Nonlinear interactions between gravitational radiation and modified Alfvén modes in astrophysical dusty plasmas

Mats Forsberg,[1] Gert Brodin,[1,2] Mattias Marklund,[1,2] Padma K. Shukla,[3] and Joachim Moortgat[4]

[1]*Department of Physics, Umeå University, SE-901 87 Umeå, Sweden*

[2]*Centre for Fundamental Physics, Rutherford Appleton Laboratory, Chilton Didcot, Oxfordshire, OX11 0QX, United Kingdom*

[3]*Institut für Theoretische Physik IV and Centre for Plasma Science and Astrophysics, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

[4]*Department of Physics and Astronomy, University of Rochester, Bausch & Lomb Hall, P.O. Box 270171, 600 Wilson Boulevard, Rochester, New York 14627-0171, USA*

(Received 15 June 2006; published 11 September 2006)

We present an investigation of nonlinear interactions between gravitational radiation and modified Alfvén modes in astrophysical dusty plasmas. Assuming that stationary charged dust grains form neutralizing background in an electron-ion-dust plasma, we obtain the three-wave coupling coefficients and calculate the growth rates for parametrically coupled gravitational radiation and modified Alfvén-Rao modes. The threshold value of the gravitational wave amplitude associated with convective stabilization is particularly small if the gravitational frequency is close to twice the modified Alfvén wave frequency. The implication of our results to astrophysical dusty plasmas is discussed.

DOI: 10.1103/PhysRevD.74.064014 PACS numbers: 04.30.Nk, 52.35.Bj, 95.30.Sf

I. INTRODUCTION

There exist several mechanisms for conversion between gravitational waves (GWs) and electromagnetic waves [1–18] in plasmas. One of the most basic processes occurs when GWs propagate across an external magnetic field, which gives rise to a linear coupling to the electromagnetic field [1], leading to the excitation of magnetohydrodynamic (MHD) waves in a plasma [3–5]. In order to excite perturbations with frequencies different from that of the GW, naturally nonlinear couplings must be considered. There exist numerous examples of such mechanisms in plasmas, giving rise to, e.g. three-wave couplings between GWs and electromagnetic waves. Wave coupling mechanisms involving GWs are studied for several different reasons. In some cases, the emphasis is on the basic theory [6–9]. In other works, the focus is on GW detectors [10–12], on cosmology [13–15], or on astrophysical applications such as binary mergers [16], gamma ray bursts [17], pulsars [18], or supernovas [19].

In the present paper, we will consider gravitational wave propagation in plasmas containing charged dust particles [20]. The latter are prominent components in many astrophysical systems and may contribute significantly to the dynamical properties of such systems [21–23]. It also has been claimed that supernovae can be significant sources of dust particles [24], although this claim is debated [25]. Previous work involving dusty plasma-gravitational wave interactions [19] have considered general relativistic versions of the dust MHD equations [20,26]. However, in cases where the dynamics is not dominated by the charged dust particles, other approximations are more useful [27]. In order to describe the modified Alfvén mode (MAM) (or the Alfvén-Rao mode [27]) that can propagate in a magnetized dusty plasma, we will apply the infinite mass approximation for immobile charged dust grains. Thus, the only force felt by the charged dust particles will be the gravitational force, which is an appropriate approximation for a broad range of Alfvén wave frequencies. Using the standard mode coupling theory [28], we then obtain the coupling coefficient describing the nonlinear interaction between two MAMs and one GW. Using these results, we consider the parametric excitation of the Rao mode in the vicinity of binary mergers. Provided that the gravitational wave frequency is close to twice the Rao cutoff frequency [27], the threshold value for the GW amplitude for the parametric interaction is much reduced compared to the ideal MHD theory [29], assuming that the limiting amplitude is determined by nondissipative stabilization.

