

Measurement of $P_\mu \xi$ in polarized muon decay

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The quantity $P_\mu^\pi \xi$, where ξ is one of the muon decay parameters and P_μ^π is the degree of muon polarization in pion decay, has been measured. The value $P_\mu^\pi \xi = 1.0003 \pm 0.0006(\text{stat.}) \pm 0.0038(\text{syst.})$ was obtained. This result agrees with previous measurements but is over a factor of 2 more precise. It also agrees with the Standard Model prediction for $P_\mu^\pi \xi$ and thus leads to restrictions on left-right symmetric models.

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I. INTRODUCTION

In the Standard Model (SM) of particle physics, positive muons decay via the weak ($V - A$) interaction into positrons plus neutrinos: $\mu \rightarrow e \nu \bar{\nu}$ through a virtual state involving W vector bosons. More generally, the amplitude for muon decay can be described in terms of a local decay matrix element, which is invariant under Lorentz transformations:

$$M = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \epsilon,\mu=R,L}} g_{\epsilon\mu}^\gamma \langle \bar{e}_\epsilon | \Gamma^\gamma | \nu \rangle \langle \bar{\nu} | \Gamma_\gamma | \mu_\mu \rangle, \quad (1)$$

where the $g_{\epsilon\mu}^\gamma$ specify the scalar, vector, and tensor couplings between μ -handed muons and ϵ -handed positrons [1]. In the SM $g_{LL}^V = 1$, and all other coupling constants are zero.

The differential decay spectrum [2] of the e^+ emitted in the decay of a polarized μ^+ can be described by four parameters— ρ , δ , η , and ξ —commonly referred to as the Michel parameters, which are bilinear combinations of the coupling constants. In the limit where the positron and neutrino masses are neglected, and radiative corrections [3]

are not explicitly included, this spectrum is given by:

$$\begin{aligned} \frac{d^2\Gamma}{dx d(\cos\theta)} &\propto 3(x^2 - x^3) + \frac{2}{3}\rho(4x^3 - 3x^2) \\ &+ P_\mu \xi \cos\theta(x^2 - x^3) \\ &+ P_\mu \xi \delta \cos\theta \frac{2}{3}(4x^3 - 3x^2), \quad (2) \end{aligned}$$

where θ is the angle between the muon polarization and the outgoing positron direction, $x = E_e/E_{\text{max}}$, $E_{\text{max}} = (m_\mu^2 + m_e^2)/2m_\mu = 52.83$ MeV, and P_μ is the degree of muon polarization. The fourth parameter, η , appears in the isotropic term when the positron mass is included in the analysis. In the SM, the Michel parameters take on the precise values $\rho = \delta = 0.75$, $\xi = 1$, and $\eta = 0$. The parameter ξ expresses the level of parity violation in muon decay, while δ parametrizes its momentum dependence.

In this experiment P_μ is the magnitude of the μ^+ polarization along the beam axis at the time of muon decay. Surface μ^+ [4], which are muons produced from π^+ decays at rest, have a polarization of magnitude P_μ^π , with a direction antiparallel to their momentum, given by a generalization of Eq. (1) for semileptonic decays. In the SM with massless neutrinos $P_\mu^\pi = 1$. In this experiment $P_\mu \xi$ is determined from the positron spectrum, and then $P_\mu^\pi \xi$ is obtained using the measurements of the muon trajectories.

SM extensions involving right-handed interactions [5] require deviations from pure $V - A$ coupling that can alter $P_\mu \xi$. Four probabilities $Q_{\epsilon\mu}(\epsilon, \mu = R, L)$ for the decay of

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a μ -handed muon into an ϵ -handed positron are given by:

$$Q_{\epsilon\mu} = \frac{1}{4}|g_{\epsilon\mu}^S|^2 + |g_{\epsilon\mu}^V|^2 + 3(1 - \delta_{\epsilon\mu})|g_{\epsilon\mu}^T|^2, \quad (3)$$

where $\delta_{\epsilon\mu} = 1$ for $\epsilon = \mu$ and $\delta_{\epsilon\mu} = 0$ for $\epsilon \neq \mu$. The probability:

$$\begin{aligned} Q_{\epsilon R} &= \frac{1}{4}|g_{LR}^S|^2 + \frac{1}{4}|g_{RR}^S|^2 + |g_{LR}^V|^2 + |g_{RR}^V|^2 + 3|g_{LR}^T|^2 \\ &= \frac{1}{2} \left[1 + \frac{1}{3}\xi - \frac{16}{9}\xi\delta \right], \end{aligned} \quad (4)$$

sets a model independent limit on any muon right-handed couplings [1,6]. A recent review of muon decay is presented in [7].

A precision measurement of muon decay can place limits on left-right symmetric (LRS) models [5]. The LRS models contain four charged gauge bosons (W_1^\pm , W_2^\pm), the photon, and two additional massive neutral gauge bosons. The W_1 and W_2 masses are m_1 and m_2 respectively, and the fields W_L and W_R are related to the mass eigenstates W_1 and W_2 through a mixing angle ζ . In these models both $V - A$ and $V + A$ couplings are present, and parity violation appears because of the difference in the mass of the vector bosons. In the SM the weak interaction contains only two left-handed vector bosons, W_L^\pm .

In the general LRS model [5],

$$\xi \approx 1 - 2 \left[\left(\frac{g_R}{g_L} \frac{m_1}{m_2} \right)^4 + \left(\frac{g_R}{g_L} \zeta \right)^2 \right], \quad (5)$$

where g_R and g_L are the right- and left-handed gauge couplings. The manifest LRS model makes the additional assumptions that $g_R = g_L$ and that the left- and right-handed quark mixing matrices are identical. In this case, P_μ^π can also be expressed in terms of m_1/m_2 and ζ , and one obtains [5]:

$$P_\mu^\pi \xi \approx 1 - 4 \left(\frac{m_1}{m_2} \right)^4 - 4\zeta^2 - 4 \left(\frac{m_1}{m_2} \right)^2 \zeta. \quad (6)$$

Prior to TWIST, the most precise direct measurement of $P_\mu^\pi \xi$ was $1.0027 \pm 0.0079(\text{stat.}) \pm 0.0028(\text{syst.})$ [8], in agreement with the SM. A similar value has been measured using muons from kaon decay [9]. Recently the TWIST collaboration reported new measurements of $\rho = 0.75080 \pm 0.00032(\text{stat.}) \pm 0.00097(\text{syst.}) \pm 0.000023(\eta)$ [10] and $\delta = 0.74964 \pm 0.00066(\text{stat.}) \pm 0.00112(\text{syst.})$ [11]. Using the result $P_\mu^\pi \xi \delta / \rho > 0.99682$, at the 90% confidence level [12], along with the TWIST measurements of ρ and δ , an indirect limit on $P_\mu^\pi \xi$ was determined to be $0.9960 < P_\mu^\pi \xi \leq \xi < 1.0040$ (90% confidence level) [11]. In this paper a new measurement of $P_\mu^\pi \xi$ is reported.

