

Observables in effective gravitySteven B. Giddings,^{*} Donald Marolf,[†] and James B. Hartle[‡]*Department of Physics, University of California, Santa Barbara, California 93106-9530, USA*

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We address the construction and interpretation of diffeomorphism-invariant observables in a low-energy effective theory of quantum gravity. The observables we consider are constructed as integrals over the space of coordinates, in analogy to the construction of gauge-invariant observables in Yang-Mills theory via traces. As such, they are explicitly nonlocal. Nevertheless we describe how, in suitable quantum states and in a suitable limit, the familiar physics of local quantum field theory can be recovered from appropriate such observables, which we term “pseudolocal.” We consider measurement of pseudolocal observables, and describe how such measurements are limited by both quantum effects and gravitational interactions. These limitations support suggestions that theories of quantum gravity associated with finite regions of spacetime contain far fewer degrees of freedom than do local field theories.

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I. INTRODUCTION

An outstanding and central issue in the quantum mechanics of gravity is the identification and interpretation of observables, see e.g. [1] and references therein. If gravity is studied about a background with an asymptotic region, then this issue can be sidestepped, or at least postponed, by focusing attention on the S -matrix and by avoiding asking questions about local quantities within the spacetime. Such backgrounds include the interesting cases of asymptotically Minkowski and asymptotically anti-de Sitter spacetimes,¹ but not generic cosmologies. However, even in cases with an asymptotic region, restricting attention to the S -matrix leaves out critical physics; namely, the physics described by local observers within the spacetime. We are manifestly local observers within our own cosmological spacetime, and ultimately one of the goals of physics must be a precise mathematical description of the observations we make.

For many practical purposes, predictions can be made using the formalism of quantum field theory in a curved background. However, this puts aside the important problem of describing local physics in a framework consistent with the expected symmetries and properties of an effective low-energy quantum theory of gravity. Moreover, we expect such a framework to be indispensable in any attempt to describe the region near the singularity of a black hole, the early universe, or more global aspects of quantum cosmology.

In particular, in field theory, the *local* observables of the theory play a central role. However, the low-energy sym-

metries of quantum gravity apparently include diffeomorphism invariance which, as we will review, is known to preclude the existence of local observables. This leads to a well-known quandary in describing our own observations, which are accomplished within the finite spacetime volume of the laboratory or observatory. In particular, such observations take place on time and distance scales that are quite small as compared to those set by cosmology. It is clear that such observations are not fundamentally described by a global S -matrix.

Thus, this paper works towards two important goals. The first is to improve our understanding of possible constructions of diffeomorphism-invariant observables, which are the allowed observables in quantum gravity. The second is to find observables that, in appropriate circumstances, approximately reduce to local observables of field theory, or, more generally, to “nearly local” field-theory observables, such as multilocal expressions or Wilson loops.

As we will describe, in a wide class of circumstances, an approach to the problem of defining diffeomorphism-invariant observables is to define quantities that are integrals (or multiple integrals) over spacetime. This is in rough analogy to using traces (or multiple traces) to define gauge-invariant observables in Yang-Mills theory.

Given this, the next problem is to identify observables that, in an appropriate sense, reduce to the local observables of field theory. A central idea here is that such observables should be “relational.” In classical general relativity, one approach is to discuss the spacetime location of events relative to some physical reference body, such as a clock on the earth. Specifying events in this way allows one to build relational classical observables, in an approach going back to [5], such as the value of $R_{abcd}R^{abcd}$ at an event specified by its relation to the earth and to the time registered on the clock. Such quantities capture a certain sense of locality, but are nevertheless *observables*, in the sense that they define diffeomorphism-invariant functions

^{*}Electronic address: giddings@physics.ucsb.edu[†]Electronic address: marolf@physics.ucsb.edu[‡]Electronic address: hartle@physics.ucsb.edu¹For discussion of the S -matrix in anti-de Sitter space, see [2]. The status of an observable S -matrix in the de Sitter case is more controversial, but see [3,4].

on the space of classical solutions. The question then is how, and to what extent, similar correlations can be used to extract physical information in quantum gravity.

The literature contains a number of approaches to this question, see e.g. [1,5–13]. The method of defining *relational operators* has, in particular, been followed and extended to the quantum context in [5,6,9,11,14–22]. Here we pursue this direction further, and argue that this is the key to extracting physics that reduces to that of local field theory in appropriate approximations. These relational operators are to be quantum analogues of the classical relational observables discussed above.

Specifically, one of our main results will be to argue that, in an appropriate limit, certain such relational, diffeomorphism-invariant observables of quantum gravity reduce to the more familiar *local* observables of quantum field theory on a fixed spacetime background. We refer to such diffeomorphism-invariant observables as “pseudolocal.” An important point is that this reduction depends on the *state* as well as the observable in question.

Our work represents a field-theoretic generalization of similar results [18–20] previously established² for certain relational observables in various $0 + 1$ dimensional systems (reparametrization-invariant quantum mechanics). These field-theoretic observables suggest fundamental limits on locality and quantum measurement. Moreover, growth in their fluctuations with the volume of space raises interesting questions in both the quantum cosmological and asymptotically flat contexts.

In outline, we first summarize the effective field-theory approach to gravity, describing its long-distance quantum dynamics and symmetries, as well as the problem of finding observables respecting these symmetries. In Sec. III we investigate a broad class of diffeomorphism-invariant quantities that we expect to serve as observables in gravitational physics in a manner similar to Refs. [5,6,14–21]. Section IV focuses on a special subclass of these observables, the “pseudolocal” observables, which in certain approximations reduce to local observables of field theory; we do so primarily by giving illustrative examples. Section V discusses measurement theory of these observables. Section VI describes limitations on observables arising from considerations of quantum mechanics and gravity. In particular, we see how general arguments (see e.g. [5,23–26]) concerning measurements in quantum gravity manifest themselves in terms of restrictions on our relational operators. Such limitations may represent fundamental restrictions on observation, and on the domain of validity of local quantum theories. We close with a brief summary and discussion in Sec. VII.

²In [20] such pseudolocal observables in $0 + 1$ dimensional models were referred to as “almost-local” observables.

II. EFFECTIVE GRAVITY AND THE PROBLEM OF OBSERVABLES

As a fully controlled theory of quantum gravity does not yet exist, we take an agnostic position here as to the nature of this underlying fundamental theory; while, for example, string theory could well be such a theory, as yet we lack the ability to perform many calculations (particularly in the nonperturbative regime). However, whatever its dynamics, we expect the fundamental theory to reduce to quantum general relativity in non-Planckian regimes. Thus, our initial viewpoint is that we will deal with the nonrenormalizability of general relativity by treating it as an effective theory with a cutoff at $\approx \mathcal{O}(M_p)$, with a renormalization prescription specifying the infinite number of couplings determined by the more fundamental theory. Ultimately, we will find further constraints that suggest the need to supplement this cutoff with more stringent limitations on the effective theory.

While we will not be precise about the nature of the cutoff, in our view a central question is in what regime the low-energy effective theory predicts its own failure; before this one expects that the precise cutoff prescription has a negligible effect, and beyond this we will need the full quantum dynamics of the underlying theory to make predictions.

Although we do not know the fundamental description of states in quantum gravity, we expect that the cutoff theory has an effective description similar to that obtained by canonical quantization of the gravitational field. In particular, there should be a regime in which states $|\Psi\rangle$ admit an effective description in terms of functionals $\Psi[h_{ij}, \phi^r]$ of a Euclidean signature three-metric (or in greater generality a $D - 1$ -metric) and other fields ϕ^r on some surface Σ , where in the classical limit Σ will become a spacelike three-surface embedded in some four-dimensional spacetime.

The canonical formalism provides a useful perspective on the long-distance quantum dynamics of gravity. In addition to any symmetries of the matter theory, the low-energy symmetries of the theory should include diffeomorphisms, $x^\mu \rightarrow x^\mu + \xi^\mu(x^\nu)$. As a consequence, we learn from this formalism (see, e.g., [6,27]) that this dynamics should be described by a set of constraints of the form

$$\mathcal{H}|\Psi\rangle = 0, \quad \mathcal{H}_i|\Psi\rangle = 0, \quad (2.1)$$

where \mathcal{H} is the densitized scalar constraint (sometimes called the “Wheeler-DeWitt operator”),

$$\mathcal{H} = G_{ijkl}\pi^{ij}\pi^{kl} - \sqrt{h}\left[{}^3R(h) + \frac{16\pi}{M_p^2}\mathcal{H}^m(\pi_r, \phi^r, h)\right], \quad (2.2)$$

and \mathcal{H}_i are the densitized vector constraints

$$\mathcal{H}_i = \frac{16\pi}{M_p^2} (-2D_j \pi^{ij} + \mathcal{H}_i^m). \quad (2.3)$$

Here D_i is the covariant derivative on Σ compatible with h_{ij} . In the above, the superspace metric G_{ijkl} is

$$G_{ijkl} = \frac{1}{2} \left(\frac{16\pi}{M_p} \right)^2 \frac{1}{\sqrt{h}} (h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}), \quad (2.4)$$

while \mathcal{H}^m , \mathcal{H}_i^m represent contributions from the matter fields, and π^{ij} , π_r are the momenta conjugate to h_{ij} , ϕ^r . In particular, in the wave functional representation described above, π^{ij} , π_r , will act as $-i \frac{\delta}{\delta h_{ij}}$, $-i \frac{\delta}{\delta \phi^r}$. [In greater generality, initial conditions or processes that cause the Universe to branch may, in a third quantized framework [28,29], introduce nonzero terms on the right-hand side of (2.1); for further discussion see [29].] Proper definition of these operators requires an appropriate operator ordering and regularization, which we view as being supplied by our cutoff prescription.

Although the constraints \mathcal{H} , \mathcal{H}_i encode invariance under diffeomorphisms, they generate a somewhat different algebra known as the ‘‘hypersurface deformation algebra,’’

$$\begin{aligned} \left[\int_{\Sigma} N \mathcal{H}, \int_{\Sigma} M \mathcal{H} \right] &= i \frac{16\pi}{M_p^2} \\ &\quad \times \int_{\Sigma} (N \partial_i M - M \partial_i N) h^{ij} \mathcal{H}_j, \\ \left[\int_{\Sigma} N^i \mathcal{H}_i, \int_{\Sigma} M^j \mathcal{H}_j \right] &= i \frac{16\pi}{M_p^2} \int_{\Sigma} [\vec{N}, \vec{M}]^k \mathcal{H}_k, \\ \left[\int_{\Sigma} N \mathcal{H}, \int_{\Sigma} M^j \mathcal{H}_j \right] &= -i \frac{16\pi}{M_p^2} \int_{\Sigma} \mathcal{L}_{\vec{M}} N \mathcal{H}, \end{aligned} \quad (2.5)$$

where $\mathcal{L}_{\vec{M}}$ denotes the Lie derivative along the vector field M^j and $[\vec{N}, \vec{M}]$ denotes the commutator of the two vector fields. The operators $\int_{\Sigma} N^i \mathcal{H}_i$ generate diffeomorphisms of Σ and, on classical solutions, the operators $\int_{\Sigma} N \mathcal{H}$ generate displacements of the hypersurface Σ along the vector field $N n^\mu$, where n^μ is the future-pointing space-time normal to Σ . As a result, the hypersurface deformation algebra generates the same orbits in the space of classical solutions as does the diffeomorphism group [30,31]; i.e., invariance of a function on the space of solutions under the action of one algebra is equivalent to invariance under the action of the other algebra. Similarly, invariance under the constraints \mathcal{H} , \mathcal{H}_i should also en-

code diffeomorphism invariance in the low-energy effective description of quantum gravity.

