Non-Abelian vector backgrounds with restored Lorentz invariance

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The influence of vector backgrounds with restored Lorentz invariance on non-Abelian gauge field theories is studied. Lorentz invariance is ensured by taking the average over a Lorentz invariant ensemble of background vectors, which are shifting the gauge field. Thereby the propagation of fermions is suppressed over long distances. Contrary to the fermionic sector, pure gauge configurations of the background suppress the long-distance propagation of the bosons only partially, that is not in all channels beyond the leading contribution for a large number of colors.

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I. INTRODUCTION

We are studying the influence of an ensemble of constant pure gauge vector backgrounds on non-Abelian gauge field theories. As before in [1,2] they are to represent a nontrivial vacuum structure. While every single vector in the ensemble breaks Lorentz invariance, the ensemble as a whole is to be Lorentz invariant. That means, although almost every single member of the set changes under a Lorentz transformation, the set is still to be mapped onto itself. The same must also hold with respect to gauge transformations as will be discussed below.

Technically, the modification is implemented by shifting the gauge field of the original field theory by a background vector and defining correlators as an average over a Lorentz and gauge-invariant ensemble of backgrounds. In [1] that approach has been pursued for an Abelian theory and is extended to non-Abelian gauge groups in this article. Previously, in [2] the correspondence to a theory featuring a boson propagator with a modified zero mode has been investigated. There, as well, the motive is to take into account the effects of a nonempty vacuum in the construction of a field theory [2]. These investigations are motivated by the fact that the degrees of freedom of the action of quantum chromodynamics (QCD) do not appear as physical degrees of freedom [3] as opposed to those of quantum electrodynamics (QED). In [1], in the Abelian case, the propagation of fermions over long distances is damped in the presence of the ensemble of backgrounds. The same finding has been made with a modified gauge boson propagator in [2] for quarks as well as gluons. Below, we shall extend the study of the influence of an ensemble of pure gauge backgrounds beyond the Abelian case. This choice is made in order to be able to analyze the effect of the latter ensemble in comparison to the ones mixing pure gauge and non pure gauge contributions [2]. Furthermore, here, we only want to address backgrounds which do not couple to gravity through their energy density.

A comparable area of research is formed by mass-dimension two gluon condensates or their BRST invariant generalization. Of late, various aspects of these condensates have attracted attention (see, for example [4,5]). Some qualitative similarities are also recognized in the present context: in [4] it is shown that taking into account such a condensate also leads to a suppression of the propagation of partons at long distances.

In a somewhat different context vector backgrounds appear—among many other terms—in theories breaking Lorentz invariance explicitly [6–8]. These so-called standard model extensions (SME) are motivated from string [7] and noncommutative field theory [8], where Lorentz invariance can be broken spontaneously. In the case of the SME, the Lorentz invariance is not restored and one aims at constraining the parameters of these standard model extensions by comparison to data. Commonly, the parameters are considered as being perturbatively small and constant. The effective description of the spontaneous breaking of Lorentz symmetry by a model with explicit symmetry breaking, the SME, can be regarded as an extreme, in which the system remains in one background configuration during the considered physical processes; in fact during the whole data taking campaign. If one loosens this constraint the background becomes space-time dependent. If the space-time dependence should become faster than the resolution of the experiment, the Lorentz symmetry would be restored in the sense studied here. When the experiment becomes more and more rapid the ensemble will become skewed. Such situations interpolate between the two extremes. In all cases different from the one with a frozen background, the limits on the parameters become qualitatively different. In particular, the limits must be determined for the distributions describing the ensembles because the original SME parameters then are silent variables. Here, the postulate of Lorentz invariance of the theory is not to be dropped in order to be able to accomodate this aspect and the investigation along the lines of [1,2] at the same time. For the same reason also the parameters of the standard model extension other than the vector are not addressed at present. Thus, in the context of the SME, the approach which we pursue here can be viewed as a practical way of including a space-time dependence of the backgrounds. An unlimited space-time dependence and Lorentz-skewed ensembles represent objects of interest for further research, which, however, are beyond the scope of the present investigation.

Section II describes how the background is incorporated into the treatment of the theory. Section III discusses the Lorentz invariant weight functions characterising the ensembles of background vectors for non-Abelian gauge groups and the preservation of gauge invariance. Section IV investigates the influence of the backgrounds on the propagation of quarks and gluons. Section V summarizes the results.

II. INCLUDING THE BACKGROUND

The Lagrangian density of a non-Abelian $SU(N_c)$ gauge field theory is given by

$$\mathcal{L} := \mathcal{L}_F + \mathcal{L}_G \tag{1}$$

with the fermionic part

$$\mathcal{L}_F := \bar{\psi}(i\partial\!\!\!/ + A\!\!\!/ - m)\psi \tag{2}$$

and the gauge field part

$$\mathcal{L}_G := -\frac{1}{4g^2} F^{a\mu\nu}(A) F^a_{\mu\nu}(A), \tag{3}$$

where the field tensor reads

$$F^a_{\mu\nu}(A) := \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu. \tag{4}$$

ψ stands for the fermionic spinor field (in QCD the quark field) and $A = A^a T^a$ represents the gauge field (in QCD the gluon field). T^a with $a \in \{1; \ldots; N_c{}^2 - 1\}$ are generators in the fundamental representation of the $SU(N_c)$ Lie algebra. g represents the coupling constant. The Lagrangian (1) is invariant under the simultaneous gauge transformation, U, of the fields according to

$$\psi \to \psi_U := U\psi \tag{5}$$

and

$$A \to A_U := UAU^\dagger - i(\partial U)U^\dagger, \tag{6}$$

that is

$$\mathcal{L}(\psi_U, \bar{\psi}_U, A_U) = \mathcal{L}(\psi, \bar{\psi}, A) \equiv \mathcal{L}. \tag{7}$$

The partition function,

$$P := \int [d\psi][d\bar{\psi}][dA] \exp\left\{ i \int_x \mathcal{L} \right\}, \tag{8}$$

where $\int_x := \int d^4x$, inherits the same invariance properties from the Lagrangian density. The functional integration runs over all configurations and, thereby, over infinitely many which differ merely by a transformation of gauge. Therefore, in order to obtain a meaningful object, we have to factor out the infinite dimensional integral $\int[dU]$ over the gauge group, where $[dU]$ is the invariant group measure. To this end, we single out a gauge field configuration A_U by imposing the gauge-fixing condition

$$f(A_U) \stackrel{!}{=} 0. \tag{9}$$

Its incorporation into the partition function can be achieved after defining the corresponding gauge-invariant Faddeev-Popov determinant, $\Delta_f(A)$, by

$$1 \stackrel{!}{=} \int [dU] \delta[f(A_U)] \Delta_f(A), \tag{10}$$

where $\delta[f(A)]$ is a functional δ distribution. Introducing the last expression into the partition function leads to

$$P = \int [d\psi][d\bar{\psi}][dA][dU] \delta[f(A_U)] \Delta_f(A) \exp\left\{ i \int_x \mathcal{L} \right\}. \tag{11}$$

The only gauge dependent factor in the integral is the argument of the gauge-fixing condition inside the δ distribution, whence we can also replace it by A. Thus the integrand is completely independent of the gauge and the integral over the gauge group factors out

$$P = \check{P} \times \int [dU], \tag{12}$$

where

$$\check{P} := \int [d\psi][d\bar{\psi}][dA] \delta[f(A)] \Delta_f(A) \exp\left\{ i \int_x \mathcal{L} \right\} \tag{13}$$

is the partition function in the gauge $f(A) = 0$.

