Neutrino induced charged current $1\pi^+$ production at intermediate energies

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The charged current one pion production induced by ν_μ from nucleons and nuclei like ^{12}C and ^{16}O nuclei has been studied. The calculations have been done for the incoherent and the coherent processes from nuclear targets assuming the Δ dominance model and taking into account the effect of Pauli blocking, Fermi motion of the nucleon, and renormalization of Δ properties in a nuclear medium. The effect of final state interactions of pions has been taken into account. The theoretical uncertainty in the total cross sections due to various parametrizations of the weak transition form factors used in literature has been studied. The numerical results for the total cross sections are compared with the recent preliminary results from the MiniBooNE collaboration on ^{12}C and could be useful in analyzing future data from the K2K collaboration.

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I. INTRODUCTION

The study of neutrino induced pion production from nucleons and nuclei has a long history starting with the neutrino experiments performed at CERN [1] and Serpukhov [2] with the bubble chambers filled with heavy liquid like propane and freon. However in the intermediate energy region of 1–3 GeV, most of the data have been obtained from the later experiments performed at ANL [3–5] and BNL [6] with hydrogen and deuterium filled bubble chambers. Theoretically, the weak production of pions induced by neutrinos from the free nucleons have been studied by many authors [7–16] using various approaches like multipole analysis, effective Lagrangian, and quark model. Recent interest in the study of these processes has been generated by the ongoing neutrino oscillation experiments being performed at the intermediate neutrino energies by the MiniBooNE and the K2K collaborations using ^{12}C and ^{16}O as the nuclear targets in the detector [17–19]. Furthermore, many high precision neutrino experiments in the intermediate energy region of 1–3 GeV using neutrino beams from neutrino factories, superbeams, and β-beams have been recently proposed [20–26]. These experiments are planned to be performed with the nuclear targets like ^{12}C, ^{16}O, ^{40}Ar, ^{56}Fe, etc. In order to analyze these neutrino oscillation experiments, a study of neutrino induced pion production from nuclei is very important. It is, therefore, desired that various nuclear effects in the weak pion production processes induced by neutrinos be studied in the energy region of these experiments. There exist some calculations in the past where these studies have been made [27–30] which are relevant for neutrino oscillation experiments with atmospheric neutrinos. In view of the recent data on some weak pion production processes already available [18] and new data to be expected soon from the MiniBooNE and K2K collaborations, the subject has attracted much attention and many calculations have been made for these processes [31–35].

In the energy region of low and intermediate neutrino energies, the dominant mechanism of single pion production from the nucleon arises through the excitation of a baryon resonance which then decays into a nucleon and a pion. In a nucleus, the target nucleus can stay in the ground state leading to the coherent production of pions or can be excited and/or broken up leading to the incoherent production of pions. The excitation of the Δ resonance is the dominant resonance excitation at these energies contributing to one pion production and many authors have used the delta dominance model to calculate the one pion production. However, neutrino generators like NUANCE and NEUGEN which are used to model low energy neutrino nucleus interactions to analyze the neutrino oscillation experiments include higher resonance states as well [36–38]. However, these generators do not include any nuclear effects in their resonance production model for the single pion production and take into account the pion absorption effects in some *ad hoc* way [36]. These nuclear effects are quite important in the energy region of 1 GeV, corresponding to K2K and MiniBooNE experiments and should be included in the numerical codes of various neutrino generators.

In this paper, we have studied the neutrino induced charged current incoherent and coherent single pion production from ^{12}C and ^{16}O at intermediate energies relevant for the MiniBooNE and the K2K experiments using the delta dominance model developed by Oset and his collaborators [39]. In Sec. II, we describe the formalism for single π^+ production from the nucleons in the Δ dominance model and describe the nuclear medium and the final state interaction effects in Sec. III. In Sec. IV, we present and discuss the numerical results for the total cross section for π^+ production and their Q^2 distribution and compare

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them with the preliminary results available from the MiniBooNE experiment [18]. In Sec. V, we provide a summary and conclusion of our work.

II. WEAK PION PRODUCTION FROM NUCLEONS

In the intermediate energy region of about 1 GeV the neutrino induced pion production from nucleon is dominated by the Δ excitation in which a Δ resonance is excited which subsequently decays into a pion and a nucleon through the following reactions:

$$\nu_\mu(k) + p(p) \to \mu^-(k') + \Delta^{++}(p') \quad \searrow p + \pi^+, \tag{1}$$

$$\nu_\mu(k) + n(p) \to \mu^-(k') + \Delta^{+}(p') \quad \searrow n + \pi^+ \quad \searrow p + \pi^0. \tag{2}$$

In this model of the Δ dominance the neutrino induced charged current one pion production is calculated using the Lagrangian in the standard model of electroweak interactions given by

$$L = \frac{G_F}{\sqrt{2}} l_\mu(x) J^{\mu\dagger}(x) + \text{H.c.}, \tag{3}$$

where $l_\mu(x) = \bar{\psi}(k')\gamma_\mu(1-\gamma_5)\psi(k)$ and $J^\mu(x) = cos\theta_c(V^\mu(x) + A^\mu(x))$, θ_c being the Cabibbo angle.

The matrix element of the vector current V^μ and the axial vector current A^μ of the hadronic current J^μ for the Δ excitation from proton target is written as:

$$\begin{aligned}\langle \Delta^{++}|V^\mu|p\rangle = \sqrt{3}\bar{\psi}_\alpha(p')\Big(&\frac{C_3^V(q^2)}{M}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu) \\ &+ \frac{C_4^V(q^2)}{M^2}(g^{\alpha\mu} q\cdot p' - q^\alpha p'^\mu) \\ &+ \frac{C_5^V(q^2)}{M^2}(g^{\alpha\mu} q\cdot p - q^\alpha p^\mu) \\ &+ \frac{C_6^V(q^2)}{M^2} q^\alpha q^\mu\Big)\gamma_5 u(p)\end{aligned} \tag{4}$$

and

$$\begin{aligned}\langle \Delta^{++}|A^\mu|p\rangle = \sqrt{3}\bar{\psi}_\alpha(p')\Big(&\frac{C_3^A(q^2)}{M}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu) \\ &+ \frac{C_4^A(q^2)}{M^2}(g^{\alpha\mu} q\cdot p' - q^\alpha p'^\mu) \\ &+ C_5^A(q^2) g^{\alpha\mu} + \frac{C_6^A(q^2)}{M^2} q^\alpha q^\mu\Big) u(p).\end{aligned} \tag{5}$$

A similar expression is used for the Δ^+ excitation from the neutron target. Here $\psi_\alpha(p')$ and $u(p)$ are the Rarita Schwinger and Dirac spinors for the Δ and the nucleon of momenta p' and p, respectively, $q(= p' - p = k - k')$ is the momentum transfer, $Q^2(= -q^2)$ is the momentum transfer square, and M is the mass of the nucleon. C_i^V ($i =$ 3–6) are the vector and C_i^A ($i =$ 3–6) are the axial vector transition form factors. The vector form factors C_i^V ($i =$ 3–6) are determined by using the conserved vector current (CVC) hypothesis which gives $C_6^V(q^2) = 0$ and relates C_i^V ($i = 3, 4, 5$) to the electromagnetic form factors which are determined from the analysis of experimental data on the photoproduction and electroproduction of Δ's. They are generally parametrized in a dipole form [14]:

$$C_i^V(q^2) = C_i^V(0)\Big(1 - \frac{q^2}{M_V^2}\Big)^{-2}; \qquad i = 3, 4, 5, \tag{6}$$

where M_V is the vector dipole mass.

