

Origin of families of fermions and their mass matrices

A. Borštnik Bračič

Educational Faculty, University of Ljubljana, Kardeljeva ploščad 17, 1000 Ljubljana, Slovenia

N. S. Mankoč Borštnik

Department of Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia

(Received 14 June 2006; published 27 October 2006)

We are proposing a new way of describing families of quarks and leptons, using the approach unifying all the internal degrees of freedom, proposed by one of us [N. Mankoč Borštnik, Phys. Lett. B **292**, 25 (1992)]. [N. Mankoč-Borštnik, J. Math. Phys. (N.Y.) **34**, 3731 (1993)]. [N. Mankoč Borštnik, J. Math. Phys. (N.Y.) **36**, 1593 (1995)]. [N. S. Mankoč Borštnik, Mod. Phys. Lett. A **10**, 587 (1995)]. [N. S. Mankoč Borštnik and S. Fajfer, Nuovo Cimento Soc. Ital. Fis. B **112**, 1637 (1997)]. [A. Borštnik and N. S. Mankoč Borštnik, in *Proceedings to the International Workshop on “What Comes Beyond the Standard Model, Bled, Slovenia, 1998*, edited by N. Mankoč Borštnik, H. B. Nielsen, and C. Froggatt (DMFA, Založništvo, 1999), p. 52]. [N. S. Mankoč Borštnik and H. B. Nielsen, Phys. Rev. **62**, 04010 (2000)]. [N. S. Mankoč Borštnik, Int. J. Theor. Phys. **40**, 315 (2001), and references therein]. [A. Borštnik and N. S. Mankoč Borštnik, in *Proceedings to the International Workshop on “What Comes Beyond the Standard Model”, Bled 2000, 2001, 2002 Volume 2*, edited by N. Mankoč Borštnik, H. B. Nielsen, C. Froggatt, and D. Lukman (DMFA, Založništvo, 2002), p. 27 and the paper (unpublished)]. [A. Borštnik and N. S. Mankoč Borštnik, in *Proceedings to the Euroconference on Symmetries Beyond the Standard Model, Portorož, 2003* edited by N. Mankoč Borštnik, H. B. Nielsen, C. Froggatt, and D. Lukman (DMFA, Založništvo, 2003), pp. 27–51.]. Spinors, living in $d = 1 + 13$ -dimensional space, carry in this approach only the spin and interact with only the gravity through vielbeins and two kinds of the spin connection fields—the gauge fields of the Poincaré group (p^a, S^{ab}) and the second kind of the Clifford algebra objects (\tilde{S}^{ab}). All the quarks and the leptons of one family appear in one Weyl representation of a chosen handedness of the Lorentz group, if analyzed with respect to the standard model gauge groups, which are subgroups of the group $SO(1, 13)$: the right handed (with respect to $SO(1, 3)$) weak chargeless quarks and leptons and the left handed weak charged quarks and leptons (with the right handed neutrino included). A part of the starting Lagrange density of a Weyl spinor in $d = 1 + 13$ transforms right handed quarks and leptons into left handed quarks and leptons manifesting as the Yukawa couplings of the standard model. A kind of the Clifford algebra objects generates families of quarks and leptons and contributes to diagonal and off-diagonal Yukawa couplings. The approach predicts an even number of families, treating leptons and quarks equivalently (we do not study a possible appearance of Majorana fermions yet). In this paper we investigate within this approach the appearance of the Yukawa couplings within one family of quarks and leptons as well as among the families (without assuming any Higgs fields like in the standard model). We present the mass matrices for four families and investigate whether our way of generating families might explain the origin of families of quarks and leptons as well as their observed properties—the masses and the mixing matrices. Numerical results are presented in Ref. [M. Breskvar, D. Lukman, and N. S. Mankoč Borštnik, hep-ph/0606159].

DOI: [10.1103/PhysRevD.74.073013](https://doi.org/10.1103/PhysRevD.74.073013)

PACS numbers: 12.15.Ff, 04.50.+h, 12.10.–g

I. INTRODUCTION

The standard model of the electroweak and strong interactions (extended by the inclusion of the massive neutrinos) fits well all the existing experimental data. It assumes around 25 parameters and constraints, the origin of which is not yet understood. Questions like: Why has Nature chosen $SU(3) \times SU(2) \times U(1)$ to describe the charges of spinors and $SO(1, 3)$ to describe the spin of spinors? Why are the left handed spinors weak charged, while the right handed spinors are weak chargeless? Where do the Yukawa couplings (together with the weak scale and the families of quarks and leptons) come from? and many others, remain unanswered.

The advantage of the approach, unifying spins and charges [1–10], is that it might offer possible answers to the open questions of the standard electroweak model. We demonstrated in Refs. [6,8–10] that a left handed $SO(1, 13)$ Weyl spinor multiplet includes, if the representation is interpreted in terms of the subgroups $SO(1, 3)$, $SU(2)$, $SU(3)$ and the sum of the two $U(1)$'s, all the spinors of the standard model—that is the left handed $SU(2)$ doublets and the right handed $SU(2)$ singlets of (with the group $SU(3)$ charged) quarks and (chargeless) leptons. Right handed neutrinos—weak and hyper chargeless—are also included. In the gauge theory of gravity (in our case in $d = 1 + 13$ -dimensional space), the Poincaré group is gauged, leading to spin connections and vielbeins, which

determine the gravitational field [2,8,11]. There are vielbein fields and spin connection fields, which might manifest—after the appropriate compactification (or some other kind of making the rest of $d - 4$ space unobservable at low energies)—in the four-dimensional space-time as all the gauge fields of the known charges, as well as the Yukawa couplings within each family. No additional Higgs field is needed to generate masses of families and to “dress” the right handed spinors with the weak charge. It is a part of the starting Lagrangian in $d \geq 1 + 13$, which manifests in $d = 1 + 3$ as the Yukawa coupling and does what the Higgs does in the standard model. If assuming a second kind of the Clifford algebra objects, the corresponding gauge fields manifest as the Yukawa couplings among families (contributing also to Yukawa couplings within each family).

In the Refs. [9,10,12–15] it was shown, that the approach unifying spins and charges might explain the Yukawa couplings if an appropriate break of both symmetries, connected with the two kinds of the Clifford algebra objects, appears. An even number of families is predicted, in particular, the fourth family of quarks and leptons might appear under certain conditions at low energies in agreement with [16,17].

The approach seems to have, like all the Kaluza-Klein-like theories, a very serious disadvantage, namely, that there might not exist any massless, mass protected spinors, which are, after the break of symmetries, chirally coupled to the desired (Kaluza-Klein) gauge fields [18]. This would mean that there are no observable spinors at low energies. Since the idea that it is only one internal degree of freedom—the spin—and the Kaluza-Klein idea that the gravity is the only gauge field, are beautiful and attractive, we have tried hard to find any example, which would give hope to Kaluza-Klein-like theories by demonstrating that a kind of a break of symmetries leads to massless, mass protected spinors, chirally coupled to the Kaluza-Klein gauge fields, observable at low energies. We discuss in Ref. [19] such a case—a toy model of a spinor, living in $d(= 1 + 5)$ -dimensional space, which breaks into a finite disk with the boundary, which allows spinors of only one handedness. Although not yet realistic, the toy model looks promising.

In the present paper we analyze how families of quarks and leptons, and accordingly also the Yukawa couplings, appear within the approach unifying spins and charges. We comment on the type of contributions to the Yukawa couplings and discuss some general properties of the mass matrices, which follow from the assumptions of the approach, trying to find out whether the approach could show a possible answer to the questions: What is the origin of the families of quarks and leptons? What does determine the Yukawa couplings? Why only the left handed quarks and leptons carry the weak charge?

Since we do not know which way of breaking the starting symmetries of the approach is the appropriate one, and

since results of the investigation drastically depend on the way of breaking symmetries and might as well depend on nonadiabatic processes following the break of symmetries, this paper (and also [23] which represents some numerical investigations) can only be understood as an attempt to see whether the approach unifying spins and charges has a chance to explain the origin of families of quarks and leptons and their properties and to which extend might it help to understand the appearance of families and the Yukawa couplings.

We are not (yet) performing the calculations of breaking the symmetry $SO(1, 13)$ to $SO(1, 7) \times U(1) \times SU(3)$ within our approach. (Some very rough estimates can be found in Ref. [20].) The break of symmetries influences both kinds of gauge fields, although we can not yet tell indeed in which way. Therefore, we can not tell the strength of the fields which appear in the Yukawa couplings as “the vacuum expectation values” and which lead further to $SO(1, 3) \times U(1) \times SU(3)$. We only can evaluate (after making some assumptions) several relations among the spin connection fields. Using then these very preliminary relations and the known experimental data, we can make a prediction for the number of families at physical energies and discuss properties of quarks and leptons within this approach: their masses and mixing matrices. Accordingly the results can be taken only as a first step in analyzing properties of families of quarks and leptons within *the approach unifying spins and charges, which might offer a mechanism for generating families and correspondingly the Yukawa couplings*. We shall present some numerical results in Ref. [23].

In Sec. II of this paper we present the action for a Weyl spinor in $(1 + 13)$ -dimensional space within our approach and suggest a break of the symmetry $SO(1, 13)$ to $SO(1, 7) \times SO(6)$ and further. We assume that the break of $SO(1, 13)$ to $SO(1, 7) \times U(1) \times SU(3)$ does lead to massless Weyl spinors with the $U(1) \times SU(3)$ charges. The main point of this paper is to demonstrate that while one Weyl spinor representation of $SO(1, 13)$, if analyzed with respect to subgroups $SO(1, 3) \times SU(2) \times U(1) \times U(1) \times SU(3)$, contains all the spinors needed in the standard model (the right handed weak chargeless quarks and leptons and left handed weak charged quarks and leptons), *the starting action for a Weyl spinor, which carries only (two kinds of) the spin and no charges and interacts with only the gravitational field, includes the Yukawa couplings*, which transform the right handed weak chargeless spinors into left handed weak charged spinors and contribute to the mass terms just as it is suggested by the standard model, without assuming the existence of a Higgs weak charged doublet. There are, namely, the generators of the Lorentz transformations within the group $SO(1, 7)$ in our model (S^{0s} , $s = 7, 8$, for example), which take care of what in the standard model the Higgs doublet, together with γ^0 , does.

In Subsection II A we comment on a possible break of the starting symmetry $SO(1, 13)$ the internal symmetry of

which is connected by S^{ab} , while in Subsection IIB of Sec. II we discuss properties of the group $SO(1, 13)$ in terms of subgroups, which appear in the standard electro-weak model. In the same section, Subsection IIC, we present briefly the technique [12,13], which turns out to be very helpful when discussing spinor representations, since it allows to generate as well as present spinor representations and families of spinor representations in a very transparent way. In particular, the technique helps to point out very clearly how do the Yukawa couplings appear in our approach. In Subsection IID we comment on the appearance of families within our technique [2,13].

In Sec. III we discuss in details within our approach the appearance of the Yukawa couplings within one family, while in Sec. IV we discuss the number of families as well as the Yukawa couplings among the families.

In Sec. V we present, after making several assumptions and simplifications, a possible explicit expression for the mass matrices for four families of quarks and leptons in terms of the spin connection fields.

II. WEYL SPINORS IN $d = (1 + 13)$ MANIFESTING FAMILIES OF QUARKS AND LEPTONS IN $d = (1 + 3)$

We start with a left handed Weyl spinor in $(1 + 13)$ -dimensional space. A spinor carries no charges, only two

kinds of spins and interacts accordingly with only gauge gravitational fields—with spin connections and vielbeins. We assume two kinds of the Clifford algebra objects defining two kinds of the generators of the Lorentz algebra and allow accordingly two kinds of gauge fields [1–10]. One kind is the ordinary gauge field (gauging the Poincaré symmetry in $d = 1 + 13$). The corresponding spin connection field appears for spinors as a gauge field of $S^{ab} = \frac{i}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a)$, where γ^a are the ordinary Dirac operators. The contribution of these fields to the mass matrices manifests in only the diagonal terms (connecting right handed weak chargeless quarks or leptons with left handed weak charged partners within one family of spinors).

The second kind of gauge fields is in our approach responsible for the appearance of families and consequently for the Yukawa couplings among families of spinors (contributing also to diagonal matrix elements) and will be used in this paper to explain the origin of the families of quarks and leptons. The corresponding spin connection fields appear for spinors as a gauge field of \tilde{S}^{ab} ($\tilde{S}^{ab} = \frac{i}{4} \times (\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$) with $\tilde{\gamma}^a$, which are the Clifford algebra objects [2,13], like γ^a , but anticommute with γ^a .

