by Jenkins *et al.* [2]

<sup>13</sup>Note the different notation and global phases used.

TABLE XIX. Branching ratios in percent. Our central values have been evaluated with the AL1 potential.

	Branching ratio (%)								
	This work	[4]	[8]	[9]	[12]	[16]	[18]	[19]	[20]
$B_c^- \rightarrow \bar{B}^0 e \bar{\nu}_e$	$0.046^{+0.004}_{-0.007}$	0.071	0.042	0.16	0.078	0.34	0.06		0.048
$B_c^- \rightarrow \bar{B}^0 \mu \bar{\nu}_\mu$	$0.044^{+0.005}_{-0.006}$								
$B_c^- \rightarrow \bar{B}_s^0 e \bar{\nu}_e$	$1.06^{+0.05}_{-0.02}$	1.10	0.84	1.82	0.98	4.03	0.8	0.99	0.92
$B_c^- \rightarrow \bar{B}_s^0 \mu \bar{\nu}_\mu$	$1.02^{+0.04}_{-0.02}$								
$B_c^- \rightarrow \bar{B}^{*0} e \bar{\nu}_e$	$0.11^{+0.01}_{-0.01}$	0.063	0.12	0.23	0.24	0.58	0.19		0.051
$B_c^- \rightarrow \bar{B}^{*0} \mu \bar{\nu}_\mu$	$0.11^{+0.01}_{-0.02}$								
$B_c^- \rightarrow \bar{B}_s^{*0} e \bar{\nu}_e$	$2.35^{+0.14}_{-0.10}$	2.37	1.75	3.01	3.45	5.06	2.3	2.30	1.41
$B_c^- \rightarrow \bar{B}_s^{*0} \mu \bar{\nu}_\mu$	$2.22^{+0.12}_{-0.10}$								

FIG. 12.  $\Sigma_1^{(0^-)}$  (solid line) and  $\Sigma_2^{(0^-)}$  (dashed line) of the  $B_c^- \rightarrow \bar{B}^0$  and  $B_c^- \rightarrow \bar{B}_s^0$  transitions, and  $\Sigma_1^{(1^-)}$  (solid line),  $\Sigma_2^{(1^-)}$  (dashed line),  $\Sigma_2^{(1^-)}$  (dotted line), and  $\Sigma_3^{(1^-)}$  (dash-dotted line) of the  $B_c^- \rightarrow \bar{B}^{*0}$  and  $B_c^- \rightarrow \bar{B}_s^{*0}$  transitions evaluated with the AL1 potential.

that has not been noted before: the sign of the term in  $\nu^\mu$  in the last expression of Eqs. (2.9) and (2.10) in Ref. [2] should be a minus [42]. Also from Ref. [2] one would expect<sup>14</sup>  $\bar{\Sigma}_2^{(1^-)} = \bar{\Sigma}_2^{(0^-)}$  contradicting our result in Eq. (66)

<sup>14</sup>One would have to look at Eq. (2.10) in Ref. [2], even though it refers to  $B_c^+$  decay into  $D^0$ ,  $D^{*0}$ , because that is the reaction where you have antiquark decay in their case.

where we find  $\bar{\Sigma}_2^{(1^-)} = -\bar{\Sigma}_2^{(0^-)}$ . Our result is a clear prediction of the quark model and comes from the extra signs that appear due to the fact that it is the antiquark that decays (see Appendix C). This difference between quark and antiquark decay was not properly reflected in their published work [42].

How far are we from the infinite heavy quark mass limit? In Fig. 13 we show  $\Sigma_1^{(0^-)}$  of the semileptonic

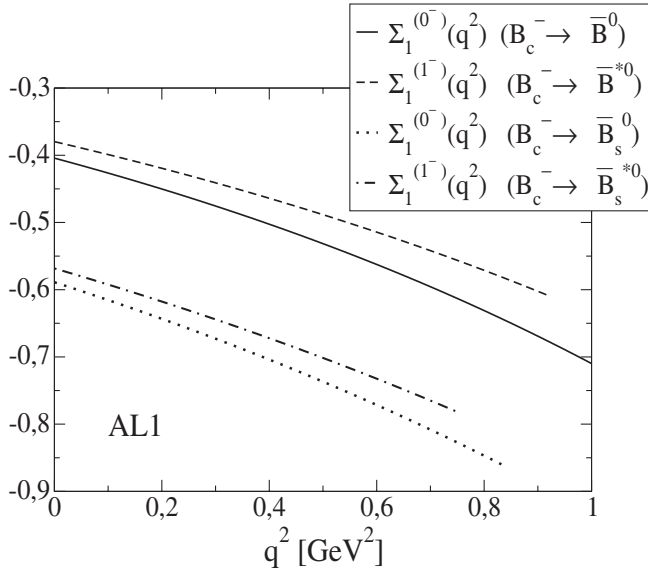
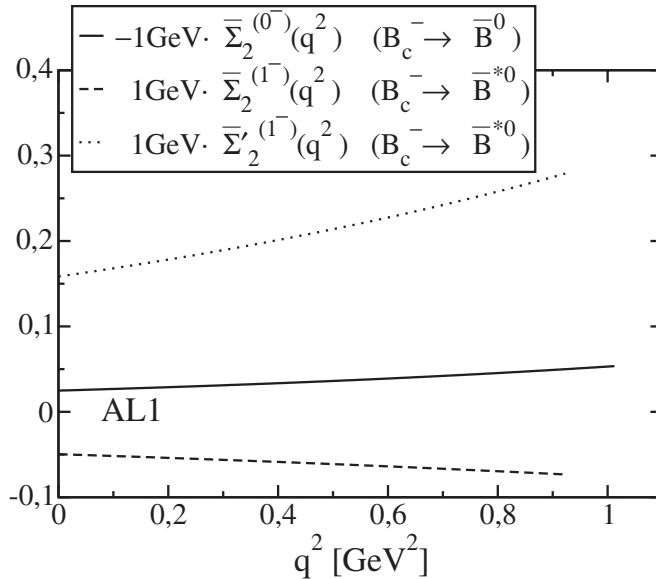


FIG. 13.  $\Sigma_1^{(0-)}$  of the semileptonic  $B_c^- \rightarrow \bar{B}^0$  (solid line) and  $B_c^- \rightarrow \bar{B}_s^0$  (dotted line) transitions, and  $\Sigma_1^{(1-)}$  of the semileptonic  $B_c^- \rightarrow \bar{B}^{*0}$  (dashed line), and  $B_c^- \rightarrow \bar{B}_s^{*0}$  (dash-dotted line) transitions evaluated with the AL1 potential.

$B_c^- \rightarrow \bar{B}^0$  and  $B_c^- \rightarrow \bar{B}_s^0$  transitions, and  $\Sigma_1^{(1-)}$  of the semileptonic  $B_c^- \rightarrow \bar{B}^{*0}$  and  $B_c^- \rightarrow \bar{B}_s^{*0}$  transitions. The differences

between the corresponding  $\Sigma_1^{(0-)}$  and  $\Sigma_1^{(1-)}$  are at the level of 10%. The differences are much more significant for  $\bar{\Sigma}_2^{(0-)}$ ,  $\bar{\Sigma}_2^{(1-)}$ , and  $\bar{\Sigma}_2'^{(1-)}$  that we show in Fig. 14. In each case the three curves shown would be the same in the infinite heavy quark mass limit. Clearly, in this case corrections on the inverse of the heavy quark masses seem to be important.