However, some authors [15,31,35,40,41] have recently proposed modified dipole form factors while others use quark models without or with some pion dynamics. In the case of dipole form factors various modifications have been proposed. For example, Lalakulich *et al.* [40] use

$$\begin{aligned}&C_i^V(q^2) = C_i^V(0)\Big(1 - \frac{q^2}{M_V^2}\Big)^{-2} D_i; \qquad i = 3, 4, 5, \\ &D_i = \Big(1 - \frac{q^2}{4M_V^2}\Big)^{-1} \quad \text{for } i = 3, 4, \\ &D_i = \Big(1 - \frac{q^2}{0.776 M_V^2}\Big)^{-1}; \qquad i = 5,\end{aligned} \tag{7}$$

while Paschos *et al.* [31] and Leitner *et al.* [35] use

$$\begin{aligned}&C_i^V(q^2) = C_i^V(0)\Big(1 - \frac{q^2}{M_V^2}\Big)^{-2} D_i; \qquad i = 3, 4, 5, \\ &D_i = \Big(1 - \frac{q^2}{4M_V^2}\Big)^{-1} \quad \text{for } i = 3, 4, 5.\end{aligned} \tag{8}$$

Similarly, the axial vector form factors are determined using partially conserved axial current (PCAC) hypothesis which gives $C_6^A(q^2) = C_5^A(q^2)\frac{M^2}{m_\pi^2 - q^2}$ and the other form factors are defined from the analysis of neutrino induced pion production from hydrogen and deuterium targets. They are generally parametrized in a modified dipole form and are given as

$$\begin{aligned}&C_i^A(q^2) = C_i^A(0)\Big(1 - \frac{q^2}{M_A^2}\Big)^{-2} D_i; \qquad i = 3, 4, 5, \\ &D_i = 1 + \frac{a_i q^2}{(b_i - q^2)}; \qquad i = 3, 4, 5, \\ &a_i = -1.21 \quad \text{and} \quad b_i = 2.0\ \text{GeV}^2\end{aligned} \tag{9}$$

by Schreiner and von Hippel [14], while Paschos *et al.* [31], Leitner *et al.* [35], and Lalakulich *et al.* [40] use

$$C_i^A(q^2) = C_i^A(0)\left(1 - \frac{q^2}{M_A^2}\right)^{-2} D_i; \qquad i = 3, 4, 5,$$
$$D_i = \left(1 - \frac{q^2}{3M_A^2}\right)^{-1}, \tag{10}$$

where M_A is the axial vector dipole mass and m_π is the pion mass.

Various parameters occurring in these form factors used by these authors are summarized in Table I.

The differential scattering cross section is given by

$$\frac{d^2\sigma}{dE_{k'} d\Omega_{k'}} = \frac{1}{64\pi^3} \frac{1}{MM_\Delta} \frac{|\mathbf{k}'|}{E_k} \frac{\frac{\Gamma(W)}{2}}{(W - M_\Delta)^2 + \frac{\Gamma^2(W)}{4.}} |\mathcal{M}|^2, \tag{11}$$

where Γ is the delta decay width and $|\mathcal{M}|^2 = \frac{G_F^2}{2} L_{\mu\nu} J^{\mu\nu}$, with

$$L_{\mu\nu} = \bar{\Sigma}\Sigma l_\mu{}^\dagger l_\nu = L_{\mu\nu}^S + iL_{\mu\nu}^A$$
$$= 8(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k \cdot k' + i\epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta),$$

and

$$J^{\mu\nu} = \bar{\Sigma}\Sigma J^{\mu\dagger} J^\nu \tag{12}$$

which is calculated with the use of spin-$\frac{3}{2}$ projection operator $P^{\mu\nu}$ defined as

$$P^{\mu\nu} = \sum_{\text{spins}} \psi^\mu \bar{\psi}^\nu$$

and is given by:

$$P^{\mu\nu} = -\frac{\not{p}' + M_\Delta}{2M_\Delta}\left(g^{\mu\nu} - \frac{2}{3}\frac{p'^\mu p'^\nu}{M_\Delta^2} + \frac{1}{3}\frac{p'^\mu \gamma^\nu - p'^\nu \gamma_\mu}{M_\Delta} - \frac{1}{3}\gamma^\mu\gamma^\nu\right). \tag{13}$$

In Eq. (11), the delta decay width Γ is taken to be an energy dependent P-wave decay width given by [39]:

$$\Gamma(W) = \frac{1}{6\pi}\left(\frac{f_{\pi N\Delta}}{m_\pi}\right)^2 \frac{M}{W} |\mathbf{q}_{\text{cm}}|^3 \Theta(W - M - m_\pi), \tag{14}$$

where

$$|\mathbf{q}_{\text{cm}}| = \frac{\sqrt{(W^2 - m_\pi^2 - M^2)^2 - 4m_\pi^2 M^2}}{2W}$$

and M is the mass of nucleon. The step function Θ denotes the fact that the width is zero for the invariant masses below the $N\pi$ threshold. $|\mathbf{q}_{\mathbf{cm}}|$ is the pion momentum in the rest frame of the resonance.

III. WEAK PION PRODUCTION FROM NUCLEI

A. Incoherent pion production

When the reactions given by Eq. (1) or (2) take place in the nucleus, the neutrino interacts with a nucleon moving inside the nucleus of density $\rho(r)$ with its corresponding momentum $\vec{p}$ constrained to be below its Fermi momentum $k_{F_{n,p}}(r) = [3\pi^2 \rho_{n,p}(r)]^{1/3}$, where $\rho_n(r)$ and $\rho_p(r)$ are the neutron and proton nuclear densities. In the local density approximation, the differential scattering cross section for a π^+ production from the proton target is written as

$$\frac{d^2\sigma}{dE_{k'} d\Omega_{k'}} = \frac{1}{64\pi^3} \int d\mathbf{r} \rho_p(\mathbf{r}) \frac{|\mathbf{k}'|}{E_k} \frac{1}{MM_\Delta} \times \frac{\frac{\Gamma(W)}{2}}{(W - M_\Delta)^2 + \frac{\Gamma^2(W)}{4.}} |\mathcal{M}|^2. \tag{15}$$

However, in the nuclear medium the properties of Δ like its mass and decay width Γ to be used in Eq. (15) are modified due to the nuclear effects. These are mainly due to the following processes.