It is our aim to include a vector background Φ into the theory by shifting the vector field

$$A \mapsto A + \Phi. \tag{14}$$

Thereafter, however, the gauge and Poincaré invariance of the resulting theory are still to be ensured. The restoration of these symmetries is achieved by performing an average over an appropriate ensemble of backgrounds. This ensemble is to be elaborated in what follows.

As mentioned in the introduction, the theory in the ensemble of backgrounds can be seen as one with a modified vacuum structure. One can express this by means of an ensemble average ($\langle \ldots \rangle_W$):

$$\langle {}_\Phi\langle 0| T\psi^n \bar{\psi}^n A^{n'} |0\rangle_\Phi \rangle_W =: \langle \Omega | T\psi^n \bar{\psi}^n A^{n'} | \Omega \rangle, \tag{15}$$

where $|\Omega\rangle$ represents the structured vacuum and $|0\rangle_\Phi$ depends on a single background. The vacuum expectation values of the modified theory $\langle \Omega | \mathcal{O} | \Omega \rangle$ are the averaged vacuum expectation values $\langle {}_\Phi\langle 0|\mathcal{O}|0\rangle|_\Phi \rangle_W$ each in a single background. In this theory the restoration of the symmetries is accomplished on the level of the partition function. If it was merely for the sake of restoring the symmetries, one could also try to do this at other stages; on the level of the observables or the action. Performing the average on the level of the observables, for example $\langle |{}_\Phi\langle 0|\mathcal{O}|0\rangle_\Phi|^2 \rangle_W$ would amount to a different theory. It would not correspond to a theory with a dressed vacuum but to one in the bare vacuum and with random background fields. For the standard model extensions the theory with the average observables would correspond to a situation, in which measurements are taken on time and length scales, which are much shorter than those on which the backgrounds change, but where the backgrounds do change between measurements. Opposed to that, the setting, which is studied here is, in the sense of the SME, the long-wavelength limit of an extension with space-time dependent vector backgrounds. Restoring the symmetries already on the level of the classical action and not the correlators would again not describe a theory with a structured quantum vacuum. Therefore, it would not describe the system that is to be studied here.

Carrying out the shift (14) leads to the Lagrangian in the background

$$\mathcal{L}^{\Phi} := \mathcal{L}_F^{\Phi} + \mathcal{L}_G^{\Phi}, \tag{16}$$

where

$$\mathcal{L}_F^{\Phi} := \bar{\psi}(i\not{\partial} + \not{A} + \not{\Phi} - m)\psi \tag{17}$$

and

$$\mathcal{L}_G = -\frac{1}{4g^2}F^{a\mu\nu}(A+\Phi)F^a_{\mu\nu}(A+\Phi). \tag{18}$$

Because of the shift (14) the gauge transformation of the gauge field (6), which, if carried out simultaneously with the gauge transformation (5) of the spinor fields, leaves invariant the Lagrangian $\mathcal{L}^{\Phi}$, becomes

$$A + \Phi \to U(A+\Phi)U^{\dagger} - i(\partial U)U^{\dagger}. \tag{19}$$

At the end, the dynamical gluon field A is still to transform according to the original prescription (6). Thus, a simultaneous transformation of the fields according to (5) and (6), and

$$\Phi \to \Phi_U := U\Phi U^{\dagger} \tag{20}$$

leaves the Lagrangian unchanged.

The gauge-fixing term $\delta[f(A)]$ and the Faddeev-Popov determinant $\Delta_f(A)$ in the partition function can be exponentiated in the standard way and included with the Lagrangian density. In background Feynman gauge this leads to the equivalent expression for the gauged partition function $\check{P}^{\Phi}$ in the background Φ:

$$\check{P}^{\Phi} := \int [d\psi][d\bar{\psi}][dA][d\chi][d\chi^*]\exp\left\{i\int_x(\mathcal{L}^{\Phi} + \mathcal{L}^{\Phi}_{\mathrm{FG}})\right\} \tag{21}$$

with the ghost fields χ and χ^* and where the additional addends in the Lagrangian density are given by

$$\mathcal{L}^{\Phi}_{\mathrm{FG}} := \mathcal{L}^{\Phi}_{\mathrm{GF}} + \mathcal{L}^{\Phi}_{\mathrm{FP}}, \tag{22}$$

where

$$\mathcal{L}^{\Phi}_{\mathrm{GF}} := -\tfrac{1}{2}[D^{ab}_{\mu}(\Phi)A^{b\mu}][D^{ac}_{\nu}(\Phi)A^{c\nu}] \tag{23}$$

and

$$\mathcal{L}^{\Phi}_{\mathrm{FP}} := [D^{ab}_{\mu}(\Phi)\chi^{b*}][D^{ac\mu}(A+\Phi)\chi^c]. \tag{24}$$

Finally, as alluded to before, the correlators of the theory are to be defined as average of correlators in a single background over an ensemble of backgrounds. This ensemble is characterized by a weight W, which is a function of the background, $W = W(\Phi)$. It must be gauge and Lorentz invariant. The details for the weight function are developed below.

By construction the additional terms (22) in the Lagrangian are BRST invariant. For gauge and matter fields the BRST transformation corresponds to a normal gauge transformation, whence the BRST invariance of $\mathcal{L}^{\Phi}$ follows from Eq. (19). For the same reason, the gauge-invariant weight W is also BRST invariant. At this point a difference of the present approach [1,2] with one where mass-dimension two gluon condensates are treated (see e.g. [4,5]) becomes apparent. Here the gauge and Lorentz invariant weight function can be expressed as a function of $\Phi^2 = \Phi^a_{\mu}\Phi^{a\mu}$ [although this is not the most general possibility (see below)]. There, a gluon field bilinear A^2 characterizes the distribution of fields. Because of the inhomogeneous gauge transformation (6) of the gauge field A this is no gauge-invariant quantity. It can be given a BRST invariant meaning by adding a ghost field bilinear [4,5], which, in Landau gauge drops out again. Still, in the two approaches similar results are obtained as has already been noted in [2] with respect to [4].