Accordingly we write the action for a Weyl (massless) spinor in $d(= 1 + 13)$ —dimensional space as follows [21]

$$S = \int d^d x \mathcal{L}, \quad \mathcal{L} = \frac{1}{2}(E \bar{\psi} \gamma^a p_{0a} \psi) + \text{H.c.} = \frac{1}{2}(E \bar{\psi} \gamma^a f_a^{\alpha} p_{0\alpha} \psi) + \text{H.c.}, \quad p_{0\alpha} = p_\alpha - \frac{1}{2} S^{ab} \omega_{ab\alpha} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\alpha}. \quad (1)$$

Here f_a^α are vielbeins (inverse to the gauge field of the generators of translations e^a_α , $e^a_\alpha f^\alpha_b = \delta_b^a$, $e^a_\alpha f^\beta_a = \delta_\alpha^\beta$), with $E = \det(e^a_\alpha)$, while $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$ are the two kinds of the spin connection fields, the gauge fields of S^{ab} and \tilde{S}^{ab} , respectively, corresponding to the two kinds of the Clifford algebra objects [10,12,13], namely γ^a and $\tilde{\gamma}^a$, with the property $\{\gamma^a, \tilde{\gamma}^b\}_+ = 0$, which leads to $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$. We shall discuss the properties of these two kinds of γ^a 's in Subsections IIC and IID. We shall neglect the gravity in $d = 1 + 3$. We shall from now on use the notation $\omega_{abc} = f^\alpha_c \omega_{ab\alpha}$ and $\tilde{\omega}_{abc} = f^\alpha_c \tilde{\omega}_{ab\alpha}$.

To see that one Weyl spinor in $d = (1 + 13)$, with the spin as the only internal degree of freedom, can manifest in four-dimensional physical space as the ordinary ($SO(1, 3)$) spinor with all the known charges of one family of quarks and leptons of the standard model, one has to analyze one Weyl spinor (we make a choice of the left handed one) representation in terms of the subgroups $SO(1, 3) \times U(1) \times SU(2) \times SU(3)$. We shall do this in Subsection IIB of this section. (The reader can see this analyses in several references, like the one in [10].)

To see that the Yukawa couplings are the part of the starting Lagrangian of Eq. (1), we rewrite the Lagrangian

in Eq. (1) as follows [10]

$$\mathcal{L} = \bar{\psi} \gamma^m \left(p_m - \sum_{A,i} g^A \tau^{Ai} A_m^{Ai} \right) \psi + \sum_{s=7,8} \bar{\psi} \gamma^s p_{0s} \psi + \text{the rest.} \quad (2)$$

Index A determines the charge groups ($SU(3)$, $SU(2)$ and the two $U(1)$'s), index i determines the generators within one charge group. τ^{Ai} denote the generators of the charge groups (expressible [8–10] in terms of S^{st} , $s, t \in 5, 6, \dots, 14$), while A_m^{Ai} , $m = 0, 1, 2, 3$, denote the corresponding gauge fields of the standard model (expressible in terms of ω_{stm}). The first part of the Lagrangian in Eq. (1) is therefore the usual Lagrangian for a spinor in $d = 1 + 3$ in the interaction with all the known gauge fields.

The second term plays the role of a mass term, since $\gamma^s p_{0s}$ has no vector index in $d = 1 + 3$ and it transforms a right handed weak chargeless spinor (for example d_R or e_R) into a left handed weak charged spinor (in this case to d_L or e_L), without changing the spin in $d = 1 + 3$ (as we shall see in Subsection IIC and can be easily recognized if one looks in Table I at the first and the seventh row or in Table II at the third and the fifth row, noticing by using

TABLE I. The 8-plet of quarks—the members of $SO(1, 7)$ subgroup, belonging to one Weyl left handed ($\Gamma^{(1,13)} = -1 = \Gamma^{(1,7)} \times \Gamma^{(6)}$) spinor representation of $SO(1, 13)$. It contains the left handed weak charged quarks and the right handed weak chargeless quarks of a particular color ($(1/2, 1/(2\sqrt{3}))$). Here $\Gamma^{(1,3)}$ defines the handedness in $(1 + 3)$ space, S^{12} defines the ordinary spin (which can also be read directly from the basic vector), τ^{13} defines the weak charge, τ^{21} defines the $U(1)$ charge, τ^{33} and τ^{38} define the color charge and τ^{41} another $U(1)$ charge, which together with the first one defines Y and Y' . The reader can find the whole Weyl representation in the Ref. [10].

i	$ \psi_i\rangle$	$\Gamma^{(1,3)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{21}	τ^{33}	τ^{38}	τ^{41}	Y	Y'
	Octet, $\Gamma^{(1,7)} = 1, \Gamma^{(6)} = -1$, of quarks										
1	u_R^{c1} $(+i)(+) (+)(+) (+)(-)(-)$	1	1/2	1	0	1/2	1/2	$1/(2\sqrt{3})$	1/6	2/3	-1/3
2	u_R^{c1} $[-i][-] (+)(+) (+)(-)(-)$	1	-1/2	1	0	1/2	1/2	$1/(2\sqrt{3})$	1/6	2/3	-1/3
3	d_R^{c1} $(+i)(+) [-][-] (+)(-)(-)$	1	1/2	1	0	-1/2	1/2	$1/(2\sqrt{3})$	1/6	-1/3	2/3
4	d_R^{c1} $[-i][-] [-][-] (+)(-)(-)$	1	-1/2	1	0	-1/2	1/2	$1/(2\sqrt{3})$	1/6	-1/3	2/3
5	d_L^{c1} $[-i](+) [-](+) (+)(-)(-)$	-1	1/2	-1	-1/2	0	1/2	$1/(2\sqrt{3})$	1/6	1/6	1/6
6	d_L^{c1} $(+i)[-] [-](+) (+)(-)(-)$	-1	-1/2	-1	-1/2	0	1/2	$1/(2\sqrt{3})$	1/6	1/6	1/6
7	u_L^{c1} $[-i](+) (+)[-] (+)(-)(-)$	-1	1/2	-1	1/2	0	1/2	$1/(2\sqrt{3})$	1/6	1/6	1/6
8	u_L^{c1} $(+i)[-] (+)[-] (+)(-)(-)$	-1	-1/2	-1	1/2	0	1/2	$1/(2\sqrt{3})$	1/6	1/6	1/6

Eq. (12) that $\gamma^0\gamma^7$ or $\gamma^0\gamma^8$ transform the first row into the seventh of Table I or the third row into the fifth of Table II—just what the Yukawa couplings with the Higgs doublet included do in the standard model formulation.

The reader will find the detail explanation in Subsections IIC and IID. It should be pointed out that no Higgs weak charge doublet is needed here to dress the right handed weak chargeless spinor with a weak charge, as S^{0s} , $s = 7, 8$, accompanied by the p_{0s} does its job.

One can always rewrite the Lagrangian from Eq. (1) in the way of Eq. (2). The question is, of course, what are the

terms, which are in Eq. (2) written under “the rest” and whether they can be assumed as negligible at “low energy world”. We have no proof that any break of symmetry, presented in Subsection IIA, leads to such an effective Lagrangian, which would after the first break (or several successive breaks) of the starting symmetry of $SO(1, 13)$ manifest any massless spinors, which would then, after further breaks, manifest in the “physical space” the masses corresponding to the Yukawa couplings of Eq. (2), while all the rest terms are negligibly small. We shall comment on this problem in Sec. VI from the point of view of the toy model, on which we try to understand how massless spin-

TABLE II. The 8-plet of leptons—the members of $SO(1, 7)$ subgroup, belonging to one Weyl left handed ($\Gamma^{(1,13)} = -1 = \Gamma^{(1,7)} \times \Gamma^{(6)}$) spinor representation of $SO(1, 13)$, is presented. It contains the left handed weak charged leptons and the right handed weak chargeless leptons, all color chargeless. The two 8-plets in Table I and Table II are equivalent with respect to the groups $SO(1, 7)$. They only differ in properties with respect to the group $SU(3)$ and $U(1)$ and consequently in Y and Y' .

i	$ \psi_i\rangle$	$\Gamma^{(1,3)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{21}	τ^{33}	τ^{38}	τ^{41}	Y	Y'
	Octet, $\Gamma^{(1,7)} = 1, \Gamma^{(6)} = -1$, of leptons										
1	ν_R $(+i)(+) (+)(+) (+)[+][+]$	1	1/2	1	0	1/2	0	0	-1/2	0	-1
2	ν_R $[-i][-] (+)(+) (+)[+][+]$	1	-1/2	1	0	1/2	0	0	-1/2	0	-1
3	e_R $(+i)(+) [-][-] (+)[+][+]$	1	1/2	1	0	-1/2	0	0	-1/2	-1	0
4	e_R $[-i][-] [-][-] (+)[+][+]$	1	-1/2	1	0	-1/2	0	0	-1/2	-1	0
5	e_L $[-i](+) [-](+) (+)[+][+]$	-1	1/2	-1	-1/2	0	0	0	-1/2	-1/2	-1/2
6	e_L $(+i)[-] [-](+) (+)[+][+]$	-1	-1/2	-1	-1/2	0	0	0	-1/2	-1/2	-1/2
7	ν_L $[-i](+) (+)[-] (+)[+][+]$	-1	1/2	-1	1/2	0	0	0	-1/2	-1/2	-1/2
8	ν_L $(+i)[-] (+)[-] (+)[+][+]$	-1	-1/2	-1	1/2	0	0	0	-1/2	-1/2	-1/2

ors can appear after the break of symmetries [19]. Since we have done no detailed work on the break of $SO(1, 13)$ (we studied this phenomena in an approximate way in the Ref. [20]), we in this paper just assume instead, that we start with the Lagrangian of Eq. (2) and then study properties of the system, described by such a Lagrange density at physical energies.

There are \tilde{S}^{ab} , which transform one family into another, without changing any property of a Weyl representation, which concerns the Lorentz group, whose generators are S^{ab} . Since \tilde{S}^{ab} and S^{ab} commute ($\{\tilde{S}^{ab}, S^{ab}\}_- = 0$), most of properties of quarks and leptons must be the same within this approach. There are, namely, only the generators of families, which define off-diagonal elements of the Yukawa couplings. But they do not at all distinguish among quarks and leptons. Since also in the diagonal matrix elements differ quarks and leptons in only one parameter times the identity, the question arises: What is then the reason for so different mixing matrices of quarks and leptons as observed? Might it be that there are the non-perturbative effects (like in the hadron case when quarks “dress nonadiabatically” into the clouds of quarks and antiquarks and the gluon field before forming a hadron) which are responsible for so different properties of quarks and leptons? Could instead be that very peculiar breaks of symmetries cause the difference in off-diagonal matrix elements for quarks and leptons? Or one must take the appearance of the Majorana fermions into account? The approach by itself gives different off-diagonal elements of mass matrices for u -quarks and d -quarks, and for ν and electrons (although it still relates them). We shall discuss this point later in this paper, as well as in Ref. [23].

A. Break of symmetries

There are several ways of breaking the group $SO(1, 13)$ down to subgroups of the standard model. One of the most probable breaks, suggested by the approach unifying spins and charges, is the following:

$$\begin{array}{c} SO(1, 13) \\ \downarrow \\ \underbrace{SO(1, 7) \otimes SU(3) \otimes U(1)} \\ \downarrow \\ SO(1, 3) \otimes U(1) \otimes SU(3) \end{array}$$

We start from a massless left handed Weyl spinor in $d = 1 + 13$. We assume that the first break of symmetries leads again to massless spinors in $d = 1 + 7$, chirally coupled with the $SU(3)$ and $U(1)$ charge to the corresponding fields, which follow from the spin connection and vielbein fields in $d = 1 + 13$. (The reader can find more about this kind of breaking the starting symmetry in Ref. [20].) We have no justification for such an assumption (except that we have shown on one toy model [19] that in that very special case such an assumption is justified). And we have

no calculation, which would help to guess the strength of the “vacuum expectation values” of the fields. The Yukawa-like terms themselves then break further the symmetry, ending up with the physical degrees of freedom.

B. Spin and charges of one left handed Weyl representation of $SO(1, 13)$

We discuss in this subsection the properties of one Weyl spinor representation when analyzing the representation in terms of subgroups of the group $SO(1, 13)$.