The lack of local observables in gravity is now clear. As first emphasized by Dirac [27], a predictive framework requires that observables commute with the generators of gauge symmetries. However, for example, given any local scalar field $\phi(x)$, this field commutes with the constraints if and only if it is invariant under all diffeomorphisms; i.e., if $\partial_\mu \phi$ vanishes identically. Similar results follow for spinor, vector, and tensor fields. Hence, local fields are not observables in theories with gravity.

Now, one could take the viewpoint that we cannot even approximately identify gauge-invariant observables until we have total control over the fundamental theory of quantum gravity. However, this seems an extreme position if there is a sensible cutoff theory of effective gravity at low energies. The reason is that observables should exist in the effective theory, and such observables should respect the low-energy gauge invariance. Put differently, we believe that we should be able to describe the low-energy observations of local observers in terms of the framework of low-energy gravity. While an exact identification of the observables of quantum gravity presumably requires the ultimate fundamental theory of quantum gravity, we expect that a framework for treating them in the low-energy theory will remain useful.

III. DIFFEOMORPHISM-INVARIANT OBSERVABLES

As reviewed above, the problem of finding quantum gravity observables is that of finding the appropriate gauge-invariant operators. Moreover, the ones capable of describing our experiences in the laboratory should reduce to the usual local observables of quantum field theory in an appropriate limit.

Beginning with the first question, in effective gravity, we seek operators that are combinations of the metric and other fields ϕ^r , which are Hermitian³, and which commute with the constraints, \mathcal{H} , \mathcal{H}_i . For example, let $\hat{\mathcal{O}}(x)$ be a local scalar observable in ordinary quantum field theory; in a scalar theory, we might consider $\hat{\mathcal{O}}(x) = \phi(x), \phi^2(x), \dots$. Such an operator is not diffeomorphism invariant, but

$$\mathcal{O} = \int d^4x \sqrt{-g} \hat{\mathcal{O}}(x) \quad (3.1)$$

³As discussed below, we will use the *induced* or *group averaging* inner product [18,32–36] on the space of physical states [i.e., those satisfying (2.1)], so that operators which are Hermitian with respect to the inner product on the auxiliary Hilbert space are automatically Hermitian on the physical Hilbert space. However, due to the complicated nature of the operators we consider, self-adjointness can be more subtle. See [37] for comments on this issue and an example of how it may be dealt with.

is clearly diffeomorphism invariant. It also commutes with the constraints \mathcal{H} , \mathcal{H}_i . The key step in this argument is that we define the time dependence of $\hat{\mathcal{O}}(x)$ in (3.1) through the Heisenberg equation of motion

$$i \frac{\partial}{\partial t} \hat{\mathcal{O}}(x) = \left[\hat{\mathcal{O}}(x), \int_{\Sigma} (N\mathcal{H} + N^i \mathcal{H}_i) \right]. \quad (3.2)$$

Thus, the analogous commutator with \mathcal{O} reduces directly to a boundary term, which vanishes under appropriate boundary conditions.⁴ It is also clear that (3.1) is invariant under spatial diffeomorphisms, and therefore that it commutes with any operator of the form $\int_{\Sigma} \tilde{N} \mathcal{H}$, where \tilde{N} is related to N in (3.2) by a spatial diffeomorphism. We may combine these observations to show that \mathcal{O} commutes with \mathcal{H} , \mathcal{H}_i . The corresponding fact is explicitly shown in a number of 0 + 1 models in [18], which paid close attention to subtleties such as the implicit appearance of N , N^i in (3.1) [through the time dependence of $\hat{\mathcal{O}}(x)$].

More generally, for a collection of matter fields ϕ^r , consider an arbitrary local scalar density formed from the fields, the metric, and their derivatives which is invariant under any gauge symmetries of the matter theory,

$$\tilde{\mathcal{O}} = F(\phi^r(x), \partial_{\mu} \phi^r(x), \dots; g_{\mu\nu}(x), \partial_{\lambda} g_{\mu\nu}(x), \dots). \quad (3.3)$$

Then

$$\mathcal{O} = \int d^4x \tilde{\mathcal{O}}(x) \quad (3.4)$$

will commute with the constraints \mathcal{H} , \mathcal{H}_i and is an observable if $\tilde{\mathcal{O}}$ is Hermitian. We refer to observables of the form (3.4) as “single-integral observables.” Clearly, we can formulate other operators that are likewise diffeomorphism invariant, but which are more complex, by considering operators that depend on more than one point. Examples would be objects such as

$$\mathcal{O} = \int d^4x \sqrt{-g} \int d^4y \sqrt{-g} f(\phi(x), \phi(y)), \quad (3.5)$$

generalizations of Wilson loops, and other such “multi-local” expressions.

⁴More discussion of boundary conditions will follow. For examples in the 0 + 1 context, see [18,20,35,38]. In particular, convergence of the integral in (3.1) [and in (3.3) below] is a subtle issue: The integral converges on what is called the auxiliary Hilbert space below, but this space may contain no normalizable states satisfying the constraints (2.1). Nevertheless, the action of \mathcal{O} on this auxiliary Hilbert space induces an action on physical states.

To describe local experiments, we will be interested in such observables which (approximately) localize in some spacetime region, and the corresponding operators will need to include physical degrees of freedom which specify this region. This connects to a perspective going back to Einstein [39], and emphasized by DeWitt [5,6], which we may paraphrase as follows: the description of the flow of time requires a self-consistent inclusion of the actual dynamical degrees of freedom that register this flow. We follow an established tradition and refer to such degrees of freedom as a *clock*, though we emphasize that the reading of this clock need not be simply related to the passage of proper time as defined by some metric, and though more generally we are interested in position information in both space and time directions. We hope that this terminology does not cause excessive confusion.

In preparation for proceeding, let us make three comments. First, we will be most interested in operators $\tilde{\mathcal{O}}$ which are composite, and such operators require a regularization in order to be defined in quantum field theory. We assume this is provided by the cutoff of the effective gravity theory, and that appropriate renormalization prescriptions are provided at that cutoff scale. Second, note that single-integral observables are precisely the operators that can be added to the action to give a local interaction term in the low-energy effective gravity theory. Finally, while the integrals in e.g. (3.1), (3.4), and (3.5) formally may extend into regions where the effective gravity description begins to fail, we assume that there are appropriate operators and states for which the contribution of such regimes is small. We will elaborate more on this point subsequently.

Before considering details of the problem of localization, we finish this section by discussing the formal role of diffeomorphism-invariant observables in a theory of gravity. In particular, we will discuss details of defining matrix elements of the above diffeomorphism-invariant observables between physical states; i.e., between states satisfying the constraints (2.1). (We will discuss the relation of such matrix elements to measurements in Sec. V.) This discussion is rather technical. The reader may wish to scan the rest of this section quickly on a first reading of the paper.

Computation of matrix elements requires an inner product on the space of physical states. Here we follow an approach described in [18,32–35,38,40] which define the *induced* or *group averaging* inner product on physical states.⁵ This inner product also agrees with certain Becchi-Rouet-Stora-Tyutin methods [42].

⁵See [38] for a brief introduction to the method and [41] for comments on how, in a minisuperspace context, the positive definite induced inner product can be related to the more familiar Klein-Gordon inner product.

As a first step, we may note that the space of functionals of the metric and fields (i.e. not necessarily satisfying the constraints) can be made into a Hilbert space via the usual Schrödinger representation inner product.⁶ However, in general *no* states satisfying the constraints (2.1) will be normalizable in this inner product. The physical inner product can at best be a “renormalized” version of the auxiliary inner product.⁷ For this reason, we follow the tradition of referring to the resulting Hilbert space as the *auxiliary* Hilbert space. We denote the corresponding (auxiliary) inner product as $\langle \Psi_2 | \Psi_1 \rangle$. States in the auxiliary Hilbert space may be expanded, for example, in terms of the basis of eigenstates $|h_{ij}, \phi^r\rangle$ of the configuration variables⁸ h_{ij}, ϕ^r .

We may usefully combine the step of solving the constraints with the step of introducing a useful inner product on the space of solutions. In particular, consider the functional integral

$$\langle h_2, \phi_2^r | \eta | h_1, \phi_1^r \rangle := \int_{h_1, \phi_1^r}^{h_2, \phi_2^r} \mathcal{D}g \mathcal{D}\phi^r e^{iS}, \quad (3.6)$$

where we have taken this integral to define the matrix elements of an object η .⁹ Here S is the action, h_i, ϕ_i^r specify data on initial and final slices, and we functionally integrate over all interpolating geometries and field configurations, with an appropriate gauge-fixing procedure. While we have written (3.6) in a covariant notation, the functional integral we have in mind is most easily defined using the canonical form of the functional integral in which $S = \int dt d^3x (N\mathcal{H} + N^i \mathcal{H}_i)$ where N, N^i are the lapse and shift.

In particular, we take the integral $\mathcal{D}g$ above to include an integral over both positive and negative lapse. An important consequence of this is that, as noted in e.g.

⁶In fact, in field theory the particular inner product used needs to be adapted to the dynamics of the theory. However, at the formal level at which we work here, all such details are taken care of by the path integral and the renormalization process.

⁷Some constraints may have both normalizable and non-normalizable solutions, in which case one expects that these two classes define different superselection sectors. One expects similar superselection rules between classes of states whose norms in the auxiliary space in some sense have different degrees of divergence. See [34,35,40,43–45].

⁸In fact, due to our desire to perform the integrals (3.1) and (3.4), we work in a Heisenberg picture in which the operators h_{ij}, ϕ^r depend on time. By $|h_{ij}, \phi^r\rangle$, we mean the eigenstate of h_{ij}, ϕ^r on some (fixed but arbitrary) reference hypersurface Σ_0 in the space of coordinates x .

⁹If the inner product on the auxiliary Hilbert space was chosen appropriately, (3.6) and linearity should at least define matrix elements $\langle \Psi_1 | \eta | \Psi_2 \rangle$ of η when $|\Psi_1\rangle, |\Psi_2\rangle$ lie in a dense subspace Φ of this Hilbert space, though $\eta | \Psi_2 \rangle$ itself may not be a normalizable state. Instead, the image of η naturally consists of linear functionals on Φ , which is sufficient for our purposes. See [46] for a discussion of the path integral, and [33–35,38,40] for a more general discussion of this point. See also [47,48] in the context of loop quantum gravity.

[49], the functional integral (3.6) satisfies the constraint equations (2.1) in *both* arguments. That is, we have

$$\langle h_2, \phi_2^r | \mathcal{H} \eta | h_1, \phi_1^r \rangle = \langle h_2, \phi_2^r | \eta \mathcal{H} | h_1, \phi_1^r \rangle = 0, \quad (3.7)$$

and similarly for \mathcal{H}_i . The operator η is often called a “rigging map”; roughly speaking, we may think of η as a functional delta function $\eta \sim \Delta[\mathcal{H}, \mathcal{H}_i]$ which enforces the entire set of constraints. We see that the image of η consists of solutions to the constraints and, in addition, we see that any state of the form $\mathcal{H} | \Psi \rangle$ is annihilated by η . Thus η is highly degenerate, and we may think of η as identifying entire equivalence classes, denoted $|\Psi\rangle$, of auxiliary states $| \Psi \rangle$ with solutions of the constraints. Thus, we may think of the equivalence classes $|\Psi\rangle$ as physical states themselves; i.e., $|\Psi\rangle = \eta | \Psi \rangle$. Note that the projection $|h, \phi^r\rangle = \eta | h, \phi^r \rangle$ results in an *overcomplete* basis of physical states.