II. BASIC EQUATIONS

The metric describing a linearized gravitational wave of arbitrary polarization propagating in the z direction on a flat background can be written as

$$ds^2 = -dt^2 + (1 + h_+)dx^2 + (1 - h_+)dy^2 + 2h_\times dxdy + dz^2. \tag{1}$$

A convenient choice is to introduce an orthonormal tetrad e_i, where $e_0 = \partial_t$ and the spatial part is written as $\nabla = (e_1, e_2, e_3) = \nabla_0 + \nabla_g$, where ∇ and ∇_g are given by

$$\nabla_0 = (\partial_x, \partial_y, \partial_z), \tag{2}$$

$$\nabla_g = -\tfrac{1}{2}(h_+\partial_x + h_\times\partial_y, h_\times\partial_x - h_+\partial_y, 0), \tag{3}$$

and $\mathbf{e}_1 = (e_1, 0, 0)$, $\mathbf{e}_2 = (0, e_2, 0)$, $\mathbf{e}_3 = (0, 0, e_3)$.

Below we will derive the evolution equations for the matter and fields in the low-frequency MHD limit, in the presence of a gravitational wave. We consider a cold plasma composed of the electrons, ions, and micron-sized charged dust particles. The momentum equation for the ions, when neglecting pressure and considering the electrons as massless fluid, reduces to

$$(\partial_t + \mathbf{v}_i \cdot \nabla)\mathbf{v}_i = \frac{q_i}{m_i c}(\mathbf{v}_i - \mathbf{v}_e) \times \mathbf{B} + \mathbf{g}_i, \tag{4}$$

where

$$\mathbf{g}_i = -\frac{1}{2}\left(1 - \frac{v_{iz}}{c}\right)[(v_{ix}\dot{h}_+ + v_{iy}\dot{h}_\times)\mathbf{e}_1 + (v_{ix}\dot{h}_\times - v_{iy}\dot{h}_+)\mathbf{e}_2] - \frac{1}{2c}[(v_{ix}^2 - v_{iy}^2)\dot{h}_+ + 2v_{ix}v_{iy}\dot{h}_\times]\mathbf{e}_3,$$

is the gravitational acceleration of the ions [30]. For low phase speed (in comparison with the speed of light) the displacement currents are neglected, and Maxwell's equations are written as

$$q_i n_i \mathbf{v}_i - e n_e \mathbf{v}_e = \frac{c}{4\pi}\nabla \times \mathbf{B} - \mathbf{j}_E, \tag{5}$$

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v}_e \times \mathbf{B}) - \mathbf{j}_B, \tag{6}$$

where

$$\mathbf{j}_E = -\tfrac{1}{2}[(E_x - B_y)\dot{h}_+ + (E_y + B_x)\dot{h}_\times]\mathbf{e}_1 + \tfrac{1}{2}[(E_y + B_x)\dot{h}_+ + (E_x - B_y)\dot{h}_\times]\mathbf{e}_2,$$
$$\mathbf{j}_B = -\tfrac{1}{2}[(E_y + B_x)\dot{h}_+ - (E_x - B_y)\dot{h}_\times]\mathbf{e}_1 - \tfrac{1}{2}[(E_x - B_y)\dot{h}_+ + (E_y + B_x)\dot{h}_\times]\mathbf{e}_2,$$

are effective currents due to the GWs, see e.g. Ref. [7].

Combining Eqs. (4) and (5) and assuming the plasma to be quasineutral with negatively charged dust, that is $q_i n_i - e n_e - q_d n_d = 0$ where $q_d > 0$, we obtain

$$(\partial_t + \mathbf{v}_i \cdot \nabla)\mathbf{v}_i = \frac{q_i}{e n_e m_i c}\left(-n_d q_d \mathbf{v}_i + \frac{c}{4\pi}\nabla \times \mathbf{B} - \mathbf{j}_E\right) \times \mathbf{B} + \mathbf{g}_i. \tag{7}$$

By using Eq. (5) we can eliminate $\mathbf{v}_e$ in (6) to obtain

$$\partial_t \mathbf{B} = \nabla \times \left\{\left(\frac{q_i n_i}{e n_e}\mathbf{v}_i - \frac{c}{4\pi e n_e}\nabla \times \mathbf{B} + \frac{\mathbf{j}_E}{e n_e}\right) \times \mathbf{B}\right\} - \mathbf{j}_B. \tag{8}$$