## VI. NONLEPTONIC $B_c^- \rightarrow BM_F$ TWO-MESON DECAYS

In this section we will evaluate decay widths for non-leptonic  $B_c^- \rightarrow BM_F$  two-meson decays where  $M_F$  is a pseudoscalar or vector meson with no  $b$  quark content, and, at this point,  $B$  represents a meson with a  $b$  quark. These decay modes involve a  $\bar{c} \rightarrow \bar{d}$  or  $\bar{c} \rightarrow \bar{s}$  transition at the quark level and they are governed, neglecting penguin operators, by the effective Hamiltonian [4,8]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \{ V_{cd} [c_1(\mu) Q_1^{cd} + c_2(\mu) Q_2^{cd}] + V_{cs} [c_1(\mu) Q_1^{cs} + c_2(\mu) Q_2^{cs}] + \text{H.c.} \}, \quad (67)$$

where  $c_1, c_2$  are scale-dependent Wilson coefficients, and  $Q_1^{cd}, Q_2^{cd}, Q_1^{cs}, Q_2^{cs}$  are local four-quark operators given by

$$Q_1^{cd} = \bar{\Psi}_c(0) \gamma_\mu (I - \gamma_5) \Psi_d(0) [V_{ud}^* \bar{\Psi}_d(0) \gamma^\mu (I - \gamma_5) \Psi_u(0) + V_{us}^* \bar{\Psi}_s(0) \gamma^\mu (I - \gamma_5) \Psi_u(0)]$$

$$Q_2^{cd} = \bar{\Psi}_c(0) \gamma_\mu (I - \gamma_5) \Psi_u(0) [V_{ud}^* \bar{\Psi}_d(0) \gamma^\mu (I - \gamma_5) \Psi_d(0) + V_{us}^* \bar{\Psi}_s(0) \gamma^\mu (I - \gamma_5) \Psi_d(0)]$$

$$Q_1^{cs} = \bar{\Psi}_c(0) \gamma_\mu (I - \gamma_5) \Psi_s(0) [V_{ud}^* \bar{\Psi}_d(0) \gamma^\mu (I - \gamma_5) \Psi_u(0) + V_{us}^* \bar{\Psi}_s(0) \gamma^\mu (I - \gamma_5) \Psi_u(0)]$$

$$Q_2^{cs} = \bar{\Psi}_c(0) \gamma_\mu (I - \gamma_5) \Psi_u(0) [V_{ud}^* \bar{\Psi}_d(0) \gamma^\mu (I - \gamma_5) \Psi_s(0) + V_{us}^* \bar{\Psi}_s(0) \gamma^\mu (I - \gamma_5) \Psi_s(0)]. \quad (68)$$

We shall work again in the factorization approximation taking into account the Fierz reordered contribution so that the relevant coefficients are not  $c_1$  and  $c_2$  but the combinations

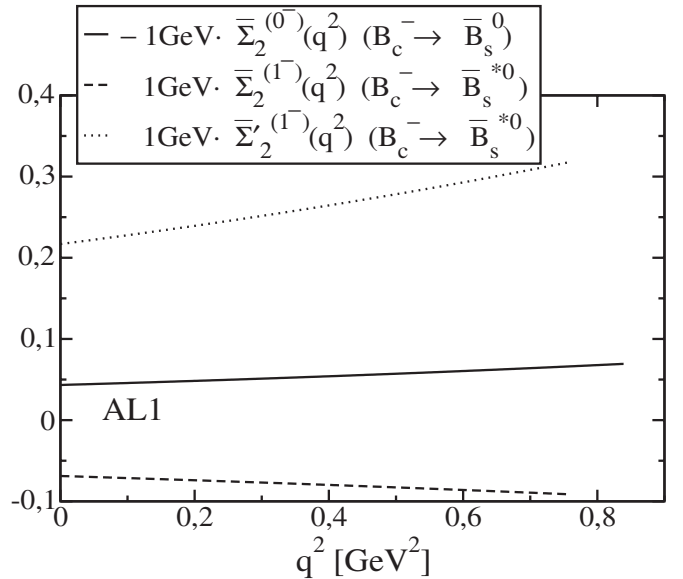


FIG. 14.  $-\bar{\Sigma}_2^{(0-)}$  (solid line) of the semileptonic  $B_c^- \rightarrow \bar{B}^0$  and  $B_c^- \rightarrow \bar{B}_s^0$  transitions, and  $\bar{\Sigma}_2^{(1-)}$  (dashed line),  $\bar{\Sigma}_2'^{(1-)}$  (dotted line) of the semileptonic  $B_c^- \rightarrow \bar{B}^{*0}$  and  $B_c^- \rightarrow \bar{B}_s^{*0}$  transitions evaluated with the AL1 potential.

$$a_1(\mu) = c_1(\mu) + \frac{1}{N_C} c_2(\mu); \quad a_2(\mu) = c_2(\mu) + \frac{1}{N_C} c_1(\mu). \quad (69)$$

The energy scale  $\mu$  appropriate in this case is  $\mu \simeq m_c$  and the values for  $a_1$  and  $a_2$  that we use are [4]

$$a_1 = 1.20; \quad a_2 = -0.317. \quad (70)$$

$$\mathbf{A.} \quad M_F = \pi^-, \rho^-, K^-, K^{*-}$$

In this case  $B$  denotes one of the  $\bar{B}^0, \bar{B}^{*0}, \bar{B}_s^0, \bar{B}_s^{*0}$ . The decay widths are

$$\begin{aligned} \Gamma &= \frac{G_F^2}{16\pi m_{B_c}^2} |V_{cd}|^2 |V_F|^2 a_1^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{\bar{B}^0, \bar{B}^{*0}}^2, m_F^2)}{2m_{B_c}} m_F^2 f_F^2 \mathcal{H}_{tt}^{B_c^- \rightarrow \bar{B}^0, \bar{B}^{*0}}(m_F^2) \quad M_F \equiv 0^- \text{ case} \\ \Gamma &= \frac{G_F^2}{16\pi m_{B_c}^2} |V_{cd}|^2 |V_F|^2 a_1^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{\bar{B}^0, \bar{B}^{*0}}^2, m_F^2)}{2m_{B_c}} m_F^2 f_F^2 (\mathcal{H}_{+1+1}^{B_c^- \rightarrow \bar{B}^0, \bar{B}^{*0}}(m_F^2) + \mathcal{H}_{-1-1}^{B_c^- \rightarrow \bar{B}^0, \bar{B}^{*0}}(m_F^2) \\ &\quad + \mathcal{H}_{00}^{B_c^- \rightarrow \bar{B}^0, \bar{B}^{*0}}(m_F^2)) \quad M_F \equiv 1^- \text{ case} \end{aligned} \quad (71)$$

TABLE XX. Decay widths in units of  $10^{-15}$  GeV, and for general values of the Wilson coefficient  $a_1$ , and branching ratios in percent for exclusive nonleptonic decays of the  $B_c^-$  meson. Our central values have been obtained with the AL1 potential.