(i) In the nuclear medium Δs decay mainly through the $\Delta \to N\pi$ channel. The final nucleons have to be above the Fermi momentum k_F of the nucleon in the nucleus thus inhibiting the decay as compared to the free decay of the Δ described by Γ in Eq. (14). This leads to a modification in the decay width of delta which has been studied by many authors [39,42–44]. We take the value given by [39] and write the modified delta decay width $\tilde{\Gamma}$ as

$$\tilde{\Gamma} = \Gamma \times F(k_F, E_\Delta, k_\Delta), \tag{16}$$

where $F(k_F, E_\Delta, k_\Delta)$ is the Pauli correction factor given by [39]:

$$F(k_F, E_\Delta, k_\Delta) = \frac{k_\Delta |\mathbf{q}_{\text{cm}}| + E_\Delta E'_{p\text{cm}} - E_F W}{2k_\Delta |\mathbf{q}'_{\text{cm}}|}, \tag{17}$$

TABLE I. Weak vector and axial vector couplings at $q^2 = 0$ and the values of M_V and M_A used in the literature.

	$C_3^V(0)$	$C_4^V(0)$	$C_5^V(0)$	$C_3^A(0)$	$C_4^A(0)$	$C_5^A(0)$	M_V (GeV)	M_A (GeV)
Schreiner and von Hippel [14], Singh *et al.* [29]	2.05	$-\frac{M}{M_\Delta} C_3^V(0)$	0.0	0.0	-0.3	1.2	0.73	1.05
Paschos *et al.* [31], Leitner *et al.* [35]	1.95	$-\frac{M}{W} C_3^V(0)$[a]	0.0	0.0	-0.25	1.2	0.84	1.05
Lalakulich *et al.* [40]	2.13	-1.51	0.48	0.0	-0.25	1.2	0.84	1.05

[a] W is the center of mass energy i.e. $W = \sqrt{(p + q)^2}$ and M_Δ is the mass of Δ.

$E_F = \sqrt{M^2 + k_F^2}$, k_Δ is the Δ momentum, and $E_\Delta = \sqrt{W + k_\Delta^2}$.

(ii) In the nuclear medium there are additional decay channels open due to two and three-body absorption processes like $\Delta N \to NN$ and $\Delta NN \to NNN$ through which Δ disappear in the nuclear medium without producing a pion, while a two-body Δ absorption process like $\Delta N \to \pi NN$ gives rise to some more pions. These nuclear medium effects on the Δ propagation are included by describing the mass and the decay width in terms of the self-energy of Δ. These considerations lead to the following modifications in the width $\tilde{\Gamma}$ and mass M_Δ of the Δ resonance,

$$\frac{\tilde{\Gamma}}{2} \to \frac{\tilde{\Gamma}}{2} - \mathrm{Im}\Sigma_\Delta \quad \text{and} \quad M_\Delta \to M_\Delta + \mathrm{Re}\Sigma_\Delta. \tag{18}$$

The expressions for the real and the imaginary parts of Σ_Δ are [39]:

$$\mathrm{Re}\,\Sigma_\Delta = 40\frac{\rho}{\rho_0}\ \mathrm{MeV} \quad \text{and}$$
$$-\mathrm{Im}\Sigma_\Delta = C_Q\left(\frac{\rho}{\rho_0}\right)^\alpha + C_{A2}\left(\frac{\rho}{\rho_0}\right)^\beta + C_{A3}\left(\frac{\rho}{\rho_0}\right)^\gamma. \tag{19}$$

In the above equation C_Q accounts for the $\Delta N \to \pi NN$ process, C_{A2} for the two-body absorption process $\Delta N \to NN$, and C_{A3} for the three-body absorption process $\Delta NN \to NNN$. The coefficients C_Q, C_{A2}, C_{A3}, and α, β, and γ are taken from Ref. [39].

With these modifications the differential scattering cross section described by Eq. (15) modifies to

$$\frac{d^2\sigma}{dE_{k'}d\Omega_{k'}} = \frac{1}{64\pi^3}\int d\mathbf{r}\rho_p(\mathbf{r})\frac{|\mathbf{k}'|}{E_k}\frac{1}{MM_\Delta} \times \frac{\frac{\tilde{\Gamma}}{2} - \mathrm{Im}\Sigma_\Delta}{(W - M_\Delta - \mathrm{Re}\Sigma_\Delta)^2 + (\frac{\tilde{\Gamma}}{2} - \mathrm{Im}\Sigma_\Delta)^2}|\mathcal{M}|^2. \tag{20}$$

For one π^+ production process $\tilde{\Gamma}$ and C_Q term in $\mathrm{Im}\Sigma_\Delta$ give contribution to the pion production. For π^+ production on the neutron target, $\rho_p(\mathbf{r})$ in the above expression is replaced by $\frac{1}{9}\rho_n(\mathbf{r})$, where the factor $\frac{1}{9}$ with ρ_n comes due to the suppression of π^+ production from the neutron target as compared to the π^+ production from the proton target through the process of Δ excitation and decay in the nucleus.

The total scattering cross section for the neutrino induced charged current one π^+ production in the nucleus is given by

$$\sigma = \frac{1}{64\pi^3}\iint d\mathbf{r}\frac{d\mathbf{k}'}{E_kE_{k'}}\frac{1}{MM_\Delta} \times \frac{\frac{\tilde{\Gamma}}{2} + C_Q(\frac{\rho}{\rho_0})^\alpha}{(W - M_\Delta - \mathrm{Re}\Sigma_\Delta)^2 + (\frac{\tilde{\Gamma}}{2} - \mathrm{Im}\Sigma_\Delta)^2} \times \left[\rho_p(\mathbf{r}) + \frac{1}{9}\rho_n(\mathbf{r})\right]|\mathcal{M}|^2. \tag{21}$$

For our numerical calculations we take the proton density $\rho_{\mathrm{p}}(r) = \frac{Z}{A}\rho(r)$ and the neutron density $\rho_{\mathrm{n}}(r) = \frac{A-Z}{A}\rho(r)$, where $\rho(r)$ is nuclear density which we have taken as 3-parameter Fermi density given by:

$$\rho(r) = \rho_0\left(1 + w\frac{r^2}{c^2}\right)\Big/\left(1 + \exp\left(\frac{r-c}{z}\right)\right)$$

and the density parameters $c = 2.355$ fm, $z = 0.5224$ fm, and $w = -0.149$ for ^{12}C and $c = 2.608$ fm, $z = 0.513$ fm and $w = -0.051$ for ^{16}O are taken from Ref. [45].

The pions produced in these processes inside the nucleus may rescatter or may produce more pions or may get absorbed while coming out from the final nucleus. We have taken the results of Vicente Vacas [46] for the final state interaction of pions which is calculated in an eikonal approximation using probabilities per unit length as the basic input. In this approximation, a pion of given momentum and charge is moved along the z-direction with a random impact parameter $\mathbf{b}$, with $|\mathbf{b}| < R$, where R is the nuclear radius which is taken to be a point where nuclear density $\rho(R)$ falls to $10^{-3}\rho_0$, where ρ_0 is the central density. To start with, the pion is placed at a point $(\mathbf{b}, z_{\mathrm{in}})$, where $z_{\mathrm{in}} = -\sqrt{R^2 - |\mathbf{b}|^2}$ and then it is moved in small steps δl along the z-direction until it comes out of the nucleus or it interacts. If $P(p_\pi, r, \lambda)$ is the probability per unit length at the point r of a pion of momentum $\mathbf{p}_\pi$ and charge λ, then $P\delta l \ll 1$. A random number x is generated such that $x \in [0, 1]$ and if $x > P\delta l$, then it is assumed that the pion has not interacted while traveling a distance δl, however, if $x < P\delta l$ then the pion has interacted and depending upon the weight factor of each channel given by its cross section it is decided whether the interaction was quasielastic, charge exchange reaction, pion production, or pion absorption [46]. For example, for the quasielastic scattering

$$P_{N(\pi^\lambda,\pi^{\lambda'})N'} = \sigma_{N(\pi^\lambda,\pi^{\lambda'})N'} \times \rho_N,$$

where N is a nucleon, ρ_N is its density, and σ is the elementary cross section for the reaction $\pi^\lambda + N \to \pi^{\lambda'} + N'$ obtained from the phase shift analysis.