Finally, the ion continuity equation is

$$\partial_t n_i + \nabla \cdot (n_i \mathbf{v}_i) = 0. \tag{9}$$

For later use it will be convenient to collect the gravitational terms on the right-hand side of the equations. Furthermore, we let the ions to be of charge $q_i = e$. Equations (7)–(9) are then written as

$$m_i n_e c(\partial_t + \mathbf{v}_i \cdot \nabla_0)\mathbf{v}_i + n_d q_d \mathbf{v}_i \times \mathbf{B} - \frac{c}{4\pi}(\nabla_0 \times \mathbf{B}) \times \mathbf{B} = \frac{c}{4\pi}(\nabla_g \times \mathbf{B}) \times \mathbf{B} + n_e c m_i \mathbf{g}_i - \mathbf{j}_E \times \mathbf{B}, \tag{10}$$

$$\partial_t \mathbf{B} - \nabla_0 \times \left[\frac{n_i \mathbf{v}_i \times \mathbf{B}}{n_e} - \frac{c(\nabla_0 \times \mathbf{B}) \times \mathbf{B}}{4\pi e n_e}\right] = \nabla_g \times \left[\frac{n_i \mathbf{v}_i \times \mathbf{B}}{n_e} - \frac{c(\nabla_0 \times \mathbf{B}) \times \mathbf{B}}{4\pi e n_e}\right] + \nabla \times \frac{\mathbf{j}_E \times \mathbf{B}}{e n_e} - \mathbf{j}_B, \tag{11}$$

and

$$\partial_t n_i + \nabla_0 \cdot (n_i \mathbf{v}_i) = -\nabla_g \cdot (n_i \mathbf{v}_i), \tag{12}$$

where the dust charge is defined by $q_d = Z_d e$. We note that in the absence of the GW-source terms, Eqs. (10)–(12) are MHD equations modified by the presence of infinitely heavy dust particles. Because of the dust charges in the quasineutrality condition, the relative contributions to the currents are modified as compared to ideal MHD. As a consequence, a characteristic frequency Ω_R called the Rao cutoff frequency [27] enters in the linear wave modes. Wave modes propagating almost perpendicular to the external magnetic will be denoted as Alfvén-Rao modes. In what follows we will be particularly interested in these modes, as they give the simplest way to introduce a characteristic time scale (Ω_R^{-1}), which is longer than the ion Larmor period, into the MHD equations. Furthermore, as we will demonstrate, these modes are particularly easy to excite when the condition $\omega_g = 2\Omega_R$ is met, where ω_g is the gravitational wave frequency.

III. DERIVATIONS OF THE COUPLED MODE EQUATIONS

In general a monochromatic wave of sufficient amplitude is subject to a number of instabilities, which can transfer the wave energy into other modes. If energy-momentum conservation allows for resonant three-wave interaction, typically this mechanism gives rise to the most rapid instability [28]. Thus, in order to study this process, a system of three weakly interacting waves will be considered: two dust MHD waves with frequencies and wave numbers $(\omega_{1,2}, \mathbf{k}_{1,2})$, and a GW with arbitrary polarization propagating parallel to the background magnetic field with the frequency and the wave number $(\omega_g, \mathbf{k}_g)$. We note here that a gravitational wave propagating in a finite angle to the magnetic field produces a linear coupling to the electromagnetic field [1]. In the next step of the calculations, such linearly induced fields will complicate the description of the nonlinear mode coupling to a large extent. Thus our motive to let $\mathbf{k}_g$ be parallel to the external magnetic field is to avoid a linear coupling between the GW and the MHD modes and be able to focus on the nonlinear phenomena. As a prerequisite to obtain the nonlinear coupling, we first study the linear modes of the system (10)–(12), omitting the gravitational contributions. Considering the plasma waves to be plane wave perturbations on background quantities such that $\mathbf{B} = B_0\hat{\mathbf{z}} + \mathbf{B}'$, $n_i = n_0 + n'$, and $\mathbf{v}_i = \mathbf{v}'$, where B_0 and n_0 are constant and the primed quantities denote the perturbations, allows us to write the linear part of (10)–(12) as