In order to facilitate a perturbative treatment of the modified theory, let us introduce current terms into the partition function $\check{P}^{\Phi}$ in a single background,

$$Z^{\Phi} := \int [dA][d\psi][d\bar{\psi}][d\chi][d\chi^*]\exp\left\{i\int_x(\mathcal{L}^{\Phi} + \mathcal{L}^{\Phi}_{\mathrm{FG}})\right\} \times \exp\left\{i\int_x(J\cdot A + \bar{\psi}\eta + \bar{\eta}\psi + \chi^*\xi + \xi^*\chi)\right\}, \tag{25}$$

with the currents J, η, $\bar{\eta}$, χ, and χ^*. Here, we will address Z^{Φ} as "generating functional," as opposed to the "partition function" without the currents ($Z^{\Phi}|_{J,\eta,\bar{\eta},\xi,\xi^*=0} = \check{P}^{\Phi}$). Let us define the two-point functions of the fermions (G^{Φ}), gauge bosons ($\Gamma^{\Phi}_{\mu\nu}$), and ghosts (Γ^{Φ}) by their respective equations of motion in the background

$$[i\not{\partial}(x) + \not{\Phi} - m]G^{\Phi}(x-y) = \delta^{(4)}(x-y), \tag{26}$$

$$[\partial(x) - i\check{\Phi}]\cdot[\partial(x) - i\check{\Phi}]\Gamma^{\Phi}_{\mu\nu}(x-y) = \delta^{(4)}(x-y)g_{\mu\nu}, \tag{27}$$

and

$$[\partial(x) - i\check{\Phi}]\cdot[\partial(x) - i\check{\Phi}]\Gamma^{\Phi}(x-y) = \delta^{(4)}(x-y), \tag{28}$$

where "$\breve{}$" indicates that the field transforms under the adjoint representation. Then we can reexpress the generating functional as

$$Z^{\Phi} = Z^{\Phi}_{\rm int} Z^{\Phi}_{A} Z^{\Phi}_{\chi} Z^{\Phi}_{\psi}, \tag{29}$$

where

$$Z^{\Phi}_{A} = \exp\left\{\frac{i}{2}\int_{x,y} J^{\mu}(x)\Gamma^{\Phi}_{\mu\nu}(x-y)J^{\nu}(y)\right\}, \tag{30}$$

$$Z^{\Phi}_{\chi} = \exp\left\{-i\int_{x,y} \xi^{*}(x)\Gamma^{\Phi}(x-y)\xi(y)\right\}, \tag{31}$$

and

$$Z^{\Phi}_{\psi} = \exp\left\{-i\int_{x,y} \bar{\eta}(x)G^{\Phi}(x-y)\eta(y)\right\}. \tag{32}$$

$Z^{\Phi}_{\rm int}$ contains all terms of third and fourth order in the dynamic fields. In background Feynman gauge it is given by

$$Z^{\Phi}_{\rm int} = \exp\left\{i\int_{x}\mathcal{L}^{\Phi}_{\rm int}\right\}, \tag{33}$$

with the interaction part of the Lagrangian density

$$\begin{aligned}\mathcal{L}^{\Phi}_{\rm int} := &-\frac{1}{2g^2}f^{abc}(\partial_{\mu}A^{a}_{\nu}-\partial_{\nu}A^{a}_{\mu})A^{b\mu}A^{c\nu}\\ &-\frac{1}{4g^2}f^{abe}f^{cde}A^{a}_{\mu}A^{b}_{\nu}A^{c\mu}A^{d\nu}\\ &-\frac{1}{g^2}f^{abe}f^{cde}\Phi^{a}_{\mu}A^{b}_{\nu}A^{c\mu}A^{d\nu}\\ &-f^{abc}[D^{ad}_{\mu}(\Phi)\chi^{d*}]\chi^{b}A^{c\mu}+\bar{\psi}T^{a}\gamma^{\mu}\psi A^{a}_{\mu}\end{aligned} \tag{34}$$

and where all fields have to be replaced by functional derivatives with respect to the corresponding currents.

III. WEIGHT CLASSIFICATION

This section is concerned with the construction of the ensemble of backgrounds and the corresponding weight function, W. At the beginning let us remember some results for the Abelian case [1] also needed for the non-Abelian theory. The Lorentz invariant ensemble of backgrounds Φ is characterized by a weight function $W'(\Phi)$ which appears in the averaging prescription ($\int_{\Phi} := \int d^4\Phi$),

$$\langle\mathcal{O}\rangle_{W'} = \int_{\Phi} W'(\Phi)\mathcal{O}. \tag{35}$$

It does not change under Lorentz transformations and is normalized in such a way that

$$\int_{\Phi} W'(\Phi) = 1. \tag{36}$$

Except for when $\Phi = 0$, which leads to the unmodified theory, all other Lorentz invariant quantities depending on the vector Φ must be functions of Φ^2. Thus every allowed weight function W' can be cast into the form:

$$W'(\Phi) = c\delta^{(4)}(\Phi) + w(\Phi^2), \tag{37}$$

where w is a normalizable function of Φ^2.

In Euclidean space $\Phi^2 = 0$ also means $\Phi = 0$, whereby that case could be encoded in $w_E(\Phi^2)$, where the subscript E marks the Euclidean space. In the following the possible contribution from $\Phi = 0$, from the unmodified theory, is to remain marked clearly and the delta term is kept explicitly. Thereby the normalization condition (36) becomes

$$\pi^2\int_0^{\infty} v\,dv\,w_E(v) = 1-c, \tag{38}$$

with $v := \Phi^2$. The Lorentz invariant weight functions can be decomposed into elementary delta weights

$$w^{E}_{\lambda}(\Phi^2) := (4\pi\lambda)^{-1}\delta(\Phi^2-\lambda), \tag{39}$$

according to

$$w_E(\Phi^2) = \int 4\pi\lambda\, d\lambda\, w^{E}_{\lambda}(\Phi^2)w_E(\lambda). \tag{40}$$