The integral over both positive and negative lapse in (3.6) also implies η is Hermitian. It thus defines an inner product, which we shall denote in the usual Dirac fashion, on the equivalence classes $|\Psi\rangle$:

$$\langle \Psi_1 | \Psi_2 \rangle := \langle \Psi_1 | \eta | \Psi_2 \rangle. \quad (3.8)$$

As discussed in [46], the inner product (3.8) defined in this way by (3.6) agrees with what is known as the *induced* (or group averaging) inner product. If (3.8) is positive definite,¹⁰ it defines a Hilbert space of physical states.

Given that $[\mathcal{O}, \mathcal{H}] = [\mathcal{O}, \mathcal{H}_i] = 0$, the observable \mathcal{O} preserves the space of physical states;¹¹ i.e.,

$$\mathcal{H} \mathcal{O} | \Psi \rangle = \mathcal{H}_i \mathcal{O} | \Psi \rangle = 0. \quad (3.9)$$

As a result, the above definitions allow us to compute the matrix element of an observable \mathcal{O} between two physical states; we can act with \mathcal{O} on state $|\Psi_1\rangle = \eta | \Psi_1 \rangle$, and then take its induced product with $|\Psi_2\rangle = \eta | \Psi_2 \rangle$, in the usual fashion:

$$\langle \Psi_2 | \mathcal{O} | \Psi_1 \rangle := \langle \Psi_2 | \mathcal{O} \eta | \Psi_1 \rangle. \quad (3.10)$$

Note that since (3.10) is the physical inner product of $\mathcal{O} \eta | \Psi_1 \rangle$ and $\eta | \Psi_2 \rangle$, the result depends only on the choice of physical states $|\Psi_1\rangle, |\Psi_2\rangle$ and not on the particular representatives $| \Psi_1 \rangle, | \Psi_2 \rangle$ of the corresponding equivalence classes.

So far we have outlined the definition of a rather broad class of operators which are manifestly nonlocal. As yet, we have made no direct contact with local observables in quantum field theory. However, in the sections below we explore how, in an appropriate approximation, certain diffeomorphism-invariant operators do indeed reduce to the local observables of ordinary quantum field theory,

¹⁰See [35,38,40,50] for known results concerning this positivity.

¹¹In fact, because η is built from \mathcal{H} and \mathcal{H}_i , \mathcal{O} commutes with η in the sense that $\mathcal{O} \eta | \Psi \rangle = \eta \mathcal{O} | \Psi \rangle$.

with one critical caveat: such a reduction depends essentially on the choice of state $|\Psi\rangle$ in combination with the choice of observable \mathcal{O} . These points are best illustrated by examples, to which we now turn.

IV. DIFFEOMORPHISM-INVARIANT OBSERVABLES AND LOCALIZATION: EXAMPLES

In the last section, we outlined the general low-energy effective framework for quantum gravity, emphasizing that observables are necessarily invariant under the constraints, and that such operators are naturally given by diffeomorphism-invariant expressions such as (3.1) and (3.5). As noted above, diffeomorphism invariant operators are not local, so that additional steps are required to mesh this discussion with our usual treatment of local physics. We attempt to fill this gap here through a treatment of a number of examples.

Before beginning, let us recall from the last section that a critical step is to define diffeomorphism-invariant observables on the *auxiliary* Hilbert space. If this can be done, then such operators naturally define observables on the physical Hilbert space as well. As a result, we may reasonably hope to separate the treatment of some issues of locality from a detailed study of, say, the constraints (2.1). For this reason, we begin our first two examples by working with diffeomorphism-invariant operators in the context of scalar field theory in the usual (unconstrained) Fock space, before considering coupling to the 3 + 1 gravitational field. In contrast, our last two examples will directly include the gravitational field, albeit in low dimensions (0 + 1 and 1 + 1) where the dynamics of gravity is somewhat trivial.

A. Scalar fields as physical coordinates: The Z model

As our first example, we discuss pseudolocal diffeomorphism-invariant observables constructed using scalar quantum field theory. This may be regarded either as a toy model that illustrates some features of interest, or as a first step toward studying pseudolocal observables in low-energy gravity coupled to a set of such scalars. In particular, we will see how one can, approximately and in an appropriate state, connect pseudolocal observables to the usual framework of local observables of quantum field theory. In doing so, the key point is that the location of the local observable is specified *relative* to a structure determined by the state in a manner determined by the particular pseudolocal observable. We believe that this example serves as a paradigm for how the local operators of field theory can be recovered in theories with diverse field content.

Our starting point is a general theory with fields ϕ^a . We work in four dimensions, although the naive generalization to higher dimensions follows trivially; we initially consider working in a flat background, but discuss aspects of curved

spacetimes shortly. To define the Z-model corresponding to the field theory, we assume that in addition to the fields ϕ^a we have four additional massless free scalar fields Z^i , $i = 0, 1, 2, 3$. For such a theory, we may consider an initial state $|\Psi_Z\rangle$ such that, in some region of spacetime,

$$\langle\Psi_Z|Z^i|\Psi_Z\rangle = \lambda\delta_\mu^i x^\mu, \quad (4.1)$$

that is, the fields have expectation values that satisfy the classical equations of motion and moreover are proportional to the background coordinates. The state of these fields therefore spontaneously breaks the Poincaré invariance of the background spacetime. In particular, we will take Ψ_Z to be a minimally excited such state, in the sense that we take the fluctuating field

$$\tilde{Z}^i = Z^i - Z_{\text{cl}}^i = Z^i - \lambda\delta_\mu^i x^\mu \quad (4.2)$$

to be in the Fock ground state.

The basic idea is that positions of local observables can be defined in a translation invariant way relative to the background expectation values (4.1). Specifically, given a local operator $O(x)$ in the theory of the ϕ^a 's, we might imagine defining operators of the form

$$\mathcal{O}_{0,\xi} = \int d^4x O(x) \delta[Z^i(x) - \xi^i] \left| \frac{\partial Z^i}{\partial x^\mu} \right|. \quad (4.3)$$

Such operators were suggested in [5], though we will treat them directly in quantum field theory without first passing to the semiclassical limit. For a classical solution of the form (4.1), the delta function picks out a definite point. Moreover, it will pick out a finite set of points in a generic perturbation of (4.1). Thus, operators of the form (4.3) qualify as pseudolocal observables.

The operator defined in (4.3) is not only Poincaré invariant, but also diffeomorphism invariant under changes of coordinates $x^\mu \rightarrow x^{\mu'}(x^\nu)$. $\mathcal{O}_{0,\xi}$ is, however, potentially problematic to define in the context of a quantum field theory due to the δ -function of quantum fields in (4.3). For this reason, we instead consider a similar but more regular operator of the form

$$\mathcal{O}_\xi = \int d^4x O(x) e^{-(1/\sigma^2)(Z^i - \xi^i)^2} \left| \frac{\partial Z^i}{\partial x^\mu} \right|, \quad (4.4)$$

where σ is a constant of mass dimension one that plays the role of a *resolution* of the operator in (4.3).

Suppose now that we evaluate the expectation value of a product of a collection of N such operators, each with different ξ_A^i , $A = 1, \dots, N$, in a state of the form (4.1). We might expect that this expectation value approximately reduces to the correlation function of a product of the operators $O(x_A^\mu)$, with locations given by

$$x_A^\mu = \frac{1}{\lambda} \delta_i^\mu \xi_A^i. \quad (4.5)$$

Let us examine this calculation more closely in order to check this statement, and also to find its limitations.

The functional integral computes the correlation function, in the state $|\Psi_Z\rangle$, time ordered with respect to parameter time,

$$\langle T(\mathcal{O}_{\xi_1} \cdots \mathcal{O}_{\xi_N}) \rangle = \int \mathcal{D}\phi^a \int_{\Psi_Z} \mathcal{D}Z e^{iS[\phi^a] + iS[Z]} \prod_A^N \mathcal{O}_{\xi_A}. \quad (4.6)$$

Here, as we have indicated, the boundary conditions on the Z integral are furnished by the state giving (4.1). We assume that the Gaussian operators in Z are determined

$$\int_{\Psi_Z} \mathcal{D}Z e^{iS[Z]} \prod_A^N e^{-(1/\sigma^2)(Z^i(x_A) - \xi^i)^2} \left| \frac{\partial Z^i}{\partial x^\mu} \right| = \int \prod_A \left(\frac{\sigma^4}{16\pi^2} d^4 \kappa_A \right) \times e^{iS[Z_{\text{cl}}]} \left[e^{-\sum_A \kappa_A^2 \sigma^2/4 + i\kappa_{A,i}(\lambda x_A^i - \xi_A^i)} e^{-i/2 \sum_{AB} \kappa_A \cdot \kappa_B G(x_A, x_B)} \left| \frac{\partial Z_{\text{cl}}^i}{\partial x^\mu} \right| M(x_A, \kappa_{A,i}) \right]. \quad (4.8)$$

Here $G(x_A, x_B)$ is the appropriate Green's function and M is a factor arising from the fluctuation part of the Jacobian. The first exponent is the classical action for Z_{cl} , the second is the contribution of the classical solution to the correlation function, and the third comes from fluctuations of the fields Z about Z_{cl} . The correlation function (4.6) then incorporates this expression as

$$\int \prod_A dx_A \langle T(\mathcal{O}(x_1) \cdots \mathcal{O}(x_N)) \rangle_\phi \times \int_{\Psi_Z} e^{iS[Z]} \mathcal{D}Z \prod_A^N e^{-(1/\sigma^2)(Z^i(x_A) - \xi^i)^2} \left| \frac{\partial Z^i}{\partial x^\mu} \right|, \quad (4.9)$$

where the notation $\langle \cdots \rangle_\phi$ denotes a correlator in the vacuum of the ϕ theory.

If we can neglect the fluctuation pieces of (4.8), this expression reduces to the usual field-theory correlator of the $\mathcal{O}(x_A)$'s, smeared over a width

$$\Delta x_A \sim \sigma/\lambda \quad (4.10)$$

about the values (4.5),

$$\langle T(\mathcal{O}_{\xi_1} \cdots \mathcal{O}_{\xi_N}) \rangle \approx \int \mathcal{D}\phi^a e^{iS[\phi^a]} [\mathcal{O}(x_1) \cdots \mathcal{O}(x_N)]. \quad (4.11)$$

The operator products $\mathcal{O}_{\xi_1} \cdots \mathcal{O}_{\xi_N}$ (without time ordering) behave similarly.