II. EXPERIMENTAL PROCEDURES

In the present experiment, highly polarized surface muons [4] were delivered, in vacuum, to the TWIST spec-

trometer [13] by the M13 channel at TRIUMF [14]. The properties of the surface muon beam are a typical rate of 2.5 kHz, a momentum of 29.6 MeV/c, and a momentum bite, $\Delta p/p \approx 1.0\%$ FWHM. Beam positrons with a typical rate of 22 kHz, and the same momentum as the muons are also a feature of the surface muon beam.

A low pressure (8 kPa dimethyl ether gas) removable beam monitoring chamber [15] provides information on the muon beam before it traverses the fringe field of the solenoid. The chamber consists of two 4 cm long modules, one to measure the position and direction of the muon beam in the horizontal (x) direction, and the other for measurements in the vertical (y) direction. The location of the final quadrupole of the M13 channel, the beam monitor, muon ranging gas degrader, trigger scintillator, and the TWIST detector are shown in Fig. 1. The beam monitor is inserted for measurement of the beam properties, and removed during data collection.

Changing the angle of the muon beam relative to the magnetic field axis gives rise to a change in the polarization. The solenoid field is found to interact with the iron of the beam line magnets, such that the muon beam is deflected off axis. The available M13 channel magnets could only partially alleviate this deflection. Figure 2(a) shows that the beam is centered in x , but not in y . The measured muon beam distribution in position and angle at the beam monitor, as shown in Fig. 2, is input into a simulation to calculate the average depolarization of the muons from the location of the beam monitor to the high-purity Al muon

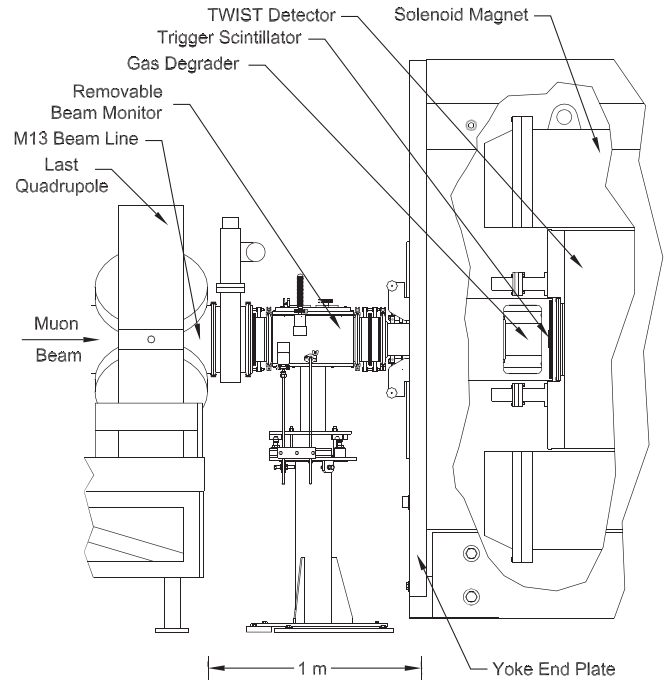


FIG. 1. Location of the last beam line quadrupole, beam monitor, gas degrader, trigger scintillator, and the TWIST solenoid.

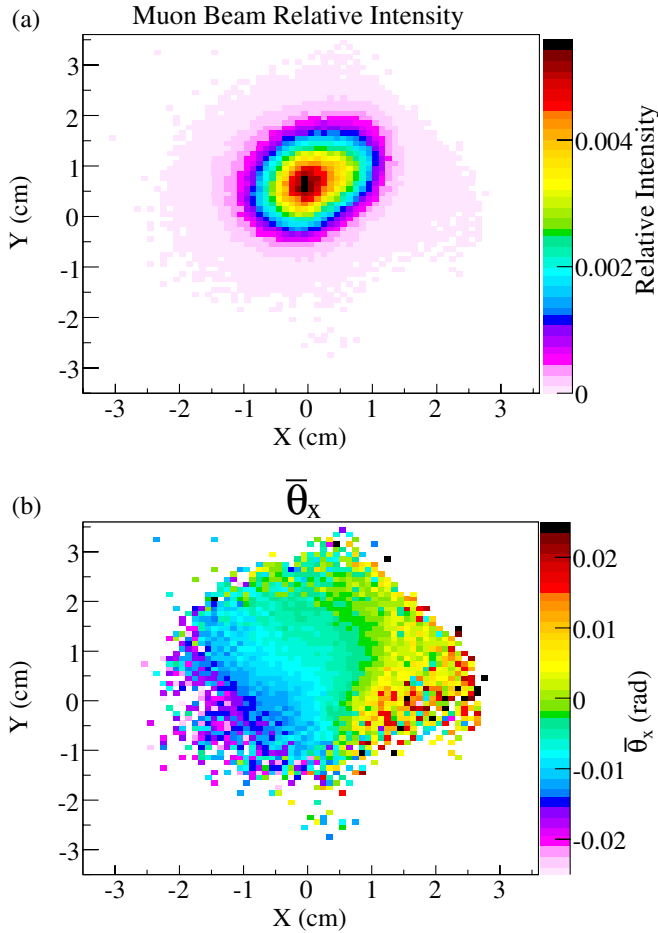


FIG. 2 (color online). Beam measurements projected to the center of the beam monitor at each 1 mm by 1 mm bin in y versus x . (a) Muon beam intensity profile. (b) Mean angle in x ($\bar{\theta}_x$).

stopping target at the center of the TWIST detector (Fig. 3). The angular distribution in x is shown in the figure, but similar distributions are measured in y and these are also included in the simulation [15]. The RMS size of the beam is 0.6 cm in both x and y , and the average angular spread of the beam is 10 mrad in x and 12 mrad in y .

The muon beam is transported in vacuum until it is inside the solenoid field, where it then passes through the gas degrader and trigger scintillator. The gas degrader is a 21.67 cm long (along z) chamber containing a mixture of He and CO₂ that can be adjusted to stop the muons in the stopping target. The plastic (Bicron BC408) disk shaped trigger scintillator has a thickness of 195 μm , a radius of 3 cm, and is located 80 cm upstream of the central stopping target.

The materials that the muons pass through, starting from the production target to the center of the stopping target, are summarized in Table I.