In Minkowski space the same basis can be used. If a time-ordered formalism is to be pursued a different basis may be useful. From the identity

$$4\pi i\delta(\Phi^2-\lambda) = S^{-}_{\lambda}(\Phi) - S^{+}_{\lambda}(\Phi), \tag{41}$$

with the time-ordered (+) and anti time-ordered (−) scalar propagators

$$S^{\pm}_{\lambda}(\Phi) = (\Phi^2-\lambda\pm i\epsilon)^{-1}, \tag{42}$$

it follows that the elementary weight function of choice is then constructed from the scalar Feynman propagator S^{+}_{λ}. For this purpose, in Eq. (40) w^{E}_{λ} is replaced by S^{+}_{λ}. Yet in Minkowski space the hyperboloid pair defined by $\Phi^2 =$ const has infinite content. Thus the minimal normalizable weight is given by a sum of three terms [1],

$$w_M(\Phi^2) = \sum_{j=1}^{3} a_j S^{+}_{\lambda_j}(\Phi), \tag{43}$$

satisfying

$$\sum_{j=1}^{3} a_j = 0, \tag{44}$$

$$\sum_{j=1}^{3} a_j\lambda_j = 0, \tag{45}$$

and the normalization condition

$$\frac{4\pi^2}{4}\sum_{j=1}^{3} a_j\lambda_j \ln\lambda_j = 1-c. \tag{46}$$

The subscript M marks the Minkowski space. On top of that, the case $\Phi = 0$ cannot be described by a function

$w_M = w_M(\Phi^2)$ and must be added in form of a delta term where required.

A. Non-Abelian

In this subsection the approach is generalized to pure gauge configurations in non-Abelian gauge groups. In the non-Abelian case the gauge field carries color. Nevertheless, the different Lorentz components of a homogeneous pure gauge background commute. The background Φ transforms homogeneously [see Eq. (20)] under gauge transformations U [2]. Therefore, we can decompose the background (for the fundamental representation) according to

$$\sum_{a=1}^{N_c^2-1} T^a \Phi_\mu^a = \Phi_\mu = \sum_{\underline{a}=1}^{N_c} \Theta^{\underline{a}} \Phi_\mu^{\underline{a}}, \tag{47}$$

where the $\Theta^{\underline{a}}$ are N_c projectors. The projector indices $\underline{a} \in \{1; \ldots; N_c\}$ are distinguished by the underline from the generator indices $a \in \{1; \ldots; N_c{}^2 - 1\}$. Only such summations over projector indices are carried out, which are marked explicitly. The projectors satisfy $\Theta^{\underline{a}}\Theta^{\underline{b}} = \delta^{\underline{a}\,\underline{b}}\Theta^{\underline{a}}$ and

$$\sum_{\underline{a}=1}^{N_c} (\Theta^{\underline{a}})_{jk} = \delta_{jk}, \tag{48}$$

where δ_{jk} is the Kronecker symbol. The projectors can be expressed as unitary transformations of $(\Theta^{\underline{a}})_{jk} = \delta^{j\underline{a}}\delta^{k\underline{a}}$. A function of the background vector Φ can be deconstructed in the basis of the projectors $\Theta^{\underline{a}}$:

$$f(\Phi) = \sum_{\underline{a}=1}^{N_c} \Theta^{\underline{a}} f(\Phi^{\underline{a}}). \tag{49}$$

The gauge dependence resides entirely in the transformations U of the projectors $\Theta^{\underline{a}}$,

$$f(\Phi) \overset{(20)}{\longrightarrow} f(\Phi_U) = \sum_{\underline{a}=1}^{N_c} U\Theta^{\underline{a}} U^\dagger f(\Phi^{\underline{a}}). \tag{50}$$

The $\Phi^{\underline{a}}$ being gauge-invariant quantities, one can study each addend separately. To this end, we use the weight function

$$W(\Phi) = \prod_{\underline{b}=1}^{N_c} W'(\Phi^{\underline{b}}) \tag{51}$$

and integrate over $\int_{\{\Phi\}} := \int_{\{\Phi^{\underline{1}}\}} \ldots \int_{\{\Phi^{\underline{N_c}}\}}$. This leads to the average [see Eq. (35)]

$$\begin{aligned} \langle f(\Phi_U)\rangle_W &\overset{(36)}{=} \sum_{\underline{a}=1}^{N_c} U\Theta^{\underline{a}} U^\dagger \int_{\Phi^{\underline{a}}} W'(\Phi^{\underline{a}}) f(\Phi^{\underline{a}}) \\ &= U\left(\sum_{\underline{b}=1}^{N_c} \Theta^{\underline{b}}\right) U^\dagger \int_{\Phi'} W'(\Phi') f(\Phi') \\ &\overset{(48)}{=} \int_{\Phi'} W'(\Phi') f(\Phi'), \end{aligned} \tag{52}$$

where W' is a weight constructed in the beginning of Sec. III and $\Phi' \in \mathbb{R}^4$. This demonstrates also that the dependence on the gauge transformation of the background drops out. Alternatively, the average can be taken with weights being functions of $\Phi_\mu^{\underline{a}}\Phi^{\underline{a}\mu}$, which is a gauge and Lorentz invariant quantity. Examples for both types of weights are studied below.

For the fundamental representation of a $U(N_c)$ gauge group all ingredients for the averaging procedure are presented above. In the case of an $SU(N_c)$ gauge group an additional constraint arises, $\sum_{\underline{a}=1}^{N_c} \Phi^{\underline{a}} = 0$, because the generators all have vanishing trace. Its incorporation leads to a coupling of the channels and thereby to a modification of Eq. (52). The vanishing of the trace can be imposed replacing the product of weights in the previous expression according to

$$\prod_{\underline{b}=1}^{N_c} W'(\Phi^{\underline{b}}) \mapsto \delta^{(4)}\left(\sum_{\underline{d}=1}^{N_c} \Phi^{\underline{d}}\right) \frac{1}{N_c} \sum_{\underline{c}=1}^{N_c} \prod_{\underline{b}\neq\underline{c}} W'(\Phi^{\underline{b}}), \tag{53}$$

where a symmetrization in the numbering of the projectors has been taken into account, as the assignment of the eigenvalues to the projectors may not play a rôle because of gauge invariance. Introducing the previous expression into Eq. (52), taking the summation over $\underline{c}$ to the front, and repeating the same steps yields

$$\langle f(\Phi_U)\rangle_W^{SU} = \int_{\Phi'} W''(\Phi') f(\Phi'), \tag{54}$$

where

$$\begin{aligned} W''(\Phi^{\underline{a}}) :=& \frac{N_c - 1}{N_c} W'(\Phi^{\underline{a}}) \\ &+ \frac{1}{N_c} \int_{\{\Phi^{\underline{b}\neq\underline{a}}\}} \delta^{(4)}\left(\sum_{\underline{d}=1}^{N_c} \Phi^{\underline{d}}\right) \prod_{\underline{b}\neq\underline{a}} W'(\Phi^{\underline{b}}), \end{aligned} \tag{55}$$

where the integrations run over the components of the background vector but the ones with projector index $\underline{a}$. For example in $SU(3)$ W'' becomes

$$W''(\Phi') \overset{SU(3)}{=} \frac{2}{3} W'(\Phi') + \frac{1}{3} \int_{\Phi''} W'(\Phi'') W'(\Phi'' + \Phi') \tag{56}$$

and in $SU(2)$

$$W''(\Phi') \overset{SU(2)}{=} W'(\Phi'), \tag{57}$$

where use has been made of the fact that $W'(\Phi')$ is always an even function of Φ'.