For a pion to be absorbed, P is expressed in terms of the imaginary part of the pion self-energy Π, i.e. $P_{\mathrm{abs}} = -\frac{\mathrm{Im}\Pi_{\mathrm{abs}}(p_\pi)}{p_\pi}$, where the self-energy Π is related to the pion optical potential [47].

B. Coherent pion production

The coherent production of pion has been calculated earlier in this model [48], where Δ resonance excitations and their decays are such that the nucleus stays in the ground state. The matrix elements for Δ excitations are calculated using the hadronic transition current given in Eqs. (4) and (5) with the nuclear modification in Δ properties as described in Eqs. (18) and (19).

With the incorporation of the nuclear medium effects as discussed in Sec. III A, the Δ-dependent hadronic factors become density dependent and the hadronic transition operator J^μ for the s-channel, is written as

$$\mathcal{J}^\mu = \cos\theta_c \int \mathcal{T}_s^\mu \frac{\mathrm{M}^2}{\mathrm{P}_s^2 - \tilde{\mathrm{M}}_\Delta^2 + i\tilde{\Gamma}\tilde{\mathrm{M}}_\Delta} \rho^s(r) e^{i(\vec{q}-\vec{p}_\pi)\cdot\vec{r}} d\vec{r}, \tag{22}$$

where P is the momentum of the Δ resonance in s-channel, $\mathcal{T}_s^\mu$ is the nonpole part of the kinematic factors involving transition form factors $\mathrm{C}_j^{\mathrm{V,A}}(q^2)$, $\rho^s(r)$ is the linear combination of proton and neutron densities incorporating the isospin factors for one pion production from proton and neutron targets. The contribution of u-channel is shown to be small [48].

In this case the final state interactions involve the interaction of the outgoing pions with the final nucleus in the ground state. This has been calculated by using a distorted wave pion wave function in the field of the final nucleus. The distortion of the pion has been calculated in the eikonal approximation [49] using a pion nucleus optical potential which is given in terms of the self-energy of pions in the nuclear matter [39] calculated in the local density approximation. The nuclear form factor corresponding to the coherent pion production is calculated using a final state pion wave function given by [48]

$$\tilde{\phi}_\pi(\vec{r}) = e^{-i\vec{p}_\pi\cdot\vec{r}} e^{-i\int_z^\infty (1/2p_\pi)\Pi(\rho(\vec{b},z'))dz'}, \tag{23}$$

where $\vec{r} = (\vec{b}, z)$. $\Pi(\rho)$ is the self-energy of pion calculated in the local density approximation of the delta hole model and is taken from Ref. [39].

The numerical results for the coherent pion production cross sections from ^{12}C are recently presented in Ref. [48]. For the sake of completeness, these are also included here in the total cross sections along with the cross sections for the incoherent pion production and are discussed in Sec. IV while comparing with the experimental results on the total one π^+ production from nuclei.

IV. RESULTS AND DISCUSSION

We have calculated the total scattering cross section for the charged current $1\pi^+$ production for the incoherent and coherent processes using different N-Δ transition form factors given by Schreiner and von Hippel [14], Paschos *et al.* [31], and Lalakulich *et al.* [40] as discussed in Sec. II. The numerical results for the total scattering cross section $\sigma(E_\nu)$ for ν_μ induced reaction on a free proton target, i.e. $\nu_\mu + p \to \mu^- + p + \pi^+$ are presented in Fig. 1 along with the experimental results from the ANL and the BNL experiments [3–6]. The various theoretical curves show the cross sections calculated using N-Δ transition form factors given by Schreiner and von Hippel [14], Paschos *et al.* [31], and Lalakulich *et al.* [40]. We find that in the neutrino energy region of 0.7–2.0 GeV the cross sections obtained with the N-Δ transition form factors given by Paschos *et al.* [31] and Lalakulich *et al.* [40] are larger than the cross sections obtained by using the Schreiner and von Hippel [14] parametrization. The uncertainty in the total cross section for $1\pi^+$ production associated due to the uncertainty in the transition form factors is seen from these figures to be about 10%–20% in this energy region.

The experimental data from ANL by Campbell *et al.* [3] are explained satisfactorily in our model. The theoretical results are within 1 standard deviation of the experimental results using weak N-Δ transition form factors of Paschos *et al.* [31] ($\chi^2_{\mathrm{pdf}} = 0.9$) and Lalakulich *et al.* [40] ($\chi^2_{\mathrm{pdf}} = 0.8$) and within 1.5 standard deviation if Schreiner and von Hippel [14] parametrization is used. The experimental results of Barish *et al.* [4] (excluding the lowest energy points) are also described satisfactorily by our model within 1 standard deviation if the form factors of Schreiner and von Hippel [14] ($\chi^2_{\mathrm{pdf}} = 0.6$) and Paschos *et al.* [31] ($\chi^2_{\mathrm{pdf}} = 0.8$) are used and within 1.2 standard deviation if the parametrization of Lalakulich *et al.* [40] is

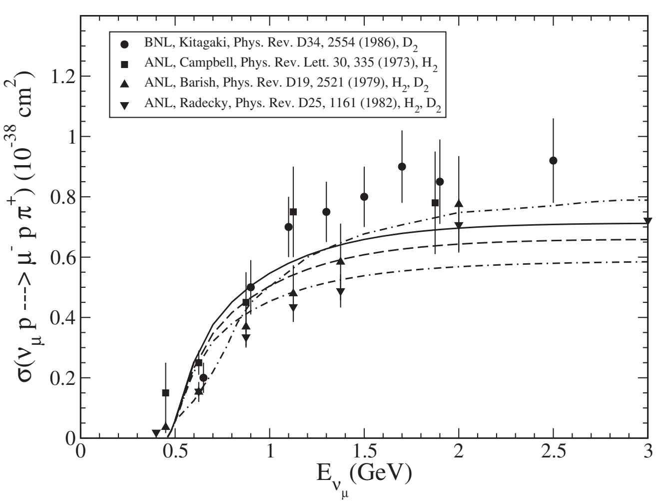

FIG. 1. Charged current one pion production cross section induced by neutrinos on proton target ($\nu_\mu + p \to \mu^- + p + \pi^+$). Experimental points are the ANL and the BNL data and the dashed-dotted line is the NUANCE cross section taken from Wascko [18]. The various theoretical curves show the cross section calculated using weak N-Δ transition form factors given by Schreiner and von Hippel [14] (double dashed-dotted line), Paschos *et al.* [31] (dashed line), and Lalakulich *et al.* [40](solid line).

used. On the other hand the experimental data from BNL by Kitagaki *et al.* [6] are higher and the experimental data from ANL by Radecky *et al.* [5] are lower than our theoretical predictions by 2–5 standard deviations depending upon the various N-Δ transition form factors used in this calculation. Clearly, better quality data on neutrino induced pion production is needed in order to determine the N-Δ transition form factors, for which various theoretical predictions exist [50] in addition to the three models considered in this work.