The TWIST spectrometer [13] is designed to measure a broad range of the positron spectrum from muon decays in a stopping target, allowing the simultaneous extraction of

the spectrum shape parameters. The spectrometer consists of 12 very thin high-precision proportional chamber (PC) planes and 44 drift chamber (DC) planes, perpendicular to the axis of a solenoid producing a magnetic field of 2 T. The upstream half of the spectrometer (22 DCs and 6 PCs) is shown in Fig. 3, and the downstream half of the detector is mirror symmetric with the upstream.

The gas particle detectors used by TWIST are mounted in a leak tight aluminum cradle that is filled with 3% N₂ and 97% He gas. Helium, rather than air, is used to reduce the scattering between measurement points and to allow the low energy muons to reach the stopping target at the center of TWIST. The spacing of the PCs and DCs is maintained by precision ground spacers made of a Russian engineered material Sital CO-113M which has a very small thermal coefficient of expansion. The spacers are pushed together by four pneumatic cylinders with a force of 1470 N each to ensure they are touching and do not move.

The DCs are used to obtain a precise measurement of the position of the e^+ as it traverses the chambers. To accomplish this a slow drift gas, dimethyl ether (DME), which has a small angle between the drift field and the electron drift direction in a nonzero magnetic field (Lorentz angle), is used. Each DC plane consists of 80 gold-plated tungsten anode sense wires of 15 μm diameter, spaced by 4 mm [16]. The cathode foil walls are 6.35 μm of aluminized Mylar, nominally placed 2 mm from the wires. The drift time for each signal is used along with a space-to-time relationship (STR) that is calculated by a GARFIELD simulation [17] to improve the determination of the momentum and angle of the decay positrons.

The purpose of the PCs is to have reasonably fast timing information to help with pattern recognition. The width of the time to digital converter (TDC) signal from the PCs is used to discriminate muons from positrons. The PCs are similar to the DCs, with the exception of the following: the PCs use CF₄/isobutane gas, a wire spacing of 2 mm, and 160 wires in each plane. The PC modules consist of four wire planes. One PC module is placed at either end of the detector stack, and the specialized target module in the center of the detector stack contains two PCs on either side of the muon stopping target.

Collection of an event is triggered by the muon scintillator whose threshold is set high enough to reduce the beam positron trigger rate but low enough to trigger on most beam muons. The fraction of triggers due to beam positrons is only 10%. Positrons in the beam are easily removed from the data since they leave signals through the full length of the detector.

Events in TWIST are recorded by LeCroy Model 1877 TDCs. The trigger and TDC readout are set up to record signals in 500 ps time bins from 6 μs before to 10 μs after a muon passes through the trigger scintillator. For each wire signal, the TDC records a pulse start and stop, and

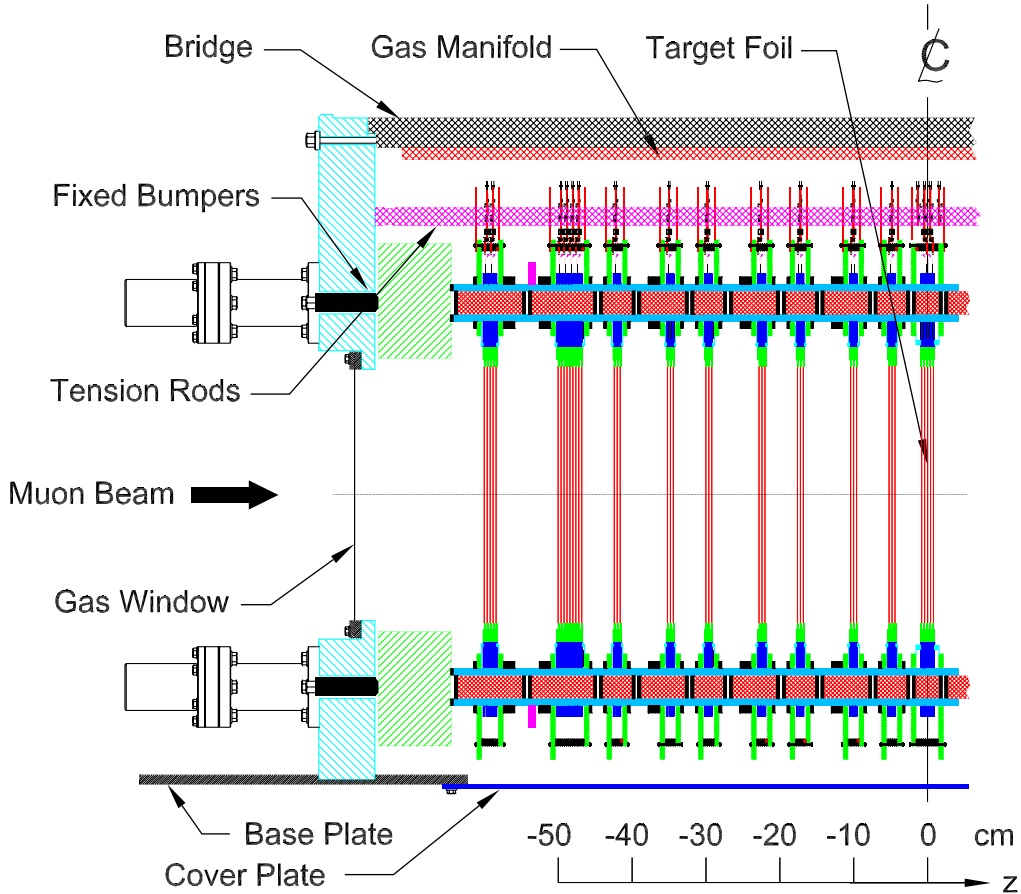


FIG. 3 (color online). Side view of the upstream half of the TWIST spectrometer planar chambers and support structure.

thus can be used to calculate a pulse width. In this configuration up to eight wire signals can be recorded for each wire in any triggered event [18]. A fixed blanking time of

TABLE I. Estimates of the material thicknesses in mg/cm^2 that the muon penetrates from the production target to the center of the muon stopping target. Surface muon momenta are such that this approximately matches the muon range for a degrader containing 50% CO_2 .

Material	Thickness (mg/cm^2)
Degrader and vacuum foils	11.91
He/ CO_2 degrader	1.95 to 42.80
Air gap (2.82 cm)	3.65
Muon scintillator	20.12
Scintillator wrap	3.03
Cradle window	0.88
PC module	9.46
Dense stack	13.41
Seven UV modules	27.80
He/ N_2 (63.8 cm)	12.25
Half target module before target	4.31
Half of $71 \mu\text{m}$ Al target	9.59
Total to center of Al target	118.36 to 159.21

$80 \mu\text{s}$ was imposed after each accepted trigger. This blank is sufficiently long to allow each TDC to finish conversion. To ensure that all TDCs have fully converted their hits, each one was required to produce a header specifying how many hits were converted even when zero hits were received.