B. Adjoint representation

The background for bosonic correlators transforms under the adjoint representation of the gauge group. The adjoint representation of the $SU(N_c)$ can be embedded in the fundamental of the $SU(N_c{}^2-1)$ or the $U(N_c{}^2-1)$, that is

$$\sum_{a=1}^{N_c{}^2-1}\Phi^a\breve{T}^a=\breve{\Phi}\stackrel{!}{=}\phi=\sum_{b=1}^{(N_c{}^2-1)^2-1}\phi^b t^b, \tag{58}$$

where $\breve{T}^a$ with $a\in\{1;\ldots;N_c{}^2-1\}$ are the generators of the adjoint representation of $SU(N_c)$ and t^b with $b\in\{1;\ldots;(N_c{}^2-1)^2-1\}$ the generators of the fundamental representation of $SU(N_c{}^2-1)$ [or $U(N_c{}^2-1)$, but then with $b\in\{0;\ldots;(N_c{}^2-1)^2-1\}$]. ϕ can be decomposed in a basis of projectors $\theta^{\underline{a}}$ with $\underline{a}\in\{1;\ldots;N_c{}^2-1\}$,

$$\phi=\sum_{\underline{a}=1}^{N_c{}^2-1}\theta^{\underline{a}}\phi^{\underline{a}}. \tag{59}$$

The projectors satisfy $\theta^{\underline{a}}\theta^{\underline{b}}=\delta^{\underline{a}\,\underline{b}}\theta^{\underline{a}}$ (no summation over $\underline{a}$). Possible representations of the projectors are unitary transformations of $(\theta^{\underline{a}})_{jk}=\delta^{j\underline{a}}\delta^{k\underline{a}}$. A function of ϕ has the decomposition

$$f(\phi)=\sum_{\underline{a}=1}^{N_c{}^2-1}\theta^{\underline{a}}f(\phi^{\underline{a}}). \tag{60}$$

Finally, the expression relevant for the adjoint representation of $SU(N_c)$ is extracted by imposing additional constraints. In general, members of the adjoint representation of $SU(N_c)$ are hermitian and antisymmetric. Therefore, the eigenvalues either vanish or come in pairs with opposite sign [9]. For the adjoint representation of $SU(2)$ embedded in $U(3)$ this means that the eigenvalues always satisfy $\phi^{\underline{1}}=-\phi^{\underline{2}}$ and $\phi^{\underline{3}}=0$. Thus,

$$f(\phi)\stackrel{SU(2)}{=}\theta^{\underline{1}}f(+\phi^{\underline{1}})+\theta^{\underline{2}}f(-\phi^{\underline{1}})+\theta^{\underline{3}}f(0). \tag{61}$$

For the adjoint representation of $SU(3)$ embedded in $U(8)$ one has $\phi^{\underline{1}}=-\phi^{\underline{2}}$, $\phi^{\underline{3}}=-\phi^{\underline{4}}$, $\phi^{\underline{5}}=-\phi^{\underline{6}}$, and $\phi^{\underline{7}}=0=\phi^{\underline{8}}$ [9]. Hence,

$$f(\phi)=(\theta^{\underline{7}}+\theta^{\underline{8}})f(0)+\sum_{\substack{\underline{a}\in\\\{1;3;5\}}}[\theta^{\underline{a}}f(+\phi^{\underline{a}})+\theta^{\underline{a+1}}f(-\phi^{\underline{a}})]. \tag{62}$$

That induces the following form for the average

$$\langle f(\phi)\rangle_W^{SU_2^{\mathrm{adj}}}=\frac{1}{3}f(0)+\frac{2}{3}\int_{\phi'}W'(\phi')f(\phi') \tag{63}$$

and

$$\langle f(\phi)\rangle_W^{SU_3^{\mathrm{adj}}}=\frac{1}{4}f(0)+\frac{3}{4}\int_{\phi'}W'(\phi')f(\phi'). \tag{64}$$

These results can be understood by noticing that the calculation for each pair of eigenvalues with opposite sign resembles the calculation for the *fundamental* representation of $SU(2)$ [see Eq. (57)]: If we express the mean as

$$\langle f(\phi)\rangle_W^{SU_{N_c}^{\mathrm{adj}}}=\int_{\{\phi\}}W(\phi)f(\phi), \tag{65}$$

where $\int_{\{\phi\}}:=\int_{\phi^{\underline{1}}}\cdots\int_{\phi^{\underline{N_c{}^2-1}}}$, the weight function W for $N_c=2$ becomes

$$\begin{aligned}W(\phi)\stackrel{SU_2^{\mathrm{adj}}}{=}&\frac{1}{3}W'(\phi^{\underline{1}})\delta(\phi^{\underline{1}}+\phi^{\underline{2}})\delta(\phi^{\underline{3}})\\&+\frac{1}{3}W'(\phi^{\underline{2}})\delta(\phi^{\underline{2}}+\phi^{\underline{3}})\delta(\phi^{\underline{1}})\\&+\frac{1}{3}W'(\phi^{\underline{3}})\delta(\phi^{\underline{3}}+\phi^{\underline{1}})\delta(\phi^{\underline{2}})\\&=\frac{1}{6}\sum_{\mathrm{perm}}W'(\phi^{\underline{1}})\delta(\phi^{\underline{1}}+\phi^{\underline{2}})\delta(\phi^{\underline{3}}).\end{aligned} \tag{66}$$

In the last expression the necessary symmetrization with respect to the projector indices is taken into account like above in the case of the fundamental representation. The sum runs over all permutations of the indices. The corresponding expression for $N_c=3$ reads

$$\begin{aligned}W(\phi)\stackrel{SU_3^{\mathrm{adj}}}{=}&\frac{1}{8!}\sum_{\mathrm{perm}}W'(\phi^{\underline{1}})W'(\phi^{\underline{3}})W'(\phi^{\underline{5}})\delta(\phi^{\underline{7}})\delta(\phi^{\underline{8}})\\&\times\delta(\phi^{\underline{1}}+\phi^{\underline{2}})\delta(\phi^{\underline{3}}+\phi^{\underline{4}})\delta(\phi^{\underline{5}}+\phi^{\underline{6}}).\end{aligned} \tag{67}$$