In Fig. 2, we show the total cross section for the incoherent [Fig. 2(a)] and the coherent [Fig. 2(b)] charged current single π^+ production from ^{12}C using the N-Δ transition form factors given by Lalakulich *et al.* [40]. We have presented the results for total scattering cross section $\sigma(E_\nu)$ without nuclear medium effects, with nuclear medium effects, with nuclear medium and pion absorption effects. For the incoherent process, we find that the nuclear medium effects lead to a reduction of around 12%–15% for neutrino energies $E_\nu = 0.7$–2 GeV. When pion absorption effects are taken into account along with the nuclear medium effects the total reduction in the cross section is around 30%–40%. For the coherent process, the nuclear medium effects lead to a reduction of around 45% for $E_\nu = 0.7$ GeV, 35% for $E_\nu = 1$ GeV, 25% for $E_\nu = 2$ GeV. The pion absorption effects taken into account along with the nuclear medium effects lead to a very large reduction in the total scattering cross section. The suppression in the total cross section due to nuclear medium and pion absorption effects in our model is found to be 80% for E_ν around 1 GeV and 70% for E_ν around 2 GeV [48]. Because of large reduction in the total cross section for the coherent process its contribution to the total charged current $1\pi^+$ production ($< 4\%$–5%) in the neutrino energy region of 1–2 GeV is found to be smaller than the predictions of the NUANCE neutrino generator [36].

We have calculated the ratio of the cross sections for inclusive charged current $1\pi^+(\mathrm{CC}1\pi^+)$ production to charged current scattering (CCQE) cross sections. For this purpose the cross section for quasielastic charged lepton production (σ(CCQE)) is calculated in this model [51,52] for the process $\nu_\mu + {}^{12}\mathrm{C} \to \mu^- + X$ using weak nucleon axial vector and vector form factors of BBBA05 (Bradford, Bodek, Budd, and Arrington) [53] with axial dipole mass $M_A = 1.05$ GeV and vector dipole mass $M_V = 0.84$ GeV. The Fermi motion and Pauli blocking effects in nuclei are included through the imaginary part of the Lindhard function for the particle hole excitations in the nuclear medium. The renormalization of the weak transition strengths are calculated in the random phase approximation (RPA) through the interaction of the p-h excitations as they propagate in the nuclear medium using a nucleon-nucleon potential described by pion and rho exchanges. The effect of the Coulomb distortion of muon in the field of final nucleus is also taken into account using a local version of the modified effective momentum approximation [51,54]. The details of the formalism and the relevant expressions for the cross section are given in Ref. [51]. We see that with the incorporation of various nuclear effects the total cross section is reduced. The reduction is energy dependent, and is quite large at lower energies. This is shown in Fig. 3. We see that with the incorporation of the various nuclear effects the total reduction in the cross section as compared to cross sections calculated without the nuclear medium modification effects is around 70% at $E_{\nu_\mu} = 200$ MeV, 45% at $E_{\nu_\mu} = 400$ MeV, 20% at $E_{\nu_\mu} = 0.8$ GeV, 18% at $E_{\nu_\mu} = 1$ GeV, and around 15% at $E_{\nu_\mu} = 1.4$ GeV. The theoretical uncertainty in the total cross sections due to various parametrizations of the electroweak form factors of the nucleon [51,53,55,56] has been studied and found to be small provided the same values for the axial vector dipole

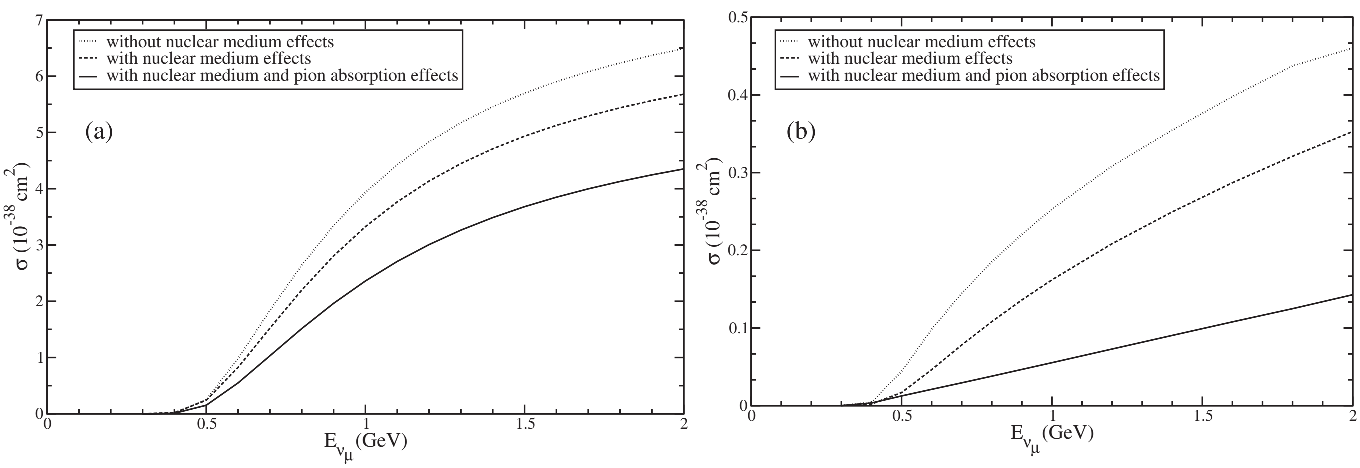

FIG. 2. Charged current one pion production cross section induced by neutrinos on ^{12}C target using the Lalakulich's [40] N-Δ weak transition form factors for (a) the incoherent and (b) the coherent processes. The dashed (dashed-dotted) line is the result with (without) the nuclear medium modification effects and the solid line is the result with the medium modification and pion absorption effects.

mass $M_A(= 1.05\ \mathrm{GeV})$ and vector dipole mass $M_V(= 0.84\ \mathrm{GeV})$ are used in all the parametrizations. Recently the K2K collaboration [57] has analyzed their low energy inclusive quasielastic lepton production data using dipole parametrization for the axial vector form factor with the axial dipole mass $M_A = 1.2$ GeV. If this value of the axial dipole mass is used then the cross section for the lepton production increases to $\sim$12% at $E_\nu = 1$ GeV as compared to the cross section calculated by using dipole parametrization with $M_A = 1.05$ GeV. Therefore, there could be an uncertainty of 10%–12% in the lepton production cross section associated with the value of M_A in the neutrino energy region considered in this work.