	$\Gamma [10^{-15} \text{ GeV}]$ This work
$B_c^- \rightarrow \bar{B}^0 \pi^-$	$1.10^{+0.14}_{-0.16} a_1^2$
$B_c^- \rightarrow \bar{B}^0 \rho^-$	$1.41^{+0.12}_{-0.19} a_1^2$
$B_c^- \rightarrow \bar{B}^0 K^-$	$0.098^{+0.012}_{-0.012} a_1^2$
$B_c^- \rightarrow \bar{B}^0 K^{*-}$	$0.038^{+0.003}_{-0.005} a_1^2$
$B_c^- \rightarrow \bar{B}^{*0} \pi^-$	$0.71^{+0.12}_{-0.11} a_1^2$
$B_c^- \rightarrow \bar{B}^{*0} \rho^-$	$5.68^{+0.55}_{-0.77} a_1^2$
$B_c^- \rightarrow \bar{B}^{*0} K^-$	$0.047^{+0.007}_{-0.007} a_1^2$
$B_c^- \rightarrow \bar{B}^{*0} K^{*-}$	$0.29^{+0.03}_{-0.04} a_1^2$
$B_c^- \rightarrow \bar{B}_s^0 \pi^-$	$34.7^{+2.0}_{-0.6} a_1^2$
$B_c^- \rightarrow \bar{B}_s^0 \rho^-$	$23.1^{+0.5}_{-0.6} a_1^2$
$B_c^- \rightarrow \bar{B}_s^0 K^-$	$2.87^{+0.13}_{-0.06} a_1^2$
$B_c^- \rightarrow \bar{B}_s^0 K^{*-}$	$0.13^{+0.01}_{-0.01} a_1^2$
$B_c^- \rightarrow \bar{B}_s^{*0} \pi^-$	$22.8^{+2.2}_{-1.0} a_1^2$
$B_c^- \rightarrow \bar{B}_s^{*0} \rho^-$	$132^{+5}_{-6} a_1^2$
$B_c^- \rightarrow \bar{B}_s^{*0} K^-$	$1.29^{+0.10}_{-0.06} a_1^2$
	Branching ratio in %
	This work [4] [8] [9] [12] [16] [18] [19]
$B_c^- \rightarrow \bar{B}^0 \pi^-$	$0.11^{+0.01}_{-0.01}$ 0.20 0.10 0.32 0.10 1.06 0.19 0.15
$B_c^- \rightarrow \bar{B}^0 \rho^-$	$0.14^{+0.02}_{-0.02}$ 0.20 0.13 0.59 0.28 0.96 0.15 0.19
$B_c^- \rightarrow \bar{B}^0 K^-$	$0.010^{+0.001}_{-0.001}$ 0.015 0.009 0.025 0.010 0.07 0.014
$B_c^- \rightarrow \bar{B}^0 K^{*-}$	$0.0039^{+0.0003}_{-0.0005}$ 0.0048 0.004 0.018 0.012 0.015 0.003
$B_c^- \rightarrow \bar{B}^{*0} \pi^-$	$0.072^{+0.012}_{-0.012}$ 0.057 0.026 0.29 0.076 0.95 0.24 0.077
$B_c^- \rightarrow \bar{B}^{*0} \rho^-$	$0.58^{+0.05}_{-0.08}$ 0.30 0.67 1.17 0.89 2.57 0.85 0.67
$B_c^- \rightarrow \bar{B}^{*0} K^-$	$0.0048^{+0.0007}_{-0.0008}$ 0.0036 0.004 0.019 0.006 0.055 0.012
$B_c^- \rightarrow \bar{B}^{*0} K^{*-}$	$0.030^{+0.002}_{-0.004}$ 0.013 0.032 0.037 0.065 0.058 0.033
$B_c^- \rightarrow \bar{B}_s^0 \pi^-$	$3.51^{+0.19}_{-0.06}$ 3.9 2.46 5.75 1.56 16.4 3.01 3.42
$B_c^- \rightarrow \bar{B}_s^0 \rho^-$	$2.34^{+0.05}_{-0.06}$ 2.3 1.38 4.41 3.86 7.2 1.34 2.33
$B_c^- \rightarrow \bar{B}_s^0 K^-$	$0.29^{+0.01}_{-0.01}$ 0.29 0.21 0.41 0.17 1.06 0.21
$B_c^- \rightarrow \bar{B}_s^0 K^{*-}$	$0.013^{+0.001}_{-0.001}$ 0.011 0.0030 0.10 0.0043
$B_c^- \rightarrow \bar{B}_s^{*0} \pi^-$	$2.34^{+0.19}_{-0.14}$ 2.1 1.58 5.08 1.23 6.5 3.50 1.95
$B_c^- \rightarrow \bar{B}_s^{*0} \rho^-$	$13.4^{+0.5}_{-0.6}$ 11 10.8 14.8 16.8 20.2 10.8 12.1
$B_c^- \rightarrow \bar{B}_s^{*0} K^-$	$0.13^{+0.01}_{-0.01}$ 0.13 0.11 0.29 0.13 0.37 0.16

and similarly for  $\bar{B}_s^0, \bar{B}_s^{*0}$  with  $|V_{cd}| \rightarrow |V_{cs}|$ ;  $\bar{B}^0, \bar{B}^{*0} \rightarrow \bar{B}_s^0, \bar{B}_s^{*0}$ .  $V_F = V_{ud}$  or  $V_F = V_{us}$  depending on whether  $M_F = \pi^-, \rho^-$  or  $M_F = K^-, K^{*-}$ ,  $f_F$  is the decay constant of the  $M_F$  meson, and the different  $\mathcal{H}_{rr}$  have been evaluated at  $q^2 = m_F^2$ . In Table XX we show the decay widths for a general value of the Wilson coefficient  $a_1$ , and the corresponding branching ratios evaluated with  $a_1 = 1.20$ . The transition  $B_c^- \rightarrow \bar{B}_s^{*0} K^{*-}$  is not allowed with the new  $B_c$  mass value from Ref. [26]. Our branching ratios for a final  $\bar{B}_s^0$  or  $\bar{B}_s^{*0}$  are in very good agreement with the results by Ivanov *et al.* [4], while for a final  $\bar{B}^0$  or  $\bar{B}^{*0}$  we are in very good agreement with the results by Ebert *et al.* [8] (with the exception of the  $B_c \rightarrow \bar{B}^{*0} \pi^-$  decay).

### B. $M_F = \pi^0, \rho^0, K^0, K^{*0}$

Here the generic name  $B$  stands for a  $B^-$  or a  $B^{*-}$  meson. The different decay widths are given by

$$\begin{aligned} \Gamma &= \frac{G_F^2}{16\pi m_{B_c}^2} |V_{ud}|^2 |V_F|^2 a_2^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{B^-, B^{*-}}^2, m_F^2)}{2m_{B_c}} \\ &\quad \times m_F^2 \tilde{f}_F^2 \mathcal{H}_{tt}^{B_c^- \rightarrow B^-, B^{*-}}(m_F^2) \quad M_F \equiv 0^- \text{ case} \\ \Gamma &= \frac{G_F^2}{16\pi m_{B_c}^2} |V_{ud}|^2 |V_F|^2 a_2^2 \frac{\lambda^{1/2}(m_{B_c}^2, m_{B^-, B^{*-}}^2, m_F^2)}{2m_{B_c}} \\ &\quad \times m_F^2 \tilde{f}_F^2 (\mathcal{H}_{+1+1}^{B_c^- \rightarrow B^-, B^{*-}}(m_F^2) + \mathcal{H}_{-1-1}^{B_c^- \rightarrow B^-, B^{*-}}(m_F^2) \\ &\quad + \mathcal{H}_{00}^{B_c^- \rightarrow B^-, B^{*-}}(m_F^2)) \quad M_F \equiv 1^- \text{ case}, \quad (72) \end{aligned}$$

TABLE XXI. Decay widths in units of  $10^{-15}$  GeV, and for general values of the Wilson coefficient  $a_2$ , and branching ratios in percent for exclusive nonleptonic decays of the  $B_c^-$  meson. Our central values have been obtained with the AL1 potential.

	$\Gamma [10^{-15} \text{ GeV}]$						
	This work						
$B_c^- \rightarrow B^- \pi^0$	$0.54_{-0.07}^{+0.07} a_2^2$						
$B_c^- \rightarrow B^- \rho^0$	$0.71_{-0.10}^{+0.06} a_2^2$						
$B_c^- \rightarrow B^- K^0$	$35.3_{-4.9}^{+4.0} a_2^2$						
$B_c^- \rightarrow B^- K^{*0}$	$13.1_{-0.7}^{+0.9} a_2^2$						
$B_c \rightarrow B^{*-} \pi^0$	$0.35_{-0.05}^{+0.06} a_2^2$						
$B_c \rightarrow B^{*-} \rho^0$	$2.84_{-0.39}^{+0.27} a_2^2$						
$B_c \rightarrow B^{*-} K^0$	$16.9_{-2.7}^{+2.4} a_2^2$						
$B_c \rightarrow B^{*-} K^{*0}$	$103_{-13}^{+8} a_2^2$						
	Branching ratio in %						
	This work	[4]	[8]	[9]	[12]	[16]	[19]
$B_c^- \rightarrow B^- \pi^0$	$0.0038_{-0.0006}^{+0.0005}$	0.007	0.004	0.011	0.004	0.037	0.007
$B_c^- \rightarrow B^- \rho^0$	$0.0050_{-0.0007}^{+0.0004}$	0.0071	0.005	0.020	0.010	0.034	0.009
$B_c^- \rightarrow B^- K^0$	$0.25_{-0.04}^{+0.03}$	0.38	0.24	0.66	0.27	1.98	0.17
$B_c^- \rightarrow B^- K^{*0}$	$0.093_{-0.013}^{+0.006}$	0.11	0.09	0.47	0.32	0.43	0.095
$B_c \rightarrow B^{*-} \pi^0$	$0.0025_{-0.0005}^{+0.0004}$	0.0020	0.001	0.010	0.003	0.033	0.004
$B_c \rightarrow B^{*-} \rho^0$	$0.020_{-0.003}^{+0.002}$	0.011	0.024	0.041	0.031	0.09	0.031
$B_c \rightarrow B^{*-} K^0$	$0.12_{-0.02}^{+0.02}$	0.088	0.11	0.50	0.16	1.60	0.061
$B_c \rightarrow B^{*-} K^{*0}$	$0.73_{-0.10}^{+0.06}$	0.32	0.84	0.97	1.70	1.67	0.57

where  $V_F = V_{cd}$  or  $V_F = V_{cs}$  depending on whether  $M_F = \pi^0, \rho^0$  or  $M_F = K^0, K^{*0}$ ,  $\tilde{f}_F = f_F$  for  $M_F = K^0, K^{*0}$  whereas  $\tilde{f}_F = f_F/\sqrt{2}$  for  $M_F = \pi^0, \rho^0$ , with  $f_F$  the  $M_F$  meson decay constant, and the different  $\mathcal{H}_{rr}$  evaluated at  $q^2 = m_F^2$ . The latter have been obtained from the matrix elements for the effective current operator  $\bar{\Psi}_c(0) \gamma^\mu (I - \gamma_5) \Psi_u(0)$ . The decay widths, for a general value of the Wilson coefficient  $a_2$ , and the corresponding branching ratios are shown in Table XXI. With the exception of the  $B_c^- \rightarrow B^{*-} \pi^0$  case, our results are in a global good agreement with the ones by Ebert *et al.* [8].