The group $SO(1, 13)$ of the rank 7 has as possible subgroups the groups $SO(1, 3)$ (the “complexified” $SU(2) \times SU(2)$), $SU(2)$, $SU(3)$ and the two $U(1)$ ’s, with the sum of the ranks of all these subgroups equal to 7. These subgroups are candidates for describing the spin, the weak charge, the color charge and the two hyper charges, respectively, (only one is needed in the standard model, two in $SO(10)$ models). The generators of these groups can be written in terms of the generators S^{ab} as follows

$$\tau^{Ai} = \sum_{a,b} c_{ab}^{Ai} S^{ab}, \quad \{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Aijk} \tau^{Ak}. \quad (3)$$

We could count the two $SU(2)$ subgroups of the group $SO(1, 3)$ in the same way as the rest of subgroups. Instead we shall use $A = 1, 2, 3, 4$, to represent only the subgroups describing charges and f^{Aijk} to describe the corresponding structure constants. Coefficients c_{ab}^{Ai} , with $a, b \in \{5, 6, \dots, 14\}$, have to be determined so that the commutation relations of Eq. (3) hold [5].

The weak charge ($SU(2)$ with the generators τ^{1i}) and one $U(1)$ charge (with the generator τ^{21}) content of the compact group $SO(4)$ (a subgroup of $SO(1, 13)$) can be demonstrated when expressing

$$\begin{aligned} \tau^{11} &:= \frac{1}{2}(S^{58} - S^{67}), & \tau^{12} &:= \frac{1}{2}(S^{57} + S^{68}), \\ \tau^{13} &:= \frac{1}{2}(S^{56} - S^{78}), & \tau^{21} &:= \frac{1}{2}(S^{56} + S^{78}). \end{aligned} \quad (4)$$

To see the color charge and one additional $U(1)$ content in the group $SO(1, 13)$ we write τ^{3i} and τ^{41} , respectively, in terms of the generators S^{ab}

$$\begin{aligned} \tau^{31} &:= \frac{1}{2}(S^{912} - S^{1011}), & \tau^{32} &:= \frac{1}{2}(S^{911} + S^{1012}), \\ \tau^{33} &:= \frac{1}{2}(S^{910} - S^{1112}), & \tau^{34} &:= \frac{1}{2}(S^{914} - S^{1013}), \\ \tau^{35} &:= \frac{1}{2}(S^{913} + S^{1014}), & \tau^{36} &:= \frac{1}{2}(S^{1114} - S^{1213}), \\ & & \tau^{37} &:= \frac{1}{2}(S^{1113} + S^{1214}), \\ \tau^{38} &:= \frac{1}{2\sqrt{3}}(S^{910} + S^{1112} - 2S^{1314}), \\ \tau^{41} &:= -\frac{1}{3}(S^{910} + S^{1112} + S^{1314}). \end{aligned} \quad (5)$$

To reproduce the standard model groups one must intro-

duce the two superpositions of the two $U(1)$'s generators as follows

$$Y = \tau^{41} + \tau^{21}, \quad Y' = \tau^{41} - \tau^{21}. \quad (6)$$

The above choice of subgroups of the group $SO(1, 13)$ manifests the standard model charge structure of one Weyl spinor of the group $SO(1, 13)$, with one additional hyper charge.

We may very similarly proceed also with the generators \tilde{S}^{ab} by assuming that a kind of a break makes the starting $SO(1, 13)$ group to manifest in terms of some $\tilde{\tau}^{\tilde{A}i}$ like

$$\tilde{\tau}^{\tilde{A}i} = \sum_{a,b} \tilde{c}^{\tilde{A}i}_{ab} \tilde{S}^{ab}, \quad \{\tilde{\tau}^{\tilde{A}i}, \tilde{\tau}^{\tilde{B}j}\}_- = i\delta^{\tilde{A}\tilde{B}} \tilde{f}^{\tilde{A}ijk} \tilde{\tau}^{\tilde{A}k}. \quad (7)$$

We shall try to guess the way of breaking through the comparison of the results with the experimental data in [16,17,23].

C. Spinor representation in terms of Clifford algebra objects

In this subsection we briefly present our technique [12] for generating spinor representations in any dimensional space. The advantage of this technique is simplicity in using it and transparency in understanding detailed prop-

erties of spinor representations. We also show how families of spinors enter into our approach [2,13].

We start by defining two kinds of the Clifford algebra objects, γ^a and $\tilde{\gamma}^a$, with the properties

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+, \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0. \quad (8)$$

The operators $\tilde{\gamma}^a$ are introduced formally as operating on any Clifford algebra object B from the left hand side, but they also can be expressed in terms of the ordinary γ^a as operating from the right hand side as follows

$$\tilde{\gamma}^a B := i(-)^{n_B} B \gamma^a, \quad (9)$$

with $(-)^{n_B} = +1$ or -1 , when the object B has a Clifford even or odd character, respectively.

Accordingly two kinds of generators of the Lorentz transformations follow, namely $S^{ab} := (i/4) \times (\gamma^a \gamma^b - \gamma^b \gamma^a)$ and $\tilde{S}^{ab} := (i/4)(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$, with the property $\{S^{ab}, \tilde{S}^{cd}\}_- = 0$.

We define a basis of spinor representations as eigenstates of the chosen Cartan subalgebra of the Lorentz algebra $SO(1, 13)$, with the operators S^{ab} and \tilde{S}^{ab} in the two Cartan subalgebra sets, with the same indices in both cases.

By introducing the notation

$$\begin{aligned} (\pm i) &:= \frac{1}{2}(\gamma^a \mp \gamma^b), & [\pm i] &:= \frac{1}{2}(1 \pm \gamma^a \gamma^b), & \text{for } \eta^{aa} \eta^{bb} &= -1, \\ (\pm) &:= \frac{1}{2}(\gamma^a \pm i\gamma^b), & [\pm] &:= \frac{1}{2}(1 \pm i\gamma^a \gamma^b), & \text{for } \eta^{aa} \eta^{bb} &= 1, \end{aligned} \quad (10)$$

it can be shown that

$$\begin{aligned} S^{ab}(k) &= \frac{k}{2}(k), & S^{ab}[k] &= \frac{k}{2}[k], \\ \tilde{S}^{ab}(k) &= \frac{k}{2}(k), & \tilde{S}^{ab}[k] &= -\frac{k}{2}[k]. \end{aligned} \quad (11)$$

The above binomials are all eigenvectors of the generators S^{ab} , as well as of \tilde{S}^{ab} .

We further find

$$\begin{aligned} \gamma^a(k) &= \eta^{aa}[-k], & \gamma^b(k) &= -ik[-k], \\ \gamma^a[k] &= (-k), & \gamma^b[k] &= -ik\eta^{aa}(-k) \end{aligned} \quad (12)$$

and

$$\begin{aligned} \tilde{\gamma}^a(k) &= -i\eta^{aa}[k], & \tilde{\gamma}^b(k) &= -k[k], \\ \tilde{\gamma}^a[k] &= i(k), & \tilde{\gamma}^b[k] &= -k\eta^{aa}(k). \end{aligned} \quad (13)$$

Using the following useful relations

$$(\overset{ab}{k})^\dagger = \eta^{aa}(\overset{ab}{-k}), \quad [\overset{ab}{k}]^\dagger = [\overset{ab}{k}], \quad (14)$$

we may define

$$\langle \overset{ab}{k} | \overset{ab}{k} \rangle = 1 = \langle [\overset{ab}{k}] | [\overset{ab}{k}] \rangle. \quad (15)$$

We shall later make use of the relations

$$\begin{aligned} (\overset{ab}{k})(\overset{ab}{k}) &= 0, & (\overset{ab}{k})(\overset{ab}{-k}) &= \eta^{aa}[\overset{ab}{k}], & [\overset{ab}{k}][\overset{ab}{k}] &= [\overset{ab}{k}], \\ [\overset{ab}{k}][\overset{ab}{-k}] &= 0, & (\overset{ab}{k})[\overset{ab}{k}] &= 0, & \overset{ab}{k} &= (k), \\ (\overset{ab}{k})[\overset{ab}{-k}] &= (k), & [\overset{ab}{k}](\overset{ab}{-k}) &= 0, \end{aligned} \quad (16)$$

as well as the relations, following from Eqs. (13) and (16),

$$\begin{aligned} (\overset{ab}{k})(\overset{ab}{k}) &= 0, & (\overset{ab}{-k})(\overset{ab}{k}) &= -i\eta^{aa}[\overset{ab}{k}], \\ (\overset{ab}{-k})[\overset{ab}{-k}] &= i(-k), & (\overset{ab}{k})[\overset{ab}{-k}] &= 0, \\ (\overset{ab}{k})[\overset{ab}{k}] &= i(k), & (\overset{ab}{-k})[\overset{ab}{k}] &= 0, & (\overset{ab}{-k})(\overset{ab}{-k}) &= 0, \\ (\overset{ab}{k})(\overset{ab}{-k}) &= -i\eta^{aa}[\overset{ab}{-k}]. \end{aligned} \quad (17)$$

Here

$$\begin{aligned}
 \binom{ab}{\pm i} &= \frac{1}{2}(\tilde{\gamma}^a \mp \tilde{\gamma}^b), & \binom{ab}{\pm} &= \frac{1}{2}(\tilde{\gamma}^a \pm i\tilde{\gamma}^b), \\
 \binom{ab}{\pm i} &= \frac{1}{2}(1 \pm \tilde{\gamma}^a \tilde{\gamma}^b), & \binom{ab}{\pm} &= \frac{1}{2}(1 \pm i\tilde{\gamma}^a \tilde{\gamma}^b).
 \end{aligned} \tag{18}$$

The reader should notice that γ^a 's transform the binomial $\binom{ab}{k}$ into the binomial $\binom{ab}{-k}$, whose eigenvalue with respect to S^{ab} changes sign, while $\tilde{\gamma}^a$'s transform the binomial $\binom{ab}{k}$ into $\binom{ab}{k}$ with unchanged eigenvalue with respect to S^{ab} . We define the operators of handedness of the group $SO(1, 13)$ and of the subgroups $SO(1, 3)$, $SO(1, 7)$, $SO(6)$ and $SO(4)$ as follows

$$\begin{aligned}
 \Gamma^{(1,13)} &= i2^7 S^{03} S^{12} S^{56} \dots S^{13\ 14}, \\
 \Gamma^{(1,3)} &= -i2^2 S^{03} S^{12}, \\
 \Gamma^{(1,7)} &= -i2^4 S^{03} S^{12} S^{56} S^{78}, \\
 \Gamma^{(1,9)} &= i2^5 S^{03} S^{12} S^9\ 10\ S^{11\ 12} S^{13\ 14}, \\
 \Gamma^{(6)} &= -2^3 S^9\ 10\ S^{11\ 12} S^{13\ 14}, \\
 \Gamma^{(4)} &= 2^2 S^{56} S^{78}.
 \end{aligned} \tag{19}$$

We shall represent one Weyl left handed spinor as products of binomials $\binom{ab}{k}$ or $\binom{ab}{[k]}$, which are eigenvectors of the members of the Cartan subalgebra set. We make the following choice of the Cartan subalgebra set of the algebra S^{ab} and \tilde{S}^{ab}

$$\begin{aligned}
 S^{03}, S^{12}, S^{56}, S^{78}, S^9\ 10, S^{11\ 12}, S^{13\ 14}, \\
 \tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^9\ 10, \tilde{S}^{11\ 12}, \tilde{S}^{13\ 14}.
 \end{aligned} \tag{20}$$

We are now prepared to make a choice of a starting basic vector of one Weyl representation of the group $SO(1, 13)$, which is the eigenstate of all the members of the Cartan subalgebra (Eq. (20)) and is left handed ($\Gamma^{(1,13)} = -1$)

$$\begin{aligned}
 & {}^{03\ 12\ 56\ 78\ 9\ 10\ 11\ 12\ 13\ 14} \\
 & (+i)(+)|\psi\rangle \\
 & = \left(\frac{1}{2}\right)^7 (\gamma^0 - \gamma^3)(\gamma^1 + i\gamma^2)(\gamma^5 + i\gamma^6)(\gamma^7 + i\gamma^8) \\
 & \quad \times |(\gamma^9 + i\gamma^{10})(\gamma^{11} - i\gamma^{12})(\gamma^{13} - i\gamma^{14})|\psi\rangle.
 \end{aligned} \tag{21}$$

The signs “|” and “||” are to point out the $SO(1, 3)$ (up to |), $SO(1, 7)$ (up to ||) and $SO(6)$ (after ||) substructure of the starting basic vector of the left handed multiplet of $SO(1, 13)$, which has $2^{14/2-1} = 64$ vectors. Here $|\psi\rangle$ is any vector, which is not transformed to zero and therefore we shall not write down $|\psi\rangle$ any longer. One easily finds that the eigenvalues of the chosen Cartan subalgebra elements of S^{ab} and \tilde{S}^{ab} (Eq. (20)) are $(+i/2, 1/2, 1/2, 1/2, 1/2, -1/2, -1/2)$ and $(+i/2, 1/2, 1/2, 1/2, 1/2, -1/2, -1/2)$, respectively. This state has with respect to the operators S^{ab} the following properties: With respect to

the group $SO(1, 3)$ is a right handed spinor ($\Gamma^{(1,3)} = 1$) with spin up ($S^{12} = 1/2$), it is weak chargeless (it is an $SU(2)$ singlet— $\tau^{13} = 0$) and it carries a color charge (it is the member of the $SU(3)$ triplet with $(\tau^{33} = 1/2, \tau^{38} = 1/(2\sqrt{3}))$, it has $\tau^{21} = 1/2$ and $\tau^{41} = 1/6$ and correspondingly the two hyper charges equal to $Y = 2/3$ and $Y' = -1/3$, respectively. We further find according to Eq. (19) that $\Gamma^{(4)} = 1$ (the handedness of the group $SO(4)$, whose subgroups are $SU(2)$ and $U(1)$), $\Gamma^{(1,7)} = 1$ and $\Gamma^{(6)} = -1$. The starting vector (Eq. (21)) can be recognized in terms of the standard model subgroups as the right handed weak chargeless u -quark carrying one of the three colors.