Fluctuations correct this expression. One can estimate their sizes by expanding Z as in (4.2) and extracting the leading (quadratic) term. Equivalently, without the Jacobian factor $|\frac{\partial Z}{\partial x}|$, the requirement that they be small follows immediately from the form of (4.8),

in some regularization scheme, by a set of operator boundary conditions, which we assume preserves the correct semiclassical limit for the Gaussian. A convenient way to evaluate this expression is to Fourier transform,

$$e^{-(1/\sigma^2)(Z^i - \xi^i)^2} = \frac{\sigma^4}{16\pi^2} \int d^4 \kappa e^{-[(\kappa^2 \sigma^2)/4] + i\kappa_i(Z^i - \xi^i)}. \quad (4.7)$$

We then write Z^i as a classical piece plus fluctuation piece, as in (4.2), and functionally integrate over \tilde{Z}^i to find

$$\frac{1}{\sigma^2} \langle T(\tilde{Z}^i(x_A) \tilde{Z}^j(x_B)) \rangle = \frac{\delta^{ij}}{\sigma^2} G(x_A, x_B) \sim \frac{\delta^{ij}}{\sigma^2} \frac{1}{(x_A - x_B)^2} \ll 1. \quad (4.12)$$

Including contributions of the Jacobian, we also find the conditions

$$\frac{1}{\sigma \lambda} \langle T(\partial \tilde{Z}^i(x_A) \tilde{Z}^j(x_B)) \rangle \sim \frac{\delta^{ij}}{\sigma^2} \frac{1}{(x_A - x_B)^2} \frac{\sigma}{\lambda} \frac{1}{(x_A - x_B)} \ll 1$$

$$\frac{1}{\lambda^2} \langle T(\partial \tilde{Z}^i(x_A) \partial \tilde{Z}^j(x_B)) \rangle \sim \frac{\delta^{ij}}{\lambda^2} \frac{1}{(x_A - x_B)^4} \ll 1. \quad (4.13)$$

One can begin to understand these conditions by considering working in an effective theory¹² with a momentum cutoff Λ . In such a theory, there is effectively a bound

$$\frac{1}{|x_A - x_B|} \lesssim \Lambda. \quad (4.14)$$

Saturating this bound gives the tightest constraint from (4.12): $\sigma \gg \Lambda$. The Gaussian uncertainty (4.10) in the positions x_A is determined by σ and λ . The field momentum λ should be bounded by Λ^2 , for validity of the cutoff theory. These statements then translate into a lower bound on the uncertainty in x_A :

$$\Delta x \gg \frac{1}{\Lambda}. \quad (4.15)$$

¹²However, it is interesting to note that, even for our highly composite operators (4.4), the correlators of operator products (without time ordering) are well-defined and approximate correlators of $\phi(x_1) \cdots \phi(x_N)$ without any such cutoff, so long as the theory of the ϕ -field is itself well defined. In particular, \mathcal{O}_ξ is a densely defined operator on our Fock space. These results will be presented in a forthcoming paper.

This result is sensible: in the context of the cutoff theory, the maximum distance resolution is the inverse of the cutoff. These results are readily generalized to other dimensions.

The constraints (4.12) and (4.13) are due to basic quantum uncertainty in the definition of the position using the relation to the state of the Z fields. While they have been derived directly only in this model, we expect similar results to hold for an arbitrary model in which the location at which an observable is being computed is determined by a physical dynamical clock or position variable localized in the region being investigated.¹³ The reason is that they follow simply from the uncertainty principle and from the properties of a theory with a cutoff.

We now complete our discussion of the Z model by making a few comments on the generalization to include a dynamical metric. Note that both the Z fields and the ϕ^a 's will couple to the metric. We can consider a combined state of the Z field and metric such that the behavior of the Z fields is approximately classical; the weakest version of this is simply that the expectation values of the Z 's vary monotonically, and that their fluctuations are small. In this case, the Z 's approximately define temporal and spatial location, in a manner analogous to the above discussion. In fact, in such a case, the operators (4.4) are already diffeomorphism invariant. In some cases one might also want to consider a similar but different set of diffeomorphism-invariant operators,

$$\mathcal{O}_{g,\xi} = \int d^4x \sqrt{-g} O(x) e^{-(1/\sigma^2)(Z^i - \xi^i)^2}, \quad (4.16)$$

where the determinant in (4.4) has been replaced by $\sqrt{-g}$. Such observables also approximately localize, subject to constraints analogous to (4.12) and (4.13).

However, in the case of dynamical geometry, one does not expect (4.1) to provide a viable classical background over an arbitrarily large region. In particular, the constant energy density will backreact on the geometry. It is therefore natural to consider states in which the Z -fields approximate (4.1) only over some spacetime region Ω which is bounded in space (though which need not be bounded in time if the physics provides a way to keep the Z -fields from dispersing). The Z -fields would then be essentially in their vacuum state outside of Ω . In this context, we say that only the region Ω has been ‘‘instrumented’’ with our dynamical reference background.

One expects to be able to use the operators (4.4) to determine position within the region Ω . We will further discuss constraints that arise from the incorporation of

¹³However, so long as the region studied is not the entire universe, we leave open the possibility that pseudolocal observables may exist for which the clock and position degrees of freedom are kept at some distance from the region under investigation. Such ‘‘remote sensing’’ observables are particularly relevant to spacetimes with an asymptotic region.

gravity in Sec. VI, but one effect which must now be taken into account arises directly from the scalar sector in the region Ω^c which forms the complement of Ω ; i.e., from the region *outside* of the original region Ω . The effect of this region can be modeled by simply computing correlators of \mathcal{O}_ξ in the vacuum state $|0\rangle$. One can easily arrange that, in $|0\rangle$, the integrand of (4.4) has vanishing expectation value, by shifting the operator. Thus, Ω^c does not contribute to the expectation value of \mathcal{O}_ξ . Nevertheless, it will in general contribute to the expectation value of $\mathcal{O}_{\xi_1} \mathcal{O}_{\xi_2}$; i.e., to correlators of pseudolocal observables, and thus to the fluctuations of pseudolocal observables about their expectation values.

When considering a fixed observable \mathcal{O}_ξ , it is clear that for sufficiently large Ω^c the resulting noise will overwhelm our desired signal. In particular, our signal will be overwhelmed in an infinite volume universe. When the volume of space is merely very large (but finite), this effect will place fundamental limits on the accuracy with which any given \mathcal{O}_ξ reduces to a local observable in a given region. However, since the fluctuations involve the operator $e^{-(1/\sigma^2)(Z^i - \xi^i)^2}$, they are exponentially small in the parameter $(\xi^i)^2$. Thus, such limits need not be especially stringent in practice and can be further suppressed by using operators that effectively enforce more conditions. On the other hand, they raise interesting questions concerning the infinite volume limit and the connection to, for example, the S -matrix. They may also play an interesting role for universes which experience sufficiently long periods of rapid growth, and, in particular, in eternal inflation scenarios.

1. Generalizations

An important overall goal of this work is to understand some approximation of the types of observations we make, for example, at particle accelerators such as the LHC. The Z model captures some aspects of such observations, in particular, their localization, but in reality experimental apparatuses are quite complex and involve detectors which are very complicated excited states above the vacuum. Working towards actual physical measurements, one may wish to consider more complicated operators than those in (4.4) and (4.16). One first step is to separate the *timing* function from the *observing* function, for example, by considering both the Z fields and additional degrees of freedom comprising a detector. A candidate class of diffeomorphism-invariant observables is of the form

$$\mathcal{O}_{g,\xi} = \int d^4x \sqrt{-g} O(x) m(x) e^{-(1/\sigma^2)(Z^i - \xi^i)^2}. \quad (4.17)$$

Here $m(x)$ is an operator acting on the detector. Concretely, $O(x)$ might be an operator annihilating a photon, with $m(x)$ describing the consequent excitation of an atom (or more complicated ensemble). One might choose the Z operators to merely provide approximate location information, which, for example, could be much less accurate than the

time scale associated with the spacing between the detector's energy levels. Clearly there are further extensions of increasing complexity.

B. $\psi^2\phi$ model

We next consider another field-theory example which illustrates some of the features of pseudolocal observables. Specifically, consider a theory of two massive noninteracting scalar fields, ψ and ϕ . In this case, an example of a generalized observable is the diffeomorphism-invariant operator

$$\mathcal{O}_{\psi^2\phi} = \int d^4x \sqrt{-g} \psi^2(x) \phi(x), \quad (4.18)$$

which has the virtue of being simpler than the Gaussians of the Z model, as well as renormalizable.

Despite the simplicity of such operators, localized information about ϕ can be obtained by encoding this information in the state of the ψ -field. This is a second paradigm for recovery of local operators from diffeomorphism-invariant operators. For example, begin by working about a flat background, and suppose that we are interested in extracting an N -point function of the field ϕ from a correlation function of the operators (4.18). We do so by considering ψ states corresponding to incoming and outgoing wave packets. These are defined in terms of wave packet creation operators, which, for a given wave packet function f , take the form

$$a_f^\dagger = i \int_\sigma dn^\mu f^* \vec{\partial}_\mu \psi. \quad (4.19)$$

Specifically, consider the in-state

$$|f_1, \dots, f_K\rangle = \prod_K a_{f_K}^\dagger |0\rangle, \quad (4.20)$$

and likewise for an out-state with L creation operators. Our interest lies in correlators of the form

$$\langle f_1, \dots, f_L | (\mathcal{O}_{\psi^2\phi})^N | f_1, \dots, f_K \rangle. \quad (4.21)$$

Let us choose K and L even, with $K + L = 2N$, and moreover choose the in-states such that each pair of incoming wave packets f_{2i-1}, f_{2i} overlaps in some definite spacetime region near $x^\mu = x_i^\mu$, and likewise for pairs of outgoing wave packets, but no other pair has substantial overlap in any region of spacetime. In that case, (4.21) reduces to an expression of the form

$$\begin{aligned} & \langle f_1, \dots, f_L | (\mathcal{O}_{\psi^2\phi})^N | f_1, \dots, f_K \rangle \\ & \approx C \langle 0 | \phi(x_1) \cdots \phi(x_N) | 0 \rangle. \end{aligned} \quad (4.22)$$

One can thus approximately extract local observables from expectation values of products of \mathcal{O} 's. In the infinite volume limit there is, however, a subtlety; due to fluctuations, the \mathcal{O} 's are not well-defined operators on the Hilbert space of states. This problem apparently can be suppressed for

finite large volume through careful choice of operators. It does, however, raise possibly fundamental issues that could be relevant in quantum cosmology, and may have implications, for example, in the context of interpreting eternal inflation.

More generally, one could also consider the case of a dynamical metric. In this situation, one should generalize the states (4.20) to states solving the Wheeler-DeWitt equation (2.1) which correspond to incoming (or outgoing) wave packets coupled to the metric. To the extent to which such states can be defined, one expects to recover a relationship of the form (4.22).

The distinction between the Z -model and the $\psi^2\phi$ model lies in the specific position information being parametrized in the operator variables in the Z -model, but in the quantum state in the $\psi^2\phi$ model. In particular, in the Z -model we defined a four-parameter family of operators \mathcal{O}_ξ , where for a given choice of state we may dial the parameters ξ^i in order to sample the physics in different regions of the spacetime. In contrast, we defined only *one* operator $\mathcal{O}_{\psi^2\phi}$ in the $\psi^2\phi$ model. There, in order to sample ϕ -physics in different spacetime regions, one must adjust the state of the ψ -field. Nevertheless, in both models it is the interplay between the chosen observable and a particular class of quantum states which leads to localization.