Actually, it is equivalent and more economic to use the unsymmetrized weight function and symmetrize the projectors instead. Noticing this also makes the average's invariance under gauge transformations of the background more apparent. Like for the fundamental representation, the gauge dependence of the unaveraged quantity with respect to gauge transformations of the background resides entirely in the projectors,

$$f(\phi)\to f(\phi_U)\stackrel{(60)}{=}\sum_{\underline{a}=1}^{N_c{}^2-1}U\theta^{\underline{a}}U^\dagger f(\phi^{\underline{a}}). \tag{68}$$

For the next step, it is not of importance whether we consider only adjoint $SU(N_c)$ transformations or fundamental $SU(N_c{}^2-1)$ transformations. The symmetrization of the projectors,

$$f(\phi_U) \overset{\text{sym}}{\mapsto} \sum_{\underline{a}=1}^{N_c{}^2-1} U\Big(\frac{1}{N_c{}^2-1}\sum_{\underline{b}=1}^{N_c{}^2-1}\theta^{\underline{b}}\Big)U^\dagger f(\phi^{\underline{a}})$$
$$= \frac{1}{N_c{}^2-1}\sum_{\underline{a}=1}^{N_c{}^2-1} UU^\dagger f(\phi^{\underline{a}})$$
$$= \frac{1}{N_c{}^2-1}\sum_{\underline{a}=1}^{N_c{}^2-1} f(\phi^{\underline{a}}), \tag{69}$$

leads to an expression, which does not depend on gauge transformations of the background.

In Eq. (72) a common mean over the factors of the generating functional transforming under the fundamental and the adjoint representation of the gauge group, respectively, is taken. For this reason a connection has to be established between the eigenvalues $\Phi^{\underline{a}}$, $\underline{a} \in \{1;\ldots;N_c\}$ of the fundamental representation and $\phi^{\underline{b}}$, $\underline{b} \in \{1;\ldots;N_c{}^2-1\}$ of the adjoint representation. (Of the latter only N_c are independent as has been discussed above.) This link comes into play if correlators are to be calculated that involve bosonic and fermionic fields simultaneously. In $SU(3)$ one has the connection [9]:

$$\begin{aligned} 2(\phi^1_\mu)^2 &= (\Phi_\mu)^2[1-\cos(\vartheta_\mu)],\\ 2(\phi^3_\mu)^2 &= (\Phi_\mu)^2[1+\cos(\vartheta_\mu-\pi/3)],\\ 2(\phi^5_\mu)^2 &= (\Phi_\mu)^2[1+\cos(\vartheta_\mu+\pi/3)],\end{aligned} \tag{70}$$

with

$$1+\cos^3(\vartheta_\mu) = 18(\Phi^2_\mu)^{-3}\prod_{a=1}^{3}(\Phi^a_\mu)^2. \tag{71}$$

This relationship holds separately for each Lorentz component, which is why one does not sum over the index μ anywhere in the previous equations. These relations allow to construct the corresponding weight for one representation from the weight function for the other.

IV. TWO-POINT FUNCTIONS

As described above, the background is incorporated into the theory by translating the gauge field A by the background Φ, $A \mapsto A + \Phi$, and afterwards taking the mean over the ensemble of these vectors and thus restoring Lorentz invariance as well as gauge invariance with respect to gauge transformations of the background. Doing so leads to the modified generating functional:

$$Z := \langle Z^\Phi\rangle_W, \tag{72}$$

with Z^Φ [see Eq. (29)] the generating functional in a single background, Φ. A principal ingredient of the modified generating functional and thereby of the modified theory are the two-point correlators [see Eqs. (30)–(32)] which are studied in the following. They also serve to establish the link to the results presented in Ref. [1].

With the usually made assumption that all other condensates be absent [2,4] the fermionic propagator in the background obeys the equation of motion (26). Its solution in the pure gauge background Φ is given by

$$G^\Phi(z) = e^{i\Phi\cdot z}G_0(z), \tag{73}$$

where $G_0(z)$ is the solution of the free equation.

Without further spontaneous symmetry breaking, that is with $\langle A\rangle = 0$, the equation of motion for the gluon propagator in background-field Feynman gauge is given by Eq. (27). In the same gauge the ghost propagator obeys the equation of motion (28). Therefore, one has in background field Feynman gauge [10]:

$$\Gamma^\Phi_{\mu\nu}(x-y) = g_{\mu\nu}\Gamma_0(x-y) \tag{74}$$

and it is sufficient to study one of the two propagators, for example, the ghost propagator. In the pure gauge background Φ one has then

$$\Gamma^\Phi(z) = e^{i\check{\Phi}\cdot z}\Gamma_0(z), \tag{75}$$

with $\Gamma_0(z)$ the solution of the background-free equation and where "ˇ" indicates that $\check{\Phi}$ transforms under the adjoint representation.

As had already been seen in the Abelian case [1] the Fourier phases of the propagators in the background Φ lead to the averaging procedure being identical to a Fourier transformation of the weight function. For this reason, in coordinate space the n-point functions of the modified theory are the n-point functions of the theory without background multiplied by an n-point function which is the Fourier transformed weight. The latter is a genuine n-point function. That means that even if in the original theory the higher correlators factorize into lower ones, they will not in the modified theory.

There are other situations where such field configurations play a rôle. The zero components of aforesaid Fourier phases resemble chemical potentials [1]. The spatial components are similar to what one encounters for twisted boundary conditions on compact spaces [11]. In a color superconductor a constant field configuration, corresponding to a zero field tensor, serves to restore its color neutrality [12].

A. Euclidean space

Let us compute the average of the fermionic propagator G_Φ [see Eq. (73)] in Euclidean space for a $U(N_c)$ gauge group. In the first step we decompose the vector Φ in the basis of the projectors $\Theta^{\underline{a}}$ and likewise the phase factor $[e^{i\Phi\cdot z} = f(\Phi)]$. Afterwards, we utilize the explicit expression for the average and repeat the steps carried out previously in Eq. (52):

$$\langle G^{\Phi}(z)\rangle_W^E \overset{(73)}{=} \langle e^{i\Phi\cdot z}G_0(z)\rangle_W^E = G_0(z)\left\langle \sum_{\underline{a}=1}^{N_c} e^{iz\cdot\Phi^{\underline{a}}}\Theta^{\underline{a}}\right\rangle_W^E \overset{(52)}{=} G_0(z)\sum_{\underline{a}=1}^{N_c}\Theta^{\underline{a}}\int_{\Phi^{\underline{a}}} W'(\Phi^{\underline{a}})e^{iz\cdot\Phi^{\underline{a}}} \overset{(48)}{=} G_0(z)\int_{\Phi'} W'(\Phi')e^{i\Phi'\cdot z} \quad (76)$$