To obtain all the basic vectors of one Weyl spinor, one only has to apply on the starting basic vector of Eq. (21) the generators S^{ab} . All the quarks and the leptons of one family of the standard model appear in this multiplet (together with the corresponding anti quarks and anti leptons). We present in Table I all the quarks of one particular color (the right handed weak chargeless u_R, d_R and the left handed weak charged u_L, d_L , with the color $(1/2, 1/(2\sqrt{3}))$ in the standard model notation). They all are members of one $SO(1, 7)$ multiplet.

We want the reader to recognize that if $\gamma^0\gamma^7$ or $\gamma^0\gamma^8$ or $\gamma^0\gamma^7$ or $\gamma^0\gamma^8$ or $\gamma^0(-)$ is applied on the first row in Table I one obtains (taking into account Eqs. (12) and (16)) the seventh row of the same table. That means that $\gamma^0\gamma^7$ or $\gamma^0\gamma^8$ or $\gamma^0(-)$ transforms a weak chargeless right handed u_R quark into a weak charged left handed u_L quark, without changing any other property of the starting u_R (that is the spin or the color charge). (They do exactly what in the standard model the Higgs is used to do, namely, when “dressing” the u_R with the weak charge, it enables a nonzero contribution to the mass term.)

In Table II we present the leptons of one family of the standard model. All the leptons belong to the same multiplet with respect to the group $SO(1, 7)$. They are color chargeless and differ accordingly from the quarks in Table I in the second $U(1)$ charge and in the color charge. The quarks and the leptons are equivalent with respect to the group $SO(1, 7)$.

Again one can notice, similarly as in the case of quarks, that $\gamma^0\gamma^7$ or $\gamma^0\gamma^8$ or $\gamma^0(+)$ transforms (see Eqs. (12) and (16)), for example, a right handed weak chargeless e_R lepton with the spin $\frac{1}{2}$ (the third row in Table II) into a left handed weak charged e_L lepton with the same spin (the fifth row in Table II).

D. Appearance of families

While the generators of the Lorentz group S^{ab} , with a pair of (ab) , which does not belong to the Cartan subalgebra (Eq. (20)), transform one vector of one Weyl representation into another vector of the same Weyl representation, transform the generators \tilde{S}^{ab} (again if the pair (ab) does not

belong to the Cartan set) a member of one family into the same member of another family, leaving all the other quantum numbers (determined by S^{ab}) unchanged [1,2,4,7,8,10]. This is happening since the application of

γ^a (from the left) changes the operator $\overset{ab}{+}$ (or the operator $\overset{ab}{+i}$) into the operator $\overset{ab}{-}$ (or the operator $\overset{ab}{-i}$, respectively), see Eqs. (12) and (13), while the operator $\tilde{\gamma}^a$ (which is understood, up to a factor $\pm i$, as the application of γ^a from the right-hand side) changes $\overset{ab}{+}$ (or $\overset{ab}{+i}$) into $\overset{ab}{+}$ (or into $\overset{ab}{+i}$, respectively), without changing the eigenvalues of the Cartan subalgebra set of the operators S^{ab} . Similarly γ^a transforms $\overset{ab}{\pm}$ into $\overset{ab}{\mp}$, while $\tilde{\gamma}^a$ transforms $\overset{ab}{\pm}$ into $\overset{ab}{\pm}$.

Bellow, as an example, the application of \tilde{S}^{01} on the state of Eq. (21) (up to a constant) is presented:

$$\begin{aligned} \tilde{S}^{ac}(k)(l) &= \frac{i}{2} \eta^{aa} \eta^{cc} \overset{ab}{k} \overset{cd}{l}, & \tilde{S}^{ad}(k)(l) &= \frac{1}{2} l \eta^{aa} \overset{ab}{k} \overset{cd}{l}, & \tilde{S}^{bc}(k)(l) &= \frac{1}{2} k \eta^{cc} \overset{ab}{k} \overset{cd}{l}, & \tilde{S}^{bd}(k)(l) &= -\frac{i}{2} k l \overset{ab}{k} \overset{cd}{l}, \\ \tilde{S}^{ac}[k][l] &= -\frac{i}{2} \overset{ab}{k} \overset{cd}{l}, & \tilde{S}^{ad}[k][l] &= \frac{1}{2} l \eta^{cc} \overset{ab}{k} \overset{cd}{l}, & \tilde{S}^{bc}[k][l] &= \frac{1}{2} k \eta^{aa} \overset{ab}{k} \overset{cd}{l}, & \tilde{S}^{bd}[k][l] &= \frac{i}{2} k l \eta^{aa} \eta^{cc} \overset{ab}{k} \overset{cd}{l}, \\ \tilde{S}^{ac}(k)[l] &= -\frac{i}{2} \eta^{aa} \overset{ab}{k} \overset{cd}{l}, & \tilde{S}^{ad}(k)[l] &= \frac{1}{2} l \eta^{aa} \eta^{cc} \overset{ab}{k} \overset{cd}{l}, & \tilde{S}^{bc}(k)[l] &= -\frac{1}{2} k \overset{ab}{k} \overset{cd}{l}, & & \\ \tilde{S}^{bd}(k)[l] &= -\frac{i}{2} k l \eta^{cc} \overset{ab}{k} \overset{cd}{l}, & \tilde{S}^{ac}[k](l) &= \frac{i}{2} \eta^{cc} \overset{ab}{k} \overset{cd}{l}, & \tilde{S}^{ad}[k](l) &= \frac{1}{2} l \overset{ab}{k} \overset{cd}{l}, & & \\ \tilde{S}^{bc}[k](l) &= -\frac{1}{2} k \eta^{aa} \eta^{cc} \overset{ab}{k} \overset{cd}{l}, & \tilde{S}^{bd}[k](l) &= \frac{i}{2} k l \eta^{aa} \overset{ab}{k} \overset{cd}{l}. & & & & \end{aligned} \quad (22)$$

III. MASS MATRICES IN THE APPROACH UNIFYING SPINS AND CHARGES—TERMS WITHIN EACH FAMILY

We are now prepared to look at the terms, which manifest as the Yukawa couplings in our approach unifying spins and charges. Let us at first neglect the terms $\tilde{S}^{ab} \tilde{\omega}_{abc}$ (Eqs. (1) and (2)) and let us see how does the ordinary Poincaré gauging gravity contribute to the Yukawa couplings. It can contribute to the matrix elements within families only.

Let us treat with the Lagrange density (Eq. (2)) one Weyl spinor of a particular handedness—say the left handed one ($\Gamma^{(1+13)} = -1$)—after the breaks of symmetries of $SO(1, 13)$ have already occurred, leaving massless coloured and colourless $SO(1, 7)$ multiplets. In Table I the quark multiplet of a particular $SU(3)$ and $U(1)$ charges is presented ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$ and $\tau^{41} = 1/6$), while in Table II the colorless lepton multiplet is presented ($\tau^{33} = 0$, $\tau^{38} = 0$ and $\tau^{41} = -1/2$).

The Lagrange density (Eq. (2)) manifests the part which demonstrates the gauge couplings of a spinor with the

$$\begin{aligned} & \overset{03}{+i} \overset{12}{+} \overset{56}{+} \overset{78}{+} \overset{9}{+} \overset{10}{+} \overset{11}{+} \overset{12}{+} \overset{13}{+} \overset{14}{-} \\ & \overset{03}{+i} \overset{12}{+} \overset{56}{+} \overset{78}{+} \overset{9}{+} \overset{10}{+} \overset{11}{+} \overset{12}{+} \overset{13}{-} \overset{14}{-}. \end{aligned} \quad (22)$$

One can easily see that both vectors of (22) describe a right handed u -quark of the same color. They are equivalent with respect to the operators S^{ab} . They only differ in properties, determined by the operators \tilde{S}^{ab} and accordingly also with respect to the Cartan subalgebra set ($\tilde{S}^{03}, \dots, \tilde{S}^{1314}$). The first row vector has the following eigenvalues of this Cartan subalgebra set \tilde{S}^{ab} ($i/2, 1/2, 1/2, 1/2, 1/2, -1/2, -1/2$), while the corresponding eigenvalues of the vector in the second row are ($-i/2, -1/2, 1/2, 1/2, 1/2, -1/2, -1/2$). Therefore, the operators \tilde{S}^{ab} can be used to generate families of quarks and leptons of the standard model.

We present here some useful relations, concerning families

color, the weak and the hyper charges gauge fields and it also manifests the mass term

$$- \mathcal{L}_Y = \psi^\dagger \gamma^0 \gamma^s p_{0s} \psi. \quad (24)$$

As pointed out in Subsect. II C the terms with $\gamma^0 \gamma^7$ or $\gamma^0 \gamma^8$ or $\gamma^0 \overset{78}{\mp}$, since

$$\gamma^7 = (\overset{78}{+} + \overset{78}{-}), \quad \gamma^8 = -i(\overset{78}{+} - \overset{78}{-}), \quad (25)$$

transform a right handed weak chargeless spinors with the spin $\pm 1/2$ (like it is the u_R quark from the first row in Table I or the e_R electron from the third row in Table II) into a left handed weak charged spinors with the spin $\pm 1/2$ (in our example into the u_L quark from the seventh row in Table I or the e_L electron from the fifth row in Table II). It is, namely, that $\gamma^0 \overset{78}{-}$ if applied on u_R (or ν_R) transforms it to u_L (or ν_L) ($\overset{78}{-} \overset{78}{+} = -[\overset{78}{-}]$, Eq. (16)), and $\gamma^0 \overset{78}{+}$ if applied on d_R (or e_R) transforms it to d_L (or e_L) ($\overset{78}{+} \overset{78}{-} = \overset{78}{+}$, Eq. (16)), while $\gamma^0 \overset{78}{+}$ gives zero if being applied on

u_R or ν_R ($(+)(+)^{78} = 0$, Eq. (16)) and so does $\gamma^{0(-)^{78}}$ ($(-)[-]^{78} = 0$, Eq. (16)), if applied on d_R or e_R . We may write

$$\begin{aligned} -\mathcal{L}_Y &= \psi^\dagger \gamma^0 \{ (+)^{78} p_{0+} + (-)^{78} p_{0-} \}, \\ p_{0\pm} &= p_{05} \mp i p_{06}, \end{aligned} \quad (26)$$

recognizing, according to Eq. (16) and our discussions just above, that *the term with the fields A_y^+ , $y = Y, Y'$, contributes only to the masses of the d -quarks and the electrons, while A_y^- , $y = Y, Y'$, contributes only to the masses of the u -quarks and the neutrinos.*

As one can rearrange the first term in the Lagrangian of Eq. (2), with $m \in \{0, 1, 2, 3\}$, to manifest the standard model structure

$$\begin{aligned} -\mathcal{L}_f &= \bar{\psi} \gamma^m \left\{ p_m - \frac{g}{2} (\tau^+ W_m^+ + \tau^- W_m^-) \right. \\ &\quad + \frac{g^2}{\sqrt{g^2 + g'^2}} Q' Z_m + \frac{g g'}{\sqrt{g^2 + g'^2}} Q A_m \\ &\quad \left. + \sum_i g^3 \tau^{3i} A_m^{3i} + g' A_m^{Y'} \right\} \psi, \end{aligned}$$

with

$$\begin{aligned} Q &= \tau^{13} + Y = S^{56} + \tau^{41}, \\ Q' &= \tau^{13} - \left(\frac{g'}{g}\right)^2 Y = \frac{1}{2} \left(1 - \left(\frac{g'}{g}\right)^2\right) S^{56} \\ &\quad - \frac{1}{2} \left(1 + \left(\frac{g'}{g}\right)^2\right) S^{78} - \left(\frac{g'}{g}\right)^2 \tau^{41}, \end{aligned} \quad (27)$$

(assuming that—due to an appropriate break of symmetries—the term $A_m^{Y'}$ is nonobservable at physical energies, not yet), so can one rearrange also the mass term of the Lagrange density of Eq. (24), so that the fields $A_s^{A_i}$, $s \in \{5, 6, 7, 8\}$, instead of $\omega_{\mu s}$, appear in \mathcal{L}_Y .