$$\mathbf{v}' = -i\frac{\Omega_R}{\omega}\mathbf{v}' \times \hat{\mathbf{z}} - \frac{C_A}{\sqrt{4\pi n_0 m_i}}\frac{[k_z\mathbf{B}' - B_z'\mathbf{k}]}{\omega}, \quad (13)$$

$$\mathbf{B}' = \sqrt{4\pi n_0 m_i}C_A\frac{[(\mathbf{k}\cdot\mathbf{v}')\hat{\mathbf{z}} - k_z\mathbf{v}']}{\omega} + \frac{i m_i c C_A}{e\sqrt{4\pi n_0 m_i}} \times \frac{k_z\mathbf{k}\times\mathbf{B}'}{\omega}, \quad (14)$$

$$n' = \frac{n_0}{\omega}\mathbf{k}\cdot\mathbf{v}', \quad (15)$$

where $\Omega_R = n_d q_d B_0/m_i(n_0 - Z_d n_d)c$ is the Rao cutoff frequency and $C_A = n_0 B_0/(n_0 - Z_d n_d)\sqrt{4\pi n_0 m_i}$ is the Alfvén speed. The frequency matching is

$$\omega_g = \omega_1 + \omega_2, \quad (16)$$

and since the gravitational dispersion relation reads $\omega_g = ck_g$ and $C_A \ll c$, the wave number matching can be approximated by

$$\mathbf{k}_g = \mathbf{k}_1 + \mathbf{k}_2 \Rightarrow \mathbf{k}_1 \approx -\mathbf{k}_2. \quad (17)$$

Thus we may consider the excitation of MHD wave modes with wave vectors that are almost perpendicular to the GW (or the external magnetic field). In particular we choose a coordinate system such that $k_y = 0$ and let $|k_{1,2z}| \ll |k_{1,2x}|$, which allows the linear eigenmodes of the system to be represented by the following eigenvector

$$\begin{pmatrix} v_x' \\ v_y' \\ B_z' \\ n' \end{pmatrix} = v_x'\begin{pmatrix} 1 \\ i\frac{\Omega_R}{\omega} \\ \frac{C_A\sqrt{4\pi n_0 m_i}}{\omega}k_x \\ \frac{n_0}{\omega}k_x \end{pmatrix}. \quad (18)$$

The dispersion relation is now readily obtained and can be expressed as

$$\omega^2 = \Omega_R^2 + C_A^2k^2, \quad (19)$$

which is the Alfvén-Rao mode [27], that reduces to the compressional Alfvén (or fast magnetosonic) wave in the limit of zero dust density. We note that the dispersion relation (19), together with (16) and (17), implies $\omega_g = 2\omega_1 = 2\omega_2$.

Next, assuming the dust MHD waves and the GWs to be plane waves with weakly varying amplitudes, we write $h_{+,\times} = \tilde{h}_{+,\times}(t)e^{i(\mathbf{k}_g\cdot\mathbf{z}-\omega_g t)} + \text{c.c.}$ and $\psi' = \tilde{\psi}(t)e^{i(\mathbf{k}_{1,2}\cdot\mathbf{r}-\omega_{1,2}t)} + \text{c.c.}$, where c.c. stands for complex conjugate, and ψ' represents any component of $\mathbf{B}'$, $\mathbf{v}'$, and n'. Making use of the linear eigenvector (18) as approximations in the nonlinear terms in the system (10)–(12), and keeping only the resonant part to second order in the amplitudes, we obtain the coupled mode equations [28]

$$\partial_t\tilde{v}_{1,2x} = i\frac{\omega_g}{4}\left(1 + \frac{4\Omega_R^2}{\omega_g^2}\right)\tilde{v}_{2,1x}^*\tilde{h}_+ + 2\Omega_R\tilde{v}_{2,1x}^*\tilde{h}_\times. \quad (20)$$

For the GWs we obtain, by using Einstein's equations linearized in h_+, $h_\times$, and keeping only the resonant part of the energy-momentum tensor,

$$\partial_t\tilde{h}_+ = i\frac{\kappa}{\omega_g}m_i n_0\left(1 + \frac{4\Omega_R^2}{\omega_g^2}\right)\tilde{v}_{1x}\tilde{v}_{2x}, \quad (21)$$

and

$$\partial_t\tilde{h}_\times = -4\kappa m_i n_0\frac{\Omega_R}{\omega_g^2}\tilde{v}_{1x}\tilde{v}_{2x}, \quad (22)$$

for the $+$ and $\times$ polarization, respectively.