For this measurement a $71 \pm 1 \mu\text{m}$ thick, 99.999% pure, Al target is used to stop muons. The tracks of the selected muons are required to have at least one signal in the PC immediately before the Al target and no signals in the PC immediately after the Al target. The average z from the last chamber plane fired by the muons (\bar{z}_μ) is used in a feedback loop to control the fractions of He and CO_2 in a gas degrader, in order to stop $97.0 \pm 1.5\%$ of the selected muons in the Al target at the center of the solenoid. Some muons that stop in the CF_4 /isobutane or Mylar foils in the vicinity of the Al target could not be separated and were included in the measurement.

III. DATA ANALYSIS

TWIST determines the Michel parameters by fitting two-dimensional distributions of reconstructed experimental decay positron momenta and angles with distributions of reconstructed simulated data [11].

The nominal fiducial region adopted for this analysis requires $p < 50$ MeV/c, $|p_z| > 13.7$ MeV/c, $p_T < 38.5$ MeV/c, and $0.50 < |\cos\theta| < 0.84$. The fiducial cuts, while intentionally chosen to be conservative, are related to physical limitations of the TWIST detector. The 50 MeV/c momentum cut rejects events that are near the region utilized in the energy calibration. It is also important to avoid the region very close to the end point to minimize the sensitivity of the Michel parameter fits to details of the simulation that may affect the momentum resolution. The longitudinal momentum constraint eliminates events with helix pitch near the 12.4 cm periodicity in the wire chamber spacing. The transverse momentum constraint ensures that all decays are well confined within the wire chamber volume. The angular constraint removes events at large $|\cos\theta|$ that have worse resolution and events at small $|\cos\theta|$ that experience large energy loss and multiple scattering. These limits were fixed early in the analysis. The value of $P_{\mu\xi}^{\pi}$ was found to change by less than 0.0001 when the fiducial boundaries were moved by $\pm 2\%$ in momentum cut values and $\pm 10\%$ in $|\cos\theta|$ cut values.

The decay positrons spiral through the chambers producing signals on the wires, which are recorded by the TDCs. These helical tracks are subsequently reconstructed and analyzed to determine the positron energy and angular distributions. Determining the positron momentum and angle is done with a χ -squared fit to a helical track that includes the drift time information from each cell. The helix fits also include positron multiple scattering in the χ^2 calculation using the procedure from [19]. The efficiency of the track fitting is $\geq 99.5\%$ within the nominal fiducial region used for spectrum fitting [10,11]. The number of degrees of freedom (DOF) varies with track angle, since higher angle tracks will pass through more DC cells per plane. For the average DOF, the most probable χ^2/DOF is $29.7/28 = 1.06$, but there is a long tail to larger values because the non-Gaussian tails of the multiple scattering distributions are not treated properly in the χ^2 calculation and because we only use approximate STRs. The momentum resolution is typically 100 keV/c, and the $\cos\theta$ resolution is about 0.005 [10].

In event selection, the PC and DC signals are examined to identify events in which the muon stopped in the target, then decayed at least $1.05 \mu\text{s}$, and no more than $9 \mu\text{s}$, later. The delay insures that the PC and DC signals associated with the muon and decay positron do not overlap. Events are rejected if a second muon enters the spectrometer, or if a beam positron passes through the spectrometer within $1.05 \mu\text{s}$ of either the muon arrival or decay time. Additional cuts include the muon flight time through the M13 beam line and a requirement that the muon stopping location be within 2.5 cm of the detector axis. All events that pass these cuts are analyzed to reconstruct the decay positron kinematics.

After track fitting, $\sim 2\%$ of the events contain additional tracks in coincidence with the decay. Extra tracks can arise from beam particles that are not resolved in time, events that scatter within the detector leading to two reconstructed track segments, and events that include delta rays or decay positrons that backscatter from material outside the detector volume. Algorithms were developed to identify events with backscatters and reject coincident beam particles [11].

The energy calibration of the decay positrons is obtained from a fit to the end point of the spectrum. The fit for the end point is done separately from the fit for the muon decay parameters, and to avoid bias the data from the region of momentum used in the end point fit are excluded from the determination of the muon decay parameters. The end point fit function is a slope with an edge ($f(x) = ax + b$ for $x \leq 1$, and $f(x) = 0$ for $x > 1$) convoluted with a Gaussian σ . The parameters a , b , and σ all depend on $\cos\theta$ [20]. The end point of the muon decay spectrum and sections of the two-dimensional end point fit function for the bins within the fiducial region with the smallest upstream and downstream angles are shown in Fig. 4. The difference in yield between upstream and downstream emphasizes the asymmetry of polarized muon decay. The corrected momentum p_{ec} is given by:

$$p_{ec} = p_{rec} \left(1 + \frac{\beta}{p_{edge}} \right)^{-1} + \frac{\alpha}{|\cos\theta|}, \quad (7)$$

where p_{rec} is the reconstructed momentum, $\cos\theta$ is the reconstructed cosine of the decay positron angle, p_{edge} is the maximum positron momentum, β defines the momentum scale related to the magnitude of the spectrometer magnetic field, and $\alpha = (\alpha_u, \alpha_d)$ is the zero angle energy loss for upstream (u) or downstream (d) decay positron tracks. This simple form was chosen because of the planar geometry of the wire chambers, such that the amount of

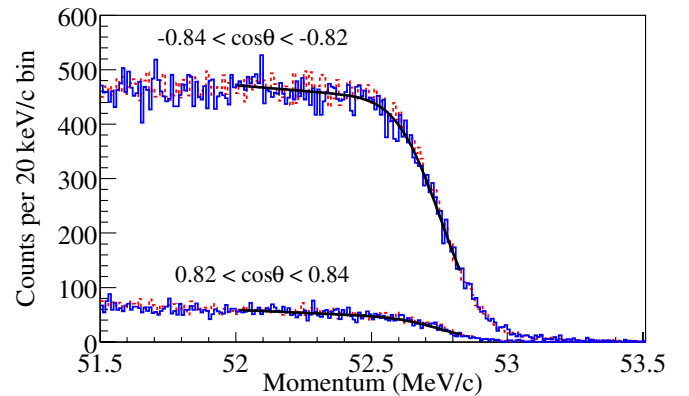


FIG. 4 (color online). Sections of the 2-dimensional end point fit function of the muon decay spectrum for the bins within the fiducial region containing the smallest upstream and downstream angles. The data are shown as a solid line, the matching simulation set is shown as a dashed line, and the end point fit functions are shown as smooth curves.

material the positron passes through increases linearly as $1/\cos\theta$.