As has already been seen in Eq. (52) the dependence on gauge transformations of the background drops out. Subsequently, we substitute Eq. (37) with $c = 0$ for W' and choose Eq. (39) for w before carrying out the integral to obtain

$$\langle G_\Phi(z)\rangle_W^E = \frac{\sin\sqrt{\lambda z^2}}{\sqrt{\lambda z^2}}G_0(z). \quad (77)$$

The fraction involving the sine function arises from the Fourier transformation of $\delta(\Phi^2 - \lambda)$. If, instead, we are considering an $SU(N_c)$ gauge group, we have to replace the last line of Eq. (76) with the expression from Eq. (54). Choosing the same function for $W'(\Phi')$ yields

$$\langle G_\Phi(z)\rangle_W^{E,SU} = G_0(z)\left[\frac{N_c-1}{N_c}\frac{\sin\sqrt{\lambda z^2}}{\sqrt{\lambda z^2}} + \frac{1}{N_c}\left(\frac{\sin\sqrt{\lambda z^2}}{\sqrt{\lambda z^2}}\right)^{N_c-1}\right]. \quad (78)$$

The Fourier transformation has transformed the multiple convolution in the second line of Eq. (55) into a power.

For large distances $\sqrt{z^2}$, the propagators (77) and (78) are damped with respect to the free one. At short distances $\sqrt{z^2}$ the free propagator is recovered. In momentum space this manifests itself in the on-shell pole being removed and a pole proportional to $1/\sqrt{k^2}$ being introduced [1]. The elementary fermions are no longer part of the spectrum of freely propagating particles.

This result resembles the one for the tree-level propagator in [2,4]. However, there, as opposed to here, configurations of the gauge field were taken into account that lead to a nonvanishing field tensor. Copying the steps that lead to Eqs. (77) and (78), respectively, the weight used there $W_{\rm HP}(\Phi) \sim \exp(-\Phi^2/\Lambda^2)$ but constrained to zero field tensors gives $[U(N_c)]$

$$\langle G_\Phi(z)\rangle_{\rm HP}^E = \exp[-z^2\Lambda^2/4]G_0(z), \quad (79)$$

or in an $SU(N_c)$ gauge group

$$\langle G_\Phi(z)\rangle_{\rm HP}^{E,SU} = \{(1 - N_c{}^{-1})\exp[-z^2\Lambda^2/4] + N_c{}^{-1}\exp[-z^2\Lambda^2(N_c-1)/4]\}G_0(z). \quad (80)$$

For a large number of colors, N_c, the last expression reduces to the $U(N_c)$ result. Again the propagation over large distances $\sqrt{z^2}$ is suppressed while the free propagator is obtained for $\sqrt{z^2} \to 0$.

As Eq. (73) is satisfied by every fermion propagator which is a singlet of the color group, the previous result holds for all those correlators, too.

In background-field Feynman gauge for the present investigation it is sufficient to study either the gluon or the ghost propagator. Let us take the latter. Calculating the average of the ghost propagator (75) according to Eq. (64) with the elementary weight (39), that is choosing $c = 0$, yields

$$\langle \Gamma_\Phi(z)\rangle_W^{E,SU_3^{\rm adj.}} = \frac{1}{4}\Gamma_0(z)\left(1 + 3\frac{\sin\sqrt{\lambda z^2}}{\sqrt{\lambda z^2}}\right). \quad (81)$$

The free propagator is recovered for short distances $\sqrt{z^2}$. Six of eight channels are suppressed at large distances $\sqrt{z^2}$ while the two belonging to the zero eigenvalues remain freely propagating. Contrary to the fermions, which belong to the fundamental representation of the gauge group, the ghosts and gluons can be kept from propagating only partially. In momentum space the suppressed channels behave like the fermionic propagators in the background, that is the on-shell pole is absent and a pole proportional to $1/\sqrt{k^2}$ present. These results hold for every ghost/gluon propagator that is a color singlet.

In [2] for gluons in background configurations with, in general, nonzero field tensor and evaluated at leading order in $N_c{}^{-1}$ no remainder of freely propagating bosons is found. Already for fundamental reasons this cannot be due to the specific form of the weight. This can also be seen directly because, again, two free channels remain in the spectrum if it is used within the present framework:

$$\langle \Gamma_\Phi(z)\rangle_{\rm HP}^{E,SU_3^{\rm adj.}} = \tfrac{1}{4}\Gamma_0(z)\{1 + 3\exp[-z^2\Lambda^2/4]\}. \quad (82)$$

As the number of eigenvalue channels in the adjoint representation equals $N_c{}^2 - 1$ and the multiplicity of zero eigenvalues is $N_c - 1$ the zeros are of subleading importance for large values of N_c and go unnoticed in an analysis based on the leading order of an expansion in $N_c{}^{-1}$. If the analysis in [2] was extended to subleading orders in $N_c{}^{-1}$, the above could mean that freely propagating channels would be found.

Nevertheless, there is still the second difference that in Ref. [2] configurations with nonzero field tensors are taken into account. In that case the Lorentz components of the background vector do not commute and a simultaneous diagonalisation of all components is impossible. Then products of matrices could occur which do not belong to the adjoint representation, which is not closed with respect to matrix multiplication of its elements. Thereby the necessarily existing zero eigenvalues of the adjoint representation might be avoided and the propagation of the gluons be stopped entirely.

In the situation under investigation here, however, the behavior of the $N_c - 1$ photonlike gluons is fundamentally different from that of the rest, and the leading order of an expansion in $N_c{}^{-1}$ does have different characteristics than higher orders or the full result. For these reasons investigations based on large-N_c arguments or models of strong interactions with several "photons" should be treated with the necessary caution.

B. Minkowski space

Repeating the steps in Eq. (76), but with the weight function (43) and taking into account the Minkowski metric leads to the following form for the average fermion propagator:

$$\langle G_\Phi(z)\rangle_W^M = \sum_{j=1}^{3} a_j s^+_{\lambda_j}(z) G_0(z), \tag{83}$$

with the Fourier transformed scalar Feynman propagator

$$s^+_\lambda(z) := 4\pi^2\sqrt{\lambda}\frac{K_1(\sqrt{\lambda}\sqrt{-z^2 + i\epsilon})}{\sqrt{-z^2 + i\epsilon}}, \tag{84}$$

which arises after integration over the components of the background and where K_1 stands for a modified Bessel function. In an $SU(N_c)$ gauge group the result becomes [see Eq. (54)]:

$$\langle G_\Phi(z)\rangle_W^{M,SU} = G_0(z)\left\{\frac{N_c - 1}{N_c}\sum_{j=1}^{3} a_j s^+_{\lambda_j}(z) + \frac{1}{N_c}\left[\sum_{j=1}^{3} a_j s^+_{\lambda_j}(z)\right]^{N_c-1}\right\}. \tag{85}$$

For $z^2 = 0$ the previous expressions reduce to the free propagator. For large absolute values of z^2 the propagator is suppressed at least proportionally to $|z^2|^{-3/4}$ [1] with respect to the free propagator. Therefore, the fermions cannot propagate over arbitrarily large distances.