Noting that the gravitational wave energy density can be written as

$$E_g \equiv W_g(|h_+|^2 + |h_\times|^2) = \frac{\omega_g^2}{2\kappa}(|h_+|^2 + |h_\times|^2), \quad (23)$$

and the Alfvén-Rao wave energy density as

$$E_{1,2} \equiv W_{1,2}|v_{1,2x}|^2 = \frac{1}{2}m_i n_0(|v_{1,2x}|^2 + |v_{1,2y}|^2) + \frac{|B_{1,2z}|^2}{8\pi} = m_i n_0|v_{1,2x}|^2, \quad (24)$$

we can deduce three independent conservation laws from the coupled mode equations (22), (21), and (22). For the total wave energy, i.e.

$$\frac{d(E_g + E_1 + E_2)}{dt} = 0, \quad (25)$$

the difference in Alfvén-Rao wave quanta, i.e.

$$\frac{d(N_1 - N_2)}{dt} = 0, \quad (26)$$

and the sum of wave quanta, i.e.

$$\frac{d(2N_g + N_1 + N_2) = 0}{dt}, \quad (27)$$

where we have introduced the number density of gravitational wave quanta $N_g = E_g/\hbar\omega_g$ and of the Alfvén-Rao wave quanta $N_{1,2} = E_{1,2}/\hbar\omega_{1,2}$. The existence of these conservation laws are equivalent to the Manley-Rowe relations [28].

For simplicity, we have made the derivation of the coupled mode equations considering only time-dependent amplitudes. However, we note that a generalization to allow for weakly space-dependent amplitudes can be made by the simple substitution $\partial_t \to \partial_t + \mathbf{v}_g \cdot \nabla$ for each mode [28], i.e. for a general slowly varying amplitude the coupled mode equations reads

$$(\partial_t + v_{g1,2}\partial_x)\tilde{v}_{1,2x} = i\frac{\omega_g}{4}\left(1 + \frac{4\Omega_R^2}{\omega_g^2}\right)\tilde{v}^*_{2,1x}\tilde{h}_+ + 2\Omega_R\tilde{v}^*_{2,1x}\tilde{h}_\times, \tag{28}$$

$$(\partial_t + c\partial_z)\tilde{h}_+ = i\frac{\kappa}{\omega_g}m_i n_0\left(1 + \frac{4\Omega_R^2}{\omega_g^2}\right)\tilde{v}_{1x}\tilde{v}_{2x}, \tag{29}$$

and

$$(\partial_t + c\partial_z)\tilde{h}_\times = -4\kappa m_i n_0\frac{\Omega_R}{\omega_g^2}\tilde{v}_{1x}\tilde{v}_{2x}, \tag{30}$$

where $v_{g1,2} = k_{1,2x}C_A^2/\omega_{1,2}$, where $k_{1,2x}$ is the x component of the wave vector for modes 1 and 2, respectively [31].

IV. SUMMARY AND DISCUSSION

We have considered the nonlinear interaction between the modified Alfvén (or the Alfvén-Rao) mode and gravitational waves in a magnetized dusty plasma. In order to describe this process, dust MHD equations incorporating the effects of the gravitational waves have been derived. In particular, we have focused on the case where a gravitational wave of arbitrary polarization propagates parallel to the magnetic field. We have then calculated the three-wave coupling coefficients for MHD waves propagating almost perpendicular to the magnetic field, in which case the latter wave modes obey the Alfvén-Rao dispersion relation (19). From the coupled mode equations we note that the GW can parametrically excite Alfvén-Rao modes, which grow as $\exp(\gamma t)$, where γ depends on Ω_R, ω_g, $\tilde{h}_{+,\times}$ etc. Furthermore, it can be seen that γ is roughly independent of the GW polarization, and of the order $\gamma \sim |\tilde{h}_{+,\times}|\omega_g$. This estimate is the same as for the decay into high-frequency waves, see Ref. [2], or MHD waves, see e.g. Refs. [3,29]. The highest growth rate may be reached for approximately monochromatic gravitational waves, which could be produced by compact binaries close to merging, see e.g. [29], or during the black-hole ringdown [32].