The numerical values of the total cross sections for $1\pi^+$ production shown in Figs. 2(a) and 2(b) and the total cross sections for inclusive quasielastic lepton production shown in Fig. 3 have been used to calculate the ratio $r = \frac{\sigma(\mathrm{CC}1\pi^+)}{\sigma(\mathrm{CCQE})}$ which is shown in Fig. 4, for the various parametrizations for N-Δ transition form factors given by Schreiner and von Hippel [14], Paschos *et al.* [31], and Lalakulich *et al.* [40]. We also show in this figure the experimental results for this ratio reported by the MiniBooNE collaboration [18]. We see that the theoretical predictions for the cross sections in our model are in satisfactory agreement with the experimental results for the ratio and are described within 1 standard deviation for the parametrization of N-Δ transition form factors considered in this work except for the parametrization of Schreiner and von Hippel [14] form factors for which $\chi^2_{\mathrm{pdf}} = 1.6$. We would like to emphasize that the nuclear medium and pion absorption effects in pion production processes as shown in Fig. 2 and the nuclear medium effects on the inclusive quasielastic process as shown in Fig. 3, play an important role in bringing about

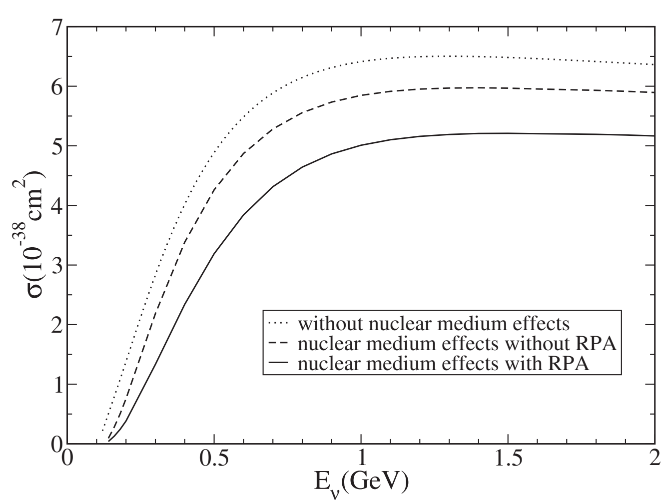

FIG. 3. Quasielastic charged current lepton production cross section induced by neutrinos on ^{12}C target using BBBA05 [53] weak nucleon form factors. The dotted line is the result for the free case and the solid line (dashed line) is the result with nuclear medium effects including RPA (without RPA).

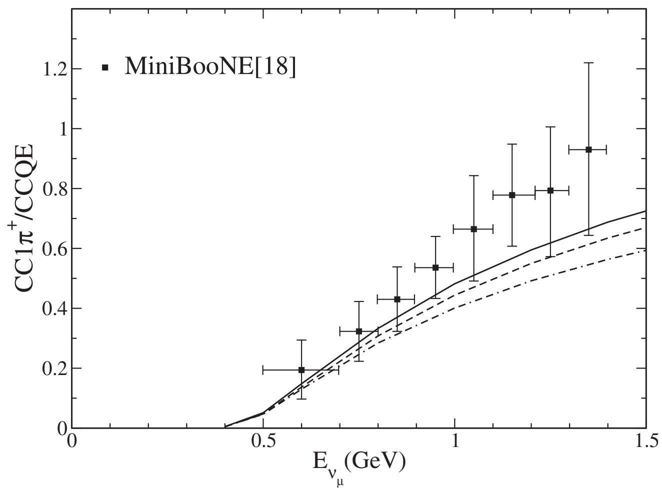

FIG. 4. $\frac{\sigma(\mathrm{CC}1\pi^+)}{\sigma(\mathrm{CCQE})}$ for the ν_μ induced reaction on ^{12}C. The experimental points are taken from Wascko [18]. The theoretical curves are obtained by using Schreiner and von Hippel [14] (double dashed-dotted line), Paschos *et al.* [31] (dashed line), and Lalakulich *et al.* [40] (solid line) weak N-Δ transition form factors for C.C.$1\pi^+$ production and Bradford *et al.* [53] weak nucleon form factors for CCQE.

this agreement. For a given choice of the electroweak nucleon form factors in the quasielastic sector, there is a theoretical uncertainty of 10%–20% in this ratio due to the use of various parametrizations for the N-Δ transition form factors shown in Table I. There is a further uncertainty of 2%–3% in this ratio due to the various electroweak nucleon form factors used in the calculations of the total cross section for the quasielastic production if the world average of $M_A = 1.05$ GeV is used.

In Fig. 5, we have shown the variation in the total cross section for the charged current $1\pi^+$ production for ν_μ induced reaction in ^{12}C due to the variation in the axial vector dipole mass M_A in the N-Δ transition form factors using the parametrization given by Lalakulich *et al.* [40]. The results are shown for $M_A = 1.0$ GeV, $M_A = 1.1$ GeV, and $M_A = 1.2$ GeV. We find that a 20% change in M_A results in a change of around 20% in the cross section which increases with M_A. In this figure, we have also shown the results predicted by the NUANCE [36] and NEUGEN [37] neutrino event generators. These theoretical results are compared with the experimental results reported by the MiniBooNE collaboration. These cross sections are obtained by multiplying the experimental ratio $r = \frac{\sigma(\mathrm{CC}1\pi^+)}{\sigma(\mathrm{CCQE})}$ given in Fig. 4 with the theoretical cross section for quasielastic production given by the model of Smith and Moniz [58] which does not include the effect of nuclear medium modifications due to RPA correlations in the quasielastic cross sections. These results agree quite well with our results for $1\pi^+$ production cross section, shown by dashed-dotted lines, when we do not include the nuclear medium modifications due to RPA correlations in

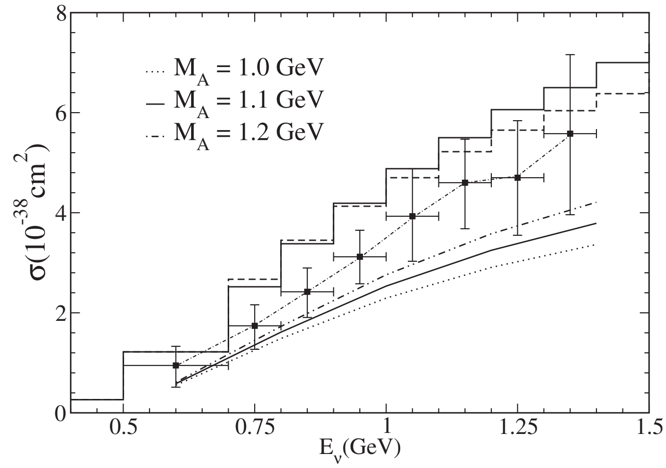

FIG. 5. $\sigma(\mathrm{CC}1\pi^+)$ for ν_μ induced reaction on ^{12}C. The dashed (solid) stairs are the cross sections from NEUGEN (NUANCE) Monte Carlo event simulation and the experimental points shown by a solid dot with error bars are the MiniBooNE results [18]. The theoretical curves show the $\mathrm{CC}1\pi^+$ cross section using Lalakulich *et al.* [40] weak N-Δ transition form factors for the various values of M_A. The dashed-dotted line is $\sigma(\mathrm{CC}1\pi^+)$ obtained by using the central value of the experimental results for the ratio $r = \frac{\sigma(\mathrm{CC}1\pi^+)}{\sigma(\mathrm{CCQE})}$ [18] (experimental points shown in Fig. 4) and $\sigma(\mathrm{CCQE})$ calculated in our model without RPA effects.