As a final observation, notice that the operator (4.18) can naturally be added to the Lagrangian with a coupling constant to give an interacting theory. In this case, we may compute expectation values of the form (4.22) by differentiating the path integral with respect to λ . More discussion of this kind of relation between single-integral diffeomorphism-invariant observables and interaction terms in a Lagrangian will be given in Sec. V, where this will provide part of the connection to the traditional notion of ‘‘measurement’’ of observables.

C. String theory and two-dimensional gravity

The general framework we have described can also be illustrated in the context of string theory, in which the string is viewed as a model for two-dimensional gravity. While there are no propagating gravitational degrees of freedom in $1 + 1$ dimensions, diffeomorphism invariance nevertheless plays a crucial role.

To begin, let us recall that, at the perturbative level, string scattering amplitudes are computed as the expectation values of vertex operators \mathcal{V}_i ,

$$\left\langle \prod_i \mathcal{V}_i \right\rangle, \quad (4.23)$$

which are defined as a functional integral over geometries and fields. The vertex operators \mathcal{V}_i should be diffeomorphism invariant, and, in particular, typically take the form

$$\mathcal{V}_i = \int d^2\sigma \tilde{\mathcal{V}}_i, \quad (4.24)$$

where $\tilde{\mathcal{V}}_i$ are densities of the appropriate weight. Thus, the vertex operators of string theory are diffeomorphism-invariant observables in the two-dimensional gravity theory on the world sheet.

One might ask to what extent the world-sheet fields can be used to give conditionals defining position, as in the Z-model of Sec. IVA. For example, in the context of the bosonic string, vertex operators of the form

$$\tilde{\mathcal{V}} = e^{ik \cdot X} \quad (4.25)$$

are commonly considered, where X^μ , $\mu = 0, \dots, D-1$ are the world-sheet scalar fields. However, in order for the correlator (4.23) to be well defined in the critical theory with $D = 26$, the vertex operators (4.24) must be both diffeomorphism and Weyl invariant, implying the momenta k^μ must satisfy the constraint $k^2 = 8$; i.e., they must satisfy the mass-shell condition of the target-space tachyon. This means that one cannot treat the different components of k^μ as independent integration variables, and produce sharp Gaussians as in (4.7).

Relaxation of the condition $k^2 = 8$ leads to explicit dependence on the conformal part of the metric, ϕ , where we work in conformal gauge,

$$ds^2 = e^\phi \hat{g}_{ab} d\sigma^a d\sigma^b. \quad (4.26)$$

Here \hat{g}_{ab} is a background metric, which fixes the conformal equivalence class. Since for the critical string the action is independent of the conformal factor, the expression (4.23) is no longer well defined. This situation changes for the noncritical theory, $D \neq 26$, where quantum effects induce the Liouville action for ϕ ,

$$S_L = \frac{25-D}{48\pi} \int d^2\sigma \sqrt{\hat{g}} \left(\frac{1}{2} \hat{g}^{ab} \partial_a \phi \partial_b \phi + \hat{R} \phi \right). \quad (4.27)$$

In general dimension, a matter operator $\mathcal{W}_i[X]$ of definite conformal dimension Δ_i receives a gravitational dressing, so that, instead of $\mathcal{W}_i[X]$ itself, it is the operator

$$\tilde{\mathcal{V}}_i = e^{\alpha_i \phi} \mathcal{W}_i, \quad (4.28)$$

which transforms as a density of weight one. Here

$$\alpha_i = \frac{25-D}{12} \left[1 \pm \sqrt{\frac{1-D+24\Delta_i}{25-D}} \right]. \quad (4.29)$$

Once again, the dependence of α_i on k restricts our ability to define Gaussians of the X^μ fields.

In either critical or noncritical cases, however, it appears possible in a long-distance approximation to use the operators $\int \tilde{\mathcal{V}}_i$ in analogy to the Z-model to specify location and time information. One could write an expression such as

$$\int_{-1/L}^{1/L} \prod_\mu dk^\mu e^{ik_\nu \cdot (X^\nu - \xi^\nu)} e^{\alpha(k)\phi}, \quad (4.30)$$

or, in the critical case, replace $\alpha(k)\phi$ by a term proportional to X^{25} and the combined squares of the independent momenta. For $L \gg 1$, the k dependence in $\alpha(k)$ is small, and on scales $X \gg L$ one might anticipate this expression approximates a delta function concentrated at $X^\mu \approx \xi^\mu$, which in turn could be used to specify world-sheet position.

In the critical case, this can, in particular, be illustrated by working about a background corresponding to a string wound on a noncontractible cycle, of the form $X^0 = p\tau$, $X^1 = w\sigma$. For $25 > D > 1$, additional subtleties arise as one must deal with the so-called $c = 1$ barrier. Dynamics in this regime is poorly understood, but it is believed that one encounters a phase such that the geometry is a branched polymer. Thus, while the general framework we describe plays a role here, one will not necessarily have a phase in which the two-dimensional geometries have clean semiclassical behavior and permit the existence of useful clocks. We presume this is a feature unique to two-dimensional physics, which typically has large fluctuations on all scales, and based on empirical observation, do not expect such a limitation on our discussion of four-dimensional physics. Indeed, due to the branched polymer structure (and in contrast to the higher-dimensional case), it is not even clear what form of local physics one might wish to recover.

D. Cosmological observables

Since cosmology is an important domain in which to describe observation, we briefly comment on how the approach outlined above may be used to define relevant observables. In particular, in the cosmological context, one is interested in describing observables at different times in some cosmological evolution. Objects of particular interest include correlators of the inflaton and information about the temperature and geometry of the early universe.

Some of the information of interest requires only locality in time. For example, if we are interested in the temperature of the universe at the end of inflation, we might begin by studying the energy density at the time when the effective cosmological constant drops to some level well below the GUT scale. In the minisuperspace truncation, we might describe this using as a time variable the radius of the universe. This radius is of course not locally defined, but quantities such as the curvature are, and allow us to generalize the idea beyond minisuperspace. In particular, in the case where the universe is spatially compact, we may investigate such quantities through observables of the form

$$\mathcal{O}_\tau = \int d^4x \sqrt{-g} O(x) f_\tau(R), \quad (4.31)$$

where $O(x)$ is a local scalar operator and $f_\tau(R)$ is a sharply

peaked function of the spacetime scalar curvature R with peak near some value τ , which thus serves as an approximate time label.¹⁴ The observable (4.31) roughly corresponds to the value of the observable O at the given value of R . In states where we expect the universe to be very homogeneous, there is no need to attempt to resolve spatial information, or even to localize (4.31) in space.

For example, in the case of the energy density, we might use a quantity such as

$$O(x) = T^{\alpha\beta} \frac{\partial_\alpha R \partial_\beta R}{\sqrt{-|\partial R|^2}}, \quad (4.32)$$

where $T^{\alpha\beta}$ is the stress energy tensor. Here the symbol $|\partial R|^2$ denotes the norm of the covector $\partial_\alpha R$. In the case mentioned above, the value of τ might be chosen to correspond to an effective cosmological constant at some level below the GUT scale, and the observable (4.31) then roughly corresponds to the total energy of the universe at the given value of R . Of course, this depends on both the total volume and on the energy density. To recover information about the energy density (and thus the temperature) alone, we might divide by, e.g.,

$$O_V = \int d^4x \sqrt{-g} f_\tau(R) \sqrt{-|\partial R|^2}. \quad (4.33)$$

A similar quotient, defined through some limiting procedure, might serve as a useful pseudolocal probe of temperature in cases where the universe is nearly homogeneous, but is not spatially compact.

In short, one may adapt to our framework the common idea (see, e.g. [6]) that one may use the ‘‘size’’ of the universe to label times when the universe is nearly homogeneous. This idea has often been implemented in the minisuperspace truncation, which amounts to using a toy $0 + 1$ model. In this context, operators analogous to \mathcal{O}_τ were studied in greater depth in [18–20].

We emphasize that, by using the local notion of the spacetime curvature scalar R instead of the nonlocal notion of the size of the universe, our definition (4.31) can make sense even in the presence of inhomogeneities (in which case it merely gives a spatial average of the desired energy density). Thus, we expect that \mathcal{O}_τ will define an operator in quantum $3 + 1$ effective gravity.

On the other hand, in the context of homogeneous cosmologies, we expect information about inflaton correlators to be encoded in more complicated observables, which are *not* of the single-integral type. The point is that one needs a means of specifying the separation between the two operators in a two-point function in a context where the one-point functions are independent of

position on the homogeneous slice. This suggests one should build an operator of the form¹⁵

$$\mathcal{O}_{\tau,\Delta} = \int d^4x d^4y \sqrt{-g} \sqrt{-g} f_{\tau,\Delta}(x, y) \phi(x) \phi(y) \quad (4.34)$$

which samples the bilocal operator $\phi(x)\phi(y)$ only when the two points x and y have some physically specified separation Δ . This can be done by, for example, using an operator $f(x, y)$ whose classical limit is sharply peaked when x, y are separated by a geodesic of length Δ lying in the surface in which the scalar curvature $R(x)$ takes the value τ . For example, one might take

$$f_{\tau,\Delta}(x, y) = f_\tau(R(x)) f_\tau(R(y)) f_\Delta(s(x, y)), \quad (4.35)$$

where $f_a(b)$ is the sampling function from (4.31) and $s(x, y)$ is any functional of the metric which approximates the geodesic distance between x and y when (i) the quantum state is sufficiently semiclassical and approximates a universe that is spatially homogeneous in a neighborhood of some spacelike slice Σ and (ii) x and y are both located on Σ .¹⁶ The resulting operators are complicated; we assume a renormalization scheme for such operators can be specified in a low-energy effective theory of quantum gravity.

E. General comments

The examples we have outlined show how relational data may be encoded in a combination of state and diffeomorphism-invariant observables, and, in particular, allow specification of position information. Many other examples of these basic principles may be considered. In particular, there is no obvious in-principle obstacle to constructing such operators purely out of gravitational data, say by constructing objects relating the values of different curvature invariants.

Note also that a definition of observables, such as that described above, is useful for characterizing the physical states of a theory with dynamical gravity. Given a physical state Ψ satisfying the Wheeler-DeWitt equation (2.1), the above observables may be used to formulate projectors onto solutions with definite attributes. This follows by virtue of the statement that an operator of the form

$$\delta(\mathcal{O} - a) \quad (4.36)$$

(or more precisely a projector onto a spectral interval of \mathcal{O}), which projects onto states in which \mathcal{O} takes value a , commutes with the constraints if \mathcal{O} does. Thus, combina-

¹⁴More generally, one may wish to use an appropriate *spatial average* over the curvature; precise specification of such a prescription is more complicated, but similar to the construction of ‘‘bilocal’’ operators that will be described below.

¹⁵Bilocal and other diffeomorphism-invariant quantities have also been employed in simplicial quantum gravity based on Regge calculus [51], and in two-dimensional gravity in [52].

¹⁶The operator $f_\Delta(s(x, y))$ may, in turn, be defined at least on some open set of such auxiliary states by computing the result on the classical metric corresponding to one such state and then expanding $f_\Delta(s(x, y))$ as a power series in the metric.

tions of such projectors can be used to specify attributes of the physical states in terms of values of the observables.