The end point fit parameter β is highly correlated with $\alpha_{\text{sum}} = \alpha_{\mu} + \alpha_d$. In the simulation the momentum is measured without bias at the few keV/c level, the magnetic field is measured with an NMR probe, and the magnetic field map has been determined to better than 0.2 mT in the tracking region. For this reason the end point fits are done with the value of β set to zero and assigned an uncertainty consistent with the momentum fit and field map accuracy. A 12 keV/c difference between the data and simulation α_{sum} values was observed, and this is corrected by applying the energy calibration.

The TWIST simulation is based on GEANT 3.21 [21]. The simulation contains virtually all the components of the spectrometer with which a muon or a decay positron could interact. The output exactly mimics the binary files generated by the data acquisition system. Details of the simulation have been presented previously [10,11].

In bringing the muon beam to a stop in the Al target at the center of the TWIST spectrometer, the muons are depolarized by the combined effect of multiple scattering, interaction with the fringe field of the spectrometer, and interactions when stopped in the high-purity Al target. Thus, the polarization of the muon with respect to the z axis when it decays is lower than its polarization with respect to the muon momentum when it was produced in pion decay. To obtain an absolute measurement of $P_{\mu}^{\pi}\xi$, data are fit to a simulation that includes effects of fringe field depolarization and material depolarization.

The main factors that influence the difference between P_{μ}^{π} and P_{μ} are the transport of the muon spins in the various regions of magnetic field and models for the muon depolarization in materials. Transport of the muon spins from the beam measurement location to the muon stopping target is done using a classical fourth order Runge-Kutta using the Nystrom algorithm [22], of the Bargmann-Michel-Telegdi [23] equation. The inherent accuracy of this numerical integration is excellent, but depends on the accuracy of the knowledge of the input beam parameters and the magnetic field map. For this reason, the beam monitoring chamber measurements were used to generate the muon beam for the simulation.

Ideally the magnetic field is meant to be uniform and aligned with the axis of the muon beam momentum (and antialigned with the muon spin), which is labeled as the $+z$ coordinate in TWIST. The field shaping elements of the magnet were input into a finite element analysis which models the magnetic field in order to create a field map that included the radial components of the field as well as the longitudinal components at each of the positions at which the longitudinal component was measured.

The magnetic field of the solenoid was mapped using a specially constructed mapping tool which used Hall probes, and NMR magnetometers to measure the magnetic

field. The component of the magnetic field along z was mapped in the uniform field region and in the beam monitoring entrance region. A field map was generated that matches the measurements to 0.2 mT in 2.0 T in the uniform region, but deviates up to 4% in the entrance region at $z < -200$ cm. The effects of the steel from the last two quadrupoles of the M13 beam line were studied using field maps from the finite element analysis that included or excluded the iron of the last three beam line elements. The map produced from this study was used to estimate a systematic error on $P_{\mu}^{\pi}\xi$ due to the uncertainty in the magnetic field in the fringe field region.

A blind analysis was implemented by utilizing hidden Michel parameters ρ_H , δ_H , and ξ_H to generate the simulated decay rate. The decay rate can be written as:

$$\frac{d^2\Gamma}{dx d(\cos\theta)} \Big|_{\rho_H, \delta_H, \xi_H} + \sum_{\lambda=\rho, \xi, \delta} \frac{\partial}{\partial \lambda} \left[\frac{d^2\Gamma}{dx d(\cos\theta)} \right] \Delta \lambda,$$

because the decay spectrum is linear in the shape parameters. The simulation spectrum was fit to the data spectrum by adjusting $\Delta\rho$, $\Delta\xi$, and $\Delta\xi\delta$.

An alternate analysis scheme, used only to compare relative polarizations, was developed using an integral asymmetry defined as the difference between the number of forward and the number of backward decays divided by their sum. To obtain a polarization estimate the forward and backward sums were done inside the fiducial region described earlier in this section and normalized using integrals of Eq. (2) with the SM values of the Michel parameters inserted.

IV. EVALUATION OF SYSTEMATICS

The leading systematic uncertainties in this measurement of $P_{\mu}^{\pi}\xi$ arise from the potential sources of muon depolarization. These include depolarization due to multiple scattering in the production target and a 3 μm beam line foil, fringe field depolarization, and interactions with material while the muon is propagating through the detector and after stopping.

The depolarization in the production target is due to multiple scattering of the muons while exiting the target. An estimate of the depolarization in the small angle approximation is $(\theta_{\text{space}}^{\text{RMS}})^2/2$, where $\theta_{\text{space}}^{\text{RMS}}$ is the width of the multiple scattering angular distribution [6]. The muons in the beam arise from a maximum depth of 0.003 cm of graphite, which produces a depolarization of 0.2×10^{-3} which is taken as the systematic uncertainty in $P_{\mu}^{\pi}\xi$ due to multiple scattering in the production target.

The systematic uncertainty in the fringe field depolarization is estimated from the different settings used in data taking for the second dipole element (B2) in the M13 channel. The beam parameters were measured for two different B2 settings, both before and after the data collection. The relative changes in angle and position between

the nominal B2 value (94.4 mT) and B2 + 0.5% settings are similar for the two periods, but the absolute numbers for the average beam angles are quite different. This could be due to changes in the performance of the beam monitoring chamber or to its alignment to the beam line. To determine the sensitivity of the polarization to beam position and angle, a simulated beam was scanned in position and angle and the polarization was found to depend quadratically on the input variables. Using this parameterization, the predicted polarizations for the four characterization runs are shown in Table II. The beam for the third row of Table II is illustrated in Fig. 2. The larger of the differences in predicted polarization for a given B2 setting (0.0033) is adopted as an estimate of the uncertainty due to limits of reproducibility.

Uncertainties due to deconvolution of the beam angle measurement, modeling of the shape of the solenoid fringe field, and beam size reproduction also contribute to the final quoted systematic uncertainty of 0.0034 due to fringe field depolarization.

The depolarization of the muons while they propagate through the detector and interact with the detector materials is negligibly small for muons with kinetic energy between 4 MeV and 100 keV [24]. Simulations show that greater than 99% of muons stopping in our Al target have more than 100 keV kinetic energy. Muonium formation is thus suppressed by ensuring that the majority of the muons have sufficient energy entering the Al target. For those muons that do not stop in the Al target, the Paschen-Back effect minimizes but does not eliminate the depolarization due to muonium formation. Most of the muons stop in the high-purity Al target, where they can interact with conduction electrons. These electrons create a hyperfine magnetic field at the site of the muon, which can be considered as a fluctuating local field with a correlation time $\tau_c \simeq 10^{-13}$ s in Al [25]. This interaction results in a Korringa depolarization rate [26,27] that has an exponential form, and does not depend on the magnetic field. Significant depolarization rates of $\lambda > 0.001 \mu\text{s}^{-1}$ have been measured for muons in Cd, Sn, Pb, As, Sb, and Bi [27]. The authors explained the measured depolarization rates to be due to Korringa depolarization because the λ values increase with temperature as predicted.