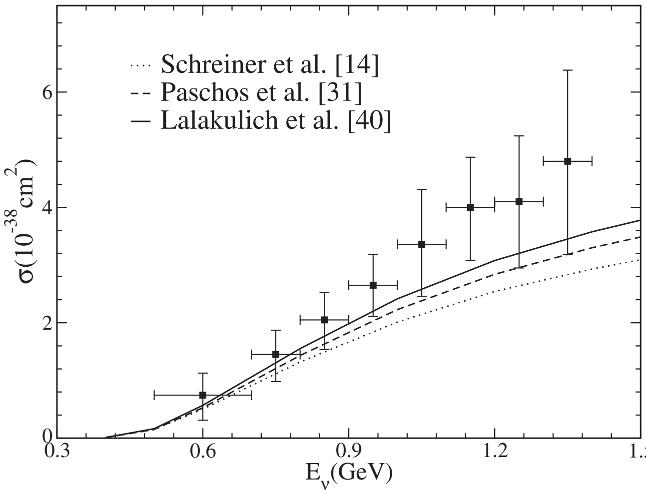

FIG. 6. $\sigma(\mathrm{CC}1\pi^+)$ for ν_μ induced reaction on ^{12}C. The theoretical curves show the cross sections for the various weak N-Δ transition form factors. The experimental points show $\sigma(\mathrm{CC}1\pi^+)$ obtained by using the experimental results for the ratio $r = \frac{\sigma(\mathrm{CC}1\pi^+)}{\sigma(\mathrm{CCQE})}$ [18] (experimental points shown in Fig. 4) and $\sigma(\mathrm{CCQE})$ calculated in our model with RPA effects.

the quasielastic cross sections. However, when the nuclear medium modification effects due to RPA correlations in the quasielastic production cross section shown in Fig. 3 are used to calculate the total cross section for $1\pi^+$ production by multiplying it by the ratio r (shown in Fig. 4) the cross sections are reduced. This is shown in Fig. 6. We see that the experimental results for the total $1\pi^+$ cross sections are now explained satisfactorily with the various parametrizations of N-Δ transition form factors within 1 standard deviation except for the parametrization of Schreiner and von Hippel for which $\chi^2_{\mathrm{pdf}} = 1.4$.

It can be seen from Fig. 5, that the theoretical predictions for the total charged current $1\pi^+$ production cross sections by the neutrino generators like NUANCE [36] and NEUGEN [37] overestimate the experimental cross sections as they do not include the nuclear effects appropriately which are known to reduce the cross sections. For example, the nuclear effects lead to a reduction of 30%–40% for the dominant process of incoherent production in this energy region as shown in Fig. 2(a), which is large compared to 10% reduction considered in the $T = 3/2$ channel in the NUANCE generator [36].

One may also consider the contribution of higher resonance excitations to $1\pi^+$ production in this energy region, for which there exist very few calculations in literature [31,40,59,60]. It has been shown by Paschos *et al.* [31] that the total cross section for neutrino induced excitation of higher resonances like Roper (1440), $S_{11}(1535)$, and $D_{13}(1520)$ is quite small. In an earlier calculation, Alvarez-Ruso *et al.* [59] have studied weak electroexcitation of Roper and recently Valverde and Vicente Vacas [60] have studied neutrino induced excitation of Roper and consequent pion production processes through excitation of this resonance. We have used these results to estimate one pion production in the energy region $E_\nu < 1.5$ GeV considered in this paper. It is found that the contribution to one pion production through the excitation of Roper resonance is around 2%–4% and the contribution of other higher resonance excitations to one pion production is quite negligible. Therefore, higher mass resonances are not expected to make any important contributions to pion production in this energy region.

In Fig. 7, we have presented the results for the differential cross section $\langle \frac{d\sigma}{dQ^2} \rangle$ vs Q^2 for charged current $1\pi^+$ production for the incoherent process averaged over the MiniBooNE and K2K spectrum for ν_μ induced reaction in ^{12}C [Fig. 7(a) for MiniBooNE) and ^{16}O [Fig. 7(b) for K2K). The various curves show the results with the nuclear medium modification and final state interaction effects and obtained by using the different N-Δ transition form factors given by Schreiner and von Hippel [14], Paschos *et al.* [31], and Lalakulich *et al.* [40]. We find that for the incoherent process in the peak region, $\langle \frac{d\sigma}{dQ^2} \rangle$ obtained by using Paschos *et al.* [31] and Lalakulich *et al.* [40] N-Δ transition form factors are, respectively, 4%–5% and 10% larger than the differential cross section obtained by using Schreiner and von Hippel [14] N-Δ transition form factors. In the inset of these figures we have also shown the effect of nuclear medium and pion absorption on $\langle \frac{d\sigma}{dQ^2} \rangle$ using N-Δ

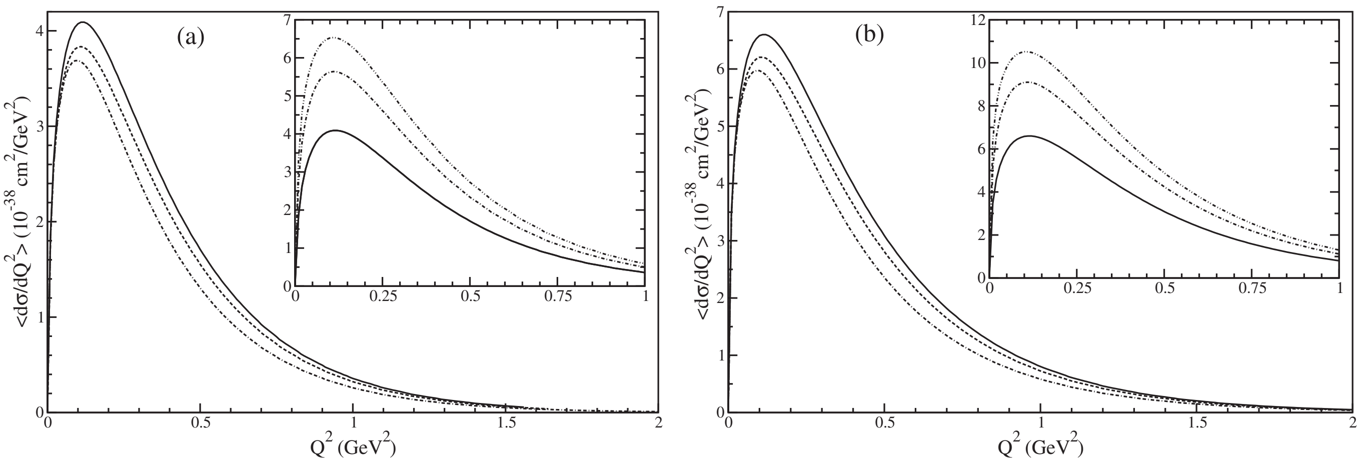

FIG. 7. $\langle \frac{d\sigma}{dQ^2} \rangle$ vs Q^2 for ν_μ induced reaction on ^{12}C averaged over the MiniBooNE spectrum [Fig. 7(a)] and on ^{16}O averaged over the K2K spectrum [Fig. 7(b)] for the incoherent process. The various curves are the differential cross sections for the charged current $1\pi^+$ production with nuclear medium and final state interaction effects and calculated by using Schreiner and von Hippel [14] (double dashed-dotted line), Paschos *et al.* [31] (dashed line), and Lalakulich *et al.* [40] (solid line) weak N-Δ transition form factors. In the inset we have also shown the nuclear medium modification effects on $\langle \frac{d\sigma}{dQ^2} \rangle$ vs Q^2 averaged over the MiniBooNE and K2K spectra using the Lalakulich's [40] N-Δ weak transition form factors. The dashed-dotted (dashed double dotted) line is the result with (without) the nuclear medium modification effects and the solid line is the result with the medium modification and pion absorption effects.

transition form factors given by Lalakulich *et al.* [40]. We find that for the incoherent process, the nuclear medium effects lead to a reduction in the differential cross section of around 14% in the peak region. When nuclear medium and final state interaction effects are taken into account the total reduction in the cross section is around 38%.