Like in the Abelian case [1] all terms for the $U(N_c)$ gauge group correspond to contributions of scalars to the self energy of the fermion without external legs, which indicates that no freely propagating particles are described. For the $SU(N_c)$ gauge group the first terms have the same interpretation. Only the last one corresponds to a sum over multiple $(N_c - 1)$-loop contributions of scalars to the self energy of the fermion once more without external legs. Thus, what was said concerning the absence of freely propagating particles remains valid.

Computing the average of the ghost propagator (75) with the weight (43) in $SU(3)$ yields

$$\langle \Gamma^\Phi(z)\rangle_W^{M,SU_3^{\mathrm{adj.}}} = \frac{\Gamma_0(z)}{4}\left[1 + 3\sum_{j=1}^{3} a_j s^+_{\lambda_j}(z)\right]. \tag{86}$$

For small absolute values of z^2 the free propagator is reobtained. At large absolute values of z^2 two $[N_c - 1]$ of the eight $[N_c{}^2 - 1]$ channels remain unaltered and correspond to freely propagating gluons.

The terms for which the propagation over long distances is suppressed again do resemble the contribution of scalars to the self energy of the considered particle—which now is the gluon/ghost—without external legs.

The observations for ghosts and gluons remain the same in all background-field Lorenz gauges. For those the ghost propagator (75) does not change. In the gluon propagator only the free part $\Gamma_0^{\mu\nu}(z)$ changes, which leaves the envelope function, that is the Fourier transformed weight function, unchanged.

Also in Minkowski space the above findings are valid for every propagator that is a color singlet. The consequences remain the same as in Euclidean space.

Finally, let us consider examples for gauge-invariant quantities, that is observables and how they are altered by the background. For this purpose, regard observables of the form

$$F(x, y) := \mathrm{tr}\{Q_1(x)G(x - y)Q_2(y)G(y - x)\}, \tag{87}$$

where $G(x - y) := \langle G_\Phi(x - y)\rangle_W$ and where Q_1, Q_2 are gauge-invariant operators which are to commute with local gauge transformations. Then, because of the cyclic property of the trace F is gauge invariant:

$$\begin{aligned} F(x, y) \to F_U(x, y) &:= \mathrm{tr}\{Q_1(x)U(x)G(x - y)U^\dagger(y)Q_2(y)U(y)G(y - x)U^\dagger(x)\} \\ &= \mathrm{tr}\{Q_1(x)U^\dagger(x)U(x)G(x - y)Q_2(y)U^\dagger(y)U(y)G(y - x)\} \\ &= \mathrm{tr}\{Q_1(x)G(x - y)Q_2(y)G(y - x)\} = F(x, y). \end{aligned} \tag{88}$$

After putting the explicit expressions for the propagators into F, we obtain

$$\begin{aligned} F(x, y) &= \mathrm{tr}\{Q_1(x)\tilde{W}(x - y)G_0(x - y)Q_2(y)\tilde{W}(y - x)G_0(y - x)\} = |\tilde{W}(x - y)|^2\,\mathrm{tr}\{Q_1(x)G_0(x - y)Q_2(y)G_0(y - x)\} \\ &=: |\tilde{W}(x - y)|^2 \times F_0(x, y), \end{aligned} \tag{89}$$

where $\tilde{W}$ represents the Fourier transformed weight function,

$$\tilde{W}(x-y) := \int_{\{\Phi\}} e^{i\Phi\cdot(x-y)} W(\Phi). \tag{90}$$

In Eq. (89) its absolute square appears inside a manifestly gauge-invariant quantity. This demonstrates that the modification caused by the ensemble of backgrounds is independent of the gauge.

V. SUMMARY

Non-Abelian gauge field theories have been studied whose correlators are defined with an additional average over a Lorentz and gauge-invariant ensemble of homogeneous pure gauge vector backgrounds. The background vector, Φ, acts as an additive contribution to the gauge field. The orginal theory is obtained by setting $\Phi = 0$. Modified contributions are characterized by normalisable functions of the Lorentz invariant quantity Φ^2. The construction can be seen as a method for taking into account a nontrivial vacuum structure.

The background ensemble's presence keeps the fermions from propagating freely over large distances; in Euclidean as well as in Minkowski space. The on-shell pole is absent in the dressed fermion propagator. At high momenta or short distances the particles can still propagate approximately freely. This finding coincides with the one for Abelian gauge groups [1] and the one obtained admitting nonzero field tensors [2,4]. Thus for fermions the pure gauge configurations of the background also block the free propagation in the case of non-Abelian charges.

For the gluons the propagation over long distances can be stopped in most but not all of the (color-)channels [13]. Technically, those remaining free reflect the $N_c - 1$ zero eigenvalues of elements of the adjoint representation under which the gluons transform. As the total number of channels equals ${N_c}^2 - 1$ the undamped channels are subleading in a large-N_c expansion. In [2] the propagation was hindered completely but then, there, the investigation was carried out to leading order in ${N_c}^{-1}$ and, additionally, nonzero field tensors were admitted. At leading order in ${N_c}^{-1}$ also in the present setting the free propagation is stopped. Nevertheless, the finite field tensors in [2] might also be of influence.

The results in the present article for the propagators in pure gauge backgrounds do not only hold for the correlators to all orders in the background and otherwise at tree-level but for every color singlet correlator. In Minkowski space the contributions which do not describe freely propagating particles resemble contributions of scalars to the self energy of the particle without external legs. This fact confirms the interpretation concerning the nonpropagation.

The interpretation of the present result in the sense of an extension of the standard model (SME) where Lorentz invariance is broken by a background but restored, as alluded to before, would lead to rather different constraints on the SME parameters. Most importantly the ususal coefficients—here the components of the vector Φ—become silent variables. The limits have to be determined for the weight functions. A suppression of the propagation over long distances of the quarks and of most of the gluonic channels, however, would surely also be discovered in many Lorentz-skewed ensembles of backgrounds. In these ensembles it is also possible to change the characteristics of the weight function by redefining the elementary spinors [14]. In the Lorentz invariant case this ambiguity can be avoided.

The feature that the propagation over short distances remains essentially unchanged while it is modified over long distances is shared by noncommutative field theories preserving Lorentz invariance [15].

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