TABLE II. Average beam positions and angles from beam monitoring measurements taken at different times, along with the simulation estimates of the muon polarization. The first (last) two entries are from before (after) the data collection.

B2 (mT)	\bar{x} (cm)	$\bar{\theta}_x$ (mrad)	\bar{y} (cm)	$\bar{\theta}_y$ (mrad)	P_{μ}^{sim}
94.4	0.07	-5.9	0.97	7.0	0.9929
94.9	0.85	-1.1	0.87	-5.0	0.9955
94.4	0.06	-6.7	0.73	-11.2	0.9941
94.9	0.94	-1.5	0.64	-19.2	0.9922

Jodidio *et al.* [12] measured a depolarization rate of $(0.43 \pm 0.34) \times 10^{-3} \mu\text{s}^{-1}$ for their Al target at 1.1 T. This rate is about 2.5σ smaller than the $(1.55 \pm 0.28) \times 10^{-3} \mu\text{s}^{-1}$ observed in this experiment. The difference could partly be due to the 2.5 to 5.5% of the muons that stop in the gas before the stopping target. The functional form of the depolarization in gases is unknown, thus an assumption that there is no unseen rapid depolarization is made. The spin relaxation, with mean lattice-site residence time τ_c , is given approximately by the Kubo-Tomita expression [28], which reduces to Gaussian (exponential) forms for $\tau_c \rightarrow \infty$ ($\tau_c \rightarrow 0$).

The difference between Gaussian and exponential extrapolations of the integral asymmetry measurement, as shown in Fig. 5, is 2.4×10^{-3} . Data before 1.05 μs are not considered because of possible contamination of late TDC signals from muons for upstream decay positrons. Changing the cutoff between 900 ns and 1100 ns has negligible effect on the asymmetry measurement extrapolated to zero time. Half the difference between the two different extrapolations is the *correction* applied to the simulation to data fits, because the simulation was generated with a Gaussian form, while in reality the shape is most likely a linear combination of a Gaussian and exponential. An estimate of the extrapolation uncertainty is half the difference between the Gaussian and exponential extrapolations.

Other systematic uncertainties were studied by employing the fitting technique described in the data analysis section. In this case the fits are of experimental data (or simulation) samples, taken with a systematic parameter set at an exaggerated level, to data (or simulation) taken under ideal conditions. The difference measured, or sensitivity, expresses the changes in the spectrum shape caused by the systematic effect in terms of the changes in the Michel parameters. Systematic uncertainties in the measurement of P_{μ}^{ξ} are summarized in Table III.

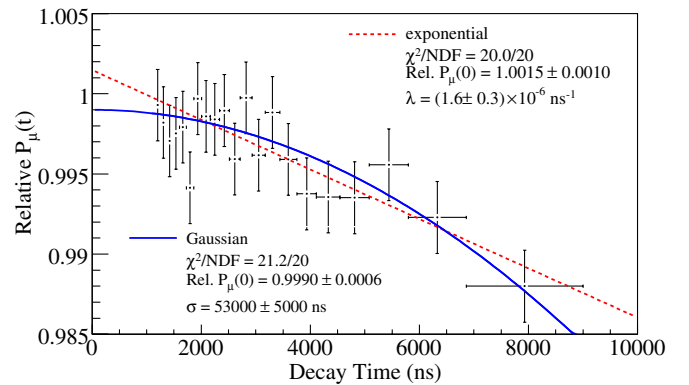


FIG. 5 (color online). Extrapolation to zero decay time of relative muon polarization, estimated using the decay positron integral asymmetry described in the text. The extrapolation function is fit to data: with an exponential shown as a dashed line, and as a Gaussian shown as a solid line.

TABLE III. Contributions to the systematic uncertainty for $P_{\mu}^{\pi}\xi$.

Effect	Uncertainty
Depolarization in fringe field (ave)	0.0034
Depolarization in stopping material (ave)	0.0012
Chamber response (ave)	0.0010
Spectrometer alignment	0.0003
Positron interactions (ave)	0.0003
Depolarization in production target	0.0002
Momentum calibration	0.0002
Upstream-downstream efficiency	0.0002
Background muon contamination (ave)	0.0002
Beam intensity (ave)	0.0002
Michel parameter η	0.0001
Theoretical radiative corrections	0.0001

The chamber response systematic uncertainty in $P_{\mu}^{\pi}\xi$ comes from the uncertainty due to the PC and DC response. Six sources of uncertainty, described in the following paragraphs, are: time variations in the wire-to-wire timing, changes to the STR from density variations, chamber foil bulge due to changes in differential pressure between the chamber gas and cradle helium, the asymmetry of the wire positions relative to the foils due to construction, and a dead zone due to a recovery time after a muon passes through the chambers.

To estimate the possible time variation in the wire-to-wire time zero (t_0), calibration runs are taken at the beginning and end of the run period. To measure the sensitivity of the muon decay parameters to t_0 variations, a calibration scaled by 10 times the difference in the beginning and end of run time calibrations is used. The contribution to the chamber response systematic uncertainty in $P_{\mu}^{\pi}\xi$ due to t_0 variations, obtained from fitting the exaggerated time shift analysis spectrum to a nominal spectrum, is 0.89×10^{-3} . A second calibration scaled by 5 times the difference in beginning and end of run, shows that this systematic uncertainty scales linearly.

The shape of the chamber cathode foils depends on the differential pressure between the chamber gas and the He gas of the cradle holding all of the chambers. The differential pressure of the chambers is monitored and controlled by the gas system. To measure the level of foil bulge, several nominal data runs are taken at different differential pressures. The average helix fit drift time difference, for tracks that go through the center of the chambers, relative to the average helix fit drift times for tracks that go through the radially distant part of the chamber, is sensitive to the foil bulge. Run to run monitoring of this fit time difference shows that the variation in foil bulge is controlled to better than $50 \mu\text{m}$. To measure the sensitivity of the muon decay parameters to the chamber foil bulge, a simulation generated with the STR calibration changed as if the foils were moved outward by $500 \mu\text{m}$ is used. The contribution to

the chamber response systematic uncertainty, in $P_{\mu}^{\pi}\xi$, due to time variations in the foil bulge, is estimated to be 0.22×10^{-3} .