## VII. SUMMARY

We have made a comprehensive and exhaustive study of exclusive semileptonic and nonleptonic two-meson decays of the  $B_c$  meson within a nonrelativistic quark model. We have left out semileptonic processes involving a  $b \rightarrow u$  transition at the quark level to avoid known deficiencies both at high and low  $q^2$  transfers [3]. For similar reasons we have only considered two-meson nonleptonic decay channels that include a  $c\bar{c}$  or  $B$  meson. Our model respects HQSS constraints in the infinite heavy quark mass limit but hints at sizable corrections away from that limit for some form factors. Unfortunately such corrections have not been worked out in perturbative QCD as they have for heavy-light mesons [43].

To check the sensitivity of our results to the interquark interaction, we have used five different quark-quark potentials. Most observables change only at the level of a few percent when changing the interaction. There is another source of theoretical uncertainty in the use of nonrelativistic kinematics in the evaluation of the orbital wave functions and the construction of our states in Eq. (1). While this is a very good approximation for the  $B_c$  itself, it is not necessarily so for mesons with a light quark. We nevertheless think that, to a certain extent, the ignored relativistic effects are contained in an effective way in the free parameters of the interquark interaction, which are fitted to experimental data.

Our results for the observables analyzed are in a general good agreement (whenever comparison is possible) with the results obtained within the quasipotential approach to the relativistic quark model of Ebert *et al.* [7,8].

The branching ratios for the leptonic  $B_c^- \rightarrow c\bar{c}$  and  $B_c^- \rightarrow \bar{B}$  decays are also in reasonable agreement with the relativistic constituent quark model results of Ivanov *et al.* [4,5].

For the nonleptonic  $B_c^- \rightarrow \eta_c M_F^-$  and  $B_c^- \rightarrow J/\Psi M_F^-$  two-meson decay channels with  $M_F^- = \pi^-, \rho^-, K^-, K^{*-}$ , we find also reasonable agreement (better for the  $J/\Psi$  channel) with the Bethe-Salpeter calculation by El-Hady *et al.* [12] and the light front calculation by Anisimov *et al.* [19], while our results are a factor of 2 smaller than the ones by Ivanov *et al.* [4] and Chang *et al.* [9–11], the latter obtained within the nonrelativistic approach to the Bethe-Salpeter equation. For the two-meson decay channels with



$\chi_{c0}$ ,  $\chi_{c1}$ ,  $h_c$ ,  $\chi_{c2}$ , or  $\Psi(3836)$  as the final  $c\bar{c}$  meson and  $M_F = \pi^-, \rho^-, K^-, K^{*-}$  our results are generally a factor of 2 smaller than the ones of Ivanov *et al.* [4], whereas for some channels ( $\chi_{c0}\pi^-, \chi_{c0}\rho^-, h_c\pi^-, h_c\rho^-, \chi_{c2}\pi^-$ , and  $\chi_{c2}\rho^-$ ) we find very good agreement with the results by Chang *et al.* [9–11]. The disagreement with Ivanov *et al.* extends to the two-meson decay channels with a final  $c\bar{c}$  and  $D$  mesons. There we find good agreement with the results by El-Hady *et al.* [12] and the ones obtained within the sum rules of QCD and nonrelativistic QCD by Kiselev [16].

As for the two-meson nonleptonic decay channels  $B_c^- \rightarrow \bar{B}M_F$  with  $M_F = \pi^-, \rho^-, K^-, K^{*-}$  we are in a reasonable good agreement with the results by Anisimov *et al.* [19]. For the case of a final  $\bar{B}_s^0$  or  $\bar{B}_s^{*0}$ , the agreement with the results by Ivanov *et al.* [4] is very good. For the  $B_c^- \rightarrow B^-M_F$  case with  $M_F = \pi^0, \rho^0, K^0, K^{*0}$ , and apart from the results by Ebert *et al.* [8], we find no global agreement with other calculations, neither do they agree with each other.

From the above comparison, one sees that there are different models producing sometimes very different results for the same observables. Accurate experimental data will shed light on this issue.

## ACKNOWLEDGMENTS

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## APPENDIX A: $\epsilon_{(\lambda)}^\mu(\vec{P})$ POLARIZATION VECTORS

Different sets of polarization vectors used in this paper:

(i)  $\vec{P} = \vec{0}$  case:

$$\text{Spin or helicity bases} \quad \begin{cases} \epsilon_{(+1)}^\mu(\vec{P}) = \left(0, -\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right) \\ \epsilon_{(-1)}^\mu(\vec{P}) = \left(0, \frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right) \\ \epsilon_{(0)}^\mu(\vec{P}) = (0, 0, 0, 1) \end{cases} \quad (\text{A1})$$

(ii)  $\vec{P} = |\vec{P}|\vec{k}$  case:

$$\text{Spin or helicity bases} \quad \begin{cases} \epsilon_{(+1)}^\mu(\vec{P}) = \left(0, -\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right) \\ \epsilon_{(-1)}^\mu(\vec{P}) = \left(0, \frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right) \\ \epsilon_{(0)}^\mu(\vec{P}) = \left(\frac{|\vec{P}|}{m}, 0, 0, \frac{E(\vec{P})}{m}\right) \end{cases} \quad (\text{A2})$$

(iii)  $\vec{P} = -|\vec{P}|\vec{k}$  case:

$$\begin{aligned} \text{Spin base} \quad \begin{cases} \epsilon_{(+1)}^\mu(\vec{P}) = \left(0, -\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right) \\ \epsilon_{(-1)}^\mu(\vec{P}) = \left(0, \frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right) \\ \epsilon_{(0)}^\mu(\vec{P}) = \left(-\frac{|\vec{P}|}{m}, 0, 0, \frac{E(\vec{P})}{m}\right) \end{cases} & \quad \text{Helicity base} \quad \begin{cases} \epsilon_{(+1)}^\mu(\vec{P}) = \left(0, \frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right) \\ \epsilon_{(-1)}^\mu(\vec{P}) = \left(0, -\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right) \\ \epsilon_{(0)}^\mu(\vec{P}) = \left(\frac{|\vec{P}|}{m}, 0, 0, -\frac{E(\vec{P})}{m}\right). \end{cases} \end{aligned} \quad (\text{A3})$$

## APPENDIX B: EXPRESSION FOR THE $V_{(\lambda)}^\mu(|\vec{q}|)$ , $V_{T(\lambda)}^\mu(|\vec{q}|)$ AND $A^\mu(|\vec{q}|)$ , $A_{(\lambda)}^\mu(|\vec{q}|)$ , $A_{T(\lambda)}^\mu(|\vec{q}|)$ MATRIX ELEMENTS

Here we give general expressions valid for transitions between a pseudoscalar meson  $M_I$  at rest with quark content  $q_{f_1}\bar{q}_{f_2}$  and a final  $M_F$  meson with total angular momentum and parity  $J^\pi = 0^-, 0^+, 1^-, 1^+, 2^-, 2^+$ , three-momentum