The charge Q is conserved, as seen in Eq. (27), if we assume that no terms with either γ^5 or γ^6 or τ^{3i} contribute to the mass term (except in the diagonal form S^{56}).

Since all the operators in Eq. (24) are to be applied on right handed spinors, which are weak chargeless objects (as seen from Table I and II), the part with $\sum_i \tau^{3i} A_s^{3i}$ contributes zero to mass matrices and we shall leave it out. We also expect, that at the “observable” energies the contribution of the components p_s of momenta, with $s = 5, 6, \dots$, are negligible. Accordingly we neglect also this term.

It then follows

$$\begin{aligned} -\mathcal{L}_Y &= -\psi^\dagger \gamma^0 \sum_{y=Y, Y'} \{ (+)^{78} y A_+^y + (-)^{78} y A_-^y \} \psi \\ &\quad + \text{terms with } \tilde{S}^{ab} \tilde{\omega}_{abc}, \end{aligned} \quad (28)$$

with $A_\pm^y = -(A_7^y \mp i A_8^y)$ and $y = Y, Y'$.

IV. MASS MATRICES IN THE APPROACH UNIFYING SPINS AND CHARGES—TERMS WITHIN AND AMONG FAMILIES

We have seen in Subsection II D that while the operators S^{ab} transform the members of one Weyl representation among themselves, the operators \tilde{S}^{ab} transform one member of a family into the same member of another family, changing nothing but the family index. Each spinor basic vector has accordingly two indices: one index tells to which family a spinor belongs, another index tells which member of a particular family a spinor represents.

There are two types of terms in the Lagrange density of Eq. (1), which contribute to the mass matrices. We have studied in the previous Sec. III only the terms, determined by the generators of the Lorentz group (S^{ab}) and the corresponding gauge fields. After making a few assumptions we ended up with quite a simple expression for the contribution to *the masses of quarks and leptons within one family.*

The assumption, that there are two kinds of gauge fields connected with two kinds of the generators of the Lorentz transformations, is new [2,7,8,10] and requires accordingly additional cautions, when using it. On the other hand, the idea of the existence of *two kinds of Clifford algebra objects leads to the concept of families* and is therefore too exciting not to be used to try to describe families of quarks and leptons and to see what can the approach say about families of quarks and leptons.

As already said, the two kinds of generators are in our approach accompanied by the two kinds of gauge fields, gauging S^{ab} and \tilde{S}^{ab} , respectively. We shall assume that breaks of the symmetries of the Poincaré group in $d = 1 + 13$ influence this additional spin connection field as well. Since we do not know, how these breaks could occur for any of these two kinds of degrees of freedom, the calculations which will follow can be understood only as an attempt to study these degrees of freedom, that is families of quarks and leptons, within the proposed approach and not yet as a reliable prediction of properties of families of this approach.

On the other hand, many a property of families, presented in this paper, not connected with the break of symmetries, can follow from any concept of construction of families, if operators for generating families commute with the generators of spins and charges and if the generators are accompanied by gauge fields. What our proposal might offer in addition to the general concept of families is the prediction of the number of families and possible relations among matrix elements of mass matrices due to connected effects followed by breaks of symmetries.

Our approach suggests an even number of families. Namely, the number of all the orthogonal basic states is in our approach for a particular d equal to 2^d . Since we start with one Weyl spinor and neither S^{ab} nor \tilde{S}^{ab} change the Clifford oddness or evenness of basic states, we stay with

2^{d-2} basic states. Each Weyl representation has $2^{d/2-1}$ members. For any d there are accordingly $2^{d/2-1}$ families. After the assumption that some spontaneous breaks of symmetries lead to massless spinors in $d = (1 + 7)$ -dimensional space, in which spinors carry the $SU(3)$ and $U(1)$ charges (the two $U(1)$'s, one from $SO(6)$ and another from $SO(1, 7)$, being connected), there are accordingly only $2^{8/2-1} = 8$ families at most, seen at low energies. The Yukawa-like terms further break symmetry, leading to $Q = S^{56} + \tau^{41}$ and $SO(1, 3)$, $SU(3)$ as conserved quantities and to massive spinors in the physical space with $d = (1 + 3)$.

Since the application of \tilde{S}^{ab} generates the equivalent representations with respect to the Lorentz group ($\{S^{ab}, \tilde{S}^{cd}\}_- = 0$ and accordingly $\tilde{S}^{ab}(k) = \frac{k}{2}(k)$, while $\tilde{S}^{ab}[\bar{k}] = \frac{k}{2}[\bar{k}]$, see Eqs. (11)), it follows that if we know properties with respect to \tilde{S}^{ab} for one of basic states of a Weyl spinor, we know them for the whole Weyl spinor—*up to the influence of the break of symmetries* and up to the fact that the contribution to the Yukawa couplings of the two kinds of generators are in our approach related.

To demonstrate properties of families we shall make use of the first state in Table I and II. The two tables differ only in the $SU(3)$ and $U(1)$ charges (both kinds originate in $SO(6)$) and these two charges do not concern the $SO(1, 7)$ part. Accordingly we shall tell only families, connected with $SO(1, 7)$.

$$\begin{array}{ll}
I. \begin{matrix} 03 & 12 & 56 & 78 \\ (+i)(+)|(+)(+)|| \dots \end{matrix} & II. \begin{matrix} 03 & 12 & 56 & 78 \\ [+i][+]|(+)(+)|| \dots \end{matrix} \\
III. \begin{matrix} 03 & 12 & 56 & 78 \\ (+i)(+)|[+][+]|| \dots \end{matrix} & IV. \begin{matrix} 03 & 12 & 56 & 78 \\ [+i][+]|[+][+]|| \dots \end{matrix} \\
V. \begin{matrix} 03 & 12 & 56 & 78 \\ [+i](+)|(+)[+]|| \dots \end{matrix} & VI. \begin{matrix} 03 & 12 & 56 & 78 \\ (+i)[+]|(+)[+]|| \dots \end{matrix} \\
VII. \begin{matrix} 03 & 12 & 56 & 78 \\ (+i)[+]|+|| \dots \end{matrix} & VIII. \begin{matrix} 03 & 12 & 56 & 78 \\ [+i](+)|+|| \dots \end{matrix}
\end{array} \quad (29)$$

The diagonal terms, to which $S^{\text{st}}\omega_{\text{st}'}$ contribute, depend on a state of the Weyl representation, as we have commented in Sec. III, and distinguish between quarks and leptons, as well as between the u and the d quarks and between the electrons and the neutrinos. The diagonal and the off-diagonal elements, to which $\tilde{S}^{ab}\tilde{\omega}_{abs}$ contribute, distinguish between the u and the d quarks and between the electrons and the neutrinos, due to the factor (\pm) (determined by the ordinary γ^a operators), but do not distinguish between the quarks and the leptons. However, the way of breaking symmetries might influence the commutation relations among both kinds of generators. (We could, in some rough estimation, take these effects into account for instance just by putting an additional index to gauge fields $\tilde{\omega}_{abc}$. We shall discuss such possibilities in the paper which

will present the numerical results.) The diagonalization of the u (ν) mass matrix leads accordingly to a different transformation matrix than the diagonalization of d (e) mass matrix. The mixing matrix for quarks is correspondingly not the unit matrix (as expected, if it should agree with the experimental data) but might not differ from the mixing matrix for leptons.

Let us now write down the whole expression for the Yukawa couplings, with $\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}$ included

$$-\mathcal{L}_Y = \psi^\dagger \gamma^0 \{ \begin{matrix} 78 \\ (+) \end{matrix} p_{0+} + \begin{matrix} 78 \\ (-) \end{matrix} p_{0-} \} \psi,$$

$$\text{with } p_{0\pm} = (p_7 \mp ip_8) - \frac{1}{2}S^{ab}\omega_{ab\pm} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\pm}, \quad (30)$$

$$\omega_{ab\pm} = \omega_{ab7} \mp i\omega_{ab8}, \quad \tilde{\omega}_{ab\pm} = \tilde{\omega}_{ab7} \mp i\tilde{\omega}_{ab8}.$$

We shall rewrite diagonal matrix elements, to which \tilde{S}^{ab} contribute, in a similar way as we did in the previous sections for the contribution of S^{ab} . We therefore introduce the appropriate superposition of the operators \tilde{S}^{ab}

$$\begin{aligned}
\tilde{N}_3^\pm &:= \frac{1}{2}(\tilde{S}^{12} \pm i\tilde{S}^{03}), \\
\tilde{\tau}^{13} &:= \frac{1}{2}(\tilde{S}^{56} - \tilde{S}^{78}), \\
\tilde{Y} &= \tilde{\tau}^{41} + \tilde{\tau}^{21}, \\
\tilde{Y}' &= \tilde{\tau}^{41} - \tilde{\tau}^{21}, \\
\tilde{\tau}^{21} &:= \frac{1}{2}(\tilde{S}^{56} + \tilde{S}^{78}), \\
\tilde{\tau}^{41} &:= -\frac{1}{3}(\tilde{S}^{910} + \tilde{S}^{1112} + \tilde{S}^{1314}).
\end{aligned} \quad (31)$$

We allow also terms with \tilde{S}^{mn} , $m, n = 0, 1, 2, 3$, which in diagonal matrix elements of a mass matrix appear as \tilde{N}_3^\pm . Taking into account that

$$\begin{aligned}
-\frac{1}{2}S^{\text{st}}\omega_{\text{st}\pm} &= YA_\pm^Y + Y'A_\pm^{Y'} + \tau^{13}A_\pm^{13}, \\
-\frac{1}{2}\tilde{S}^{\text{st}}\tilde{\omega}_{\text{st}\pm} &= \tilde{Y}\tilde{A}_\pm^{\tilde{Y}} + \tilde{Y}'\tilde{A}_\pm^{\tilde{Y}'} + \tilde{\tau}^{13}\tilde{A}_\pm^{13}, \\
-\frac{1}{2}\tilde{S}^{mn}\tilde{\omega}_{mn\pm} &= \tilde{N}^{+3}\tilde{A}_\pm^{+3} + \tilde{N}^{-3}\tilde{A}_\pm^{-3},
\end{aligned} \quad (32)$$

with the pairs $(m, n) = (0, 3), (1, 2); (s, t) = (5, 6), (7, 8)$ belonging to the Cartan sub algebra and $\Omega_\pm = \Omega_7 \mp i\Omega_8$, where Ω_7, Ω_8 stay for any of the above fields, we find

$$\begin{aligned}
A_\pm^{13} &= -(\omega_{56\pm} - \omega_{78\pm}), \\
A_\pm^Y &= -\frac{1}{2}(A_\pm^{41} + (\omega_{56\pm} + \omega_{78\pm})), \\
A_\pm^{Y'} &= -\frac{1}{2}(A_\pm^{41} - (\omega_{56\pm} + \omega_{78\pm})), \\
\tilde{A}_\pm^{13} &= -(\tilde{\omega}_{56\pm} - \tilde{\omega}_{78\pm}), \\
\tilde{A}_\pm^{\tilde{Y}} &= -\frac{1}{2}(\tilde{A}_\pm^{41} + (\tilde{\omega}_{56\pm} + \tilde{\omega}_{78\pm})), \\
\tilde{A}_\pm^{\tilde{Y}'} &= -\frac{1}{2}(\tilde{A}_\pm^{41} - (\tilde{\omega}_{56\pm} + \tilde{\omega}_{78\pm})), \\
\tilde{A}_\pm^{\tilde{N}_3^+} &= -(\tilde{\omega}_{12\pm} - i\tilde{\omega}_{03\pm}), \\
\tilde{A}_\pm^{\tilde{N}_3^-} &= -(\tilde{\omega}_{12\pm} + i\tilde{\omega}_{03\pm}),
\end{aligned} \quad (33)$$

where the fields A_\pm^y , $y = 13, 41, Y, Y'$, and $\tilde{A}_\pm^{\tilde{y}}$, $\tilde{y} = \tilde{N}_3^+, \tilde{N}_3^-, 13, 41, \tilde{Y}, \tilde{Y}'$, are uniquely expressible with the corre-

spending spin connection fields. Let us repeat that $\omega_{abc} = f^{\alpha}_c \omega_{ab\alpha}$ and $\tilde{\omega}_{abc} = f^{\alpha}_c \tilde{\omega}_{ab\alpha}$.