Such a specification of states is quite similar in spirit to the conditional probability interpretation, advanced in [7]. Kuchař [1] has argued that this suffers from a *reductio ad absurdum*; a counterargument has recently been proposed in [12]. However, note that the “projection operators” of the latter reference do not in fact act as such. In contrast, projection operators defined according to (4.36) (or the more precise spectral interval version) are indeed projectors, and lead to a different approach to defining probabilities.

V. DIFFEOMORPHISM-INVARIANT OBSERVABLES AND MEASUREMENT

A. Measurement: Generalities

The examples of the preceding section have illustrated how certain “pseudolocal” diffeomorphism-invariant observables reduce to the usual local observables of quantum field theory. As we have seen, this property is critically dependent on the state(s) in which the observables are evaluated.

Associated with the usual observables of quantum field theory (QFT) is a theory of measurement, see e.g., [53]. One assumes the existence of an appropriate measuring apparatus, whose coupling to the quantum system is capable of measuring the eigenvalues of the operator in question. In this section, we describe some aspects of measurement theory for relational observables.

In the gravitational setting we have seen that, though one must be aware of important infrared issues, the requirements of diffeomorphism invariance can nevertheless be satisfied by integrating over the entire spacetime. In order to define localized operators, one must also include a reference framework. Specifically, localized information about some degrees of freedom can be recovered by constructing operators which explicitly refer both to those particular degrees of freedom (which we may call the “target” degrees of freedom) *and* to other dynamical degrees of freedom; the additional degrees of freedom can specify the location at which the target degrees of freedom are to be sampled. In some cases, these additional degrees of freedom might be thought of as providing an abstract background of “clocks and rods” against which to localize the target degrees of freedom, though of course this background will be dynamic and will be influenced by the target degrees of freedom. Moreover, in any context where one would consider a local measurement to have taken place (e.g., in a specific laboratory), it is natural to include degrees of freedom describing the measuring apparatus, and, in fact, it is natural to use the apparatus itself to specify the spacetime regime in which the target degrees of freedom are sampled. Specifically, the sampling occurs at the location of the apparatus and during the time interval in which the apparatus is switched on.

This fits with the broader perspective that in a fully quantum-mechanical framework, there should be no sharp distinction between the observed system and the measuring apparatus—they are both quantum systems, with some coupling between them. In this context, a simple viewpoint is that *measurement is correlation with a subsystem that can be understood as a measuring apparatus*: a measurement is performed when the system being observed and the measuring apparatus are allowed to interact, and form correlations between their degrees of freedom. This is a general notion for quantum systems. One more specifically can speak of a Copenhagen measurement situation, in which the Copenhagen formulation of quantum mechanics can be reproduced.¹⁷ A measurement framework is Copenhagen to the extent it can be thought of as describing a quantum system interacting with a classical measuring device. Several critical aspects play a role. First, the Hilbert space should decompose into states of the system and states of the measuring device. Second, the system variables and the corresponding variables of the measuring device should be exactly correlated, so that the measurement is good. Third, the combined system should decohere, so that consistent probabilities can be assigned to the different alternative results of measurements. Finally, the measuring device should form stable records that are robust against fluctuations and further inspection. As we will discuss below, such conditions can be satisfied when the measuring apparatus has a large number of degrees of freedom.

B. Measurement and relational observables

Although they are nonlocal, a connection between measurement and correlation can nevertheless emerge from a treatment of relational diffeomorphism-invariant observables. However, the correlations we desire will typically arise only in special states of the system. This is a standard feature of measurement situations (see e.g. [53]), but is especially prominent here since, as described in Sec. IV, the state plays a key role in the recovery of the notion of locality itself.

Thus, and in line with the above discussion, a link between measurement and relational observables arises when specific conditions hold. The first is that the state and dynamics must allow an approximate division of the degrees of freedom of the universe into the measured (target) system and the measuring device; these may possibly be supplemented by other degrees of freedom irrelevant to the discussion. Second, the coupling between these two systems should be weak, in a sense to be described shortly. Since we describe the measurement within effective low-energy gravity, the coupling must be diffeomor-

¹⁷For further discussion of this idea, see e.g. [54–59]. In [54] such Copenhagen measurement situations were referred to as “ideal measurement situations.”

phism invariant. Furthermore, if the effective description of the coupled system is local, the coupling must provide a term in the action which is an integral of a local density. Thus, this coupling is precisely given by a single-integral observable.

Before proceeding, we pause to clarify one conceptual point. In practice, measurement always occurs within some given physical system. For example, our laboratories are filled with devices which, together with their couplings to any target systems, are described by the standard model of particle physics. In particular, the laboratory technician has no freedom to adjust any coupling constants of the standard model. However, it is often useful to give a low-energy effective description of these devices in which their construction from standard model fields is not explicit. Of course, in resonance with our recurring theme, such an effective description is valid only when the full system (i.e., the standard model fields) is in an appropriate state, and interesting features of the effective description can depend on the details of the state (e.g., whether the device is “on” or “off”). This state dependence gives rise to coupling constants in an effective description which *are* under the control of the technician. As a result, measure-

ment theory is typically discussed in terms of deforming the action of some (typically uncoupled) system of target and apparatus by introducing some new coupling between them. We will pursue this approach below.

Specifically, given an action S , let us consider its perturbation by a single-integral observable \mathcal{O} of the form (3.1); i.e., we deform the Lagrangian through

$$\mathcal{L} \rightarrow \mathcal{L}' = \mathcal{L} + f\hat{\mathcal{O}}, \quad (5.1)$$

where f is a small parameter, and \mathcal{O} and $\hat{\mathcal{O}}$ are related as in (3.1). Such a perturbation of the action leads to a shift in the inner product (3.6), inducing new correlations between the target system and apparatus.

To find this shift, first note that the functional integral in (3.6) will in general be defined over some *fixed* range of parameter time; one then integrates over all geometries interpolating between the endpoint field configurations in this parameter time interval. For example, one may take this parameter range to be $(0, 1)$, and this defines the limits on the integral determining the action in (3.6). In this case, the change of (3.6) under the perturbation (5.1) is

$$\begin{aligned} \delta\langle h_2, \phi_2^r \parallel \eta \parallel h_1, \phi_1^r \rangle &= if \int_{h_1, \phi_1^r}^{h_2, \phi_2^r} \mathcal{D}g \mathcal{D}\phi^r e^{iS} \int_0^1 dt d^3x \sqrt{-g} i f \langle h_2, \phi_2^r \parallel \eta \hat{\mathcal{O}}(t, x) \parallel h_1, \phi_1^r \rangle - \delta\{\langle h_2, \phi_2^r \parallel \eta \parallel h_1, \phi_1^r \rangle\} \\ &- \langle h_2, \phi_2^r \parallel \delta\{\langle h_1, \phi_1^r \rangle\} \rangle, \end{aligned} \quad (5.2)$$

with

$$\delta\{\langle h_2, \phi_2^r \parallel \eta \parallel h_1, \phi_1^r \rangle\} := if \int_{h_1, \phi_1^r}^{h_2, \phi_2^r} \mathcal{D}g \mathcal{D}\phi^r e^{iS} \int_1^\infty dt d^3x \sqrt{-g} \hat{\mathcal{O}}(t, x), \quad (5.3)$$

$$\langle h_2, \phi_2^r \parallel \delta\{\langle h_1, \phi_1^r \rangle\} \rangle := if \int_{h_1, \phi_1^r}^{h_2, \phi_2^r} \mathcal{D}g \mathcal{D}\phi^r e^{iS} \int_{-\infty}^0 dt d^3x \sqrt{-g} \hat{\mathcal{O}}(t, x). \quad (5.4)$$

In expression (5.3), the integral is over paths which begin at $t = 0$, advance in t to the far future and then return to $t = 1$. Expression (5.4) is similar. The construction is analogous to that used in the $\langle in|in \rangle$ formalism.

Let us assume that contributions to (5.3) and (5.4) come only from regions far from the Planckian regime. For example, we expect this to hold for operators $\hat{\mathcal{O}}(x, t)$ which, on classical solutions approximate to $|\Psi_1\rangle$, $|\Psi_2\rangle$, happen to be supported in such regions of spacetime. It is now clear that (5.3) and (5.4) may be interpreted as changes in the states $\eta \parallel h_1, \phi_1^r \rangle$ and $\eta \parallel h_2, \phi_2^r \rangle$ when these states are held fixed at, respectively, late and early times, perhaps as they emerge from a region of Planck-scale physics.¹⁸

¹⁸We make the implicit assumption that, for states of interest, regimes of (t, x) contributing to (5.3) and (5.4) are not separated by intervening Planck-scale physics. This, in particular, requires exclusion of evaporating black holes and phenomena such as bouncing universes.

More generally, we can superpose the quantities (5.2) to find the change in the inner product between two arbitrary auxiliary states, $\langle \Psi_2 \parallel \eta \parallel \Psi_1 \rangle$. With the above understanding of boundary conditions, we may describe this as the change in the physical inner product $\langle \Psi_2 | \Psi_1 \rangle$. That is, we define $\delta\langle \Psi_2 | \Psi_1 \rangle$ to be $\delta\langle \Psi_2 \parallel \eta \parallel \Psi_1 \rangle$ where the auxiliary states $\parallel \Psi_1 \rangle$, $\parallel \Psi_2 \rangle$ are chosen so that $\delta\{\langle \Psi_2 \parallel \eta \parallel \Psi_1 \rangle\}$ and $\langle \Psi_2 \parallel \delta\{\parallel \Psi_1 \rangle\} \rangle$ are both small; we make no definition of $\delta\langle \Psi_2 | \Psi_1 \rangle$ when such a choice is not possible.

Thus, we have derived a diffeomorphism-invariant version of the Schwinger variational principle [60,61] relating the change in this inner product to the matrix element of our diffeomorphism-invariant observable,

$$\delta\langle \Psi_2 | \Psi_1 \rangle = \langle \Psi_2 | \mathcal{O} | \Psi_1 \rangle. \quad (5.5)$$

Said differently, we take the initial and final states $|\Psi_i\rangle$, $i = 1, 2$, to be specified in terms of data associated with a region undisturbed by the interaction $\hat{\mathcal{O}}(x, t)$, where possible. This data is encoded through the choice of auxiliary

states $|\Psi_i\rangle$, for which $|\Psi_i\rangle = \eta|\Psi_i\rangle$ and for which $\delta|\Psi_i\rangle$ as defined above is small. A more complete way of stating this is to say that we start with a notion of asymptotic physical states, in some basis, in both past and future, analogously to what we do in the Lehmann-Szymanzik-Zimmerman framework in field theory. We assume that the perturbation (5.1) has a negligible effect on the form of the “in” states in the past, or on the form of the “out” states in the future. Of course, complete specification of the states involves physics at the Planck scale, so here we must make the assumption (which we consider reasonable, based on simple examples) that we are working in a sufficiently semiclassical regime that we can specify the states in the effective theory and that the operator in question in effect turns off in the past and future.