$-|\vec{q}|\vec{k}$ , and quark content  $q_{f'_1}\bar{q}_{f_2}$ . In the transition it is the quark that changes flavor. The phases of the wave functions are the ones chosen in Eqs. (5)–(7). We generally have

$$\begin{aligned}\mathcal{V}^\mu(|\vec{q}|) - \mathcal{A}^\mu(|\vec{q}|) &= \sqrt{2m_I 2E_F(-\vec{q})}_{\text{NR}} \langle M_F(J^\pi), \lambda - |\vec{q}|\vec{k} | J_{f'_1 f_1}^{\mu}(0) | M_I(0^-), \vec{0} \rangle_{\text{NR}} \\ &= \sqrt{2m_I 2E_F(-\vec{q})} \int d^3 p \sum_{s'_1} \sum_{s_1, s_2} (\hat{\phi}_{(s'_1, f'_1), (s_2, f_2)}^{(M_F(J^\pi), \lambda)}(\vec{p}))^* \hat{\phi}_{(s_1, f_1), (s_2, f_2)}^{(M_I(0^-))} \left( \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} \right) \\ &\quad \times \frac{1}{\sqrt{2E_{f'_1} 2E_{f_1}}} \bar{u}_{s'_1, f'_1} \left( -\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} - \vec{p} \right) \gamma^\mu (I - \gamma_5) u_{s_1, f_1} \left( \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} - \vec{p} \right),\end{aligned}\quad (\text{B1})$$

where  $\mathcal{V}^\mu(\mathcal{A}^\mu)$  represent any of the  $V^\mu(A^\mu)$ ,  $V_{(\lambda)}^\mu(A_{(\lambda)}^\mu)$ , or  $V_{T(\lambda)}^\mu(A_{T(\lambda)}^\mu)$ , and where  $E_{f'_1}$  and  $E_{f_1}$  are shorthand notations for  $E_{f'_1}(-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} - \vec{p})$  and  $E_{f_1}(\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} - \vec{p})$ , respectively. Defining also  $\hat{E}_{f'_1} = E_{f'_1} + m_{f'_1}$  and  $\hat{E}_{f_1} = E_{f_1} + m_{f_1}$ , we arrive at the following final expressions:

(i) Case  $J^\pi = 0^-$

$$\begin{aligned}V^0(|\vec{q}|) &= \sqrt{2m_I 2E_F(-\vec{q})} \int d^3 p \frac{1}{4\pi} (\hat{\phi}_{f'_1, f_2}^{(M_F(0^-))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} \right| \right) \\ &\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left( 1 + \frac{(-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} - \vec{p}) \cdot (\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} - \vec{p})}{\hat{E}_{f'_1} \hat{E}_{f_1}} \right) \\ V^3(|\vec{q}|) &= \sqrt{2m_I 2E_F(-\vec{q})} \int d^3 p \frac{1}{4\pi} (\hat{\phi}_{f'_1, f_2}^{(M_F(0^-))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} \right| \right) \\ &\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left( \frac{\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z}{\hat{E}_{f_1}} + \frac{-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z}{\hat{E}_{f'_1}} \right)\end{aligned}\quad (\text{B2})$$

(ii) Case  $J^\pi = 0^+$

$$\begin{aligned}A^0(|\vec{q}|) &= \sqrt{2m_I 2E_F(-\vec{q})} \int d^3 p \frac{1}{4\pi |\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(0^+))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} \right| \right) \\ &\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left( \frac{\vec{p} \cdot (\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} - \vec{p})}{\hat{E}_{f_1}} + \frac{\vec{p} \cdot (-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} - \vec{p})}{\hat{E}_{f'_1}} \right) \\ A^3(|\vec{q}|) &= \sqrt{2m_I 2E_F(-\vec{q})} \int d^3 p \frac{1}{4\pi |\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(0^+))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} \right| \right) \\ &\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left\{ p_z \left( 1 - \frac{(-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} - \vec{p}) \cdot (\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} - \vec{p})}{\hat{E}_{f'_1} \hat{E}_{f_1}} \right) + \frac{1}{\hat{E}_{f'_1} \hat{E}_{f_1}} \left[ \left( -\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z \right) \right. \right. \\ &\quad \left. \left. \times \vec{p} \cdot \left( \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} - \vec{p} \right) + \left( \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z \right) \vec{p} \cdot \left( -\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}|\vec{k} - \vec{p} \right) \right] \right\}\end{aligned}\quad (\text{B3})$$

(iii) Case  $J^\pi = 1^-$ 

$$\begin{aligned}
V_{\lambda=-1}^{(1^-)1}(|\vec{q}|) &= \frac{-i}{\sqrt{2}} \sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi} (\hat{\phi}_{f'_1, f_2}^{(M_F(1^-))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left( -\frac{\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z}{\hat{E}_{f_1}} + \frac{-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z}{\hat{E}_{f'_1}} \right)
\end{aligned} \tag{B4}$$

and similarly

$$\begin{aligned}
A_{\lambda=0}^{(1^-)0}(|\vec{q}|) &= i \sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi} (\hat{\phi}_{f'_1, f_2}^{(M_F(1^-))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left( \frac{\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z}{\hat{E}_{f_1}} + \frac{-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z}{\hat{E}_{f'_1}} \right) \\
A_{\lambda=-1}^{(1^-)1}(|\vec{q}|) &= \frac{i}{\sqrt{2}} \sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi} (\hat{\phi}_{f'_1, f_2}^{(M_F(1^-))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left( 1 + \frac{2p_x^2 - (-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p}) \cdot (\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p})}{\hat{E}_{f'_1} \hat{E}_{f_1}} \right) \\
A_{\lambda=0}^{(1^-)3}(|\vec{q}|) &= i \sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi} (\hat{\phi}_{f'_1, f_2}^{(M_F(1^-))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left( 1 + \frac{2(-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z)(\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z)}{\hat{E}_{f'_1} \hat{E}_{f_1}} - \frac{(-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p}) \cdot (\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p})}{\hat{E}_{f'_1} \hat{E}_{f_1}} \right)
\end{aligned} \tag{B5}$$

(iv) Case  $J^\pi = 1^+$

$$\begin{aligned}
V_{\lambda=0}^{(1^+, S_{q\bar{q}}=0)0}(|\vec{q}|) &= i\sqrt{3}\sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(1^+, S_{q\bar{q}}=0))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} p_z \left( 1 + \frac{(-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p}) \cdot (\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p})}{\hat{E}_{f'_1} \hat{E}_{f_1}} \right) \\
V_{\lambda=0}^{(1^+, S_{q\bar{q}}=1)0}(|\vec{q}|) &= -i\sqrt{\frac{3}{2}}\sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(1^+, S_{q\bar{q}}=1))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \frac{|\vec{q}|(p_z^2 - \vec{p}^2)}{\hat{E}_{f'_1} \hat{E}_{f_1}} \\
V_{\lambda=-1}^{(1^+, S_{q\bar{q}}=0)1}(|\vec{q}|) &= -i\sqrt{\frac{3}{2}}\sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(1^+, S_{q\bar{q}}=0))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} p_x^2 \left( \frac{1}{\hat{E}_{f_1}} + \frac{1}{\hat{E}_{f'_1}} \right) \\
V_{\lambda=-1}^{(1^+, S_{q\bar{q}}=1)1}(|\vec{q}|) &= i\frac{\sqrt{3}}{2}\sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(1^+, S_{q\bar{q}}=1))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left( \frac{p_y^2 + p_z^2 + p_z |\vec{q}| \frac{m_{f'_1}}{m_{f'_1} + m_{f_2}}}{\hat{E}_{f'_1}} - \frac{p_y^2 + p_z^2 - p_z |\vec{q}| \frac{m_{f_2}}{m_{f'_1} + m_{f_2}}}{\hat{E}_{f_1}} \right) \\
V_{\lambda=0}^{(1^+, S_{q\bar{q}}=0)3}(|\vec{q}|) &= i\sqrt{3}\sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(1^+, S_{q\bar{q}}=0))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} p_z \left( \frac{\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z}{\hat{E}_{f_1}} + \frac{-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z}{\hat{E}_{f'_1}} \right) \\
V_{\lambda=0}^{(1^+, S_{q\bar{q}}=1)3}(|\vec{q}|) &= -i\sqrt{\frac{3}{2}}\sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(1^+, S_{q\bar{q}}=1))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} (p_x^2 + p_y^2) \left( \frac{1}{\hat{E}_{f_1}} - \frac{1}{\hat{E}_{f'_1}} \right) \tag{B6}
\end{aligned}$$

and similarly

$$\begin{aligned}
A_{\lambda=-1}^{(1^+, S_{q\bar{q}}=0)1}(|\vec{q}|) &= -i\sqrt{\frac{3}{2}}\sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(1^+, S_{q\bar{q}}=0))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \frac{p_y^2 |\vec{q}|}{\hat{E}_{f_1} \hat{E}_{f'_1}} \\
A_{\lambda=-1}^{(1^+, S_{q\bar{q}}=1)1}(|\vec{q}|) &= i\frac{\sqrt{3}}{2}\sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(1^+, S_{q\bar{q}}=1))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left\{ p_z \left( 1 - \frac{(-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p}) \cdot (\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p})}{\hat{E}_{f'_1} \hat{E}_{f_1}} \right) + \frac{m_{f_2} - m_{f'_1}}{m_{f'_1} + m_{f_2}} \frac{p_x^2 |\vec{q}|}{\hat{E}_{f'_1} \hat{E}_{f_1}} \right\} \tag{B7}
\end{aligned}$$