In Fig. 8, we have presented the results for the coherent process and shown the effect of nuclear medium and pion absorption effects on $\langle \frac{d\sigma}{dQ^2} \rangle$ averaged over the MiniBooNE and K2K spectrum for ν_μ induced reaction in ^{12}C [Fig. 8(a) for MiniBooNE) and ^{16}O [Fig. 8(b) for K2K) using N-Δ transition form factors given by Lalakulich *et al.* [40]. We find that the reduction in the differential scattering cross section $\langle \frac{d\sigma}{dQ^2} \rangle$ in the peak region, when nuclear medium effects are taken into account is around 35% and the total reduction is 85% when the pion absorption effect is also taken into account. The uncertainty due to the use of various parametrizations of the transition form factors is small in the case of the coherent process as it is dominated by the low Q^2 behavior of the form factor $C_5^A(Q^2)$ which is fixed by the generalized Goldberger Treiman relation at $Q^2 = 0$.

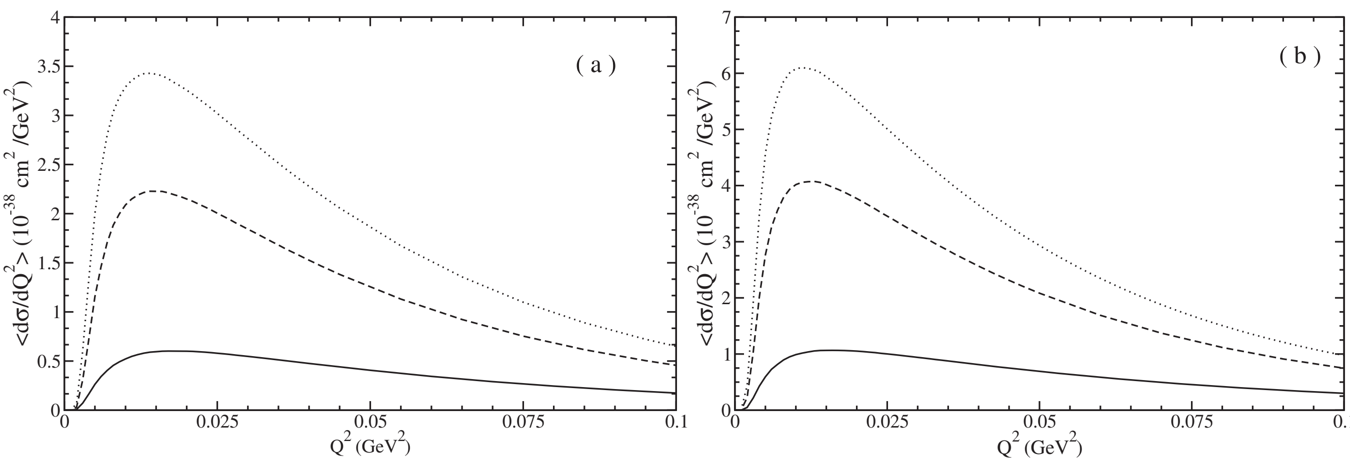

FIG. 8. $\langle \frac{d\sigma}{dQ^2} \rangle$ vs Q^2 for ν_μ induced reaction on ^{12}C averaged over the MiniBooNE spectrum [Fig. 8(a)] and on ^{16}O averaged over the K2K spectrum [Fig. 8(b)] for the coherent process using the Lalakulich's [40] N-Δ weak transition form factors. The dashed-dotted (dashed double-dotted) line is the result with (without) nuclear medium modification effects and the solid line is the result with medium modification and pion absorption effects.

V. SUMMARY AND CONCLUSION

We have studied neutrino induced charged current $1\pi^+$ production from proton, ^{12}C, and ^{16}O at the intermediate neutrino energies relevant for the MiniBooNE and the K2K experiments. The energy dependence of the total scattering cross sections for the charged current one pion production induced by ν_μ is studied. We have done the calculations for the incoherent and coherent production of pions from nuclear targets in the Δ dominance model which incorporates the modification of the mass and the width of Δ resonance in the nuclear medium and takes into account the final state interaction of pions with the final nucleus. We have presented the results for the total cross section for $1\pi^+$ production from ^{12}C and studied the energy dependence of the ratio of single π^+ production to the quasielastic reaction. The results have been compared with the preliminary results available from the MiniBooNE experiment. We have also presented the numerical results for Q^2 distribution, i.e. $\langle \frac{d\sigma}{dQ^2} \rangle$ in ^{12}C and ^{16}O averaged over the MiniBooNE and K2K spectra, respectively.

From this study we conclude that:

(1) The total cross sections for neutrino induced $1\pi^+$ production from free proton are closer to the π^+ production cross sections obtained by the ANL experiment and are smaller than the π^+ production cross sections obtained by the BNL experiment in the intermediate energy region. In this energy region, there is a 10%–20% theoretical uncertainty in the total cross section due to use of various parametrization of N-Δ transition form factors.

(2) The total cross sections for $1\pi^+$ production is dominated by the incoherent process. The contribution of the coherent pion production is about 4%–5% in the energy region of 0.7–1.4 GeV.

(3) In the neutrino energy region of 0.7–1.4 GeV, the results for the ratio of cross section of $1\pi^+$ production to the quasielastic lepton production is described quite well for $E_\nu < 1.0$ GeV, when nuclear effects in both the processes are taken into account. However, for energies higher than $E_\nu > 1.0$ GeV, the theoretical value of the ratio underestimates the experimental value.

(4) The role of nuclear medium effects is quite important in bringing out the good agreement between the theoretical and experimental value of the ratio for the total cross sections for $1\pi^+$ production and quasielastic lepton production for neutrino energies up to 1.0 GeV. For $E_\nu = 1$ GeV, the nuclear medium effects reduce the charged current quasielastic scattering cross section by 18%, while $1\pi^+$ production cross section is reduced by 40%.

(5) The results for $\langle \frac{d\sigma}{dQ^2} \rangle$ vs Q^2 in ^{12}C and ^{16}O averaged over the MiniBooNE and K2K spectra have been presented for the incoherent and coherent charged current one pion production with various N-Δ transition form factors. We have also presented the results for the nuclear medium and the final state interaction effects on the Q^2 distribution.

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