Our basic picture is then that the left side of Eq. (5.5) can, in these circumstances, be related to the result of a measuring process; this then provides a measurement interpretation of the matrix element on the right side of this equation. Specifically, start with the assumption that the state is such that there is a clean division between target system and measuring apparatus, with only a weak interaction between them. For example, we might consider the situation where the target system corresponds to one of the fields, which we call $\phi(x)$. A concrete example to bear in mind is that the field describing the system might be, e.g. the muon field, whereas the measuring apparatus is constructed from electrons, protons, etc. The Wheeler-DeWitt wave function should be linear combinations of auxiliary states of the form

$$|\Psi_A\rangle = |\alpha, a, h_A\rangle = |\alpha\rangle_\phi |a\rangle_m |h_A\rangle_h, \quad (5.6)$$

where the factors are states $|\alpha\rangle_\phi$ of the target system, $|a\rangle_m$ of the measuring device, and $|h_A\rangle_h$ of the metric (and possibly other degrees of freedom). The state of the metric then becomes correlated to that of the system and measuring device through the Wheeler-DeWitt equation (2.1); i.e., in the corresponding physical state $\eta|\Psi_A\rangle$. The interaction between the system and measuring device will typically be of the form of a single-integral diffeomorphism-invariant operator,

$$S_i = f\mathcal{O} = f \int d^4x \sqrt{-g} O(\phi(x)) m(x), \quad (5.7)$$

where $O(\phi(x))$ is a local operator constructed from the field ϕ and $m(x)$ is an operator acting on the state of the measuring device.

Working about a background which is sufficiently semiclassical (which presumably requires gravity to be weakly coupled), the inner product (3.6) of states of the form (5.6) is approximated by matrix elements of a so-called “deparametrized theory,” in which the constraints have been solved and one finds an “external time” which plays the same role as time in ordinary quantum field theory (or, for that matter, in nonrelativistic quantum mechanics). This

external time may arise either from clock degrees of freedom in the measuring apparatus, or from the metric background. Work along these lines has a long history; see, e.g., [6,8,62,63], and in particular [20] for a careful discussion in terms of pseudolocal observables (in the $0+1$ context).

If U is the evolution operator of the target system and measuring device in the deparametrized theory, the relation takes the form

$$\langle\Psi_B|\Psi_A\rangle \approx \langle\beta, b|U|\alpha, a\rangle e^{iS[\mathbf{g}_{cl}]}, \quad (5.8)$$

where the states on the right-hand side lie in the deparametrized theory (so that the clock degrees of freedom no longer appear in the state). The assumption that the system and measuring device are weakly coupled justifies the approximation in (5.2) of truncating to linear order in the coupling f , so that (5.5) may be written

$$\begin{aligned} \langle\beta, b|(U-1)|\alpha, a\rangle &= if \int d^4x \sqrt{-g_{cl}} \langle\beta|O(\phi(x))|\alpha\rangle \\ &\times \langle b|m(x)|a\rangle + \mathcal{O}(f^2), \end{aligned} \quad (5.9)$$

which agrees with the interaction typically used to discuss measurement of the local field-theory observable $O(\phi)$ in the spacetime region in which the device $m(x)$ is active. Physically, the measurement proceeds through the establishment of correlations of the ϕ system with the measuring device. If the device is sufficiently classical, a Copenhagen measurement results.

While we have outlined the connection to measurement as if the degrees of freedom of the measured system are a different type of field than those of the target system, the discussion generalizes readily to the situation where both measured system and measuring device have the same constituents, e.g. electrons. In this case the decomposition (5.6) corresponds to factoring the auxiliary Hilbert space into a product of Hilbert spaces corresponding to distinct degrees of freedom of the electron field, and likewise the two operators in (5.7) are operators that act on the two different sets of degrees of freedom. (The general interaction/single-integral observable will be a sum of such terms.)

In short, the fact that (5.9) approximates (5.5) makes it clear that, just as in more familiar (e.g., [53]) discussions of measurement, when the states and observables are of a specific form, measuring devices become correlated with states of the target system in such a way that the outcome of the measurement is given by the matrix elements of the pseudolocal observable \mathcal{O} . As in the case of measurement theory in the presence of an external time, one may also ask about the degree to which such correlations may be viewed as *Copenhagen* measurements; i.e., measurements to which the Copenhagen interpretation of quantum mechanics can be consistently applied. This question is examined in the following subsection, and again in Sec. VI, where constraints imposed by gravity are discussed.

C. The Copenhagen measurement approximation: Large N

Having described measurements of (single-integral) diffeomorphism-invariant observables, one may also ask to what extent such measurements can approximate Copenhagen measurements. In particular, we expect to precisely recover the needed properties of *decoherence* and *stability* only in the case of measuring devices comprised of infinitely many degrees of freedom (here we may also wish to include other variables describing the environment as part of the measuring device; these can be important for ensuring decoherence). In a later section, we will discuss gravitational constraints on numbers of degrees of freedom, but for the moment let us consider more generally the limitations imposed if the number of degrees of freedom of a measuring device is finite. Thus, diffeomorphism invariance will not play a direct role in the discussion below.

For illustration, we consider the Coleman-Hepp model [64,65]; for other examples making use of an “environment,” see e.g. [55–59]. This is a quantum-mechanical model for a device that measures the state of a two-state quantum system, for example, the spin of an electron. The measuring device consists of N two-state spins. Let the states of the “electron” be denoted $|+\rangle$, $|-\rangle$, and states of the measuring device be of the form $|\uparrow\downarrow\cdots\uparrow\rangle$. An explicit Hamiltonian can be written down, but all we need is the result of its evolution: a general state of the two-state system (combined with some specific initial state for the measuring device) evolves into a perfectly correlated state,

$$\alpha|+\rangle + \beta|-\rangle \rightarrow \alpha|+\rangle|\uparrow^N\rangle + \beta|-\rangle|\downarrow^N\rangle. \quad (5.10)$$

Thus, the system variables and measurement variables are indeed perfectly correlated.

The limitations arising from a finite number of degrees of freedom are manifest in the conditions of decoherence and stability. For the state on the left-hand side of (5.10), interference effects are important for computing the expectation values of many operators, such as, e.g. σ_x or σ_y . In the decoherent histories formulations of quantum mechanics (see, e.g. [54] and references therein), the corresponding statement is that a typical set of alternative histories will not decohere. Of course, the state on the right-hand side of (5.10) is also a quantum state for which interference can be measured, but as N grows this becomes increasingly difficult, as the phase information becomes distributed over a larger number of degrees of freedom. Thus, as N gets large, interference effects are suppressed for operators involving only a finite number of spins, or, equivalently, typical sets of alternative histories decohere. To make this more precise, the only operators that are sensitive to interference between the two components of the composite state (5.10) are composite operators that act on *all* of the $N + 1$ degrees of freedom:

$$\langle \uparrow^N | \langle + | O | - \rangle | \downarrow^N \rangle \neq 0 \quad (5.11)$$

only for an operator O that flips all the spins, e.g.

$$O = \sigma_y^{\text{system}} \prod_{i=1}^N \sigma_y^i. \quad (5.12)$$

In the “classical” limit of $N \rightarrow \infty$, no operator O acting on a finite number of spins is sensitive to this interference. Likewise, stability improves with increasing N . Real systems are difficult to isolate, and generic small perturbation terms in the Hamiltonian, e.g. due to interactions with the environment or other effects, will typically randomly flip individual spins. However, if the probability to flip a single spin in a given time interval is $\gamma < 1$, the probability to flip more than half the spins of the measuring device, and thus spoil the measurement, is $\gamma^{N/2}$ which vanishes as $N \rightarrow \infty$.

We see that at infinite N the expected classical behavior is recovered, but for finite N there are limitations on the extent to which one can achieve a classical measurement. Put more descriptively, if we make an observation of one alternative, but then via a quantum or other fluctuation, our brain transitions into a state corresponding to a different alternative, we ultimately reach a different conclusion about the outcome of the measurement. Such fluctuations are always in principle possible for finite systems. This suggests that any such measurement has an intrinsic uncertainty that falls exponentially with the number of degrees of freedom of the measuring apparatus,

$$\Delta \approx e^{-cN}, \quad (5.13)$$

where the constant c depends on the details of the apparatus. Similarly, quantum interference effects mean that the measurement will fail to be Copenhagen also at order e^{-cN} . Reference [25] has previously emphasized the importance of uncertainties of this magnitude, and made a similar estimate from quantum tunneling. We will discuss gravitational restrictions on this number of degrees of freedom in the next section.

Finally, a remaining source of uncertainty is the limited resolution provided by a system with a finite number of bits. Whenever one attempts to measure what might be a continuous parameter, using an N -bit device, one expects that the result stored has an uncertainty of the form $\Delta \sim 2^{-N}$.

To summarize this section, we see that in cases where there is a decomposition into target system and measuring device degrees of freedom, along with remaining metric and other degrees of freedom, such that interactions between the system, measuring device, and other degrees of freedom are weak, and such that the measuring device is well approximated as a classical measuring device, one can recover measurements of a quantum system, with the results corresponding to matrix elements of appropriate pseudolocal diffeomorphism-invariant observables. In such circumstances relational observables can be given a

clear interpretation in terms of measurement, but such an interpretation does not follow in the case of more general dynamics and states.

VI. OBSERVABLES: LIMITATIONS

The preceding sections have described how useful diffeomorphism-invariant observables may be constructed in an effective low-energy quantum theory of gravity, and argued that, in some circumstances, these observables reduce to local observables of standard QFT. However, we also found limitations on recovering QFT observables from our diffeomorphism-invariant observables. Some of these arise from basic quantum properties, and were touched upon in Sec. IVA. However, it appears that additional limitations arise when we take into account the coupling to a dynamical metric. In this section we examine both kinds of constraints more completely, and discuss their possible role as fundamental limitations on the structure of physical theories.

A. Example of the Z model

We begin by investigating constraints on observables in the context of the Z model. Recall that we argued that the diffeomorphism-invariant observables of the model approximately reproduce the local observables of QFT, but with limitations on the spatial resolution of the QFT operators. These limitations stem from two sources. First, the position resolution of the operator in (4.4) is limited by the value of σ ; recall that a nonzero σ is required to regularize the operator. Second, when we use the variables ξ to fix the spatial coordinates, we find that fluctuations become strong and we lose control when the resulting separation between two operators is too small. The resolution Δx is limited by the large fluctuations (4.12) of the Z fields at small separations $|x_1 - x_2|$. Together, these two features limit the resolution at which we can independently measure separate degrees of freedom of the field ϕ . Specifically, the physics of two separate local operators at x_1, x_2 is reproduced only when the separation between the operators satisfies

$$|x_1 - x_2| \gtrsim \max\left(\frac{\sigma}{\lambda}, \frac{1}{\sigma}, \frac{1}{\lambda|x_1 - x_2|}\right). \quad (6.1)$$

Here the first condition follows from (4.10) and the fact that we wish to separately resolve the two observables, while the second condition follows from (4.12), and the third from (4.13). Note that the first two conditions imply the third, so that (4.13) does not play a key role in the discussion. In order for fluctuations to be under control, we find from (4.12) that the dominant contribution to this uncertainty must be that of σ/λ .

In order to minimize this uncertainty, one wishes to maximize the Z -field gradient λ . In doing so, however, we should bear in mind that we are ultimately working in a field theory with a cutoff. The maximum value for the field momentum is thus determined by the cutoff as $\lambda \lesssim \Lambda^2$.

Moreover, the minimum value of $|x_1 - x_2|$ should likewise be $1/\Lambda$, and (4.12) thus imposes the constraint $\sigma \gtrsim \Lambda$. The net result is that the fundamental limitation on the resolution is given in terms of the cutoff by

$$\Delta x \gtrsim \frac{1}{\Lambda}, \quad (6.2)$$

as discussed in Sec. IVA, and as expected.