(v) Case  $J^\pi = 2^-$ 

$$\begin{aligned}
V_{T\lambda=0}^{(2^-)0}(|\vec{q}|) &= i\sqrt{\frac{15}{2}}\sqrt{2m_l 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|^2} (\hat{\phi}_{f'_1, f_2}^{(M_F(2^-))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_l(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \frac{p_z(p_x^2 + p_y^2)|\vec{q}|}{\hat{E}_{f'_1} \hat{E}_{f_1}} \\
V_{T\lambda=+1}^{(2^-)1}(|\vec{q}|) &= i\frac{\sqrt{5}}{2}\sqrt{2m_l 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|^2} (\hat{\phi}_{f'_1, f_2}^{(M_F(2^-))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_l(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left\{ (p_z^2 - p_x^2) \left( \frac{-p_z - \frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}|}{\hat{E}_{f'_1}} - \frac{-p_z + \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}|}{\hat{E}_{f_1}} \right) - p_z p_y^2 \left( \frac{1}{\hat{E}_{f'_1}} - \frac{1}{\hat{E}_{f_1}} \right) \right\} \\
V_{T\lambda=0}^{(2^-)3}(|\vec{q}|) &= i\sqrt{\frac{15}{2}}\sqrt{2m_l 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|^2} (\hat{\phi}_{f'_1, f_2}^{(M_F(2^-))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_l(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} p_z(p_x^2 + p_y^2) \left( \frac{1}{\hat{E}_{f'_1}} - \frac{1}{\hat{E}_{f_1}} \right)
\end{aligned} \tag{B8}$$

and similarly

$$\begin{aligned}
A_{T\lambda=+1}^{(2^-)1}(|\vec{q}|) &= i\frac{\sqrt{5}}{2}\sqrt{2m_l 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|^2} (\hat{\phi}_{f'_1, f_2}^{(M_F(2^-))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_l(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \\
&\quad \times \left\{ (p_z^2 - p_y^2) \left( 1 - \frac{(-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p}) \cdot (\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p})}{\hat{E}_{f'_1} \hat{E}_{f_1}} \right) - p_z p_x^2 |\vec{q}| \frac{m_{f'_1} - m_{f_2}}{m_{f'_1} + m_{f_2}} \frac{1}{\hat{E}_{f'_1} \hat{E}_{f_1}} \right\}
\end{aligned} \tag{B9}$$

(vi) Case  $J^\pi = 2^+$ 

$$\begin{aligned}
V_{T\lambda=+1}^{(2^+)1}(|\vec{q}|) &= i\frac{\sqrt{3}}{2}\sqrt{2m_l 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(2^+))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_l(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left( \frac{p_y^2 - p_z^2 - p_z |\vec{q}| \frac{m_{f'_1}}{m_{f'_1} + m_{f_2}}}{\hat{E}_{f'_1}} - \frac{p_y^2 - p_z^2 + p_z |\vec{q}| \frac{m_{f_2}}{m_{f'_1} + m_{f_2}}}{\hat{E}_{f_1}} \right)
\end{aligned} \tag{B10}$$

and similarly

$$\begin{aligned}
A_{T\lambda=0}^{(2^+)0}(|\vec{q}|) &= \frac{-i}{\sqrt{2}} \sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(2^+))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left( \frac{p_x^2 + p_y^2 - 2p_z^2 - 2p_z |\vec{q}| \frac{m_{f'_1}}{m_{f'_1} + m_{f_2}}}{\hat{E}_{f'_1}} + \frac{p_x^2 + p_y^2 - 2p_z^2 + 2p_z |\vec{q}| \frac{m_{f_2}}{m_{f'_1} + m_{f_2}}}{\hat{E}_{f_1}} \right) \\
A_{T\lambda=+1}^{(2^+)1}(|\vec{q}|) &= i \frac{\sqrt{3}}{2} \sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(2^+))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left\{ p_z \left( 1 - \frac{(-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p}) \cdot (\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p})}{\hat{E}_{f'_1} \hat{E}_{f_1}} \right) + \frac{4p_z p_x^2 - p_x^2 |\vec{q}| \frac{m_{f_2} - m_{f'_1}}{m_{f'_1} + m_{f_2}}}{\hat{E}_{f'_1} \hat{E}_{f_1}} \right\} \\
A_{T\lambda=0}^{(2^+)3}(|\vec{q}|) &= -i\sqrt{2} \sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \frac{1}{4\pi|\vec{p}|} (\hat{\phi}_{f'_1, f_2}^{(M_F(2^+))}(|\vec{p}|))^* \hat{\phi}_{f_1, f_2}^{(M_I(0^-))} \left( \left| \vec{p} - \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} \right| \right) \\
&\quad \times \sqrt{\frac{\hat{E}_{f'_1} \hat{E}_{f_1}}{4E_{f'_1} E_{f_1}}} \left\{ p_z \left( 1 - \frac{(-\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p}) \cdot (\frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| \vec{k} - \vec{p})}{\hat{E}_{f'_1} \hat{E}_{f_1}} \right) + \frac{1}{\hat{E}_{f'_1} \hat{E}_{f_1}} \left[ 2p_z \left( -\frac{m_{f'_1}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z \right) \right. \right. \\
&\quad \left. \left. \times \left( \frac{m_{f_2}}{m_{f'_1} + m_{f_2}} |\vec{q}| - p_z \right) + (p_x^2 + p_y^2) \left( -p_z + \frac{m_{f_2} - m_{f'_1}}{2(m_{f'_1} + m_{f_2})} |\vec{q}| \right) \right] \right\}. \tag{B11}
\end{aligned}$$

### APPENDIX C: TRANSITIONS INVOLVING ANTIQUARKS

When it is the antiquark that suffers the decay, the expressions are modified as described below for a general transition between a pseudoscalar meson  $M_I$  at rest with quark content  $q_{f_1} \bar{q}_{f_2}$  and a final  $M_F$  meson with total angular momentum and parity  $J^\pi = 0^-, 0^+, 1^-, 1^+, 2^-, 2^+$ , three-momentum  $-|\vec{q}| \vec{k}$ , and quark content  $q_{f'_1} \bar{q}_{f'_2}$ . We have

$$\begin{aligned}
\mathcal{V}^\mu(|\vec{q}|) - \mathcal{A}^\mu(|\vec{q}|) &= \sqrt{2m_I 2E_F(-\vec{q})}_{\text{NR}} \langle M_F(J^\pi), \lambda - |\vec{q}| \vec{k} | J^{f_2 f'_2 \mu}(0) | M_I(0^-), \vec{0} \rangle_{\text{NR}} \\
&= -\sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \sum_{s'_2} \sum_{s_1, s_2} (\hat{\phi}_{(s_1, f_1), (s'_2, f'_2)}^{(M_F(J^\pi, \lambda))}(\vec{p}))^* \hat{\phi}_{(s_1, f_1), (s_2, f_2)}^{(M_I(0^-))} \left( \vec{p} + \frac{m_{f_1}}{m_{f_1} + m_{f'_2}} |\vec{q}| \vec{k} \right) \\
&\quad \times \frac{(-1)^{s_2 - s'_2}}{\sqrt{2E_{f'_2} 2E_{f_2}}} \bar{v}_{s_2, f_2} \left( \frac{m_{f_1}}{m_{f_1} + m_{f'_2}} |\vec{q}| \vec{k} + \vec{p} \right) \gamma^\mu (I - \gamma_5) v_{s'_2, f'_2} \left( -\frac{m_{f'_2}}{m_{f_1} + m_{f'_2}} |\vec{q}| \vec{k} + \vec{p} \right), \tag{C1}
\end{aligned}$$