Surveys of the constructed chambers show that the spacing between the wire plane and cathode foils is not the same on both sides of the wire. On average the cathodes are found to be shifted by about $150 \mu\text{m}$ from their nominal 2 mm spacing from the wire planes. With the solenoid field on, the position of the cathode foils was also determined using normal decay positron data. Using the bulge calibration data, from runs with different bulges, a relationship between a fit time difference to a foil shift is obtained. STR files were changed half plane by half plane to match the measured shifts. A fit of the muon decay distribution, from the shifted cathode foil simulation to a nominal simulation, results in a contribution to the chamber response systematic uncertainty in $P_{\mu}^{\pi}\xi$ of 0.2×10^{-3} .

History plots of the density variation for each data set show that the largest RMS change in density is $\pm 0.7\%$. To estimate the effect of density changes, a simulation was generated with the temperature changed by 10% (from 300 K to 270 K). A scaled fit of the changed temperature simulation set to a nominal simulation set is used to estimate the addition to the chamber response systematic uncertainty in $P_{\mu}^{\pi}\xi$ of 0.17×10^{-3} .

As the muon passes through the upstream half of the detector, it deposits large ionization in the chambers. The resulting electron avalanches near the wires take time to recover as the positive ions drift back to the cathode planes. These dead zones produce an upstream-downstream efficiency difference for decay positrons. For each muon hit the wire efficiency versus distance away from the muon hit is estimated as a function of time after the muon track. A determination of the muon decay parameter sensitivities to the muon dead zone is made by introducing an exaggerated dead zone into the simulation which removes 11.4 times as many hits as are expected in normal data. A conservative estimate of the contribution to the chamber response systematic uncertainty in $P_{\mu}^{\pi}\xi$ due to the muon dead zone is 0.01×10^{-3} .

The systematic uncertainty in $P_{\mu}^{\pi}\xi$ due to spectrometer alignment comes from how well the chamber translations in x , y , z and angle are corrected to match their true positions, and the degree to which the misalignment between the magnetic field axis to the chamber axis is treated.

The systematic uncertainty in $P_{\mu}^{\pi}\xi$ due to positron interactions includes four effects: a discrepancy between the simulation and data energy loss, hard and intermediate interactions, multiple scattering, and backscattering from material outside the detector.

The systematic uncertainty in $P_{\mu}^{\pi}\xi$ due to the momentum calibration comes from two contributions. One contribution is due to how well the end point energy calibration can be determined. The other factor is how well the measured magnetic field map represents reality.

The systematic uncertainty in $P_\mu^\pi \xi$ due to a difference in the upstream and downstream efficiencies is estimated from the mean number of hits used in fitting the upstream and downstream tracks. An upstream-downstream difference is expected from the different angular distributions, and is reproduced by the simulation. The observed discrepancy between data and simulation of 0.18 hits/track is a measure of the efficiency difference.

To estimate the sensitivity of the muon decay parameters, due to a difference in upstream versus downstream efficiency, a data set was analyzed with 5% of its downstream DC hits thrown away before analysis. This upstream-downstream efficiency change, compared to a normal analysis of the same data, produces a change in $P_\mu^\pi \xi$ of $(1.9 \pm 0.9) \times 10^{-3}$. The lowered efficiency run downstream track fits have 1.8 fewer hits/track relative to the standard run. An estimate of the systematic uncertainty in $P_\mu^\pi \xi$ due to a difference in upstream and downstream efficiency is ergo 0.2×10^{-3} .

The systematic uncertainty in $P_\mu^\pi \xi$ due to background muon contamination comes from muons which are born in pion decay in the vicinity of the gas degrader.

The beam intensity systematic uncertainty in $P_\mu^\pi \xi$ comes from changes in the beam rate which affects the rate of coincident particles.

Theoretical uncertainties in the radiative corrections introduce a small systematic uncertainty in $P_\mu^\pi \xi$.

Several of the systematic uncertainties could vary from data set to data set and are denoted by (ave), and are considered data set-dependent when calculating the weighted average value of $P_\mu^\pi \xi$. For example, the effect of positron interactions on upstream and downstream decay positrons changes when the mean muon stopping location is adjusted; thus the systematic uncertainty in $P_\mu^\pi \xi$ due to positron interactions is set-dependent.

V. RESULTS

The result for $P_\mu^\pi \xi$ presented here uses a data sample consisting of 2×10^9 events recorded in Fall 2004. This data sample includes eight data sets, of which seven were used for the extraction of $P_\mu^\pi \xi$. Simulations to fit each of the seven data sets used different beam characterization profiles, derived from beam measurements performed after the data collection, which matched different conditions under which the data were recorded. The remaining data set was used to determine the detector response using decay positrons from muons stopping in the trigger scintillator and the first few chamber planes (far upstream), as described in [10,11].

Five sets of data were taken with the beam steered nominally. One data set had the muon beam stopping with the Bragg peak centered in the target (stop $\frac{1}{2}$). Two sets, which were separated in time by a few days, were taken with the muon Bragg peak shifted to 3/4 of the way

TABLE IV. Results for $P_\mu^\pi \xi$. Each fit has 1887 DOF. Statistical and set-dependent systematic uncertainties are shown. A description of the data sets is in the text.

Data Set	$P_\mu^\pi \xi \pm \text{stat} \pm \text{syst}$	χ^2
B2 + 0.5%	$1.0023 \pm 0.0015 \pm 0.0037$	2007
PC5 stop	$1.0055 \pm 0.0030 \pm 0.0038$	1906
stop $\frac{1}{2}$	$1.0015 \pm 0.0014 \pm 0.0037$	1876
stop $\frac{3}{4}$ A	$0.9961 \pm 0.0014 \pm 0.0037$	1900
high rate	$0.9997 \pm 0.0019 \pm 0.0037$	1932
aperture	$0.9978 \pm 0.0018 \pm 0.0037$	1896
stop $\frac{3}{4}$ B	$1.0009 \pm 0.0019 \pm 0.0037$	1841

through the Al stopping target (stop $\frac{3}{4}$ A, B). One set was taken with a muon beam size-limiting aperture (aperture), and one set was taken with the beam rate increased (high rate).

Two sets of data were collected with the beam displaced by changing the last bending magnet (B2) field by +0.5% from nominal. One of the data sets (B2 + 0.5%) had the muon Bragg peak centered in the stopping target, while in the other set (PC5 stop), the muons were stopped relatively far upstream in order to increase the relative fraction of muons stopping in gas. All of these data sets, using different beam characterization profiles that matched the different conditions, were used in this determination of $P_\mu^\pi \xi$.