The operators, which contribute to nondiagonal terms in mass matrices, are superpositions of \tilde{S}^{ab} and can be written in terms of nilpotents

$$\begin{matrix} \overline{ab} & \overline{cd} \\ \overline{(k)} & \overline{(l)}, \end{matrix} \quad (34)$$

with indices (ab) and (cd) which belong to the Cartan sub algebra indices (Eq. (20)). We may write accordingly

$$\begin{aligned} & \sum_{(a,b)} -\frac{1}{2} \sum_{\pm}^{78} \tilde{S}^{ab} \tilde{\omega}_{ab\pm} \\ & = - \sum_{(ac),(bd),k,l}^{78} \begin{matrix} \overline{ac} & \overline{bd} \\ \pm & (k) \end{matrix} \overline{(l)} \tilde{A}_{\pm}^{kl}((ac), (bd)), \end{aligned} \quad (35)$$

where the pair (a, b) in the first sum runs over all the indices, which do not characterize the Cartan sub algebra, with $a, b = 0, \dots, 8$, while the two pairs (ac) and (bd) denote only the Cartan sub algebra pairs (for $SO(1, 7)$ we only have the pairs (03),(12); (03),(56) ; (03),(78); (12),(56); (12),(78); (56),(78)); k and l run over four possible values so that $k = \pm i$, if $(ac) = (03)$ and $k = \pm 1$ in all other cases, while $l = \pm 1$. The fields $\tilde{A}_{\pm}^{kl}((ac), (bd))$ can then be expressed by $\tilde{\omega}_{ab\pm}$ as follows

$$\begin{aligned} \tilde{A}_{\pm}^{++}((ab), (cd)) &= -\frac{i}{2} \left(\tilde{\omega}_{ac\pm} - \frac{i}{r} \tilde{\omega}_{bc\pm} - i \tilde{\omega}_{ad\pm} - \frac{1}{r} \tilde{\omega}_{bd\pm} \right), \\ \tilde{A}_{\pm}^{--}((ab), (cd)) &= -\frac{i}{2} \left(\tilde{\omega}_{ac\pm} + \frac{i}{r} \tilde{\omega}_{bc\pm} + i \tilde{\omega}_{ad\pm} - \frac{1}{r} \tilde{\omega}_{bd\pm} \right), \\ \tilde{A}_{\pm}^{-+}((ab), (cd)) &= -\frac{i}{2} \left(\tilde{\omega}_{ac\pm} + \frac{i}{r} \tilde{\omega}_{bc\pm} - i \tilde{\omega}_{ad\pm} + \frac{1}{r} \tilde{\omega}_{bd\pm} \right), \\ \tilde{A}_{\pm}^{+-}((ab), (cd)) &= -\frac{i}{2} \left(\tilde{\omega}_{ac\pm} - \frac{i}{r} \tilde{\omega}_{bc\pm} + i \tilde{\omega}_{ad\pm} + \frac{1}{r} \tilde{\omega}_{bd\pm} \right), \end{aligned} \quad (36)$$

with $r = i$, if $(ab) = (03)$ and $r = 1$ otherwise. We simplify the index kl in the exponent of fields $\tilde{A}_{\pm}^{kl}((ac), (bd))$ to \pm , omitting i .

Any break of symmetries in the \tilde{S}^{ab} sector would cause relations among the corresponding $\tilde{\omega}_{ab\pm}$. Namely, if $-\frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\pm} = \tilde{\tau}^{Ai} \tilde{A}_{\pm}^{Ai}$ is not just a unitary transformation of basic states, but means due to a break of symmetries that, let us say, a particular $\tilde{A}^{Ai} = 0$, then this can only happen, if $\tilde{\omega}_{ab\pm}$ are related.

The Lagrange density, representing the mass matrices of fermions (the Yukawa couplings in the standard model) (Eq. (30)), can be rewritten as follows

$$\begin{aligned} \mathcal{L}_Y &= \psi^+ \gamma^0 \left\{ (+) \left(\sum_{y=Y, Y'}^{78} y A_+^y + \sum_{\tilde{y}=\tilde{N}_3^+, \tilde{N}_3^-, \tilde{\tau}^{13}, \tilde{Y}, \tilde{Y}'} \tilde{y} \tilde{A}_+^{\tilde{y}} \right) \right. \\ &+ (-) \left(\sum_{y=Y, Y'}^{78} y A_-^y + \sum_{\tilde{y}=\tilde{N}_3^+, \tilde{N}_3^-, \tilde{\tau}^{13}, \tilde{Y}, \tilde{Y}'} \tilde{y} \tilde{A}_-^{\tilde{y}} \right) \\ &+ (+) \sum_{\{(ac)(bd)\}, k, l}^{78} \begin{matrix} \overline{ac} & \overline{bd} \\ (k) & (l) \end{matrix} \tilde{A}_+^{kl}((ac), (bd)) \\ &+ (-) \sum_{\{(ac)(bd)\}, k, l}^{78} \begin{matrix} \overline{ac} & \overline{bd} \\ (k) & (l) \end{matrix} \tilde{A}_-^{kl}((ac), (bd)) \left. \right\} \psi, \end{aligned} \quad (37)$$

with pairs $((ac), (bd))$, which run over all the members of the Cartan sub algebra, while $k = \pm i$, if $(ac) = (03)$, otherwise $k = \pm 1$ and $l = \pm 1$. The terms $\tilde{A}_{\pm}^{kl}((ac), (bd))$ are expressible in terms of $\tilde{\omega}_{ab\pm}$ as presented in Eq. (36), while any break of symmetries relates $\tilde{\omega}_{ab\pm}$ in a very particular way. We omitted the term with τ^{13} , as well as the terms p_{\pm} , since, as already explained in the previous section, the first one when being applied on the right handed spinors contributes zero, while for the second ones we assume that at low energies their contribution is negligible.

The mass matrix, which follows from these Lagrange density and depends strongly on all possible breaks of symmetries, is in general not Hermitian.

Let us now repeat the assumptions we have made up to now. They are either the starting assumptions of our approach unifying spins and charges, or we made them to be able to connect the starting Lagrange density at low energies with the observable phenomena.

a.i. We use the approach, unifying spins and charges, which assumes, that in $d = 1 + 13$ massless spinors carry two types of spins: the ordinary (in $d = 1 + 13$) one, which we describe by $S^{ab} = \frac{i}{4} \times (\gamma^a \gamma^b - \gamma^b \gamma^a)$ and the additional one, described by $\tilde{S}^{ab} = \frac{i}{4} (\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$. The two types of the Clifford algebra objects anti commute $\{\gamma^a, \tilde{\gamma}^b\}_+ = 0$. Spinors carry no charges in $d = 1 + 13$. The operators S^{ab} determine (after an appropriate break of symmetries) at low energies the ordinary spin in $d = 1 + 3$ and all the known charges, while \tilde{S}^{ab} generate families of spinors. Accordingly spinors interact with only the gravitational fields, the gauge fields of the Poincaré group (p_{α}, S^{ab}) , and the gauge fields of the operators \tilde{S}^{ab} ($p_{0a} = f_a^{\alpha} p_{\alpha} - \frac{1}{2} \times (S^{cd} \omega_{cda} + \tilde{S}^{cd} \tilde{\omega}_{cda})$)

a.ii. The break of symmetries of $SO(1, 13)$ into $SO(1, 7) \times SU(3) \times U(1)$ occurs in a way that only massless spinors in $d = 1 + 7$ with the charge $SU(3)$ and $U(1)$ survive, with the one $U(1)$ from $SO(1, 7)$ and the next $U(1)$ from $SO(6)$ aligned, while $S^{5a}, S^{6a}, a \neq 5, 6$, do not contribute to the Yukawa-

like terms, so that $Q = \tau^{41} + S^{56}$ is conserved in $d = 1 + 3$.

- a.iii. The break of symmetries influences both: the Poincaré symmetry and the symmetry described by \tilde{S}^{ab} , it might be that to some extend in a similar way. The study of both kinds of breaking symmetries stays as an open problem.
- a.iv. The terms which include p_s , $s = 5, \dots, 14$, do not contribute at low energies.

V. AN EXAMPLE OF MASS MATRICES FOR FOUR FAMILIES

Let us make, for simplicity, two further assumptions besides the four (a.i-a.iv.) ones, presented at the end of Sec. IV:

- b.i. There are no terms, which would in Eq. (29) trans-

form $(+)$ into $[+]$. This assumption (which could also be understood as a break of symmetry, which requires that terms of the type $\tilde{S}^{5a} \tilde{\omega}_{5ab}$ and $\tilde{S}^{6a} \tilde{\omega}_{6ab}$ are negligible and might be a part of requirement a.iii.) leaves us with only four families of quarks and leptons. (This assumption might be justified with a break of symmetry in the \tilde{S}^{ab} sector from $SO(1, 7)$ to $SO(1, 5) \times U(1)$, with all the contributions of the terms $\tilde{S}^{5a} \tilde{\omega}_{5ab}$ and $\tilde{S}^{6a} \tilde{\omega}_{6ab}$ equal to zero.)

- b.ii. The rough estimation will be done on “a tree level”.

Since we do not know either how does the break of symmetries occur or how does the break influence the strength of the fields ω_{abc} and $\tilde{\omega}_{abc}$, we can not really say, to which extend are the above assumptions justified. For none of them we have a justification. Also the non-perturbative effects could be very strong and the tree level might not mean a lot. But yet a simplified version can help us to understand to what conclusions might the proposed approach lead with respect to families of quarks and leptons and their properties.

Our approach (which predicts an even number of families) suggests that under the assumptions a. and b. there are the following four families of quarks and leptons

$$\begin{aligned}
 I. & \begin{matrix} 03 & 12 & 56 & 78 \\ (+i) & (+) & | & (+) & (+) \end{matrix} \parallel \dots \\
 II. & \begin{matrix} 03 & 12 & 56 & 78 \\ [+i] & [+] & | & (+) & (+) \end{matrix} \parallel \dots \\
 III. & \begin{matrix} 03 & 12 & 56 & 78 \\ [+i] & (+) & | & (+) & [+] \end{matrix} \parallel \dots \\
 IV. & \begin{matrix} 03 & 12 & 56 & 78 \\ (+i) & [+] & | & (+) & [+] \end{matrix} \parallel \dots
 \end{aligned} \tag{38}$$

We see from Table I and II that due to the properties of the nilpotents (\pm) (Eq. (16)), to the u quark (and to the ν lepton) mass matrix only the operator $(-)$ (accompanied by the fields A_-, \tilde{A}_-) contributes, while to the d quark (and to the e lepton) mass matrix only $(+)$ (accompanied by the

fields A_+, \tilde{A}_+) contributes. This means that *the off-diagonal matrix elements of the Yukawa couplings are different for u -quarks (ν) and for d -quarks (e)*, although still related, while the quarks have the same off-diagonal matrix elements as the corresponding leptons (unless some breaks of symmetries do not destroy this symmetry). Assuming that after the appropriate breaks of symmetries the fields contributing to the Yukawa couplings obtain some nonzero expectation values (which are in general related in a very particular way) and integrating the Lagrange density \mathcal{L}_Y over the coordinates and the internal (spin) degrees of freedom, we end up with the mass matrices for four families of quarks and leptons (Eq. (38)), whose structure is presented in Table III.