where  $\mathcal{V}^\mu(\mathcal{A}^\mu)$  represent any of the  $V^\mu(A^\mu)$ ,  $V_{(\lambda)}^\mu(A_{(\lambda)}^\mu)$ , or  $V_{T(\lambda)}^\mu(A_{T(\lambda)}^\mu)$ , and  $E_{f'_2}$ ,  $E_{f_2}$  are shorthand notation for  $E_{f'_2}(-\frac{m_{f'_2}}{m_{f_1} + m_{f'_2}} |\vec{q}| \vec{k} + \vec{p})$ ,  $E_{f_2}(\frac{m_{f_1}}{m_{f_1} + m_{f'_2}} |\vec{q}| \vec{k} + \vec{p})$ . We can use now that

$$v_{s, f}(\vec{p}) = (-1)^{(1/2) - s} \mathcal{C} \bar{u}_{s, f}^T(\vec{p}); \quad \bar{v}_{s, f}(\vec{p}) = -(-1)^{(1/2) - s} u_{s, f}^T(\vec{p}) \mathcal{C}^\dagger, \tag{C2}$$

where  $\mathcal{C}$  is a matrix given in the Fermi-Dirac representation that we use by

$$\mathcal{C} = i\gamma^2 \gamma^0 \tag{C3}$$

and that satisfies

$$\mathcal{C} = -\mathcal{C}^{-1} = -\mathcal{C}^\dagger = -\mathcal{C}^T; \quad \mathcal{C} \gamma_\mu^T \mathcal{C}^\dagger = -\gamma_\mu. \tag{C4}$$

Using the above information and making the change of variable  $\vec{p} \rightarrow -\vec{p}$ , we can rewrite

$$\begin{aligned}
\mathcal{V}^\mu(|\vec{q}|) - \mathcal{A}^\mu(|\vec{q}|) &= \sqrt{2m_I 2E_F(-\vec{q})} \int d^3p \sum_{s'_2} \sum_{s_1, s_2} (\hat{\phi}_{(s_1, f_1), (s'_2, f'_2)}^{(M_F(J^\pi, \lambda))}(-\vec{p}))^* \hat{\phi}_{(s_1, f_1), (s_2, f_2)}^{(M_I(0^-))} \left( -\left( \vec{p} - \frac{m_{f_1}}{m_{f_1} + m_{f'_2}} |\vec{q}| \vec{k} \right) \right) \\
&\quad \times \frac{1}{\sqrt{2E_{f'_2} 2E_{f_2}}} \bar{u}_{s'_2, f'_2} \left( -\frac{m_{f'_2}}{m_{f_1} + m_{f'_2}} |\vec{q}| \vec{k} - \vec{p} \right) \gamma^\mu (-I - \gamma_5) u_{s_2, f_2} \left( \frac{m_{f_1}}{m_{f_1} + m_{f'_2}} |\vec{q}| \vec{k} - \vec{p} \right). \tag{C5}
\end{aligned}$$



By comparison with the corresponding expressions involving quarks we find that, apart from the changes in the masses involved, there is an extra minus sign for the vector part, and, due to Clebsch-Gordan rearrangements and the fact that  $Y_{lm}(-\vec{p}) = (-1)^l Y_{lm}(\vec{p})$ , a global sign given by  $(-1)^{l_I + s_I - l_F - s_F}$  where  $l_I, s_I$  ( $l_F, s_F$ ) are the orbital and spin angular momenta of the initial (final) meson.

In any case this implies a change of sign in the relative phase between vector and axial contributions, which in its term produces a sign change in the tensor helicity components combination  $H_P$  due to the fact that  $\mathcal{H}_{+1+1}$  goes into  $\mathcal{H}_{-1-1}$  and vice versa. All other tensor helicity component combinations defined in Eq. (40) keep their signs.

A simple way of anticipating the above result is the following: the current for  $\bar{q}_{f_2}$  decay into  $\bar{q}_{f'_2}$  is

$$\bar{\Psi}_{f_2}(0)\gamma^\mu(I - \gamma_5)\Psi_{f'_2}(0). \quad (\text{C6})$$

For antiquarks the fields that play the similar role as the  $\Psi$  fields play for quarks are the charge conjugate ones  $\Psi^c$ . In

terms of the latter the above current is written as

$$\bar{\Psi}_{f_2}(0)\gamma^\mu(I - \gamma_5)\Psi_{f'_2}(0) = \bar{\Psi}_{f'_2}^c(0)\gamma^\mu(-I - \gamma_5)\Psi_{f_2}^c(0). \quad (\text{C7})$$

Now this is similar to the current for quark decay but with an extra minus sign in the vector part. Whatever other changes might come from reorderings in the wave functions, we will have an extra relative sign between the vector and axial part.

#### APPENDIX D: EXPRESSIONS FOR THE HELICITY COMPONENTS OF THE HADRON TENSOR

In this Appendix we give the expressions for the nonzero helicity components  $\mathcal{H}_{rs}$  of the hadron tensor, as defined in Eq. (36), corresponding to a  $B_c^- \rightarrow c\bar{c}$  transition. The different cases correspond to the ones discussed in the main text.

(i) Case  $0^- \rightarrow 0^-, 0^+$ :

$$\begin{aligned} \mathcal{H}_{tt}(P_{B_c}, P_{c\bar{c}}) &= \left( \frac{m_{B_c}^2 - m_{c\bar{c}}^2}{\sqrt{q^2}} F_+(q^2) + \sqrt{q^2} F_-(q^2) \right)^2 \\ \mathcal{H}_{t0}(P_{B_c}, P_{c\bar{c}}) &= \mathcal{H}_{0t}(P_{B_c}, P_{c\bar{c}}) = \lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2) \left( \frac{m_{B_c}^2 - m_{c\bar{c}}^2}{q^2} F_+^2(q^2) + F_+(q^2) F_-(q^2) \right) \\ \mathcal{H}_{00}(P_{B_c}, P_{c\bar{c}}) &= \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{q^2} F_+^2(q^2). \end{aligned} \quad (\text{D1})$$

(ii) Case  $0^- \rightarrow 1^-, 1^+$ :

$$\begin{aligned} \mathcal{H}_{tt}(P_{B_c}, P_{c\bar{c}}) &= \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{4m_{c\bar{c}}^2 q^2} \left( (m_{B_c} - m_{c\bar{c}})(A_0(q^2) - A_+(q^2)) - \frac{q^2}{m_{B_c} + m_{c\bar{c}}} A_-(q^2) \right)^2 \\ \mathcal{H}_{t0}(P_{B_c}, P_{c\bar{c}}) &= \mathcal{H}_{0t}(P_{B_c}, P_{c\bar{c}}) \\ &= \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{c\bar{c}}\sqrt{q^2}} \left[ (m_{B_c} - m_{c\bar{c}})(A_0(q^2) - A_+(q^2)) - \frac{q^2}{m_{B_c} + m_{c\bar{c}}} A_-(q^2) \right] \\ &\quad \times \left[ (m_{B_c} - m_{c\bar{c}}) \frac{m_{B_c}^2 - q^2 - m_{c\bar{c}}^2}{2m_{c\bar{c}}\sqrt{q^2}} A_0(q^2) - \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{c\bar{c}}\sqrt{q^2}} \frac{A_+(q^2)}{m_{B_c} + m_{c\bar{c}}} \right] \\ \mathcal{H}_{+1+1}(P_{B_c}, P_{c\bar{c}}) &= \left( \frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{m_{B_c} + m_{c\bar{c}}} V(q^2) + (m_{B_c} - m_{c\bar{c}}) A_0(q^2) \right)^2 \\ \mathcal{H}_{-1-1}(P_{B_c}, P_{c\bar{c}}) &= \left( -\frac{\lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{m_{B_c} + m_{c\bar{c}}} V(q^2) + (m_{B_c} - m_{c\bar{c}}) A_0(q^2) \right)^2 \\ \mathcal{H}_{00}(P_{B_c}, P_{c\bar{c}}) &= \left( (m_{B_c} - m_{c\bar{c}}) \frac{m_{B_c}^2 - q^2 - m_{c\bar{c}}^2}{2m_{c\bar{c}}\sqrt{q^2}} A_0(q^2) - \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{2m_{c\bar{c}}\sqrt{q^2}} \frac{A_+(q^2)}{m_{B_c} + m_{c\bar{c}}} \right)^2. \end{aligned} \quad (\text{D2})$$

For a  $B_c^- \rightarrow c\bar{c}$  transition (with  $B$  representing any of the  $B = \bar{B}^0, \bar{B}^{*0}, B^-, B^{*-}$ ), where it is the  $\bar{c}$  antiquark that decays, we have to change the mass of the final meson in the expressions above and take into account the changes in the form factors that derive from the discussions in Appendix C.