The spectrum fit results for the parameter $P_\mu^\pi \xi$ are presented in Table IV. At the present stage TWIST cannot provide an improved measurement of η , therefore its value is set to the global analysis value of -0.0036 [29], to constrain the other parameters better. The uncertainty of ± 0.0069 on the accepted value of η gives an uncertainty of ± 0.0001 on the final value of $P_\mu^\pi \xi$.

The average values of ρ and δ from the present fits are 0.749 and 0.753, respectively. An evaluation of the uncertainties in ρ and δ has not been performed, but if one assumes systematic uncertainties similar to the previous TWIST measurements, these values are reasonably consistent with the published values of $\rho = 0.7508$ [10] and $\delta = 0.7496$ [11].

To illustrate the quality of the fit, and how the spectrum fit distinguishes between $P_\mu^\pi \xi$ and $P_\mu^\pi \xi \delta$, the contribution to the fit asymmetry versus momentum for each of these terms and from the best fit $A(p)$ are shown in Fig. 6. Note that the total asymmetry versus momentum, $A(p)$, is:

$$A(p) = A_\xi(p) + A_{\xi\delta}(p). \quad (8)$$

where A_ξ is the asymmetry when the $P_\mu^\pi \xi \delta \cos^2 \theta \frac{2}{3} (4x^3 - 3x^2)$ contribution to the positron decay spectrum is zero, and $A_{\xi\delta}$ is the asymmetry when the $P_\mu^\pi \xi \cos \theta (x^2 - x^3)$ contribution is zero. The top panel in Fig. 6 shows the best fit asymmetry versus positron momentum, $A(p)$ with all of the fiducial cuts applied as a solid line; the contribution to the fit from the ξ term as a long-dashed line; and the

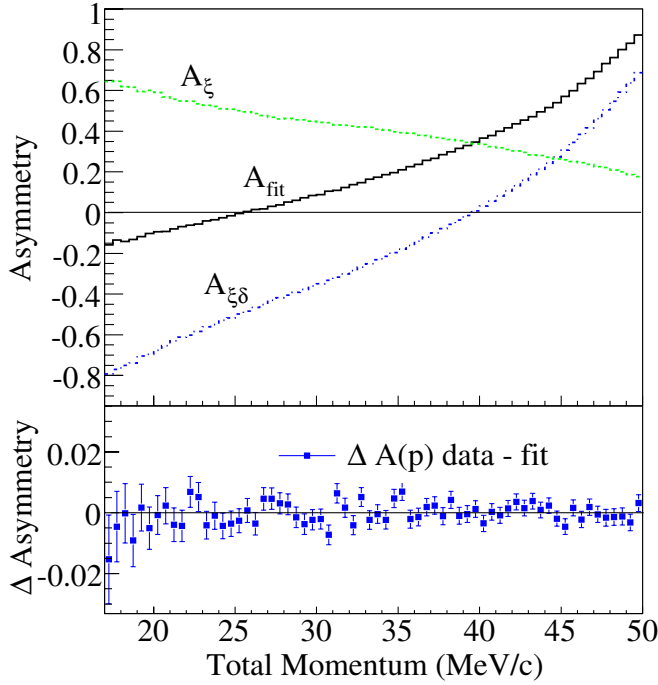


FIG. 6 (color online). The top panel shows the fit asymmetry versus positron momentum, $A(p)$, along with the contributions to the fit $A(p)$ from the ξ and $\xi\delta$ terms. The bottom panel shows the difference between the data and fit, $\Delta A(p)$.

contribution to the fit from the $\xi\delta$ term as the short-dashed line. The bottom panel shows the difference, $\Delta A(p)$ between data and fit.

VI. CONCLUSION

The value of $P_\mu^\pi \xi$ was determined to be $1.0003 \pm 0.0006(\text{stat.}) \pm 0.0038(\text{syst.})$. The central value for $P_\mu^\pi \xi$ was calculated as a weighted average using a quadratic sum of the statistical and set-dependent uncertainties for the weights. The final systematic uncertainty is a quadratic sum of the set-independent and the average values of the set-dependent systematics.

The 90% confidence limits on the LRS model parameters from this measurement are: $-0.050 < \zeta < 0.041$ and $m_2 > 360 \text{ GeV}/c^2$ in the manifest case, and $-0.061 < g_R/g_L \zeta < 0.061$ and $g_L/g_R m_2 > 325 \text{ GeV}/c^2$ in the general LRS model. The LRS model limits are shown in Fig. 7.

The central value measured is closer to the SM value than previous direct measurements, and, hence, in a global fit with all other muon decay parameter data [29] it pulls those parameters that are sensitive to $P_\mu^\pi \xi$ (Q_{RR}, Q_{LR}) closer to the SM value. However, the changes are small

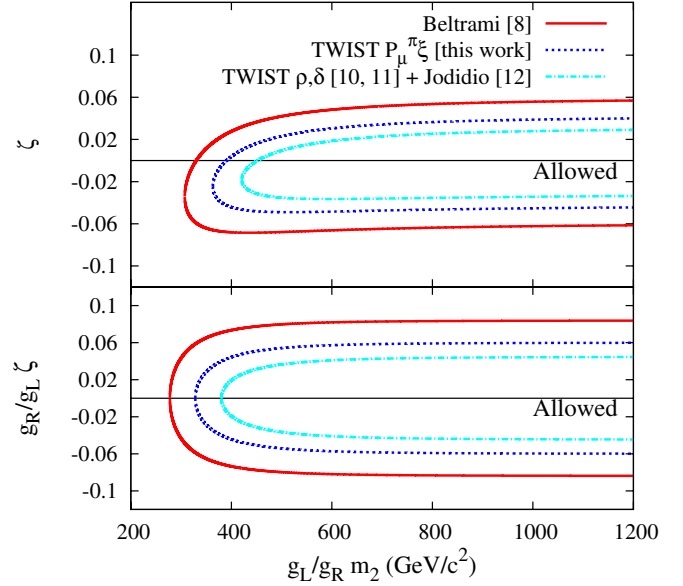


FIG. 7 (color online). The top panel shows the manifest LRS model 90% confidence limits on ζ and m_2 ($g_L/g_R = 1$) from measurements of $P_\mu^\pi \xi$. The bottom panel shows the same limits in the general LRS model case.

compared to the uncertainty on these parameters. The present result reduces the uncertainty on the direct measurement of $P_\mu^\pi \xi$ [8] by a factor of 2; it is also consistent with the SM and the value obtained indirectly [10–12]. This is TWIST's first independent measurement of $P_\mu^\pi \xi$, and prospects for reducing the main systematic uncertainties in $P_\mu^\pi \xi$ for data taken in the future with an improved beam measurement device are excellent.

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