The explicit forms of the diagonal matrix elements for the above choice of assumptions in terms of ω_{abc} , $\tilde{\omega}_{abc}$ and \tilde{A}_\pm^{41} is as follows

$$\begin{aligned}
 A_u^I &= \frac{2}{3}A_-^Y - \frac{1}{3}A_-^{Y'} + \tilde{\omega}_-^I, \\
 A_\nu^I &= -A_-^{Y'} + \tilde{\omega}_-^I, \\
 A_d^I &= -\frac{1}{3}A_+^Y + \frac{2}{3}A_+^{Y'} + \tilde{\omega}_+^I, \\
 A_e^I &= -A_+^Y + \tilde{\omega}_+^I, \\
 A_u^{II} &= A_u^I + (i\tilde{\omega}_{03-} + \tilde{\omega}_{12-}), \\
 A_\nu^{II} &= A_\nu^I + (i\tilde{\omega}_{03-} + \tilde{\omega}_{12-}), \\
 A_d^{II} &= A_d^I + (i\tilde{\omega}_{03+} + \tilde{\omega}_{12+}), \\
 A_e^{II} &= A_e^I + (i\tilde{\omega}_{03+} + \tilde{\omega}_{12+}), \\
 A_u^{III} &= A_u^I + (i\tilde{\omega}_{03-} + \tilde{\omega}_{78-}), \\
 A_\nu^{III} &= A_\nu^I + (i\tilde{\omega}_{03-} + \tilde{\omega}_{78-}), \\
 A_d^{III} &= A_d^I + (i\tilde{\omega}_{03+} + \tilde{\omega}_{78+}), \\
 A_e^{III} &= A_e^I + (i\tilde{\omega}_{03+} + \tilde{\omega}_{78+}), \\
 A_u^{IV} &= A_u^I + (\tilde{\omega}_{12-} + \tilde{\omega}_{78-}), \\
 A_\nu^{IV} &= A_\nu^I + (\tilde{\omega}_{12-} + \tilde{\omega}_{78-}), \\
 A_d^{IV} &= A_d^I + (\tilde{\omega}_{12+} + \tilde{\omega}_{78+}), \\
 A_e^{IV} &= A_e^I + (\tilde{\omega}_{12+} + \tilde{\omega}_{78+}),
 \end{aligned} \tag{39}$$

with $-\tilde{\omega}_\pm^I = \frac{1}{2}(i\tilde{\omega}_{03\pm} + \tilde{\omega}_{12\pm} + \tilde{\omega}_{56\pm} + \tilde{\omega}_{78\pm} + \frac{1}{3}\tilde{A}_\pm^{41})$. The explicit forms of nondiagonal matrix elements are written in Eq. (36). As already stated, the break of symmetries, which is not taken into account in Table III, would strongly relate vacuum expectation values of $\tilde{\omega}_{ab\pm}$.

To evaluate briefly the structure of mass matrices we make one further assumption:

- b.iii. Let the mass matrices be real and symmetric (while all the ω_{abc} and $\tilde{\omega}_{abc}$ are assumed to be real).

We then obtain for the u quarks (and neutrinos) the mass matrices as presented in Table IV.

The corresponding mass matrix for d -quarks (and electrons) is presented in Table V.

TABLE III. The mass matrices for four families of quarks and leptons in the approach unifying spins and charges, obtained under the assumptions a.i.–a.iv. and b.i.–b.ii.. The values A_{\mp}^l , $l' = I, II, III, IV$, and $\tilde{A}^{lm}((ac), (bd))$; $l, m = \pm$, determine matrix elements for the u quarks and the neutrinos, the values A_{\mp}^l , $l' = I, II, III, IV$, and $\tilde{A}_{\mp}^{lm}((ac), (bd))$; $l, m = \pm$, determine the matrix elements for the d quarks and the electrons. Diagonal matrix elements are different for quarks than for leptons and distinguish also between the u and the d quarks and between the ν and the e leptons (Eqs. (33) and (37)). They also differ from family to family. Non diagonal matrix elements distinguish among families and among (u, ν) and (d, e) . The presented matrix should be understood as a very preliminary estimate of the mass matrices of quarks and leptons.

	I_R	II_R	III_R	IV_R
I_L	A_{\mp}^I	$\tilde{A}_{\mp}^{++}((03), (12))$	$\pm\tilde{A}_{\mp}^{++}((03), (78))$	$\mp\tilde{A}_{\mp}^{++}((12), (78))$
II_L	$\tilde{A}_{\mp}^{--}((03), (12))$	A_{\mp}^{II}	$\pm\tilde{A}_{\mp}^{--}((12), (78))$	$\mp\tilde{A}_{\mp}^{--}((03), (78))$
III_L	$\pm\tilde{A}_{\mp}^{+-}((03), (78))$	$\mp\tilde{A}_{\mp}^{+-}((12), (78))$	A_{\mp}^{III}	$\tilde{A}_{\mp}^{+-}((03), (12))$
IV_L	$\pm\tilde{A}_{\mp}^{-+}((12), (78))$	$\mp\tilde{A}_{\mp}^{-+}((03), (78))$	$\tilde{A}_{\mp}^{-+}((03), (12))$	A_{\mp}^{IV}

TABLE IV. The mass matrix of four families of the u -quarks (and neutrinos) obtained within the approach unifying spins and charges and under the assumptions a.i.–a.iv., b.i.–b.iii.. Neutrinos and u -quarks distinguish in $A_u^l \neq A_{\nu}^l$. The break of symmetries, not yet taken into account, would relate $\tilde{\omega}_{ab7,8}$ and would accordingly reduce the number of free parameters.

u	I_R	II_R	III_R	IV_R
I_L	A_u^I	$\tilde{A}_u^{++}((03), (12)) =$ $\frac{1}{2}(\tilde{\omega}_{327} + \tilde{\omega}_{018})$	$\tilde{A}_u^{++}((03), (78)) =$ $\frac{1}{2}(\tilde{\omega}_{387} + \tilde{\omega}_{078})$	$-\tilde{A}_u^{++}((12), (78)) =$ $\frac{1}{2}(\tilde{\omega}_{277} + \tilde{\omega}_{187})$
II_L	$\tilde{A}_u^{--}((03), (12)) =$ $\frac{1}{2}(\tilde{\omega}_{327} + \tilde{\omega}_{018})$	$A_u^{II} = A_u^I + (\tilde{\omega}_{127} - \tilde{\omega}_{038})$	$\tilde{A}_u^{--}((12), (78)) =$ $-\frac{1}{2}(\tilde{\omega}_{277} - \tilde{\omega}_{187})$	$-\tilde{A}_u^{--}((03), (78)) =$ $\frac{1}{2}(\tilde{\omega}_{387} - \tilde{\omega}_{078})$
III_L	$\tilde{A}_u^{+-}((03), (78)) =$ $\frac{1}{2}(\tilde{\omega}_{387} + \tilde{\omega}_{078})$	$-\tilde{A}_u^{+-}((12), (78)) =$ $-\frac{1}{2}(\tilde{\omega}_{277} - \tilde{\omega}_{187})$	$A_u^{III} = A_u^I + (\tilde{\omega}_{787} - \tilde{\omega}_{038})$	$\tilde{A}_u^{+-}((03), (12)) =$ $-\frac{1}{2}(\tilde{\omega}_{327} - \tilde{\omega}_{018})$
IV_L	$\tilde{A}_u^{-+}((12), (78)) =$ $\frac{1}{2}(\tilde{\omega}_{277} + \tilde{\omega}_{187})$	$-\tilde{A}_u^{-+}((03), (78)) =$ $\frac{1}{2}(\tilde{\omega}_{387} - \tilde{\omega}_{078})$	$\tilde{A}_u^{-+}((03), (12)) =$ $-\frac{1}{2}(\tilde{\omega}_{327} - \tilde{\omega}_{018})$	$A_u^{IV} = A_u^I + (\tilde{\omega}_{127} + \tilde{\omega}_{787})$

TABLE V. The mass matrix of four families of the d -quarks and electrons. The quarks and the leptons distinguish in this approximation in $A_d^l \neq A_e^l$. Other comments are the same as in Table IV.

$d-$	I_R	II_R	III_R	IV_R
I_L	A_d^I	$\tilde{A}_d^{++}((03), (12)) =$ $\frac{1}{2}(\tilde{\omega}_{327} - \tilde{\omega}_{018})$	$-\tilde{A}_d^{++}((03), (78)) =$ $-\frac{1}{2}(\tilde{\omega}_{387} - \tilde{\omega}_{078})$	$\tilde{A}_d^{++}((12), (78)) =$ $-\frac{1}{2}(\tilde{\omega}_{277} + \tilde{\omega}_{187})$
II_L	$\tilde{A}_d^{--}((03), (12)) =$ $\frac{1}{2}(\tilde{\omega}_{327} - \tilde{\omega}_{018})$	$A_d^{II} = A_d^I + (\tilde{\omega}_{127} + \tilde{\omega}_{038})$	$-\tilde{A}_d^{--}((12), (78)) =$ $\frac{1}{2}(\tilde{\omega}_{277} - \tilde{\omega}_{187})$	$\tilde{A}_d^{--}((03), (78)) =$ $-\frac{1}{2}(\tilde{\omega}_{387} + \tilde{\omega}_{078})$
III_L	$-\tilde{A}_d^{+-}((03), (78)) =$ $-\frac{1}{2}(\tilde{\omega}_{387} - \tilde{\omega}_{078})$	$\tilde{A}_d^{+-}((12), (78)) =$ $\frac{1}{2}(\tilde{\omega}_{277} - \tilde{\omega}_{187})$	$A_d^{III} = A_d^I + (\tilde{\omega}_{787} + \tilde{\omega}_{038})$	$\tilde{A}_d^{+-}((03), (12)) =$ $-\frac{1}{2}(\tilde{\omega}_{018} + \tilde{\omega}_{327})$
IV_L	$-\tilde{A}_d^{-+}((12), (78)) =$ $-\frac{1}{2}(\tilde{\omega}_{277} + \tilde{\omega}_{187})$	$\tilde{A}_d^{-+}((03), (78)) =$ $-\frac{1}{2}(\tilde{\omega}_{387} + \tilde{\omega}_{078})$	$\tilde{A}_d^{-+}((03), (12)) =$ $-\frac{1}{2}(\tilde{\omega}_{018} + \tilde{\omega}_{327})$	$A_d^{IV} = A_d^I + (\tilde{\omega}_{127} + \tilde{\omega}_{787})$

The relation between the mass matrix of u -quarks and the mass matrix of neutrinos is, under the assumptions and simplifications made during deriving both tables, as follows: All the off-diagonal elements are for the neutrinos the same as for the u -quarks, while the diagonal matrix elements depend on the eigenvalues of Y and Y' . Accordingly, both A_{α}^l , $\alpha = u, \nu$, can be understood as independent parameters, expressible in terms of A_{\mp}^Y , $A_{\mp}^{Y'}$, and $\tilde{\omega}_{\mp}^l$.

Similarly, the relation between the mass matrix of d -quarks and the mass matrix for electrons is, under the same assumptions and simplification as used for finding the

expressions for the mass matrices for the u -quarks and the neutrinos, as follows: All the off-diagonal elements are the same for both—the d -quarks and the electrons, while the diagonal matrix elements distinguish in the eigenvalues of Y and Y' . Again, both A_{β}^l , $\beta = d, e$, can be understood as independent parameters, expressible in terms of A_{\mp}^Y , $A_{\mp}^{Y'}$, and $\tilde{\omega}_{\mp}^l$.

The requirement about reality and symmetry of mass matrices, relates $A_{\mp}^Y = A_{\mp}^Y$, $A_{\mp}^{Y'} = A_{\mp}^{Y'}$, $\tilde{\omega}_{\mp}^l = \frac{1}{2}\tilde{\omega}_{038} + \tilde{\omega}$, $\tilde{\omega}_{\mp}^l = -\frac{1}{2}\tilde{\omega}_{038} + \tilde{\omega}$, where $\tilde{\omega} = \tilde{\omega}_{127} + \tilde{\omega}_{567} + \tilde{\omega}_{787} + \frac{1}{3}\tilde{A}^{41}$ and \tilde{A}^{41} is the real part of either \tilde{A}_{\mp}^{41} or \tilde{A}_{\mp}^{41} . The

same assumption relates also off-diagonal elements for u -quarks and d -quarks (or neutrinos and electrons), as seen from both tables, so that there are 13 free parameters, which determine $4 \times 4(4 + 1)/2$ mass matrix elements, and from these mass matrices 4×4 masses of quarks and leptons and $2(4(4 + 1)/2 - 1)$ elements of the two mixing matrices should follow.

Further break of symmetries would further relate the $\tilde{\omega}_{ab\pm}$ fields, reducing strongly the number of free parameters on Table IV and V. A very peculiar boundary conditions could—when breaking symmetries—even cause differences in off-diagonal matrix elements of quarks and leptons. Also could the nonperturbative effects beyond the tree level be responsible for the differences observed in the measured properties of quarks and leptons or for what in many references are trying to achieve with additional Higgs fields. We did not take into account any Majorana fermions.

Most of the above assumptions were proposed to be able to make a rough estimation of properties of the mass matrices, predicted by the approach unifying spins and charges.

We shall present the calculations with the parameters presented in Tables IV and V in [23].

VI. CONCLUDING DISCUSSIONS

In this paper we discuss about a possible origin of the families of quarks and leptons and of their Yukawa couplings as proposed by the approach unifying spins and charges [1–10].