However, we note that the high gravitational amplitudes only exist during a limited time, and that the finite group velocity of the decay products has a stabilizing effect on the parametric process. To study this effect we consider the decay of a homogeneous intense gravitational wave which for definiteness has the polarization $\tilde{h} = \tilde{h}_+$. The amplitudes of the Alfvén-Rao modes are assumed to have the form $\tilde{v}_{1x} = \hat{v}_{1x}\exp(iKx - i\Omega t)$ and $\tilde{v}^*_{2x} = \hat{v}^*_{2x}\exp(iKx - i\Omega t)$. Inserting this ansatz latter into Eq. (28), we immediately obtain the nonlinear dispersion relation

$$(\Omega - v_{g1}K)(\Omega - v_{g2}K) = -\frac{\omega_g^2}{16}\left(1 + \frac{4\Omega_R^2}{\omega_g^2}\right)^2|\tilde{h}_+|^2. \tag{31}$$

Next, introducing $v_g = |v_{g1}| = |v_{g2}|$, we deduce from (31) that the growth rate of the Alfvén-Rao modes is

$$\gamma = \frac{1}{4}\sqrt{\omega_g^2\left(1 + \frac{4\Omega_R^2}{\omega_g^2}\right)^2|\tilde{h}_+|^2 - 16K^2v_g^2}, \tag{32}$$

provided that the second term under the root sign is smaller than the first one. While Eq. (32) formally shows that sufficiently long wavelength amplitude perturbations always are unstable, in reality there is a minimum wave number possible. This wave number $K_{\min}$ is set by the shortest nonoscillatory scale of the problem, which may be either the inverse inhomogeneity scale length $\nabla n_0/n_0$, R_{curv}^{-1}, or $(ct_p)^{-1}$, where R_{curv} is the background curvature and t_p is the pulse duration time. Note that the pulse duration here refers to the time during which the pulse has a sufficiently constant frequency, such as not to break the frequency matching condition. The threshold value on the gravitational wave amplitude for the parametric excitation now becomes

$$\tilde{h}_{+\mathrm{tre}} = \frac{8K_{\min}v_g}{\omega_g}. \tag{33}$$

Since the group velocity of the Alfvén-Rao mode approaches zero when $\omega \to \Omega_R$, we note that $\tilde{h}_{+\mathrm{tre}} \to 0$ when $\omega_g \to 2\Omega_R$. Introducing the frequency mismatch $\delta\omega = \omega_g - 2\Omega_R$, and using Eq. (33), we find that the threshold value is

$$\tilde{h}_{+\mathrm{tre}} = \frac{8\delta\omega}{\omega_g}K_{\min}C_A. \tag{34}$$

Thus, we conclude that provided the gravitational spectrum contains twice the Rao frequency, parametric excitation of MHD waves occur more easily in a dusty plasma, that may exist in astrophysical systems, see e.g. Refs. [21–24], as compared to a plasma without dust particles. As a consequence of the low threshold, GW-induced Alfvén-Rao modes may be excited at a comparatively large distance from the gravitational wave source. The aim of this study has been to shed further light on the gravitational-MHD interactions that take place in the vicinity of gravitational wave sources, such as collapsing compact binaries, quaking neutron stars, black holes during ringdown, or supernovas [33].

ACKNOWLEDGMENTS

This research was partially supported by the Swedish Research Council and the Swedish graduate school of space technology.