Thus, under purely field-theoretic considerations, we might expect to be able to choose a resolution limited only by that of the cutoff of the field theory used to specify location. Without gravity, there need not be a fundamental limitation on the size of this cutoff. Including gravity, one might expect that the Planck scale serves as a limitation on resolution. However, the inclusion of gravity also leads to additional constraints, to which we now turn.

Suppose that we couple the Z -model to the gravitational field. The Z fields serve as a source of gravity through its stress tensor,

$$T_{\mu\nu} = \frac{1}{2}[\nabla_\mu Z^i \nabla_\nu Z^i - \frac{1}{2}g_{\mu\nu}(\nabla Z^i)^2]. \quad (6.3)$$

Consider attempting to define observables throughout a spacetime region Ω , choosing a state such that (4.1) holds throughout the region. This means that the stress tensor has size

$$\langle T_{\mu\nu} \rangle \propto \lambda^2 \quad (6.4)$$

throughout Ω . If R is the linear size of the region, then the entire system undergoes gravitational collapse and our framework for defining observables breaks down if

$$\lambda^2 R^3 \gtrsim R M_p^2. \quad (6.5)$$

This simplifies to the bound

$$R\lambda \lesssim M_p \quad (6.6)$$

relating R and λ .

For example, suppose that we wish to provide Z fields which “instrument” the region Ω at the maximum resolution $1/\Lambda$ allowed in the cutoff theory. In this case, we find the bound

$$R \lesssim \frac{M_p}{\Lambda^2} \quad (6.7)$$

for the maximum sized region, given the resolution Λ . A bound of this form on the domain of validity of effective field theory has previously been proposed by Cohen, Kaplan, and Nelson in [23].

There is a similar bound involving pairs of operators. In particular, consider a correlation function of \mathcal{O}_ξ 's of the form (4.6). Suppose that we want each of the positions to be resolved at a maximum resolution $1/\Lambda$. In particular, this means that each of the operators has an energy of order Λ . Thus for two operators with a separation $|x_1 - x_2|$, gravity will become strong and our description of the observables will break down for

$$\Lambda \gtrsim |x_1 - x_2| M_p^2. \quad (6.8)$$

In fact, this bound is implied by what was termed the “locality bound” in [24,26].

Within the context of a given effective field theory, the bound (6.8) is trivially satisfied for $\Lambda < M_p$, as it can be violated only for $|x_1 - x_2| < 1/\Lambda$. However, boost invariance of the underlying theory indicates that we can create a particle with ultra-Planckian momentum by performing a sufficiently large boost on a state with sub-Planckian momentum, and one might correspondingly expect one could describe single-particle states with resolutions $1/\Lambda < 1/M_p$ using such a boost. Suppose we view such a state as being created by a pseudolocal operator. One can then ask if there is any in-principle obstacle to such a construction. The locality bound [24,26] states that there should be, since, if two such operators exceed the bound (6.8), strong quantum-gravitational backreaction cannot be ignored.

B. General discussion

While the above bounds were illustrated using our model for observables and measurements arising from our Z fields, one expects them to reflect a quite general situation. To see this, note first that constructing any kind of field configuration—whether from the metric, matter, or other fields—that has a “resolving power” $1/\Lambda$, requires working with fields with momenta $\sim \Lambda$, and hence corresponding energies. If we want to construct a “grid” from these fields, capable of this resolution throughout a region of size R , the energy of the grid is of order $\Lambda(\Lambda R)^3$. The constraint that the size of the region be greater than the Schwarzschild radius is thus the bound of [23],

$$M_p^2 R \gtrsim \Lambda(\Lambda R)^3, \quad (6.9)$$

or (6.7).

Note that this bound is surprisingly strong. If, for example, we want to instrument a region with fields capable of resolving degrees of freedom at the scale TeV^{-1} throughout the region, the maximum size region has size

$$R \sim \frac{M_p}{\text{TeV}^2}, \quad (6.10)$$

or in other words, $R \sim 1$ mm. This is not a constraint on a given single (or several) particle state in a region, which can be measured with a much smaller resolution; in practice we do so with larger detectors. But we cannot measure all of the degrees of freedom at TeV^{-1} resolution in a region larger than given by the bound, at least without accounting for black hole formation and the degrees of freedom of gravity at the Planck scale.

Likewise, merely making two measurements in a given region, each with resolution $1/\Lambda$, involves energies Λ . Absence of gravitational collapse thus means that the separation of the measurements must be greater than the

corresponding Schwarzschild radius, giving the locality bound constraint (6.8).

C. Fundamental limitations on physics?

We finish this discussion on limitations to measurement by exploring its consequences for fundamental physics. One might take the viewpoint that the constraints of this section simply arise for the kind of observables that we have described and are not fundamental constraints on the underlying physics. However, it is quite plausible that the approach we have outlined is general enough to yield the most general observables in a theory with dynamical gravity; it is not apparent that one can find *other* independent constructions of diffeomorphism-invariant operators that can play the role of observables, much less ones that reduce to QFT observables in the appropriate approximations. So a natural conjecture is that *all* observables relevant to the description of local physics in a theory with dynamical gravity arise from the kinds of observables that we have described.

Whether or not this is true, it suggests an even more interesting conclusion. For example, consider the bound (6.7) that says there is no way to simultaneously measure all of the field-theory degrees of freedom at a resolution $1/\Lambda$ in a region of size larger than given by (6.7), using only degrees of freedom inside the region. One might say that these degrees of freedom “exist,” but simply cannot all be described by observables and/or measured. But an alternative arises if we take a viewpoint which follows from the principle of parsimony: that which cannot be measured has no existence in physics; physics should be limited to describing only degrees of freedom that are at least in principle observable. Such a viewpoint was useful in the original formulation of quantum mechanics. If this principle holds here, one reaches the conclusion that the maximum number of degrees of freedom within a cube of size R^3 is

$$N(R) \sim (M_p R)^{3/2}. \quad (6.11)$$

More precisely, this is a proposal for a bound on the number of states with a nongravitational quantum field-theoretic description; such a bound was explored in [23] and earlier noted by 't Hooft [66]. It is certainly possible that, with inclusion of gravitational degrees of freedom and proper treatment of their dynamics, and of corresponding observables, a region of size R can support more degrees of freedom. For example, we would not be surprised to find the upper bound

$$N_{\text{BH}}(R) \sim (M_p R)^2 \quad (6.12)$$

corresponding to the Bekenstein-Hawking entropy of a black hole, arising from such an analysis. Indeed, [67] has even argued that (6.12) can be reached through an appropriate choice of equation of state.

Likewise, from the bound (6.8), one would conclude that there is no sense in which two independent degrees of freedom with resolution $1/\Lambda$ exist at relative separations less than given by (6.8). It was argued in [26] that such logic leads to a loophole in Hawking’s original argument [68] for information destruction by black holes.

When combined with the discussion of limitations from finite measuring apparatuses of Sec. V, such arguments for limitations on number of degrees of freedom in a finite region (or closed universe) indicate an intrinsic uncertainty in measurement. Such arguments have particular force in de Sitter space, as described in [25], which is commonly believed to have only finitely many degrees of freedom [69,70] corresponding to its finite entropy. In particular, if we work within a region of size R which has a bounded number of degrees of freedom $N(R)$, then amplitudes that can be measured by devices constructed in this region have an intrinsic uncertainty of the form (5.13). This represents an intrinsic uncertainty or imprecision above and beyond the usual uncertainties arising from quantum dynamics alone. One might draw from this the conclusion [25] that a single mathematically precise theory of de Sitter space does not exist. We consider as an alternative an analogy to quantum mechanics: once the inevitable uncertainty in momentum and position was discovered, the relevant question is what quantity can be precisely predicted, and the answer is the wave function. This begs the question: what is the analogous fundamental mathematical construction in the present context?

The reasoning we have outlined suggests the outline of a “first principles” approach, in analogy with the well-known “Heisenberg microscope discussion,” to understanding the radical thinning of degrees of freedom that is believed to occur in quantum gravity—a crucial aspect of the putative holographic principle. In short, by the above logic, what cannot be observed does not exist, and gravitational dynamics puts unexpectedly strong constraints on what can be observed. If this is the case, a very important question is to come up with a description of the degrees of freedom and dynamics that do exist, respecting these various nonlocal constraints. We expect this description to look nothing like local field theory in spacetime; ordinary local quantum field theory only emerges as an approximation to this underlying dynamics.

VII. DISCUSSION AND CONCLUSION

This paper has addressed the construction and interpretation of diffeomorphism-invariant observables of effective quantum gravity. In particular, we study operators constructed via integrals, in analogy to the construction of gauge-invariant observables in Yang-Mills theory via traces. A particularly important class of such operators are the “pseudolocal” operators, which in certain circumstances reduce to the local observables of field theory. This happens only in certain states, and the information about

location is encoded in the interplay of the operator relative to the state. Moreover, locality is only recovered in an approximation, and is in general spoiled by both quantum and gravitational effects. Thus locality is both relative and approximate.

Though single-integral pseudolocal observables experience fluctuations that grow with the infrared cutoff, for appropriate such operators (e.g., \mathcal{O}_ξ in the Z-model) this volume divergence appears with an exponentially small prefactor. Thus, in a universe of moderate volume, the effect of such fluctuations can remain small. Nevertheless, it would be very interesting to understand whether proper relational observables can be defined in the infinite volume limit. Of course, in this limit other observables exist: the S -matrix. The relationship between relational observables and the S -matrix is an interesting question for further exploration. This issue may also have interesting implications for universes with a long period of rapid growth, and, in particular, for eternal inflation scenarios.

The outline of a theory of measurement for these operators has also been presented. This theory respects the idea that there should be no fundamental separation between the measuring device and the system being measured. This theory is inherently incomplete: we can only explain how to relate matrix elements of diffeomorphism-invariant observables to results of measurement for *certain observables* and in *certain states*. In particular, a necessary condition for our discussion of measurement is the emergence of an appropriate semiclassical limit.

The further limitations that arise in the treatment of these observables may also represent intrinsic limitations on local physics. In particular, these include the statements that spatial resolution in a given region is limited by a lower bound that grows with the size of the region, and that two (or more) particles can only be measured at increasingly fine resolution if their separation increases. Moreover, these statements also suggest that the number of local quantum degrees of freedom in a finite-sized region is finite. Combined with the present discussion of measurement, this suggests an intrinsic uncertainty in measurements, above and beyond that of quantum mechanics.

A complete identification of the observables of quantum gravity clearly requires the full framework of underlying quantum-gravitational theory. We expect that there will continue to be relational observables in this context. If this is a theory of extended objects, such as strings and branes, this may suggest additional limitations on locality.

Note that our expressions for diffeomorphism-invariant and relational observables bear some formal similarity to observables constructed in noncommutative theories [71] and in open string field theory [72]. In particular, the latter take the form

$$\int v\left(\frac{\pi}{2}\right)A, \quad (7.1)$$

where A is the open string field and V is an on-shell closed string vertex operator. These share the feature that they involve an integral of a product of fields that gives an invariant. It may be that ultimately similar observables will be discovered in closed string theories, and reduce, in the effective gravity limit, to the kinds of observables we have described in this paper.

The present paper at best only outlines some of the boundaries of our knowledge of nonperturbative quantum gravity. However, even this seems a useful enterprise, and the above limitations support the statement that these boundaries reach to distances far larger than the Planck length.

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