The approach assumes that a Weyl spinor of a chosen handedness carries in $d(= 1 + 13)$ -dimensional space nothing but two kinds of spin degrees of freedom. One kind belongs to the Poincaré group in $d = 1 + 13$, another one generates families. Spinors interact with only the gravitational fields, manifested by vielbeins and spin connections, the gauge fields of the momentum p_α and the two kinds of the generators of the Lorentz group S^{ab} and \tilde{S}^{ab} , respectively. It is a part of the starting Lagrangian for spinors in $d > 4$, which manifests in $d = 1 + 3$ as all the known gauge fields and also as the Yukawa couplings (playing the role of a Higgs field in the standard model).

One Weyl spinor (we made a choice of the left handed one) in $d = 1 + 13$ manifests in $d = 1 + 3$, if analyzed with respect to the properties of the groups of the standard model, which are subgroups of $SO(1, 13)$, all the quarks and the leptons of the standard model, with the right handed neutrino included. It includes left handed weak charged quarks and leptons and right handed weak chargeless quarks and leptons in the same representation and does accordingly *answer one of the open questions of the standard model: Why only the left handed fermions carry the weak charge while the right handed ones are weak chargeless*, how can it at all happen that handedness, which

concerns only the spin (in $d = 1 + 3$), is so strongly related to a (weak) charge?

The idea of generating families with the second kind of the Clifford algebra objects (which commute with the generators of the Lorentz transformations for spinors) is new (it is indeed arising from the Grassmann algebra done by one of us [1,2]), as is new also the idea that there are the generators of the Lorentz transformations (accompanied by the spin connection fields in $d > 4$) (together with γ^0), which manifest in $d = 1 + 3$ as the Yukawa couplings within a family, transforming a right handed weak chargeless quark or lepton into a left handed weak charged one, while the second kind of the Clifford algebra objects together with the corresponding gauge fields take care of off-diagonal Yukawa couplings.

To derive the mass matrices from the starting Lagrangean—that is to calculate the Yukawa couplings of the standard model—no additional (Higgs) field is needed (which in the standard model dresses right handed spinors with a weak charge), since the part $\psi^\dagger \gamma^0 \{ \overset{78}{(+)}p_{0+} + \overset{78}{(-)}p_{0-} \} \psi$ does its job (namely it transforms a right handed weak chargeless quark or lepton into his weak charged left handed partner).

To come from the starting Lagrange density for a Weyl in $d = 1 + 13$ to the “physical world” is a great job, for which in this paper (and [23]) only the first steps are done. There are several steps, which must be solved. One of the main problems is whether or not at all a break of a symmetry can lead to massless spinors. This is the well known open problem of the Kaluza-Klein-like theories [18]. We have shown on the toy model [19] that a particular boundary condition can assure massless spinors after the break. The work, how to find the appropriate boundary conditions in more general cases (and how those boundary conditions, if at all, are connected with orbifolds), is now in progress. Another, not at all easier problem is, how to make calculations of all the adiabatic and nonadiabatic effects, which occur after the spontaneous breaks of symmetries, and for which we know from the hadron physics, for example, how difficult (and not yet solved) they are.

In order to be able to make a first step, that is to make a simple and transparent evaluation of properties of the mass matrices within the approach of one of us unifying spins and charges and to see whether our approach is at all the right way to go beyond the standard model (it is hard to believe that families appear just as a possible repetition without any special reason), we assume (relying on our experience with the toy model) that also in the case of the break of $SO(1, 13)$ down to $SO(1, 7) \times SU(3) \times U(1)$ a kind of boundary conditions assures massless octets of coloured quarks and colourless leptons. Then the masses of quarks and leptons are determined by that part of the starting Lagrangian, which manifests as the Yukawa couplings of the standard model and connects particular members of the $SO(1, 7)$ multiplet: the right handed weak

chargeless quarks and leptons with their left handed weak charged partners.

To see the strength of the fields at physical energies all the mentioned calculations of the (non)perturbative effects should be done. For rough estimations and rough predictions of the masses and mixing matrices for quarks and leptons, observed at physical energies, however, we have done several additional assumptions, approximations and simplifications, not necessary all of them physical (and some of them should soon be relaxed in further studies, although for most of them we have some explanation), like:

- (i) The requirement that the terms with S^{5a} , S^{6a} do not contribute to the mass term at low energies, assures that the charge $Q = \tau^{41} + S^{56}$ is conserved.
- (ii) The break of symmetries influences the Poincaré symmetry and the symmetries described by \tilde{S}^{ab} . (But it is assumed that the gauge symmetries connected with \tilde{S}^{ab} do not manifest as gauge fields (additional to the known charge gauge fields) in $d = 1 + 3$.) It is also assumed that there are no terms, which would in Eq. (29) transform $\begin{smallmatrix} 56 \\ (+) \end{smallmatrix}$ into $\begin{smallmatrix} 56 \\ [+] \end{smallmatrix}$ (which can be explained by the related break of symmetries in S^{ab} and \tilde{S}^{ab} sector). This assumption reduces the number of families by a factor 2. It really means that the group $SO(1, 7)$, whose generators are \tilde{S}^{ab} , is broken into $SO(1, 5) \times U(1)$ in a way that terms $\tilde{S}^{5a} \tilde{\omega}_{5ab}$ and $\tilde{S}^{6a} \tilde{\omega}_{6ab}$ bring no contribution to the mass matrices. Otherwise no additional break of symmetry was taken into account.
- (iii) We make estimations on a tree level.
- (iv) We assume the mass matrices to be real and symmetric.

We use our technique [12,13] to present spinor representations in a transparent way so that one easily sees how does a part of the covariant derivative of a spinor in $d = 1 + 13$ manifest in $d = 1 + 3$ as Yukawa couplings. We use the same technique to represent also families of spinors. Since the starting action in $d = 1 + 13$ manifests in $d = 1 + 3$ the (even number) of families and the Yukawa couplings, it offers a possible answer to the questions, why families of quarks and leptons and the corresponding Yukawa couplings manifest in nature.

We treat quarks and leptons equivalently (not taking into account Majorana neutrinos additionally). We found the off-diagonal mass matrix elements of the quarks and the

leptons strongly related [22]. We expect that these relations might very probably turn out to be too strong, since going beyond the tree level might very easily bring different contributions for quarks and leptons, as some particular breaks of symmetries also could.

If the symmetry of mass matrices as presented in this paper for four families breaks further, the relations among the parameters determining the mass matrices follow, reducing the presented number of independent parameters $\tilde{\omega}_{abc}$. In particular, an exact break of the symmetry of $SO(1, 5)$ in the \tilde{S}^{ab} sector into $SU(3) \times U(1)$ would manifest in decoupling of the fourth family from the first three (by relating $\tilde{\omega}_{abc}$ so that the corresponding matrix elements would be zero), while the break of $SO(1, 5)$ into $SU(2) \times SU(2) \times U(1)$ would manifest in decoupling of the first two families from the second two families.

For the four predicted families of quarks and leptons we present the explicit expressions for the mass matrices in the above mentioned approximations.

Since in our approach the generators of the Poincaré group and of those, generating families, commute, many a property of mass matrices, presented in this paper, would be true also for, let us say, models, in which the generators of the Poincaré group and those of generating families, commute. However, in our case breaks of symmetries in the two sectors are related and these relations might be very important for the properties of quarks and leptons.

On the other hand we should find the explanation why the additional gauge fields, connected with the \tilde{S}^{ab} sector does not manifest in $d = 1 + 3$.

We shall present numerical estimates for the Yukawa couplings after relating our results with the known experimental data in the paper [23], together with further discussions of the properties of families of quarks and leptons as following from the approach unifying spins and charges, in order to be able to see whether this approach shows the right way beyond the standard model of electroweak and color interactions.

ACKNOWLEDGMENTS

It is a pleasure to thank all the participants of the workshops entitled “What Comes Beyond the Standard Model”, taking place at Bled annually in July, starting at 1998, for many very fruitful discussions, in particular, to H. B. Nielsen.

[1] N. S. Mankoč Borštnik, Phys. Lett. B **292**, 25 (1992).
 [2] N. S. Mankoč Borštnik, J. Math. Phys. (N.Y.) **34**, 3731 (1993).

[3] N. Mankoč Borštnik, J. Math. Phys. (N.Y.) **36**, 1593 (1995).
 [4] N. S. Mankoč Borštnik, Mod. Phys. Lett. A **10**, 587

- (1995).
- [5] N. S. Mankoč Borštnik and S. Fajfer, *Nuovo Cimento Soc. Ital. Fis. B* **112**, 1637 (1997).
- [6] A. Borštnik and N. S. Mankoč Borštnik, in *Proceedings to the International Workshop on “What Comes Beyond the Standard Model, Bled, Slovenia, 1998*, edited by N. Mankoč Borštnik, H. B. Nielsen, and C. Froggatt (DMFA, Založništvo, 1999), p. 52.
- [7] N. S. Mankoč Borštnik and H. B. Nielsen, *Phys. Rev. D* **62**, 044010 (2000).
- [8] N. S. Mankoč Borštnik, *Int. J. Theor. Phys.* **40**, 315 (2001), and references therein.
- [9] A. Borštnik and N. S. Mankoč Borštnik, in *Proceedings to the International Workshop on “What Comes Beyond the Standard Model”, Bled 2000, 2001, 2002 Vol. 2*, edited by N. Mankoč Borštnik, H. B. Nielsen, C. Froggatt, and D. Lukman (DMFA, Založništvo, 2002), p. 27 and the paper (unpublished).
- [10] A. Borštnik and N. S. Mankoč Borštnik, in *Proceedings to the Euroconference on Symmetries Beyond the Standard Model, Portorož, 2003* edited by N. Mankoč Borštnik, H. B. Nielsen, C. Froggatt, and D. Lukman (DMFA, Založništvo, 2003), pp. 27–51.
- [11] M. Blagojević, *Gravitation and Gauge Symmetries* (I. of Phys. Pub, Bristol, Philadelphia, 2002), ISBN 0750307676.
- [12] N. S. Mankoč Borštnik and H. B. Nielsen, *J. Math. Phys.* (N.Y.) **43**, 5782 (2002).
- [13] N. S. Mankoč Borštnik and H. B. Nielsen, *J. Math. Phys.* (N.Y.) **44**, 4817 (2003).
- [14] A. Kleppe, D. Lukman, and N. S. Mankoč Borštnik, in *Proceedings to the 6th Workshop “What Comes Beyond the Standard Models”, Bled, 2000, 2001, 2002* edited by N. Mankoč Borštnik, H. B. Nielsen, C. Froggatt, and D. Lukman (DMFA, Založništvo, 2002), pp. 94–105.
- [15] M. Breskvar, J. Mravlje, and N. S. Mankoč Borštnik, in *Proceedings to the 7th Workshop “What Comes Beyond the Standard Models”, Bled, 2004*, edited by N. Mankoč Borštnik, H. B. Nielsen, C. Froggatt, and D. Lukman (DMFA, Založništvo, 2004), pp. 1–16.
- [16] V. A. Novikov, L. B. Okun, A. N. Royanov, and M. I. Vysotsky, hep-ph/0111028.
- [17] S. Eidelman *et al.*, *Phys. Lett. B* **592**, 1 (2004).
- [18] E. Witten, *Nucl. Phys. B* **186**, 412 (1981); Princeton Technical Report No. PRINT-83-1056, 1983 (unpublished).
- [19] N. S. Mankoč Borštnik and H. B. Nielsen, *Phys. Lett. B* **633**, 771 (2006).
- [20] N. S. Mankoč Borštnik and H. B. Nielsen, in *Proceedings to the 6th Workshop “What Comes Beyond the Standard Models”, Bled, 2000 2001 2002* edited by N. Mankoč Borštnik, H. B. Nielsen, C. Froggatt, and D. Lukman (DMFA, Založništvo, 2002), pp. 94–105.
- [21] Latin indices $a, b, \dots, m, n, \dots, s, t, \dots$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, \dots, \mu, \nu, \dots, \sigma, \tau, \dots$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index (a, b, c, \dots and $\alpha, \beta, \gamma, \dots$), from the middle of both the alphabets the observed dimensions 0, 1, 2, 3 (m, n, \dots and μ, ν, \dots), indices from the bottom of the alphabets indicate the compactified dimensions (s, t, \dots and σ, τ, \dots). We assume the signature $\eta^{ab} = \text{diag}\{1, -1, -1, \dots, -1\}$.
- [22] There are on the tree level and under our assumptions and approximations the same off-diagonal and diagonal matrix elements, which determine the orthogonal rotations of the matrices for the u -quarks and neutrinos and the d -quarks and electron into the diagonal forms and consequently also the corresponding mixing matrices for quarks are the same as for leptons. The experimental data show quite a difference in the mixing matrices of quarks and leptons.
- [23] M. Breskvar, D. Lukman, and N. S. Mankoč Borštnik, hep-ph/0606159.