[1] L. P. Grishchuk and A. G. Polnarev, *General Relativity and Gravitation*, edited by A. Held (Plenum Press, New York, 1980), Vol. 2, pp. 416–430.
[2] G. Brodin and M. Marklund, Phys. Rev. Lett. **82**, 3012 (1999).
[3] D. Papadopoulos *et al.*, Astron. Astrophys. **377**, 701 (2001).
[4] J. Moortgat and J. Kuijpers, Astron. Astrophys. **402**, 905 (2003).
[5] J. Moortgat and J. Kuijpers, Phys. Rev. D **70**, 023001 (2004).
[6] M. Servin and G. Brodin, Phys. Rev. D **68**, 044017 (2003).
[7] M. Servin *et al.*, Phys. Rev. D **67**, 087501 (2003).
[8] J. T. Mendonça, Plasma Phys. Controlled Fusion **44**, B225 (2002).
[9] A. B. Balakin *et al.*, J. Math. Phys. (N.Y.) **44**, 5120 (2003).
[10] F. Y. Li and M. X. Tang, Int. J. Mod. Phys. D **11**, 1049 (2002).
[11] G. Brodin and M. Marklund, Classical Quantum Gravity **20**, L45 (2003).
[12] R. Ballantini *et al.*, Classical Quantum Gravity **20**, 3505 (2003).
[13] D. Papadopoulos, Classical Quantum Gravity **19**, 2939 (2002).
[14] M. Marklund, P. K. S. Dunsby, and G. Brodin, Phys. Rev. D **62**, 101501(R) (2000).
[15] P. A. Hogan and E. M. O'Shea, Phys. Rev. D **65**, 124017 (2002).
[16] G. Brodin, M. Marklund, and M. Servin, Phys. Rev. D **63**, 124003 (2001).
[17] L. Vlahos *et al.*, Astrophys. J. **604**, 297 (2004).
[18] H. J. M. Cuesta, Phys. Rev. D **65**, 064009 (2002).
[19] G. Brodin, M. Marklund, and P. K. Shukla, Pis'ma Zh. Eksp. Teor. Fiz. **81**, 169 (2005) JETP Lett. **81**, 135 (2005).
[20] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (Institute of Physics, Bristol, 2002).
[21] D. A. Mendis and M. Rosenberg, Annu. Rev. Astron. Astrophys. **32**, 419 (1994).
[22] M. Horányi, T. W. Hartquist, O. Havnes, D. A. Mendis, and G. E. Morfill, Rev. Geophys. **42**, 4002 (2004).
[23] Y. K. Okamoto, H. Kataza, M. Honda, T. Yamashita, T. Onaka, J. Watanabe, T. Miyata, S. Sako, T. Fujiyoshi, and I. Sakon, Nature (London) **431**, 660 (2004).
[24] L. Dunne, S. Eales, R. Ivison, H. Morgan, and M. Edmunds, Nature (London) **424**, 285 (2003).
[25] O. Krause, S. M. Birkmann, G. H. Rieke, D. Lemke, U. Klaas, D. C. Hines, and K. D. Gordon, Nature (London) **432**, 596 (2004).
[26] G. T. Birk, A. Kopp, and P. K. Shukla, Phys. Plasmas **3**, 3564 (1996); P. K. Shukla and H. U. Rahman, *ibid.* **3**, 430 (1996).
[27] N. N. Rao, J. Plasma Phys. **53**, 317 (1995).
[28] J. Weiland and H. Wilhelmsson, *Coherent Nonlinear Interaction of Waves in Plasmas* (Pergamon Press, New York, 1977).
[29] M. Servin, G. Brodin, M. Bradley, and M. Marklund, Phys. Rev. E **62**, 8493 (2000).
[30] When including the nonlinear GW coupling, it is in principle important to separate the coordinate components (indices x, y, z) from the tetrad components (indices 1, 2, 3), since the difference is first order in h_+, $h_\times$. However, for notational convenience we let indices x, y, z denote tetrad components 1, 2, 3 in what follows.
[31] Note that the group velocities have equal magnitudes but different signs due to Eq. (17).
[32] C. Clarkson, M. Marklund, G. Betschart, and P. Dunsby, Astrophys. J. **613**, 492 (2004).
[33] C. D. Ott *et al.*, Phys. Rev. Lett. **96**, 201102 (2006).