(iii) Case  $0^- \rightarrow 2^-, 2^+$ :

$$\begin{aligned}
\mathcal{H}_{tt}(P_{B_c}, P_{c\bar{c}}) &= \frac{\lambda^2(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{24m_{c\bar{c}}^4 q^2} (T_1(q^2) + (m_{B_c}^2 - m_{c\bar{c}}^2)T_2(q^2) + q^2 T_3(q^2))^2 \\
\mathcal{H}_{t0}(P_{B_c}, P_{c\bar{c}}) &= \mathcal{H}_{0t}(P_{B_c}, P_{c\bar{c}}) \\
&= \frac{\lambda^{3/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{24m_{c\bar{c}}^4 q^2} (T_1(q^2) + (m_{B_c}^2 - m_{c\bar{c}}^2)T_2(q^2) + q^2 T_3(q^2))((m_{B_c}^2 - q^2 - m_{c\bar{c}}^2)T_1(q^2) \\
&\quad + \lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)T_2(q^2)) \\
\mathcal{H}_{+1+1}(P_{B_c}, P_{c\bar{c}}) &= \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{8m_{c\bar{c}}^2} (T_1(q^2) - \lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)T_4(q^2))^2 \\
\mathcal{H}_{-1-1}(P_{B_c}, P_{c\bar{c}}) &= \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{8m_{c\bar{c}}^2} (T_1(q^2) + \lambda^{1/2}(q^2, m_{B_c}^2, m_{c\bar{c}}^2)T_4(q^2))^2 \\
\mathcal{H}_{00}(P_{B_c}, P_{c\bar{c}}) &= \frac{\lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)}{24m_{c\bar{c}}^4 q^2} ((m_{B_c}^2 - q^2 - m_{c\bar{c}}^2)T_1(q^2) + \lambda(q^2, m_{B_c}^2, m_{c\bar{c}}^2)T_2(q^2))^2.
\end{aligned} \tag{D3}$$

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- [1] F. Abe *et al.* (CDF Collaboration), Phys. Rev. D **58**, 112004 (1998); Phys. Rev. Lett. **81**, 2432 (1998).  
[2] E. Jenkins, M. Luke, A. V. Manohar, and M. J. Savage, Nucl. Phys. **B390**, 463 (1993).  
[3] C. Albertus, J. M. Flynn, E. Hernández, J. Nieves, and J. M. Verde-Velasco, Phys. Rev. D **72**, 033002 (2005).  
[4] M. A. Ivanov, J. G. Körner, and P. Santorelli, Phys. Rev. D **73**, 054024 (2006).  
[5] M. A. Ivanov, J. G. Körner, and P. Santorelli, Phys. Rev. D **71**, 094006 (2005); J. G. Körner (private communication).  
[6] M. A. Ivanov, J. G. Körner, and P. Santorelli, Phys. Rev. D **63**, 074010 (2001).  
[7] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D **68**, 094020 (2003).  
[8] D. Ebert, R. N. Faustov, and V. O. Galkin, Eur. Phys. J. C **32**, 29 (2003).  
[9] C. H. Chang and Y. Q. Chen, Phys. Rev. D **49**, 3399 (1994).  
[10] C. H. Chang, Y. Q. Chen, G. L. Wang, and H. S. Zong, Phys. Rev. D **65**, 014017 (2002).  
[11] C. H. Chang, Y. Q. Chen, G. L. Wang, and H. S. Zong, Commun. Theor. Phys. **35**, 395 (2001).  
[12] A. Abd El-Hady, J. H. Muñoz, and J. P. Vary, Phys. Rev. D **62**, 014019 (2000).  
[13] J.-F. Liu and K.-T. Chao, Phys. Rev. D **56**, 4133 (1997).  
[14] V. V. Kiselev, A. K. Likhoded, and A. I. Onishchenko, Nucl. Phys. **B569**, 473 (2000).  
[15] V. V. Kiselev, A. E. Kovalsky, and A. K. Likhoded, Nucl. Phys. **B585**, 353 (2000).  
[16] V. V. Kiselev, hep-ph/0211021.  
[17] V. V. Kiselev, O. N. Pakhomova, and V. A. Saleev, J. Phys. G **28**, 595 (2002).  
[18] P. Colangelo, and F. De Fazio, Phys. Rev. D **61**, 034012 (2000).  
[19] A. Y. Anisimov, P. Y. Kulikov, I. M. Narodetsky, and K. A. Ter-Martirosian, Yad. Fiz. **62**, 1868 (1999) [Phys. At. Nucl. **62**, 1739 (1999)].  
[20] M. A. Nobes and R. M. Woloshyn, J. Phys. G **26**, 1079 (2000).  
[21] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989); D. Scora and N. Isgur, Phys. Rev. D **52**, 2783 (1995).  
[22] M. A. Sanchis-Lozano, Nucl. Phys. **B440**, 251 (1995).  
[23] G. Lopez Castro, H. B. Mayorga, and J. H. Muñoz, J. Phys. G **28**, 2241 (2002).  
[24] G. Lu, Y. Yang, and H. Li, Phys. Lett. B **341**, 391 (1995).  
[25] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).  
[26] A. Abulencia *et al.* (CDF Collaboration), Phys. Rev. Lett. **96**, 082002 (2006).  
[27] A. Abulencia *et al.* (CDF Collaboration), Phys. Rev. Lett. **97**, 012002 (2006).  
[28] C. Caso *et al.* (Particle Data Group), Eur. Phys. J. C **3**, 1 (1998).  
[29] M. Artuso *et al.* (CLEO Collaboration), Phys. Rev. Lett. **95**, 251801 (2005).  
[30] D. Becirevic, Ph. Boucaud, J. P. Leroy, V. Lubicz, G. Martinelli, F. Mescia, and F. Rapuano, Phys. Rev. D **60**, 074501 (1999).  
[31] W.-M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).  
[32] C. Albertus, E. Hernández, J. Nieves, and J. M. Verde-Velasco, Phys. Rev. D **71**, 113006 (2005).  
[33] D. S. Hwang and G. H. Kim, Z. Phys. C **76**, 107 (1997).  
[34] R. K. Bhaduri, L. E. Cohler, and Y. Nogami, Nuovo Cimento A **65**, 376 (1981).  
[35] B. Silvestre-Brac, Few-Body Syst. **20**, 1 (1996).  
[36] B. Silvestre-Brac and C. Semay, Internal Report No. ISN

- 93.6999, 1993.
- [37] C. Albertus, J.E. Amaro, E. Hernández, and J. Nieves, Nucl. Phys. **A740**, 333 (2004).
- [38] C. Albertus, E. Hernández, and J. Nieves, Phys. Rev. D **71**, 014012 (2005).
- [39] C. Albertus, E. Hernández, J. Nieves, and J.M. Verde-Velasco, Phys. Rev. D **72**, 094022 (2005).
- [40] J.M. Verde-Velasco, “IVth International Conference on Quarks and Nuclear Physics (QNP06), Madrid, 2006” (to be published); C. Albertus, E. Hernández, J. Nieves, and J.M. Verde-Velasco (to be published).
- [41] J.G. Körner and G.A. Schuler, Z. Phys. C **46**, 93 (1990).
- [42] E. Jenkins and A.V. Manohar (private communication).
- [43] M. Neubert, Phys. Rev. D **46**, 2212 (1992).